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# DAs ⇔ DFs

# PDAs & PDFs

- Relationship between leading-twist PDAs and valence-quark PDFs, expressed via a meson's light-front wave function (LFWF):

$$\varphi(x) \sim \int d^2k_\perp \psi(x, k_\perp^2)\,,$$

$$q(x) \sim \int d^2k_\perp |\psi(x, k_\perp^2)|^2$$

- Given that factorization of LFWF is a good approximation for integrated quantities, then at the hadronic scale, $\zeta_H$:

$$q_{\pi,K}(x;\zeta_H) \propto \varphi^q_{\pi,K}(x;\zeta_H)^2$$

  Proportionality constant is fixed by baryon number conservation

- Owing to parton splitting effects, this identity is not valid on $\zeta > \zeta_H$ .

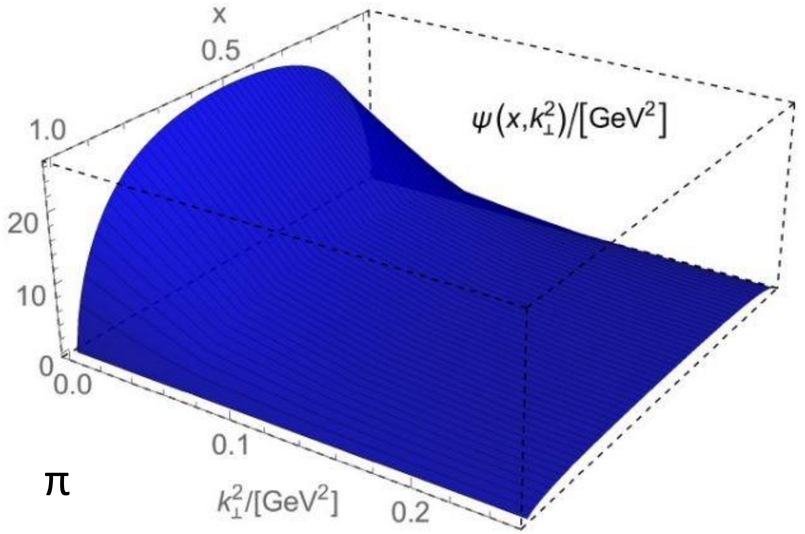
  (Think about DGLAP and ERBL regions for a GPD.)

- Nevertheless, evolution equations are known; so the connection is not lost, it just metamorphoses.

# Light Front Wave Function

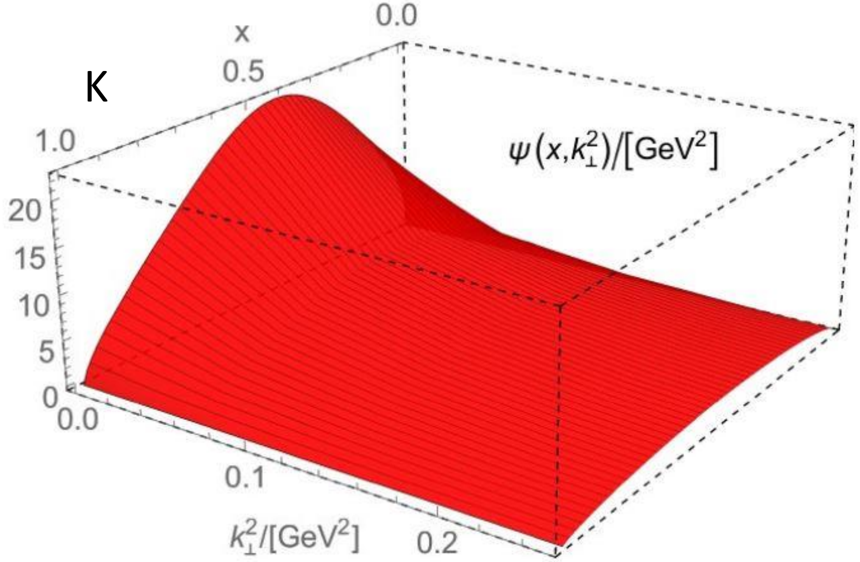
- In many respects, a hadron's LFWF is the key.
- LFWF correlates all observables
- EHM is expressed in every hadron LFWF
- The "trick" is to find a way to compute the LFWF

- Experiments sensitive to differences in LFWFs are sensitive to EHM
- Excellent examples are π & K DAs and DFs
  - Two sides of the same coin
  - Accessible via different processes
  - Independent measurements of the same thing
  - Great check on consistency



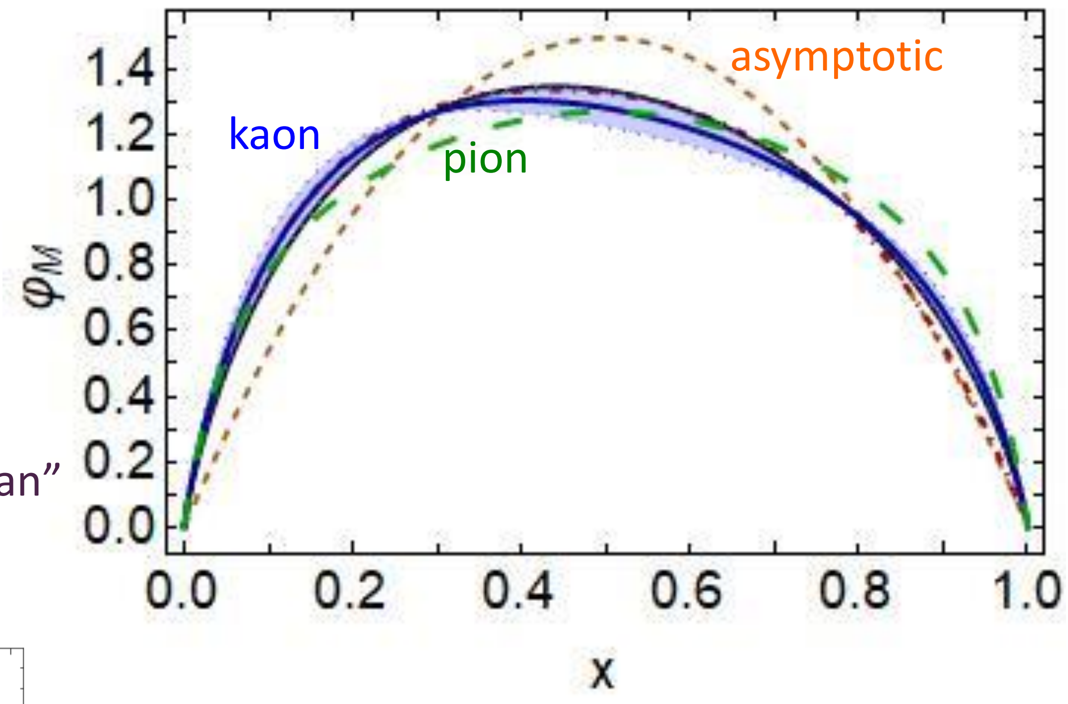

π

K



# Meson leading-twist DAs

- Continuum results exist & lQCD results arriving
- Common feature = broadening
- Origin = EHM
- NO differences between π & K if EHM is all there is
  - Differences arise from Higgs-modulation of EHM mechanism
  - "Contrasting π & K properties reveals Higgs wave on EHM ocean"



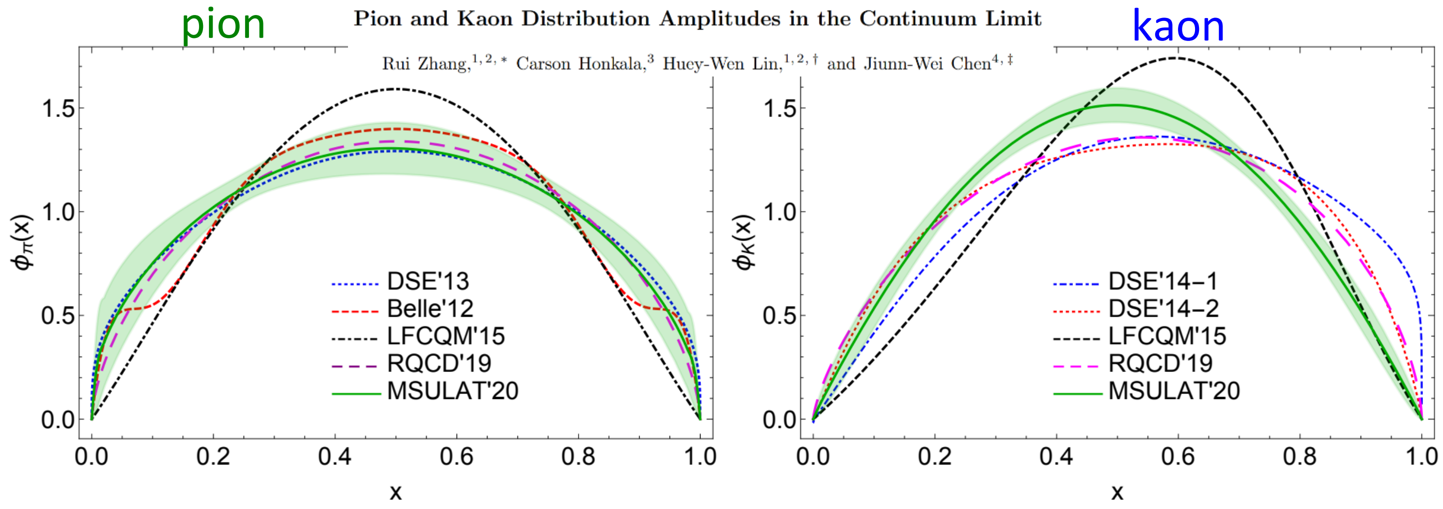




FIG. 10. Fit of the $P_z = 4\frac{2\pi}{L}$ pion (left) and kaon (right) data to the analytical form in Bjorken-$x$ space, compared with previous calculations (with only central values shown). Although we do not impose the symmetric condition $m = n$, both results for the pion and kaon are symmetric around $x = 1/2$ within error.

- Kaon DA vs pion DA
  - almost as broad
  - peak shifted to x=0.4(5)
  - $\langle \xi^2 \rangle = 0.24(1)$, $\langle \xi \rangle = 0.035(5)$
- ERBL evolution logarithmic
- Broadening & skewing persist to *<u>very</u>* large resolving scales – beyond LHC

# Pion DA & form factor

- QCD is not found in scaling … it is found in scaling violations
- Continuum predictions
  - Match existing data
  - Suggest that Jlab 12 could potentially be first to reveal scaling violations in a hard-scattering process = see QCD in a hard-scattering process
- Simulations indicate that EIC is certainly capable of doing so.
- Normalisation of the form-factor curve is a measure of the level of DA broadening; hence, size of EHM

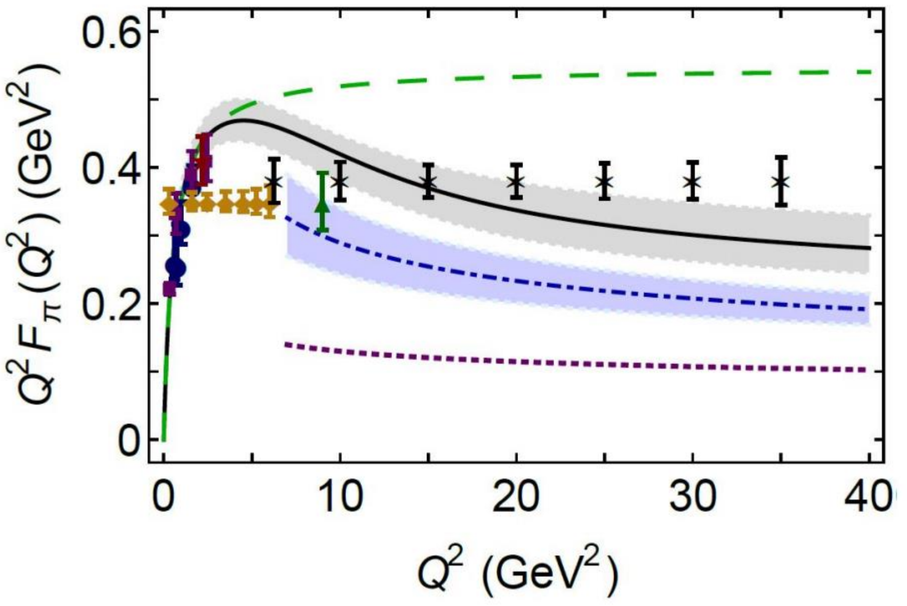



FIG. 9: Projected EIC pion form factor data as extracted from a combination of electron-proton and electron-deuteron scattering, each with an integrated luminosity of 20 fb$^{-1}$ – black stars with error bars. Also shown are projected JLab 12-GeV data from a Rosenbluth-separation technique – orange diamonds and green triangle. The long-dashed green curve is a monopole form factor whose scale is determined by the pion radius. The black solid curve is the QCD-theory prediction bridging large and short distance scales, with estimated uncertainty [41]. The dot-dashed blue and dotted purple curves represent the short-distance views [79–81], comparing the result obtained using a modern DCSB-hardened PDA and the asymptotic profile, respectively.

*Exposing strangeness: projections for kaon electromagnetic form factors, Fei Gao et al., arXiv:1703.04875 [nucl-th], Phys. Rev. D **96** (2017) 034024*

# Kaon form factor - flavour separation

$$\exists \bar{Q}_0 > \Lambda_{\rm QCD} \mid Q^2 F_K(Q^2) \overset{Q^2 > \bar{Q}_0^2}{\approx} 16\pi\alpha_s(Q^2) f_K^2 w_K^2(Q^2)$$

with [41] $f_K = 0.110\,\text{GeV}$ and, for the $K^+$:

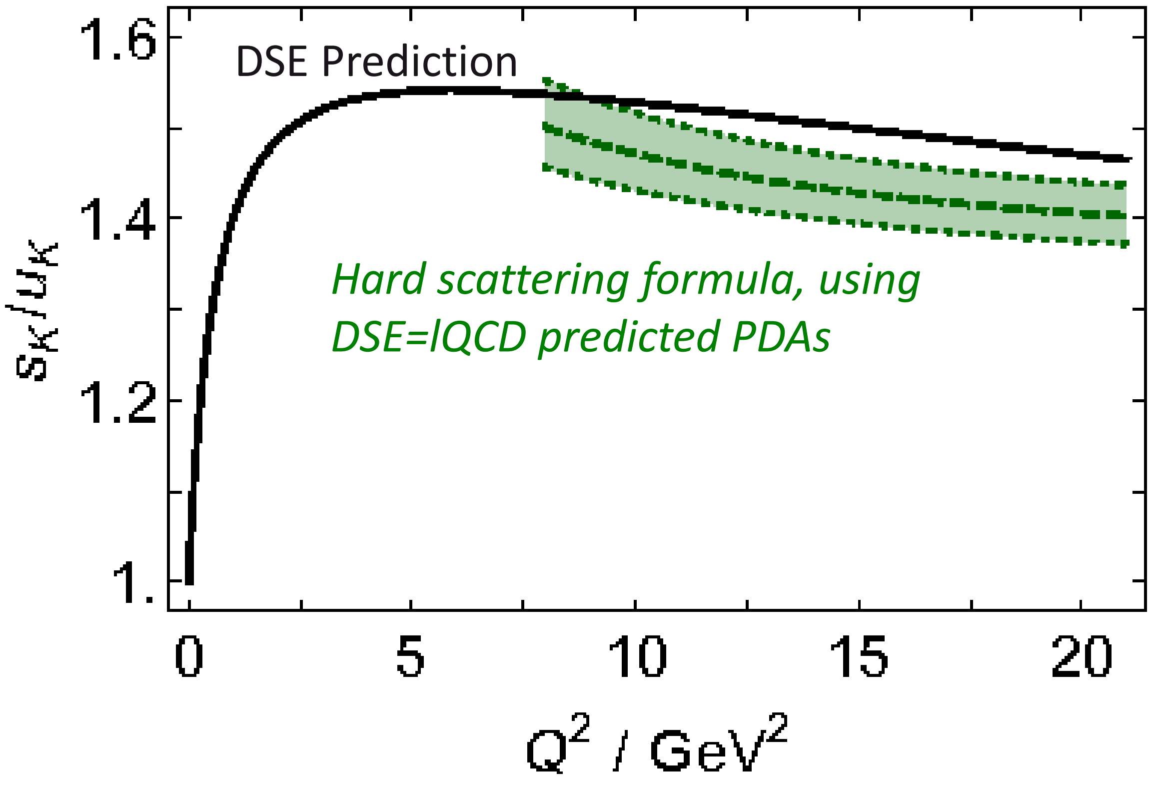
$$w_K^2 = e_{\bar{s}} w_{\bar{s}}^2 + e_u w_u^2,$$

$$w_{\bar{s}} = \frac{1}{3}\int_0^1 dx\, \frac{1}{1-x}\varphi_K(x)\,, \quad w_u = \frac{1}{3}\int_0^1 dx\, \frac{1}{x}\varphi_K(x)$$

$[\, \bar{s}\, \gamma\, s\, u_{spectator} \,/\, \bar{u}\, \gamma\, u\, s_{spectator}\,]^2 \leq 1.5$

- Current conservation: $F_{uss}(0) = F_{uus}(0)$
- Under evolution:
  $\varphi_K \to 6\,x\,(1-x) \Rightarrow \omega_{\bar{s}} \to \omega_u \Rightarrow$ Ratio $\to 1$
- Agreement between direct calculation and hard-scattering formula, using consistent PDA
- Ratio never exceeds 1.5 and

  Logarithmic approach to unity
- Typical signal of EHM-dominance in flavour-symmetry breaking, taming the large Higgs-produced current-quark mass difference:
  - *ms ~ 30 mu* $\Rightarrow M_s(0) \sim 1.25\, M_u(0)$
  - scale difference does finally become irrelevant under evolution, but only at *very* large scales

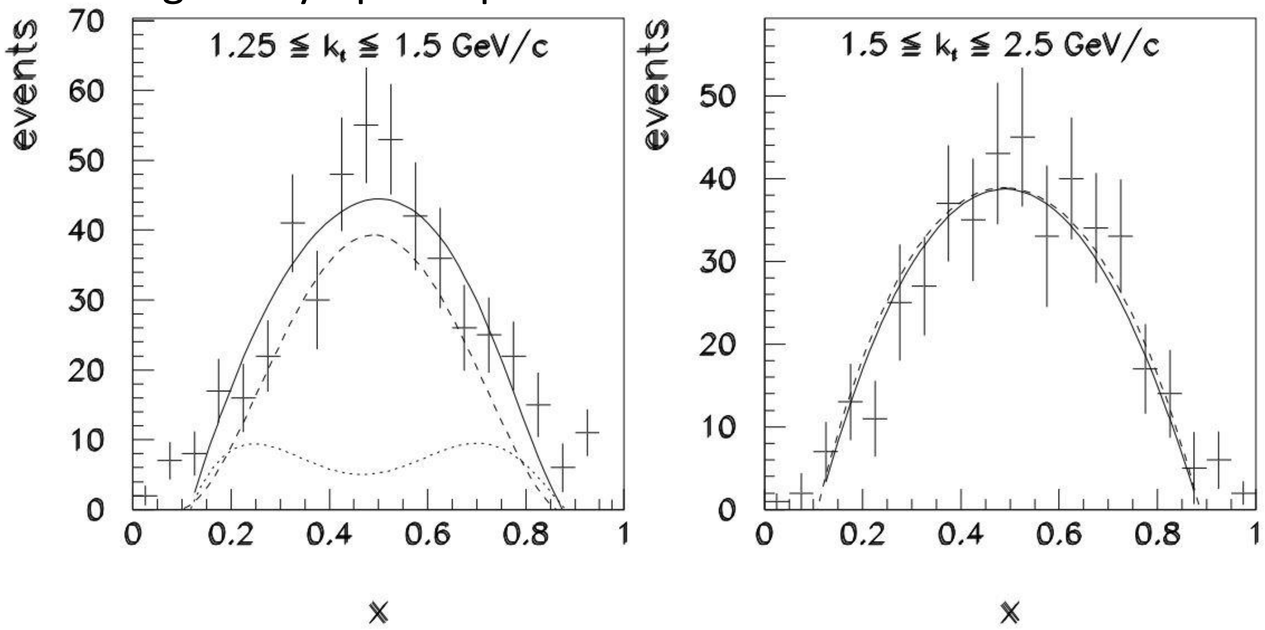


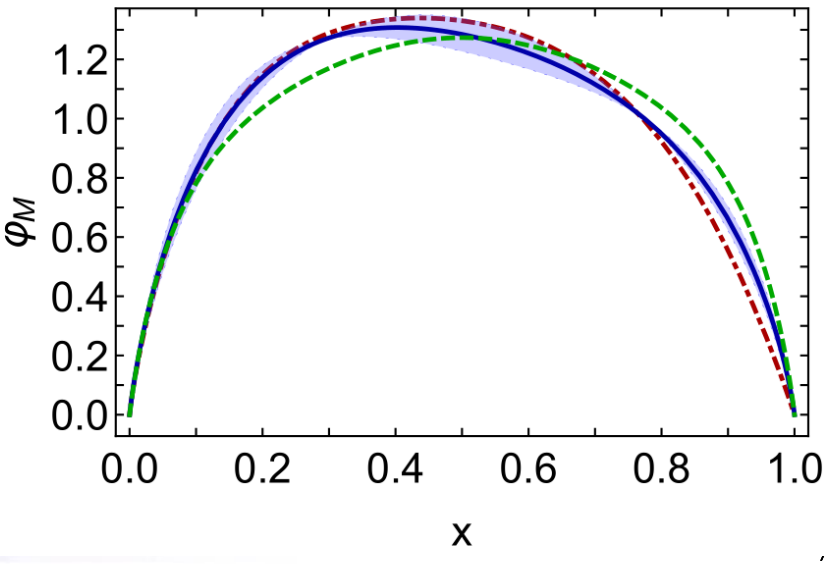
# Controversy over PDAs

- E791 Collaboration, E. Aitala *et al.*, Phys. Rev. Lett. 86, 4768 (2001).
  - Claim: $\varphi_\pi(x)$ is well represented by the asymptotic profile for $\zeta^2 > 10\ \text{GeV}^2$
- Modern continuum predictions and analyses of lQCD
  - PDAs are broadened at $\zeta^2=4\ \text{GeV}^2$
  - Evolution is logarithmic $\Rightarrow$ if true at $\zeta^2=4\ \text{GeV}^2$, then true at $\zeta^2=10\ \text{GeV}^2$
- Theory indicates that E791 conclusion cannot be correct
  - The E791 images cannot represent the same pion property
  - Not credible to assert that $\varphi_\pi(x)$ is well represented by the asymptotic distribution for $\zeta^2 > 10\ \text{GeV}^2$

Left: Nonpertubative (broadening) important

Right: Asymptotic profile sufficient




- Hard exclusive processes only sensitive to low-order PDA moments.
- Diffractive processes much better because sensitive to x-dependence?

  (check this claim)

# Meson valence-quark DFs

- Owing to these relations

$$\varphi(x) \sim \int d^2k_\perp \psi(x, k_\perp^2)\,,$$

$$q(x) \sim \int d^2k_\perp |\psi(x, k_\perp^2)|^2$$

- Broadening of DAs feeds into broadening of DFs
- Necessary consequence of EHM
- Moreover, any Higgs-boson related modulations of EHM in the DA will also be expressed in the DF
- Pion – Kaon comparisons great place to study interference between the Standard Model's two mass-generating mechanisms

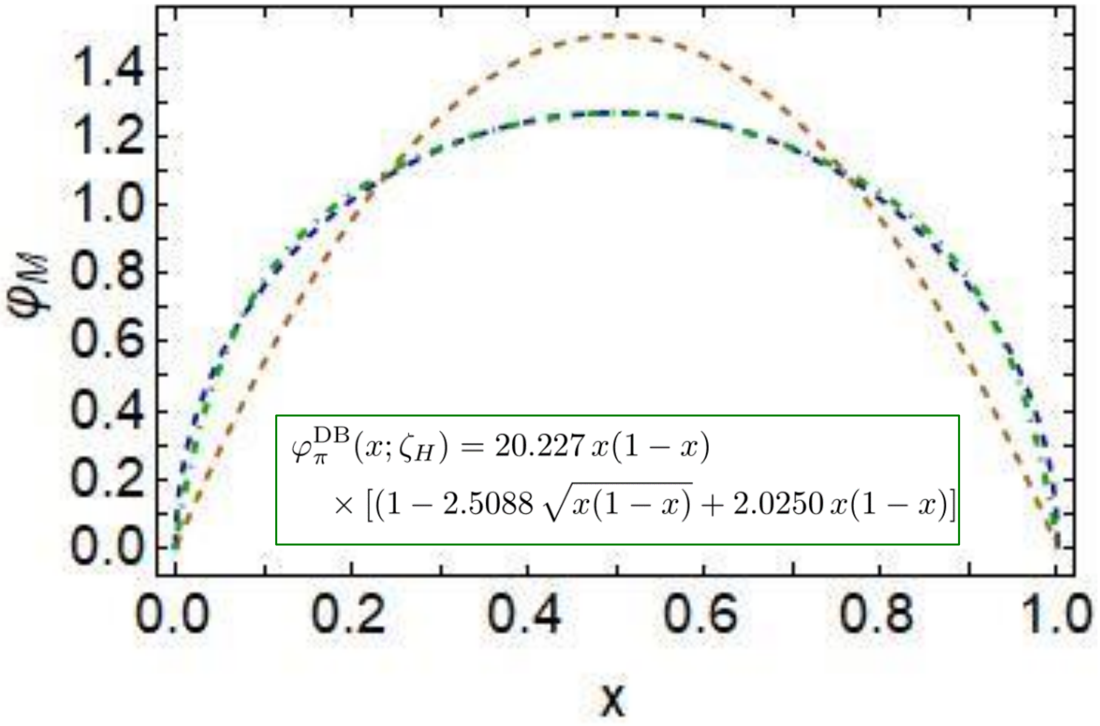
# Meson leading-twist DAs and valence-quark DFs

- Broadening need not and should not disturb the DA's endpoint behaviour
- QCD: $\varphi(x) = x\,(1-x)f(x), f(x \simeq 0) = constant_1, f(x \simeq 1) = constant_2$
- Many models that express EHM-induced broadening violate this constraint
- Typically not a problem, unless endpoint behaviour is taken too seriously
- Example AdS/QCD: $\varphi(x) = \frac{8}{\pi}\sqrt{x(1-x)}$
- Practically identical to the continuum prediction that preserves QCD constraint:

  blue dashed *vs* green dot-dashed

  – However, AdS/QCD practitioners use DA to argue for $x \simeq 1 \Rightarrow q^{\pi}(x;\zeta_{\rm H}) \propto (1-x)^1$

  – Endpoint behaviour taken "too seriously"



# Controversy over pion valence DF

- Parton model prediction for the valence-quark DF of a spin-zero meson:

$$x \simeq 1 \Rightarrow q^{\pi}(x;\zeta_{\mathrm{H}}) \propto (1-x)^2$$

- The hadronic scale is not empirically accessible in Drell-Yan or DIS processes.
  (Matter of conditions necessary for data to be interpreted in terms of distribution functions.)

- For such processes, QCD-improvement of parton model leads to the following statement:
  At any scale for which experiment can be interpreted in terms of parton distributions, then

$$x \simeq 1 \Rightarrow q^{\pi}(x;\zeta) \propto (1-x)^{\beta=2+\gamma}, \gamma > 0$$

- Simple restatement of the following:
  - The parton model gives us scaling and scaling laws.
  - QCD's gluon corrections give us scaling violations
  - Scaling violations do NOT alter the integer-number that characterises scaling powers [L&B-1980 Lepage:1980fj]
  - Certainly don't reduce 2 → 1 (or 3 → 2 for nucleon valence) – scaling violations increase power logarithmically

# Controversy over pion valence DF

- Consequence
  - Any analysis of DY or DIS (or similar) experiment which returns a value of $\beta$ <2 conflicts with QCD.
- Observation
  - All existing internally-consistent calculations preserve connection between large-$k^2$ behaviour of interaction and large-x behaviour of DF.
    - J=0 … $(1/k^2)^n \Leftrightarrow (1-x)^{2n}$
- No existing calculation with n=1 produces anything other than $(1-x)^2$
- Internally-consistent calculation that preserve RG properties of QCD,

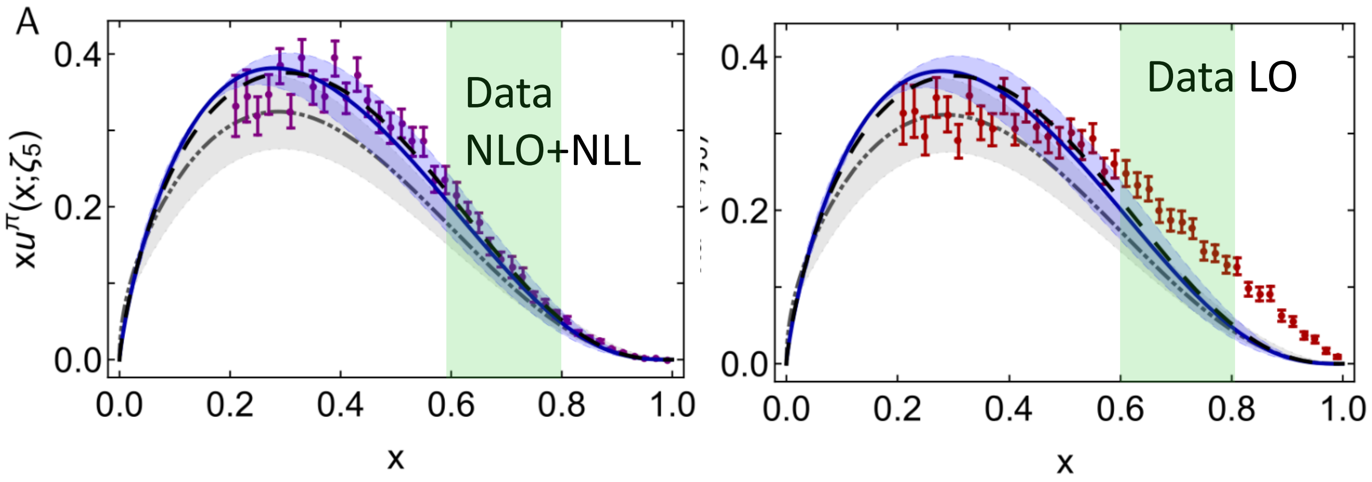
  then $2 \to 2+\gamma$, $\gamma>0$, at any factorisation-valid scale
- Controversy:
  - Ignore threshold resummation, then data analysis yields $(1-x)^{\mathbf{1}+\gamma}$
  - Include threshold resummation, then data analysis yields $(1-x)^{\mathbf{2}+\gamma}$

# Where is “2” to be seen?

- Use DSE DF … prediction … NOT fit to data
  - Within uncertainty, brackets DF points obtained in NLO+NLL analysis
    - Central curve: $\chi^2$/dof = 1.66
  - By same measure, inconsistent with LO E615
    - Central curve: $\chi^2$/dof = 19.4 – order of magnitude larger
- Valence domain begins after peak, at which point 2 x V(x) > x ( S(x)+G(x) )
- Power discriminating function – local (x-dependent) exponent:

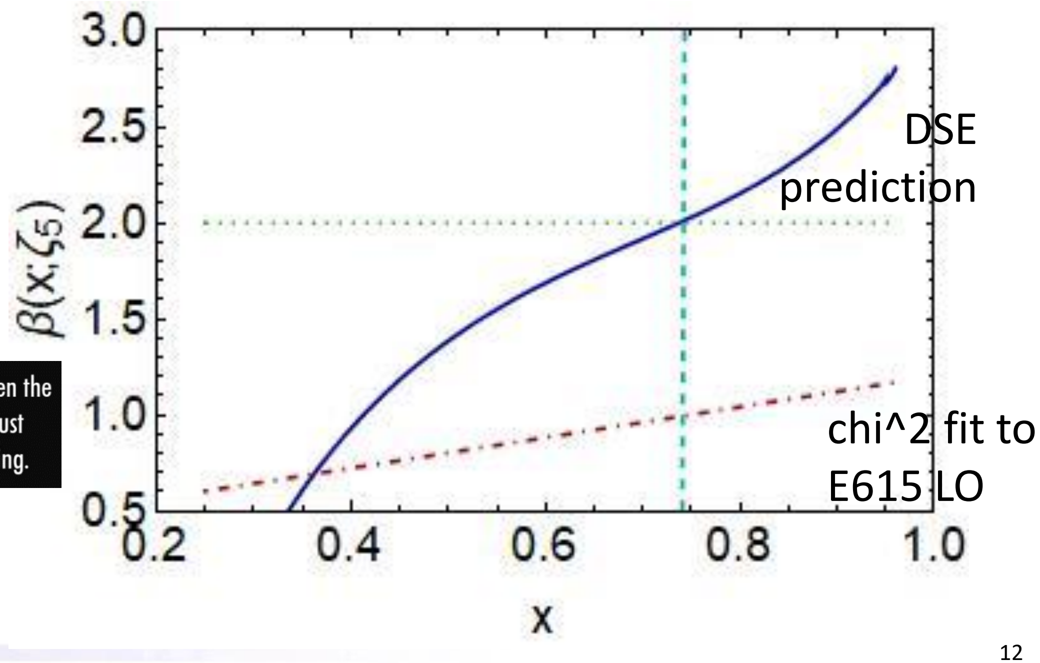
$$\beta(x) = -\frac{1-x}{q_V^\pi(x)} \frac{dq_V^\pi(x)}{dx}$$

  - “Active” power greater > 2 on x > 0.75




*Precise data & sound extraction on 0.6 < x < 0.8 sufficient to test QCD prediction: 2 ≠ 1*

Effective $\beta(x)$




This is not the end, this is not even the beginning of the end, this is just perhaps the end of the beginning.