THE TRANSVERSE STRUCTURE OF THE PION IN MOMENTUM SPACE FROM ADS/QCD MODELS

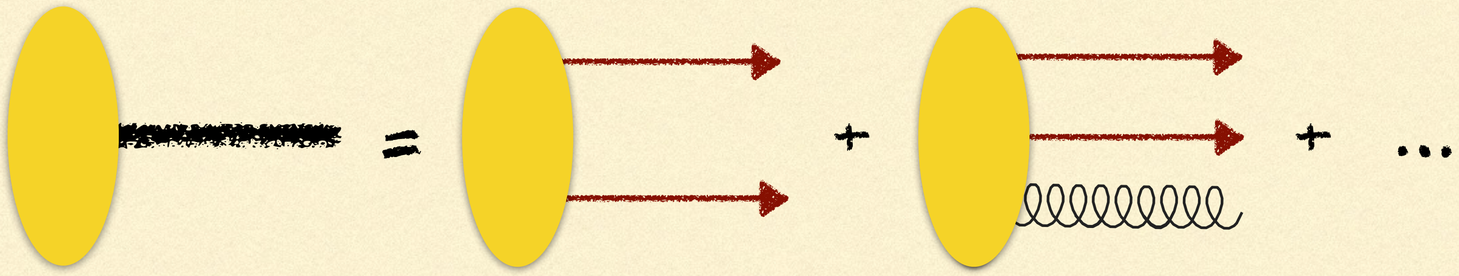
Sabrina Cotogno (sabrina.cotogno@polytechnique.edu)

in collaboration with: Alessandro Bacchetta and Barbara Pasquini (UniPV)

based on: Physics Letters B 771 (2017) 546–552

Workshop on Pion and Kaon Structure Functions at the EIC
2-5 June 2020

HADRON STRUCTURE AND LFWFS



$$|P,\Lambda\rangle = \sum_{N,\beta} \int \left[\frac{dx}{\sqrt{x}}\right]_N \left[d^2\boldsymbol{k}_T\right]_N \psi^{\Lambda}_{N,\beta}(r)|N;k_1,\cdots,k_N,\beta_1,\cdots,\beta_N\rangle$$

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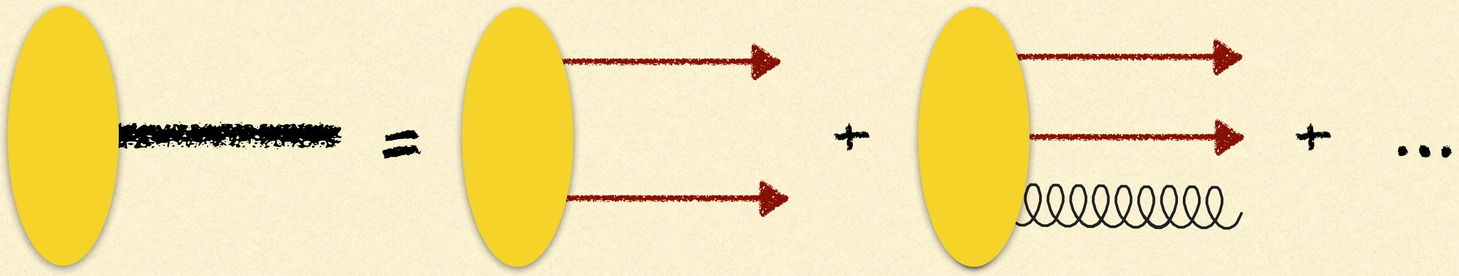

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LFWFs overlap representation formulae:

$$\sum_{\beta} \int d^2\boldsymbol{k}_T |\psi^{\Lambda}_{\beta}(x,\boldsymbol{k}_T)|^2$$

PDF

$$\sum_{\beta=\beta'} \int dx d^2\boldsymbol{k}_T \psi^{\Lambda'}_{\beta'}(x,\boldsymbol{k}'_T)\psi^{\Lambda}_{\beta}(x,\boldsymbol{k}_T)$$

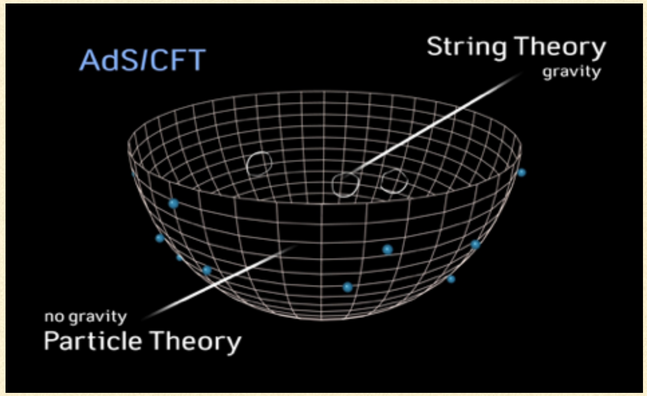
Form Factor

$$\sum_{\beta} |\psi^{\Lambda}_{\beta}(x,\boldsymbol{k}_T)|^2$$

TMD

LIGHT-FRONT HOLOGRAPHIC (LFH) QCD

String theory in a 5d Anti-de Sitter space ⟷ Conformal field theory on the 4-dimensional boundary of the AdS



[J.M. Maldacena, (1999)]

Applicability to QCD?

Soft-wall model (harmonic confining potential $U(z) \sim \kappa^2 z^2$ → **Confinement**

[Brodsky, de Téramond et al., 2004-present]

Form Factors matching

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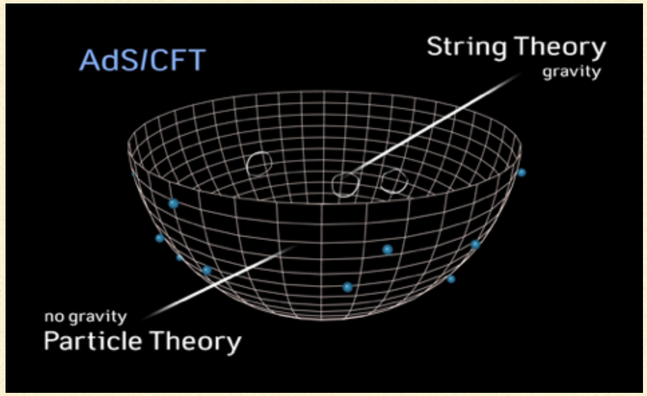

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Form Factors matching

Free current propagating in AdS space

Confining current in a warped AdS space

Valence LFWF

$$\psi^V_{q\bar{q}/\pi}(x,\ \boldsymbol{k}_T) \sim \frac{1}{\kappa\sqrt{(1-x)\,x}} e^{-\frac{1}{2}\frac{\boldsymbol{k}_T^2}{\kappa^2 x(1-x)}}$$

Effective LFWF

$$\psi^E_{q\bar{q}/\pi}(x,\ \boldsymbol{k}_T) \sim \frac{\sqrt{\log\left(\frac{1}{x}\right)}}{\kappa\,(1-x)} e^{-\frac{\log(1/x)}{(1-x)^2}\frac{\boldsymbol{k}_T^2}{2\kappa^2}}$$

IN THE LITERATURE

Theory

- Light-front holography (LFH) original approach and improvements: **Brodsky, de Téramond, Deur, Dosch, et al. (2006-present)**
- Soft-wall model for AdS/QCD: **Karch, Katz, et al. (2006)**
- Harmonic potential in LF corresponds to a linear (confinining) potential in IF: **Trawinski, et al (2014)**

Phenomenological studies

- PDFs, FFs, TMDs, GPDs, double PDFs **Brodsky, Cao, deTéramond (2011), Forshaw, Sandapen (2012), Vega, Schmidt, Gutsche, Lyubovitskij, et al (2009-2020), Chakrabarti,Mondal, et al. (2013-2019), Bacchetta, Cotogno, Pasquini (2017), Rinaldi, Traini, Vento, et al. (2017-2020), Ahmady, Sandapen, et al (2017-2020), Kaur, Dahiya(2019), Chang, Raya, Wang (2020), etc…**

MODELS OF LFWFS

Inclusion of the quark masses→ completion of the invariant mass operator

$$M^2 = \sum_i \frac{m_i^2 + \boldsymbol{k}_{Ti}^2}{x_i} = \frac{m^2 + \boldsymbol{k}_T^2}{x(1-x)}$$

[Brodsky, de Téramond et al., 2004-2015]

Valence LFWF (from bound state equation)

$$\psi^V_{q\bar{q}/\pi}(x,\ \boldsymbol{k}_T) = A \frac{4\pi}{\kappa\sqrt{(1-x)\,x}} e^{-\frac{1}{2\kappa^2}\left(\frac{m^2}{x(1-x)} + \frac{\boldsymbol{k}_T^2}{x(1-x)}\right)}$$

Effective LFWF

$$\psi^E_{q\bar{q}/\pi}(x,\ \boldsymbol{k}_T) = 4\pi A \frac{\sqrt{\log\left(\frac{1}{x}\right)}}{\kappa\,(1-x)} e^{-\frac{\log(1/x)}{(1-x)^2}\frac{\boldsymbol{k}_T^2 + m^2}{2\kappa^2}}$$

Constant A is set such that: $\int_0^1 dx \int_{-\infty}^{+\infty} \frac{d^2\boldsymbol{k}_T}{16\pi^3} |\psi^V_{q\bar{q}/\pi}(x,\ \boldsymbol{k}_T)|^2 = 1.$

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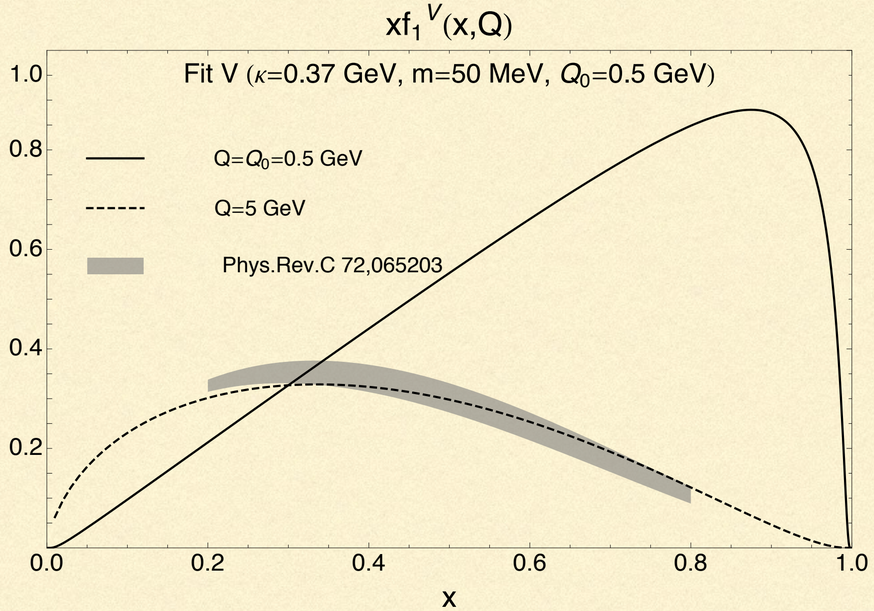
What we do: we fix the parameters of the model (m, κ, Q_0) using the experimental info on PDF and FF and calculate TMD

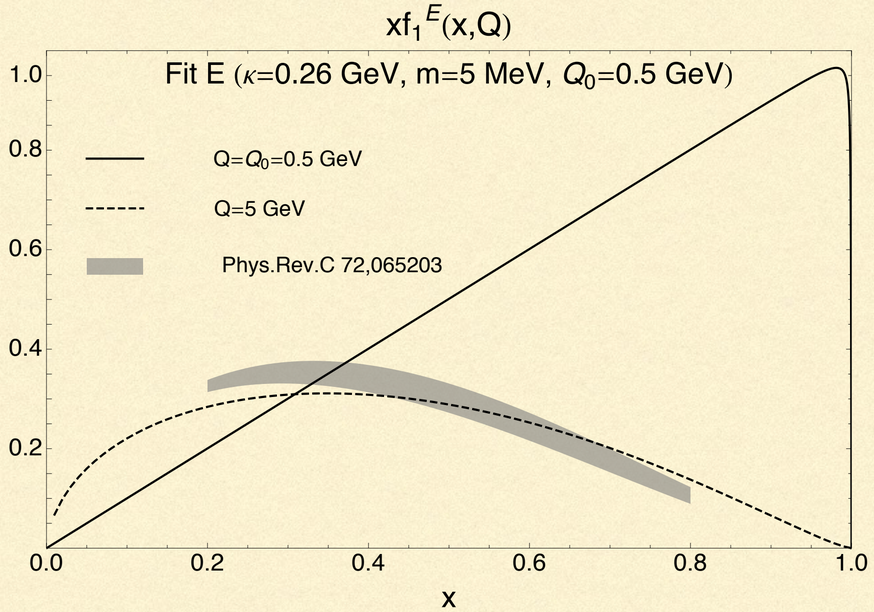
FF AND PDF OF THE PION - I

Physics Letters B 771 (2017) 546–552

$$f_1^V(x;Q_0) = A^2 e^{\left(-\frac{m^2}{\kappa^2 x} - \frac{m^2}{\kappa^2(1-x)}\right)}$$

$$f_1^E(x;Q_0) = A^2 e^{-\frac{\log(1/x)}{(1-x)^2}\frac{m^2}{\kappa^2}}$$





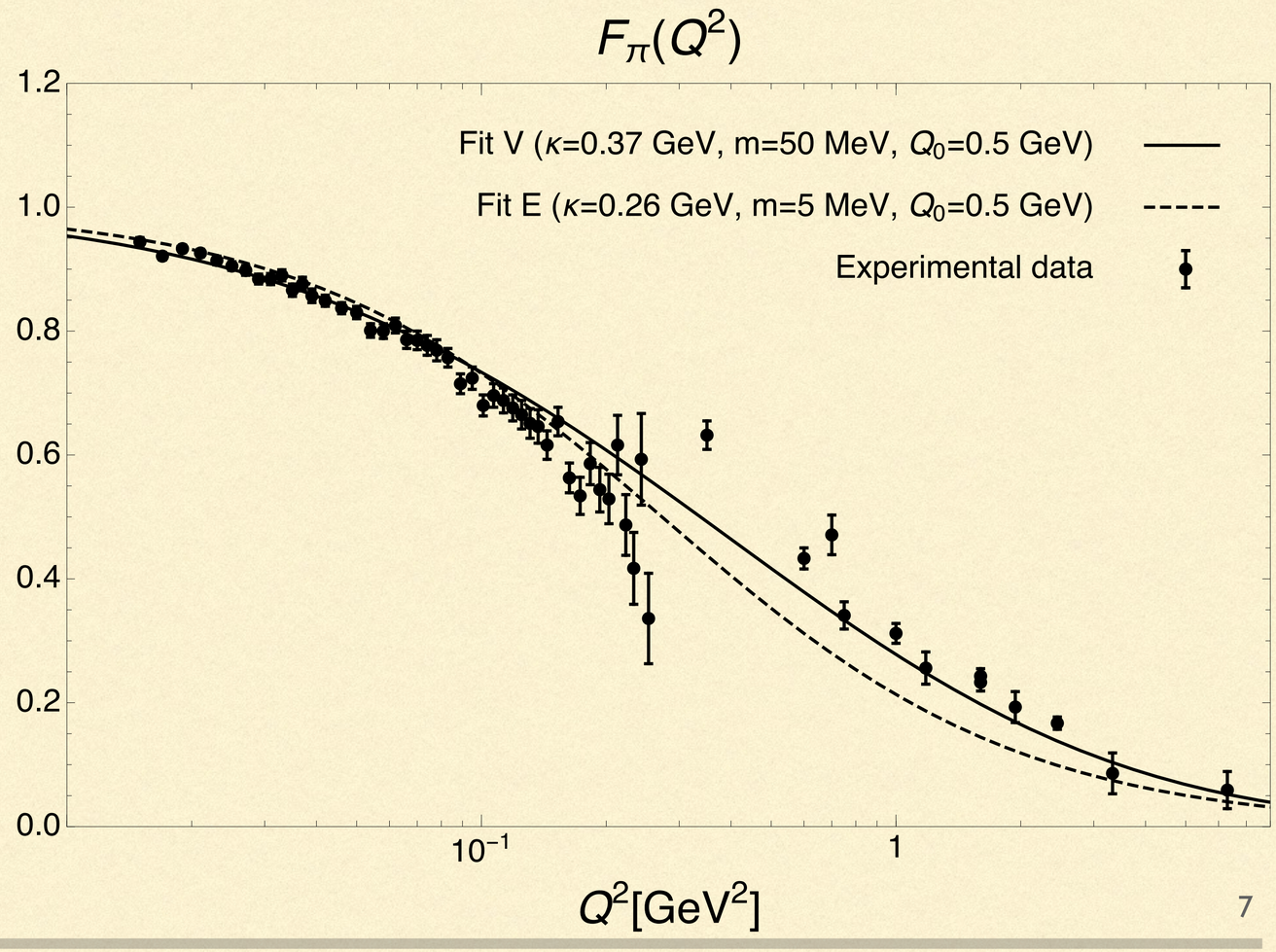
K. Wijesooriya, P. E. Reimer, and R. J. Holt Phys. Rev. C 72, 065203

FF AND PDF OF THE PION - II

Physics Letters B 771 (2017) 546–552

$$F_\pi^V(Q^2) = \int_0^1 dx A^2 e^{\left(-\frac{m^2}{\kappa^2 x} - \frac{m^2}{\kappa^2(1-x)} - \frac{Q^2(1-x)}{4\kappa^2 x}\right)}$$

$$F_\pi^E(Q^2) = \int_0^1 dx A^2 e^{-\frac{\log(1/x)}{4\kappa^2}\left(Q^2 + \frac{4m^2}{(1-x)^2}\right)}$$



S. R. Amendolia et al.,Nucl. Phys.B277168 (1986)
P. Braueln et al., Z. Phys. C 3,101 (1979)
J. Volmer et al. Phys. Rev. Lett. 86 (2001), 1713
C. J. Bebek et al.,Phys. Rev. D17,1693 (1978)

LFWF	m (GeV)	κ (GeV)	Q_0 (GeV)	$\chi^2_{\text{d.o.f.}} \left(\frac{\chi^2_{\text{FF}}+\chi^2_{\text{PDF}}}{N-N_{\text{par}}}\right)$
$\psi^V_{q\bar{q}/\pi}$	0.005 (fixed)	0.397 ± 0.003	0.500 ± 0.003	3.15
	0.200 (fixed)	0.351 ± 0.003	0.491 ± 0.003	11.76
	0.0500 ± 0.00004	0.371 ± 0.002	0.498 ± 0.002	2.25
$\psi^E_{q\bar{q}/\pi}$	0.005 (fixed)	0.261 ± 0.002	0.498 ± 0.003	5.44
	0.200 (fixed)	0.322 ± 0.002	0.630 ± 0.008	12.96
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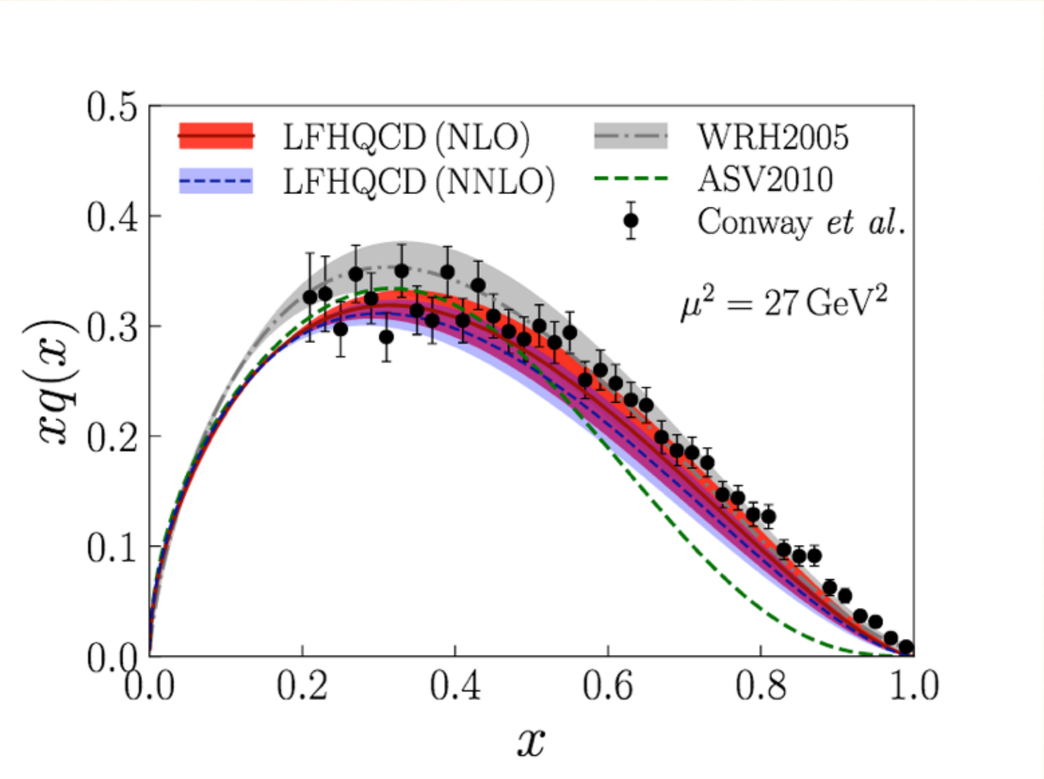
- The Effective model provides a worse description overall
- Initial scale Q_0 =0.5 GeV is lower than the predictions of the LFH QCD (~1 GeV)
- Problem with the Valence and Effective model in reproducing the pole structure of FF (solved later in the "Universal" model)

NOT IN THIS WORK: A UNIVERSAL LFWF FOR MESONS AND THE NUCLEONS

de Téramond, Liu, Sufian, Dosch, Brodsky, Deur (2018)



$\kappa = 0.548\,\mathrm{GeV} \quad \mu_0 = Q_0 = 1.06\,\mathrm{GeV}$

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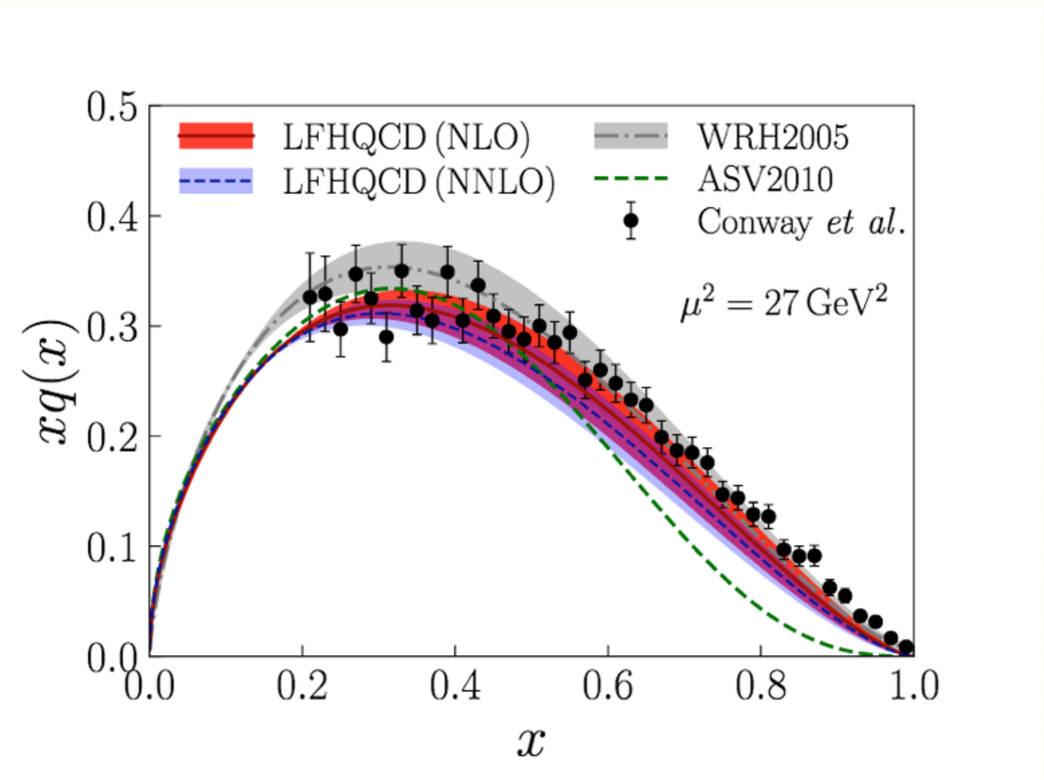


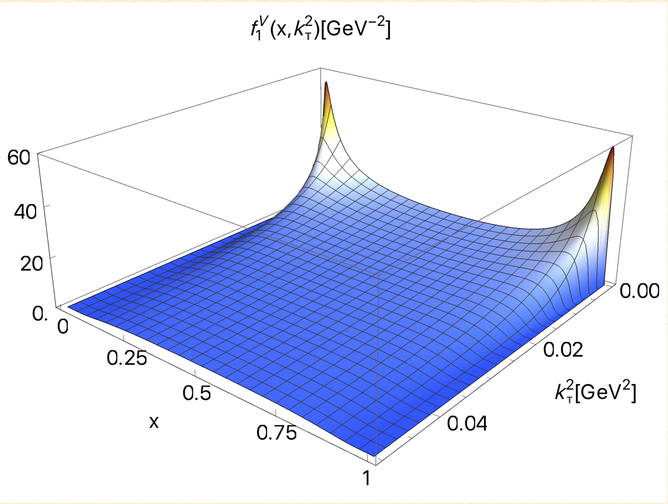
$\kappa = 0.548\,\mathrm{GeV} \quad \mu_0 = Q_0 = 1.06\,\mathrm{GeV}$

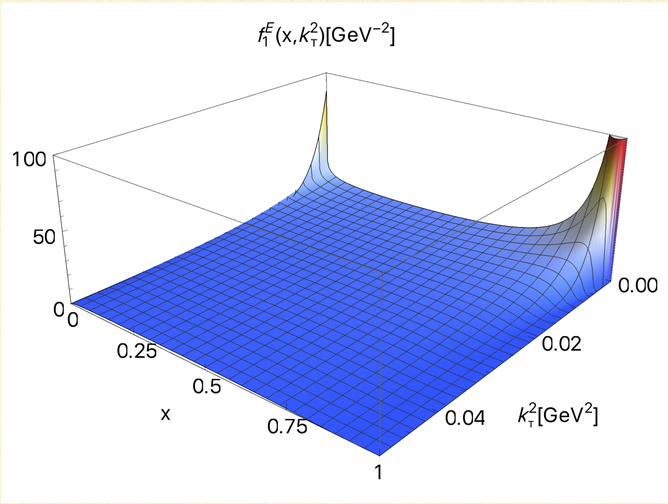
- The pole structure of the FF is restored;
- The κ parameter describes correctly the Regge trajectories, the poles in the FF pole expansion correspond to the physical ones, and the small-x behavior of the PDF is modified;
- The authors find an analytical expression for the PDF and GPD which depends on a universal function.

TRANSVERSE STRUCTURE - MODEL SCALE

Physics Letters B 771 (2017) 546–552

$$f_1^V(x, \boldsymbol{k}_T^2; Q_0) = \frac{A^2}{\pi\kappa^2 x(1-x)} e^{-\frac{\boldsymbol{k}_T^2+m^2}{\kappa^2 x(1-x)}}, \quad f_1^E(x, \boldsymbol{k}_T^2; Q_0) = \frac{A^2 \log\left(\frac{1}{x}\right)}{\pi\kappa^2(1-x)^2} e^{-\log\left(\frac{1}{x}\right)\frac{\boldsymbol{k}_T^2+m^2}{\kappa^2(1-x)^2}}$$





No spin structure: only unpolarized TMD

Spin effects constructions: Boer-Mulders TMD (Ahmady, Mondal , Sandapen, 2018-2019)

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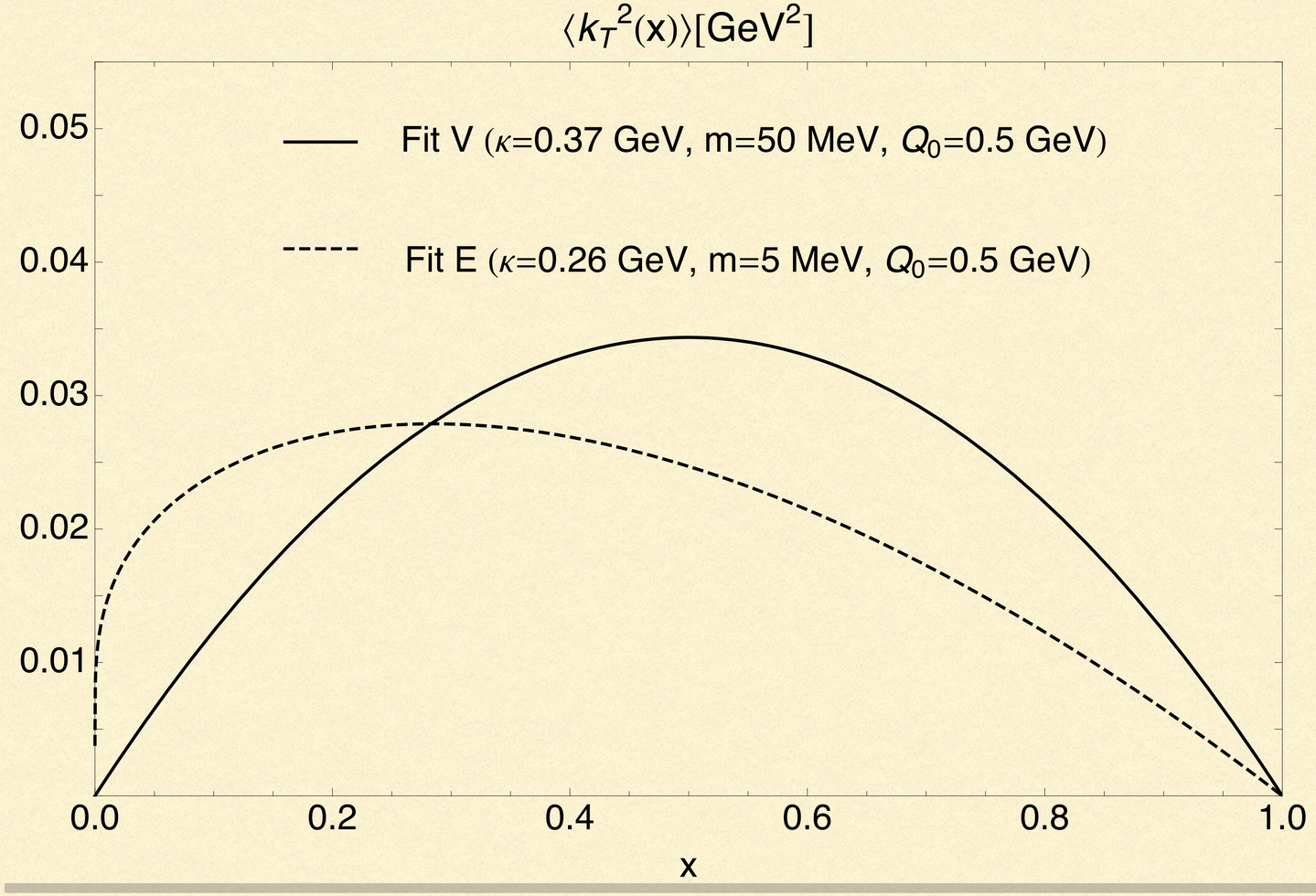

$$\langle k_T^2(x;Q_0)\rangle^V = \kappa^2 x(1-x),$$

$$\langle k_T^2(x;Q_0)\rangle^E = \frac{\kappa^2(1-x)^2}{\log(1/x)}$$

TRANSVERSE STRUCTURE - EVOLUTION

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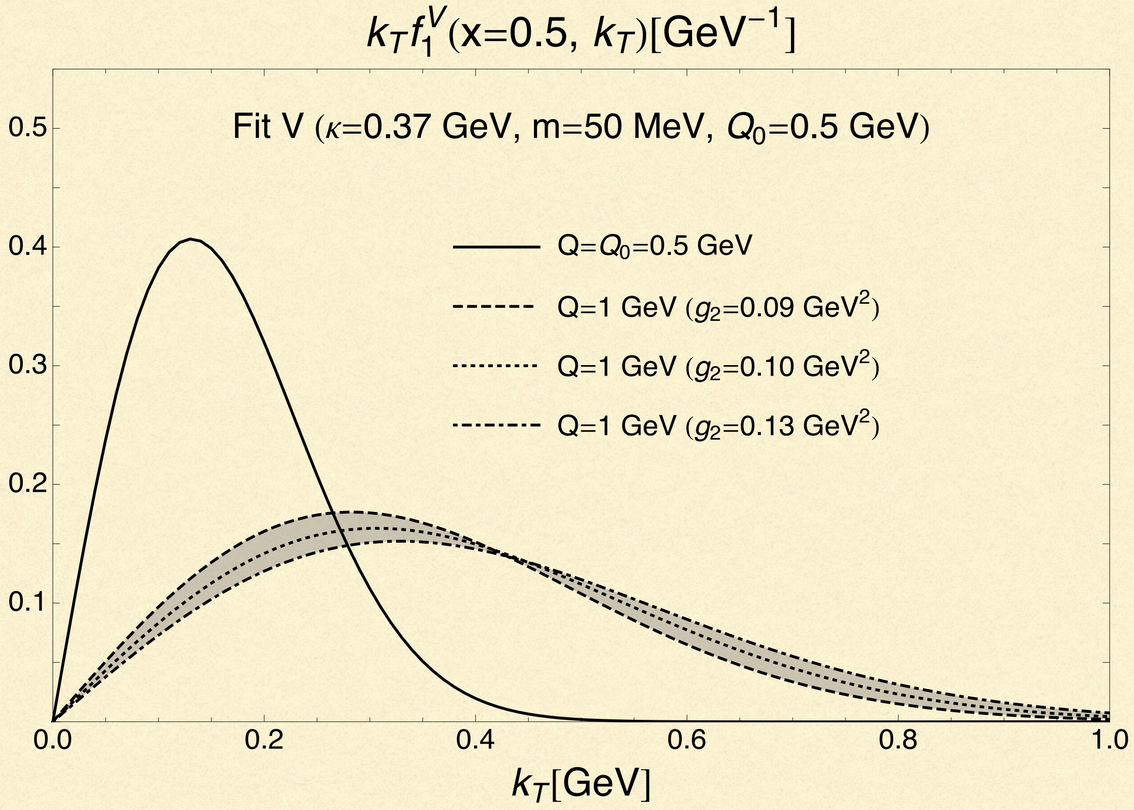

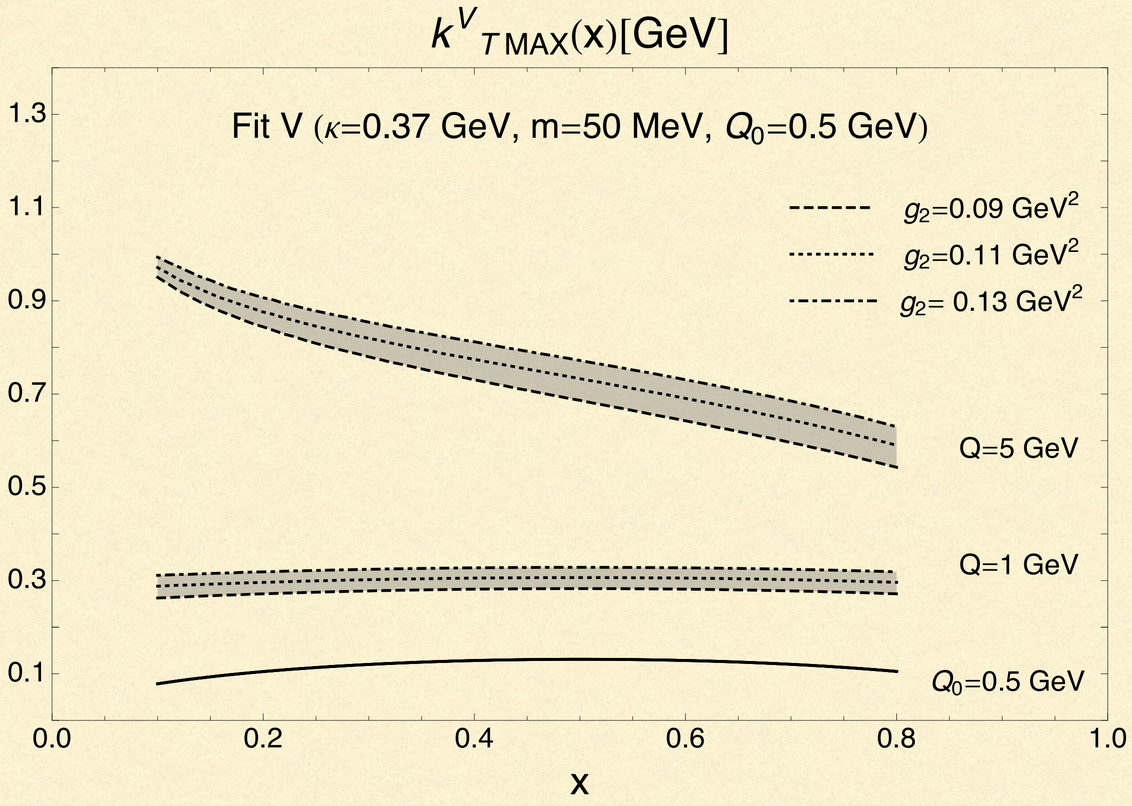

Note: Evolution'details and prescriptions as in Bacchetta, Delcarro, Pisano, Radici, Signori, (2017)

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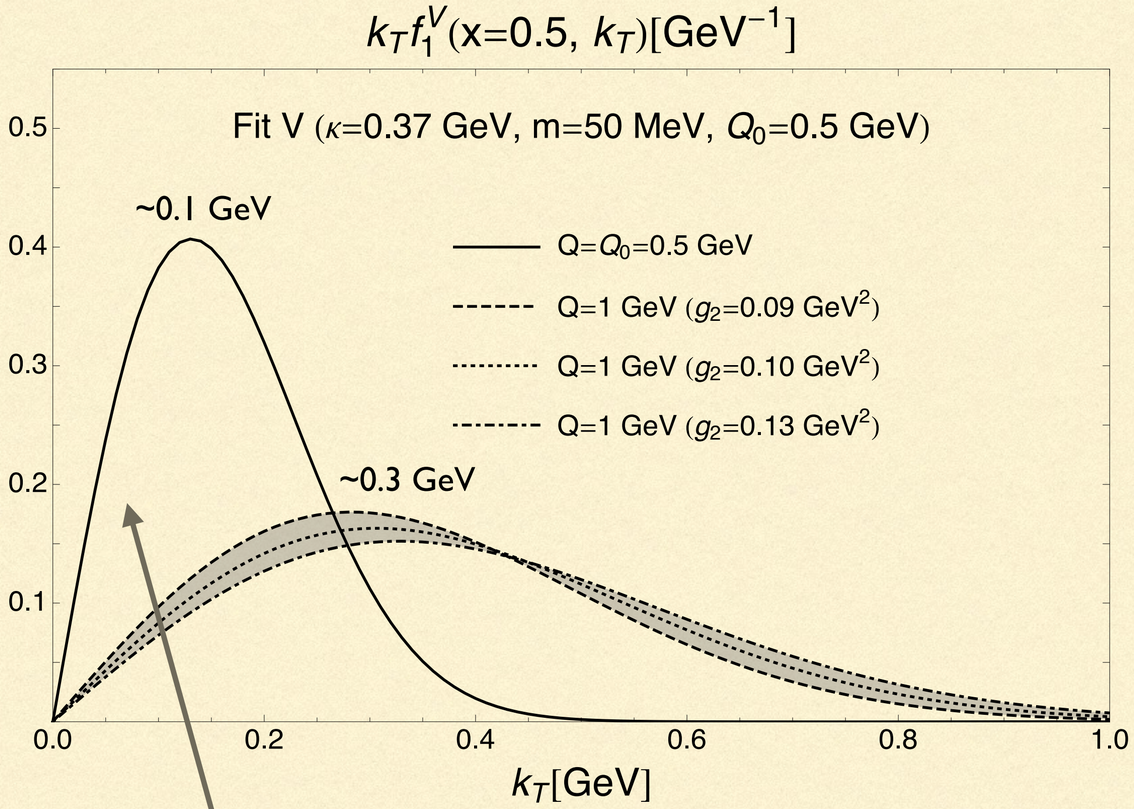

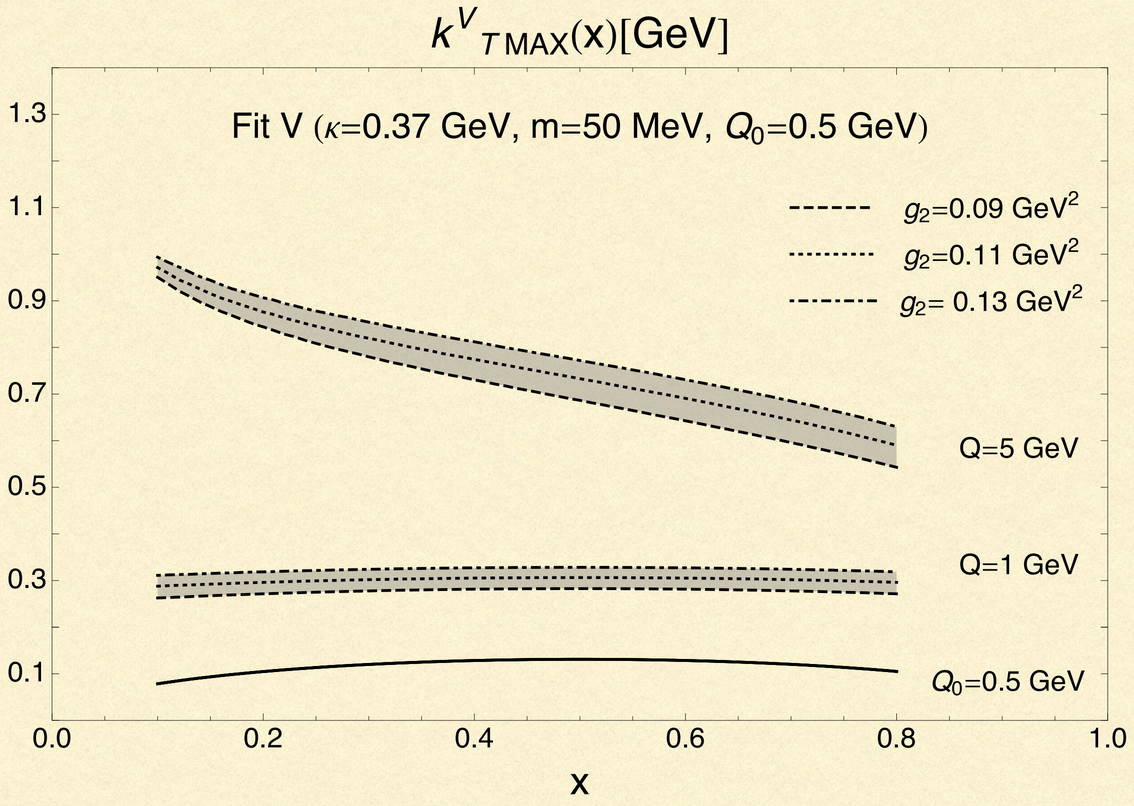

At the model scale the functions are Gaussian

Position of the max and k_T-broadening after evolution

Note: Evolution'details and prescriptions as in Bacchetta, Delcarro, Pisano, Radici, Signori, (2017)

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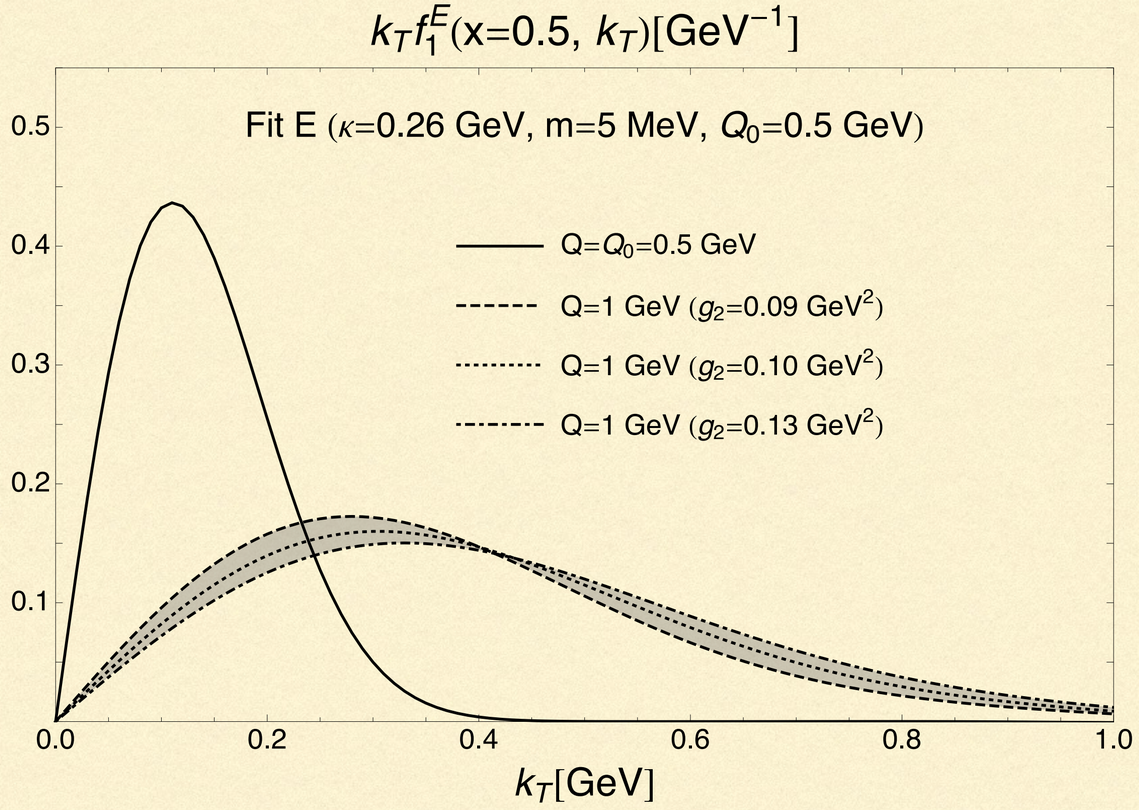
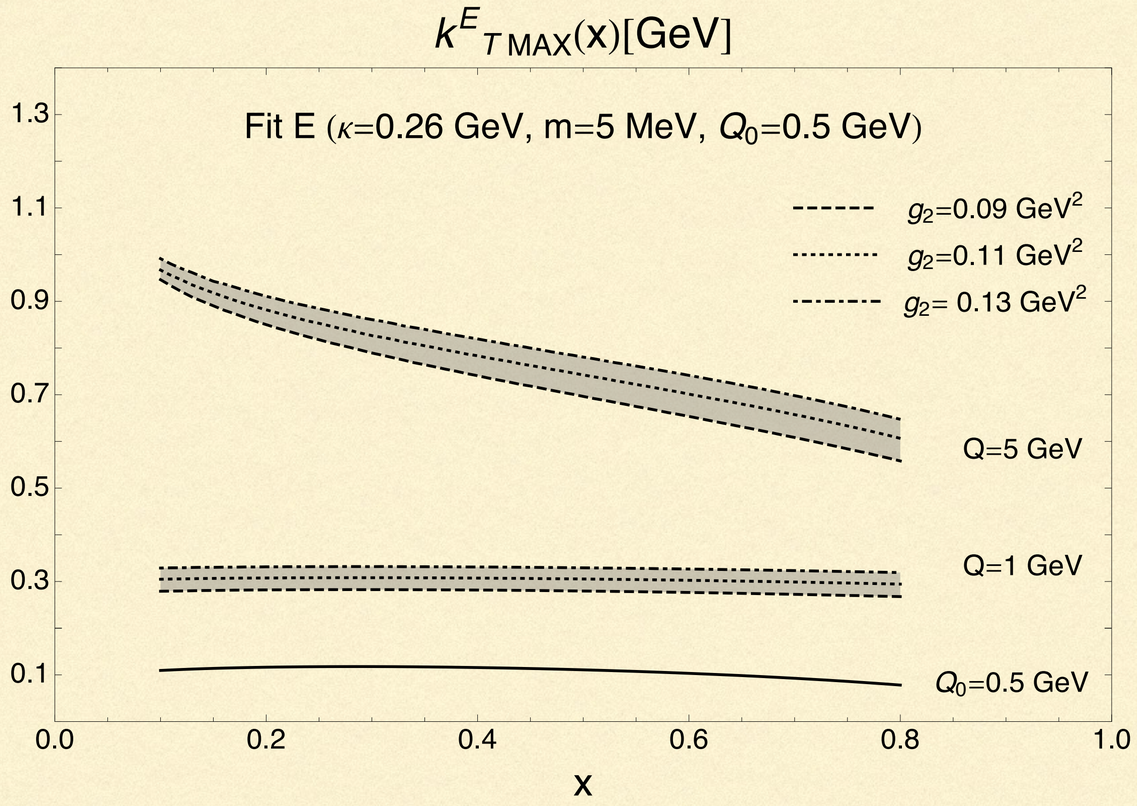
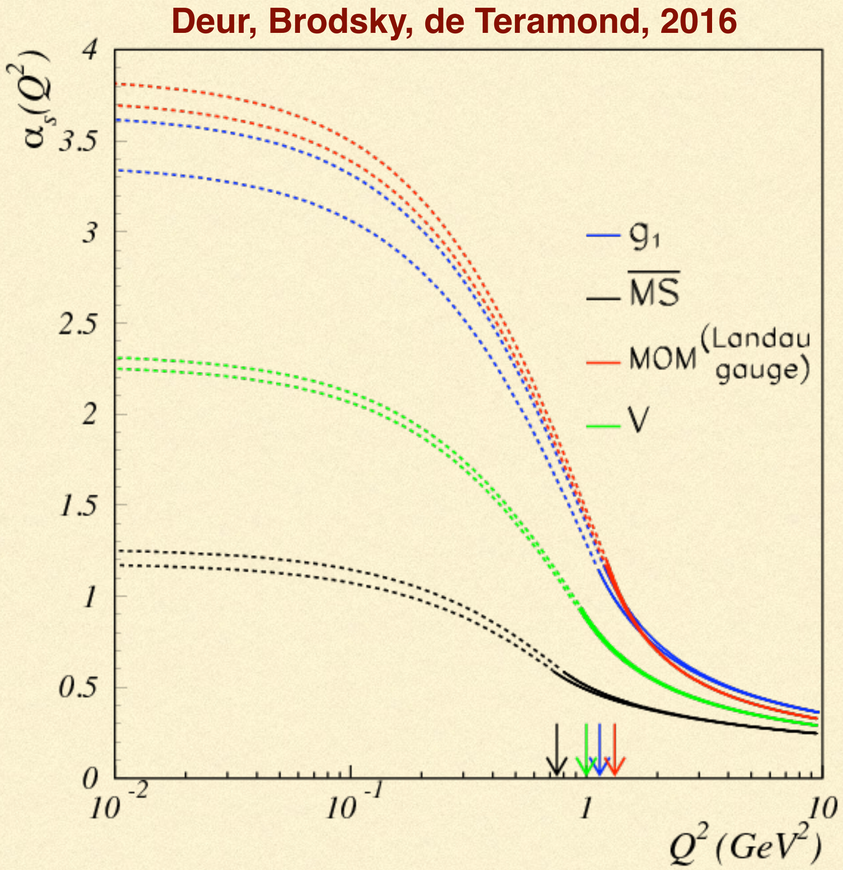




Note: Evolution'details and prescriptions as in Bacchetta, Delcarro, Pisano, Radici, Signori, (2017)

- At the scale of the model (0.5 GeV), the TMD has a Gaussian shape for both Valence and Effective LFWF.
- The mean square transverse momentum is symmetric around x=0.5 for the valence LFWF at the initial scale (0.5 GeV).
- TMD Evolution of the pion TMD, from the initial scale of 0.5 GeV to a typical experimental scale of 5 GeV, increases the width of the distributions in momentum space of almost one order of magnitude.
- The Gaussian shape is lost after Evolution.
- The x-dependence of the transverse momentum width at 5 GeV changes drastically compared to the model scale 0.5 GeV.

THE QCD RUNNING COUPLING FROM ADS/QCD MODELS



$$\alpha_s(Q) = \begin{cases} \alpha_{\mathrm{LFH}}(Q) & Q \leq Q_0 \\ \alpha_{\overline{\mathrm{MS}}}(Q) & Q > Q_0, \end{cases}$$

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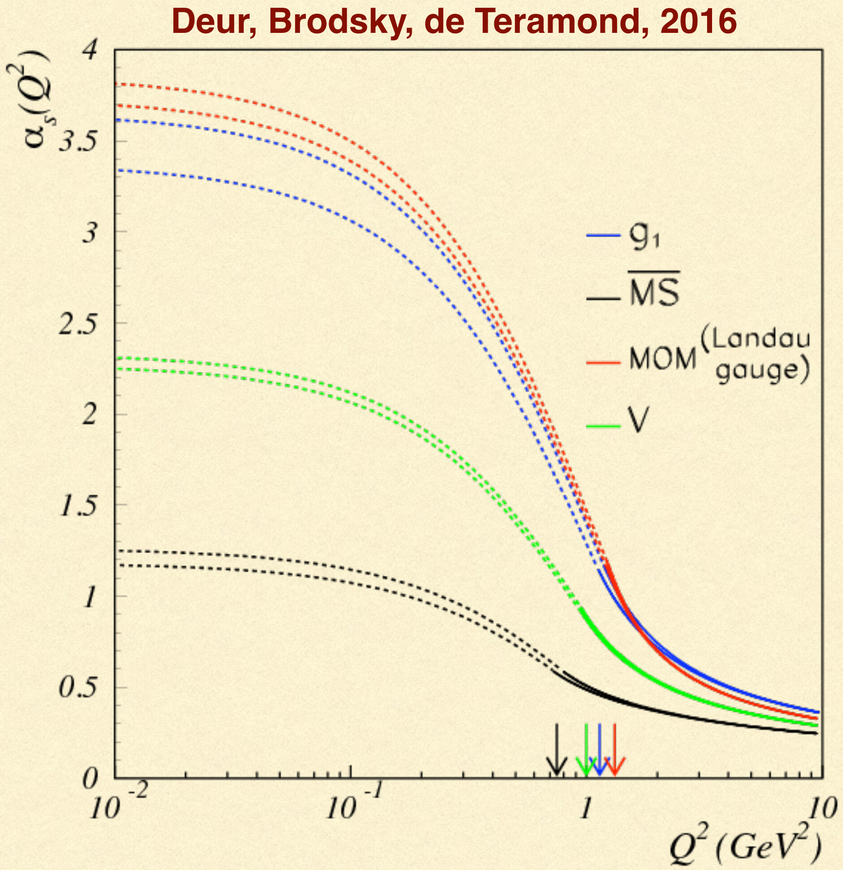

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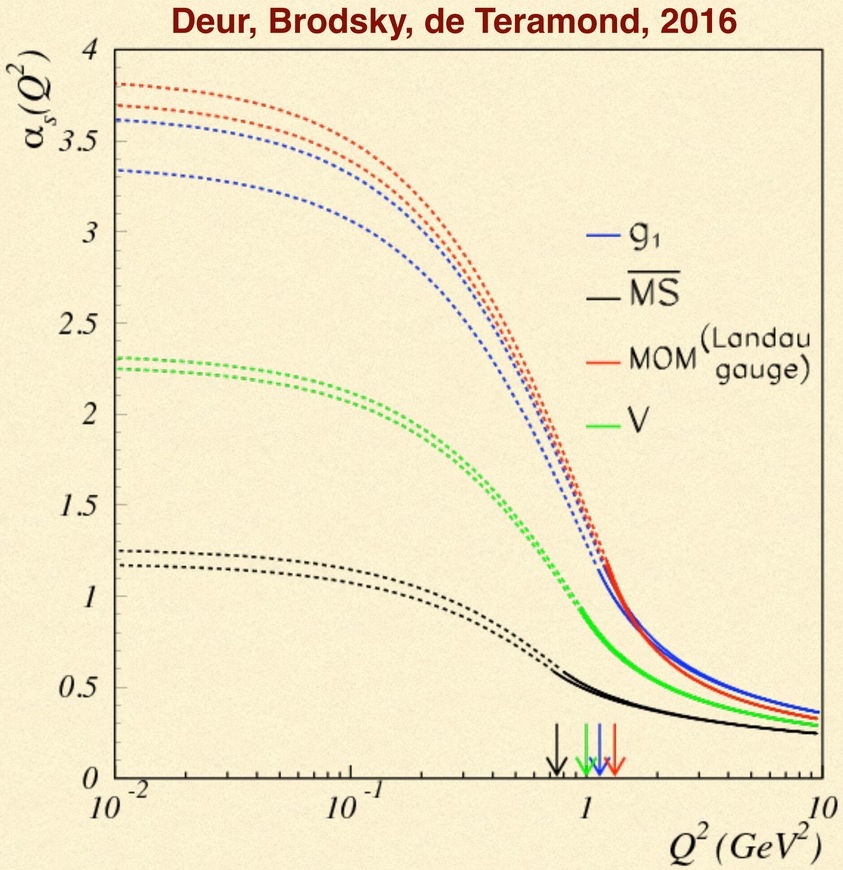
where:

$$\alpha_{\rm LFH}(Q^2) = \alpha_{\rm LFH}(0) e^{-Q^2/4\kappa^2}$$

Continuity condition:

$$\begin{cases} \alpha_{\rm LFH}(Q_0) = \alpha_{\overline{\rm MS}}(Q_0) \\ \beta_{\rm LFH}(Q_0) = \beta_{\overline{\rm MS}}(Q_0), \end{cases}$$

THE QCD RUNNING COUPLING FROM ADS/QCD MODELS



$$\alpha_s(Q) = \begin{cases} \alpha_{\rm LFH}(Q) & Q \leq Q_0 \\ \alpha_{\overline{\rm MS}}(Q) & Q > Q_0, \end{cases}$$

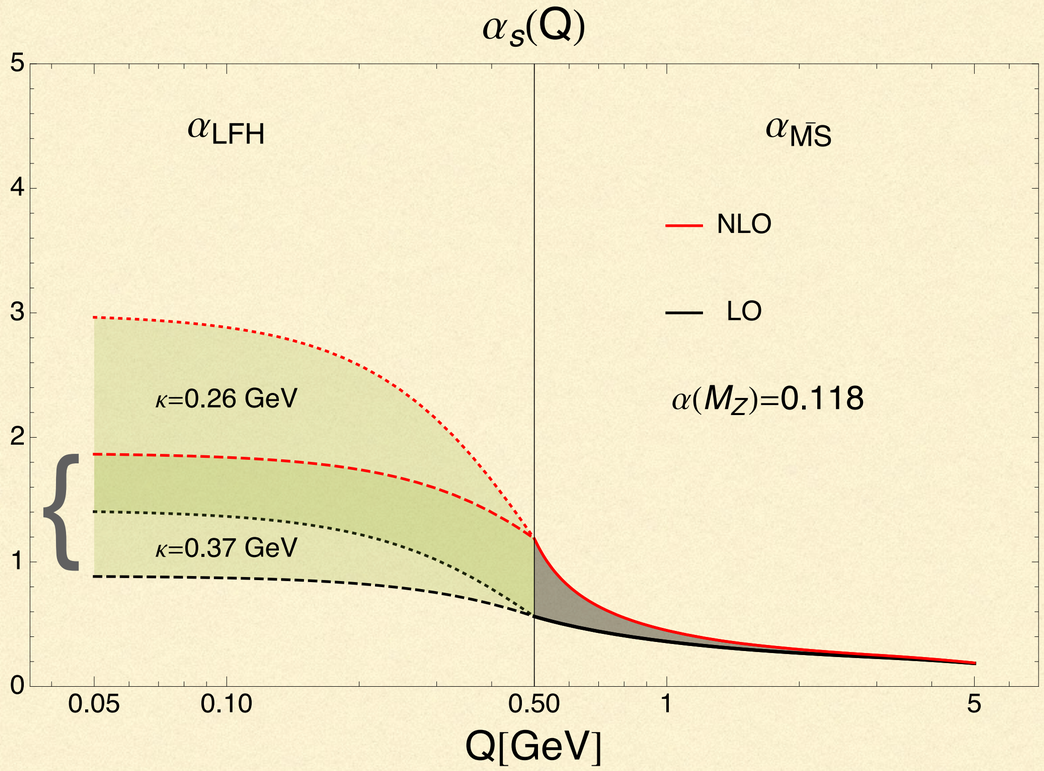
where:

$$\alpha_{\rm LFH}(Q^2) = \alpha_{\rm LFH}(0) e^{-Q^2/4\kappa^2}$$

Continuity condition: $\begin{cases} \alpha_{\rm LFH}(Q_0) = \alpha_{\overline{\rm MS}}(Q_0) \\ \beta_{\rm LFH}(Q_0) = \beta_{\overline{\rm MS}}(Q_0), \end{cases}$

- The matching with the $\overline{\rm MS}$ scheme with $\kappa = 0.51\,{\rm GeV}$ gives $Q_0^2 = 0.75\,{\rm GeV}^2$ (black line)

THE QCD RUNNING COUPLING FROM OUR STUDY

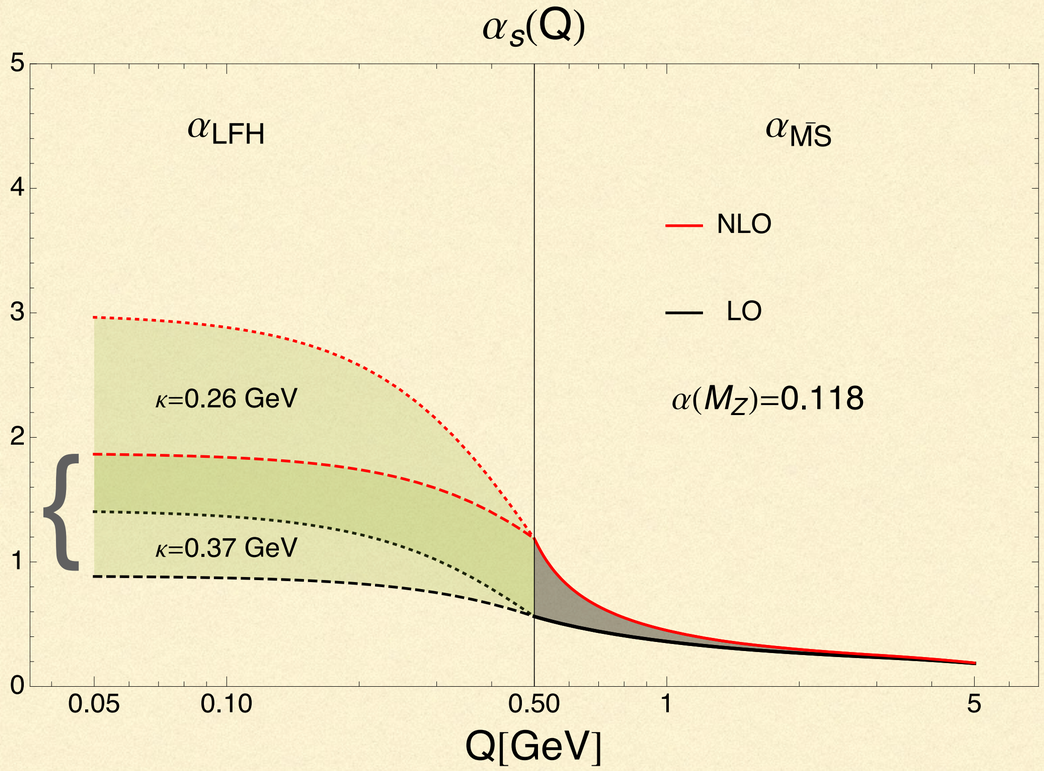


$$\alpha_s(Q) = \begin{cases} \alpha_{\text{LFH}}(Q) & Q \leq Q_0 \\ \alpha_{\overline{\text{MS}}}(Q) & Q > Q_0, \end{cases}$$

Relaxed continuity condition:

$$\alpha_{\text{LFH}}(Q_0) = \alpha_{\overline{\text{MS}}}(Q_0)$$

THE QCD RUNNING COUPLING FROM OUR STUDY



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Relaxed continuity condition:

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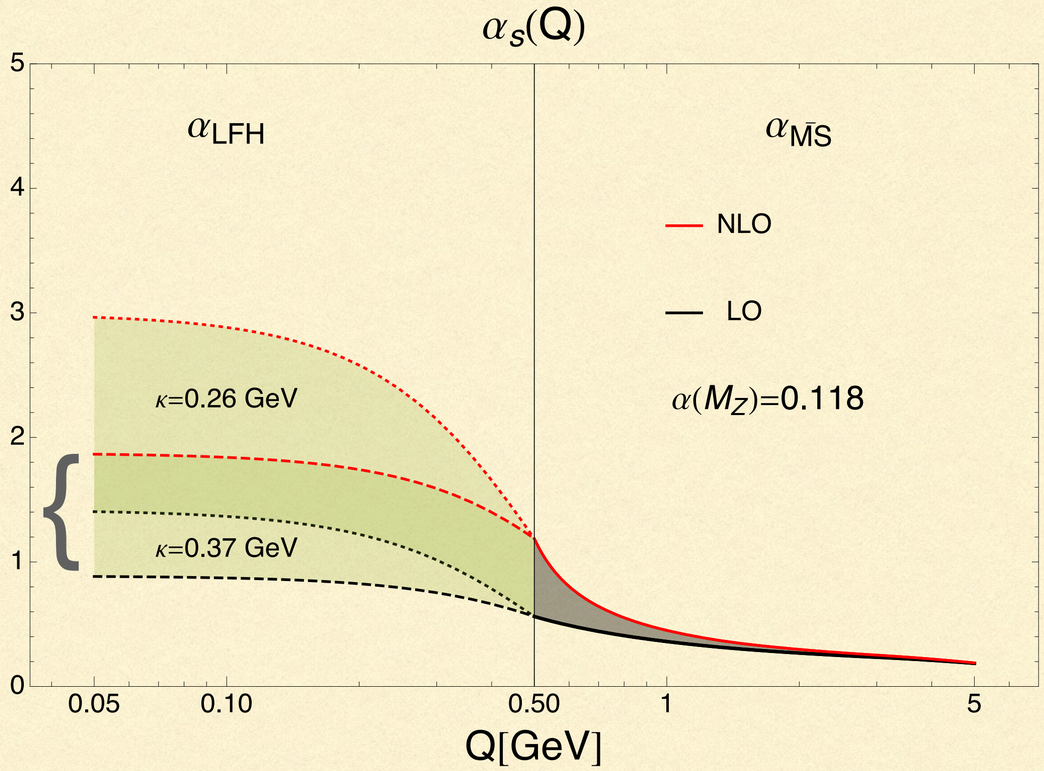
- Imposing the continuity of both α and the derivative β at the transition point Q_0 is too rigid for our approach.

THE QCD RUNNING COUPLING FROM OUR STUDY



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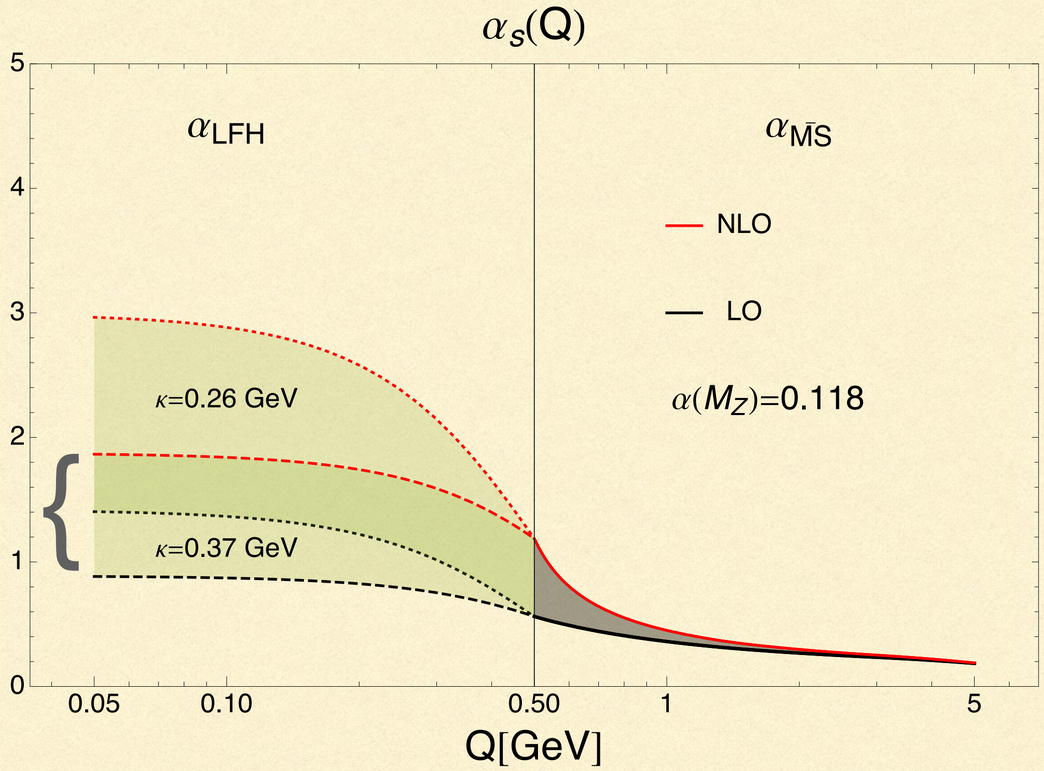
- Fixing by hand the value of $Q_0 \sim 1$ in our fit greatly deteriorates the results on the PDF

THE QCD RUNNING COUPLING FROM OUR STUDY



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Relaxed continuity condition:

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- Imposing the continuity of both α and the derivative β at the transition point Q_0 is too rigid for our approach.
- Fixing by hand the value of $Q_0 \sim 1$ in our fit greatly deteriorates the results on the PDF
- κ=0.37 GeV gives a range which is compatible with previous works

SUMMARY AND CONCLUSIONS

- We study two functional forms of pion LFWFs from LF Holographic models, with minimal modifications
- We fix the free parameters of the LFWFs using the experimental information of PDF and FF (possible to update this part with the new data)
- We obtain a predictions on the pion TMD (dependence on the non perturbative parameters used);
- We test the matching between the perturbative and non-perturbative physics deriving from this approach
- Possible update: use the new available and forthcoming data on PDF to improve the study.