Structure of the Pion and Kaon from First-Principles Calculation in Lattice QCD

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Workshop on Pion and Kaon Structure Functions at the EIC

Outline

- The challenges and opportunities for PDFs in Lattice QCD
- LaMET, quasi-PDFs, pseudo-PDFs and Good Lattice Cross Sections (LCS)
- Valence Quark PDF
- Summary

HadStruct Collaboration

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PDFs from Lattice QCD

- First Challenge:
  - Euclidean lattice precludes calculation of light-cone/time-separated correlation functions

$$q(x,\mu) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P \mid \bar{\psi}(\xi^-)\gamma^+ e^{-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)} \psi(0) \mid P \rangle$$

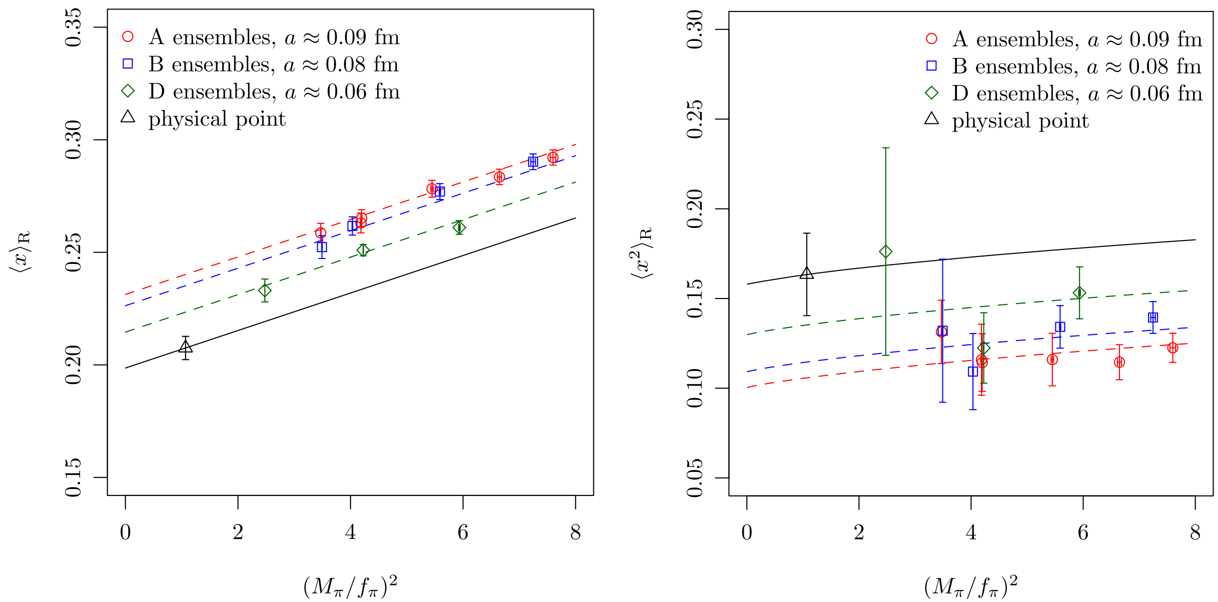
So…. …Use *Operator-Product-Expansion to formulate in terms of Mellin Moments* with respect to Bjorken x.

$$\longrightarrow \langle P \mid \bar{\psi}\gamma_{\mu_1}(\gamma_5)D_{\mu_2}\ldots D_{\mu_n}\psi \mid P\rangle \to P_{\mu_1}\ldots P_{\mu_n} a^{(n)}$$

Matrix elements of local operators that *can* be computed in Euclidean space

Moments of Pion PDF

ETM Collaboration, M.Oehm et al., PRD99, 014508 (2019)



Limited to first two moments - *limited knowledge of x-dependence*

- Second Challenge:
  - Discretised lattice: power-divergent mixing for higher moments

Moment Methods

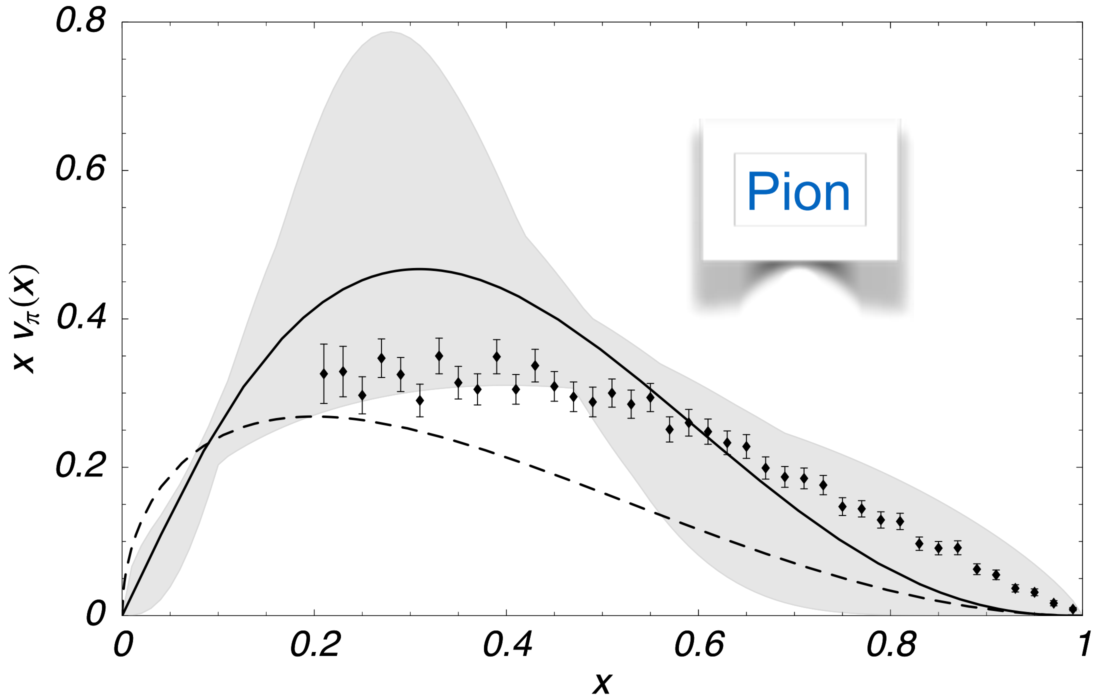
- Extended operators: Z.Davoudi and M. Savage, PRD 86,054505 (2012)
- Valence heavy quark: W.Detmold and W.Lin, PRD73, 014501 (2006)

Constraining PDFs from Moments

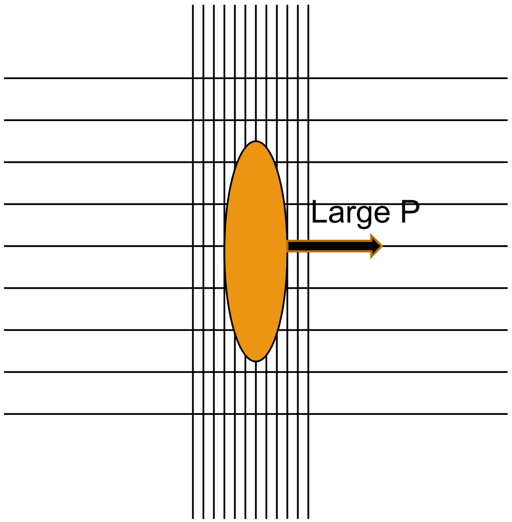
Detmold, Melnitchouk, Thomas, Mod.Phys.Lett.A18, 2681 (2003); Phys.Rev.D68, 034025 (2003)

Take-away messages:

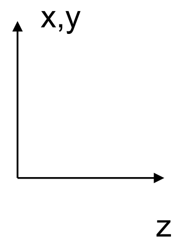
- *Need to assume parametrization*
- *More moments would be better..*



Solution….



Large-Momentum Effective Theory (LaMET)



“Equal time” correlator

X. Ji, Phys. Rev. Lett. 110, 262002 (2013).
X. Ji, J. Zhang, and Y. Zhao, Phys. Rev. Lett. 111, 112002 (2013).
J. W. Qiu and Y. Q. Ma, arXiv:1404.686.

$$q(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P \mid \bar{\psi}(z)\gamma^z e^{-ig\int_0^z dz'\, A^z(z')} \psi(0) \mid P > + \mathcal{O}((\Lambda^2/(P^z)^2), M^2/(P^z)^2))$$

See Huey-Wen Lin, Thursday

$$q(x,\mu^2,P^z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y,\mu^2) + \mathcal{O}(\Lambda^2/(P^z)^2, M^2/(P^z)^2)$$

Need for high momentum

Pseudo-PDFs

- Pseudo-PDF (pPDF) recognizing generalization of PDFs in terms of *Ioffe Time.* $\nu = p \cdot z$

A.Radyushkin, Phys. Rev. D 96, 034025 (2017)

B.Ioffe, PL39B, 123 (1969); V.Braun *et al*, PRD51, 6036 (1995)

$$M^\alpha(p, z) = \langle p \mid \bar{\psi}\gamma^\alpha U(z;0)\psi(0) \mid p\rangle$$

$p = (p^+, m^2/2p^+, 0_T)$ $\quad z = (0, z_-, 0_T)$

Ioffe-Time Distribution

$$\Downarrow$$

$$M^\alpha(z, p) = 2p^\alpha \mathcal{M}(\nu, z^2) + 2z^\alpha \mathcal{N}(\nu, z^2)$$

Ioffe-time pseudo-Distribution (pseudo-ITD) generalization to *space-like z*

Lattice "building blocks" *that of quasi-PDF approach.*

$\Downarrow$ *Lorentz covariant*

$$\mathcal{M}(\nu, z^2) = \int_{-1}^{1} dx\, e^{i\nu x}\mathcal{P}(x, z^2) \longleftarrow \textit{pseudo-PDF}$$

$$\Downarrow$$

$$f(x) = \mathcal{P}(x, 0) \underset{z_3^2 \to 0}{=} \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu e^{-i\nu x}\mathcal{M}(\nu, -z_3^2)$$

"Good Lattice Cross Sections"

$$\sigma_n(\nu, \xi^2, P^2) = \langle P \mid T\{\mathcal{O}_n(\xi)\} \mid P\rangle$$

Ma and Qiu, Phys. Rev. Lett. 120 022003

Expressed in coordinate space

Short distance scale

where

$$\sigma_n(\nu, \xi^2, P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x, \mu^2) K_n^a(x\nu, \xi^2, x^2P^2, \mu^2) + \mathcal{O}(\xi^2\Lambda^2_{\rm QCD})$$

Calculated in LQCD

Parton Distribution function

Calculated in perturbation theory ("process dependent")

$$\mathcal{O}(\xi) = \bar{\psi}(0)\Gamma W(0, 0+\xi)\psi(\xi)$$

Encompasses qPDF/pPDF

Gauge-Invariant Currents

$$\begin{aligned}\mathcal{O}_S(\xi) &= \xi^4 Z_S^2[\bar{\psi}_q\psi_q](\xi)[\bar{\psi}_q\psi](0)\\ \mathcal{O}_{V'}(\xi) &= \xi^2 Z_{V'}^2[\bar{\psi}_q\xi\cdot\gamma\psi_{q'}](\xi)[\bar{\psi}_{q'}\xi\cdot\gamma\psi](0)\end{aligned}$$

Flavor-changing

c.f. pion QDA, Bali et al., Phys. Rev. D 98, 094507 (2018)

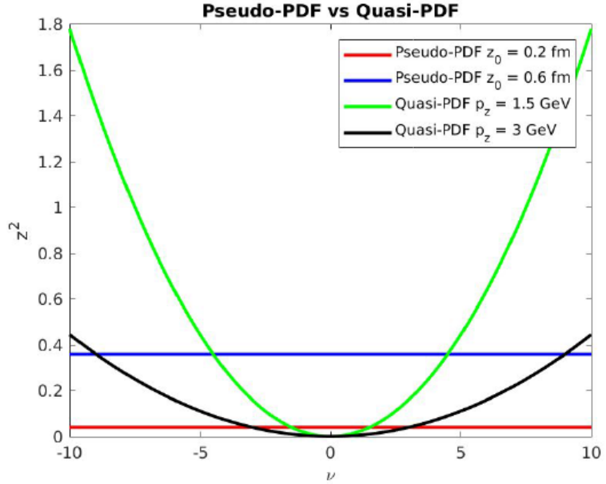
Quasi- vs Pseudo- vs LCS

- All integrals of Ioffe-Time Distribution Function
- Should yield same PDF after matching and systematic controls

Quasi-PDF

$$\mathcal{Q}(x, p_3^2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu\, e^{-i\nu x}\mathcal{M}(\nu, -\nu^2/p_3^2)$$

$$\mathcal{P}(x, -z_3^2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu\, e^{-i\nu x}\mathcal{M}(\nu, -z_3^2)$$



Pseudo-PDF and LCS

For pPDF + LCS, z sets short-distance scale.

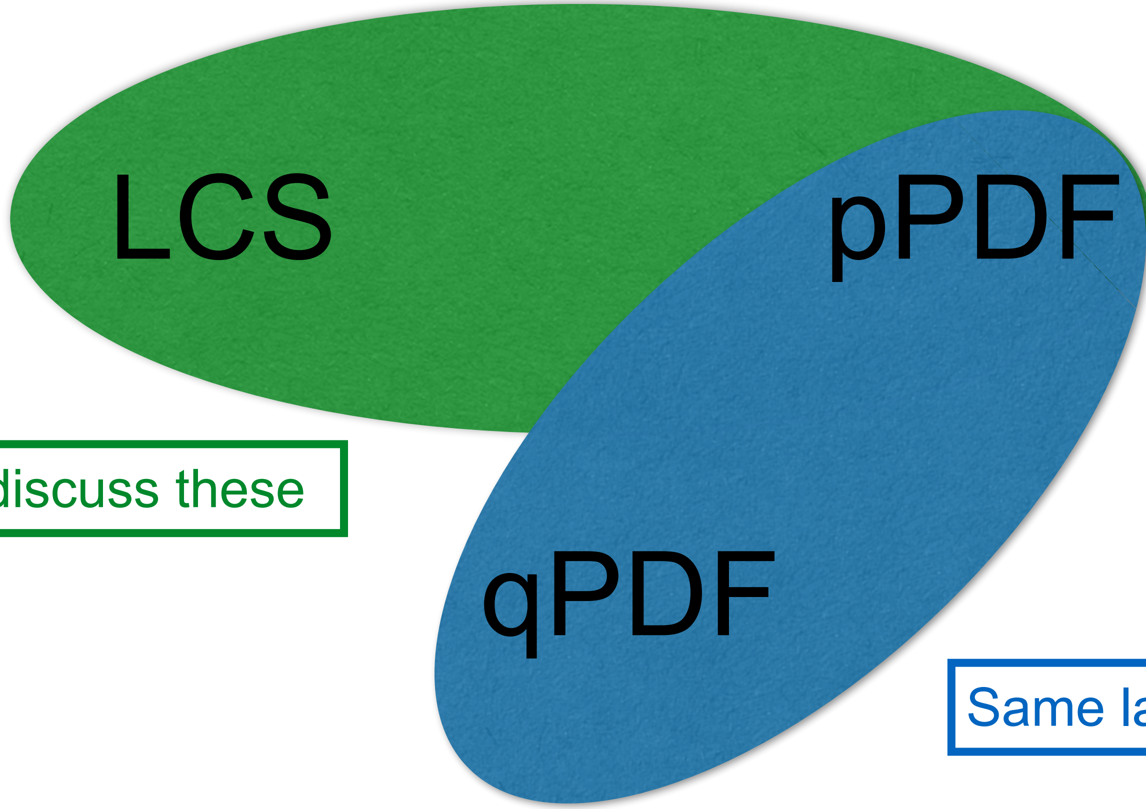
$$z << \frac{1}{\Lambda_{\mathrm{QCD}}}$$

N.B. All approaches require large momentum - but for pPDF and LCS to ensure range in Ioffe time to solve *inverse problem*.

$P \longrightarrow \sqrt{s}$ *Collision energy*

$z \longrightarrow \frac{1}{Q}$ *Hard Probe*

Analogous matching to light-cone PDFs



Colin Egerer, Friday

I will discuss these

Same lattice building blocks

Why the Pion and Kaon?

For lattice practitioners

- Pion less computationally demanding that nucleon?
  - *Larger signal-to-noise ratio*

$$C(t,\vec{p}) \equiv \sum_{\vec{x}} \langle 0 \mid \mathcal{O}(t,\vec{x})\mathcal{O}^\dagger(0,0) \mid 0\rangle e^{-i\vec{p}\cdot\vec{x}} \to e^{-E(\vec{p})t}$$

$$C_{\sqrt{\sigma^2}}(t,\vec{p}) \to \begin{cases} e^{-m_\pi t} & \text{Mesons} \\ e^{-(3m_\pi/2)t} & \text{Baryons} \end{cases}$$

- Important constraint on systematic uncertainty is understanding renormalization
  - *Operator renormalization "independent" of external states*
- Several different calculations using the different approaches
  - *LCS approach straightforward for mesons, challenging for baryons*

For experiment and phenomenology

- We are *solving* QCD
- We are studying a free, isolated pion

For all

- Pion and kaon encapsulate many of the mysteries of how QCD works

Good Lattice Cross Section

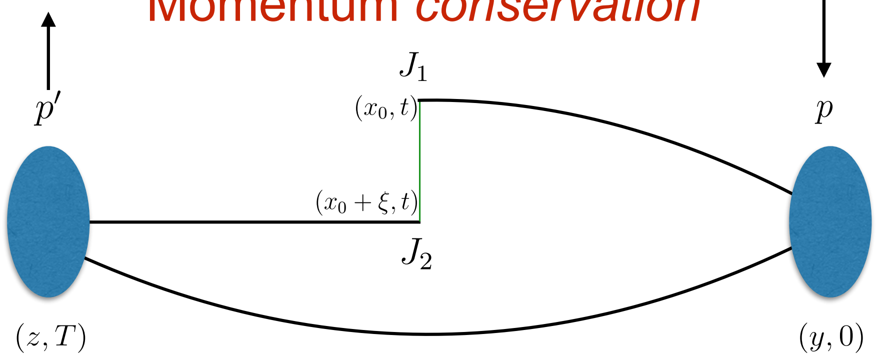
Sufian *et al.*, Phys. Rev. D 99, 074507 (2019); arXiv:2001.04960

Sequential-Source Approach

Momentum *conservation*

Momentum projection

Momentum projection



Process, i.e. current, dependent

$$\frac{1}{2}[\sigma^{\mu\nu}_{V,A}(\xi,p) + \sigma^{\mu\nu}_{A,V}(\xi,p)]$$

$$\boxed{\equiv \epsilon^{\mu\nu\alpha\beta}\xi_\alpha p_\beta T_1(\nu,\xi^2) + (p^\mu\xi^\nu - \xi^\mu p^\nu)T_2(\nu,\xi^2)}$$

Perturbative kernel:

N.B. We're inconsistent $\omega \leftrightarrow \nu$!

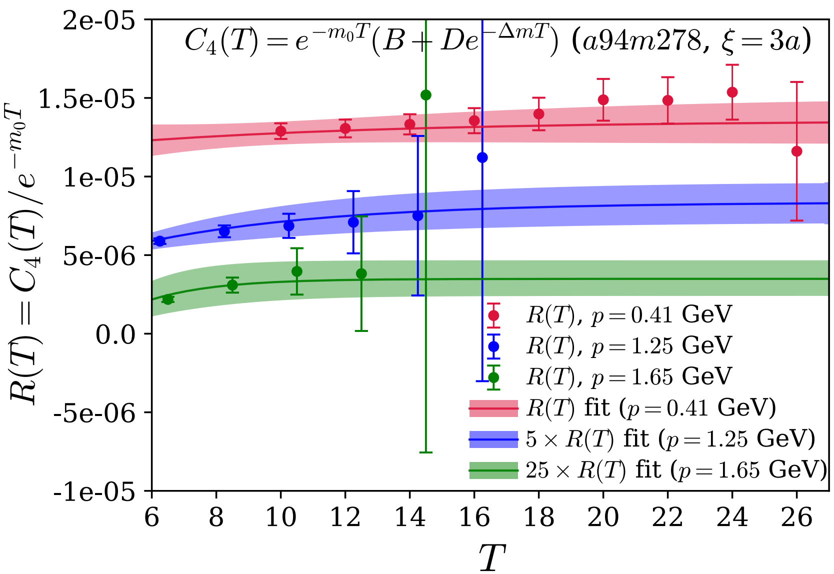
$$\widetilde{\sigma}^{q(1)}_{VA}(\widetilde{\omega},q^2) = \int_0^1 \frac{dx}{x}\,\widetilde{K}^{(1)}(x\widetilde{\omega},q^2,\mu^2)f^{(0)}_{q_v/q}(x,\mu^2) + \int_0^1 \frac{dx}{x}\,\widetilde{K}^{(0)}(x\widetilde{\omega},q^2,\mu^2)f^{(1)}_{q_v/q}(x,\mu^2).$$

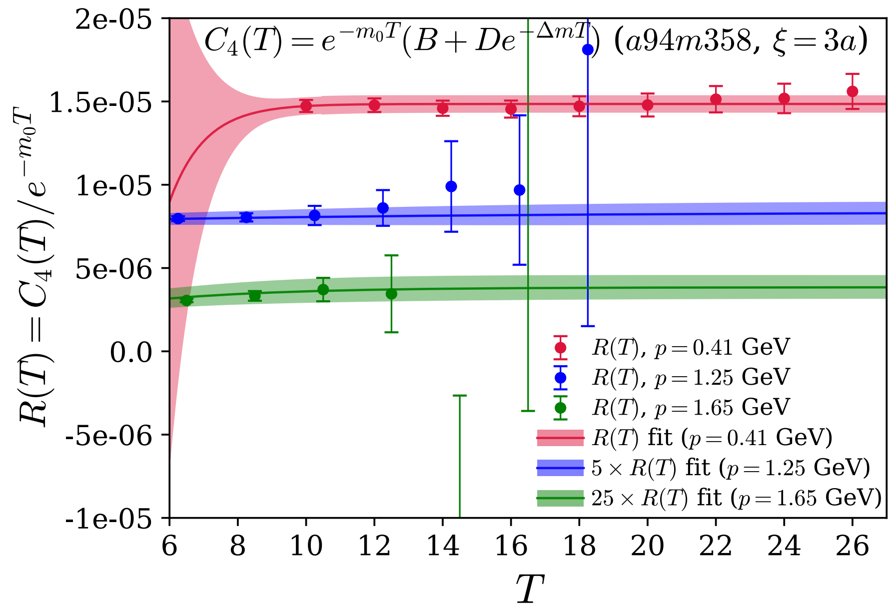
Y-Q Ma

Good Lattice Cross Section

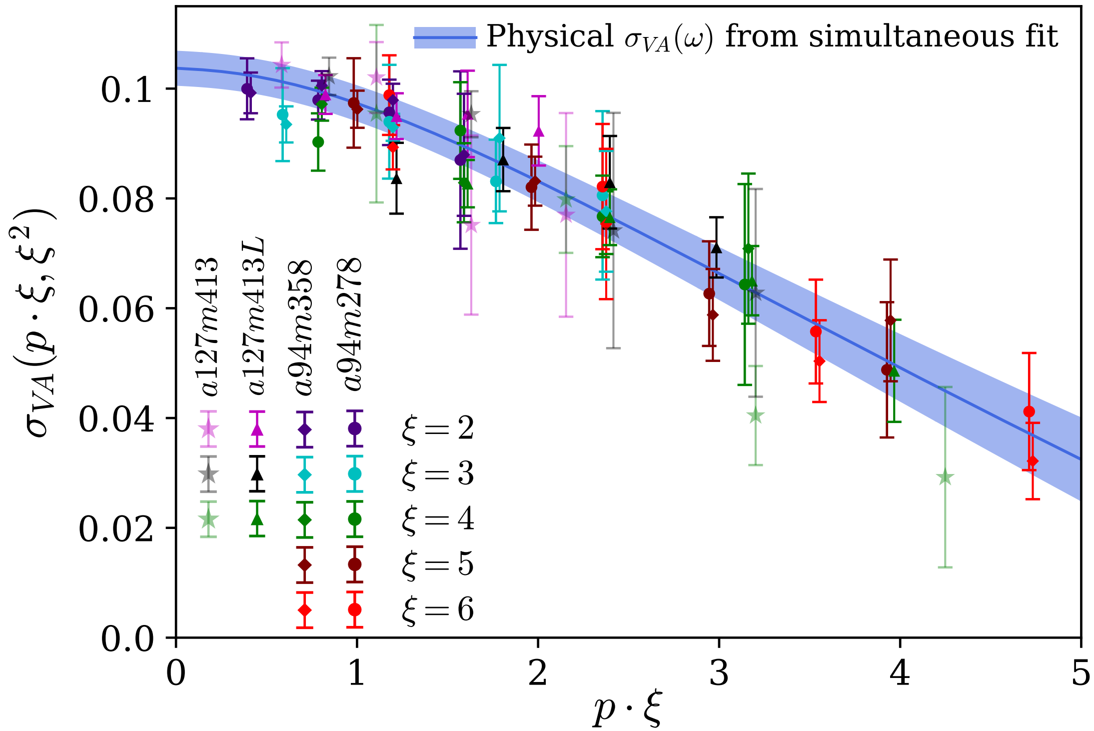
2 + 1 flavor Wilson-clover lattices

ID	a (fm)	m_π (MeV)	$L^3 \times N_t$	N_{cfg}
$a127m413$	0.127(2)	413(4)	$24^3 \times 64$	2124
$a127m413L$	0.127(2)	413(5)	$32^3 \times 96$	490
$a94m358$	0.094(1)	358(3)	$32^3 \times 64$	417
$a94m278$	0.094(1)	278(4)	$32^3 \times 64$	503





Lattice Cross Sections



"Z-expansion fit"

$$\sigma_{VA}(\omega,\xi^2) = \sum_{k=0}^{k_{\max}=4} \lambda_k \tau^k + b_1 m_\pi + b_2 a + b_3 \xi^2 + b_4 a^2 p^2 + b_5 e^{-m_\pi(L-\xi)}$$

$$\tau = \frac{\sqrt{\omega_{\text{cut}}+\omega} - \sqrt{\omega_{\text{cut}}}}{\sqrt{\omega_{\text{cut}}+\omega} + \sqrt{\omega_{\text{cut}}}}$$

Inverse problem: extract PDF

"Inverse Problem" - ill-posed inverse Fourier transform.

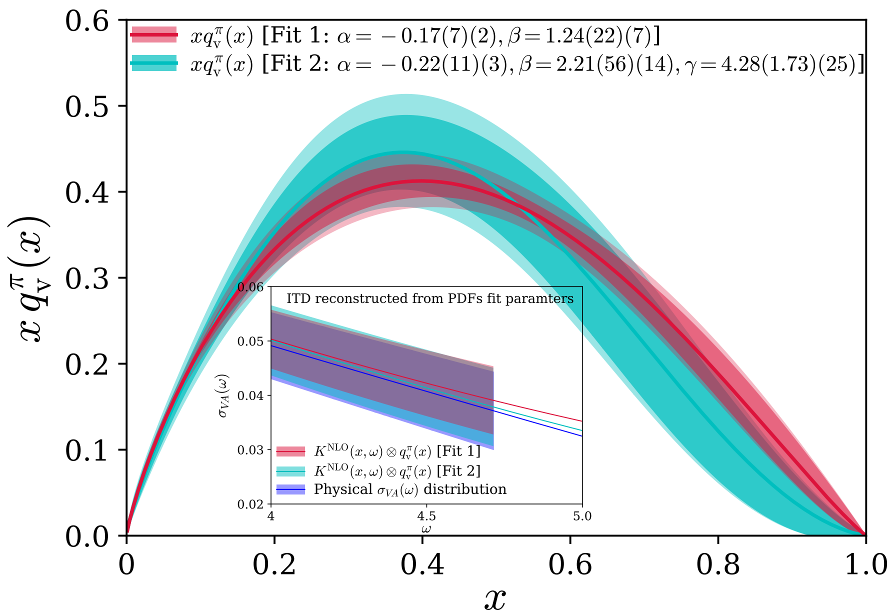
$$\sigma_n(\nu,\xi^2,P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x,\mu^2) K_n^a(x\nu,\xi^2,x^2P^2,\mu^2) + \mathcal{O}(\xi^2\Lambda_{\rm QCD}^2)$$

Calculate on Lattice

Extract PDF?

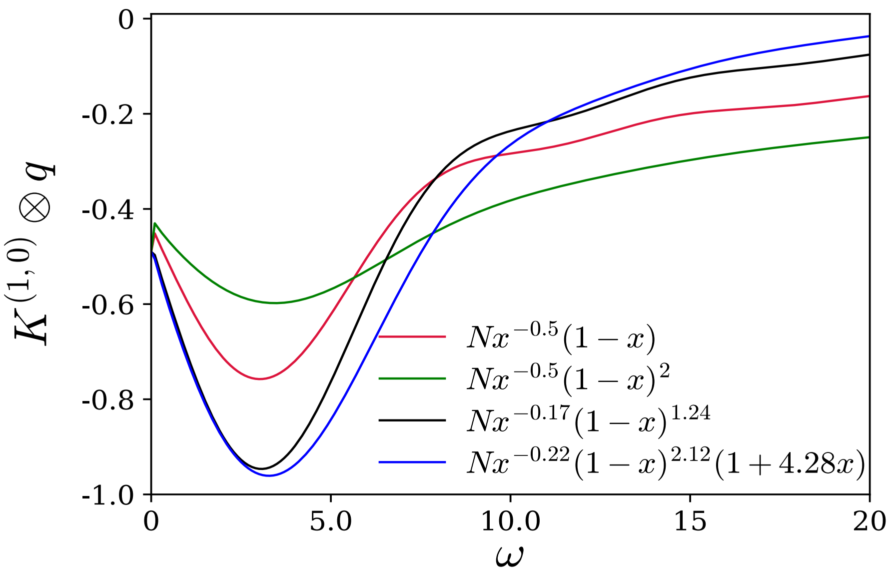
Calculate in PQCD

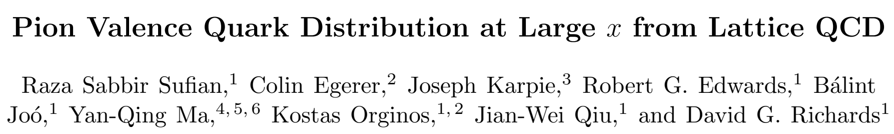
Similar challenge to global fitting community!



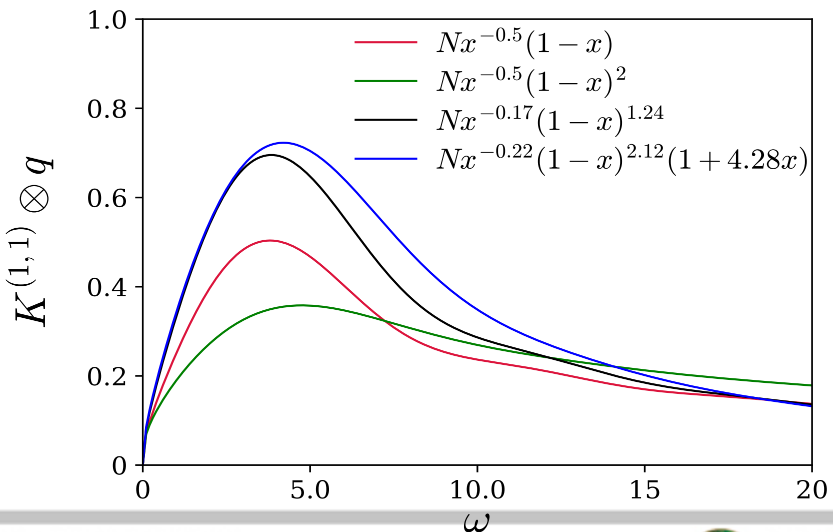
$$q_v^\pi(x) = \frac{x^\alpha(1-x)^\beta(1+\gamma x)}{B(\alpha+1,\beta+1)+\gamma B(\alpha+2,\beta+1)}$$

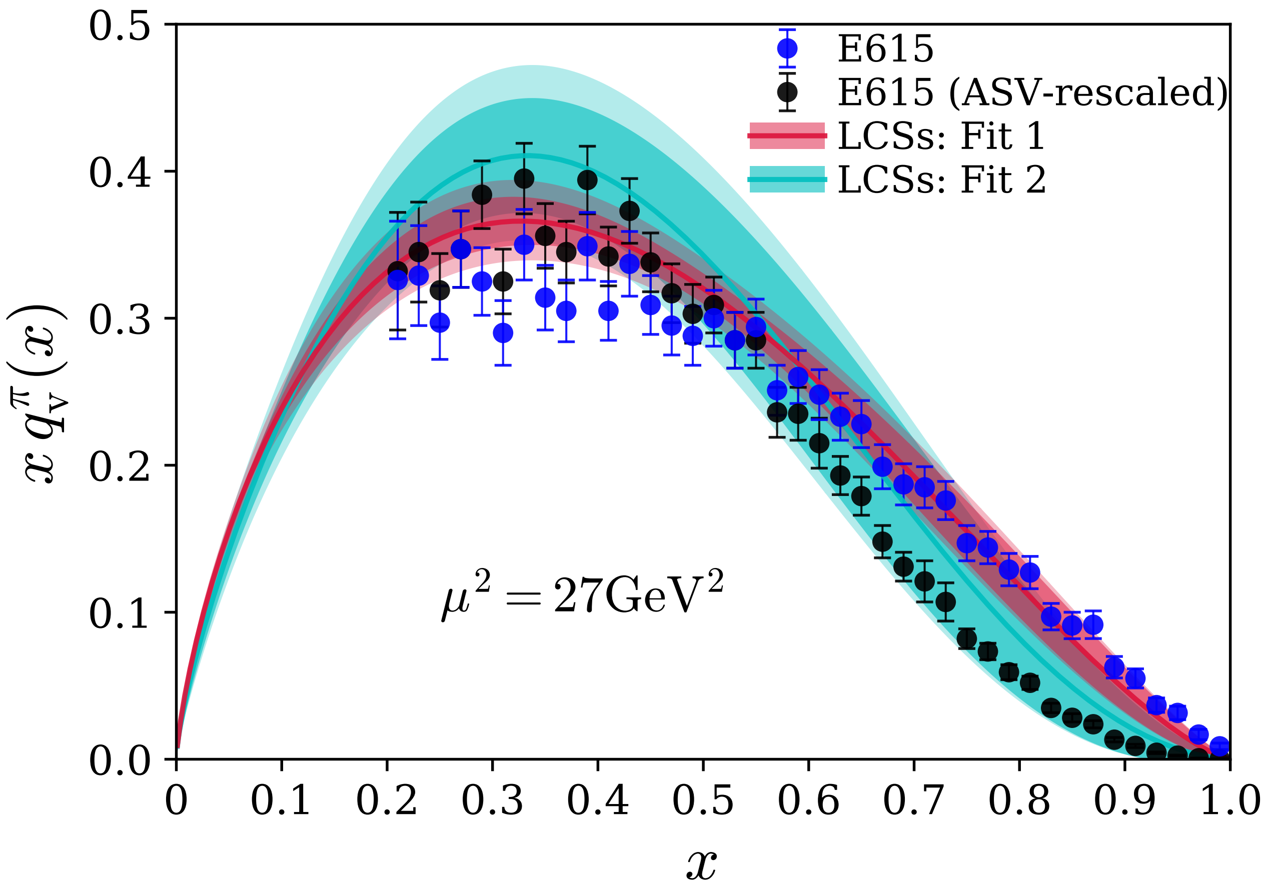
NLO term well-controlled





arXiv:2001.04960



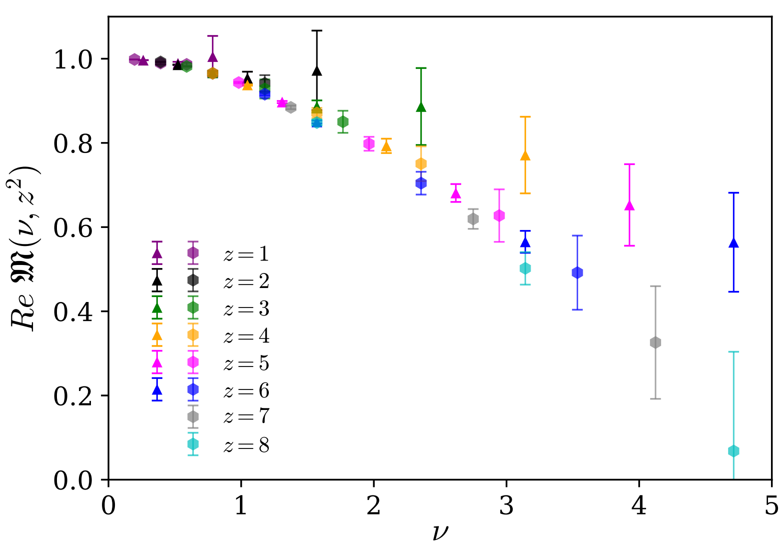


Slightly favors softer [$(1-x)^2$] fit → *need for finer resolution in Ioffe time..*

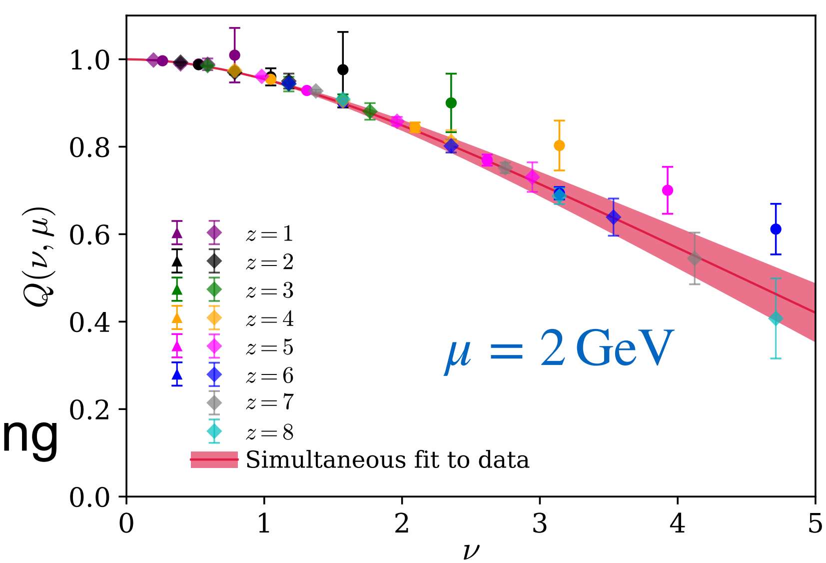
Pseudo-PDF Approach

ID	a (fm)	m_π (MeV)	β	am_l	am_s	$L^3 \times N_t$	N_{cfg}
$a127m415$	0.127(2)	415(23)	6.1	-0.280	-0.245	$24^3 \times 64$	2147
$a127m415L$	0.127(2)	415(23)	6.1	-0.280	-0.245	$32^3 \times 96$	2560

Same ensemble as LCS



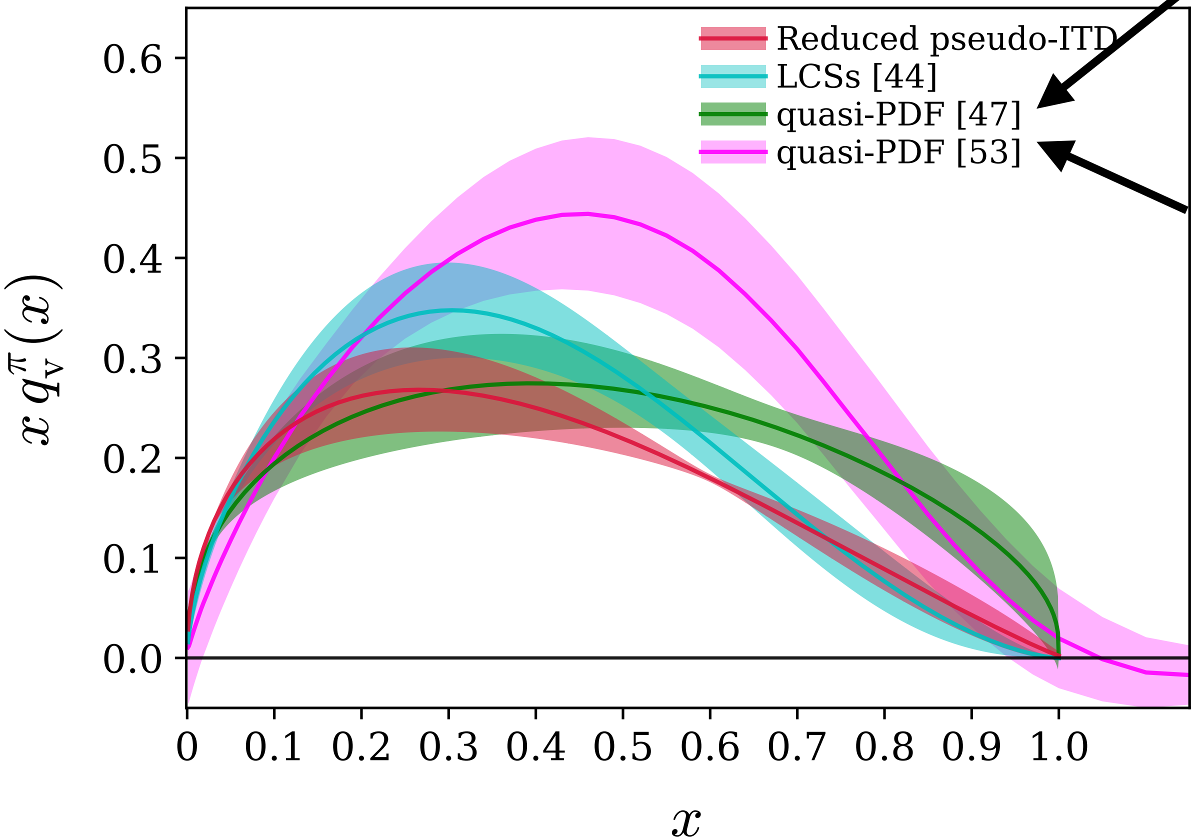
After Matching



B.Joó et al., Phys. Rev. D 100, 114512 (2019).

Pion pPDF - III

T.Izubuchi *et al.*, Phys. Rev. D 100, 034516



J-H Zhang *et al.*, Phys. Rev. D **100**, 034505

Moments

J Karpie, K Orginos, S Zafeiropoulos, arXiv:1807.10933

- Can obviate need for inverse through computation of moments

$$\mathfrak{M}(\nu, z^2) = \sum_{n=0}^{\infty} i^n \frac{\nu^n}{n!} a_n(\mu^2) K_n(\mu^2 z^2) + \mathcal{O}(z^2)$$

Mellin moments of PDF

Matching coefficient

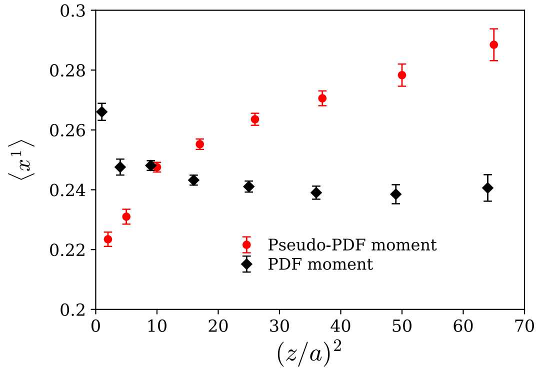
First “dedicated” lattice calculation of pseudo-PDFs - K. Orginos et al., PRD96 (2017), 094503

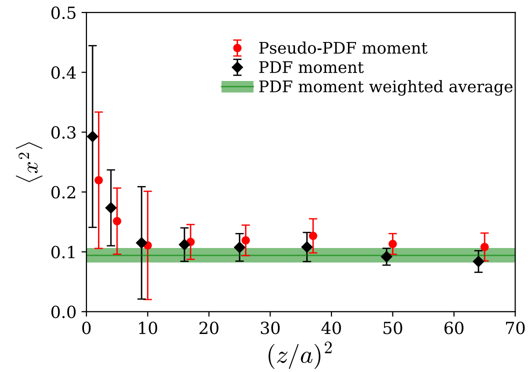
c.f. TMD calculation in Ioffe time with straight link - B. Musch et al., PRD83:094507 (2011)

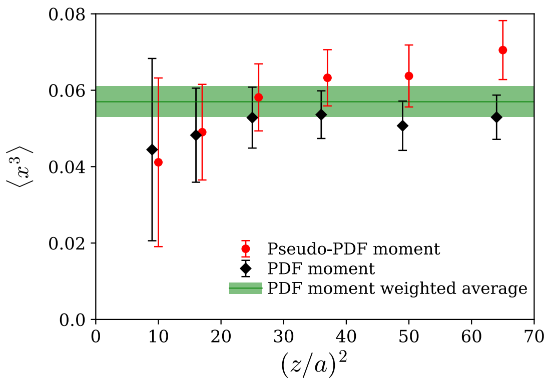
Pseudo-PDF Moments of Pion PDF

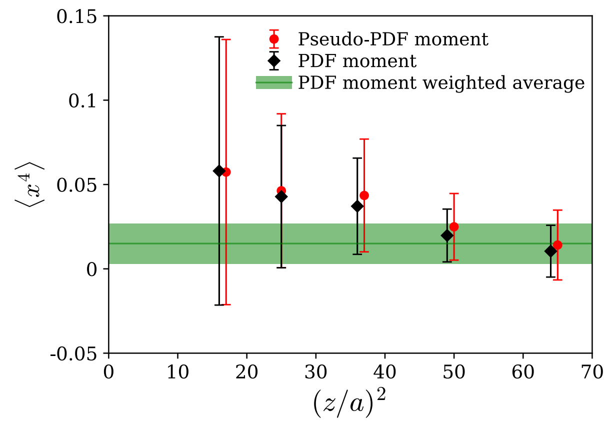
c.f. Shugert et al., arXiv:2001.11650

B.Joó et al., Phys. Rev. D 100, 114512 (2019).









Different systematics to direct calculation of local matrix elements

Going Forward…

- We have seen the importance of gluon contributions
  - Extend LCS to include gluon currents $j_{\mu\nu} = F_{\mu\rho}F^{\rho}_{\nu}$
  - Gluon pPDF: A. Balitsky *et al.*, JLAB-THY-19-3087

↓

Calculation in progress (T.Khan *et al*) in *distillation framework* - see C.Egerer

↓

Requires calculation of disconnected contributions

Kaon in progress - computationally doubles cost of calculation

Summary

- Revolution in the study of x-dependent measures of hadron structure
  - Impact global fitting community?
  - First-Principles calculation
- Pseudo-PDF/GLCS approach has a well-defined short-distance scale: factorize short-distance physics from perturbative scale.
- Solution of inverse problem: common to all attempts to extract PDFs. Appeal to global fitting community
- To control systematics
  - fine lattices - to ensure in perturbative regime
  - large momenta - to provide range in Ioffe time
  - Combined pPDF/LCS - further control over systematics
- New areas:
  - GPD's in pseudo-PDF approach: A.Radyushkin, Phys. Rev. D 100, 116011 (2019).
  - Nucleon LCS

Colin Egerer, Friday