

# Pion off-shell electromagnetic form factors

## Workshop on Pion and Kaon Structure Functions at the EIC -



J. Pacheco B. C. de Melo<sup>a</sup>

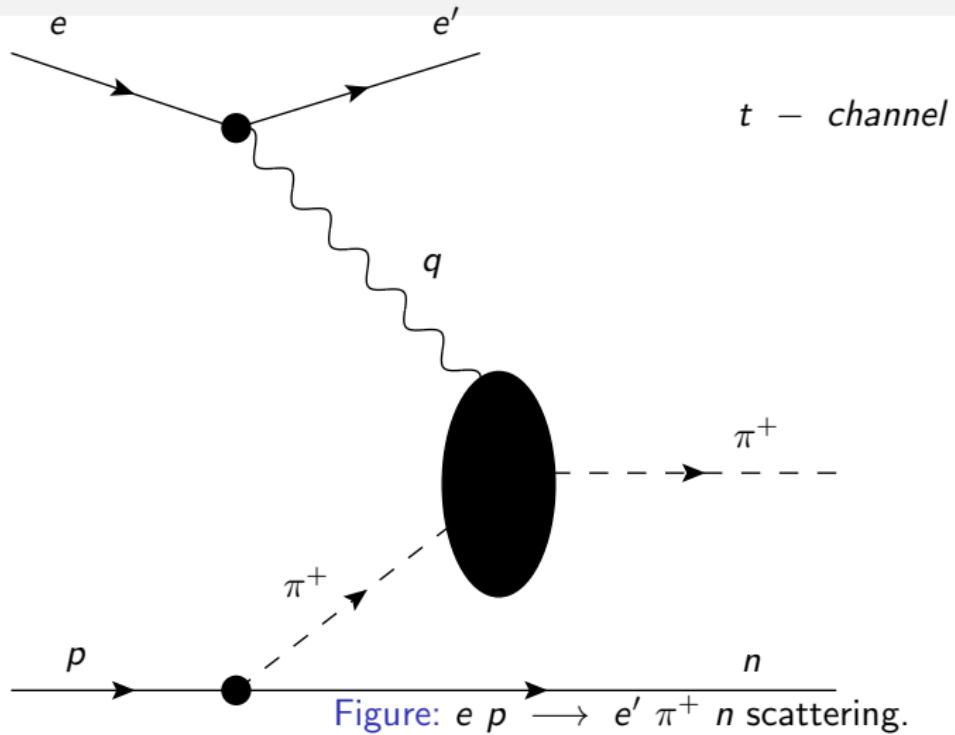
<sup>a</sup>Laboratório de Física Teórica e Computacional-LFTC, UCS (Brazil)

Collaborators: T. Frederico (ITA, Brazil), H. M. Choi (South Korea), Chueng -R. Ji (NCSU, USA) and Jurandi Leão (LFTC and IFSP)

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# Data from Experiments: Electroproduction



# Pion Electromagnetic Form Factor

- Most General Structure for the Vertex Function  $\Gamma^\mu$

$$\Gamma_\mu = e[(p + p')_\mu F_1 + (p - p')_\mu F_2]$$

and, with **Ward-Takahashi Identity-WTI**

$$q^\mu \Gamma_\mu = e \Delta_0^{-1} [\Delta(p) - \Delta(p')] \Delta_0^{-1}(p),$$

where,  $\Delta_0(p) = \frac{1}{p^2 - m^2 + i\epsilon}$ , and,  $\Delta(p) = \frac{1}{p^2 - m^2 - \Sigma(p^2) + i\epsilon}$ .

- Assuming (Standard renormalization)

$$\implies \{ \quad \Sigma(m^2) = 0$$

- **Half On-shell limit**  $\implies p'^2 = m_\pi^2$

$$\begin{aligned}\Delta_o^{-1}(p')\Delta(p') &\longrightarrow 1 \\ \Delta_0^{-1}(p')\Delta(p) &\longrightarrow 0\end{aligned}$$

So that,  $(m^2 - p^2)F_1 + q^2F_2 = m_\pi^2 - p^2$

- With the normalization  $\implies F_1(m_\pi^2, m_\pi^2, q^2 = 0) = 1$
- This gives us a relation between  $F_1$ , and  $F_2$ :

$$F_2(p'^2, p^2, q^2) = \frac{(m_\pi^2 - p^2)}{q^2} \left[ F_1(p'^2, p^2, 0) - F_1(p'^2, p^2, q^2) \right]$$

- Initial pion state off-shell mass:  $\Rightarrow p^2 = t \neq m_\pi^2$
- Final pion state on-shell:  $\Rightarrow p'^2 = m_\pi^2$
- We have the following final relation for the pion electromagnetic form factors:

$$F_2(Q^2, t) = \frac{(t - m_\pi^2)}{Q^2} [F_1(0, t) - F_1(Q^2, t)]$$

- half-on-shell  $\Rightarrow$  Pion photon Vertex:

$$\Gamma^\mu = (p'^\mu - p^\mu) F_1(Q^2, t) + q_\mu \frac{(t - m_\pi^2)}{Q^2} [F_1(0, t) - F_1(Q^2, t)]$$

- Important: The ratio of  $F_2(Q, t)$  to  $t - m_\pi^2$  is not zero in the limit  $t \rightarrow m_\pi^2$
- **However  $F_2(Q^2, t)$  goes to zero at that limit!!**
- **By definition:**

$$g(Q, t) = \frac{F_2(Q^2, t)}{t - m_\pi^2}$$

- **Off-Shell form factors "Sum Rule":**

$$F_1(Q^2, t) - F_1(0, t) + Q^2 g(Q, t) = 0$$

- Derivative in  $Q^2$  (**Master equation**):

$$\frac{\partial}{\partial Q^2} F_1(Q^2, t) + g(Q^2, t) + Q^2 \frac{\partial g(Q, t)}{\partial Q^2} = 0$$

- For  $g(Q^2 = 0, t) \implies$  **Pion electromagnetic radius**

$$g(Q^2 = 0, m_\pi^2) = -\frac{\partial}{\partial Q^2} F_1(Q^2 = 0, m_\pi^2) = \frac{1}{6} \langle r_\pi^2 \rangle$$

- even in **On-mass shell limit**  $\implies (Q^2, t = m_\pi^2)$  for  $g(Q^2, t)$   
 $\implies$  **On-mass shell solution**

$$g(Q^2, m_\pi^2) = \frac{1}{6} \langle r_\pi^2 \rangle + \alpha Q^2$$

- $\alpha \implies$  Expansion:  $\frac{\partial}{\partial Q^2} F_1(Q^2, t), \frac{\partial}{\partial Q^2} g(Q^2, t), (Q^2 \approx 0)$

- Derivative in t:

$$\frac{\partial}{\partial t} F_1(Q^2, t) - \frac{\partial F_1(0, t)}{\partial t} + Q^2 \frac{\partial g(Q^2, t)}{\partial t} = 0$$

- Again, derivative in t, Master equation:

$$\frac{\partial^2}{\partial t \partial Q^2} F_1(Q^2, t) + \frac{\partial g(Q^2, t)}{\partial t} + Q^2 \frac{\partial^2 g(Q^2, t)}{\partial t \partial Q^2} = 0$$

- $g(Q^2, t)$  is the new observable in on-shell limit
- Measurable in the electroproduction experiments

- Because no data available for  $F_1(0, t)$
- We need some model:  $\Rightarrow F_1(0, t)$ 
  - I) Covariant model
  - II) Light-front constitutive quark model (LFCQM)

# Extraction of the pion electromagnetic form factors, $F_\pi^1$ and $F_\pi^2$ from the experimental cross-section

- The electromagnetic pion form factor is extracted from the Exp. longitudinal cross section

$$\frac{d\sigma}{dt} \propto f(Q^2, W, t)$$

- Longitudinal cross-section and pion form factor

$$\frac{d\sigma_L}{dt} = \frac{16\pi}{137N} \frac{-tQ^2}{(t - m_\pi^2)^2} G_{\pi NN}^2(t) F_\pi^2(Q^2, t)$$

- Factor  $N = 32\pi (W^2 - m_p^2) \sqrt{(W^2 - m_p^2)^2 + Q^4 + 2Q^2(W^2 + m_p^2)}$

- Invariant mass of virtual-photon-nucleon  $W$

$$W = \sqrt{M_p^2 + 2M_p^2\omega - Q^2}, \text{ and } q_\gamma = (\omega, \vec{q}), \quad t = (p_\pi - q)^2$$

Ref. G. M. Huber *et al.* [Jefferson Lab Collaboration].

Phys. Rev. C78 (2008) 045203

- Pion electromagnetic form factor,  $F_\pi(Q^2, t) = F_\pi^1$  in terms of the cross-section

$$F_1^2(Q^2, t) = \frac{137N}{16\pi} \frac{1}{G_{\pi NN}^2(t)} \frac{(t - m_\pi^2)^2}{-Q^2 t} \frac{d\sigma_L}{dt}$$

- From the Cross section we can extract the function H

$$[F_1 \cdot G]^2 = H^2(Q^2, t) = \frac{137N}{16\pi} \frac{(t - m_\pi^2)^2}{-t Q^2} \frac{d\sigma_L}{dt}$$

- From  $F_1(Q^2, t)$  to  $F_2(Q^2, t)$

$$F_2(Q^2, t) = \frac{(t - m_\pi^2)}{Q^2} [F_\pi^1(t, p^2, 0) - F_\pi^1(Q^2, t)]$$

$$\implies F_1(m_\pi^2, m_\pi^2, q^2 = 0) = 1 \quad \text{Charge Normalization}$$

# Pion - Nucleon Form factor $G_{\pi NN}(t)$

- Two possible models for  $\pi - NN$  Vertex
- Choice I

$$G_{\pi NN}(t) = G_{\pi NN}(m_\pi^2) \left( \frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - t} \right)^n$$

- Choice II

$$G_{\pi NN}(t) = G_{\pi NN}(m_\pi^2) \frac{1}{\left(1 - \frac{t}{\Lambda^2}\right)^n}$$

Ref.: • G. M. Huber *et al.* [Jefferson Lab Collaboration].  
**Phys. Rev. C78 (2008) 045203.**

- R. Machleidt, K. Holinde, Ch. Ester, Phys. Rep. **149** (1987) 1.

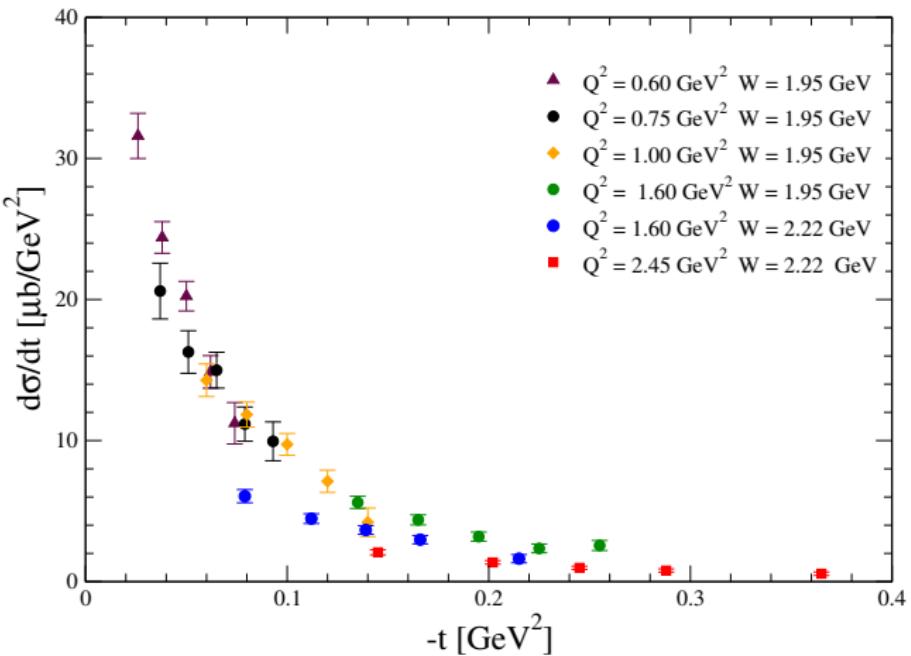
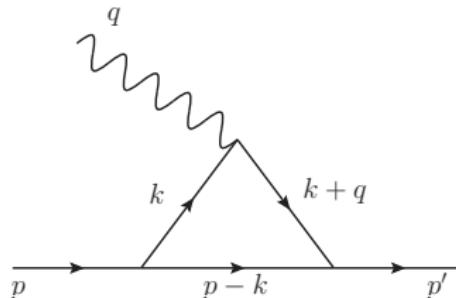


Figure: Experimental cross section,  $\sigma_I$  in function of  $t$ .  
 Data from Block et al., PRC78 (2008) 04520 2.

# Covariant Model: Manifestly covariant model



- Vertex function for the initial and final state pion:  $(p^2 = t)$  ,  $(p'^2 = m_\pi^2)$  (half off-shell regime)

$$\Gamma^\mu = iN_c g_{\pi q \bar{q}}^2 \int \frac{d^4 k}{(2\pi)^4} \frac{\text{Tr}[\gamma_5(k + q + m_q) \gamma^\mu (k + m_q) \gamma_5(k - p + m_q)]}{[k^2 - m_q^2 + i\epsilon][(k + q)^2 - m_q^2 + i\epsilon][(p - k)^2 - m_q^2 + i\epsilon]}$$

- Where  $N_c$  is the number of colors and  $g_{\pi q \bar{q}}$  coupling constant of the  $\pi q \bar{q}$  vertex

- Feynman Parametrization and dimensional regularization

$$d = (4 - 2\epsilon)$$

- Final results:  $F_1(Q^2, t)$  and  $F_2(Q^2, t)$

$$F_1(Q^2, t) = -\frac{N_c g_{\pi q \bar{q}}^2}{8\pi^2} \int_0^1 dx \int_0^x dy \left[ (1+3y) \left( \gamma - \frac{1}{\epsilon} + \frac{1}{2} + \text{Log} C \right) + \frac{\alpha}{C} \right]$$

$$F_2(Q^2, t) = -\frac{N_c g_{\pi q \bar{q}}^2}{8\pi^2} \int_0^1 dx \int_0^x dy \left[ 3(1-2x+y) \text{Log} C + \frac{2\beta - \alpha}{C} \right]$$

- Where  $\gamma$  is the Euler-Mascheroni constant and

$$\alpha = (1+y)(E^2 - m_q^2) - q \cdot E + 2yp \cdot E - yq \cdot p,$$

$$\beta = (1-x+y)(E^2 - m_q^2) + (1-2x+2y)p \cdot E + (x-y)q \cdot p$$

$$E = (x-y)q - yp, C = (x-y)(x-y-1)q^2 - y(1-y)t - 2y(x-y)q \cdot p + m_q^2,$$

and  $q \cdot p = (m_\pi^2 + Q^2 - t)/2$

- Loop correction to the charge form factor  $F_1(Q^2, t = m_\pi^2)$  vanish at  $Q^2 = 0$

- Redefine the renormalized charge form factor

$$F_1^{\text{ren.}}(Q^2, t) = 1 + [F_1(Q^2, t) - F_1(0, m_\pi^2)]$$

- Coupling  $g_{\pi q \bar{q}}$  is related with  $f_\pi$

$$\langle 0 | \bar{q} \gamma^\mu q | \pi(p) \rangle = i f_\pi p^\mu$$

- We get the Goldberg-Treiman at the quark level

$$\frac{g_{\pi q \bar{q}}}{2m_q} = \frac{F_1(0, m_\pi^2)}{f_\pi} + \mathcal{O}(\epsilon)$$

Adjusting model parameters ( $m_q, g_{\pi q \bar{q}}$ ).

Optimum ranges of quark masses,  $0.12 \leq m_q \leq 0.16$  GeV

Best fits for the coupling constants

$$g_{\pi q \bar{q}} = (1.32, 1.20, 1.11) \rightarrow (2m_q/f_\pi^{\text{Exp.}}) \text{ for } m_q = (0.12, 0.14, 0.16) \text{ GeV}$$

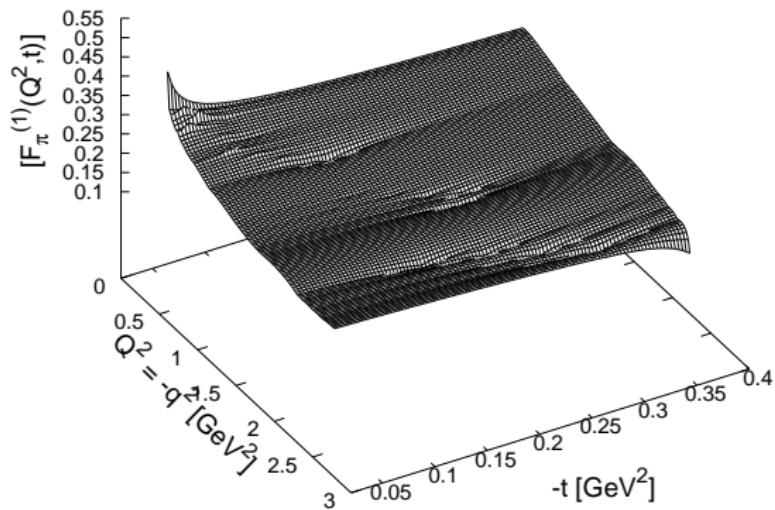
- Refs.: "Pion off-shell electromagnetic form factors: data extraction and model analysis"

H. M. Choi, T. Frederico, C. R. Ji and J. P. B. C. de Melo  
Phys. Rev. D 100, no. 11, 116020 (2019).

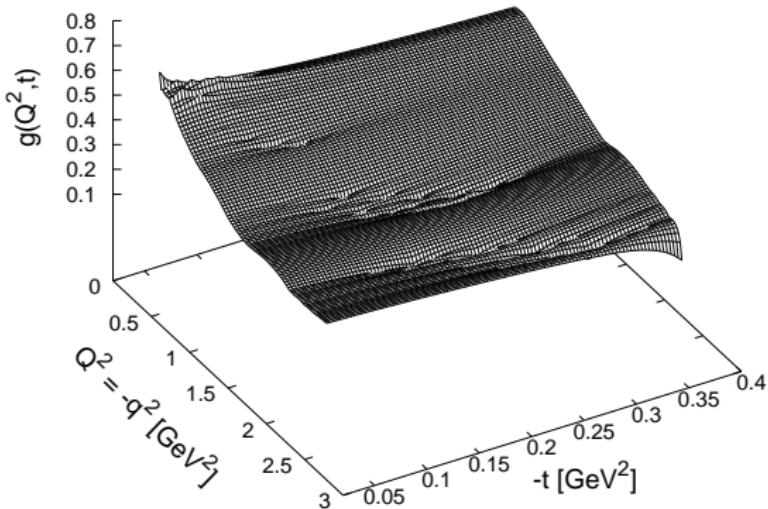
- 3D imaging of the pion off-shell electromagnetic form factors

H. M. Choi, T. Frederico, C. R. Ji and J. P. B. C. de Melo  
POS (LC2019) 035

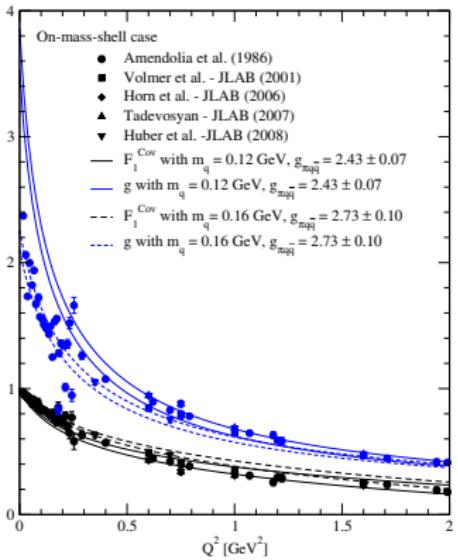
# Extraction from the experimental data



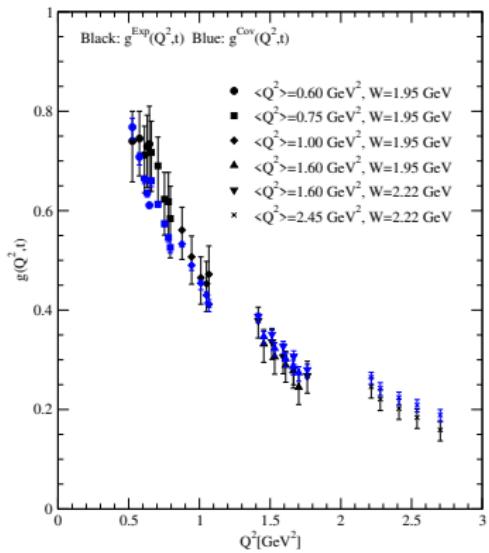
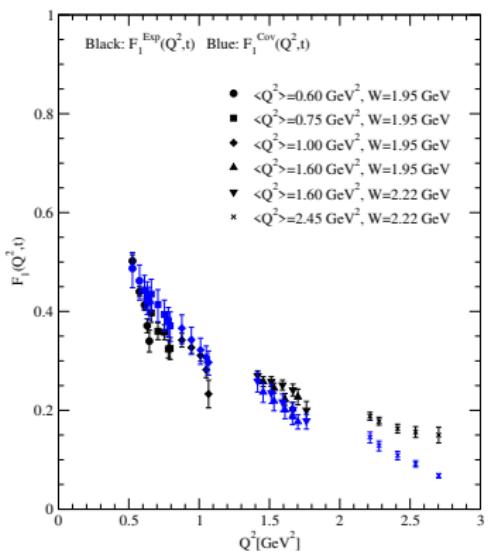
3-D electromagnetic pion form factor  $F_1(Q^2, t)$

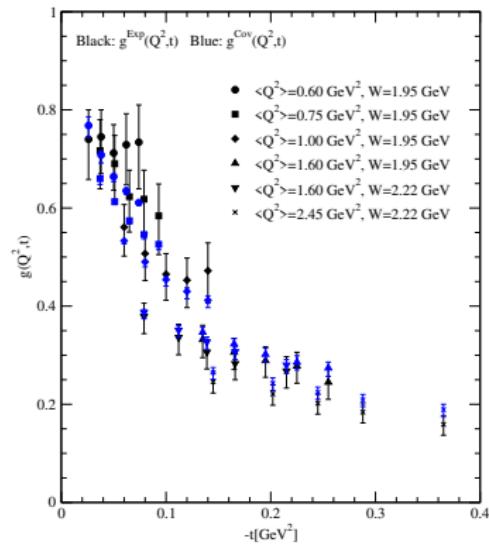
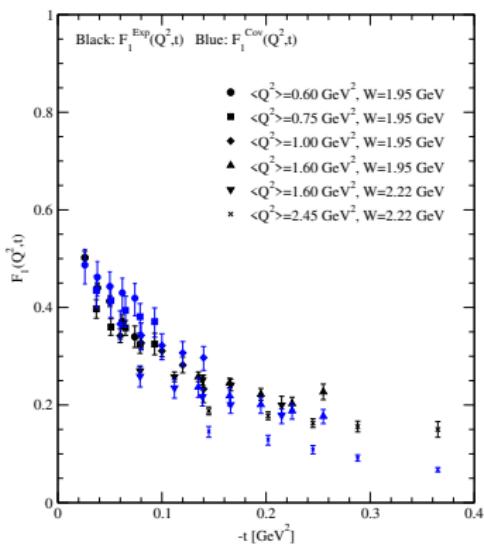


$$F_1(Q^2, t) - F_1(0, t) + Q^2 g(Q^2, t) = 0$$



$F_1(Q^2, m_\pi^2)$  (**black lines**) and  $g(Q^2, m_\pi^2)$  (**blue lines**) for the spacelike momentum transfer region  $0 \leq Q^2 \leq 2$   $\text{GeV}^2$  compared with the experimental data for  $F_1^{\text{Exp.}}$  (**black data**) and  $g^{\text{Exp.}}$  (**blue data**).  
 $g_{\pi q\bar{q}} = (1.32 \pm 0.04, 1.11 \pm 0.04)(2m_q/f_\pi^{\text{Exp.}})$ , respectively. we show only the upper and lower limits of  $g_{\pi q\bar{q}}$ .





# Light-Front Model: Non-Symmetric Vertex

- Electromagnetic current

$$J^\mu = -i2e \frac{m^2}{f_\pi^2} N_c \int \frac{d^4 k}{(2\pi)^4} Tr \left[ S(k) \gamma^5 S(k-p') \gamma^\mu S(k-p) \gamma^5 \right] \Gamma(k, p') \Gamma(k, p)$$

- Quark propagator  $\Rightarrow S(p) = \frac{1}{p-m+i\epsilon}$
- Numbers Colors  $\Rightarrow N_c = 3$  and Factor 2 from the isospin algebra
- Non-symmetric Vertex Function

$$\Gamma^{NSY}(k, p) = \left[ \frac{N}{((p-k)^2 - m_R^2 + i\epsilon)} \right]$$

$\Rightarrow$  Ref.: J. de Melo, H. Naus, H. and T. Frederico, Phys. Rev.C59 (1999)  
2278

E. Silva, J. P. de Melo, B. El-Bennich, and Victor Filho, Phys. Rev. C86  
(2012) 038202

# Pion Light-Front Wave Function

$$\Psi(x, k_\perp, p^+, \vec{p}_\perp) \propto \left[ \frac{1}{(1-x)(m_{0-}^2 - \mathcal{M}^2(m_q^2, m_R^2))(m_{0-}^2 - \mathcal{M}^2(m_q^2, m_{\bar{q}}^2))} \right]$$

here:

$$\mathcal{M}^2(m_a^2, m_b^2) = \frac{k_\perp^2 + m_a^2}{x} + \frac{(p - k)_\perp^2 + m_b^2}{(1-x)} - p_\perp^2$$

- In the case of quarks mass  $\Rightarrow$  Free Mass Operator
- $m_{0-} \iff$  Mass of the bound state

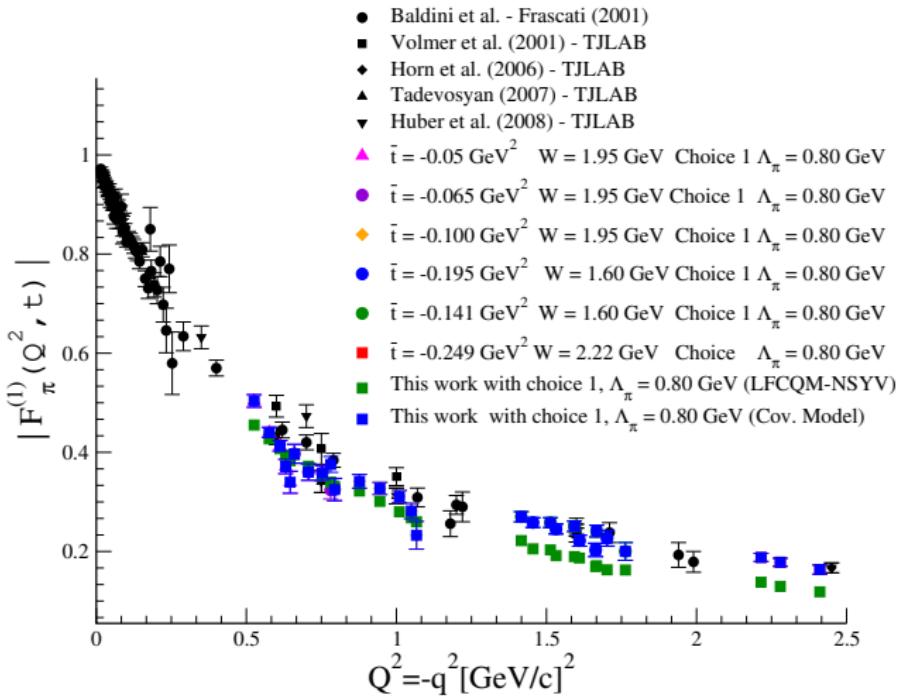
- Some Results from LFCQM -(with NSY-Vertex)

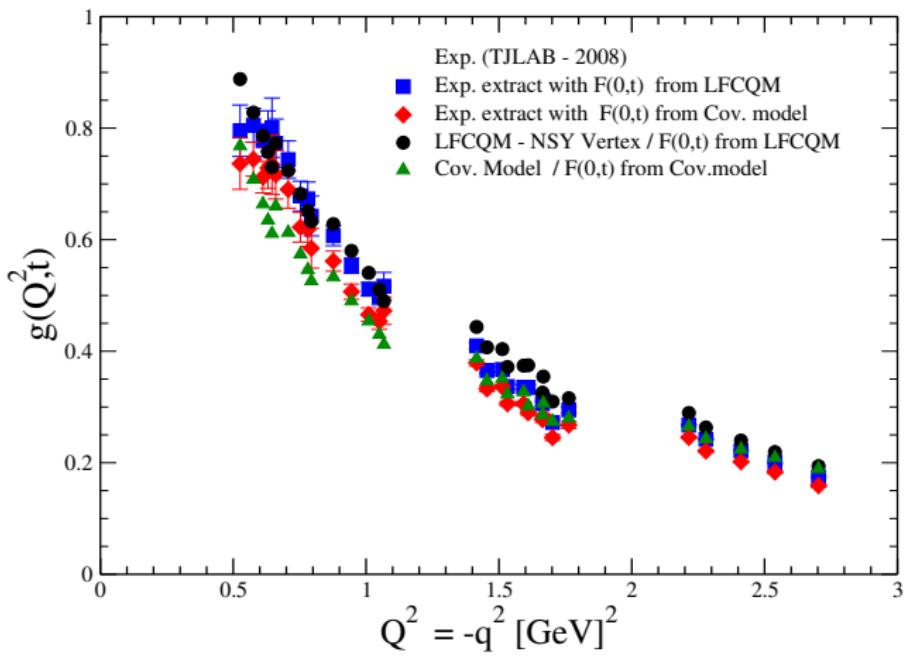
$\implies$  **Parameters:**  $m_R = 1.00 \text{ GeV}$ ,  $m_u = m_d = 0.220 \text{ GeV}$ ,  
 $m_s = 0.510 \text{ GeV}$

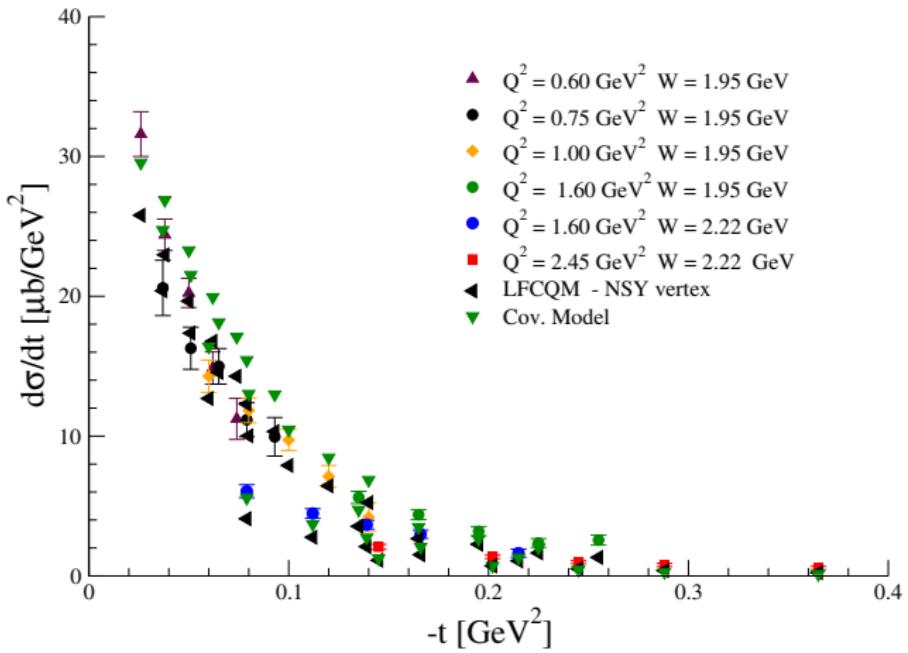
$f_\pi = 93.1 \text{ MeV}$	$(f_\pi^{\text{exp.}} = 92.42 \text{ MeV})$
$f_K = 126.9 \text{ MeV}$	$(f_K^{\text{exp.}} = 110.4 \text{ MeV})$
$\langle r_\pi \rangle = 0.679 \text{ fm}$	$(r_\pi^{\text{exp.}} = 0.672 \text{ fm})$
$\langle r_K \rangle = 0.636 \text{ fm}$	$(r_\pi^{\text{exp.}} = 0.560 \text{ fm})$

- Ref.:

- de Melo, Naus, H., Frederico, T., Phy. Rev. C59 (1999) 2278.
- E. Silva, de Melo, B. El-Bennich, V. S. Filho, Phy. Rev. C86 (2012) 038202
- The Review of Particle Physics (2019) M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018) and 2019 update







- $F_1(Q^2, t)$  extracted from experiments: Cross section
- Choose  $G_{\pi NN}(t)$  with two different parametrizations
- Models to  $F_1(0, t)$
- Use  $F_1(Q^2, t)$  to extract  $F_2(Q^2, t)$
- $F_2(Q^2, t) \neq 0$  no matter what  $G_{\pi NN}(t)$  is used
- Define  $g(Q^2, t)$  from  $F_2(Q^2, t)$
- Models to calculated,  $F_1(Q^2, t)$  and  $g(Q^2, t)$
- Comparison with experimental data
- Our sum rule (master equation) and theoretical formulation for the half-off-shell form factor are completely independent from any assumptions (pole vs. non-pole contributions) in extracting the  $F_1(Q^2, t)$  from the longitudinal cross section
- If experimentalists provide the longitudinal cross section with and without non-pole contributions, then we can estimate how much the non-pole contributions affect our data extraction for the half-off-shell form factors  $F_1(Q^2, t)$  and  $F_2(Q^2, t)$

# Next

- Continue to study the model dependence
- Correction from excited states  $m_{\pi^*}$  and "some parametrization" to  $G_{\pi^*NN}^*(t) \Rightarrow$  extract  $F_{\pi^*NN}^*(Q^2, t)$  from data?
- Compute  $G_{\pi^*NN}$  from some model
- Forbidden region of extrapolation is much larger for the Kaon than for the Pion application of our sum rule (master equation) to the Kaon form factor would be more interesting

# Thanks to the Organizers

## EIC 2020

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- FAPESP , CNPq and CAPES

Thanks (Obrigado)!!

