Pion-induced Drell-Yan and pion TMD distribution

Alexey Vladimirov
Regensburg University

Workshop on Pion and Kaon Structure Functions at the EIC
I present the analysis of $q_T$-spectrum of pion-induced DY within TMD factorization

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Main aim: Preparation for COMPASS $\pi$DY

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**Plan of talk**

- Reminder TMD factorization
- $\pi$DY data TMD data
- Problems with normalization of fixed target DY
- TMDs for pion!
TMD factorization formula for DY with TMD evolution

\[
\frac{d\sigma}{dx dz dQ^2 dq_T} = \sum_{f f'} H_{f f'} \left( \frac{Q}{\mu} \right) \int d^2 b \, e^{i(b \cdot q_T)} \, F_{f \leftarrow p}(x_1, b, \mu, \zeta) F_{f' \leftarrow \pi}(x_2, b, \mu, \zeta)
\]

TMD evolution (usual solution)

\[
\frac{d\sigma}{dx dz dQ^2 dq_T} = \sum_{f f'} H_{f f'} \left( \frac{Q}{\mu} \right) \int \cdots e^{\int_{\mu_0}^{Q} \gamma_V - D \ln \left( \frac{\zeta}{\zeta_0} \right)} F_{f \leftarrow p}(x_1, b, \mu_0, \zeta_0) F_{f' \leftarrow \pi}(x_2, b, \mu_0, \zeta_0)
\]

\[
e^{\int_{\mu_0}^{Q} \gamma_V - D \ln \left( \frac{\zeta}{\zeta_0} \right)} = \exp \left( \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left( \Gamma_{cusp}(\mu') \ln \left( \frac{\mu^2}{\sqrt{\zeta}} \right) - \gamma_V(\mu') \right) - D(b, \mu_0) \ln \left( \frac{\zeta}{\zeta_0} \right) \right)
\]
TMD factorization formula for DY with TMD evolution

$$\frac{d\sigma}{dxdzdQ^2d^2q_T} = \sum_{ff'} H_{ff'}\left(\frac{Q}{\mu}\right) \int d^2b e^{i(b\cdot q_T)} F_{f\leftarrow p}(x_1, b, \mu, \zeta) F_{f'\leftarrow \pi}(x_2, b, \mu, \zeta)$$

TMD evolution (optimal solution)

“ζ-preservation”

$$\frac{d\sigma}{dxdzdQ^2d^2q_T} = \sum_{ff'} H_{ff'}\left(\frac{Q}{\mu}\right) \int d^2b e^{i(b\cdot q_T)} \left(\frac{Q^2}{\zeta\mu[D]}\right)^{-2\mathcal{D}(b, \mu)} F_{f\leftarrow p}(x_1, b) F_{f'\leftarrow \pi}(x_2, b)$$

- Clean separation of TMDs from non-perturbative evolution (TMDs are defined at the point with $\mathcal{D} = 0$)
- Solution is made in terms of non-perturbative $\mathcal{D}$
- Simple and fast expression for TMD evolution factor (just an algebraic function)
- Simpler expression for perturbative matching for TMDs
TMD factorization formula (in $\zeta$-prescription)

\[
\frac{d\sigma}{dxdzdQ^2d^2q_T} = \sum_{ff'} H_{ff'}\left(\frac{Q}{\mu}\right) \int d^2b e^{i(b\cdot q_T)} \left(\frac{Q^2}{\zeta_\mu[D]}\right)^{-2D(b,\mu)}
\]

Rapidity anomalous dimension
\[
D \sim \langle 0|F_{+b}[\text{staple link}]|0\rangle
\]

[AV,2003.02288]

Each data-point is a product (convolution) of three independent non-perturbative functions

Functions do not “cross-talk” and could be modeled independently

Each function is responsible for a separate kinematic variable

- Rapidity AD: $D \rightarrow Q$ and $b$
- TMD N1: $F_1 \rightarrow x_1$ and $b$
- TMD N2: $F_2 \rightarrow x_2$ and $b$
Universality of TMDs

Global fit SV19
unpolarized DY + SIDIS
[1912.06532]

- DY: LHC, Tevatron, FermiLab, RHIC
- SIDIS: HERMES, COMPASS
- Large energy coverage (2<Q<150 GeV) = decorrelation of RAD and TMDs
- NNLO matching + N$^3$LO evolution

TMD evolution and proton TMD PDF is known

To describe $\pi$DY one needs only $\pi$TMD PDF
Model for $\pi$TMD PDF

$$F_{q\rightarrow \pi}(x, b) = \sum_f \int_x^1 \frac{dy}{y} C_{q\rightarrow f}(y, \mathbf{L}_\mu) f_{1,f\rightarrow \pi} \left( \frac{x}{y}, \mu \right) f_{NP}(x, b)$$

- NNLO matching to collinear distributions
- Collinear PDF is JAM18pionPDFnlo
- NP-part can be Gauss/Exponent, with three parameters $a_1, a_2, a_3$

$$f_{NP}(x, b) = \exp\left( -\frac{(a_1 + (1 - x)^2 a_2)b^2}{\sqrt{1 + a_3 b^2}} \right)$$
\[ \pi \text{DY } q_T\text{-data} \]

\textbf{not too much...}

<table>
<thead>
<tr>
<th>Experiment</th>
<th>(\sqrt{s}) [GeV]</th>
<th>(Q) [GeV]</th>
<th>(x_F)</th>
<th>(N_{pt})</th>
<th>corr.err.</th>
<th>Typical stat.err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>E537 (Q-diff.)</td>
<td>15.3</td>
<td>(4.0 &lt; Q &lt; 9.0) in 10 bins</td>
<td>(-0.1 &lt; x_F &lt; 1.0)</td>
<td>60/146</td>
<td>8%</td>
<td>(\sim 20%)</td>
</tr>
<tr>
<td>E537 ((x_F)-diff.)</td>
<td>15.3</td>
<td>(4.0 &lt; Q &lt; 9.0) in 11 bins</td>
<td>(-0.1 &lt; x_F &lt; 1.0)</td>
<td>110/165</td>
<td>8%</td>
<td>(\sim 20%)</td>
</tr>
<tr>
<td>E615 (Q-diff.)</td>
<td>21.8</td>
<td>(4.05 &lt; Q &lt; 13.05) in 10(8) bins</td>
<td>(0.0 &lt; x_F &lt; 1.0)</td>
<td>51/155</td>
<td>16%</td>
<td>(\sim 5%)</td>
</tr>
<tr>
<td>E615 ((x_F)-diff.)</td>
<td>21.8</td>
<td>(4.05 &lt; Q &lt; 8.55) in 10 bins</td>
<td>(0.0 &lt; x_F &lt; 1.0)</td>
<td>90/159</td>
<td>16%</td>
<td>(\sim 5%)</td>
</tr>
<tr>
<td>NA3</td>
<td>16.8, 19.4, 22.9</td>
<td>(4.1 &lt; Q &lt; 8.5) (y &gt; 0(?))</td>
<td>(-)</td>
<td>(-)</td>
<td>15%</td>
<td>(-)</td>
</tr>
</tbody>
</table>

The usual TMD cut \((q_T/Q < 0.25)\)

Data selected for fit \textbf{reason:}

\begin{itemize}
  \item E537 has too large uncertainties
  \item NA3 is available only as a plot
  \item It is better to use \(x\)-differential data, since \(Q\)-dependence is known.
\end{itemize}
Huge normalization deficit
Corr. uncertainty = 16%

E615 4.05 < Q < 8.55 GeV

\[ \chi^2 / N_p = 0.67 + 0.77 = 1.44 \]
Possible sources of discrepancy

- Nuclear effects: multiplication by $R(x)$ could give $\sim 10\%$ effect (not enough $+x$-dependence)
- Effect of $\pi$PDF uncertainty: significant at a point (up to 20%) but only 2-3% in normalization
- Threshold logarithms: could be but the main disagreement is at $x \sim 0.2$ and decreases to $x \sim 0.7$
- Power corrections: unknown (but I would not expect more than 5%)
- Model bias: ?? (I don’t think so)
- Resonance contamination: The bins go down to $3 - 4$ GeV, they could be contaminated by $J/\psi$ or $\psi'$ resonances. However, typically, in this case theory overshoot the data.

Conclusion:
I do not see theory sources that would cover 50% normalization gap.
The deficit in normalization is typical for TMD description of fixed-target experiments. However, it is usually 10-20% (not 50%).

Such problem does not exist for collider experiments.

$4.8 < Q < 8.2$
Data is worse
Corr. uncertainty = 8% 
$\chi^2/N_p = 0.50 + 0.11 = 0.61$

$\chi^2_{E537}/N_p = 0.50 + 0.11 = 0.61$  $\langle d/\sigma \rangle$ = 22.9%
πDY-data from NA3 abd E537 does not show such anomalous behavior

Artemide v2.01

\[ \frac{d\sigma}{dq_T} \] [pb]

- \( s = 357 \text{ GeV}^2 \)
- \( 4.3 < Q < 8.5 \text{ GeV} \)
- \( 0. < x_F < 1. \)

Normalization by theory
\[ a_1 = 0.17 \pm 0.11 \pm 0.03, \quad a_2 = 0.48 \pm 0.34 \pm 0.06, \quad a_3 = 2.15 \pm 3.25 \pm 0.32. \]

**Position-space**

\[ F_{d\ell-\pi^-}(x, b_T) \]

**Momentum-space**

\[ F_{d\ell-\pi^-}(x, k_T) \]

Only statistical uncertainties are shown
Pion is “narrower” in the momentum space.
Conclusion

- The analysis of $\pi$DY is made within the best available TMD-framework
  - NNLO matching, $N^3$LO evolution
  - Other components (proton TMD, NP TMD evolution) are from the global analysis SV19
  - Numerics is done by artemide
- Pion TMDs are extracted from the existing data
  - Normalization problem of $q_T$-dependent E615-data
  - Rest available data (lower quality) have no problem with normalization
- The extraction $Vpion19$ is available as a part of default artemide distribution
  
  github.com/VladimirovAlexey/artemide-public

Prospects

- The road to consistent global analysis ($\pi$DY angular modulations by COMPASS)
- Looking forward for COMPASS unpolarized $\pi$DY
- ...
Backup slides
\[ \chi^2/N_p = 0.92 + 4.63 = 5.54 \]
\[ \langle d/\sigma \rangle = 59.6\% \]

\[ q_T \times 10^{-2} \]

0.0 0.5 1.0 1.5 2.0 2.5

0 2 4 6 8

\[ d\sigma/dQ dq_T [\text{nb/GeV}^2] \]

E615 0.0 < x_F < 1.0

\[ 100 \Delta N(\%) \]

\[ Q[\text{GeV}] \]
\[ \frac{d\sigma}{dQdq_T} \text{ [nb/GeV}^2\text{]} \]

E537 \(-0.1 < x_T < 1.0\)

\[ \chi^2_{E537}/N_p = 0.85 + 0.12 = 0.97 \quad \langle d/\sigma \rangle = 15.3\% \]