Exploring pion structure with Minkowski space dynamical model

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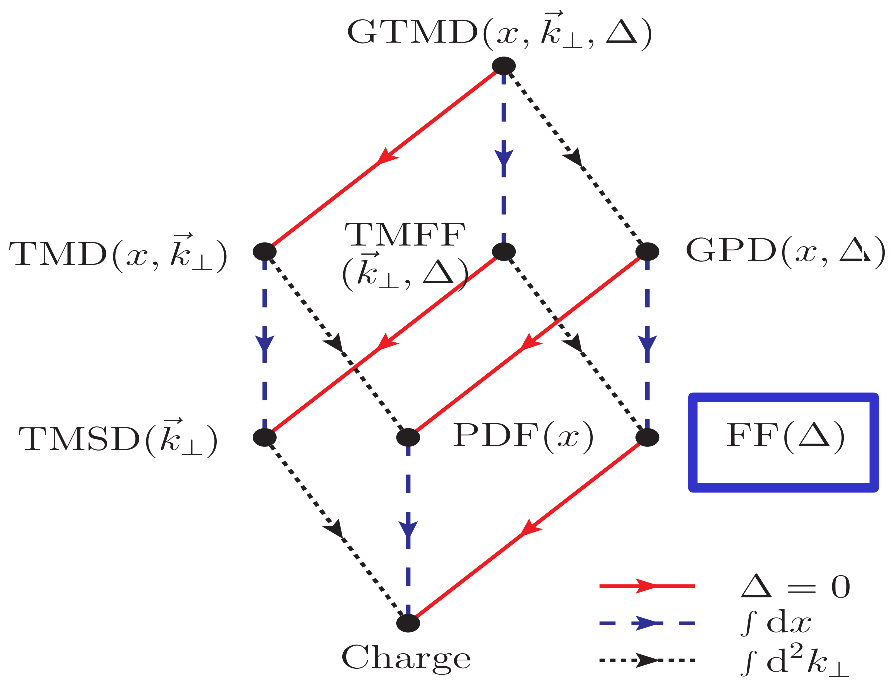
Center for Frontiers in Nuclear Science
Workshop series

Workshop on Pion and Kaon Structure Functions at the EIC
2-5 June 2020 online

Motivation

Observables associated with the hadron structure



Lorcé, Pasquini, Vanderhaeghen JHEP05(2011)041

- TMD, PDFs, SL and TL form factors (pion)…

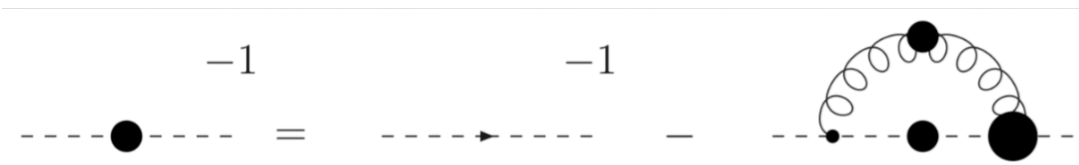
Outline

- quark-gluon vertex from LQCD data and the quark Gap Equation
- quark-antiquark BSE for the pion (Minkowski space/LF wave function)
- Pion Electromagnetic form-factor
- Summary

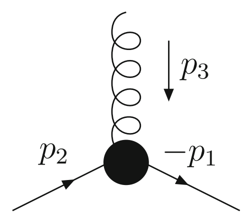
The Quark-Gap Equation and the Quark-Gluon Vertex

Spontaneous Chiral symmetry breaking & pion as a Goldstone boson
(origin of the nucleon mass – "constituent quarks", Roberts, Maris, Tandy, Cloet, Maris...)

Schwinger-Dyson eq.
Quark propagator



Quark-gluon vertex



$$\Gamma_\mu^a(p_1, p_2, p_3) = g\, t^a\, \Gamma_\mu(p_1, p_2, p_3)$$

$$\Gamma_\mu(p_1, p_2, p_3) = \Gamma_\mu^{(L)}(p_1, p_2, p_3) + \cancel{\Gamma_\mu^{(T)}(p_1, p_2, p_3)}$$

Longitudinal component

$$\Gamma_\mu^{\mathrm{L}}(p_1,p_2,p_3) = -i\Big(\lambda_1\,\gamma_\mu \;+\; \lambda_2\,(\not p_1 - \not p_2)\,(p_1-p_2)_\mu \\ +\; \lambda_3\,(p_1-p_2)_\mu \;+\; \lambda_4\,\sigma_{\mu\nu}\,(p_1-p_2)^\nu\Big)$$

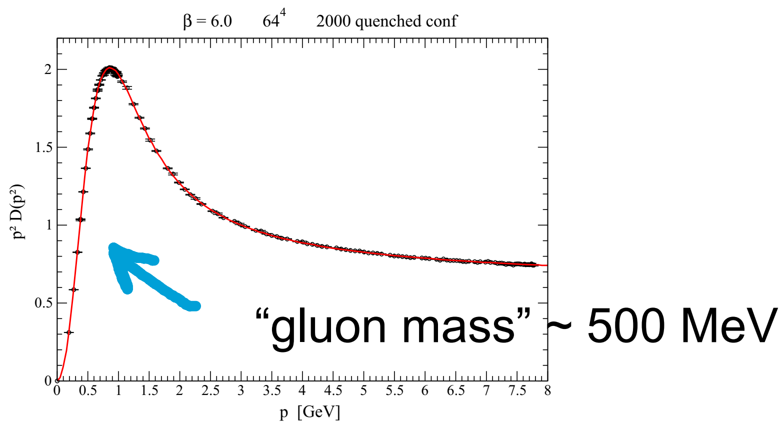
Rojas, de Melo, El-Bennich, Oliveira, Frederico, JHEP 1310 (2013) 193

INPUTS FROM LQCD in Landau gauge: SL momenta

Gluon propagator

$$D^{ab}_{\mu\nu}(q) = -i\,\delta^{ab}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) D(q^2)$$

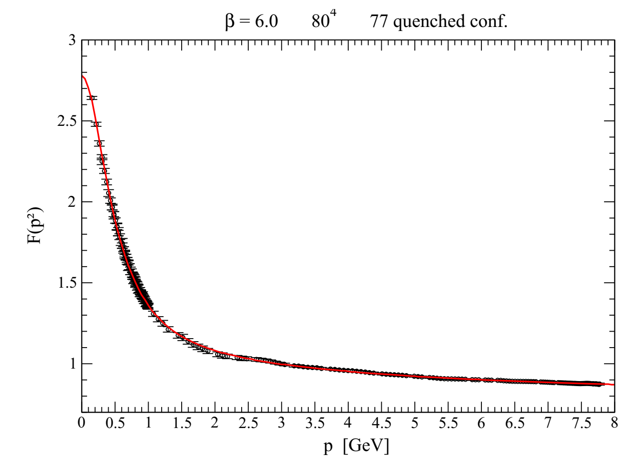
Dudal, Oliveira, Silva, Ann. Phys. **397**, 351 (2018)



Ghost propagator

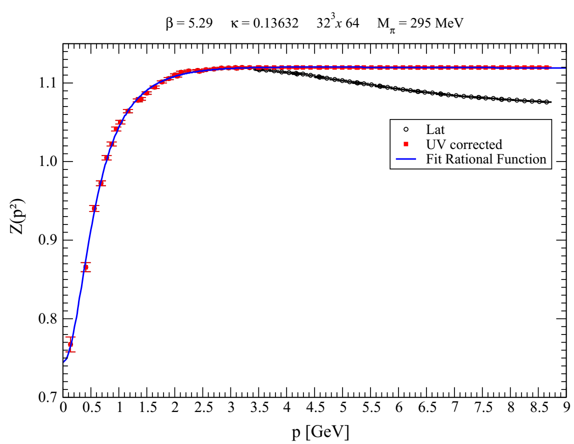
$$D_{gh}(p^2) = \frac{F(p^2)}{p^2}$$

Duarte, Oliveira, Silva, PRD 94 (2016) 014502



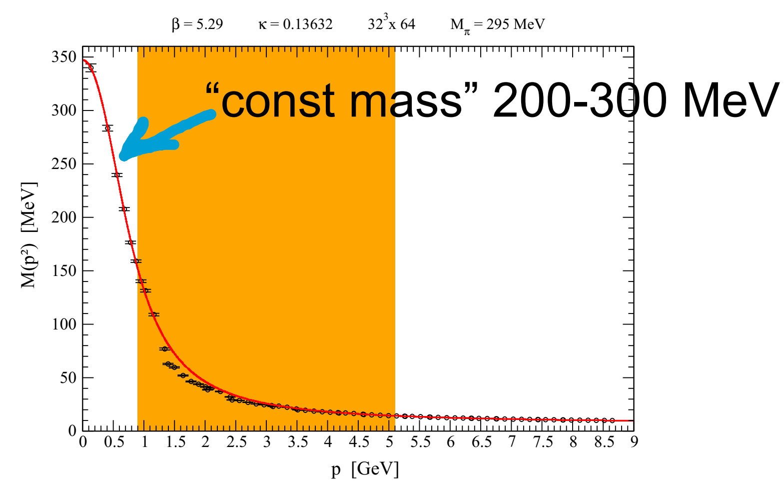
Quark propagator

Oliveira, Silva, Skullerud and Sternbeck, PRD 99 (2019) 094506



$$i\,Z(p^2)\,\frac{\not p + M(p^2)}{p^2 - M^2(p^2)}$$

$$i/\left(A(p^2)\not p - B(p^2)\right)$$



Parametrizations summarized in Oliveira, Paula, Frederico, de Melo, EPJ C 79 (2019) 116

Slanov-Taylor identity & Quark-Ghost Kernel

Davydychev, Osland, Saks, PRD **63**, 014022 (2000)

$$p_3^\mu \; \Gamma_\mu(p_1,\,p_2,\,p_3) =$$
$$F(p_3^2)\left[S^{-1}(-p_1)\,H(p_1,\,p_2,\,p_3) - \overline{H}(p_2,\,p_1,\,p_3)\,S^{-1}(p_2)\right]$$

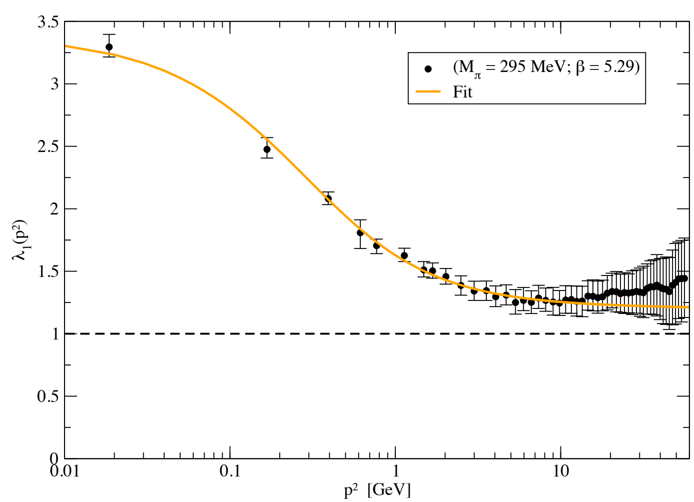
$$H(p_1,\,p_2,\,p_3) = X_0\,\mathbb{I}_D + X_1\,\not{p}_1 + X_2\,\not{p}_2 + X_3\,\sigma_{\alpha\beta}\,p_1^\alpha\,p_2^\beta,$$
$$\overline{H}(p_2,\,p_1,\,p_3) = \overline{X}_0\,\mathbb{I}_D - \overline{X}_2\,\not{p}_1 - \overline{X}_1\,\not{p}_2 + \overline{X}_3\,\sigma_{\alpha\beta}\,p_1^\alpha\,p_2^\beta$$

$X_i \equiv X_i(p_1,\,p_2,\,p_3)$ and $\overline{X}_i \equiv X_i(p_2,\,p_1,\,p_3)$.

$\lambda_i(X_i)$ Aguilar, Papavassiliou, PRD **83**, 014013 (2011)

Parametrizations of X from: Oliveira, Paula, Frederico, de Melo EPJC **78**(7), 553 (2018) & EPJC 79 (2019) 116

soft gluon limit from LQCD
Oliveira, et al Acta Phys. Polon. Supp. (2016) 363



Ansatz for the longitudinal form factors based on the STI:

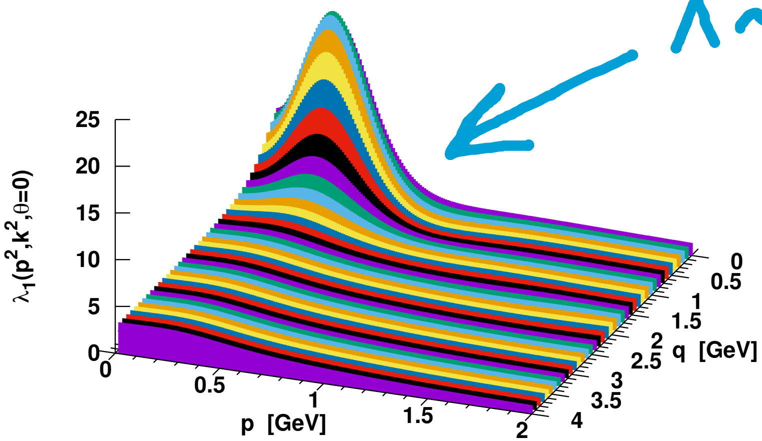
Oliveira, Frederico, de Paula, EPJC 80 (2020) 484

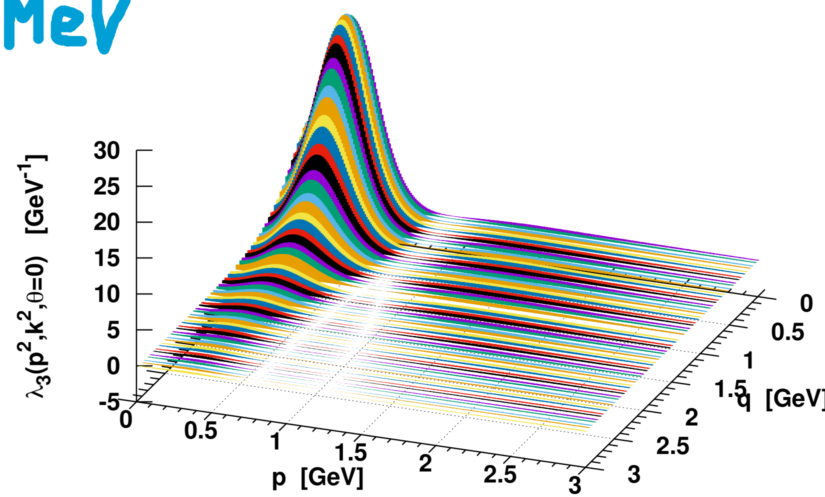
$$
\begin{aligned}
\lambda_1 &= \frac{F(q^2)}{2}\Bigg\{2\,B(p^2)\,D\left(\frac{p^2+k^2}{2}\right)\,Y_1(q^2) \\
&\quad + A(p^2)\left[X_0(q^2) + \left(p^2 - p\cdot k\right)D\left(\frac{p^2+k^2}{2}\right)\,Y_3(q^2)\right]\Bigg\} \\
&\quad + (p \leftrightarrow k)\,, \\
\lambda_2 &= \frac{F(q^2)}{2(k^2-p^2)}A(p^2)\Bigg\{\left(p^2 + p\cdot k\right)D\left(\frac{p^2+k^2}{2}\right)\,Y_3(q^2) \\
&\quad - X_0(q^2)\Bigg\} + (p \leftrightarrow k)\,, \\
\lambda_3 &= \frac{F(q^2)}{p^2-k^2}\Bigg\{A(p^2)\left[p^2 + p\cdot k\,\right]D\left(\frac{p^2+k^2}{2}\right)\,Y_1(q^2) \\
&\quad + B(p^2)\,X_0(q^2)\Bigg\} + (p \leftrightarrow k), \\
\lambda_4 &= -\frac{F(q^2)}{2}D\left(\frac{p^2+k^2}{2}\right)\left\{A(p^2)\,Y_1(q^2) + B(p^2)\,Y_3(q^2)\right\} \\
&\quad - (p \leftrightarrow k)\,, \qquad (29)
\end{aligned}
$$

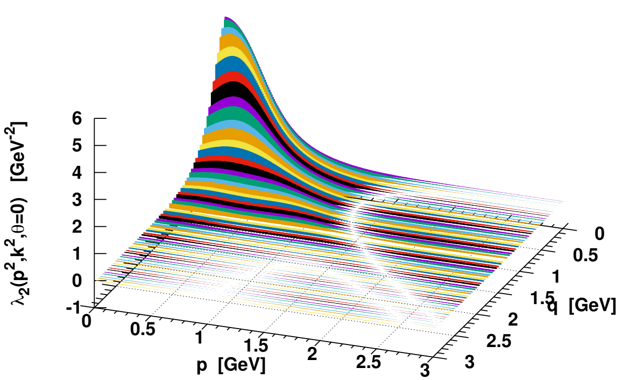
- Padé approximants
- Error minimization ~ 2-4%
- simulating annealing

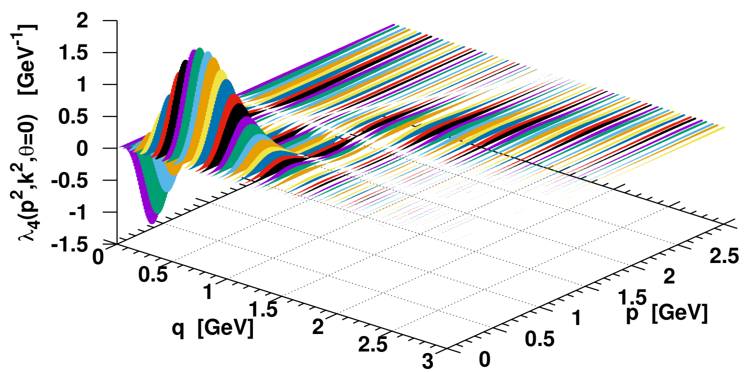
$\alpha_s = 0.22$ and all propagators renormalised at $\mu = 4.3\,\mathrm{GeV}$

$\lambda \sim 300\mathrm{MeV}$



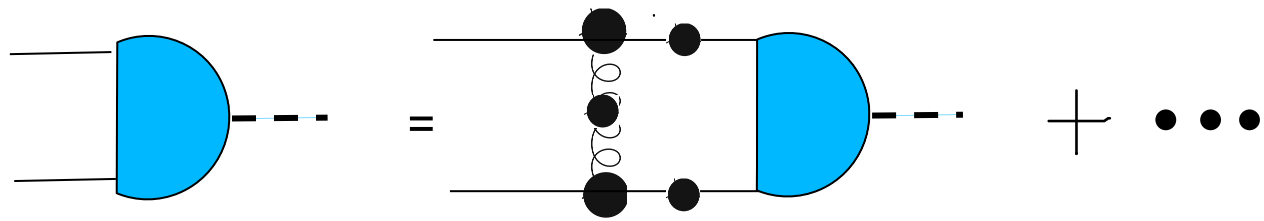






Oliveira, Frederico, de Paula, EPJC 80 (2020) 484

BSE quark-antiquark & pion model



Ladder approximation (L): suppression of XL for Nc=3
[A. Nogueira, CR Ji, Ydrefors, TF, PLB777(2017) 207]

- constituent quark mass ~ 200 – 300 MeV
- Vector exchange
  Feynman gauge
  μ ~ 500 MeV

$$iK_V^{(Ld)\mu\nu}(k,k') = -ig^2 \frac{g^{\mu\nu}}{(k-k')^2-\mu^2+i\epsilon}$$

- quark-gluon vertex form-factor
  Λ ~ 300 MeV

$$\lambda_1 \gamma_\mu \qquad F(q) = \frac{\mu^2-\Lambda^2}{q^2-\Lambda^2+i\epsilon}$$

SOLUTION IN MINKOWSKI SPACE

Main Tool: Nakanishi Integral Representation (NIR)

"Parametric representation for any Feynman diagram for interacting bosons, with a denominator carrying the overall analytical behavior in Minkowski space" (Nakanishi 1962)

Bethe-Salpeter amplitude

$$\Phi(k,p) = \int_{-1}^{1} dz' \int_{0}^{\infty} d\gamma' \frac{g(\gamma',z')}{(\gamma' + \kappa^2 - k^2 - p.kz' - i\epsilon)^3}$$

$$\kappa^2 = m^2 - \frac{M^2}{4}$$

Bosons: Kusaka and Williams, PRD 51 (1995) 7026;

Light-front projection: integration in k^- Carbonell&Karmanov EPJA27(2006)1;EPJA27(2006)11;

TF, Salme, Viviani PRD89(2014) 016010,…

Fermions (0^-): Carbonell and Karmanov EPJA 46 (2010) 387;

de Paula, TF,Salmè, Viviani PRD 94 (2016) 071901;

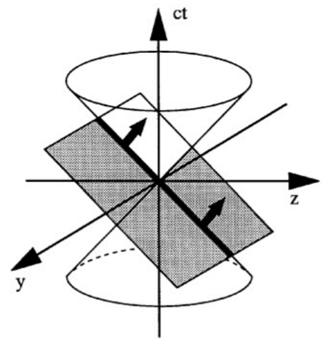
de Paula, TF, Pimentel, Salmè, Viviani, EPJC 77 (2017) 764

$$\Phi(k,p) = S_1\phi_1 + S_2\phi_2 + S_3\phi_3 + S_4\phi_4$$

$$S_1 = \gamma_5 \quad S_2 = \frac{1}{M}\not{p}\gamma_5 \quad S_3 = \frac{k\cdot p}{M^3}\not{p}\gamma_5 - \frac{1}{M}\not{k}\gamma_5 \quad S_4 = \frac{i}{M^2}\sigma_{\mu\nu}p^\mu k^\nu\gamma_5$$

Generalized Stietjes transform and the LF valence wave function

Carbonell, TF, Karmanov PLB769 (2017) 418 (bosons)



$$\boxed{\Psi_i(\gamma, z; \kappa^2) = \int_0^\infty d\gamma' \frac{g_i(\gamma', z; \kappa^2)}{[\gamma + \gamma' + m^2 z^2 + (1 - z^2)\kappa^2]^2}}$$

$$\gamma = k_\perp^2 \qquad z = 2x - 1$$

$$\begin{tikzcd}
\chi \arrow[rr, "\text{LF projection}"] & & \Psi_2 \arrow[dl, "\text{Inverse Stieltjes transform}"] \\
& g \arrow[ul, "\text{NIR}"] &
\end{tikzcd}$$

J.H. Schwarz, J. Math. Phys. 46 (2005) 014501,

UNIQUENESS OF THE NAKANISHI REPRESENTATION

PHENOMENOLOGICAL APPLICATIONS from the valence wf → BSA!

- Kernel of the LF projected pion BSE with NIR
- end-point singularities in the k^- integration (zero-modes)

T.M. Yan , Phys. Rev. **D 7**, 1780 (1973)

$$\mathcal{I}(\beta, y) = \int_{-\infty}^{\infty} \frac{dx}{\left[\beta x - y \mp i\epsilon\right]^2} = \pm \frac{2\pi i \; \delta(\beta)}{\left[-y \mp i\epsilon\right]}$$

→ Kernel with delta and its derivative!

End-point singularities– more intuitive: can be treated by the pole-dislocation method
de Melo et al. NPA631 (1998) 574C, PLB708 (2012) 87

Valence wave function

W. de Paula, JHA Nogueira, E. Ydrefors, G. Salmè in preparation

Normalization: $$i\, N_c \int \frac{d^4k}{(2\pi)^4} \left[\phi_1\phi_1 + \phi_2\phi_2 + b\phi_3\phi_3 + b\phi_4\phi_4 - 4\; b\phi_1\phi_4 - 4\frac{m}{M}\phi_2\phi_1\right] = -1$$

Valence probability: $$P_{\rm val} = \frac{N_c}{16\,\pi^2}\int_{-1}^{1} dz \int_0^\infty d\gamma\; \left[|\psi^{\uparrow\downarrow}(\gamma,z)|^2 + |\psi^{\uparrow\downarrow}(\gamma,z)|^2\right]$$

$$\psi_{\uparrow\downarrow}(\gamma,z) = \psi_2(\gamma,z) + \frac{z}{2}\psi_3(\gamma,z) + \frac{1}{M^3}\int_0^\infty d\gamma'\; \frac{\partial g_3(\gamma',z)/\partial z}{[\gamma+\gamma'+z^2m^2+(1-z^2)\kappa^2]}$$

$$\psi_{\uparrow\uparrow}(\gamma,z) \equiv \frac{\sqrt{\gamma}}{M}\,\psi_4(\gamma,z)\,,$$

$$\psi_i(\gamma,z) = -\frac{1}{M}\int_0^\infty d\gamma' \frac{g_i(\gamma',z)}{[\gamma+\gamma'+m^2z^2+(1-z^2)\kappa^2]^2}$$

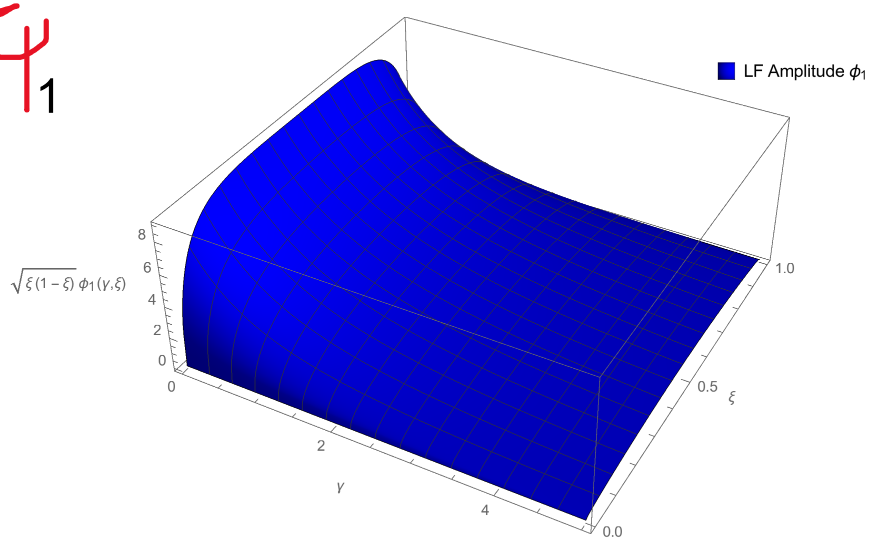
$$\gamma\; = \; k_\perp^2 \;\; \text{and} \;\; z \; = \; 2\xi - 1$$

Light-front amplitudes

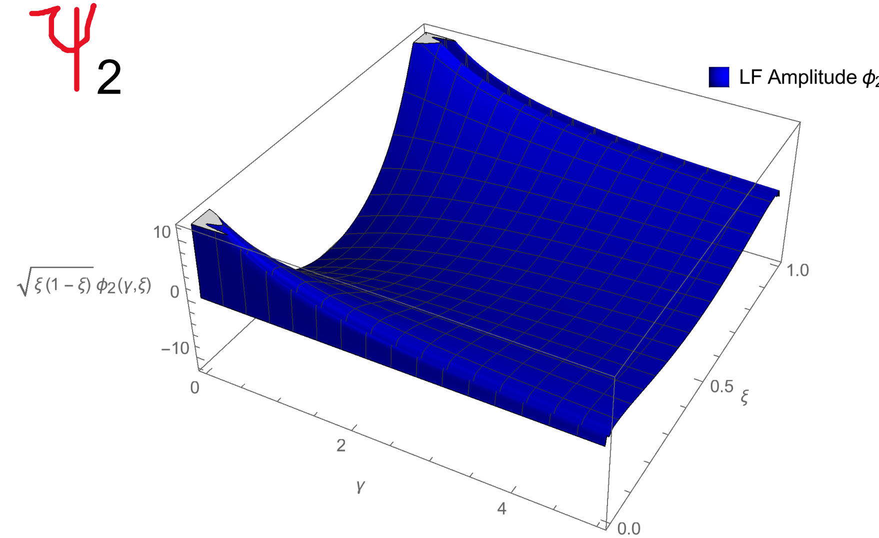
B/m	M_π (MeV)	g^2	μ (MeV)	Λ/m	m (MeV)	p_{val}
1.35	140	26.718	430	1.0	215	0.68

Kernel has similar magnitude with LQCD form-factor ~ 50%

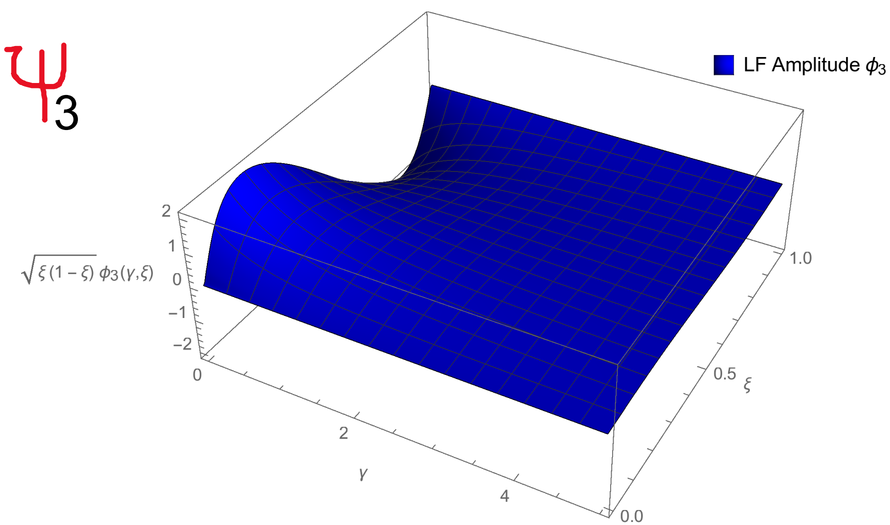
ψ_1



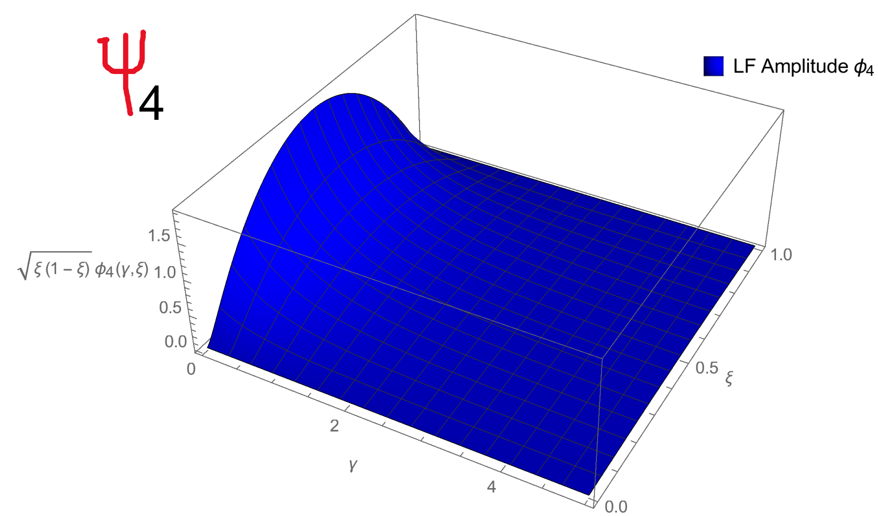
ψ_2



ψ_3



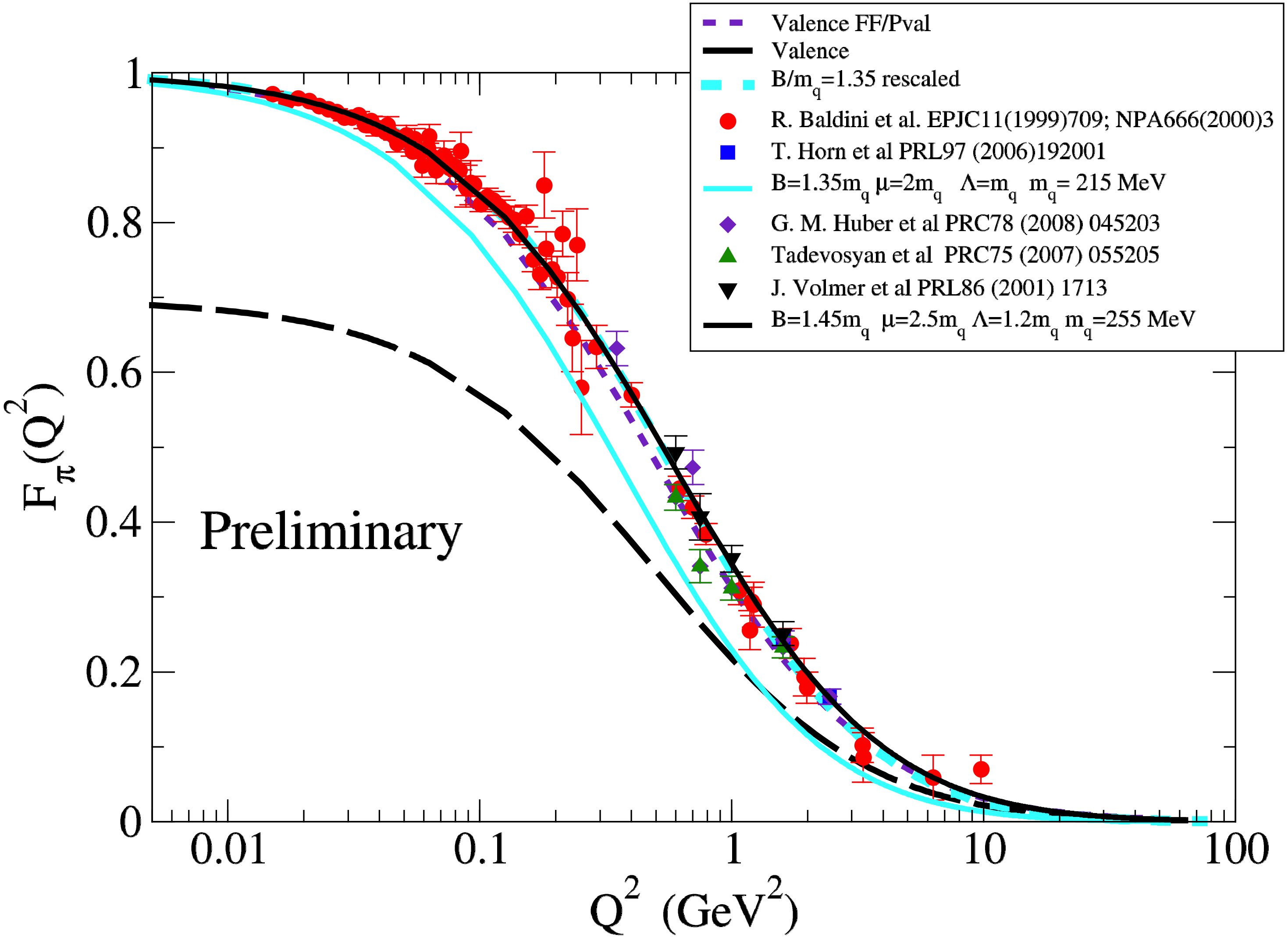
ψ_4



Pion EM Form Factor

Alvarenga Nogueira, de Paula, TF, Mezrag, Ydrefors, Salmè in preparation

$$F(Q^2) = \frac{N_c}{2^5\pi^2 N_{Q_0}} \sum_{i,j=1}^{4} \int_0^\infty d\gamma \int_{-1}^{1} dz\, g_i(\gamma,z) \int_0^\infty d\gamma' \int_{-1}^{1} dz'\, g_j(\gamma',z') \int_0^1 dv\, v^2(1-v)^2\, c_{ij}$$



$$F_\pi(Q^2) = \sum_n F_n(Q^2) = F_{val}(Q^2) + F_{nval}(Q^2)$$

$q\bar{q}$+gluons

$$r_\pi^2 = P_{val}\, r_{val}^2 + (1 - P_{val})\, r_{nval}^2$$

B = 1.45m_q m_{glue}= 2.5m_q Λ=1.2m_q m_q=255 MeV

r_π (fm)	r_{val} (fm)	r_{nval} (fm)
0.661	0.709	0.537

0.657 ± 0.003 fm   B. Ananthanarayan, I. Caprini, D. Das, Phys. Rev. Lett. 119 (2017) 132002

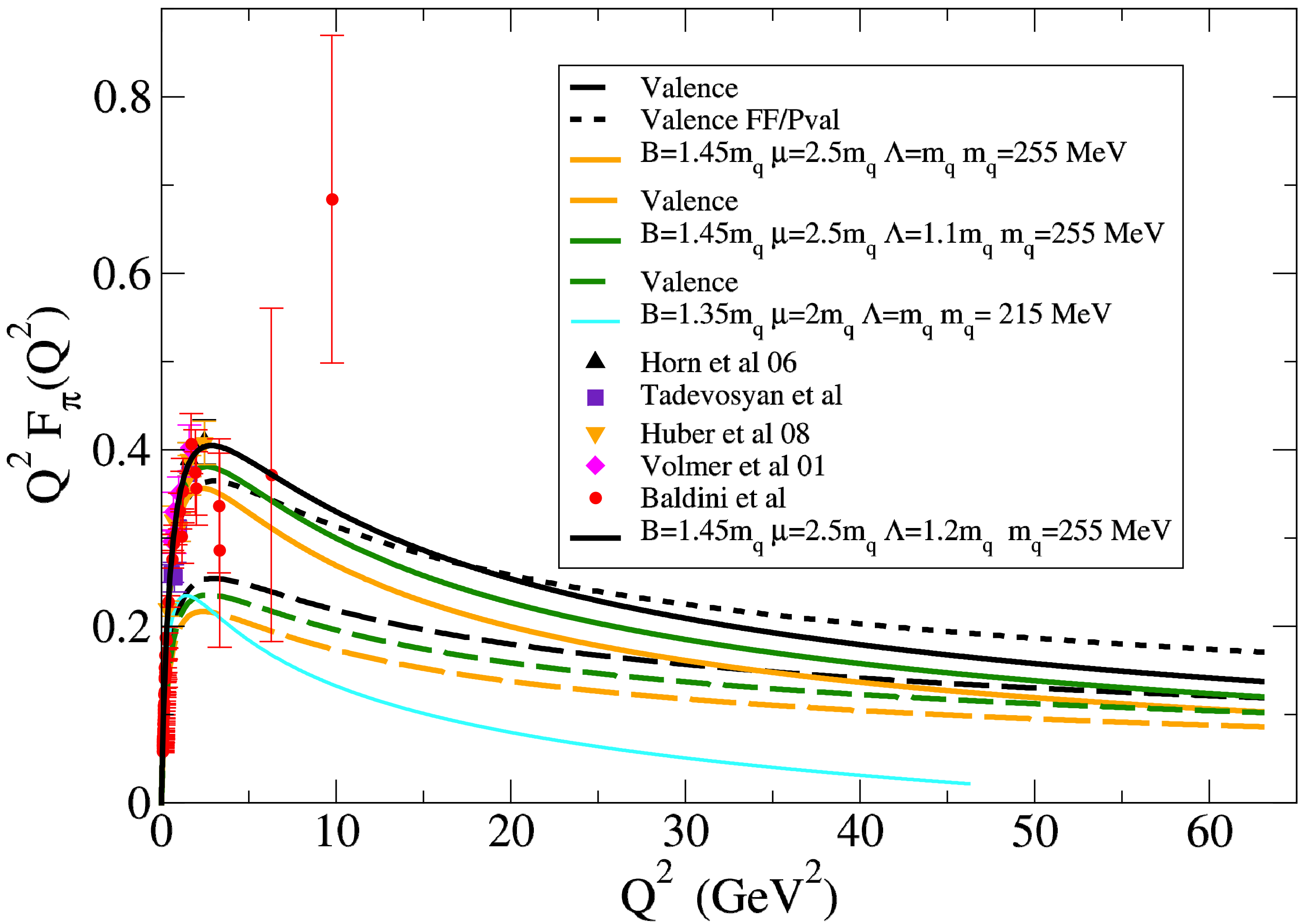
70%

Valence

higher Fock-components → large virtuality→ more compact

30%

Preliminary

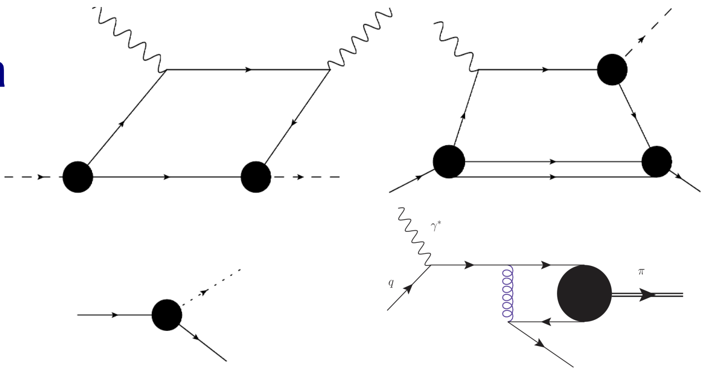


Summary

- **QCD inspired fermionic BSE model**
- **Solution in Minkowski space via Nakanishi Int. Representation;**
- **pion: LF amplitudes, SL FF**
- **pion: PDF (talk by Giovanni Salmè)**

Future ...

- **Self-energies, Landau gauge, quark-gluon vertex: ingredients from LQCD**
- **Confinement & quark-gluon vertex?**
- **Beyond the pion, kaon, D, B, rho…, and the nucleon**
- **TL FF, GPDs (DGLAP&ERBL),**
- **GTMDs (DGLAP&ERBL),**
- **Fragmentation Functions...**

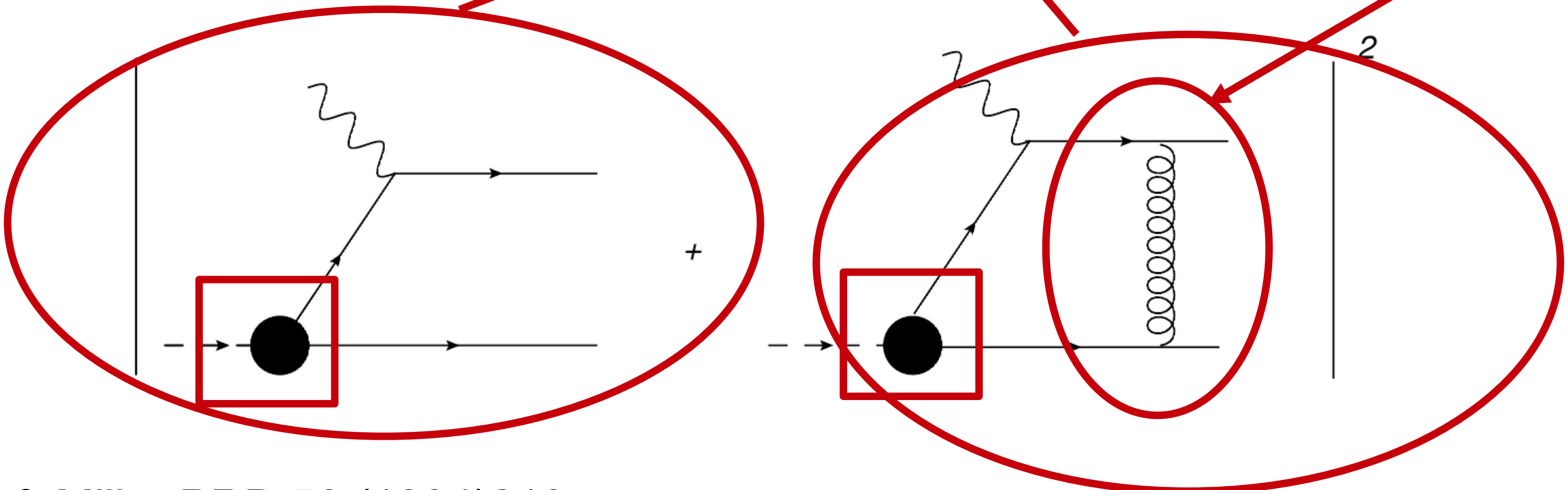


THANK YOU!

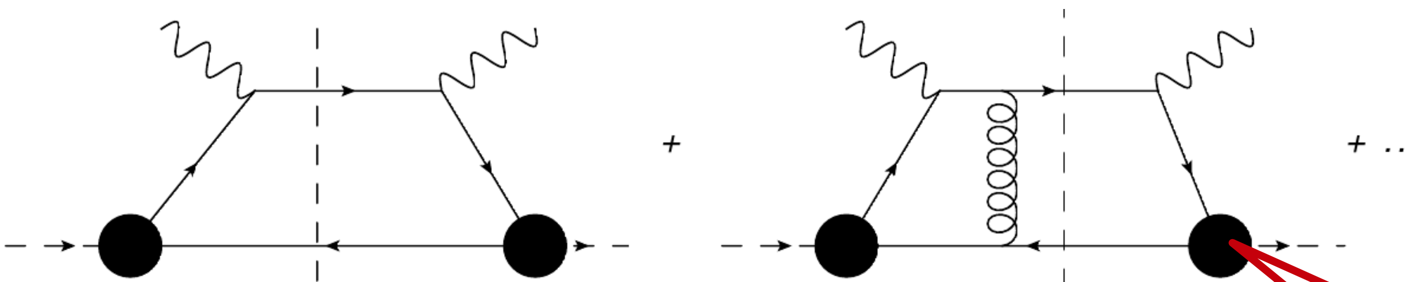
LIA/CNRS - SUBATOMIC PHYSICS: FROM THEORY TO APPLICATIONS

Schematic view: TMDs & PDFs

FSI gluon exchange: T-odd



TF & Miller PRD 50 (1994)210



$$q^2 = q^+q^- - q^2_T$$

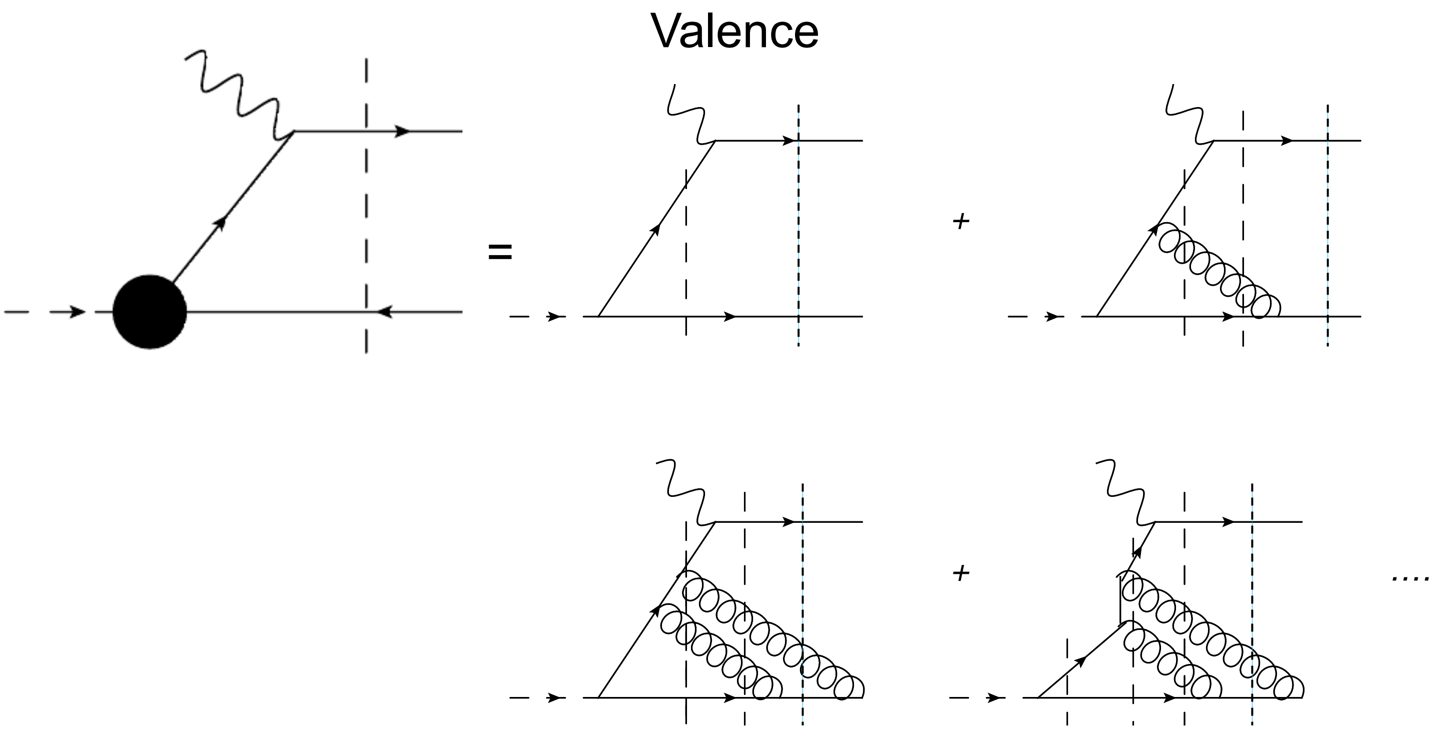
$$q^+ = q^0+q^3 \quad q^- = q^0-q^3$$

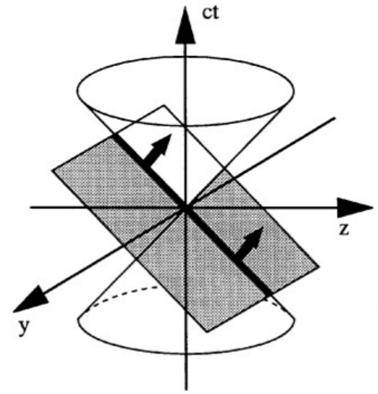
$q^- \longrightarrow$ *infty*
DIS

Bethe-Salpeter Amplitude @ x^+=0

Bethe-Salpeter amplitude: beyond the valence states

Light-front projection





- **higher Fock-components**
- **gluon radiation = initial state interaction (ISI)**
- **gluon radiation in the final state (FSI)**

Sales, TF, Carlson,Sauer, PRC 63, 064003 (2001); Marinho, TF, Pace,Salme,Sauer, PRD 77, 116010 (2008)

Generalized Stietjes transform and the LF valence wave function II

Carbonell, TF, Karmanov PLB769 (2017) 418

$$f(\gamma) \equiv \int_0^\infty d\gamma' \, L(\gamma, \gamma') g(\gamma') = \int_0^\infty d\gamma' \, \frac{g(\gamma')}{(\gamma' + \gamma + b)^2}$$

denoted symbolically as $f = \hat{L}\, g$.

$$g(\gamma) = \hat{L}^{-1} f = \frac{\gamma}{2\pi} \int_{-\pi}^{\pi} d\phi \; e^{i\phi} \; f(\gamma \, e^{i\phi} - b).$$

J.H. Schwarz, J. Math. Phys. 46 (2005) 014501,