# Asymmetries in Pion-Induced Drell-Yan with Polarized Protons

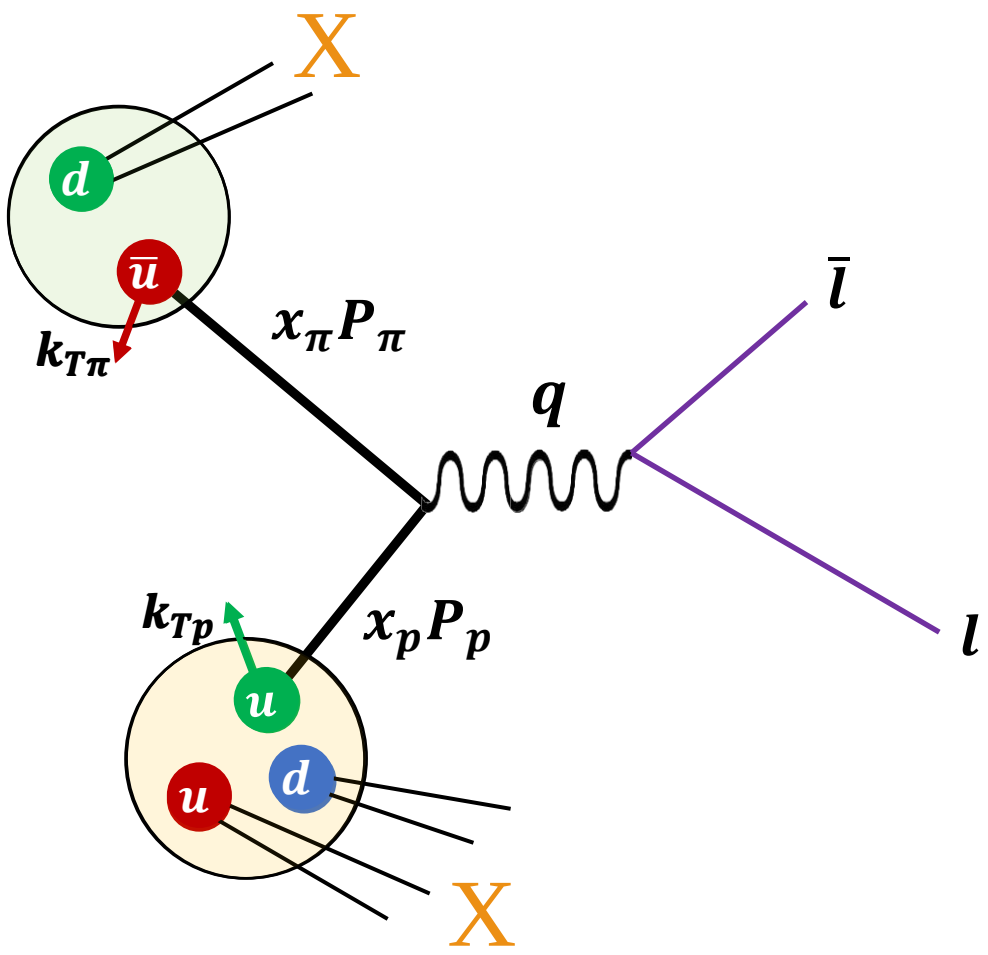
Center for Frontiers in Nuclear Science
Workshop series

$\pi$ & $K$ Structure Functions at the EIC

**Saman Bastami** *University of Connecticut*

Leonard Gamberg, Bakur Parsamyan, Barbara Pasquini, Alexei Prokudin, Peter Schweitzer

arXiv:2005.14322v1 [hep-ph]




**Motivations:**

- Full calculation of pion-DY asymmetries
- Interpret COMPASS data on pion-DY
- Test CQMs with COMPASS and predict for future

We work in TMD factorization framework and at tree level

$$F \propto \int \sum_a \sigma_H \,.\, f_\pi^{\bar{a}}(x_\pi, k_{T\pi}) \,.\, f_p^a(x_p, k_{Tp})$$

In Collins-Soper frame, the cross section is parameterized by six structure functions S. Arnold et al 0809.2262v2

$$F_{UU}^{1} = \mathcal{C}[\, f_{1,\pi}^{\bar{a}} \;, f_{1,p}^{a} \,]$$

$$F_{UT}^{\sin\phi_S} = \mathcal{C}[\omega\, f_{1,\pi}^{\bar{a}}, f_{1T,p}^{\perp a} \,]$$

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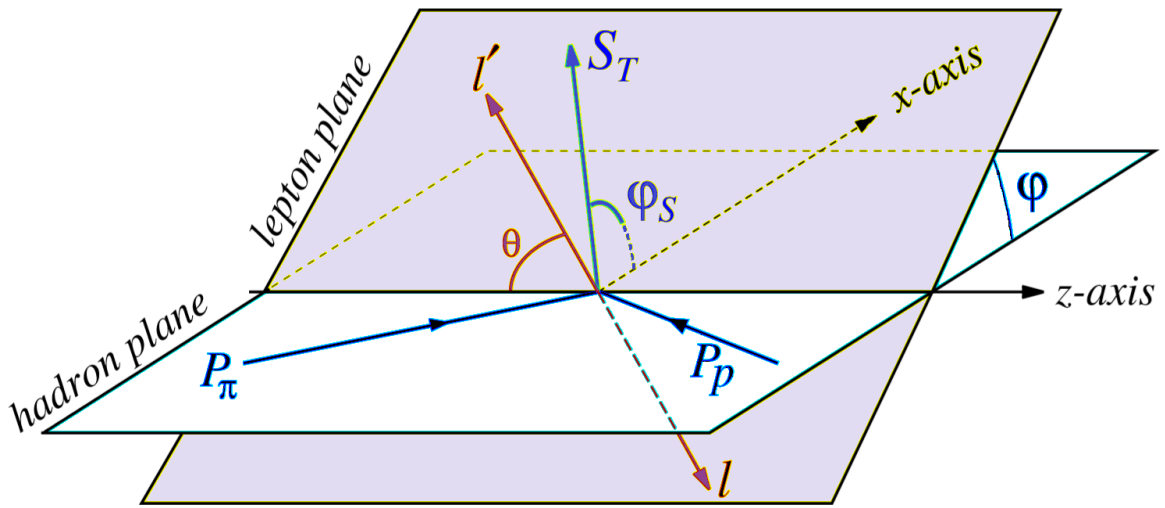


$$\mathcal{C}\left[\omega\, f_\pi^{\bar{a}}\, f_p^{a}\right] = \frac{1}{N_C}\sum_a e_a^2 \int d^2k_{T\pi}\; d^2k_{Tp}\; \delta^2\left(q_T - k_{T\pi} - k_{Tp}\right)\; \omega\, f_\pi^{\bar{a}}(x_\pi, k_{T\pi})\, f_p^{\bar{a}}\left(x_p, k_{Tp}\right)$$

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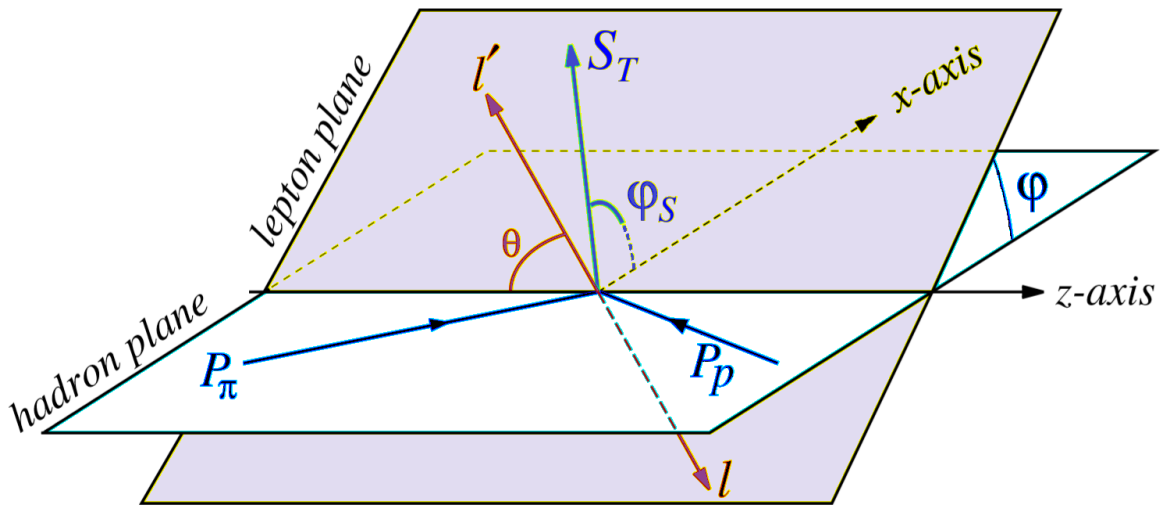


Definition: $$A_{AB}^{w(\phi,\phi_S)} = \frac{F_{AB}^{w(\phi,\phi_S)}}{F_{UU}^{1}}$$

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Gaussian Ansatz for the transverse momentum dependent part:

$$f_h^a(x, k_\perp^2) = f_h^a(x) \; \frac{e^{-k_{Th}^2/\langle k_{Th}^2\rangle_{f_h}}}{\pi \left\langle k_{Th}^2 \right\rangle_{f_h}}$$

$$f_h^{(n)a}(x, k_\perp^2) = \left(\frac{k_{Th}^2}{2\, M_h^2}\right)^n f_h^a(x, k_\perp^2)$$

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DATA-BASED KNOWLEDGE

$f_{1,p}^{a}$ $f_{1,\pi}^{a}$ $f_{1T,p}^{\perp a}$ $h_{1,p}^{a}$ $h_{1,p}^{\perp a}$ $h_{1T,p}^{\perp a}$ $h_{1L,p}^{\perp a}$ $h_{1,\pi}^{\perp a}$

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Extractions and parametrizations are available for all except for pion Boer-Mulders $\boldsymbol{h_{1,\pi}^{\perp}}$ and Kotzinian-Mulders function $\boldsymbol{h_{1L,p}^{\perp}}$.

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$\boldsymbol{h}_{1L,p}^{\perp}$ can be obtained from transversity by WW-type approximation. Bastami et al [1807.10606].

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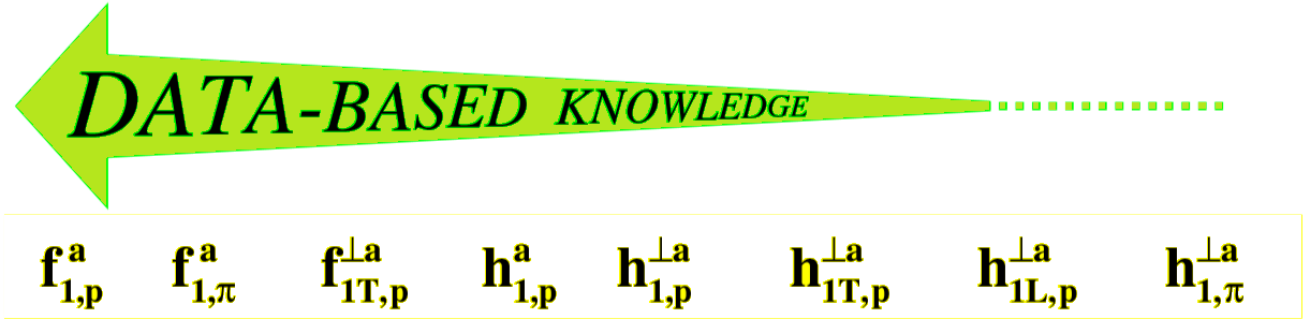
We need model calculations at least for $\boldsymbol{h}_{1,\pi}^{\perp}$ .
Model calculation are available for all.

Approaches :

- Using parametrizations only
- Using models only
- Using both



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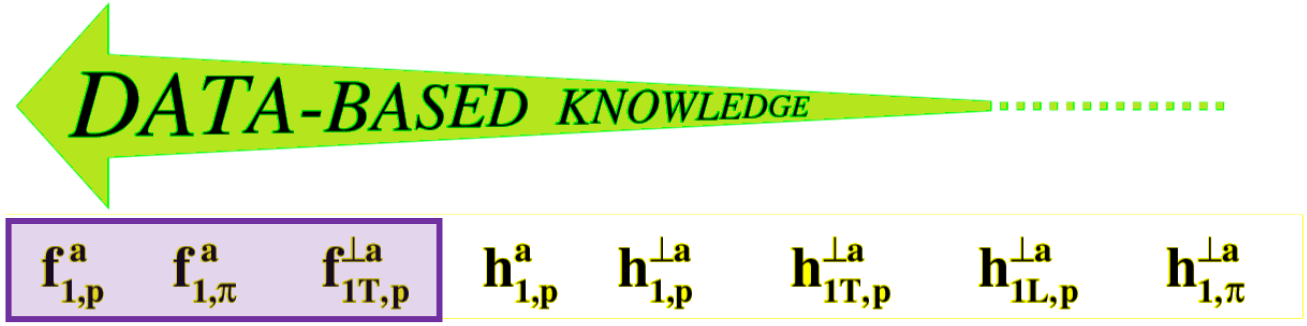

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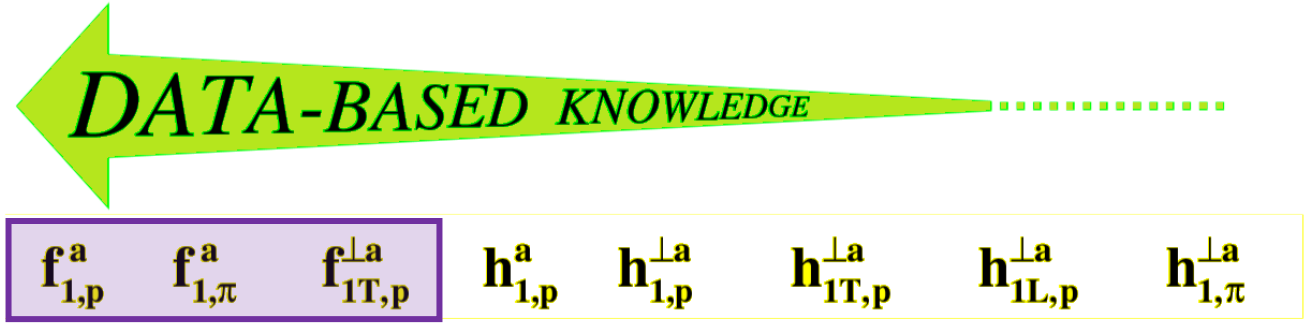


## CQM Frameworks:

- **Light front constituent quark model (LFCQM)**
- **Spectator Model (SPM)**

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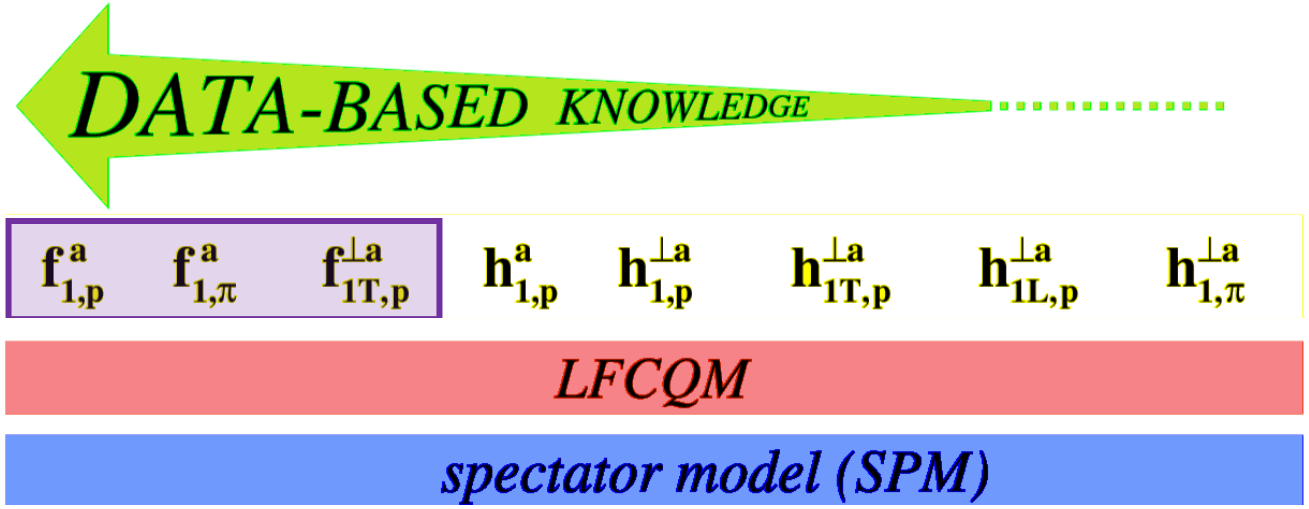


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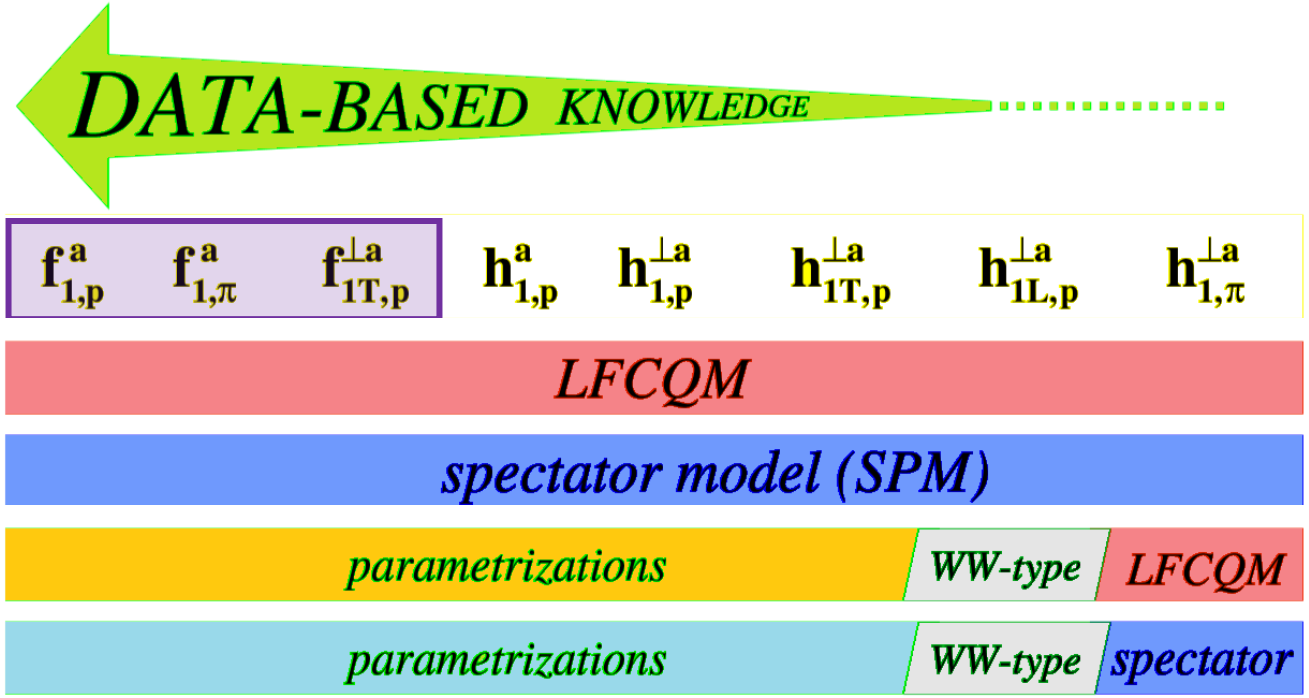


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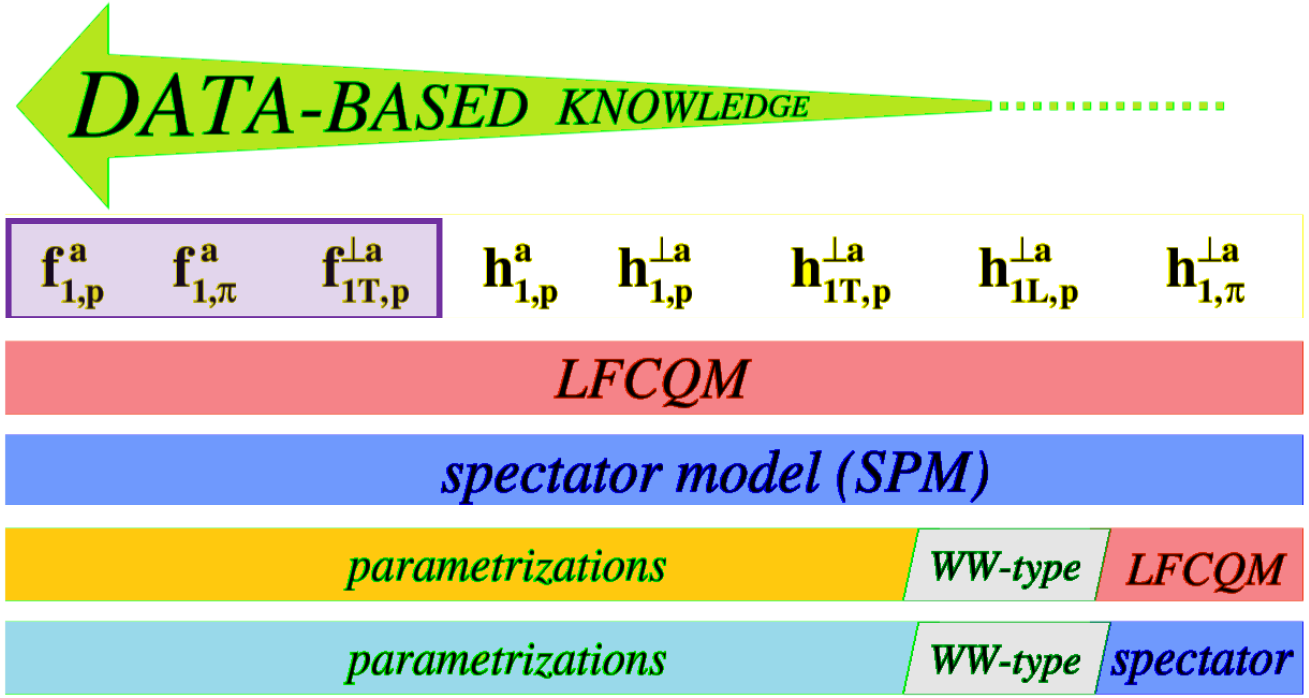


CQM Frameworks:

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Models need to be evolved from very low initial scale to $Q \approx 5.3\ GeV$ (COMPASS)

Should not forget about $\langle q_T^2 \rangle$

## Light front constituent quark model (LFCQM)

B. Pasquini, P. Schweitzer 1406.2056v1

- decomposes hadronic state to $N$-parton light front wave functions into light-cone Fock space,
- decomposes the wave functions into orbital angular momentum eigenstates,
- uses Gaussian ansatz for momentum dependence,
- fits parameters to observed properties of proton and pion.

## Spectator Model (SPM)

L. Gamberg *et al* 0708.0324

- using IRA factorizes the cross section into Born c.s. and quark correlation functions,
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- models TMDs as linear combinations of axial vector and scalar diquark contributions in SU(2),
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$$\mu_0 = 460\ MeV$$

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$$\mu_0 = 260\ MeV$$

## We utilized a pragmatic approach to evolve TMDs to $Q^2 \approx 28\ GeV^2$

- Solve TMD evolution eq. at an initial scale $Q_0 = 5.3\ GeV$ in impact parameter space $b_T$
- Parameterize TMD using a Gaussian in $b_T$-space

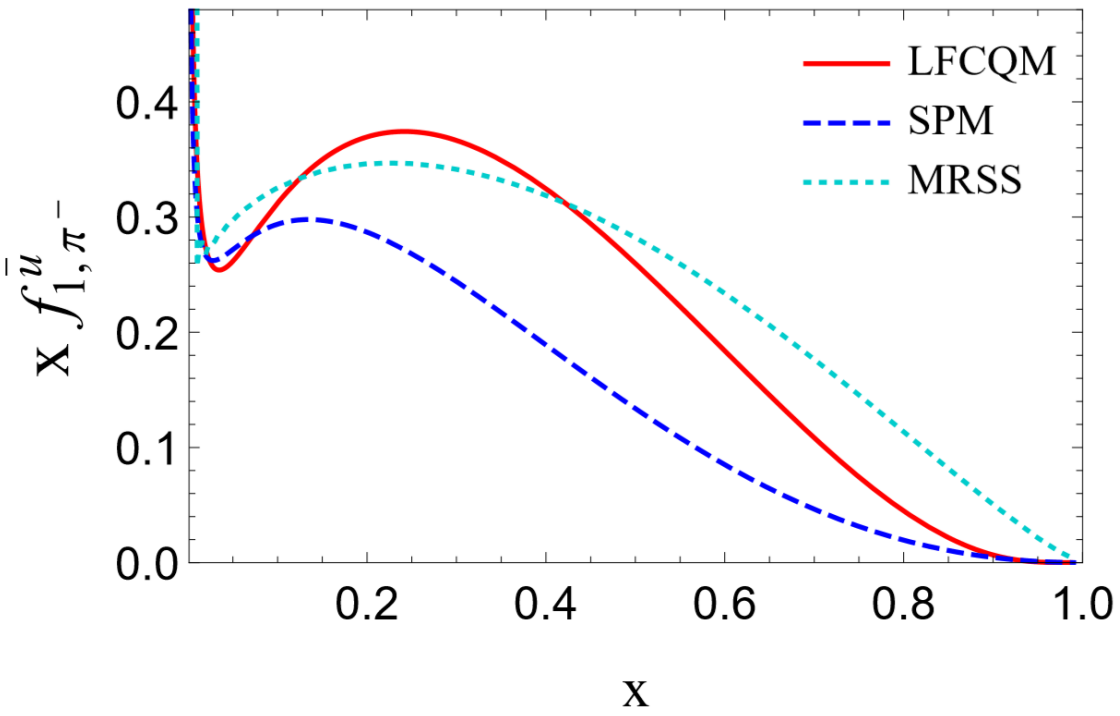
$$\tilde{f}_h^a(x_h, \boldsymbol{b}_T, Q_0, Q_0^2) = f_h^a(x_h, Q_0)\, e^{-\frac{\langle k_{Th}^2 \rangle_{f_h}}{4} \boldsymbol{b}_T^2}$$

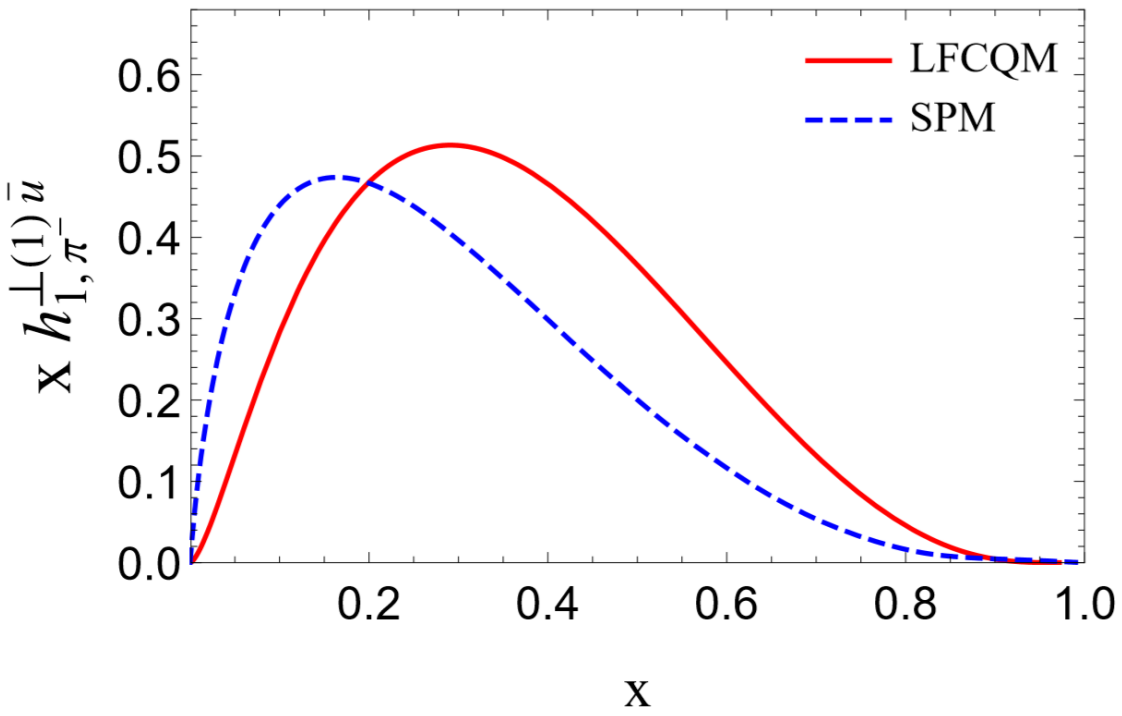
- Use exact DGLAP for $f_{1,h}^a(x_h)$ and $h_{1,p}^a(x)$ and approximate DGLAP for the rest of the collinear functions.

S. Bofi et al [0903.1271]
B. Pasquini and P. Schweitzer [1103.5977] , [1406.2056]
M. Miyama, S. Kumano [9508246]
M. Hirai, M. Miyama, S. Kumano [9707220]

# $\pi^-$ unpolarized and Boer-Mulders collinear functions

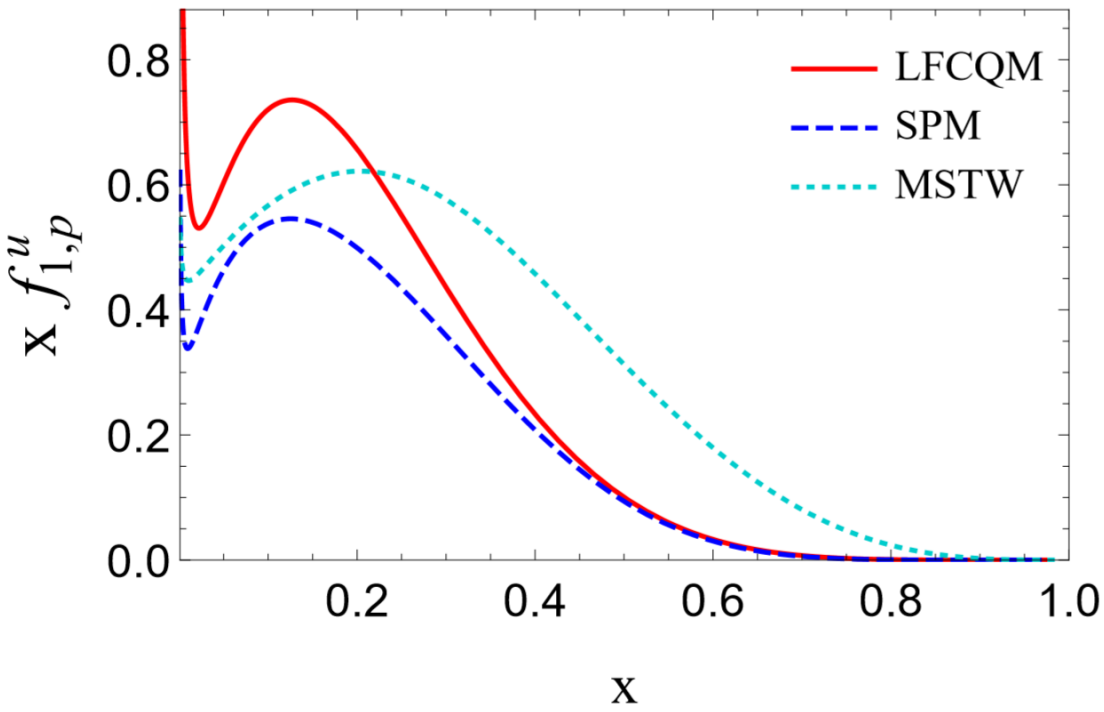
MRSS : P. J, Sutton Phys. Rev. D45 (1992) 2349–2359

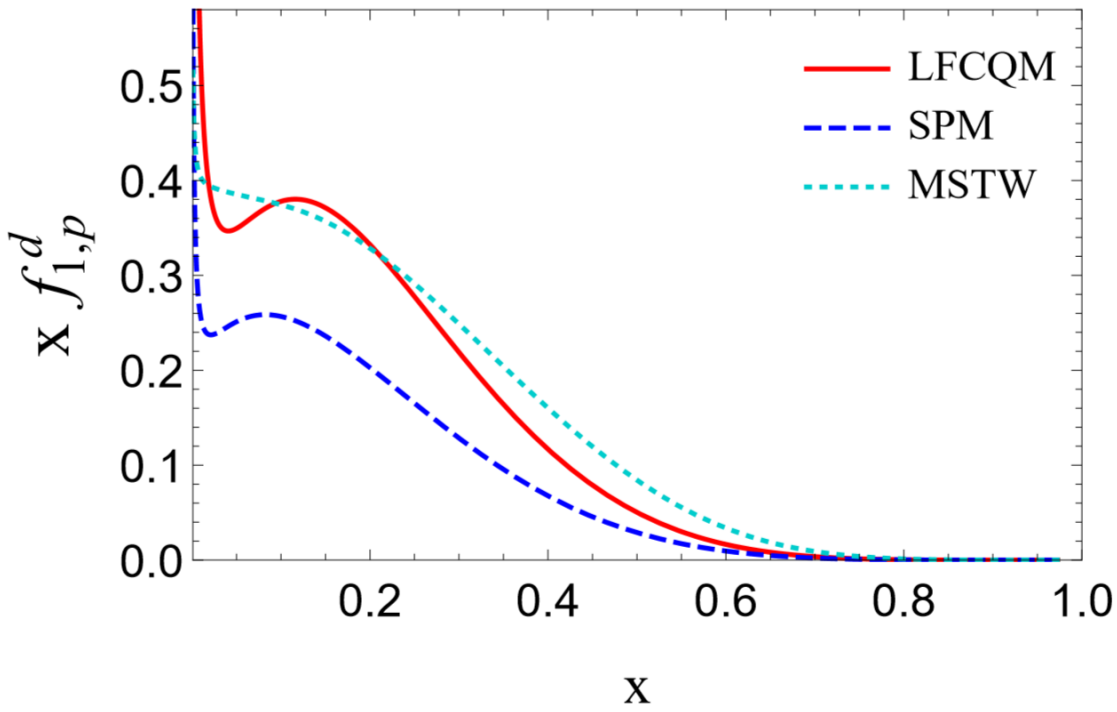


# Proton unpolarized collinear functions for $u$ and $d$ quarks

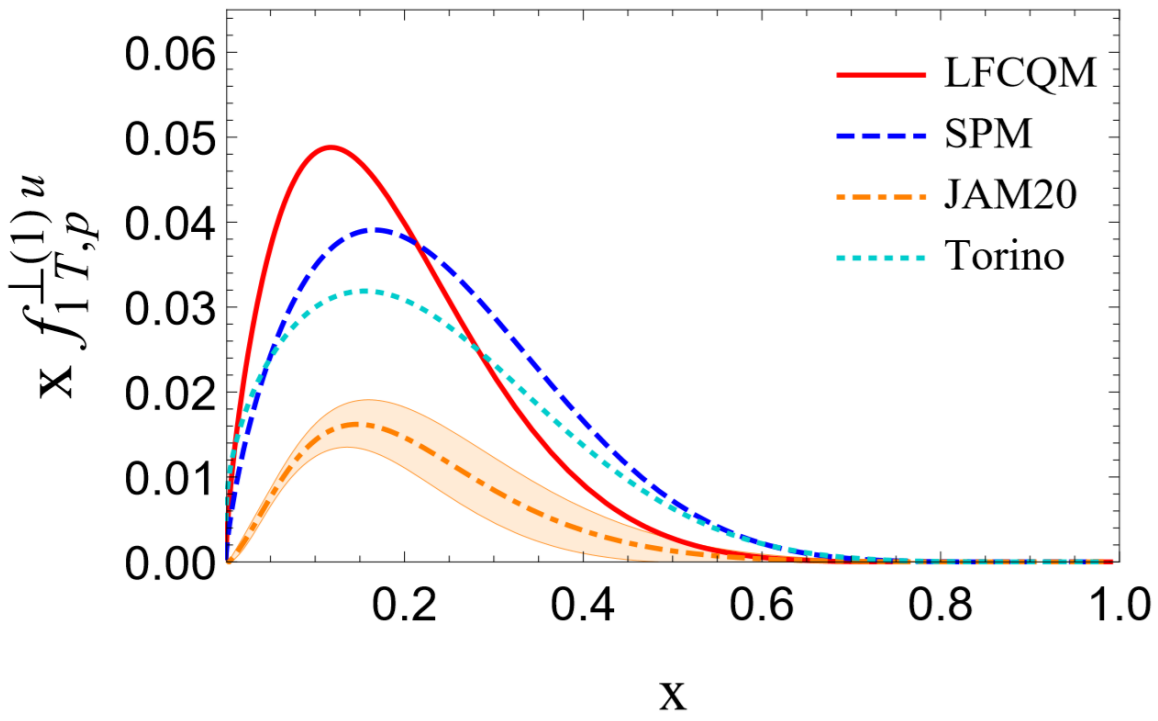
MSTW : A. D. Martin et al [0901.0002]

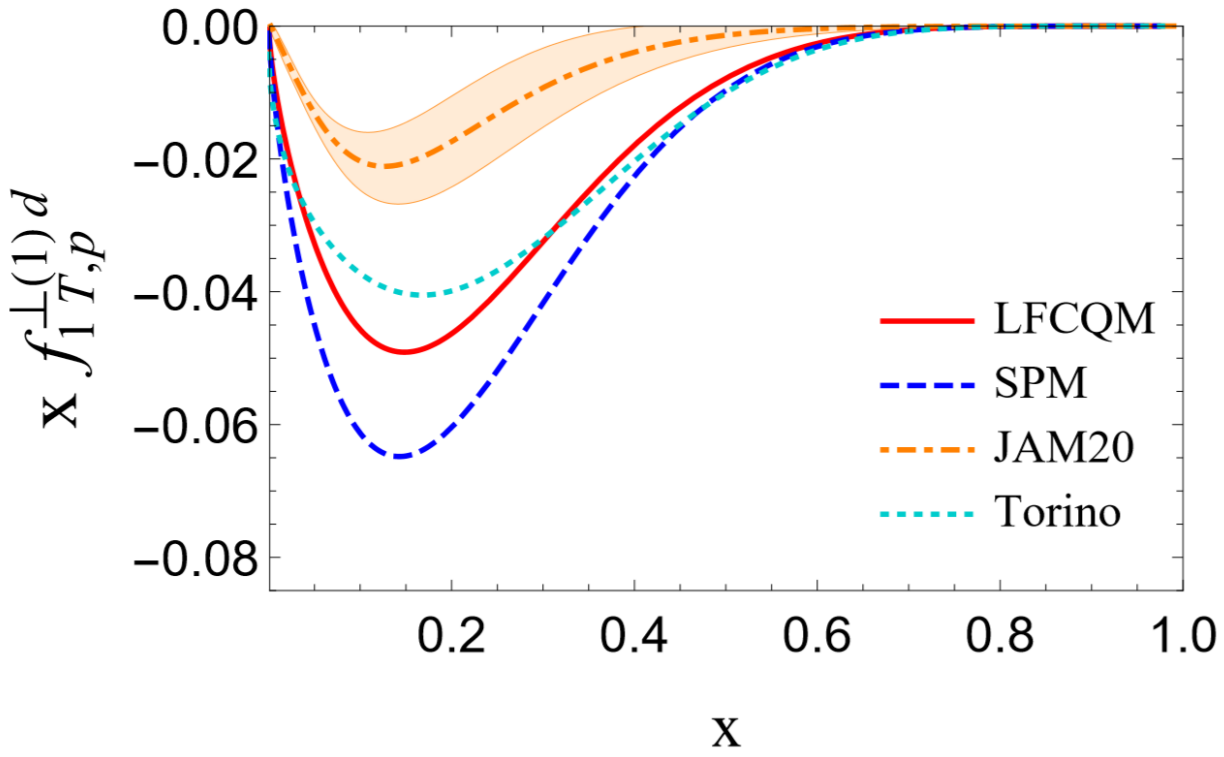


# Proton Sivers collinear functions for $u$ and $d$ quarks

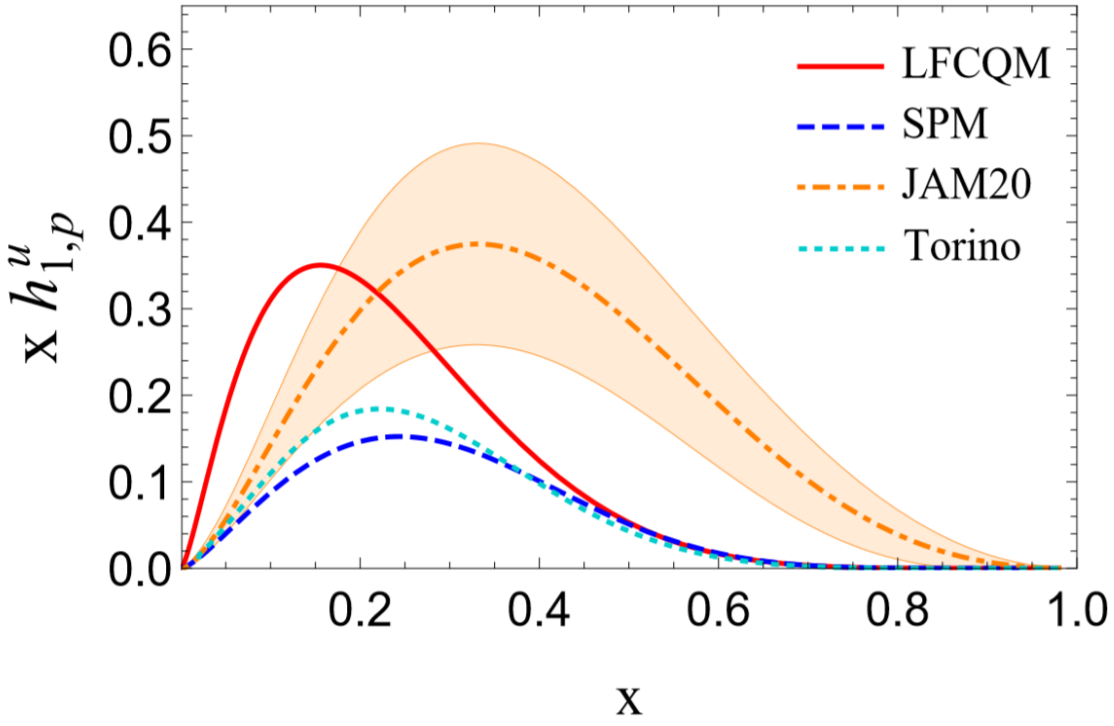
JAMM20: J. Camarrota *et al* [2002.08384]
Torino : M. Anselmino *et al* [1107.4446]

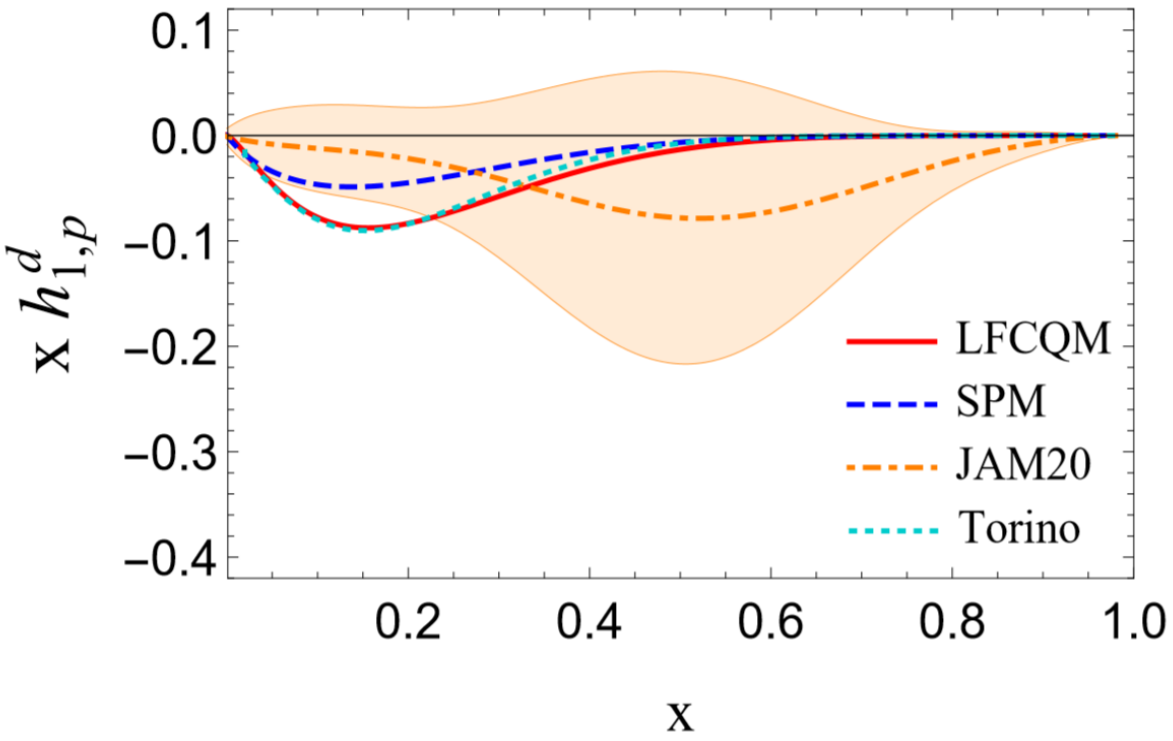


# Proton transversity collinear functions for $u$ and $d$ quarks

JAMM20: J. Camarrota *et al* [2002.08384]
Torino : M. Anselmino *et al* [1303.3822]

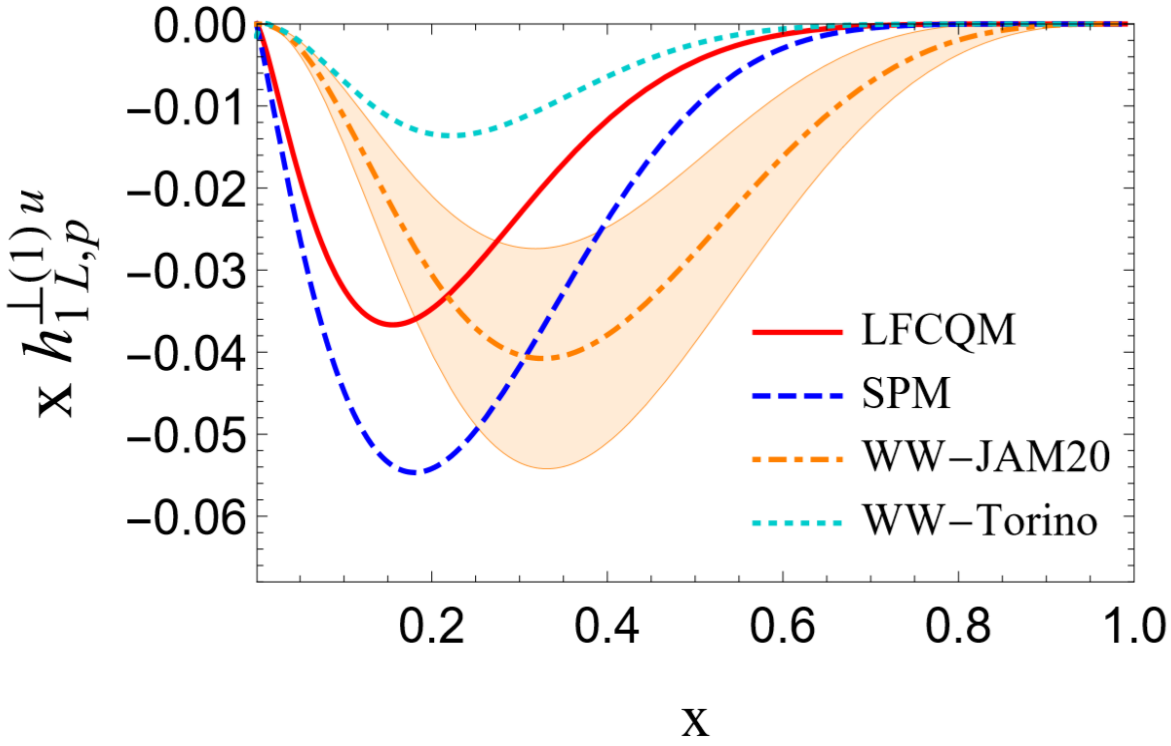


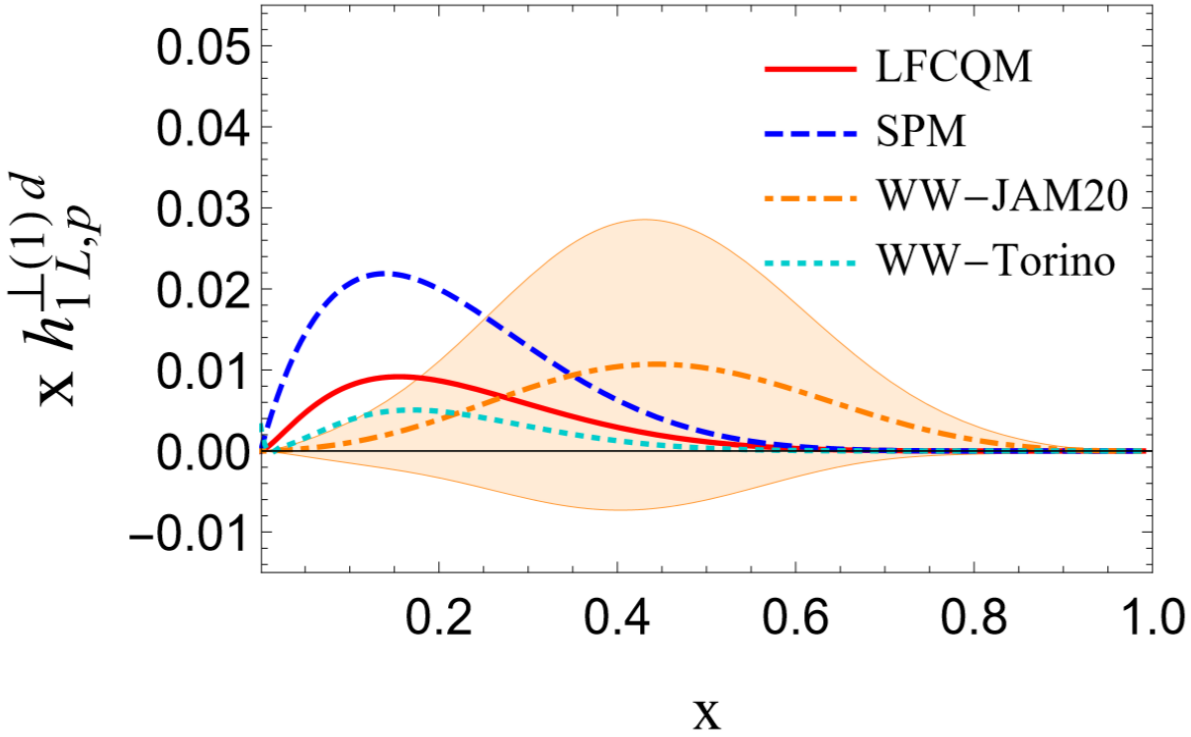



# Proton Kotzinian-Mulders functions for $u$ and $d$ quarks

WW-JAMM20: J. Camarrota *et al* [2002.08384]
WW-Torino : M. Anselmino *et al* [1303.3822]

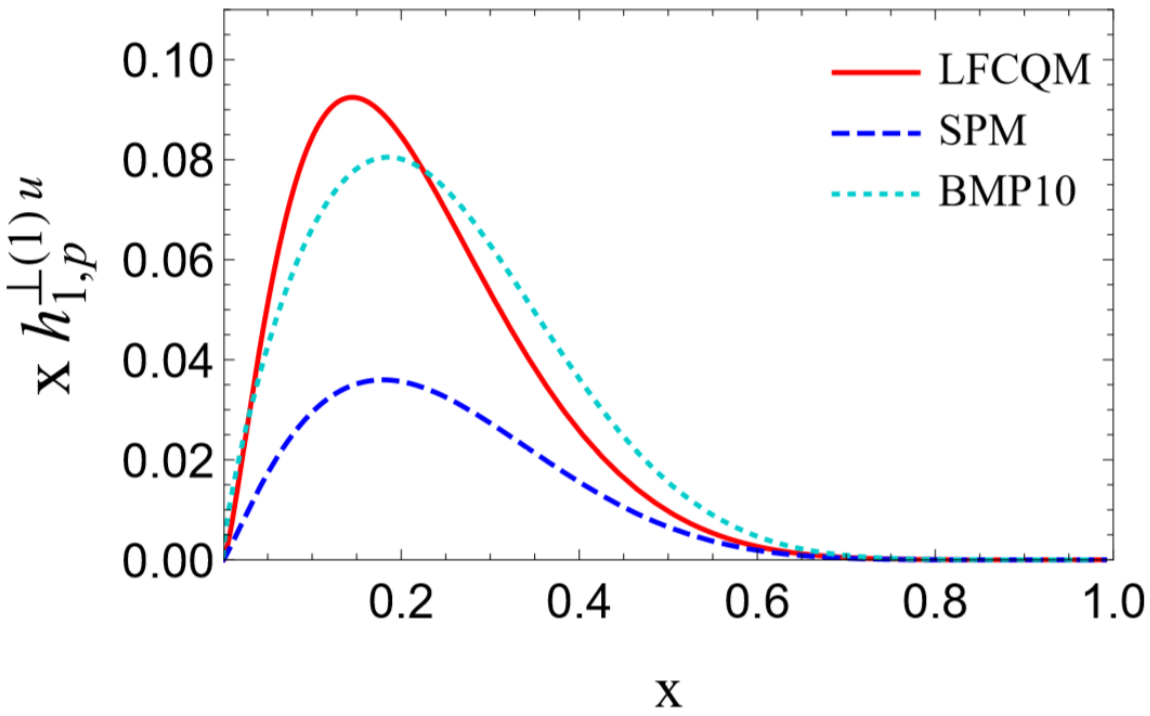
$$h_{1L}^{\perp(1)}(x) \approx -x^2 \int_x^1 \frac{dy}{y^2}\, h_1(y)$$

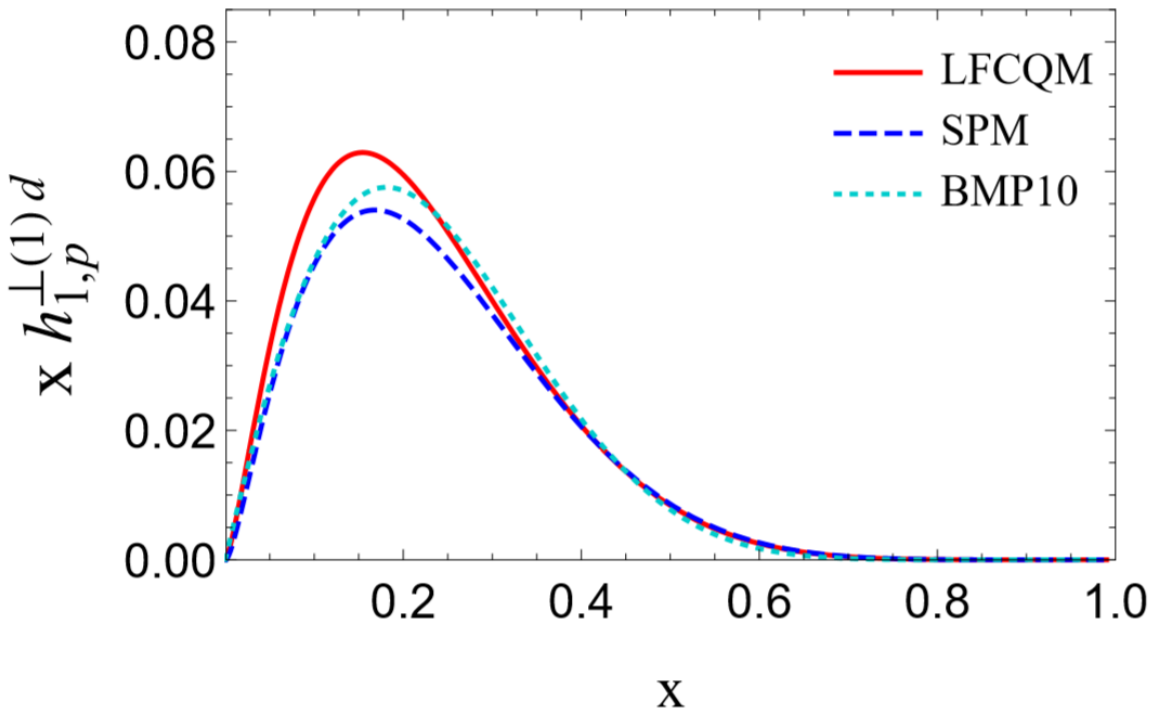


# Proton Boer-Mulders collinear functions for $u$ and $d$ quarks

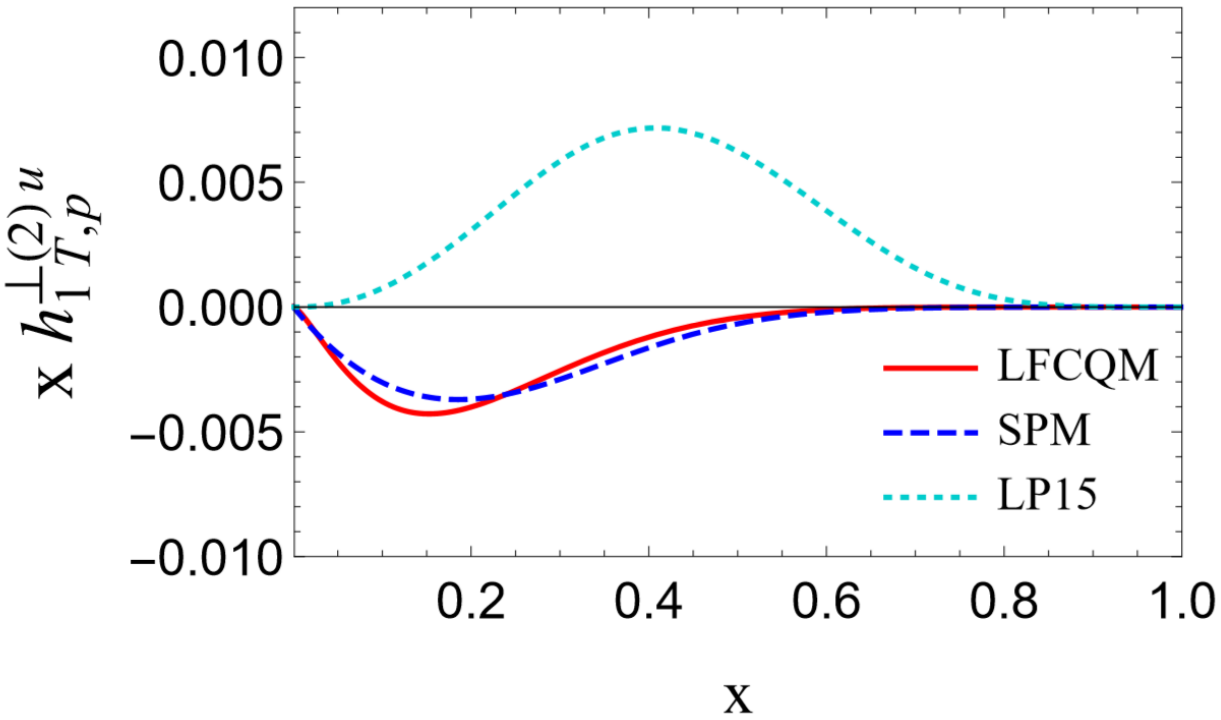
BMP10 : V. Barone, S. Melis and A. Prokudin [0912.5194]

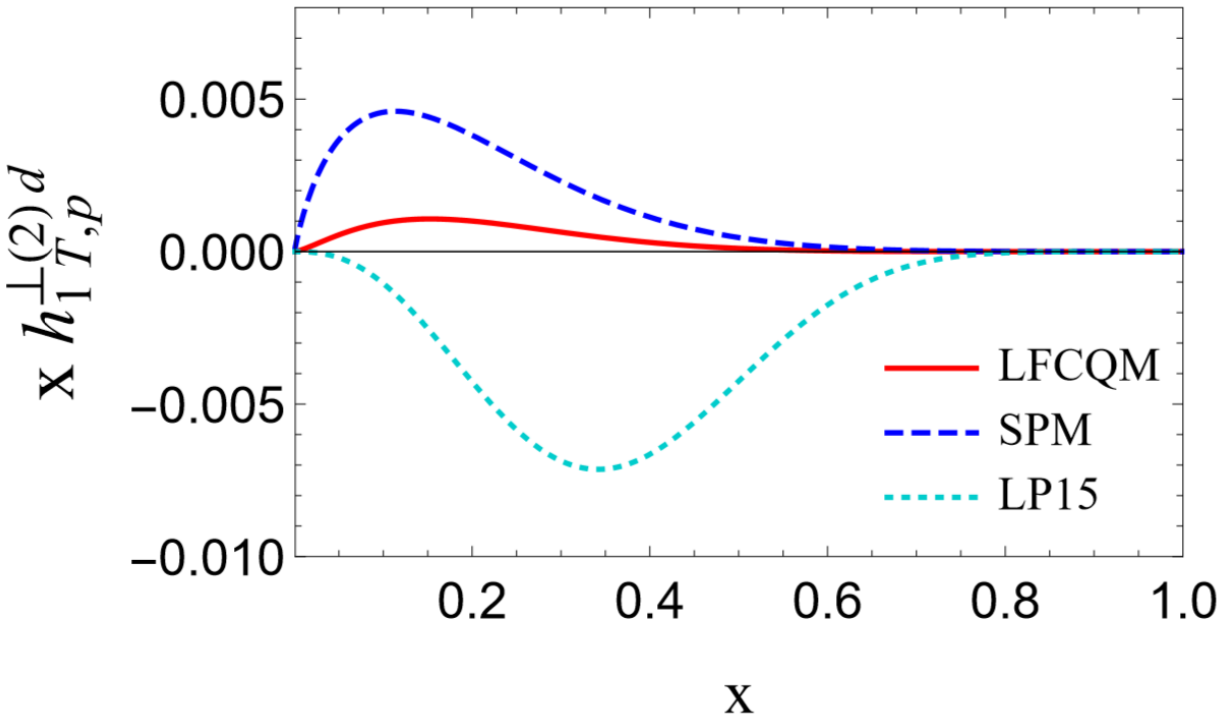


# Proton Pretzelosity collinear functions for $u$ and $d$ quarks

LP15 : C. Lefky, A. Prokudin [1411.0580]



## The Gaussian widths need to be evolved too

- At initial scale of the models $\langle q_T^2 \rangle = \langle k_{T\pi}^2 \rangle_{f_{1,\pi}} + \langle k_{Tp}^2 \rangle_{f_{1,p}} \approx 0.18$ applies only to $F_{UU}^1$
- $\langle q_T^2 \rangle$ is expected to broaden with energy and to be narrower for polarized TMDs
- $\langle q_T^2 \rangle \approx 1.5\ GeV^2$ at COMPASS energies for $F_{UU}^1$ based on P. Schweitzer *et al* [1003.2190]
- Broadened width for $F_{UU}^{\cos\phi_S} \approx 1.2\ GeV^2$ according to B. Pasquini, P. Schweitzer [1406.2056]

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- Our consensus:

$$\langle q_T^2 \rangle \approx \begin{cases} 1.5\ GeV^2 & for\ F_{UU}^1 \\ 1.3\ GeV^2 & for\ F_{UT}^{\sin\phi_S} \\ 1.2\ GeV^2 & other\ cases \end{cases}$$

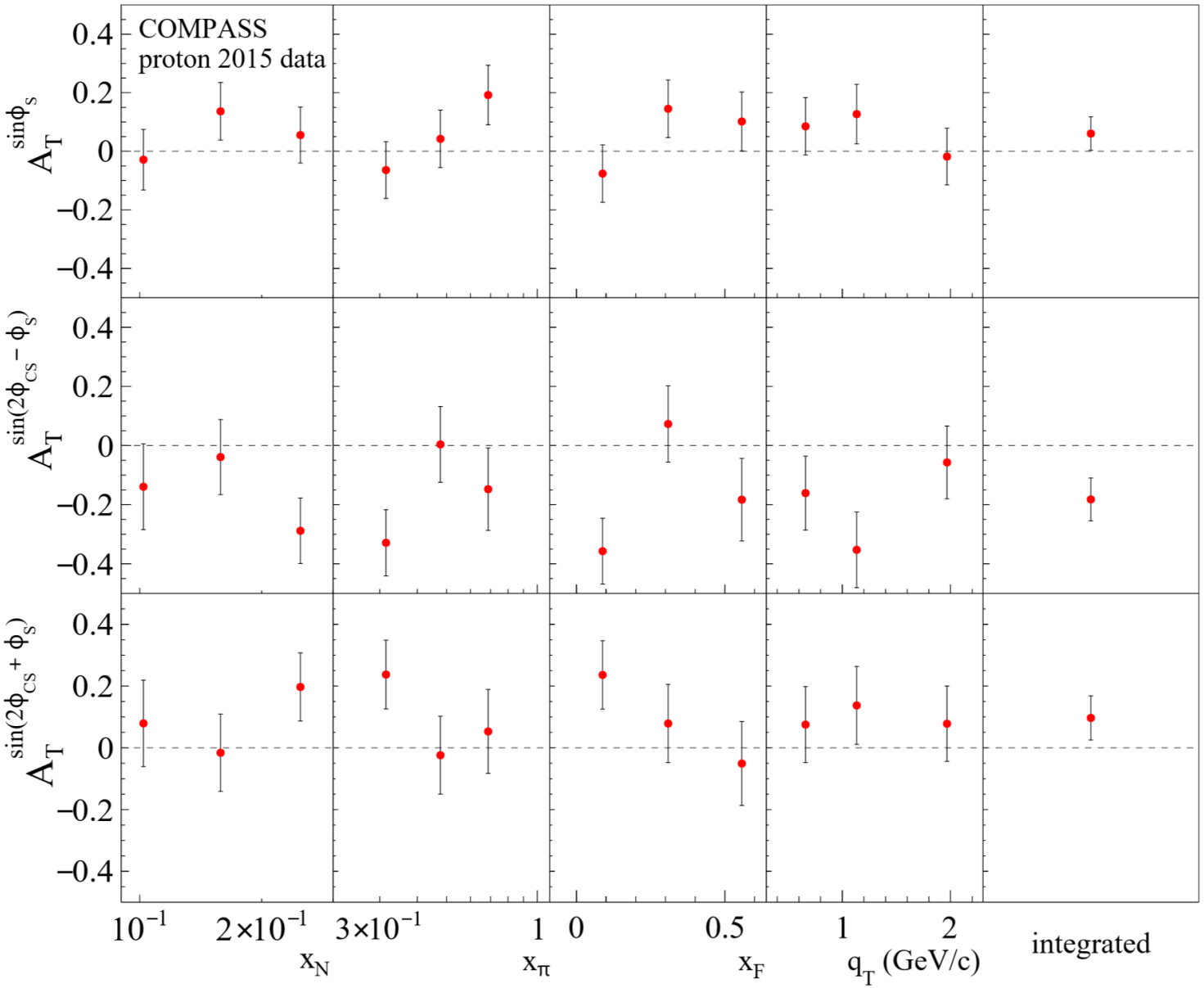

HALFWAY

# Recent COMPASS results on pion-induced DY

$E = 190\ GeV$

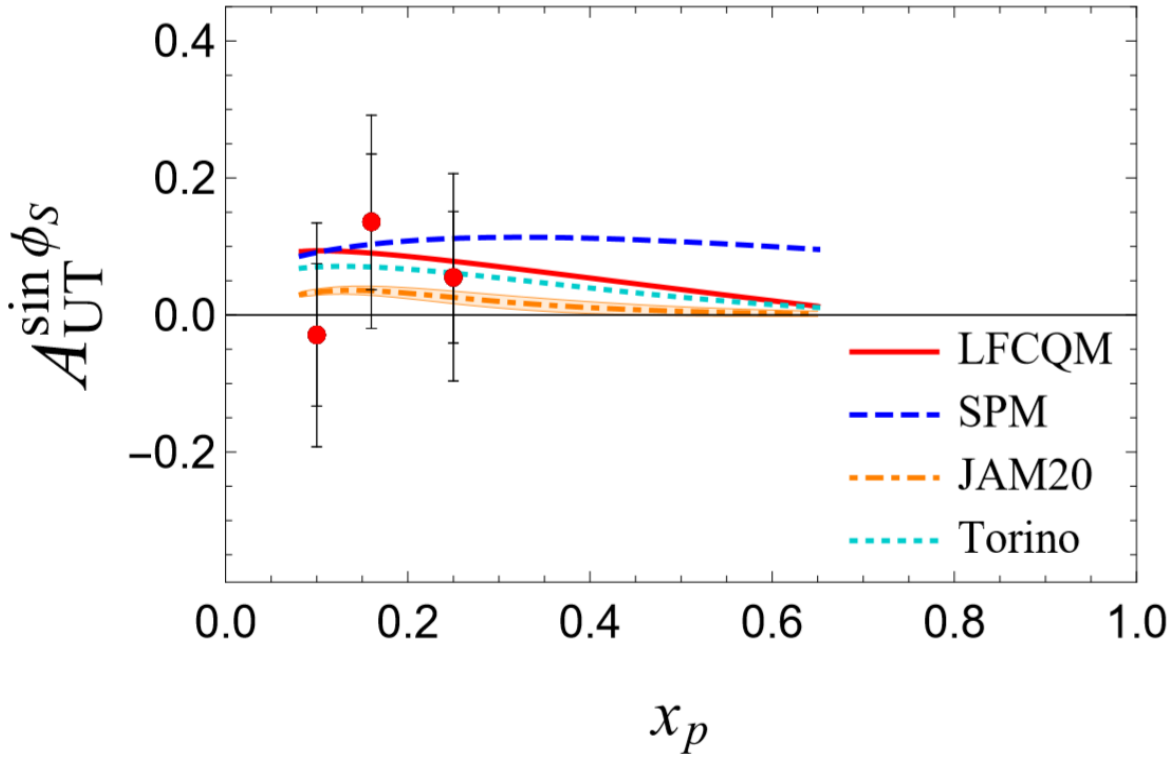
$4.3 < Q < 8.5\ GeV$

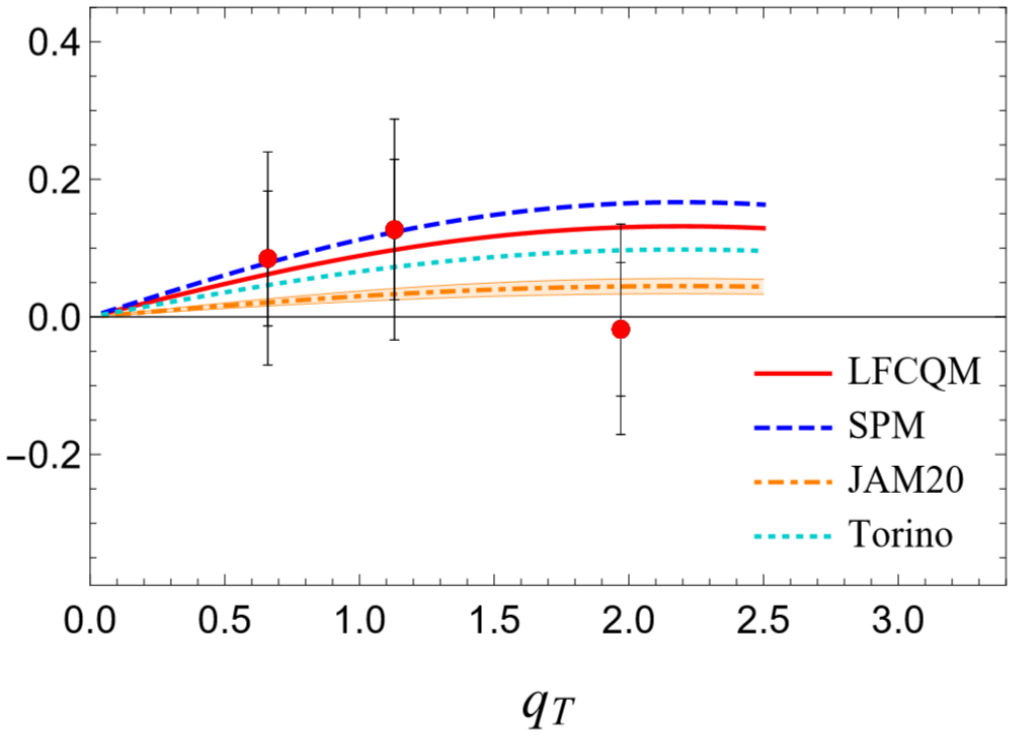
COMPASS collaboration,
M. Aghasyan *et al* Phys. Rev. Lett. 119 (2017) 112002, [1704.00488]



$$F_{UT}^{\sin\phi_S} = \frac{2\,M_p}{N_C}\sum_a e_a^2\; f_\pi^{\bar{a}}(x_\pi)\; f_{1,p}^{\perp(1)\,a}(x_p)\left(\frac{q_T}{\langle q_T^2\rangle}\right)\frac{e^{-q_T^2/\langle q_T^2\rangle}}{\langle q_T^2\rangle}$$

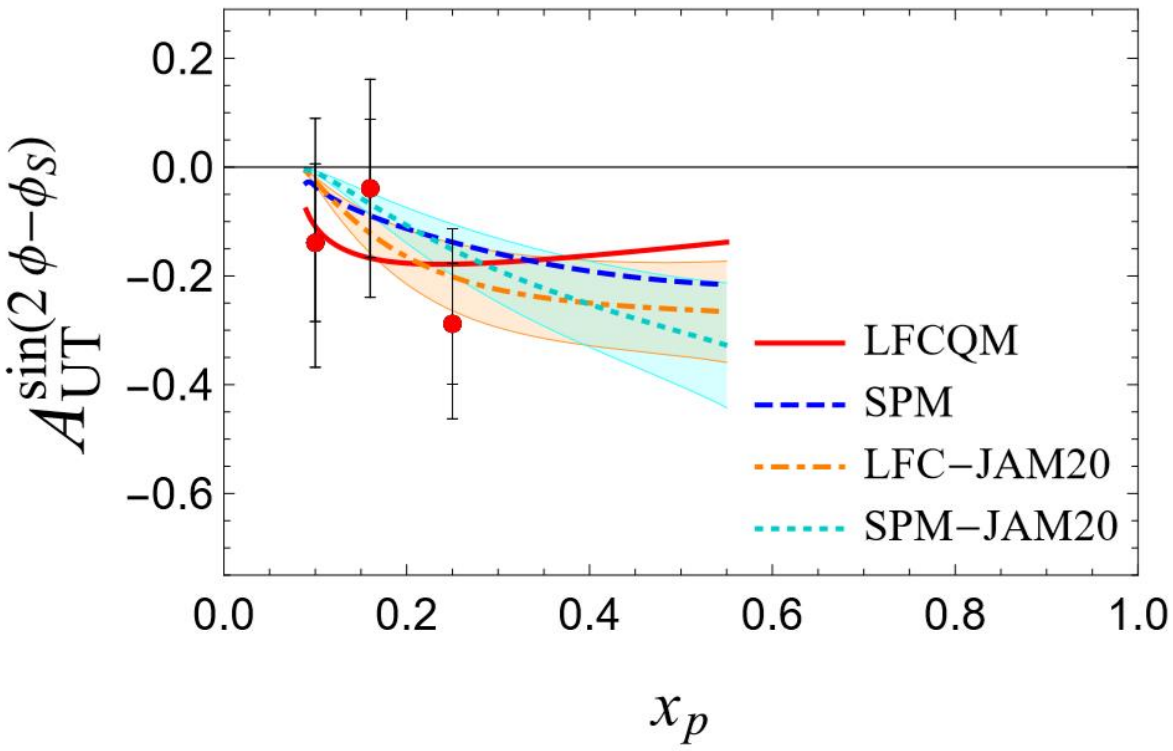
Sivers flips sign according to COMPASS
And according to our calculations!

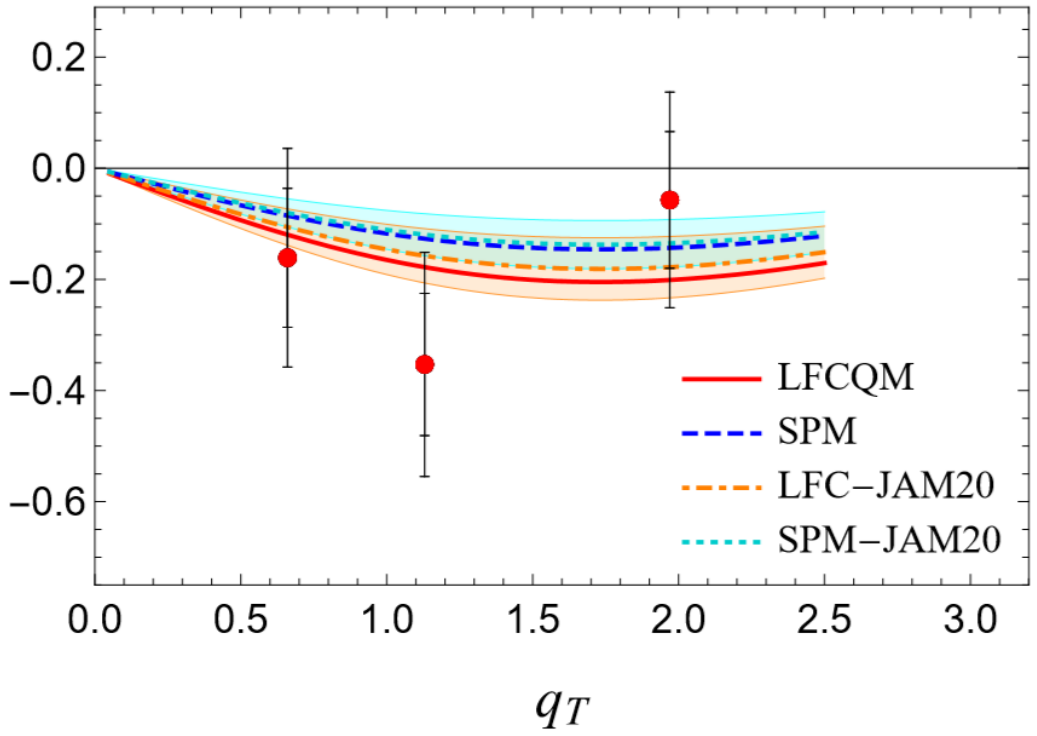


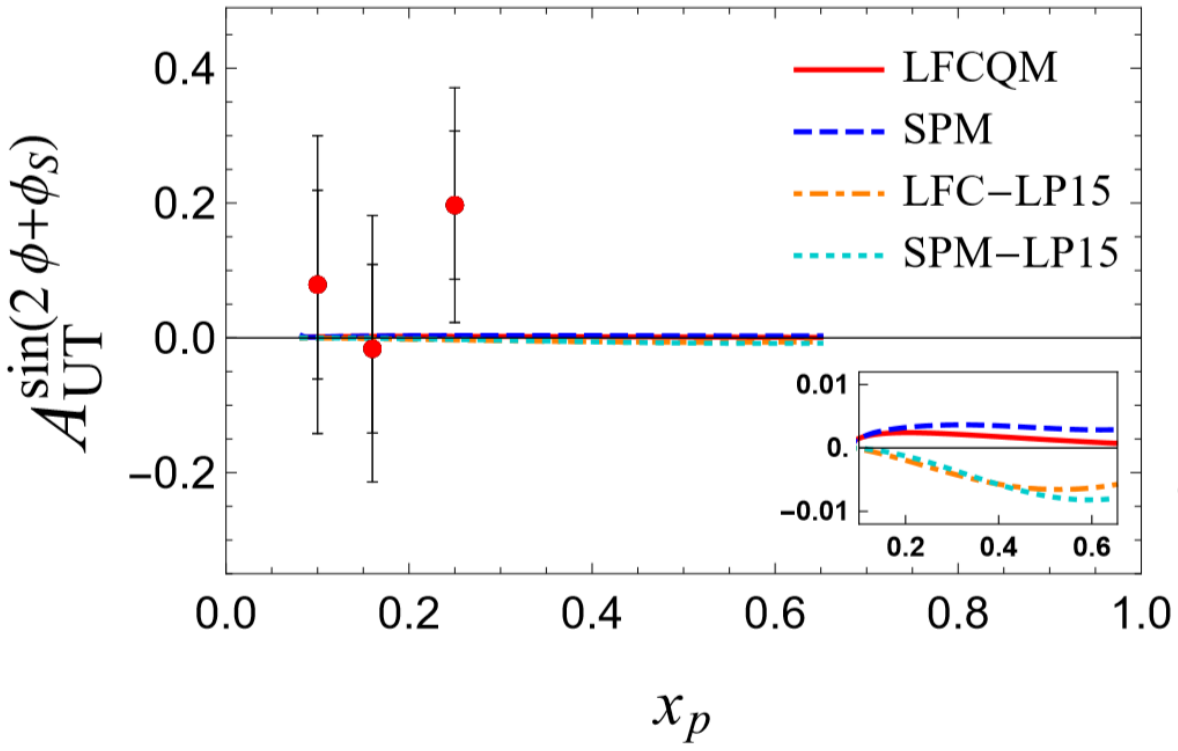
$$F_{UT}^{\sin(2\phi-\phi_S)} = -\frac{2\,M_\pi}{N_C}\sum_a e_a^2\; h_{1,\pi}^{\perp(1)\,\bar{a}}(x_\pi)\; h_{1,p}^{a}(x_p)\left(\frac{q_T}{\langle q_T^2\rangle}\right)\frac{e^{-q_T^2/\langle q_T^2\rangle}}{\langle q_T^2\rangle}$$

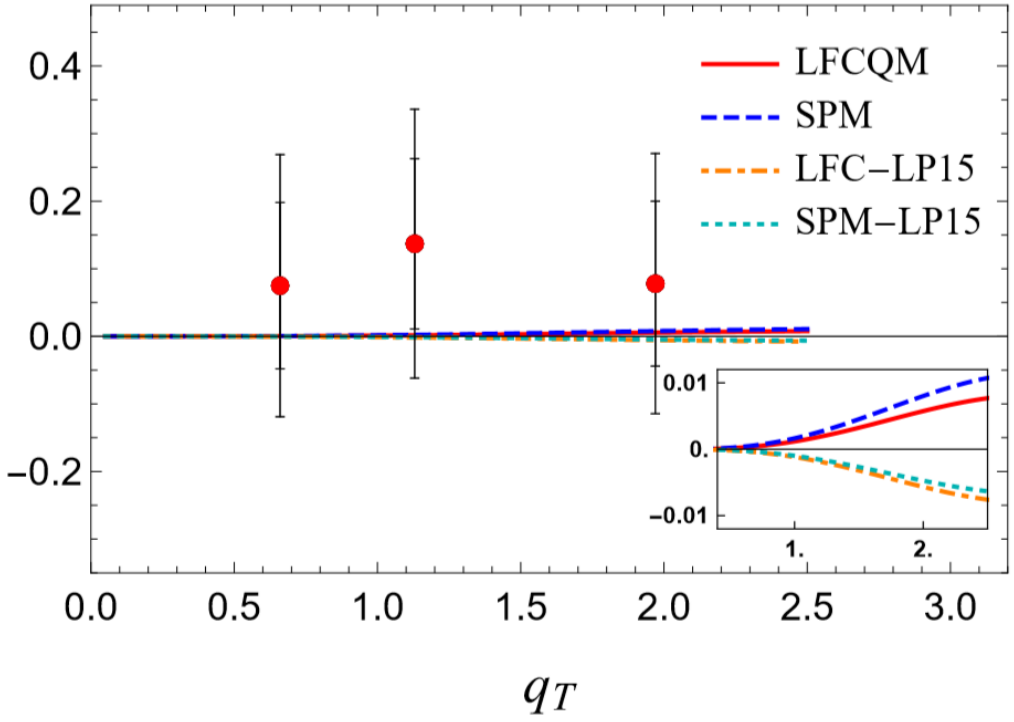
$$h_{1,\pi}^{\perp(1)\,\bar{u}} > 0$$



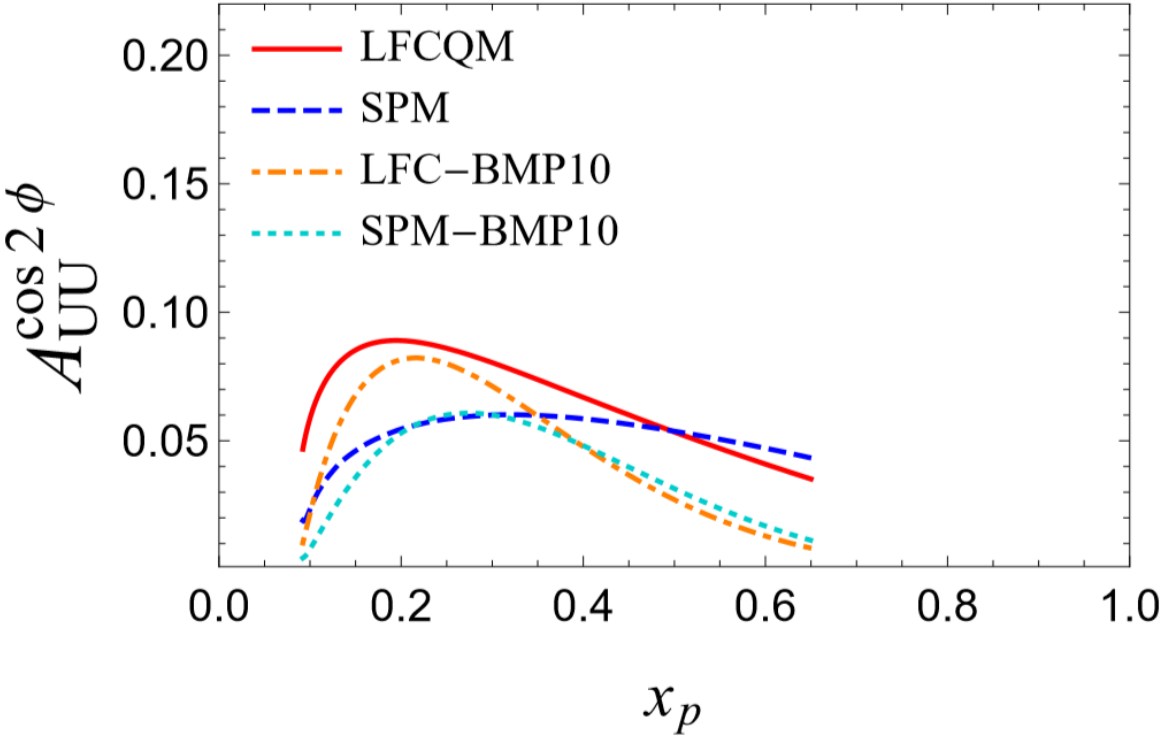
$$F_{UT}^{\sin(2\phi+\phi_S)} = -\frac{2\,M_p\,M_\pi^2}{N_C}\sum_a e_a^2\;h_{1,\pi}^{\perp(1)\,\bar{a}}(x_\pi)\,h_{1T,p}^{\perp(2)\,a}(x_p)\left(\frac{q_T}{\langle q_T^2\rangle}\right)^3\frac{e^{-q_T^2/\langle q_T^2\rangle}}{\langle q_T^2\rangle}$$

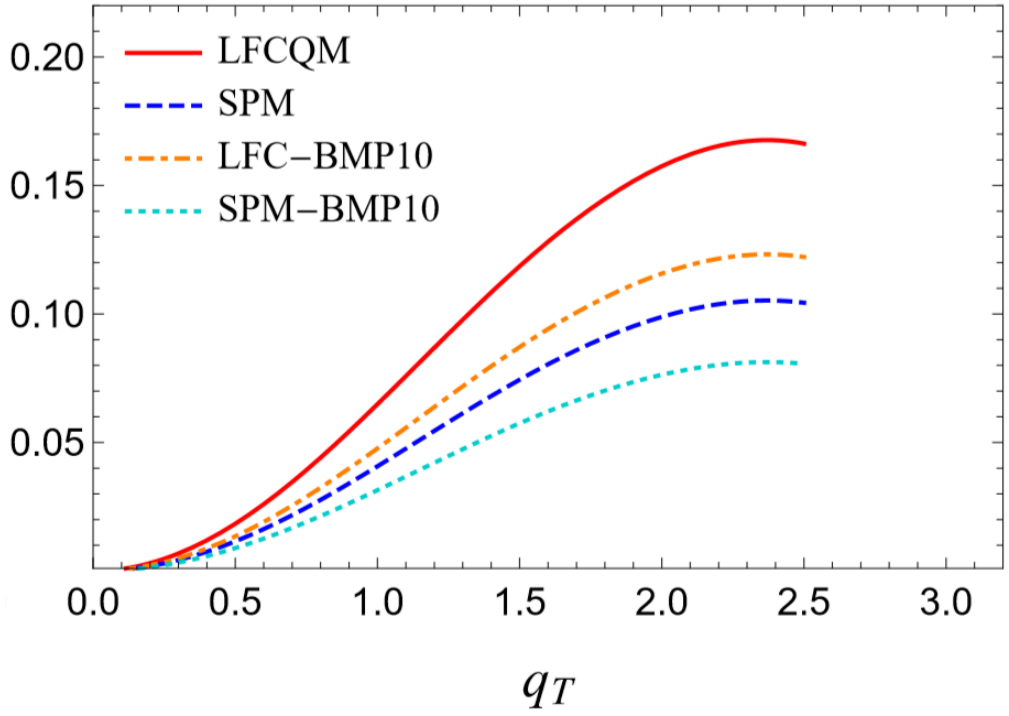


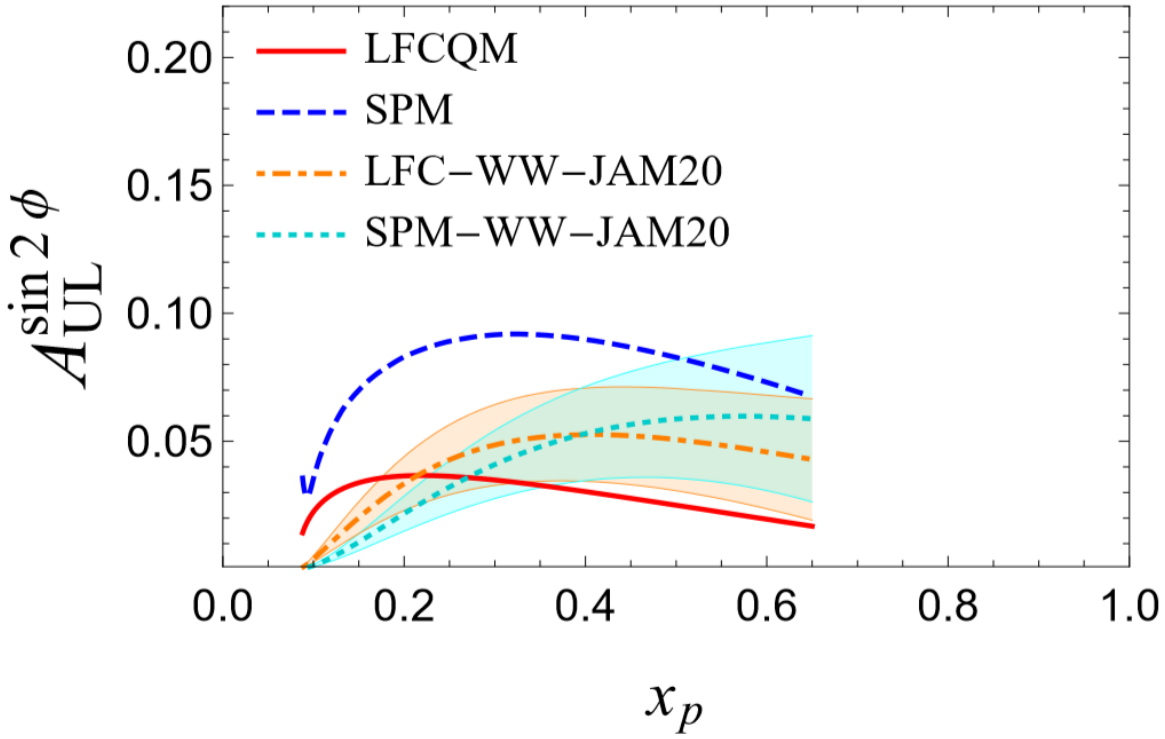
$$F_{UU}^{\cos\phi_S} = \frac{4\,M_p M_\pi}{N_C} \sum_a e_a^2 \; h_{1,\pi}^{\perp(1)\,\bar{a}}(x_\pi)\, h_{1,p}^{\perp(1)\,a}(x_p) \left(\frac{q_T}{\langle q_T^2\rangle}\right)^2 \frac{e^{-q_T^2/\langle q_T^2\rangle}}{\langle q_T^2\rangle}$$

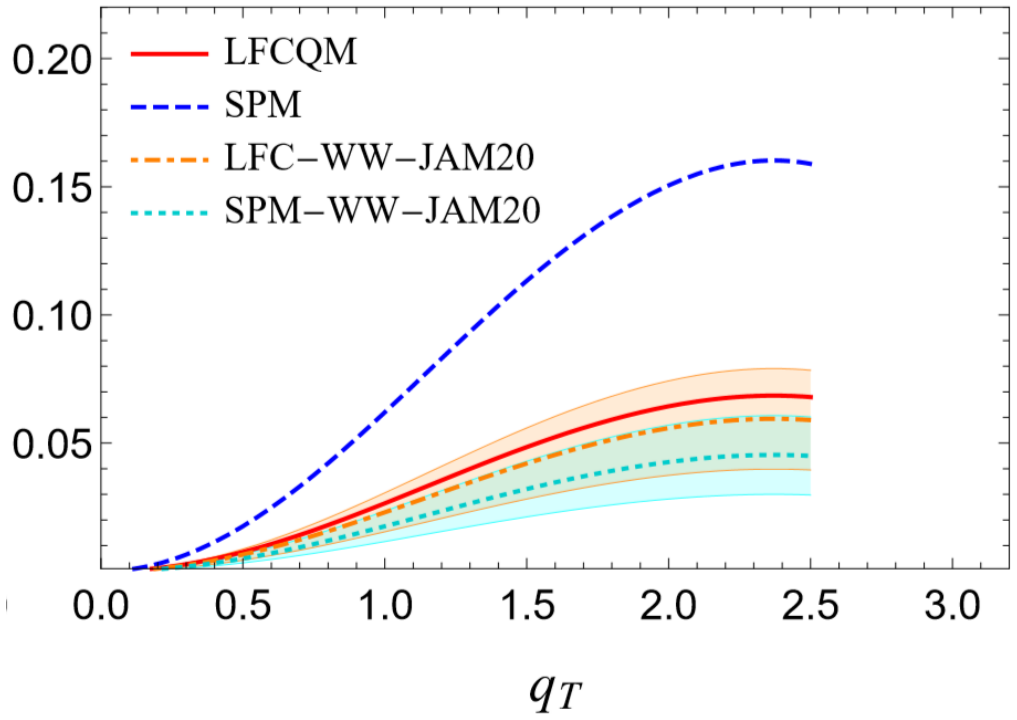
$$h_{1,\,p}^{\perp(1)\,u} > 0$$



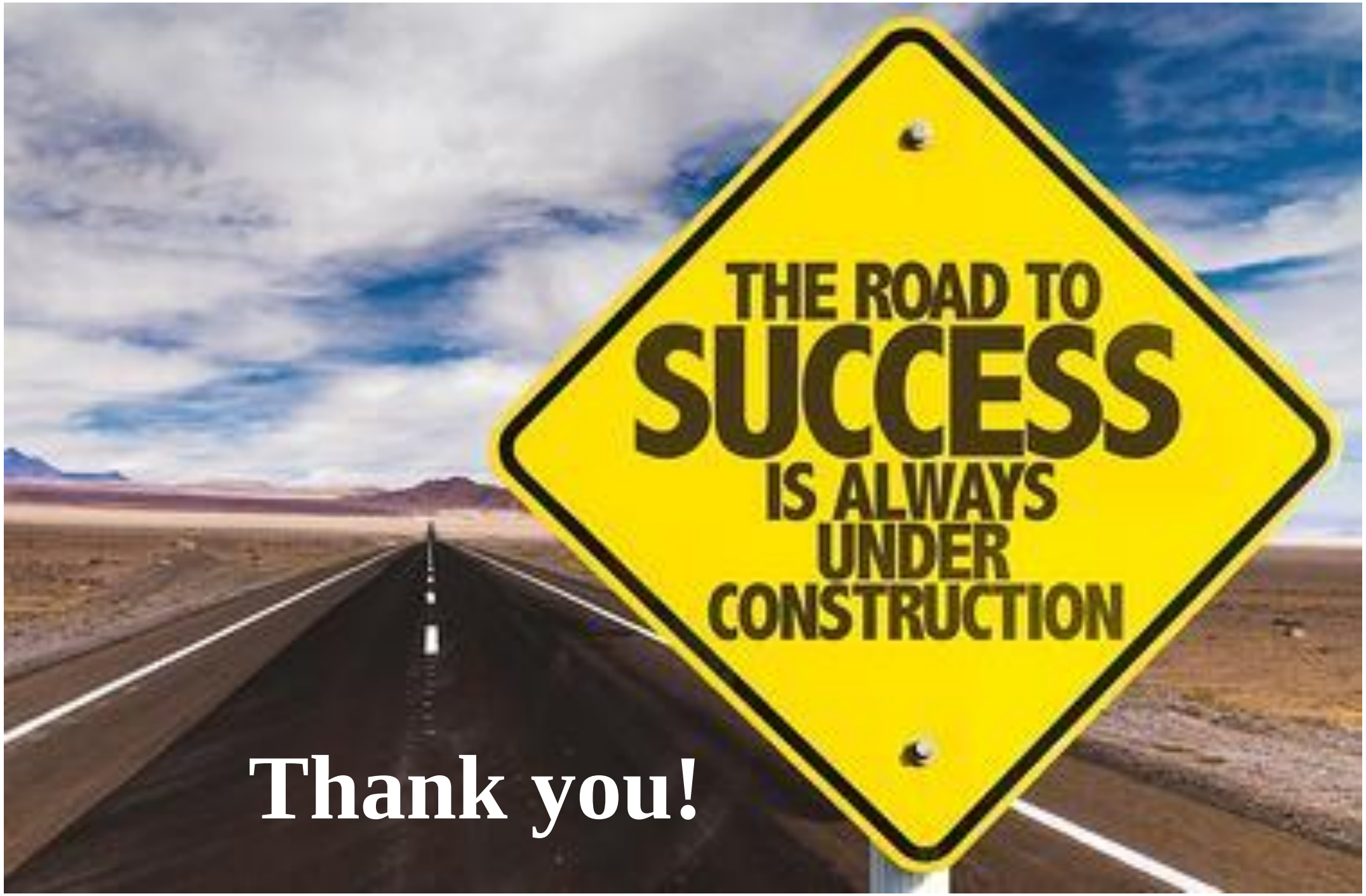
$$F_{UL}^{\sin 2\phi} = \frac{4\, M_p M_\pi}{N_C} \sum_a e_a^2 \; h_{1,\pi}^{\perp(1)\,\bar{a}}(x_\pi)\, h_{1L,\,p}^{\perp(1)\,a}(x_p) \left(\frac{q_T}{\langle q_T^2 \rangle}\right)^2 \frac{e^{-q_T^2/\langle q_T^2 \rangle}}{\langle q_T^2 \rangle}$$



# Summary

- All $\pi^- p$-DY asymmetries calculated at leading-twist to help interpret COMPASS. data on polarized DY.
- LFCQM and SPM went under a complete quantitative test and showed $20\text{-}40\%$. accuracy for most of the TMDs.
- Using different approaches we explored the model dependence of the predictions.
- The spread of the theoretical results are smaller than data uncertainties.
- Data favors Boer-Mulders $\bar{u}$-distribution in $\pi^-$, is positive.
- Proton Boer-Mulders $u$-distribution is positive too.
- More precise future data will help more stringent test for CQMs.




# Thank you!