3-DIMENSIONAL IMAGING OF PION AND KAON ON THE LIGHT FRONT

—based on DSEs study

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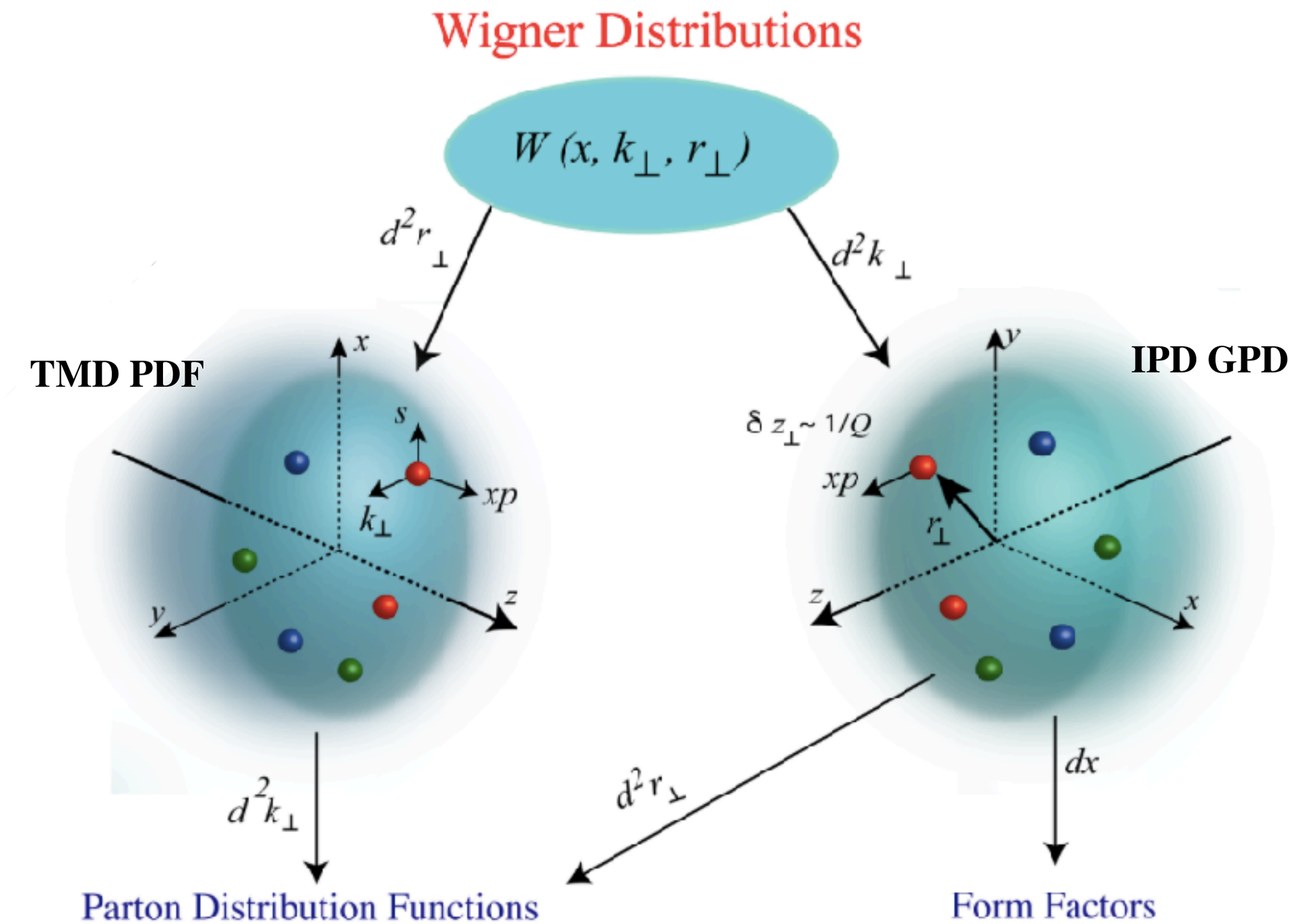
3-D imaging: GPDs & TMDs



GPDs

- 1-D correlation function & Collinear parton distribution functions

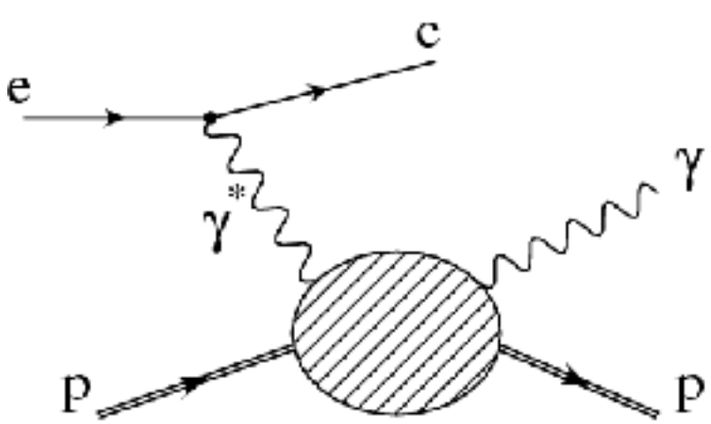
$$\phi_{ij}(x) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P|\bar{\psi}_j(0)\psi_i(z)|P\rangle\Big|_{z^+=z_\perp=0} \longrightarrow [\phi_{ij}\gamma^+] = f(x)$$

- 3-D correlation function & GPD

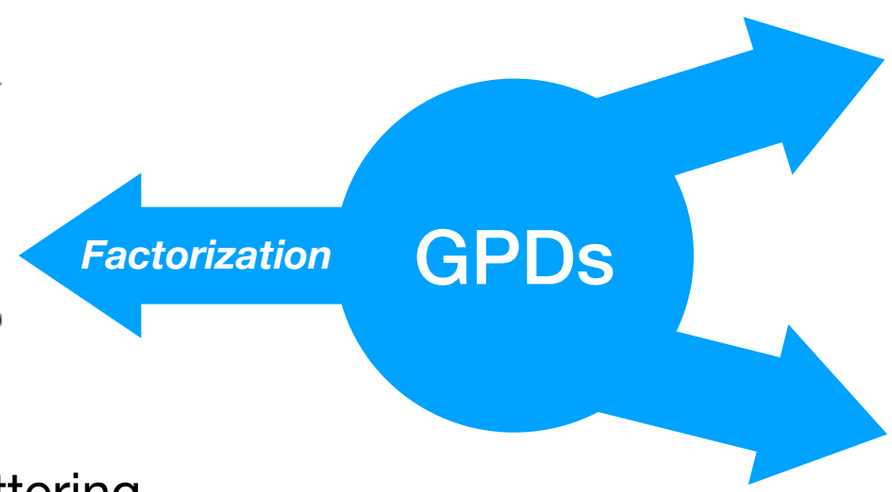
$$\phi_{ij}(x,\xi,\Delta^2) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle P-\frac{\Delta}{2}\middle|\bar{\psi}_i(0)\psi_j(z)\middle|P+\frac{\Delta}{2}\right\rangle\Big|_{z^+=z_\perp=0}$$

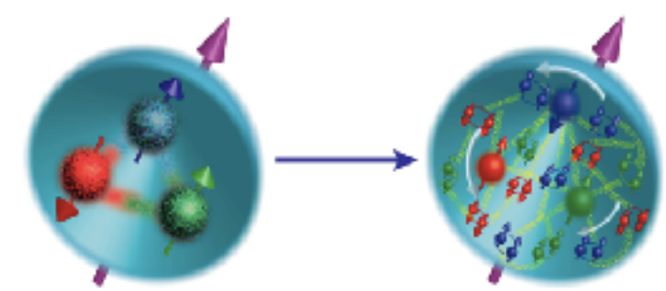
$$[\phi_{ij}\gamma^+] = \frac{1}{2P^+}\left[H^q(x,\xi,t)\bar{u}(P+\tfrac{\Delta}{2})\gamma^+u(P-\tfrac{\Delta}{2}) + ...\right]$$

- GPDs

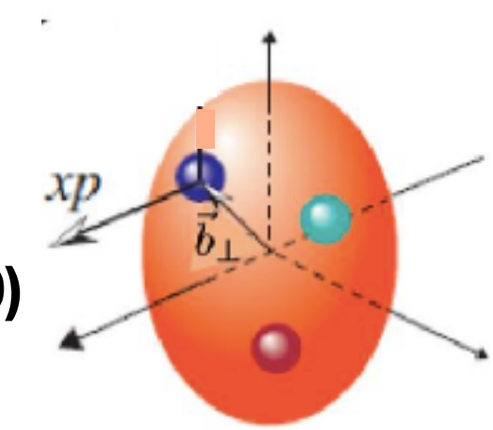


Deeply virtual Compton scattering





AM(Ji 1997)



IPD GPD (Burkardt 2000)

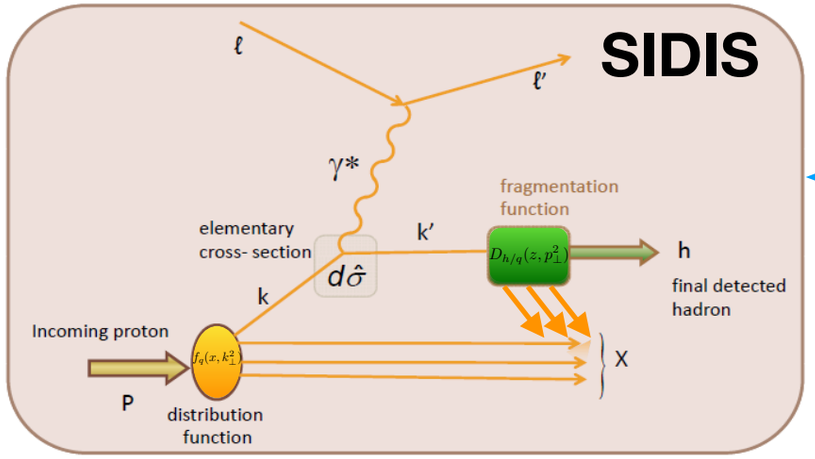
TMD PDFs

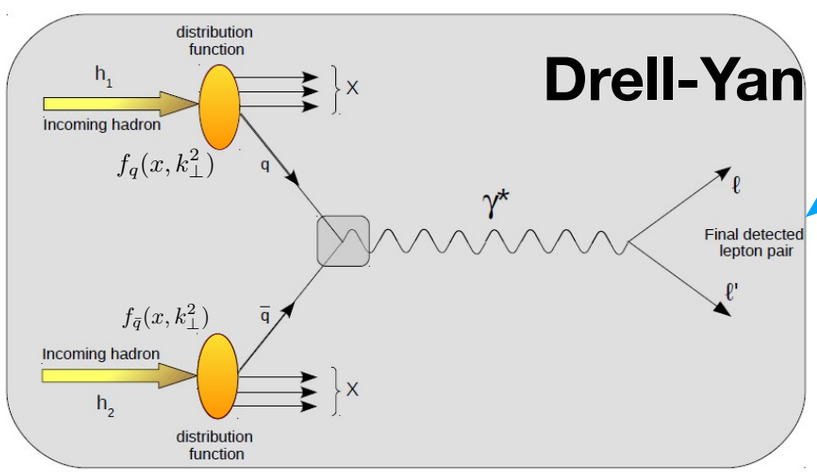
- The TMD correlation function

$$\Phi_{ij}(x,\boldsymbol{k}_\perp,S) = \int \frac{\mathrm{d}z^-\mathrm{d}^2\boldsymbol{z}_\perp}{(2\pi)^3} e^{i(k^+z_- - \boldsymbol{k}_\perp\cdot\boldsymbol{z}_\perp)} \langle P,S|\overline{\psi}_j(0)\psi_i(z)|P,S\rangle\big|_{z^+=0},$$

- The TMD PDFs (leading twist)

$$\begin{aligned}\Phi(x,\boldsymbol{k}_\perp,S) \;=\; \frac{1}{2}\Big\{ & f_1\not{n}_+ - f_{1T}^\perp\frac{\epsilon_T^{ij}\boldsymbol{k}_\perp^i S_\perp^j}{M}\not{n}_+ + \Lambda g_{1L}\gamma_5\not{n}_+ + \frac{(\boldsymbol{k}_\perp\cdot\boldsymbol{S}_\perp)}{M}g_{1T}\gamma_5\not{n}_+ + h_{1T}\frac{[\not{S}_\perp,\not{n}_+]}{2}\gamma_5 \\ & + \Lambda h_{1L}^\perp\frac{[\not{k}_\perp,\not{n}_+]}{2M}\gamma_5 + \frac{(\boldsymbol{k}_\perp\cdot\boldsymbol{S}_\perp)}{M}h_{1T}^\perp\frac{[\not{k}_\perp,\not{n}_+]}{2M}\gamma_5 + ih_1^\perp\frac{[\not{k}_\perp,\not{n}_+]}{2M}\Big\},\end{aligned}$$





hadron \ quark	*unpolarized*	*chiral*	*transverse*
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_{1T}, h_{1T}^\perp$

- 3-D momentum distribution and spin-orbit correlations.
- The sign change of Sivers and Boer-Mulders functions in SIDIS and Drell-Yan is a fundamental test for QCD.

(Collins, PLB 2002)

Nonperturbative QCD

QCD

$$\mathcal{L}_{\mathrm{QCD}} = \bar{q}_i \gamma^\mu (i\partial_\mu - g_s t^a A^a_\mu - m_i) q_i - \frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_s f^{abc} A^b_\mu A^c_\nu \qquad \alpha_s = \frac{g_s^2}{4\pi}$$

Calculation

Nonperturbative QCD methods

1. ADS/QCD
2. Dyson-Schwinger equations.
3. Effective theories and models, e.g., NJL model...
4. Light front QCD.
5. Lattice QCD.

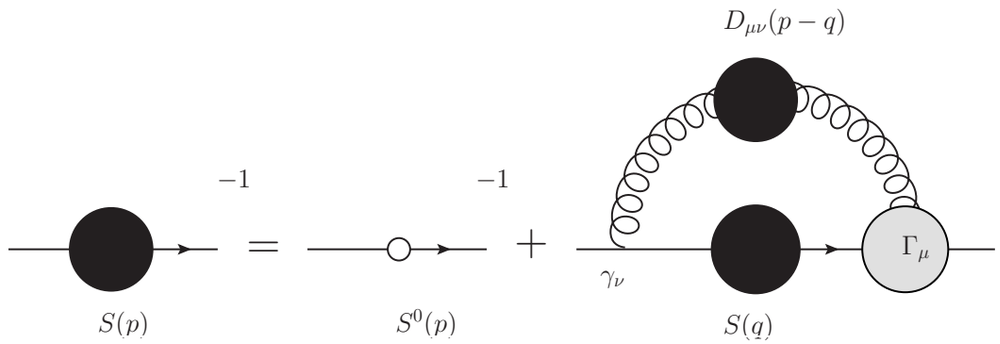
etc...

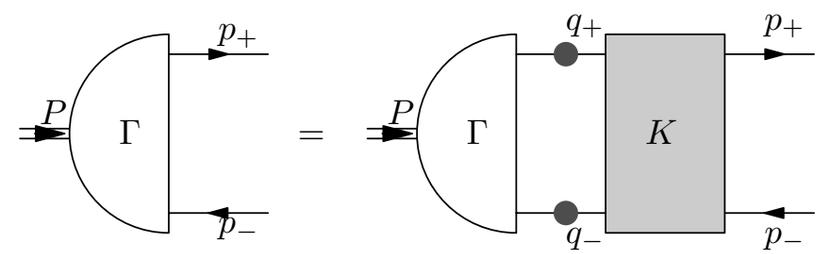
Transverse momentum dependent distributions (TMD)
3-D tomography in the momentum space.

Generalized parton distributions (GPD)
3-D picture of hadrons in the mixed spatial-momentum space.

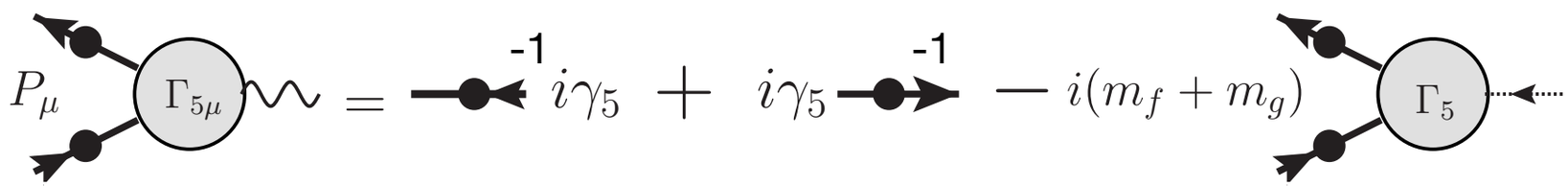
DSE & symmetry preserving

- The Pion&Kaon wave function can be solved by aligning the quark DSE and hadron BSE.

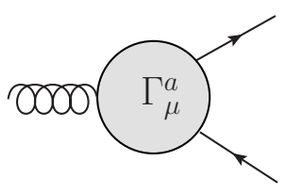




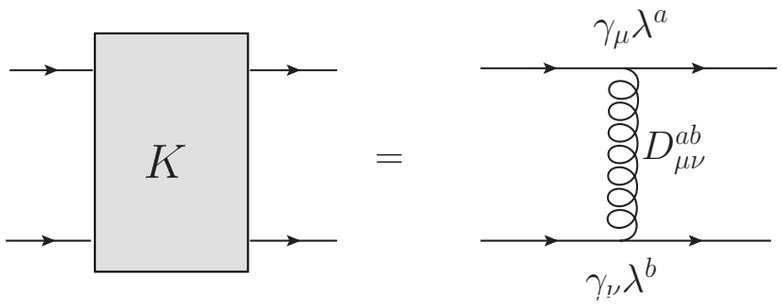
- To solve these equations, truncation is needed for the vertex and scattering kernel. A physically reasonable truncation scheme should respect QCD's (nearly) chiral symmetry, namely, the Axial-Vector Ward-Takahashi Identity



- The simplest manifestation is the Rainbow-Ladder truncation



$= \quad \gamma_\mu\lambda^a \quad$ AND



Beyond Rainbow-Ladder

- Inhomogeneous BSE

$$\Gamma_{5\mu}(k;P) = Z_2\gamma_5\gamma_\mu$$
$$- Z_2\int_{dq} \mathcal{G}(k-q)\, D^{\rm free}_{\rho\sigma}(k-q)\frac{\lambda^a}{2}\,\gamma_\alpha S(q_+)\times\Gamma_{5\mu}(q;P)S(q_-)\frac{\lambda^a}{2}\,\tilde{\Gamma}_\beta(q_-,k_-)$$
$$+ Z_1\int_{dq} g^2 D_{\alpha\beta}(k-q)\,\frac{\lambda^a}{2}\,\gamma_\alpha S_f(q_+)\times\frac{\lambda^a}{2}\Lambda_{5\mu\beta}(k,q;P)$$

When $P^2\to -m_\pi^2\,,\ \Gamma^j_{5\mu}(k;P)\sim\frac{r_A P_\mu}{P^2+m_\pi^2}\Gamma^j_\pi(k;P)$ (Lei Chang et al, PRL2009)

- Beyond-RL kernel

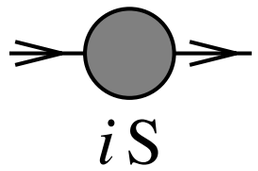
$$\Gamma_\mu(p_1,p_2) = \Gamma^{\rm BC}_\mu(p_1,p_2) + \Gamma^{\rm acm}_\mu(p_1,p_2)$$ (Lei Chang et al, PRL2011,PRC2012)

$$2\Lambda_{5\beta(\mu)} = [\tilde{\Gamma}_\beta(q_+,k_+) + \gamma_5\tilde{\Gamma}_\beta(q_-,k_-)\gamma_5]\times\frac{1}{S^{-1}(k_+)+S^{-1}(-k_-)}\Gamma_{5(\mu)}(k;P)$$
$$+\Gamma_{5(\mu)}(q;P)\frac{1}{S^{-1}(-q_+)+S^{-1}(q_-)}\times[\gamma_5\tilde{\Gamma}_\beta(q_+,k_+)\gamma_5+\tilde{\Gamma}_\beta(q_-,k_-)]$$

- The study of pion and kaon is well established in DSEs. There is no more parameter for tuning. TMDs and GPDs pose a new challenge.

TMDs & GPDs: Light-front approach

DSEs:



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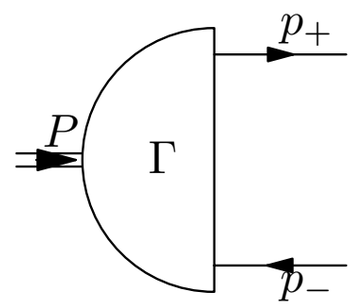
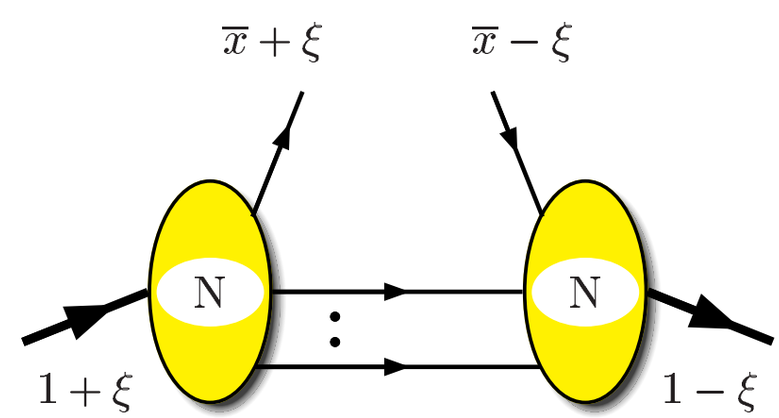


LFQCD:

Light front wave functions + overlap representation

TMD & GPD



BSE approach to LFWF

- To calculate the LFWFs, the standard way is to diagonalize the light-cone Hamiltonian. However, this is very challenging in QCD. In practice, light-cone Hamiltonian models are employed (light-front potential, holographic QCD, NJL model....)

- An alternative way to calculate the LFWFs.

  "...he ('t Hooft) did not use the light–cone formalism and which nowadays might be called standard. Instead, he started from covariant equations... The light–cone Schrodinger equation was then obtained by projecting the Bethe–Salpeter equation onto hyper-surfaces of equal light–cone time. In this way, one avoids to explicitly derive the light–cone Hamiltonian, which, as explained above, can be a tedious enterprise in view of complicated constraints one has to solve..." (Thomas Heinzl)

  What we do: solve the BS equation first and then project the BS wave functions onto the light front!

- A synergy between Lagrangian formalism (DSE) and Hamiltonian formalism (LF QCD).

  Advantage: In the DSEs, one can selectively sum infinitely many diagrams (which potentially incorporates many higher Fock states) and conveniently preserves the symmetries of the Lagrangian.

LFWFs & Bethe-Salpeter wave function

Fock state & LFWFs

LFWFs

$$|\pi^+(P)\rangle = |\pi^+(P)\rangle_{l_z=0} + |\pi^+(P)\rangle_{|l_z|=1}$$

$$|\pi^+(P)\rangle_{l_z=0} = i\int \frac{d^2k_\perp}{2(2\pi)^3}\frac{dx}{\sqrt{x\bar{x}}}\psi_0(x,k_\perp^2)\frac{\delta_{ij}}{\sqrt{3}}\frac{1}{\sqrt{2}}[b^\dagger_{u\uparrow i}(x,k_\perp)d^\dagger_{d\downarrow j}(\bar{x},\bar{k}_\perp) - b^\dagger_{u\downarrow i}(x,k_\perp)d^\dagger_{d\uparrow j}(\bar{x},\bar{k}_\perp)]|0\rangle,$$

$$|\pi^+(P)\rangle_{|l_z|=1} = i\int \frac{d^2k_\perp}{2(2\pi)^3}\frac{dx}{\sqrt{x\bar{x}}}\psi_1(x,k_\perp^2)\frac{\delta_{ij}}{\sqrt{3}}\frac{1}{\sqrt{2}}[k_\perp^- b^\dagger_{u\uparrow i}(x,k_\perp)d^\dagger_{d\uparrow j}(\bar{x},\bar{k}_\perp) + k_\perp^+ b^\dagger_{u\downarrow i}(x,k_\perp)d^\dagger_{d\downarrow j}(\bar{x},\bar{k}_\perp)]|0\rangle,$$

BS WFs & LFWFs

$$\langle 0|\bar{d}_+(0)\gamma^+\gamma_5 u_+(\xi^-,\xi_\perp)|\pi^+(P)\rangle = i\sqrt{6}P^+\psi_0(\xi^-,\xi_\perp),$$

$$\langle 0|\bar{d}_+(0)\sigma^{+i}\gamma_5 u_+(\xi^-,\xi_\perp)|\pi^+(P)\rangle = -i\sqrt{6}P^+\partial^i\psi_1(\xi^-,\xi_\perp).$$

(M. Burkardt et al, PLB 2002)

LFWFs & BS wave function:

Realistic BS wave function

Project on to the light front
(light front time $\xi^+ = 0$)

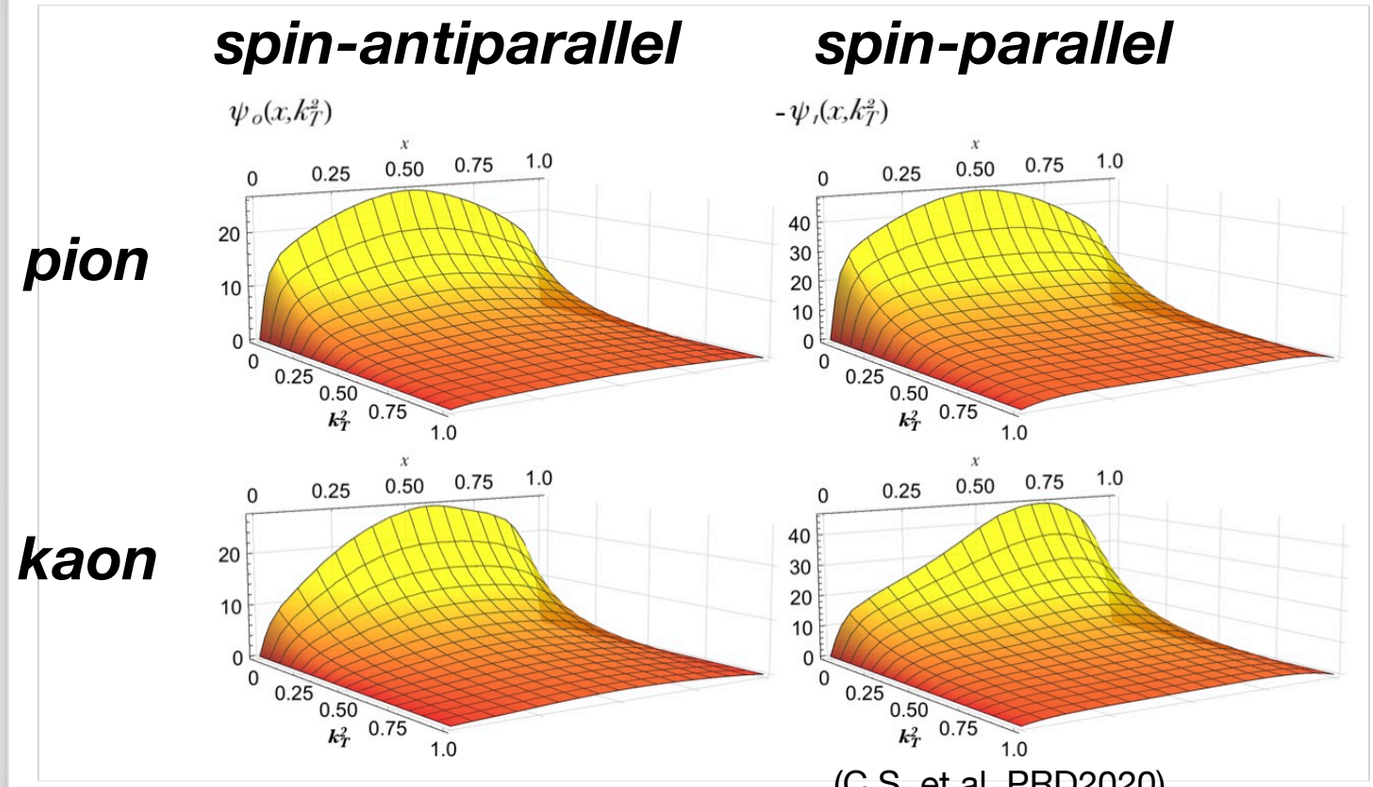
$$\psi_0(x,\boldsymbol{k}_T^2) = \sqrt{3}\,i\int\frac{dk^+dk^-}{2\pi}\times \mathrm{Tr}_D\left[\gamma^+\gamma_5\chi(k,p)\right]\delta\left(x\,p^+ - k^+\right),$$

$$\psi_1(x,\boldsymbol{k}_T^2) = -\sqrt{3}\,i\int\frac{dk^+dk^-}{2\pi}\frac{1}{\boldsymbol{k}_T^2}\times \mathrm{Tr}_D\left[i\sigma_{+i}\boldsymbol{k}_T^i\,\gamma_5\,\chi(k,p)\right]\delta\left(x\,p^+ - k^+\right),$$

(C. Mezrag et al, FBSY 2016)

LFWFs: $\psi_0(x, k_\perp^2)$ & $\psi_1(x, k_\perp^2)$

- Obtained from parameterized realistic BS wave functions.
- ψ0 (spin-antiparallel) and ψ1 (spin-parallel) are comparable in strength, suggesting the spin parallel component also has considerable contribution. Highly relativistic system.
- Strong support at infrared k_T, a consequence of the DCSB which generates significant strength in the infrared of BS wave function.
- At ultraviolet of k_T, ψ0 scale as $1/k_T^2$ and ψ1 scale as $1/k_T^4$, as has been predicted by pQCD. (one-gluon exchange dominance.)
- SU(3) flavor symmetry breaking effect: u/d and s quark mass difference masked by DCSB.



(C.S. et al, PRD2020)

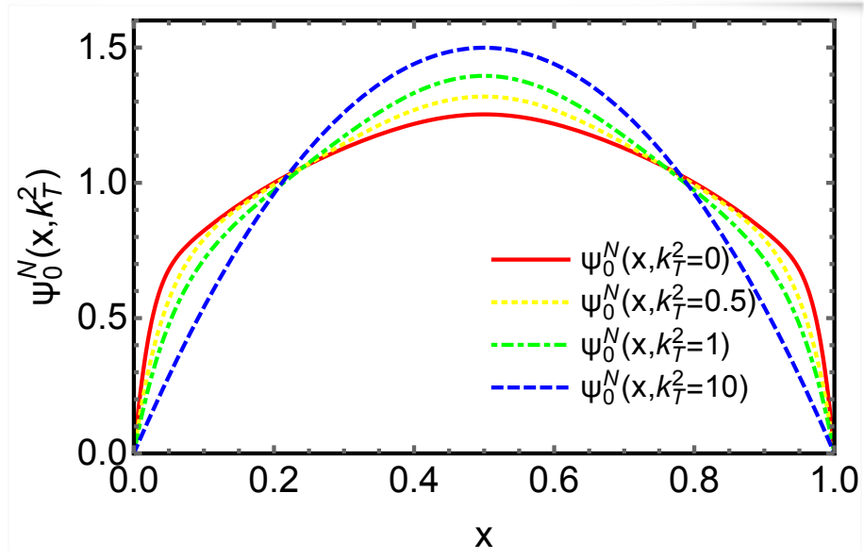


FIG. 2. Pion's spin-anti-parallel LFWF $\psi_0(x, \boldsymbol{k}_T^2)$ at different values of $\boldsymbol{k}_T^2$, normalized to $\psi_0^N(x, \boldsymbol{k}_T^2) = \frac{\psi_0(x,\boldsymbol{k}_T^2)}{\int_0^1 dx \psi_0(x,\boldsymbol{k}_T^2)}$.

Unpolarized TMD PDF

- TMD overlap representation

$$f_{1,\pi}(x, \mathbf{k}_\perp^2) = |\psi_{\uparrow\downarrow}(x, k_\perp^2)|^2 + k_\perp^2 |\psi_{\uparrow\uparrow}(x, k_\perp^2)|^2$$

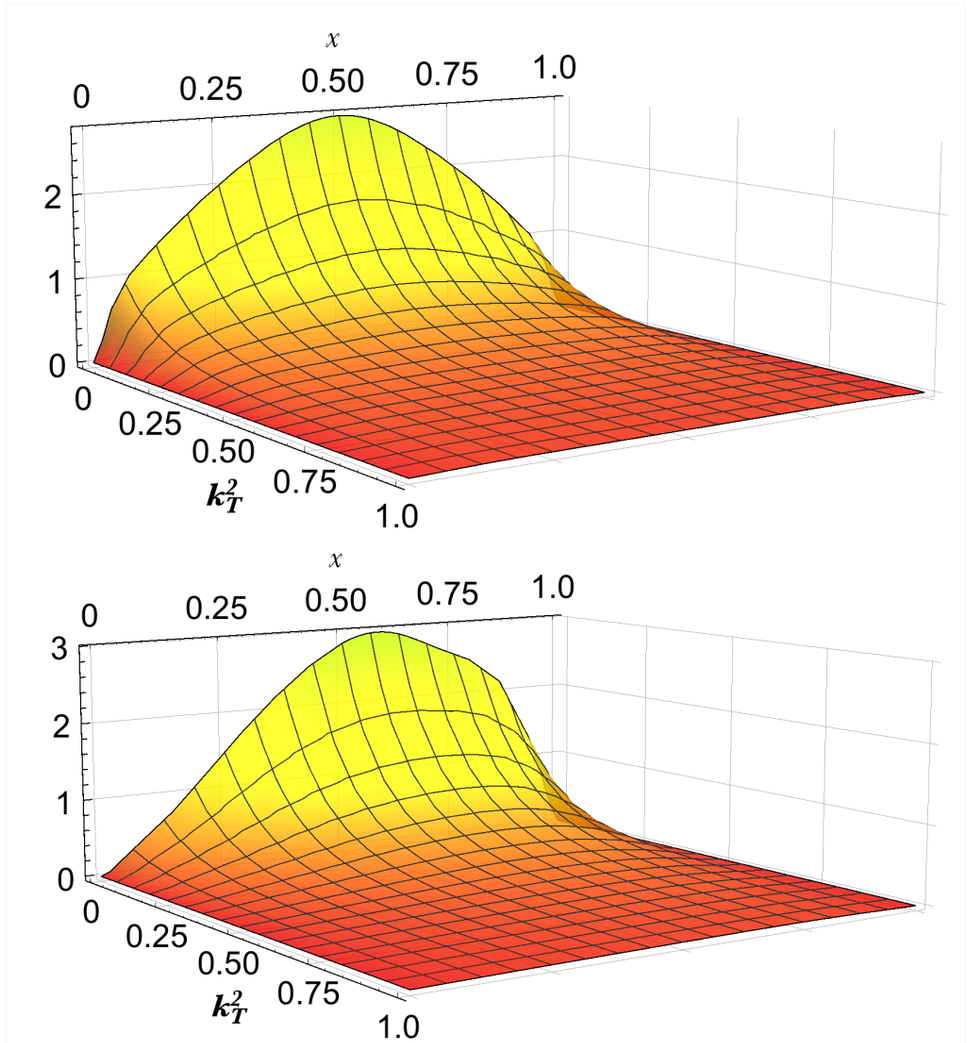
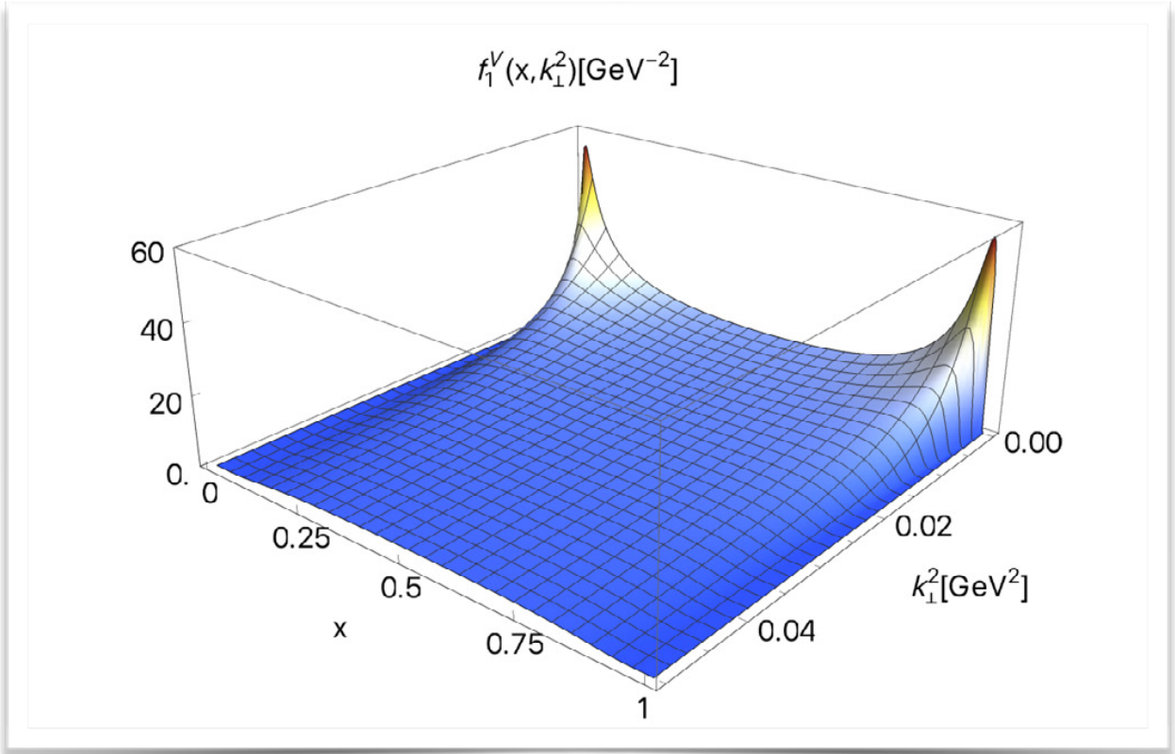


FIG. 7. The unpolarized TMD $f^d_{1;\pi}(x, \boldsymbol{k}_T^2)$ of pion (upper panel) and $f^s_{1;K}(x, \boldsymbol{k}_T^2)$ of kaon (lower panel).

DSE & LF

- Significant strength at low k_T , resembles Gaussian-like form.
- The TMD of kaon is slightly broader than pion.
- Smoother as compared to holographic QCD.



Holographic QCD(A Bacchetta, et al, PLB2017)

TMD evolution

- The TMD evolution is more conveniently done in coordinate space.

Renormalization group (RG) equation:

$$\mu^2 \frac{d}{d\mu^2} F_{f\leftarrow h}(x,\vec{b};\mu,\zeta) = \frac{1}{2}\gamma_F^f(\mu,\zeta) F_{f\leftarrow h}(x,\vec{b};\mu,\zeta),$$

$$\zeta \frac{d}{d\zeta} F_{f\leftarrow h}(x,\vec{b};\mu,\zeta) = -\mathcal{D}^f(\mu,\vec{b}) F_{f\leftarrow h}(x,\vec{b};\mu,\zeta).$$

Anomalous Dimension

TMD PDF in the coordinate space

The scale μ is the standard RG scale, with the additional rapidity factorization scale ζ to regularize the light-cone divergence arising from Wilson lines. They were usually chosen to be the same order of scattering scale.

Solution:

$$F_{f\leftarrow h}(x,\vec{b};\mu_f,\zeta_f) = \exp[\int_P (\gamma_F^f(\mu,\zeta)\frac{d\mu}{\mu} - \mathcal{D}^f(\mu,\vec{b})\frac{d\zeta}{\zeta})] F_{f\leftarrow h}(x,\vec{b};\mu_i,\zeta_i)$$

TMD evolution:

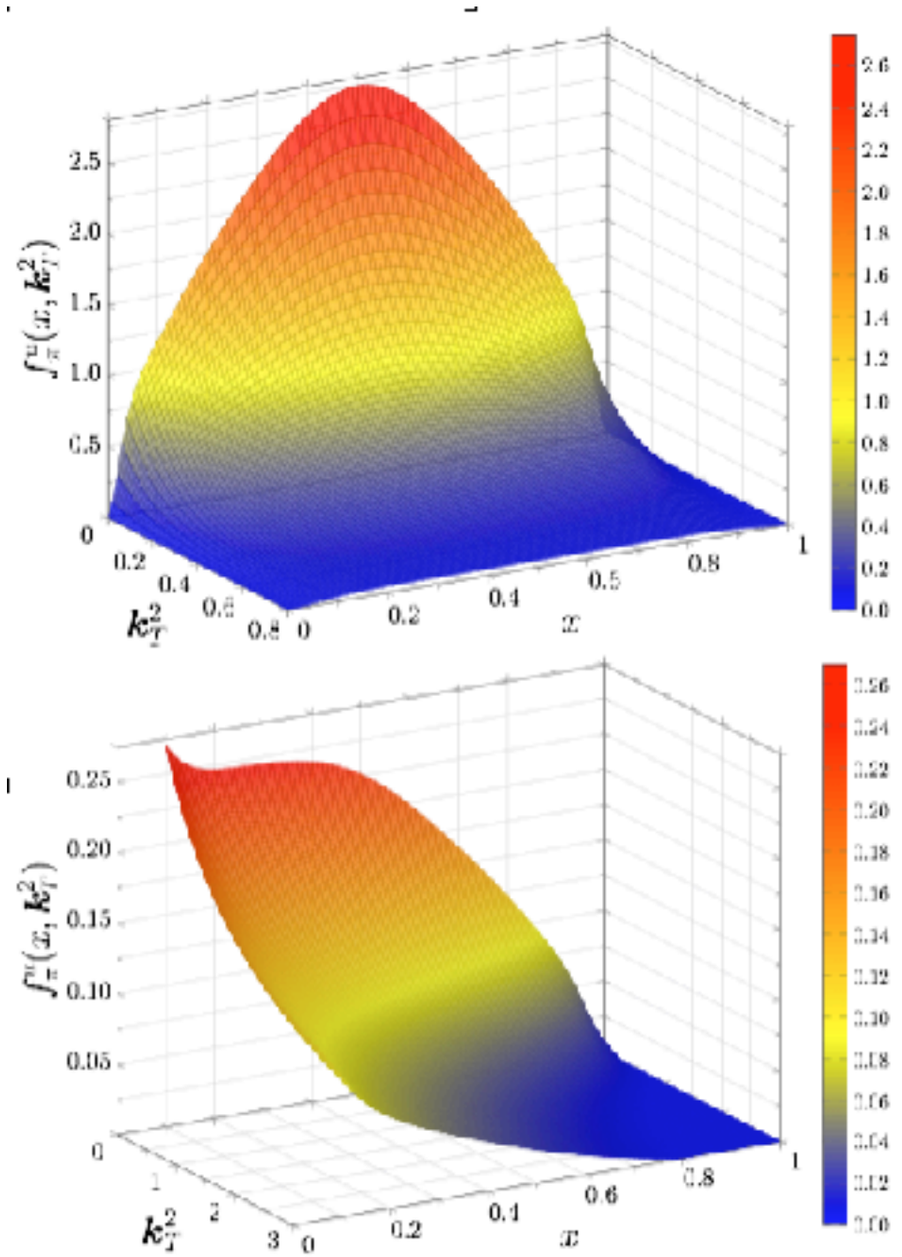
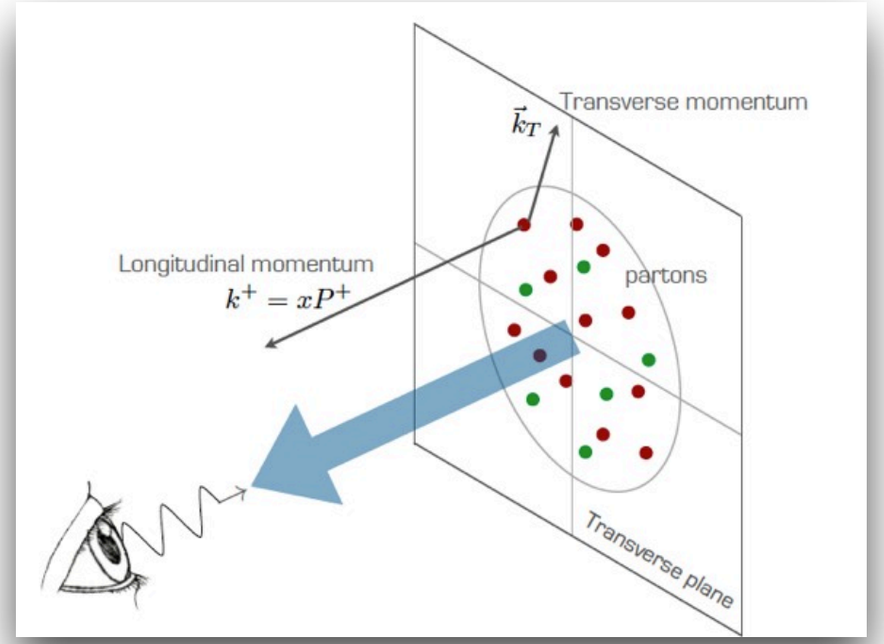


Figure 2. *Upper panel:* DSE result using the DCSB-improved kernel for the time-reversal even u-quark TMD of the pion, $f_\pi^u(x, k_T^2)$, at the model scale of $\mu_0^2 = 0.52\,\mathrm{GeV}^2$. *Lower panel:* Analogous result evolved to a scale of $\mu = 6\,\mathrm{GeV}$ using TMD evolution with the b^* prescription and $g_2 = 0.09\,\mathrm{GeV}$ [43]. The TMDs are given in units of GeV^{-2} and k_T^2 in GeV^2.



- Evolution leads broader k_T.
- The evolved TMD PDF at smaller x is significantly broader than that at large x (Non-factorizable x and k_T dependence).

Drell-Yan Process

Experiment (E615)

Transverse momentum dependence parameterized by function $P(q_T; x_F, m_{\mu\mu})$

$$\frac{d^3\sigma}{dx_\pi dx_N dq_T} = \frac{d^2\sigma}{dx_\pi dx_N} P(q_T; x_F, m_{\mu\mu}).$$

$$q^0 = \frac{\sqrt{s}}{2}(x_\pi + x_N)$$

$$q^3 = \frac{\sqrt{3}}{2}(x_\pi - x_N)$$

"Experimental study of muon pairs produced by 252-GeV pions on tungsten", Conway, J.S. et al. Phys.Rev. D39 (1989) 92-122.

Theory

$$\frac{d^3\sigma}{dx_\pi dx_N dq_T} \propto |q_T| F^1_{uu}(x_\pi, x_N, q_T)$$

(leading twist)

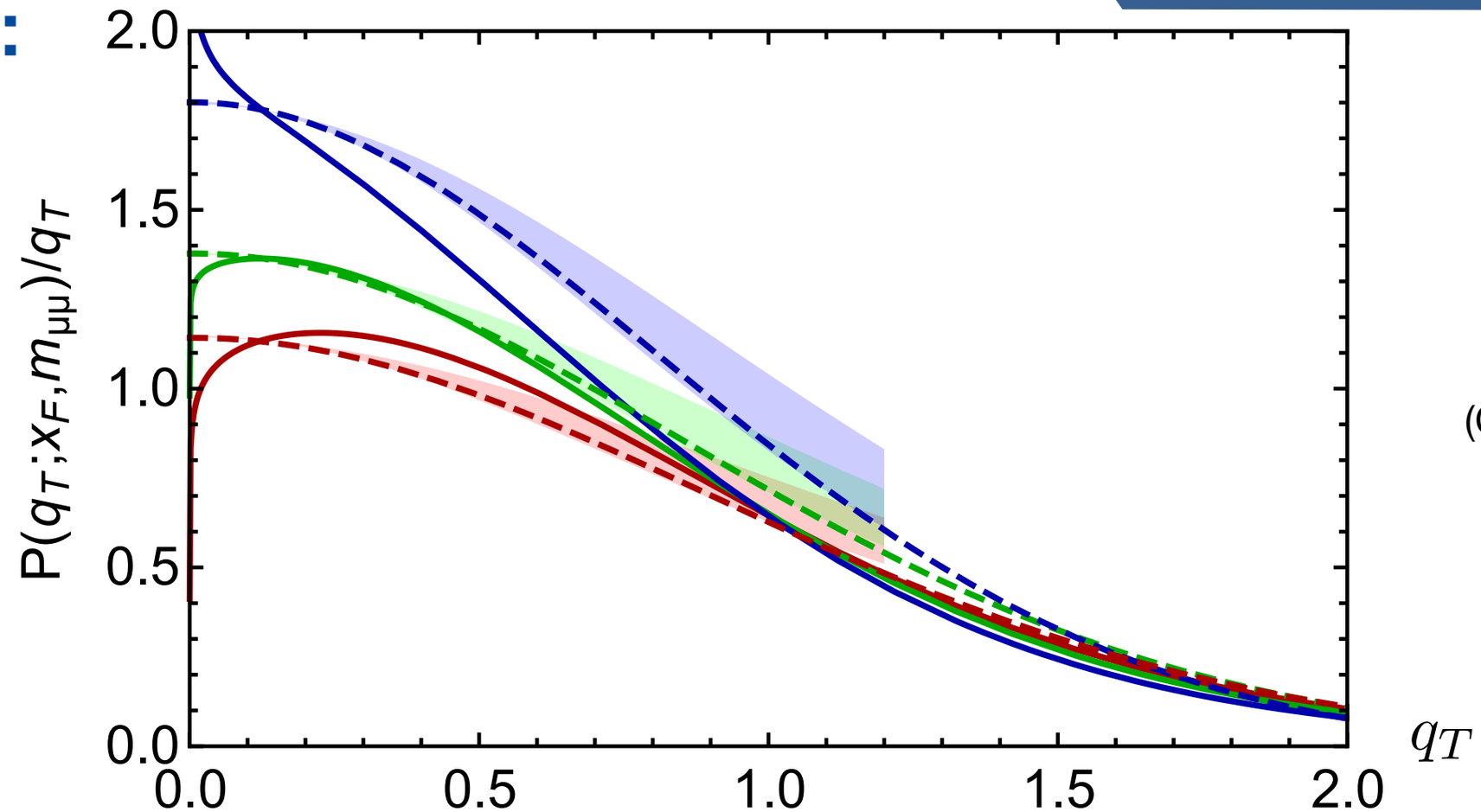
TMD formalism: $F^1_{UU}(x_1, x_2, q_T) = \frac{1}{N_c} \sum_a e_a^2 \int d^2\boldsymbol{k}_{1\perp} d^2\boldsymbol{k}_{2\perp} \delta^{(2)}(\boldsymbol{q}_T - \boldsymbol{k}_{1\perp} - \boldsymbol{k}_{2\perp}) f^{\bar{a}}_{1,\pi}(x_1, \boldsymbol{k}^2_{1\perp}) f^a_{1,N}(x_2, \boldsymbol{k}^2_{2\perp}).$

offered by DSEs&evolution

borrow from global analysis

Examine: $P(q_T; x_F, m_{\mu\mu}) \propto |q_T| F^1_{UU}(q_T; x_F, \tau)$

E615:



(C.S. et al, PRL2019)

The fitting function $P(q_T; x_F, m_{\mu\mu})/q_T$ at $x_F = 0.0$ (red solid), 0.25 (green solid) and 0.5 (blue solid). The band colored bands are our results based on b*-prescription, with upper boundary corresponding to $g_2 = 0.09$ and lower boundary for $g_0 = 0.0$. The dashed lines are obtained following ζ-prescription where g_2 is found to be consistent with zero at NNLL/NNLO.

- Our results using two evolution schemes generally agree with E615 measurement. In particular, when the non-perturbative Sudakov factor goes to zero as suggested by ζ-prescription at higher order. (The deviation is less than 10%for x_F =0 and 0.25, and increases to 30% at most for $x_F = 0.5$.) Alessandro Bacchetta, et al, JHEP2017
  Alexey Vladimirov, et al, EPJC2018

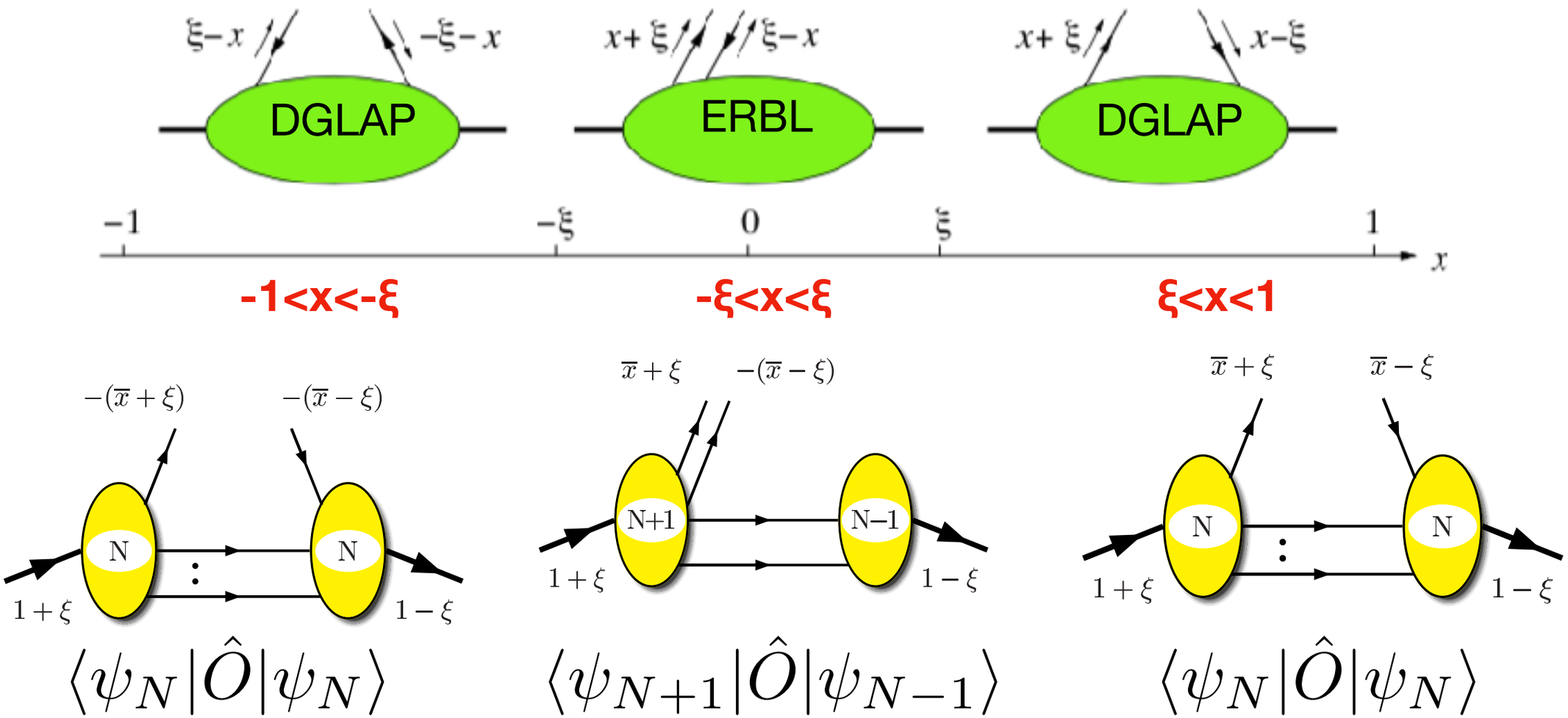
- Deviation grows as x_F goes larger, TMD formalism less valid.

GPD: overlap representation

- At leading twist, the pion has one GPD:

$$H_\pi^q(x,\xi,t) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p_2|\bar{\psi}^q(-\frac{z}{2})\gamma^+\psi^q(\frac{z}{2})|p_1\rangle|_{z^+=z_\perp=0}$$

- There are two regions, ERBL (|x|<|ξ|) and DGLAP (|x|>|ξ|) region, named after their evolution in limiting cases



M. Diehl, et al, NPB2001

IPD GPD

GPD at zero skewness H(x,ξ=0,Δ)

Fourier Transform

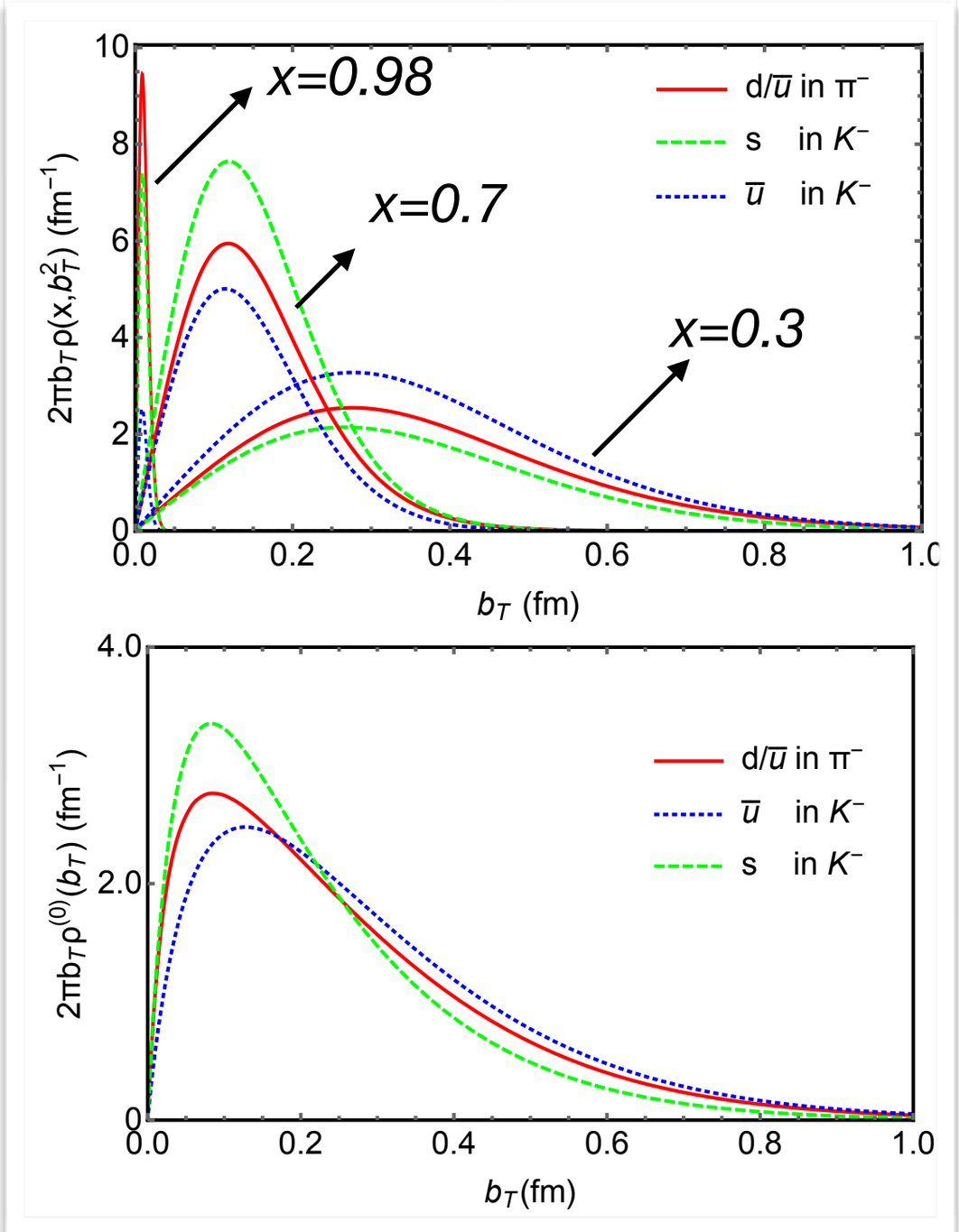
Impact Parameter Dependent GPD ρ(x,bT)

Probability density interpretation

x: longitudinal momentum fraction

b_T: transverse spatial separation ~ $(1-x)(r^1_T - r^2_T)$

- All distributions peek at the center of impact parameter (note the plot has been multiplied with b_T)
- heavier s quark is more localized as compared to light u/d quark by ~20%.
- Valence distribution $\rho^{(0)}(b_T) = \rho_q^{(0)}(b_T) - \rho_{\bar{q}}^{(0)}(b_T)$ is scale-independent, as $H(x,0,\Delta_T)$ evolution is independent of Δ_T.

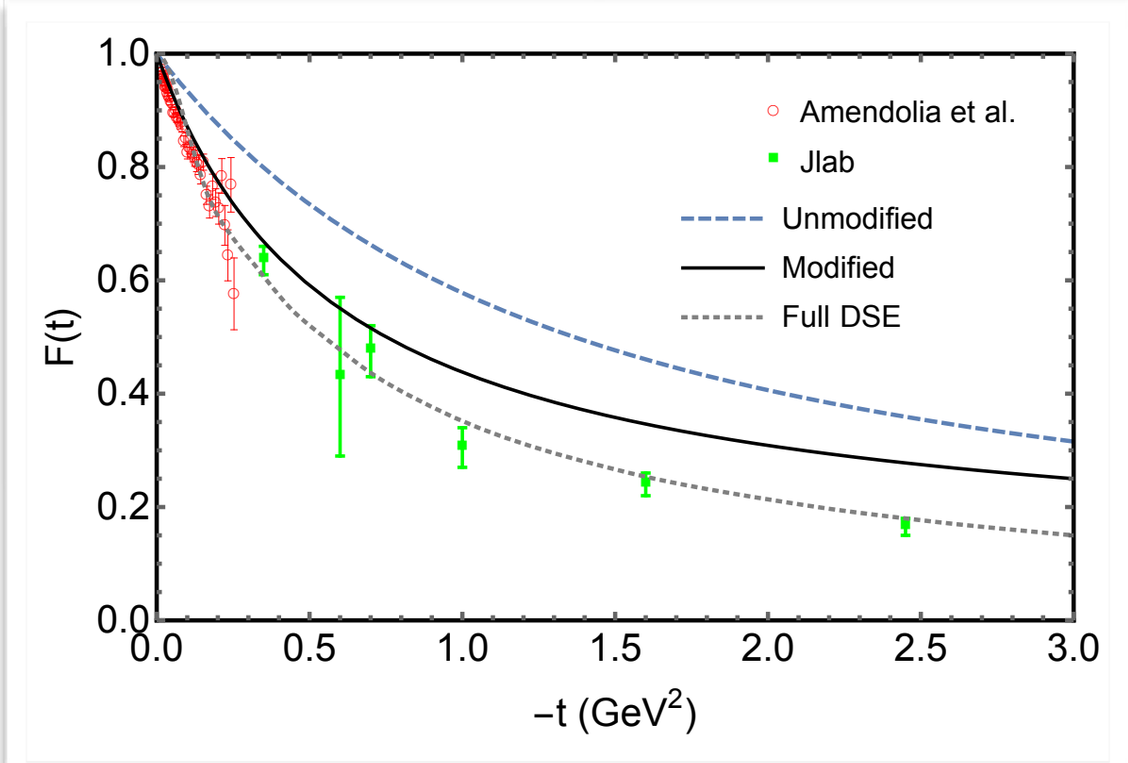


EMFF

- EMFF is the Zero-th moment of GPD

$$F_M(t) = \int_{-1}^{1} dx[e_u H_M^u(x,\xi,t) + e_d H_M^d(x,\xi,t)],$$

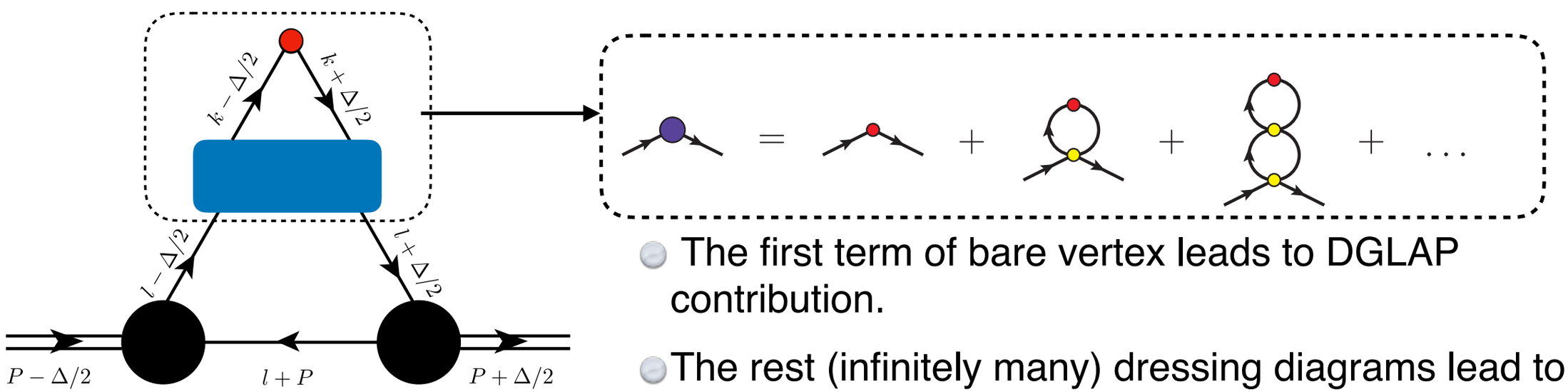
- But the curve generally overshoots the data. No such problem for a covariant calculation (with conventional Feynman diagram). Essentially a problem from Fock state truncation.



- A hidden ERBL region is found in NJL model.

$$H'_d(x,0,t) = H_d(x,0,t) + \delta(x)\tilde{F}_\rho(t)\int_{-1}^{1} dy H_{I=1}(y,0,t),$$

$$\delta(x):\ |x| < |\xi| \text{ as } |\xi| \to 0$$



- The first term of bare vertex leads to DGLAP contribution.
- The rest (infinitely many) dressing diagrams lead to the hidden ERBL contribution.

Gravitational Form Factor

GFF: $$\langle \pi^+(p')|\Theta^{\mu\nu}(0)|\pi^+(p)\rangle = \frac{1}{2}[P^\mu P^\nu \Theta_2(t) + (g^{\mu\nu}q^2 - q^\mu q^\nu)\Theta_1(t)].$$

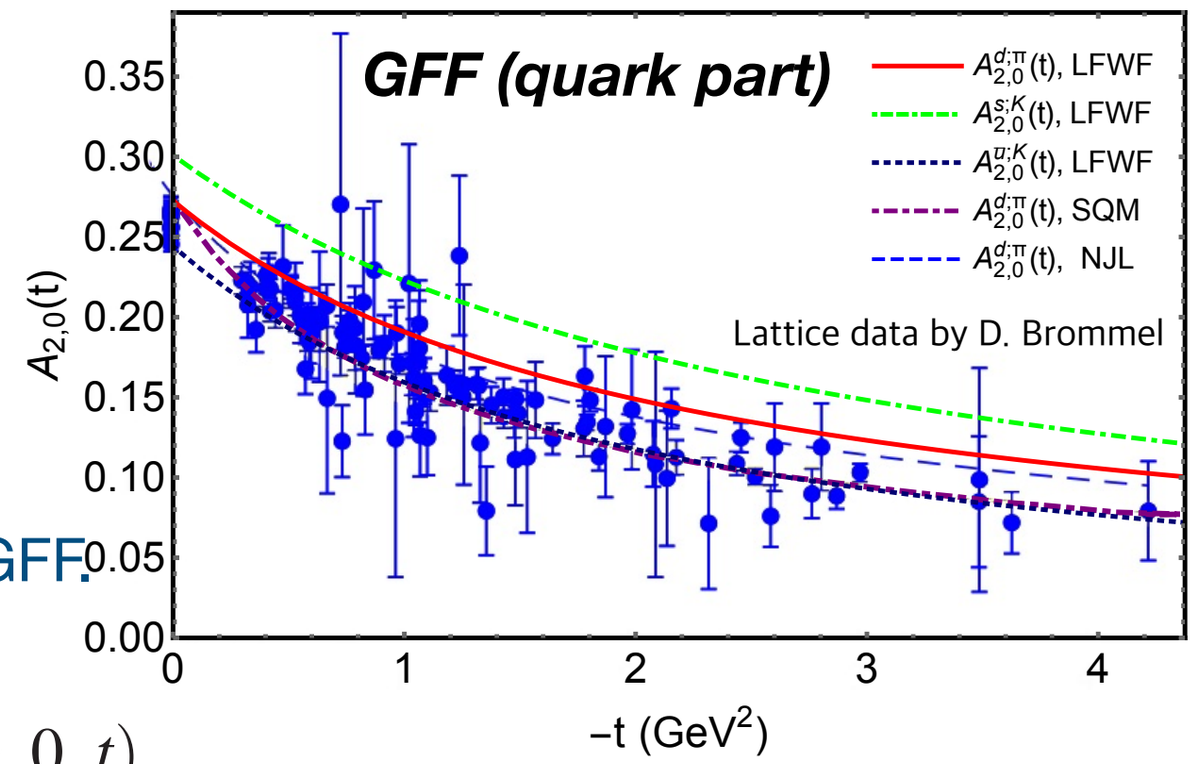
EMT $$\Theta^{\mu\nu} = \sum_{q=u,d\ldots} \bar{q}(x)(\gamma^\mu\partial^\nu + \gamma^\nu\partial^\mu)q(x) + (gluons)$$

GPD x-moments:

$$\int_{-1}^{1} dx x H'^q(x,0,t) = A^q_{2,0}(t)$$

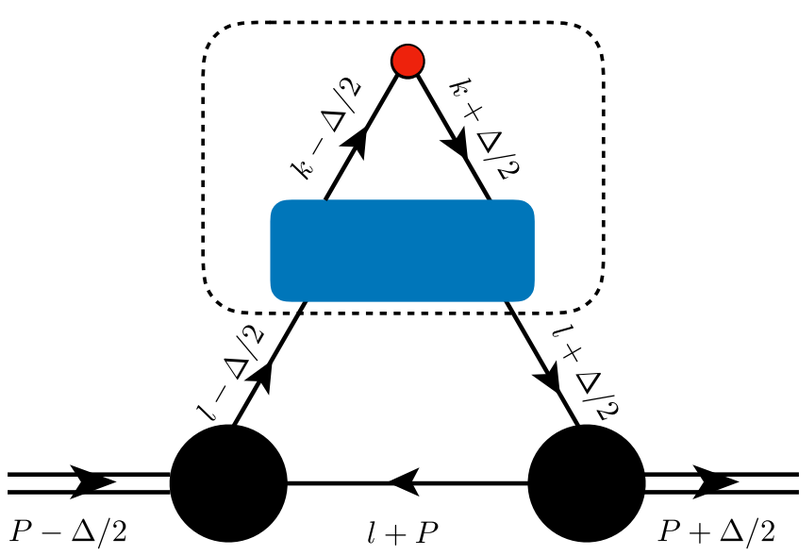
$$A^q_{2,0}(t) = \frac{1}{2}\Theta^q_1(t)$$

- The hidden ERBL region doesn't contribute to GFF.



$$H'_d(x,0,t) = H_d(x,0,t) + \delta(x)\tilde{F}_\rho(t)\int_{-1}^{1} dy H_{I=1}(y,0,t),$$

(C.S. et al, PRD2020)



	EMFF	GFF	TMD PDF	IPD GPD
Contribute:	Yes	No	No	No

A universal description within leading LFWFs truncation.

Conclusions

- LFWFs can be obtained from Bethe-Salpeter wave functions, rendering a variety of light front distributions calculable.

- In a realistic calculation, the spin-parallel LFWF of pion and kaon contributes considerably, exhibiting a highly relativistic system. Less contribution for heavy quark system (in preparation).

- Existing data can be described using realistic leading Fock state LFWFs.

- Many more to explore with DSEs.


Thank
You