DAs ⇔ DFs
PDAs & PDFs

- Relationship between leading-twist PDAs and valence-quark PDFs, expressed via a meson's light-front wave function (LFWF):

\[ \varphi(x) \sim \int d^2 k_\perp \psi(x, k_\perp^2), \]

\[ q(x) \sim \int d^2 k_\perp |\psi(x, k_\perp^2)|^2 \]

- Given that factorization of LFWF is a good approximation for integrated quantities, then at the hadronic scale, \( \zeta_H \):

\[ q_{\pi,K}(x; \zeta_H) \propto \varphi_{\pi,K}^q(x; \zeta_H)^2 \]

Proportionality constant is fixed by baryon number conservation

- Owing to parton splitting effects, this identity is not valid on \( \zeta > \zeta_H \).

(Think about DGLAP and ERBL regions for a GPD.)

- Nevertheless, evolution equations are known; so the connection is not lost, it just metamorphoses.
Light Front Wave Function

➢ In many respects, a hadron’s LFWF is the key.
➢ LFWF correlates all observables
➢ EHM is expressed in every hadron LFWF
➢ The “trick” is to find a way to compute the LFWF

➢ Experiments sensitive to differences in LFWFs are sensitive to EHM
➢ Excellent examples are π & K DAs and DFs
  – Two sides of the same coin
  – Accessible via different processes
  – Independent measurements of the same thing
  – Great check on consistency
Meson leading-twist DAs

- Continuum results exist & IQCD results arriving
- Common feature = broadening
- Origin = EHM
- NO differences between $\pi$ & $K$ if EHM is all there is
  - Differences arise from Higgs-modulation of EHM mechanism
  - “Contrasting $\pi$ & $K$ properties reveals Higgs wave on EHM ocean”

Kaon DA vs pion DA
  - almost as broad
  - peak shifted to $x=0.4(5)$
  - $\langle \xi^2 \rangle = 0.24(1), \langle \xi \rangle = 0.035(5)$

ERBL evolution logarithmic

Broadening & skewing persist to very large resolving scales – beyond LHC
Pion DA & form factor

- QCD is not found in scaling ... it is found in scaling violations
- Continuum predictions
  - Match existing data
  - Suggest that Jlab 12 could potentially be first to reveal scaling violations in a hard-scattering process = see QCD in a hard-scattering process
- Simulations indicate that EIC is certainly capable of doing so.
- Normalisation of the form-factor curve is a measure of the level of DA broadening; hence, size of EHM

![Graph](image)

FIG. 9: Projected EIC pion form factor data as extracted from a combination of electron-proton and electron-deuteron scattering, each with an integrated luminosity of 20 fb$^{-1}$ – black stars with error bars. Also shown are projected JLab 12-GeV data from a Rosenbluth-separation technique – orange diamonds and green triangle. The long-dashed green curve is a monopole form factor whose scale is determined by the pion radius. The black solid curve is the QCD-theory prediction bridging large and short distance scales, with estimated uncertainty [41]. The dot-dashed blue and dotted purple curves represent the short-distance views [79–81], comparing the result obtained using a modern DCSB-hardened PDA and the asymptotic profile, respectively.
Kaon form factor - flavour separation

\[
\langle 0 | Q_0 | \Lambda_{QCD} \rangle \approx Q^2 F_K(Q^2) \approx 16 \pi \alpha_s(Q^2) f_K^2 w_K^2(Q^2)
\]

with \( f_K = 0.110 \text{ GeV} \) and, for the \( K^+ \):

\[
w_K^2 = \epsilon_s w_s^2 + \epsilon_u w_u^2,
\]

\[
w_s = \frac{1}{3} \int_0^1 dx \frac{1}{1-x} \varphi_K(x), \quad w_u = \frac{1}{3} \int_0^1 dx \frac{1}{x} \varphi_K(x)
\]

- Current conservation: \( F_{uus}(0) = F_{uus}(0) \)
- Under evolution:
  \( \varphi_K \to 6 \times (1-x) \Rightarrow \omega_\bar{s} \to \omega_u \Rightarrow \text{Ratio} \to 1 \)
- Agreement between direct calculation and hard-scattering formula, using consistent PDA
- Ratio never exceeds 1.5 and logarithmic approach to unity
- Typical signal of EHM-dominance in flavour-symmetry breaking, taming the large Higgs-produced current-quark mass difference:
  - \( m_s \approx 30 \text{ mu} \Rightarrow M_s(0) \approx 1.25 M_u(0) \)
  - scale difference does finally become irrelevant under evolution, but only at very large scales

Craig Roberts. \( \pi & K \) structure - window onto EHM
Controversy over PDAs

  - Claim: $\phi_\pi(x)$ is well represented by the asymptotic profile for $\zeta^2 > 10 \text{ GeV}^2$
- Modern continuum predictions and analyses of lQCD
  - PDAs are broadened at $\zeta^2 = 4 \text{ GeV}^2$
  - Evolution is logarithmic ⇒ if true at $\zeta^2 = 4 \text{ GeV}^2$, then true at $\zeta^2 = 10 \text{ GeV}^2$
- Theory indicates that E791 conclusion cannot be correct
  - The E791 images cannot represent the same pion property
  - Not credible to assert that $\phi_\pi(x)$ is well represented by the asymptotic distribution for $\zeta^2 > 10 \text{ GeV}^2$

- Hard exclusive processes only sensitive to low-order PDA moments.
- Diffractive processes much better because sensitive to $x$-dependence?
  (check this claim)
Meson valence-quark DFs

\[ \varphi(x) \sim \int d^2 k_\perp \psi(x, k_\perp^2), \]

\[ q(x) \sim \int d^2 k_\perp |\psi(x, k_\perp^2)|^2 \]

- Owing to these relations
- Broadening of DAs feeds into broadening of DFs
- Necessary consequence of EHM
- Moreover, any Higgs-boson related modulations of EHM in the DA will also be expressed in the DF
- Pion – Kaon comparisons great place to study interference between the Standard Model’s two mass-generating mechanisms
Meson leading-twist DAs and valence-quark DFs

- Broadening need not and should not disturb the DA's endpoint behaviour
- QCD: \( \phi(x) = x (1 - x) f(x), f(x \approx 0) = \text{constant}_1, f(x \approx 1) = \text{constant}_2 \)
- Many models that express EHM-induced broadening violate this constraint
- Typically not a problem, unless endpoint behaviour is taken too seriously
- Example AdS/QCD: \( \phi(x) = \frac{8}{\pi} \sqrt{x(1-x)} \)
- Practically identical to the continuum prediction that preserves QCD constraint:
  - blue dashed vs green dot-dashed
  - However, AdS/QCD practitioners use DA to argue for \( x \approx 1 \Rightarrow q^\pi(x; \zeta_H) \propto (1 - x)^1 \)
  - Endpoint behaviour taken “too seriously”
Controversy over pion valence DF

- Parton model prediction for the valence-quark DF of a spin-zero meson:
  \[ x \simeq 1 \Rightarrow q^\pi(x; \zeta_H) \propto (1 - x)^2 \]

- The hadronic scale is not empirically accessible in Drell-Yan or DIS processes.
  (Matter of conditions necessary for data to be interpreted in terms of distribution functions.)

- For such processes, QCD-improvement of parton model leads to the following statement:
  At any scale for which experiment can be interpreted in terms of parton distributions, then
  \[ x \simeq 1 \Rightarrow q^\pi(x; \zeta) \propto (1 - x)^{\beta=2+\gamma}, \gamma > 0 \]

- Simple restatement of the following:
  - The parton model gives us scaling and scaling laws.
  - QCD’s gluon corrections give us scaling violations
  - Scaling violations do NOT alter the integer-number that characterises scaling powers [L&B-1980 Lepage:1980fj]
  - Certainly don’t reduce 2 → 1 (or 3 → 2 for nucleon valence) – scaling violations increase power logarithmically
Controversy over pion valence DF

Consequence
- Any analysis of DY or DIS (or similar) experiment which returns a value of $\beta < 2$ conflicts with QCD.

Observation
- All existing internally-consistent calculations preserve connection between large-$k^2$ behaviour of interaction and large-$x$ behaviour of DF.
  - $J=0 \ldots (1/k^2)^n \leftrightarrow (1-x)^{2n}$
- No existing calculation with $n=1$ produces anything other than $(1-x)^2$
- Internally-consistent calculation that preserve RG properties of QCD, then $2 \rightarrow 2+\gamma$, $\gamma > 0$, at any factorisation-valid scale

Controversy:
- Ignore threshold resummation, then data analysis yields $(1-x)^{1+\gamma}$
- Include threshold resummation, then data analysis yields $(1-x)^{2+\gamma}$
Where is “2” to be seen?

- Use DSE DF ... prediction ... NOT fit to data
  - Within uncertainty, brackets DF points obtained in NLO+NLL analysis
    - Central curve: $\chi^2$/dof = 1.66
  - By same measure, inconsistent with LO E615
    - Central curve: $\chi^2$/dof = 19.4 – order of magnitude larger

- Valence domain begins after peak, at which point $2 \times V(x) > x (S(x)+G(x))$

- Power discriminating function – local (x-dependent) exponent:
  
  $$\beta(x) = - \frac{1 - x}{q^\pi_V(x)} \frac{dq^\pi_V(x)}{dx}$$

  - “Active” power greater > 2 on $x > 0.75$

Precise data & sound extraction on $0.6 < x < 0.8$
sufficient to test QCD prediction: $2 \neq 1$