

Light meson form factors from exclusive
measurements

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Workshop on Pion and Kaon Structure
Functions at the EIC

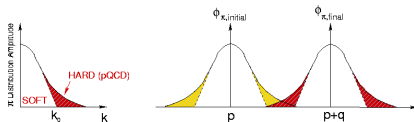
Outline

- Charged Meson Form Factors
- JLab 12 GeV data
- EIC Measurement and Simulation

Charged Meson Form Factors

- Simple $q\bar{q}$ valence structure of mesons makes them an excellent testing ground
- Pion form factor, F_π , is the overlap integral -

$$F_\pi(Q^2) = \int \phi_\pi^*(p) \phi_\pi(p+q) dp$$

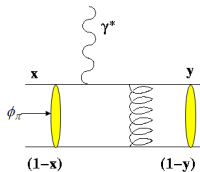


- Meson wave function can be split into ϕ_π^{soft} ($k < k_0$) and ϕ_π^{hard} , the hard tail
 - Can treat ϕ_π^{hard} in pQCD, cannot with ϕ_π^{soft}
- Study of Q^2 dependence of form factor focuses on finding description of hard and soft contributions

The Pion in pQCD 1/2

- At very large Q^2 , F_π can be calculated using pQCD via -

$$F_\pi(Q^2) = \frac{4_F \alpha_s(Q^2)}{Q^2} \left| \sum_{n=0}^{\infty} a_n \left(\log \left(\frac{Q^2}{\Lambda^2} \right) \right)^{-\gamma_n} \right|^2 \left[1 + O \left(\alpha_s(Q^2), \frac{m}{Q} \right) \right]$$



The Pion in pQCD 2/2

- At asymptotically high Q^2 ($Q^2 \rightarrow \infty$), the pion distribution amplitude becomes -

$$\phi_\pi(x) \rightarrow \frac{3f_\pi}{\sqrt{n_c}} x(1-x)$$

- With $f_\pi = 93 \text{ MeV}$, the $\pi^+ \rightarrow \mu^+ \nu$ decay constant
- F_π takes the form -

$$Q^2 F_\pi \rightarrow 16\pi\alpha_s(Q^2)f_\pi^2$$

- This only relies on asymptotic freedom in QCD, i.e. $(\partial\alpha_s/\partial\mu) < 0$ as $\mu \rightarrow \infty$
- $Q^2 F_\pi$ should behave as $\alpha_s(Q^2)$, even for moderately large Q^2
- Pion form factor seems to be the best tool for experimental study of the nature of the quark-gluon coupling constant renormalisation

Eqns - G.P. Lepage, S.J. Brodsky, PLB 87, p359, 1979 | Closing Statement - A.V. Efremov, A.V. Radyushkin PLB 94, p245, 1980

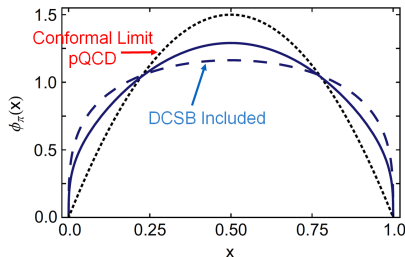
Implications for Pion Structure 1/2

- Previous pQCD derivation used normalisation of F_π based on the conformal limit of the pion's twist 2-PDA -

$$\phi_\pi^{cl}(x) = 6x(1-x)$$

- Gives F_π that are “too small”
- Incorporating the DCSB effects yields Pion PDA -

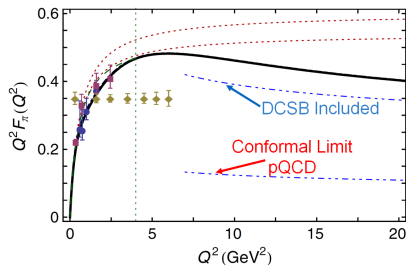
$$\phi_\pi(x) = \frac{8}{\pi} \sqrt{x(1-x)}$$



L. Chang, et al., PRL110(2013) 132001

Implications for Pion Structure 2/2

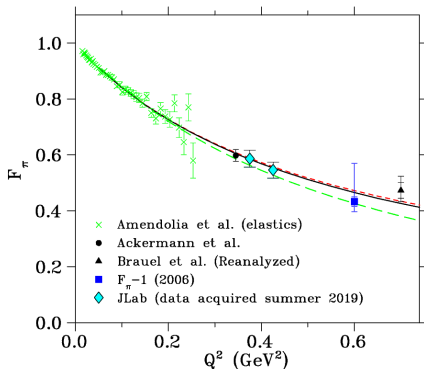
- Using this $\phi_\pi(x)$ in the pQCD expression brings the F_π calculation much closer to the data
- Underestimates the full computation by $\sim 15\%$ for $Q^2 \geq 8 \text{ GeV}^2$



L. Chang, et al., PRL111(2013) 141802

Measurement of F_π - Low Q^2

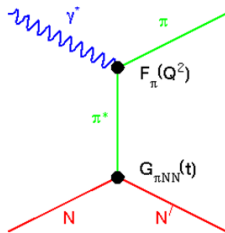
- At low Q^2 , F_π can be measured model independently
 - High energy elastic π^- scattering from atomic electrons in H
- CERN SPS used 300 GeV pions to measure F_π up to $Q^2 = 0.25 \text{ GeV}^2$
- Used data to extract pion charge radius -
 $r_\pi = 0.657 \pm 0.012 \text{ fm}$
- Maximum accessible Q^2 approximately proportional to pion beam energy
 - $Q^2 = 1 \text{ GeV}^2$ requires 1 TeV pion beam (!)



Amendolia, et al., NPB 277(1986) p168, P. Brauel, et al., ZPhysC (1979), p101, H. Ackerman, et al., NPB137 (1978), p294

Measurement of F_π - Larger Q^2

- To access higher Q^2 , must measure F_π indirectly
 - Use the “pion cloud” of the proton via pion electroproduction
 $p(e, e'\pi^+)n$
- At small $-t$, the pion pole process dominates the longitudinal cross section, σ_L
- In the Born term model, F_π^2 appears as -
$$\frac{d\sigma_L}{dt} \propto \frac{-tQ^2}{(t - m_\pi^2)} g^2(t) F_\pi^2(Q^2, t)$$
- Drawbacks of this technique -
 - Isolating σ_L experimentally challenging
 - Theoretical uncertainty in F_π extraction
→ Model dependent



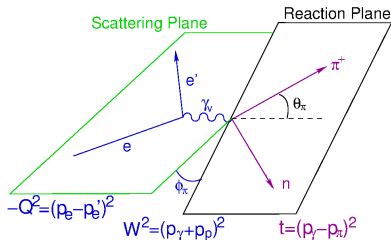
Measurement of F_π at JLab

- The physical cross section for the electroproduction process is given by -

$$2\pi \frac{d^2\sigma}{dtd\phi} = \epsilon \frac{d\sigma_L}{dt} + \frac{d\sigma_T}{dt} + \sqrt{2\epsilon(\epsilon+1)} \frac{d\sigma_{LT}}{dt} \cos\phi + \epsilon \frac{d\sigma_{TT}}{dt} \cos 2\phi,$$

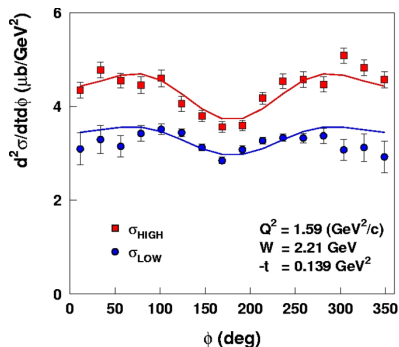
$$\epsilon = \left(1 + 2 \frac{(E_e - E_{e'})^2 + Q^2}{Q^2} \tan^2 \frac{\theta_{e'}}{2} \right)^{-1}$$

- $\epsilon \rightarrow$ Virtual photon polarisation
- L-T separation required to isolate σ_L from σ_T
- Need data at lowest $-t$ possible, σ_L has maximum pole contribution here



Measuring $\frac{d\sigma_L}{dt}$ at JLab

- Rosenbluth separation required to isolate σ_L
 - Fix W , Q^2 and $-t$, measure cross section at two beam energies
 - Carry out simultaneous fit at two different ϵ values to determine interference terms
- Careful control of point-to-point systematics crucial, $1/\Delta\epsilon$ error amplification in σ_L
- Spectrometer acceptance, kinematics and efficiencies must all be carefully studied and understood



T. Horn, et al., PRL 97(2006) 192001

Extracting F_π at JLab

- Only reliable approach for extracting F_π from σ_L is to use a model that incorporates the π^+ production mechanism and the spectator nucleon
- JLab F_π experiments use the VGL Regge model
 - Reliably describes σ_L across a wide kinematic domain
- Ideally, want a better understanding of the model dependence of the result
- There has been considerable recent interest
 - T.K. Choi, K.J. Kong, B.G. Yu, arXiv 1508.00969
 - T. Vrancx, J. Ryckebusch, PRC 89(2014)025203
 - M.M. Kaskulov, U. Mosel, PRD 81(2010)045202
 - S.V. Goloskokov, P.Kroll, EPJC 65(2010)137
- We aim to publish our experimentally measured cross section data so that updated values of F_π can be extracted as the models improve

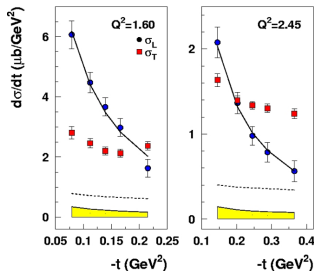
VGL - Vanderhaeghen-Guidal-Laget Model - Vanderhaeghen, Guidal, Laget, PRC 57(1998) 1454

$F_\pi(Q^2)$ from JLab Data

VGL model incorporates π^+ production mechanism and spectator neutron effects

- Feynman propagator - $\frac{1}{t-m_\pi^2}$ replaced by π and ρ Regge propagators
- Represents the exchange of a **series** of particles, compared to a **single** particle
- Free parameters - $\Lambda_\pi, \Lambda_\rho$ - Trajectory cutoff parameters
- **At small $-t$, σ_L only sensitive to F_π**

$$F_\pi = \frac{1}{1 + Q^2/\Lambda_\pi^2}$$



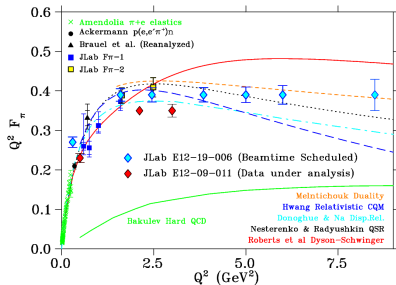
Error bars indicate statistical and random (pt-pt) systematic uncertainties in quadrature. Yellow band indicates the correlated (scale) and partly correlated (t-corr) systematic uncertainties.

$\Lambda_\pi^2 = 0.513, 0.491 \text{ GeV}^2, \Lambda_\rho^2 = 1.7 \text{ GeV}^2$

T. Horn, et al., PRL 97(2006) 192001

Current and Projected JLab F_π Data

- JLab 12 GeV program includes measurements of F_π to higher Q^2
- No other facility worldwide can perform this measurement
- New overlap points at $Q^2 = 1.6, 2.45$ will be closer to pole to constrain $-t_{min}$ dependence
- Check π^+/π^- ratios at modest Q^2 to test t -channel dominance



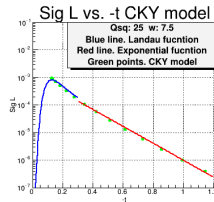
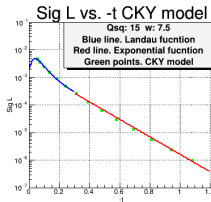
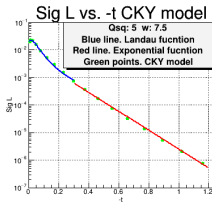
- New low Q^2 point will provide best comparison of the electroproduction extraction of F_π vs elastic $\pi + e$ data

DEMP Studies at the EIC

- Measurements of the $p(e, e'\pi^+n)$ reaction at the EIC have the potential to extend the Q^2 reach of F_π measurements even further
- A challenging measurement however
 - Need good identification of $p(e, e'\pi^+n)$ triple coincidences
 - Conventional L-T separation not possible \rightarrow would need lower than feasible proton energies to access low ϵ
- Utilise new EIC software framework to assess the feasibility of the study with updated design parameters
 - Feed in events generated from a DEMF event generator

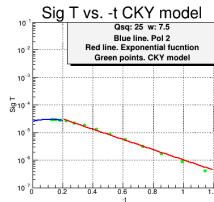
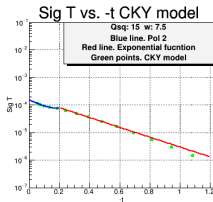
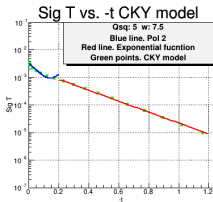
DEMP Event Generator

- Want to examine **exclusive** reactions
 - $p(e, e'\pi^+n)$ **exclusive reaction** is reaction of interest
 $\rightarrow p(e, e'\pi^+)X$ SIDIS events are background
- Generator uses Regge-based $p(e, e'\pi^+)n$ model from T.K. Choi, K.J. Kong and B.G. Yu (CKY) - arXiv 1508.00969
 - MC event generator created by parametrising CKY σ_L, σ_T for $5 < Q^2 < 35$, $2 < W < 10$, $0 < -t < 1.2$



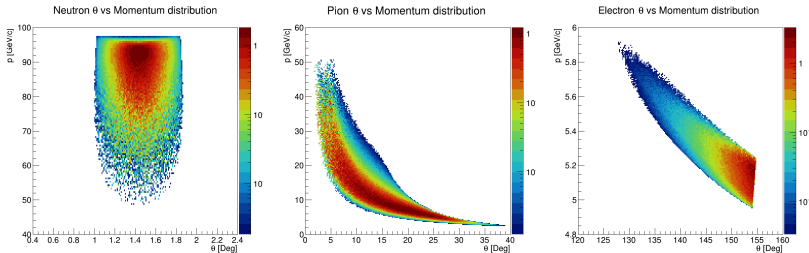
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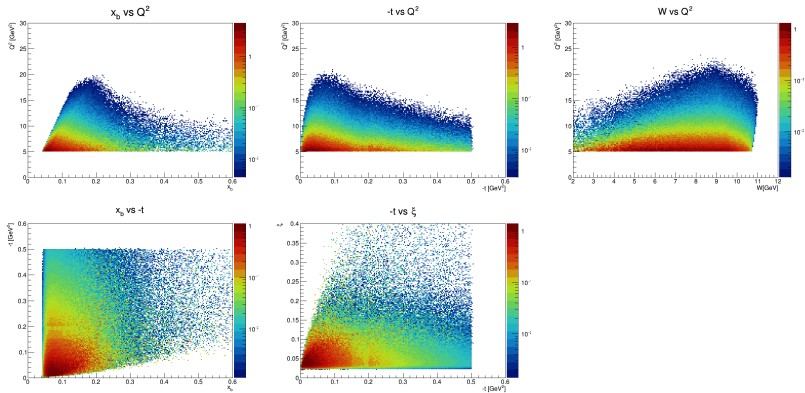
DEMP Acceptance for $-t < 0.5 \text{ GeV}^2$

- $5(e^-)$ on $100(p)$ GeV collisions, 25 mrad crossing angle
- Events weighted by cross section
- No smearing



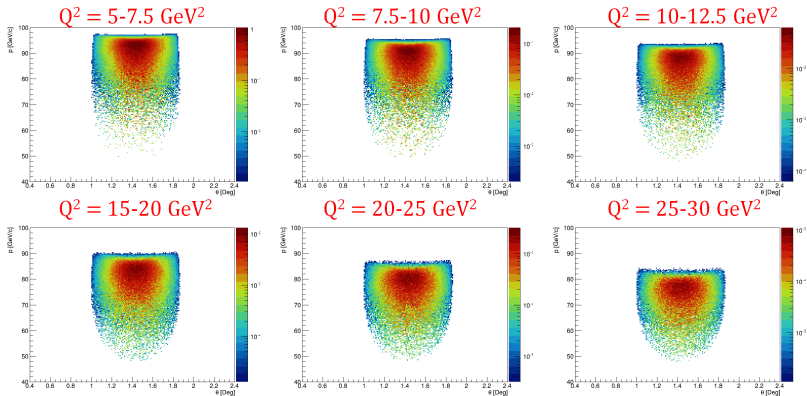
- Neutrons within 0.2° of outgoing proton beam, offset is due to the crossing angle ($25 \text{ mrad} \approx 1.4^\circ$)

DEMP Kinematic Coverage - 5 on 100

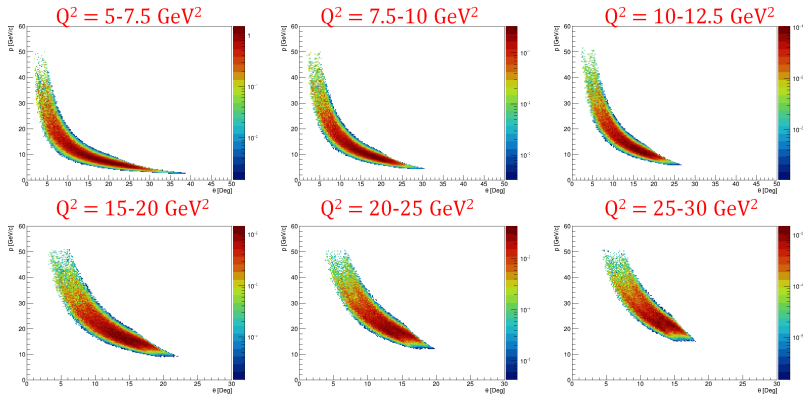


ξ = skewnees

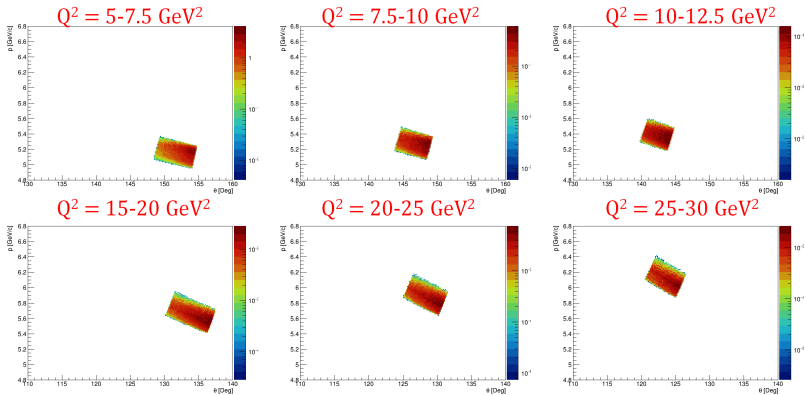
Neutron Acceptance Across Q^2 - 5 on 100



Pion Acceptance Across Q^2 - 5 on 100



Electron Acceptance Across Q^2 - 5 on 100

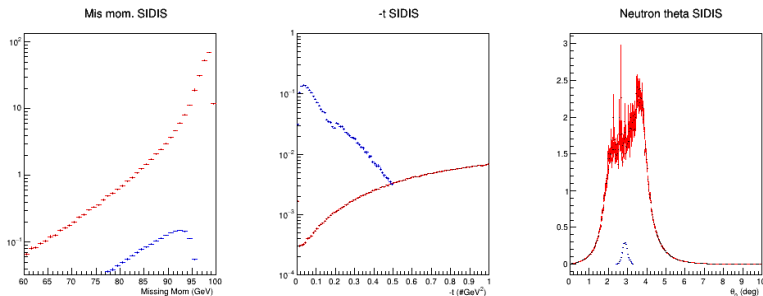


Background Events

- Want to isolate a clean sample of $p(e, e'\pi^+n)$ events by detecting the neutron
- **SIDIS $p(e, e'\pi^+)X$ events a large source of background**
 - Utilised the EIC SIDIS event generator by Duke University to generate SIDIS background events
 - [/work/eic/evgen_DUKE/e5p100 on the JLab farm](#)
- Both the DEMP and SIDIS generators produce LUND format files that can be interpreted within the EIC software container

DEMP vs SIDIS Kinematics

- DEMP events are $e'\pi^+n$ triple coincidence
- SIDIS events are $e'\pi^+$ double coincidence, p_{miss} reconstructed
 - $p_{miss} = |\underline{p}_e + \underline{p}_p - \underline{p}_{e'} - \underline{p}_{\pi^+}|$



- SIDIS events overwhelm foreground exclusive events, *but* distributed over wider momentum range and at larger $-t$
- Note - Plots from earlier study with smearing included

Isolating σ_L from σ_T in an e-p Collider

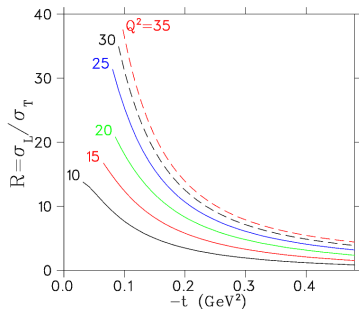
- For a collider -

$$\epsilon = \frac{2(1-y)}{1+(1-y)^2} \quad \text{with} \quad y = \frac{Q^2}{x(s_{tot} - M_N^2)}$$

- y is the fractional energy loss
- **Systematic uncertainties in σ_L magnified by $1/\Delta\epsilon$**
 - Ideally, $\Delta\epsilon > 0.2$
- To access $\epsilon < 0.8$ with a collider, need $y > 0.5$
 - Only accessible at small s_{tot}
 - **Requires low proton energies (~ 10 GeV), luminosity too low**
- Conventional L-T separation not practical, need another way to determine σ_L

σ_L Isolation with a Model

- QCD scaling predicts $\sigma_L \propto Q^{-6}$
and $\sigma_T \propto Q^{-8}$
- At the high Q^2 and W accessible at the EIC, phenomenological models predict $\sigma_L \gg \sigma_T$ at small $-t$
- Can attempt to extract σ_L by using a model to isolate dominant $d\sigma_L/dt$ from measured $d\sigma_{UNS}/dt$
- Critical to confirm the validity of the model used!



Predictions are assuming $\epsilon > 0.9995$ with the kinematic ranges seen earlier

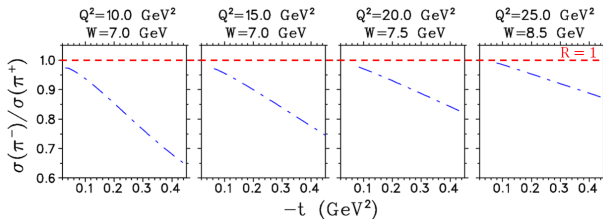
T.Vrancx, J. Ryckebusch, PRC 89(2014)025203

Model Validation via π^-/π^+ ratios

- Measure exclusive $^2H(e, e'\pi^+n)n$ and $^2H(e, e'\pi^-p)p$ in same kinematics as $p(e, e'\pi^+n)$
- π t -channel diagram is purely isovector \rightarrow G-Parity conserved

$$R = \frac{\sigma[n(e, e'\pi^-p)]}{\sigma[p(e, e'\pi^+n)]} = \frac{|A_V - A_S|^2}{|A_V + A_S|^2}$$

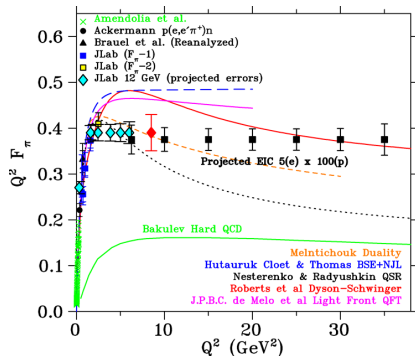
- R will be diluted if σ_T not small or if there are significant non-pole contributions to σ_L
- Compare R to model expectations



EIC Kinematic Reach

Assumptions

- $5(e^-)$ on $100(p)$
- $\int \mathcal{L} = 20 \text{ fb}^{-1} \text{ yr}^{-1}$
- Clean identification of $p(e, e' \pi^+ n)$
- Syst.Unc:
2.5% pt-pt, 12% scale
- $R = \sigma_L / \sigma_T = 0.013 - 0.14$
at lowest $-t$ from VR model
- $\delta R = R \text{ Syst.Unc}$ in model
subtraction to isolate σ_L
- π pole dominance at small
 $-t$ confirmed in $^2\text{H } \pi^+ / \pi^-$
ratios



- Results look promising, but
need further studies and
further energy combinations

Outlook and Future Plans

- Higher Q^2 data on F_π vital for our understanding of hadronic physics
 - Pion properties connected to DCSB
 - F_π is our best hope of observing QCD's transition from confinement-dominated physics to perturbative QCD
- Measurement of F_π at the EIC will be challenging
 - Conventional L-T separation not possible
 - Should be possible to use a model to separate σ_L from the unseparated cross section
 - Can use π^-/π^+ ratio in $e + d$ collisions to validate model
 - Process files through full geant simulation, process other beam energy combinations, contribute to yellow report
- Building on our current event generator, may examine possibility of a Kaon event generator based on VR model
 - Could attempt to measure F_K in a similar manner
 - Further challenges to address for such a study!

Thanks for listening, any questions?



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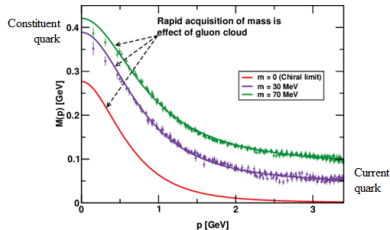
NSERC
CRSNG

S.J.D. Kay, G.M. Huber, Z. Ahmed, Daniele Binosi, Huey-Wen Lin, Timothy Hobbs,
Arun Tadepalli, Rachel Montgomery, Paul Reimer, David Richards, Rik Yoshida, Craig Roberts, Thia Keppel,
John Arrington, Lei Chang, Ian L. Pegg, Jorge Segovia, Carlos Ayerbe Gayoso, Wenliang Li, Yulia Furletova,
Dmitry Romanov, Markus Diefenthaler, Richard Trotta, Tanja Horn, Rolf Ent, Tobias Frederico

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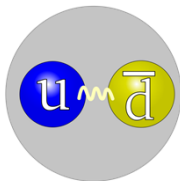
Recent Theoretical Advances

- Have a much better understanding of how **D**ynamical **C**hiral **S**ymmetry **B**reaking (**DCSB**) generates hadron mass
- Evolution of the current-quark of pQCD into constituent quark was observed as its momentum becomes smaller
- The constituent quark mass arises from a cloud of low momentum gluons attaching themselves to the current quark
- Non-perturbative effect that generates a quark mass from nothing, occurs in even in the chiral ($m = 0$) limit

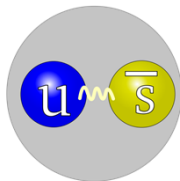


M.S. Bhagwat, et al., PRC 68(2003) 015203, L. Chang, et al., Chin.J.Phys. 49(2011)955

A 2nd Test Case - The Charged Kaon



π^+



K^+

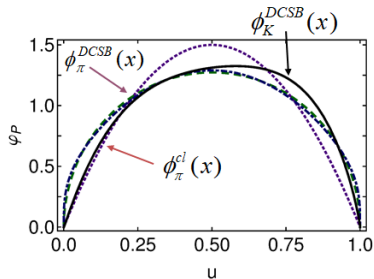
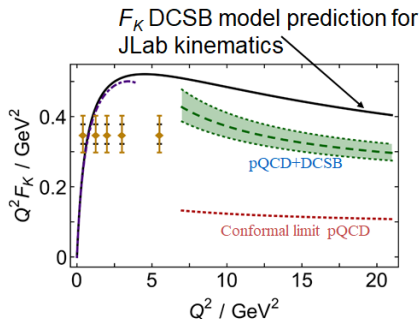
- In the hard scattering limit, pCQD predicts F_π and F_K will behave similarly -

$$\frac{F_K(Q^2)}{F_\pi(Q^2)} \rightarrow \frac{f_K^2}{f_\pi^2}$$

- Should compare the magnitude and Q^2 dependences of both form factors

Effects of DCSB on K^+ Properties

- K^+ PDA is also broad, concave and asymmetric
- Heavier s quark carries more bound state momentum than the u quark, shift is less than one might expect based on the difference in current quark masses.



C. Shi, et al., PRD 92 (2015) 014035, F. Guo, et al., PRD 96(2017) 034024 (Full calculation)

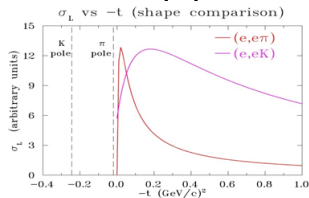
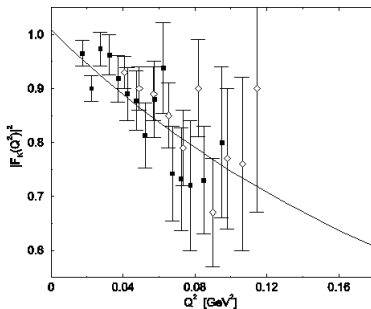
F_K Measurement at JLab

- Similar to F_π , elastic K^+ scattering from electrons used to determine F_K at low Q^2
- Can “kaon cloud” of the proton be used in the same way as the pion to extract F_K from electroproduction?
- Kaon pole further from kinematically allowed region

$$\frac{d\sigma_L}{dt} \propto \frac{-tQ^2}{(t - m_K^2)} g_K^2(T) F_K^2(Q^2, t)$$

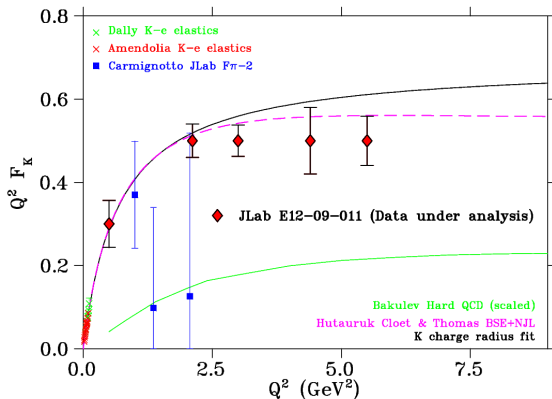
- Issues are being explored and tested in JLab E12-09-011

Amendolia, et al., PLB178(1986)435



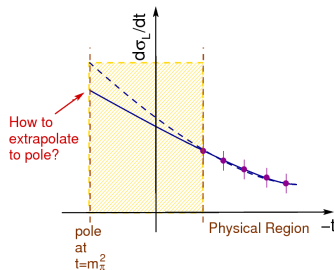
F_K Measurement at JLab - Projections

- Points with projected errors shown below
- Data has all been acquired and analysis is in progress
- y positioning of points arbitrary



Chew-Low Method to determine F_π

- $p(e, e'\pi^+)n$ data obtained away from $t = m_\pi^2$ pole
 - “Chew Low” extrapolation method - must know analytical dependence of $d\sigma_L/dt$ in unphysical region
 - Extrapolation method last used in 1972 by Devenish and Lyth
 - Very large systematic uncertainties
 - Failed to produce a reliable result
 - Different polynomial fits equally likely in physical region
 - Form factor values divergent when extrapolated
- We do not use the Chew-Low method

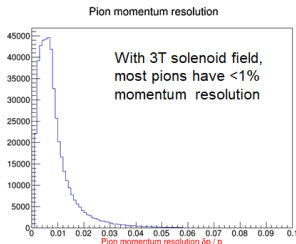
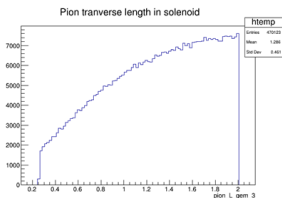
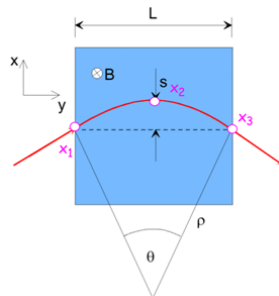


Old Momentum Resolution Estimate

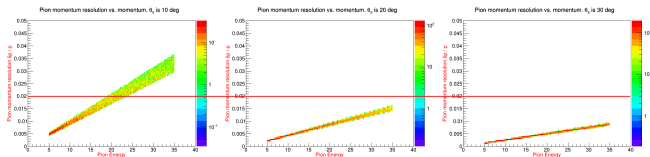
- Intrinsic momentum resolution from n equidistant measurements

$$\frac{\delta p}{p} = \frac{p}{0.3B} \frac{\sigma_{r\phi}}{L^2} \sqrt{\frac{720}{n+4}}$$

- R. L. Gluckstern, NIM24(1963), p381
- B = central field (T), $\sigma_{r\phi}$ = position resolution (m), L = length of transverse path through field (m), N = number of measurements
- Assumed $n = 5$, $B = 3$ T



Old π Momentum Resolution with 3 T Solenoid



- Pion momentum resolution suffers when the pion is emitted at a shallow angle to the solenoidal field
- To simplify the MC study, assumed $\delta p/p = 2\%$ for all angles, for both pion and electron
 - Typical π^+ angles: 7 – 30°
 - Typical e^- angles: 25 – 45°