LQCD 3D Meson Structure Prospects

Workshop on Pion & Kaon Structure Functions at the EIC

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On Behalf of the HadStruct Collaboration

WILLIAM & MARY

Center for Frontiers in Nuclear Science
Workshop series

Jefferson Lab
Thomas Jefferson National Accelerator Facility

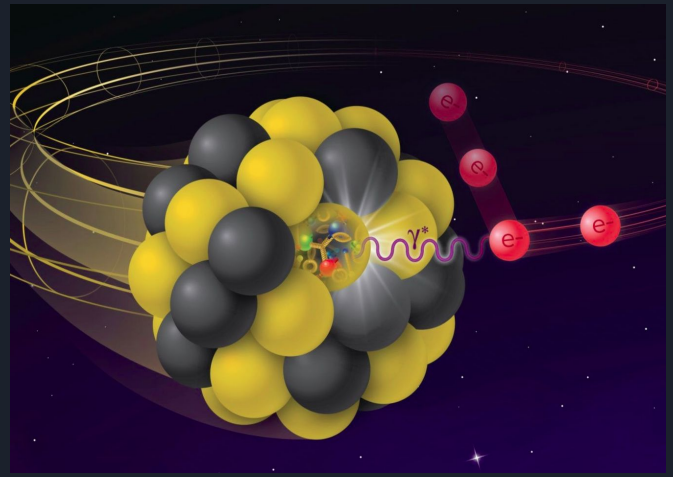
The EIC - An Essential Window into Hadron Structure

- Emergent hadrons from quarks/gluons

$$\mathcal{L}_{\mathrm{QCD}} = \overline{\psi}_i \left[i \left(\gamma^\mu D_\mu \right)_{ij} - m\delta_{ij} \right] \psi_j - \frac{1}{4} F^c_{\mu\nu} F^{\mu\nu c}$$

- "A machine that will unlock the secrets of the strongest force in Nature"
  - confinement/saturation
  - 3D-imaging + proton spin puzzle

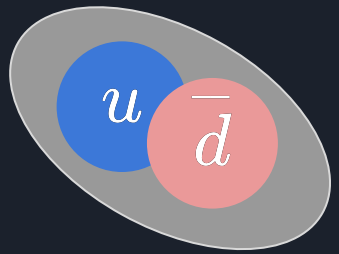
- GPDs - experimental probes
  - e.g. DVCS, exclusive meson production,...
  - challenging to fully constrain
  - timely non-perturbative methods



Physics Today 73, 3, 22 (2020)

Roadmap

- The EIC & Hadron Structure
- The light-cone & Lattice QCD
  - factorizable matrix elements
- Pseudo-Ioffe-time Distributions
  - results in pion/nucleon
- 3D Meson structure from pseudo-distributions
- Numerics
  - Distillation
  - generalized pseudo-distributions
- Outlook



Towards the Extraction of GPDs from First-Principles Lattice QCD

$$\int \frac{\mathrm{d}\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle P'| T\overline{\psi}\left(0, \tfrac{\xi^-}{2}, \mathbf{0}_T\right) \frac{\gamma^+}{2} W\left(\xi^-/2, -\xi^-/2\right) \psi\left(0, -\tfrac{\xi^-}{2}, \mathbf{0}_T\right) |P\rangle$$

- ➔ All light-cone physics is lost in a Euclidean spacetime

$$g^{\mu\nu} = \mathrm{diag}\left(-1,-1,-1,-1\right) \quad \Longrightarrow \quad z^2 = 0 \ ?!$$

- ➔ Moments calculations & the OPE
  - ◆ Power-divergent mixing/gauge noise for high moments
- ➔ Led [historically] to quasi/pseudo-PDF (forward-limit)

See *D. Richards* Wed. Talk

See *H.-W. Lin* Thur. Talk

$$\tilde{q}\left(x, P_3^2\right) = \int_{-\infty}^{\infty} \frac{\mathrm{d}z}{4\pi} e^{-ixzP_3} \langle P| \overline{\psi}\left(z\right) \gamma^0 W\left(z\right) \psi\left(0\right) |P\rangle$$

X. Ji, Phys. Rev. Lett. 110, 262002 (2013), arXiv:1305.1539 [hep-ph]

$$\mathcal{P}\left(x, z^2\right) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{d}\nu\, e^{-ix\nu} \mathcal{M}\left(\nu, z^2\right)$$

A. V. Radyushkin, Phys. Rev. D 96, 034025 (2017)

Pseudo-Ioffe-time Distributions (pITDs)

A. V. Radyushkin, Phys. Rev. D96, 034025 (2017), arXiv:1705.01488 [hep-ph]

- ➔ A single-hadron matrix element of a slightly different character

pITDs

$$M^{\alpha}(z,p) = \langle h(p)|\,\overline{\psi}(z)\,\gamma^{\alpha} W(z,0;A)\,\psi(0)\,|h(p)\rangle = 2p^{\alpha}\mathcal{M}_{\mathrm{p}}\left(\nu,z^2\right) + z^{\alpha}\mathcal{M}_{\mathrm{z}}\left(\nu,z^2\right)$$

- ➔ Leading twist : well-chosen kinematics

$$p^{\mu} = \left(p^0,0,0,p_3\right) \qquad z^{\mu} = \left(0,0,0,z_3\right)$$

- ➔ A short-distance factorization to ITD

Perturbatively computable matching coefficients

$$\mathcal{M}\left(\nu,z^2\right) = \sum_{a=q,\overline{q},g} C_a\left(z^2\mu^2,\alpha_s\right) \otimes \mathcal{I}_a\left(\nu,\mu^2\right) + h.t.$$

T. Izubuchi, et al., Phys.Rev. D98 (2018) no.5, 056004
A. Radyushkin, Phys.Lett. B781 (2018) 433-442
A. Radyushkin, Phys. Rev. D 98 (2018) no.1, 014019
J.-H. Zhang, et al., Phys.Rev. D97 (2018) no.7, 074508

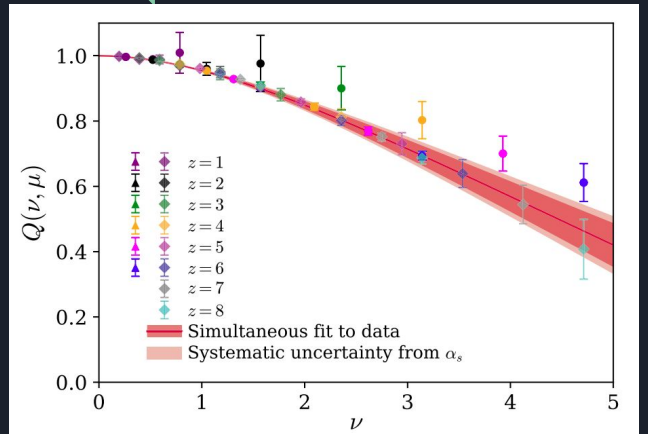
- ➔ Wilson line UV-divergent for spacelike - z^2
- ➔ Compute $M^{\alpha}(z,p)$ but instead analyze

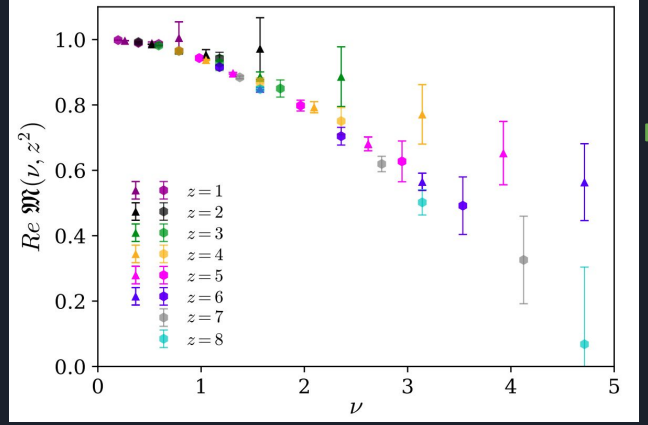
$$\mathfrak{M}\left(\nu,z^2\right) \equiv \frac{\mathcal{M}_{\mathrm{p}}\left(\nu,z^2\right)}{\mathcal{M}_{\mathrm{p}}\left(0,z^2\right)}$$

K. Orginos, A. Radyushkin, J. Karpie, and S. Zafeiropoulos, Phys. Rev. D96, 094503 (2017)

Pseudo-Distributions & Pion Valence PDF

B. Joó et. al., Phys. Rev. D100, 114512 (2019), arXiv:1909.08517 [hep-lat]



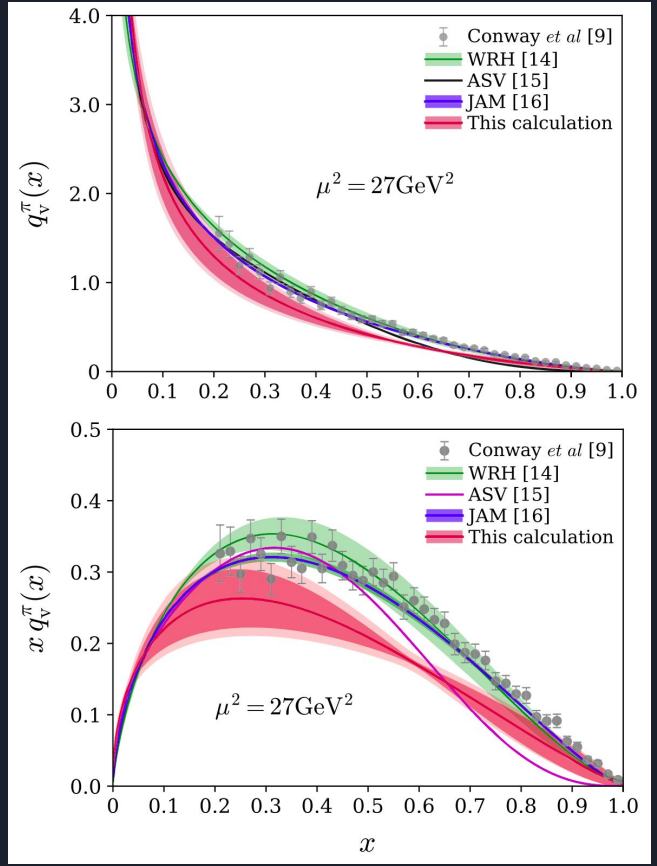


$$q_{\rm v}^{\pi}(x) = \frac{x^{\alpha}(1-x)^{\beta}}{B(1+\alpha,1+\beta)}$$

$$\text{Fit}\,(\mu = 2\ \text{GeV})$$

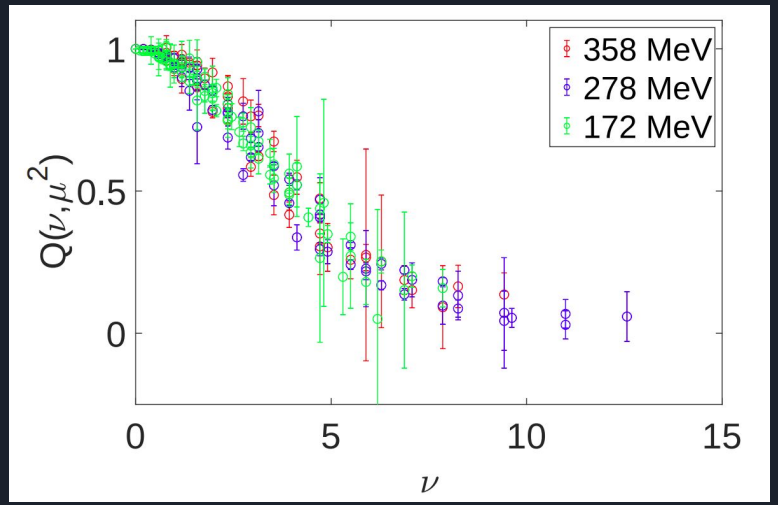
$$Q(\nu,\mu^2) = \int_0^1 \mathrm{d}x\, \cos(\nu x)\, q_{\rm v}^{\pi}(x,\mu^2)$$

- Lattice data of different z^2 evolved to common scale and matched to $2\ \text{GeV} \in \overline{\text{MS}}$



Pseudo-Distributions & Nucleon Valence PDF

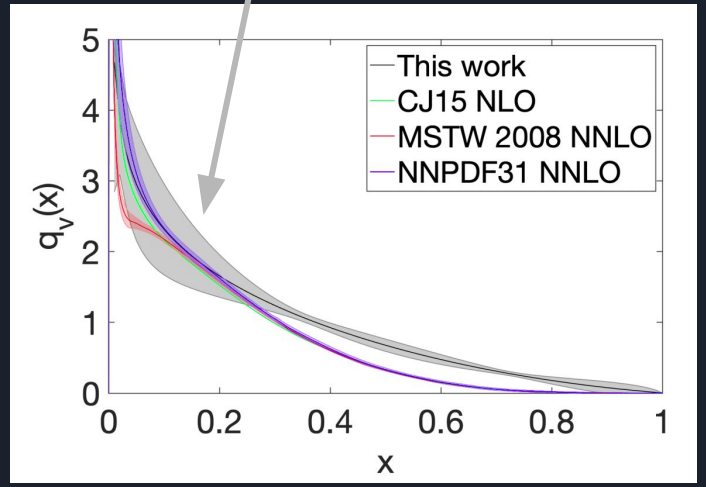
B. Joo', J. Karpie, K. Orginos et. al., arXiv:2004.01687 [hep-lat]



$$q_v^N\left(x,\mu^2,m_\pi\right) = q_v^N\left(x,\mu^2,m_0\right) + a\Delta m_\pi + b\Delta m_\pi^2$$

- [Real] ITD matched to $2\ \mathrm{GeV} \in \overline{\mathrm{MS}}$ from reduced pITD results on various ensembles

$$q_v^N\left(x\right) = \mathcal{N} x^\alpha (1-x)^\beta \left(1 + c\sqrt{x} + dx\right)$$



3D Structure of Hadrons from LQCD

A. Radyushkin, Phys. Rev. D100, 116011 (2019), arXiv:1909.08474 [hep-ph]
A. Radyushkin, arXiv:1912.04244 [hep-ph]

➔ Off-forward matrix elements and the pseudo-distribution formalism

$$F^q = \frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi} e^{ixP^+z^-} \langle p'|\,\overline{\psi}(-\tfrac{1}{2}z)\gamma^\alpha\psi(\tfrac{1}{2}z)\,|p\rangle\,|_{z^+=0,\mathbf{z}_T=0}$$

π

$$\langle p_2|\,\overline{\psi}(-\tfrac{z}{2})\gamma^\alpha \hat{E}(-\tfrac{z}{2},\tfrac{z}{2};A)\psi(\tfrac{z}{2})\,|p_1\rangle = 2\mathcal{P}^\alpha \int_{-1}^{1} \mathrm{d}x\, e^{-ix(\mathcal{P}z)} H\left(x,\xi,t;\mu^2\right)$$

➔ Compute modified spacelike matrix elements

$$\mathrm{M}^\alpha\left(p_2,p_1,z\right) \equiv \langle h(p_2)|\,\overline{\psi}\left(0\right)\boxed{\frac{\tau^3}{2}}\Gamma^\alpha W\left(0,z;A\right)\psi\left(z\right)|h\left(p_1\right)\rangle$$

Isovector projection - numerically cheaper

➔ Double Ioffe-time pseudo-distributions

$$\mathrm{M}^\alpha\left(p_2,p_1,z\right) = \frac{\left(p_2+p_1\right)^\alpha}{2} M\left(\nu_2,\nu_1,t;z^2\right) + z^\alpha N\left(\nu_2,\nu_1,t;z^2\right)$$

$$\nu_i \equiv -(p_i\cdot z)$$
$$t = \left(p_1-p_2\right)^2$$

Recent Community Progress

a (fm)	L (fm)	m_π (MeV)	ξ	μ (GeV)
~ 0.12	~ 3	~ 310	0	4

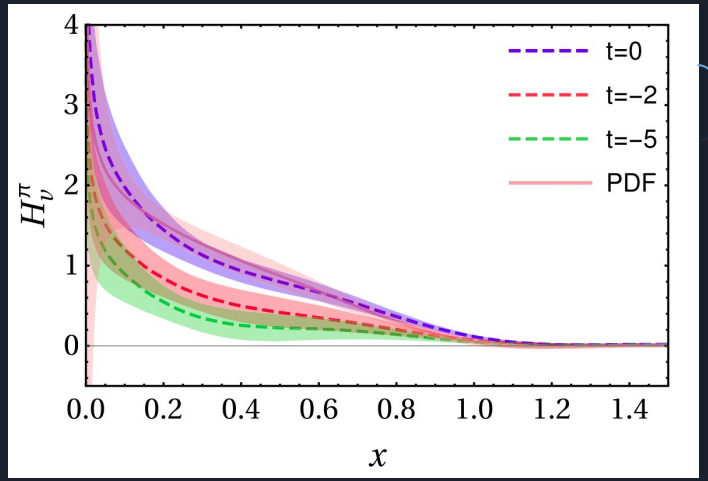
$$\frac{1}{2P^0}\langle\pi\left(P+\frac{\Delta}{2}\right)|\,\overline{q}\left(\frac{z}{2}\right)\gamma^t W\left(z/2,-z/2\right)q\left(-\frac{z}{2}\right)|\pi\left(P-\frac{\Delta}{2}\right)\rangle$$

$$\widetilde{H}^{\pi}_{u-d}\left(x,\xi,t,P^z,\tilde{\mu}\right)=\int\frac{\mathrm{d}zP^z}{2\pi}e^{ixP^z z}\tilde{h}\left(z,P^z,\xi,t,\tilde{\mu}\right)$$

Renormalize
UV Wilson line divergence

$$\widetilde{H}^{\pi}_{u-d,R}\left(x,\xi,t,P^z,p^R_z,\mu_R\right)=C\left(\frac{x}{y},\frac{\xi}{y},\frac{\mu_R}{p^R_z},\frac{yP^z}{\mu},\frac{yP^z}{p^R_z}\right)\otimes H^{\pi}_{u-d}\left(y,\xi,t,\mu\right)+\mathcal{O}\left(\frac{m^2_\pi}{P^2_z},\frac{\Lambda^2_{\mathrm{QCD}}}{P^2_z}\right)$$

Y.-S. Liu et. al., Phys. Rev.D 100 (2019) 3, 034006, arXiv:1902.00307 [hep-ph]



$\times\left(\frac{2\pi}{L}\right)^2$

J.-W. Chen, H.-W. Lin, J.-H. Zhang, Nucl. Phys.B 952 (2020) 114940, arXiv:1904.12376 [hep-lat]

True distribution recovered in infinite momentum limit; Finite corrections via LaMET

Some Practical Considerations/Definitions

- Well-chosen kinematics + $\Gamma^\alpha = \gamma^0$

$$p_i^\mu = \{E_1, (-1)^{i-1}\Delta_\perp/2, P_i\}$$
$$z^\mu = (0,0,0,z_3)$$

Removal of pure higher-twist contribution

$$z^\alpha N\left(\nu_2, \nu_1, t; z^2\right)$$

- *Pseudo-Generalized Ioffe-time Distribution* (pGITD)

$$M\left(\nu_2, \nu_1, t; z^2\right) \mapsto \boxed{\mathcal{M}\left(\nu, \xi, t; z_3^2\right)}$$

$$\xi = \frac{(p_1 z)-(p_2 z)}{(p_1 z)+(p_2 z)} = \frac{\nu_1-\nu_2}{\nu_1+\nu_2} = \frac{P_1-P_2}{P_1+P_2}$$
$$\nu = \frac{\nu_1+\nu_2}{2}$$

- `Technicality:` matching w.r.t. "symmetric" operator

$$\begin{aligned}\langle p_2|\,\overline{\psi}(0)\cdots\psi(z)\,|p_1\rangle &= e^{-i(p_1 z)/2+i(p_2 z)/2}\,\langle p_2|\,\overline{\psi}(-z/2)\cdots\psi(z/2)\,|p_1\rangle \\ &= e^{i\xi\nu}\,\langle p_2|\,\overline{\psi}(-z/2)\cdots\psi(z/2)\,|p_1\rangle \\ &= e^{i\xi\nu}\,\widetilde{\mathcal{M}}\end{aligned}$$

Compute in LQCD

Matching valid here

Matching pGITD to GITD

A. Radyushkin, Phys. Rev. D100, 116011 (2019), arXiv:1909.08474 [hep-ph]
A. Radyushkin, Phys. Rev. D98, 014019 (2018), arXiv:1801.02427 [hep-ph]

- ➔ Manage space-like Wilson line UV-divergences

K. Orginos, A. Radyushkin, J. Karpie, and S. Zafeiropoulos, Phys. Rev. D96, 094503 (2017)

$$\widetilde{\mathfrak{M}}\left(\nu,\xi,t,z_3^2\right) \equiv \frac{\widetilde{\mathcal{M}}\left(\nu,\xi,t,z_3^2\right)}{\widetilde{\mathcal{M}}\left(0,0,0,z_3^2\right)}$$

Observed to dramatically reduced higher-twist $\mathcal{O}\left(z^2\right)$ effects in forward cases

- ➔ NLO matching of pGITD to light-cone GITD

$$\bar{u} = 1 - u$$

Scale dep. of pGITD

$$\widetilde{\mathfrak{M}}\left(\nu,\xi,t,z_3^2\right) = \widetilde{\mathcal{I}}\left(\nu,\xi,t,\mu^2\right) - \frac{\alpha_s C_F}{2\pi}\int_0^1 \mathrm{d}u\,\widetilde{\mathcal{I}}\left(u\nu,\xi,t,\mu^2\right)$$

$$\times\left\{\ln\left[\frac{e^{2\gamma_E+1}}{4}z_3^2\mu^2\right]\left(\left[\frac{2u}{1-u}\right]_+\cos\left(\bar{u}\xi\nu\right)+\frac{\sin\left(\bar{u}\xi\nu\right)}{\xi\nu}-\frac{\delta\left(\bar{u}\right)}{2}\right)\right.$$

$$\left.+4\left[\frac{\ln\left(1-u\right)}{1-u}\right]_+\cos\left(\bar{u}\xi\nu\right)-2\frac{\sin\left(\bar{u}\xi\nu\right)}{\xi\nu}+\delta\left(\bar{u}\right)\right\}+\mathcal{O}(z^2\Lambda_{\mathrm{QCD}}^2)$$

Matching to $\overline{\mathrm{MS}}$

An Ill-posed Inverse

- ➔ Direct approach -- invert GITD/GPD relation

$$H\left(x,\xi,t;\mu^2\right)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}\nu\, e^{-i\nu x}\widetilde{\mathcal{I}}\left(\nu,\xi,t,\mu^2\right)$$

$N_\nu \sim \mathcal{O}\left(30\right)$

- ➔ Advanced reconstruction methods
  - Backus-Gilbert, NNs, …

J. Karpie, K. Orginos, A. Rothkopf, and S. Zafeiropoulos, JHEP 04, 057 (2019), arXiv:1901.05408 [hep-lat]

- ➔ Supply extra, *physically-motivated*, information
  - ◆ parametrize structure function
    - e.g. $q_{\mathrm{v}}^{\pi}\left(x\right)$ smooth, $0<x<1$ support
    - $q_{\mathrm{v}}^{\pi}\left(x\right)=Nx^{\alpha}\left(1-x\right)^{\beta}\mathcal{P}\left(x\right)$
  - ◆ GPDs: e.g. polynomiality $\langle x^n\rangle_{H^q}\sim\mathcal{O}\left(\xi^{n+1}\right)$

A. V. Radyushkin, Phys. Rev. D 59, 014030 (1999)

Analogous challenge faced by global fitting community!

$$Q\left(\nu,\mu^2\right)=\int_0^1\mathrm{d}x\,\cos\left(\nu x\right)q_{\mathrm{v}}^{\pi}\left(x,\mu^2\right)$$

- ➔ When to numerically perform convolution?

$$\widetilde{\mathfrak{M}}\left(\nu,\xi,t,z_3^2\right)=K\left(\xi\nu,z^2\mu^2;\alpha_s\right)\otimes\widetilde{\mathcal{I}}\left(\nu,\xi,t,\mu^2\right)$$

vs.

$$\widetilde{\mathfrak{M}}\left(\nu,\xi,t,z_3^2\right)=\mathcal{K}\left(x\nu,\xi\nu,z^2\mu^2;\alpha_s\right)\otimes H\left(x,\xi,t,\mu^2\right)$$

Numerics...

- ➔ Limits of factorizable matrix element methods
  - ★ quasi-GPD $\mathcal{O}\left(\frac{m_\pi^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{P_z^2}\right)$
  - ★ pseudo-GITD $\mathcal{O}\left(z^2 \Lambda_{\rm QCD}^2\right)$

High Momenta Essential!

- ➔ Standard sequential operator methods costly
  - ◆ new inversions: interpolator constructions, Γ^μ & q^2
- ➔ Excited-state contamination, signal-to-noise, $O(4) \longrightarrow H(4)$

$$C(t)/\sigma^2_{C(t)} \sim e^{-\left(E_H - \frac{3}{2} m_\pi\right) t}$$

$$C(t) = \langle \mathcal{O}(t)\, \mathcal{O}^\dagger(0) \rangle \sim e^{-E_H t}$$

$$\sigma^2_{C(t)} = \langle C(t)\, C^\dagger(t) \rangle - \langle C(t) \rangle^2 \sim e^{-3 m_\pi t}$$

Numerics...

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- Distillation

Distillation

M. Peardon et al., Phys. Rev. D80, 054506 (2009), arXiv:0905.2160 [hep-lat]

$$J_{\sigma,n_\sigma} = e^{\sigma\nabla^2} = \sum_\lambda e^{-\sigma\lambda} |\lambda\rangle\langle\lambda|$$

- ➔ Low-mode approximation to some gauge-covariant smearing kernel (e.g. $\mathcal{D}ist = \sum_{i=1}^{N} |\lambda\rangle\langle\lambda|$)
- ➔ Considering the Jacobi-smearing kernel

$$-\nabla^2_{ab}(\vec{x},\vec{y};t) = 6\delta_{xy}\delta_{ab} - \sum_{j=1}^{3}\left[\tilde{U}_j(\vec{x},t)_{ab}\,\delta_{x+\hat{j},y} + \tilde{U}_j^\dagger(\vec{x}-\hat{j},t)_{ab}\,\delta_{x-\hat{j},y}\right]$$

$$-\nabla^2\xi^{(k)} = \lambda^{(k)}\xi^{(k)}$$

- ➔ Define Distillation of $\mathrm{rk}(\mathcal{D}ist) = N \ll N_c \times V_3$

$$\square(\vec{x},\vec{y};t)_{ab} = \sum_{k=1}^{N}\xi_a^{(k)}(\vec{x},t)\,\xi_b^{(k)\dagger}(\vec{y},t)$$

- ➔ Admits extended basis of interpolators [GEVP]
  - low-lying meson spectrum — R. Briceno et al., Phys.Rev.D 97 (2018) 5, 054513; J. Dudek et. al., Phys.Rev.D 88 (2013) 9, 094505; J. Dudek et al., Phys.Rev.D 87 (2013) 3, 034505
  - exotic hadrons — J. Dudek, et. al., Phys. Rev.D83, 111502 (2011); L. Liu, et. al., JHEP 07, 126 (2012)

When the Dust Settles

$$C_{ij}^{2\mathrm{pt}}(t) = \langle \mathcal{O}_i(t)\, \overline{\mathcal{O}}_j(0) \rangle$$

$$C_{ij,\Gamma}^{3\mathrm{pt}}(t,\tau) = \langle \mathcal{O}_i(t)\, \mathcal{J}(\tau)\, \overline{\mathcal{O}}_j(0) \rangle$$

$$\mathcal{O}_M^\dagger(\vec{p}, t_j) = e^{-ip\cdot y} S_M^{\alpha\beta} (\overline{u}\Box)^\alpha \Gamma_M (\Box d)^\beta (t)$$

➔ Factorization of correlation functions

$$C_M^{2\mathrm{pt}}(t',t) = \mathrm{Tr}\left[\Phi^M(t')\, \tau(t',t)\, \Phi^M(t)\, \tau(t,t')\right]$$

$$C_{u-d}^{3\mathrm{pt}}(t',\tau,t) = \mathrm{Tr}\left[\Phi^M(t')\, \underline{\mathcal{J}(t',\tau,t)}\, \Phi^M(t)\, \gamma_5 \tau^\dagger(t',t)\, \gamma_5\right]$$

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$$\Phi_{\mu\nu}^{(i,j)M}(t) = \xi^{(i)\dagger}(t)\,\mathcal{D}^M(t)\,\xi^{(j)}(t)\,S_{\mu\nu}^M$$

$$\tau_{\alpha\beta}^{kl}(t',t) = \xi^{(k)\dagger}(t')\,M_{\alpha\beta}^{-1}(t',t)\,\xi^{(l)}(t)$$

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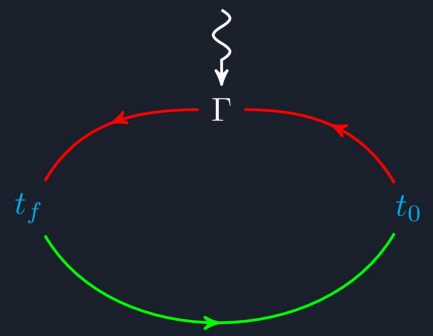

- ➔ Inversion cost amortized across physics observables!
  - ◆ extended interpolator basis
- ➔ Explicit momentum projections at each time slice

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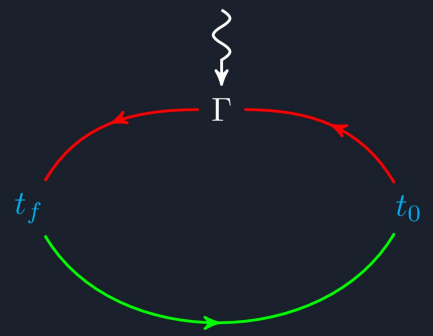


- ➔ Inversion cost amortized across physics observables!
  - ◆ extended interpolator basis
- ➔ Explicit momentum projections at each time slice
- ➔ Distillation at high-momenta feasible

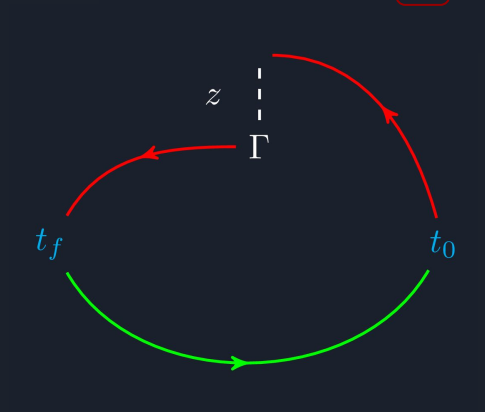
$$|\vec{p}| \sim 3\ \mathrm{GeV}$$

C.f. G. S. Bali et al. Phys. Rev. D93, 094515 (2016), arXiv:1602.05525 [hep-lat]

Sketch of Pseudo-GITD Computation

➔ Production only requires non-local "genprops"

$$\tilde{\tau}^{ij,\Gamma}_{\mathrm{pGITD}}\left(t_f, t_0; \tau, \vec{z}; \vec{q}\right) = \boxed{\sum_{\vec{z}_0}} e^{i\vec{q}\cdot\vec{z}_0} \xi^{(i)\dagger}\left(t_f\right) D^{-1}\left(t_f; \vec{z}+\vec{z}_0, \tau\right) \Gamma W\left(\vec{z}+\vec{z}_0, \vec{z}_0; \tau\right) D^{-1}\left(\vec{z}_0, \tau; t_0\right) \xi^{(j)}\left(t_0\right)$$



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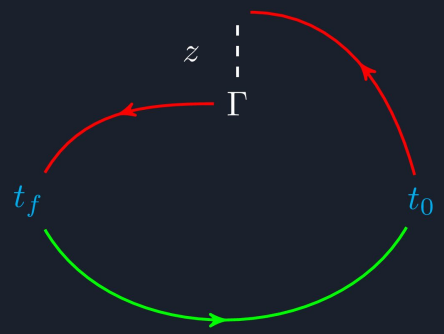

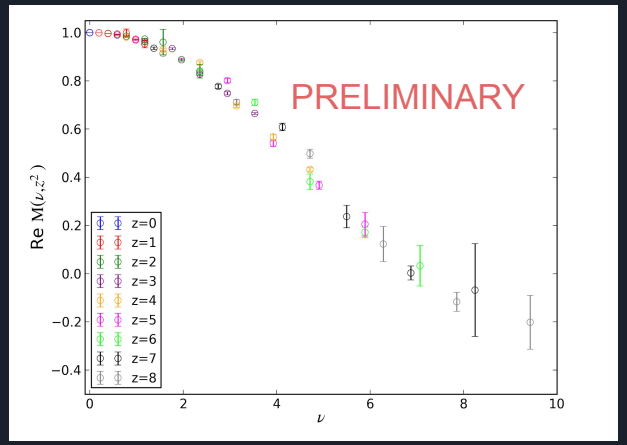

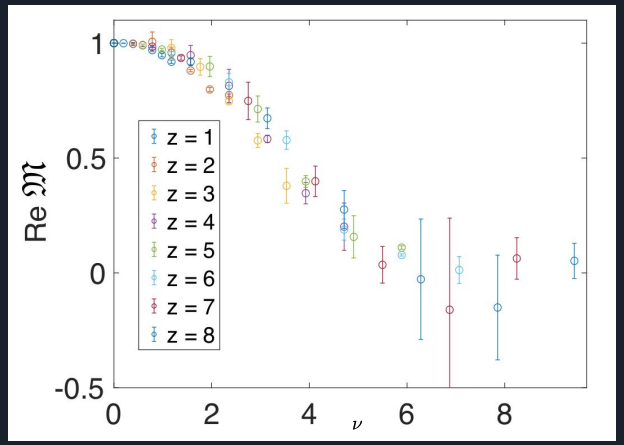

B. Joó, J. Karpie, K, Orginos et al., Phys. Rev. **D100**, 114512 (2019)

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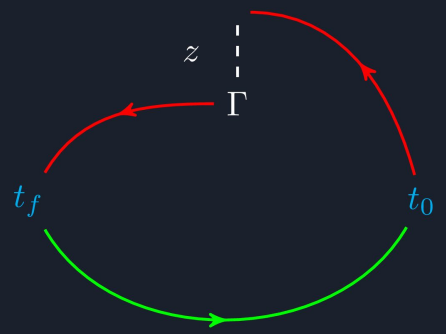
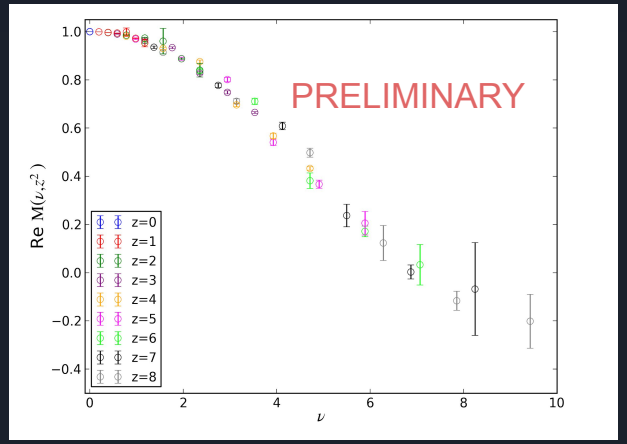
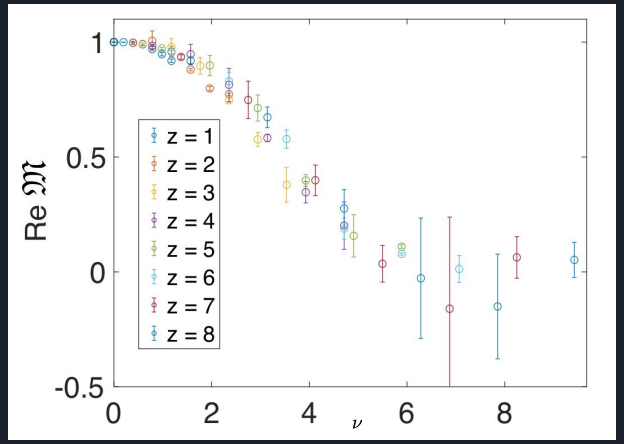






B. Joó, J. Karpie, K, Orginos et al., Phys. Rev. **D100**, 114512 (2019)

➔ Summation method to improve matrix element extraction

C. Bouchard et.al ,Phys. Rev. D 96, no. 1, 014504 (2017)

$$C^{AB}_{3\mathrm{pt}}\left(\vec{p}, \vec{q}; t_f, t_0, \tau; \vec{z}\right) = \sum_{\tau} \mathrm{Tr}\langle \Phi^A\left(\vec{p}, t_f\right) \tilde{\tau}\left(t_f, t_0; \tau, \vec{z}; \vec{q}\right) \Phi^B\left(\vec{p}, t_0\right) \tau^\dagger\left(t_f, t_0\right)\rangle$$

Ongoing Pseudo-GITD Production

➔ Target ensemble:

ID	a (fm)	m_π (MeV)	$L^3 \times N_t$	$N_{\rm cfg}$	$N_{\rm vec}$
$a94m358$	$0.094(1)$	$358(3)$	$32^3 \times 64$	350	64

➔ Genprop production well underway

- computing: $\{\gamma^\mu, |z| \leq 8, N_{Q^2} = 19, N_{\rm vec} = 64\}$
- recall,

$$\xi = \frac{(p_1 z) - (p_2 z)}{(p_1 z) + (p_2 z)} = \frac{P_1 - P_2}{P_1 + P_2}$$

P_i^z fixes ξ

$+$

z_3 fixes ν

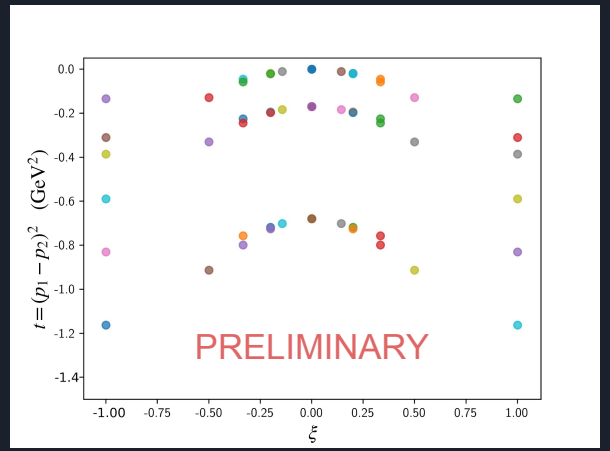
- broad longitudinal momenta $P_i^z \in \frac{2\pi}{L}\mathbb{Z}_7$

➔ Goals:

π $\quad H^\pi_{u-d}(x, \xi, t)$

K $\quad H^K(x, \xi, t)$

N $\quad H^N_{u-d}(x, \xi, t), E^N_{u-d}(x, \xi, t), \& \tilde{H}, \tilde{E}$



Summary

- Light-cone physics inaccessible directly in LQCD
  - factorizable matrix elements
    - LaMET -- quasi-PDFs/GPDs
    - Ioffe-time pseudo-distributions (pITDs/pGITDs)
    - Two-current correlators -- Lattice "Cross Sections"
- Encouraging pion/nucleon PDF results from pITDs (pseudo-PDFs)
  - Distillation ⇢ improved sampling, excited-state control
  - vehicle to isolate observables simultaneously
- Guidance for EIC effort to map hadrons tomographically
- Excited to share results in coming months...

See *H.-W. Lin* Thur. Talk

See *D. Richards* Wed. Talk

See *J.-W. Qiu* Tues. Talk

THANK YOU!

Backups...

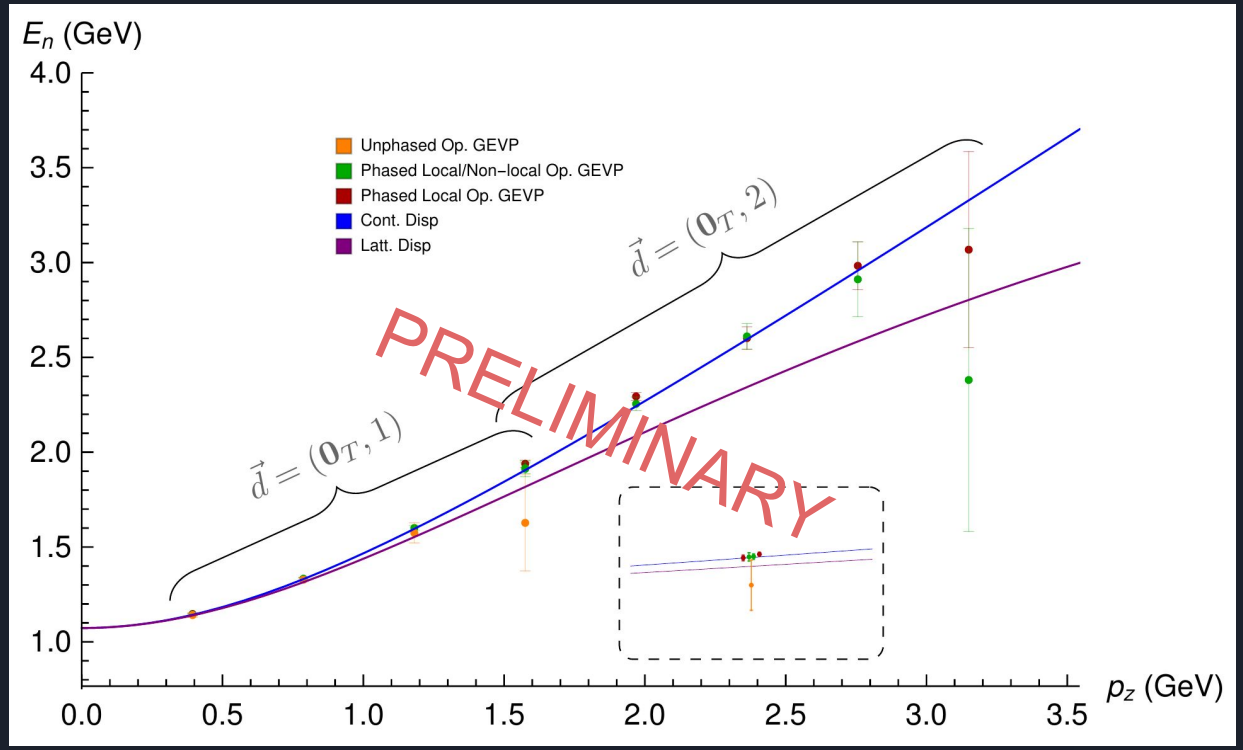
NLO Matching Kernel - pITDs to ITDs

- Can match reduced pITD at many scales to $\overline{MS}$ ITD in a single step

$$\mathfrak{M}\left(\nu, z^2\right) = \mathcal{I}\left(\nu, \mu^2\right) + \frac{\alpha_s}{2\pi} C_F \int_0^1 \mathrm{d}u \left[\ln\left(z^2\mu^2 \frac{e^{2\gamma_E+1}}{4}\right) B\left(u\right) + D\left(u\right)\right] \mathcal{I}\left(u\nu, \mu^2\right) + h.t.$$

- Where $B\left(u\right)$ is the Altarelli-Parisi kernel and $D\left(u\right)$ is a scale independent term

$$B\left(u\right) = \left[\frac{1+u^2}{1-u}\right]_+ \qquad D\left(u\right) = \left[4\frac{\ln(1-u)}{1-u} - 2\left(1-u\right)\right]_+$$



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$a94m358$	0.094(1)	358(3)	$32^3 \times 64$	100	4	64

Variational Method

- Exploit redundancy of interpolators in a symmetry channel
- Optimal linear combination to project onto $|\mathbf{n}\rangle$

$$C(t)\, v_{\mathbf{n}}(t,t_0) = \lambda_{\mathbf{n}}(t,t_0)\, C(t_0)\, v_{\mathbf{n}}(t,t_0)$$

$$C_{ij}(t') = \langle \mathcal{O}_i(t')\, \mathcal{O}_j^\dagger(0)\rangle$$

$$v_{\mathbf{n}'}^\dagger C(t_0)\, v_{\mathbf{n}} = \delta_{\mathbf{n}',\mathbf{n}}$$

- Fixed t_0 and solved for $t > t_0$
- Solutions yield (organized by $|\lambda_n(t,t_0)|$)
  - "Principal correlator" $\lambda_{\mathbf{n}}(t,t_0) \sim e^{-E_{\mathbf{n}}(t-t_0)}$
  - Interpolator weights $\mathcal{O}_{\mathbf{n}}^{\mathrm{opt}\,\dagger} = \sum_i v_{\mathbf{n}}^i(t,t_0)\, \mathcal{O}_i^\dagger$