Family Resemblances among QCD's Close Relatives

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Outline

- Large N QCD – why it's interesting to a lattice person
- Mesonic observables
- Baryons and their rotor spectrum
- Flavor $SU(3)$ spectroscopy
- Alternate large N constructions

See Phys. Rev. D86, 034508 (arXiv:1205.0235); D89, 014506 (arXiv:1308.4114;) D90, 014505 (arXiv:1404.2301)
Just a fun project for the "post-QCD" era

Why is large-N lattice QCD interesting?

- It's the source of all qualitative understanding of hadronic physics
- "No free parameters for QCD" since the coupling runs.
- 't Hooft (1974): Treat $1/N$, where N is the number of colors, as a small number
- Familiar scaling story for mesons as $\bar{q}q$ objects with masses independent of N
- Extensive successful (pre-lattice) phenomenology for matrix elements (B_K, etc) and their regularities
- An interesting set of stories for baryons, too
- Since 1974, MANY other large-N stories
  - Higher representation fermions – Corrigan - Ramond, AS2 rep fermions
  - Orientifold equivalence, volume reduction, etc etc
- Composite Higgs phenomenology often uses large-N to extrapolate predictions

And now we can simulate these systems, rather easily, and see how well large-N worked.

Fun, easy projects!

Baryons in large N

- Mass $\propto N$ (for fundamentals, to $N(N-1)/2$ for AS2)
- Hartree approximation: 2-body interaction $V \sim g^2/N$, many little interactions, central limit theorem
  - baryon size is N independent
- Large-N QCD is EFT of mesons
  - Coupling is $1/F_\pi^2 \sim 1/N$ (fundamentals)
  - Baryon mass $\sim N$ or 1/coupling – soliton-like – Skyrme model as an explicit phenomenology
- Either way, mass formulas in $1/N$ – tested
- Spin - isospin locking for flavor $SU(2)$: $J = I$ for $J = N/2,\ N/2 - 1. \ldots$ – tested
- Mass relations imply stringent matrix element relations and vice versa – so far, not tested

Technical issues for lattice simulations

I am using an NCOL version of the Milc code written by Svetitsky, Shamir and me for BSM studies

- $N = 3$, 5, 7 with fundamentals
- I did quenched approximation, first
  - Easy to justify for large N
  - Have since done dynamical fermion sims for $N = 4$ AS2
- Tune bare couplings to match lattice spacings (from the potential)
- Observe this matches $\lambda = g^2 N$, too
- Match to same volumes, roughly same quark masses
- Compare the usual lattice-y dimensionless ratios of things
- Need baryon correlators for arbitrary N – a long story with much room for improvement
- Other little technical issues

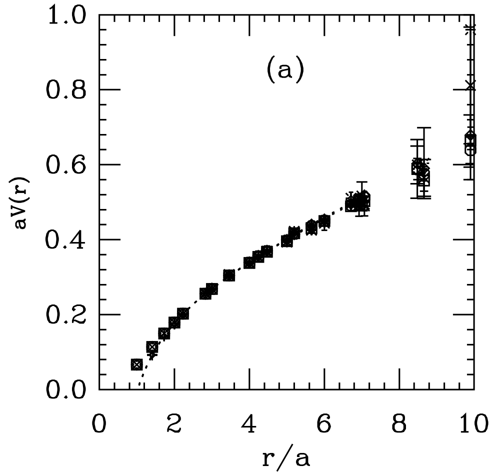
But – large N isn't about small m_q or even $a \to 0$, so it's easy

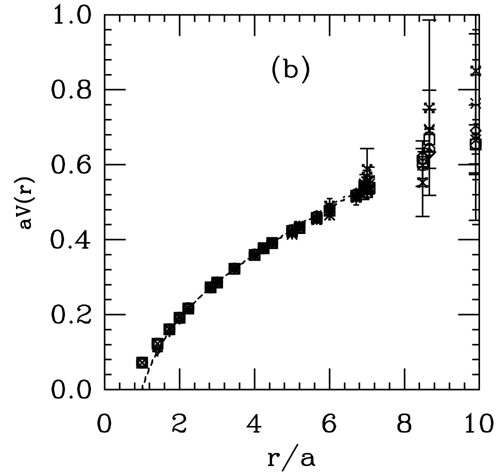
Potentials set the scale

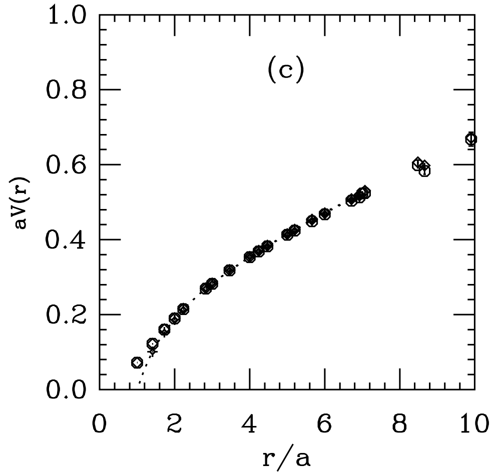
- $16^3 \times 32$ volume
- Match Sommer parameter, $r_0 = 0.5$ fm ($r_0^2 F(r_0) = 1.65$) or $r_1 = 0.3$ fm
- Lattice spacing $a = 0.08$ fm or $1/a = 2400$ MeV
- $r_0\sqrt{\sigma} = 0.5$ fm$\times 440$ MeV=1.12

Fubdamental rep fermion data sets

N	3	5	7
lattices	80	120	160
β	6.0175	17.5	34.9
$\lambda = g^2 N$	2.99	2.85	2.80
r_1/a	3.88(2)	3.77(2)	3.89(2)
$r_0\sqrt{\sigma}$	1.174(4)	1.172(3)	1.168(2)



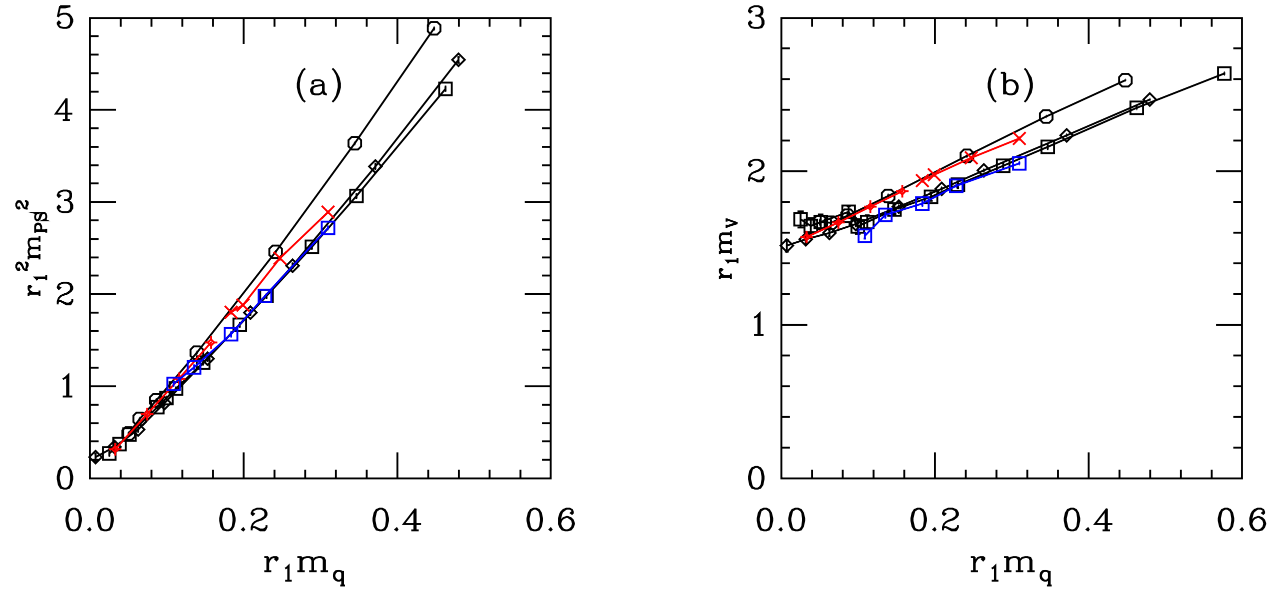




Potentials for $SU(3)$, $SU(5)$, $SU(7)$. Can you tell which is which?

Meson spectroscopy

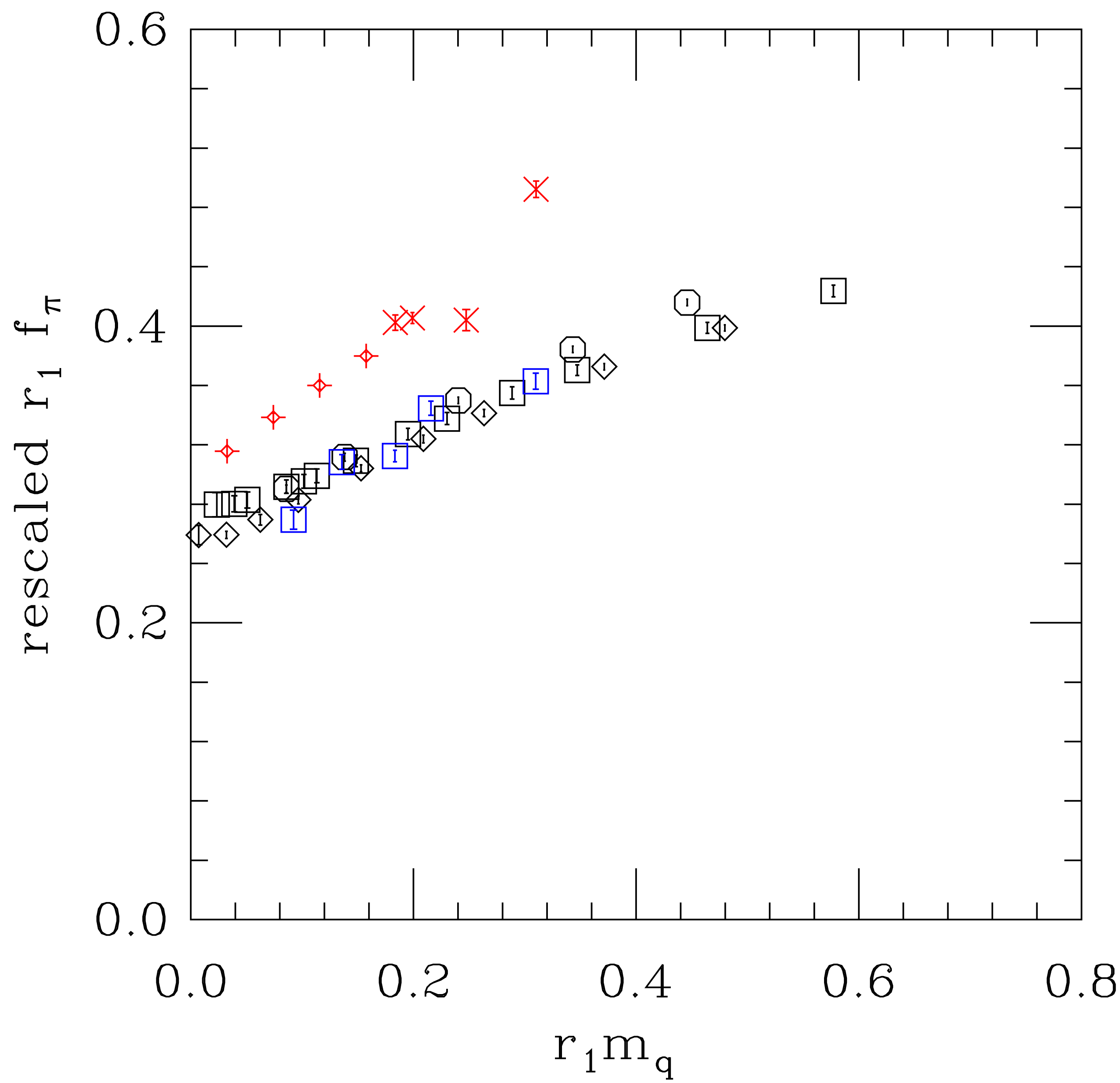
Meson masses should be N-independent



$N-$ independence, also seen by everybody else, over much wider N range (Bali et al) (a) pseudoscalar mass squared (b) vector ($1/r_1 \sim 650$ MeV)

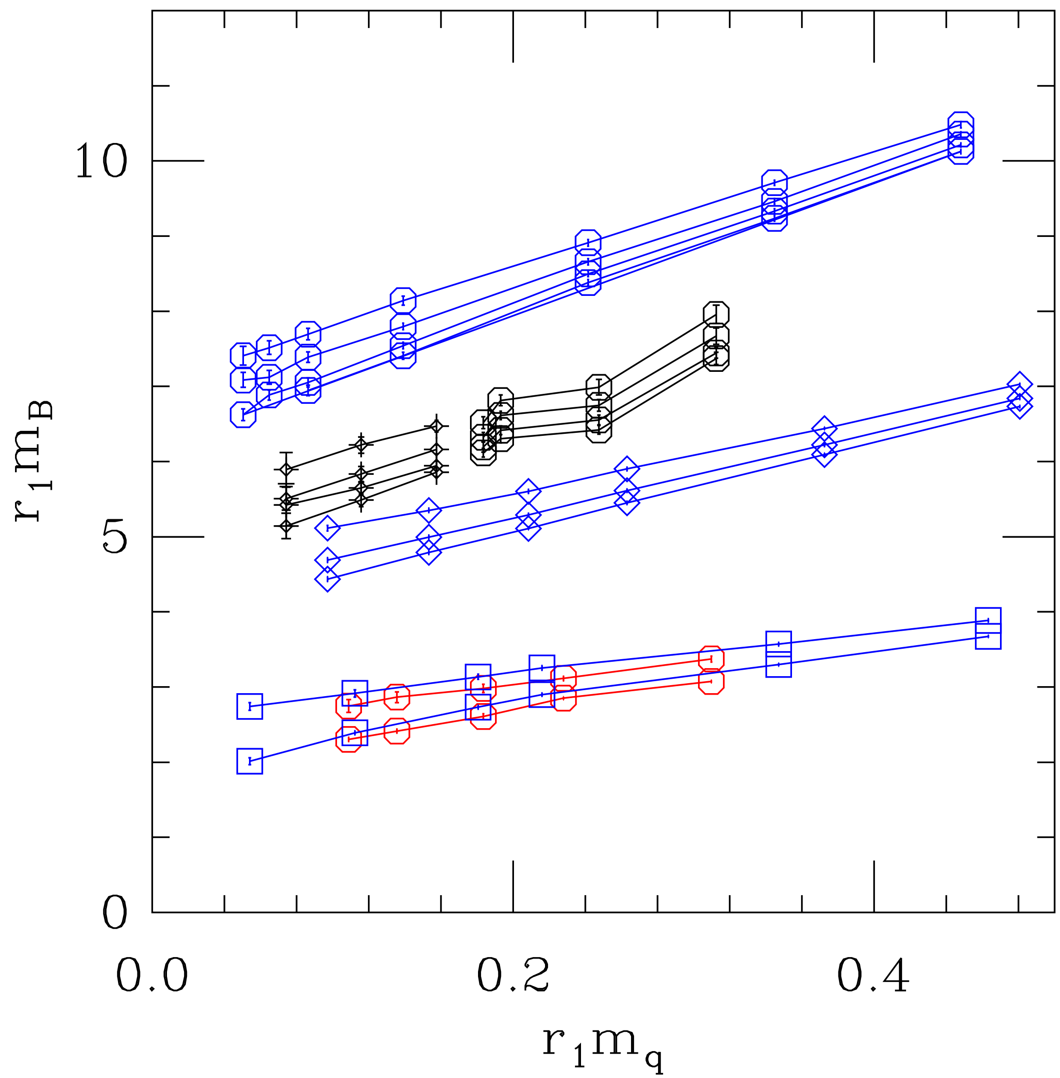
Black squares, diamonds, octagons for quenched $SU(3)$, $SU(5)$, $SU(7)$ fundamentals, red crosses for $SU(4)$ AS2; the fancy diamonds are the PQ data. Blue squares $SU(3)$ with two dynamical, fundamental flavors.

Decay constants $\langle 0|V|h\rangle \propto 1/\sqrt{N} \times N \propto \sqrt{N}$ for fundamentals, $\propto N$ for AS2



Pseudoscalar decay constant scaled to $SU(3)$ fundamentals by dividing by appropriate power of $N/3$

Baryon spectroscopy



The blue data are from the top quenched $SU(7)$,SU(5) and $SU(3)$ data. The red octagons are $SU(3)$ with dynamical fermions. The black lines are the six-quark baryons in $SU(4)$ AS2, octagons for dynamical and fancy diamonds for partially quenched.

Baryons – theory

The Rotor spectrum: large N baryon masses generically obey

$$M(N, J) = Nm_0 + \frac{J(J+1)}{N}B + \ldots \tag{1}$$

m_0 and B are m_q dependent, so do comparisons versus m_q

Test the $J(J+1)$: ratios of mass differences are pure numbers

$$\Delta(J_1, J_2, J_3) = \frac{M(N, J_2) - M(J_3)}{M(N, J_1) - M(N, J_3)}, \tag{2}$$

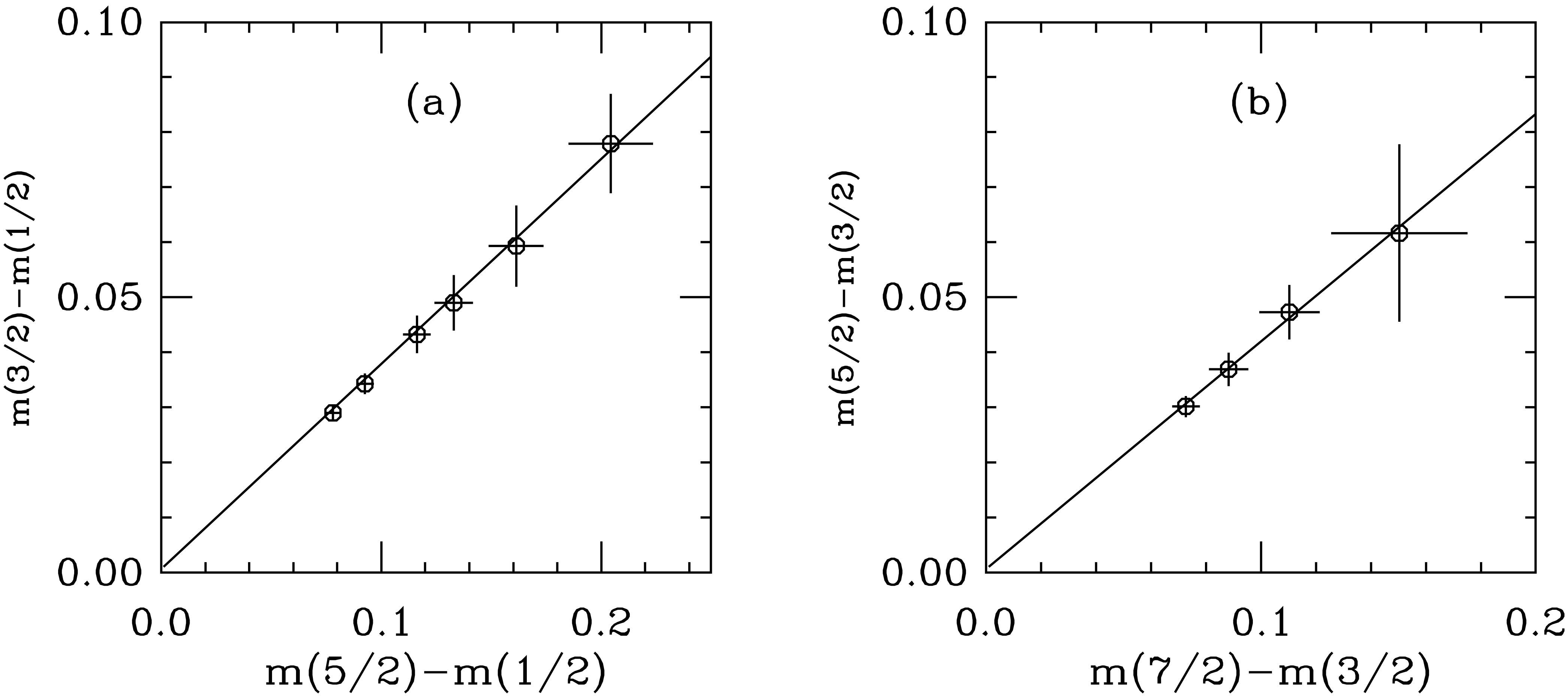
Exposing B: from the bottom of the multiplet

$$M(N, 3/2) - M(N, 1/2) = \frac{3B}{N} \tag{3}$$

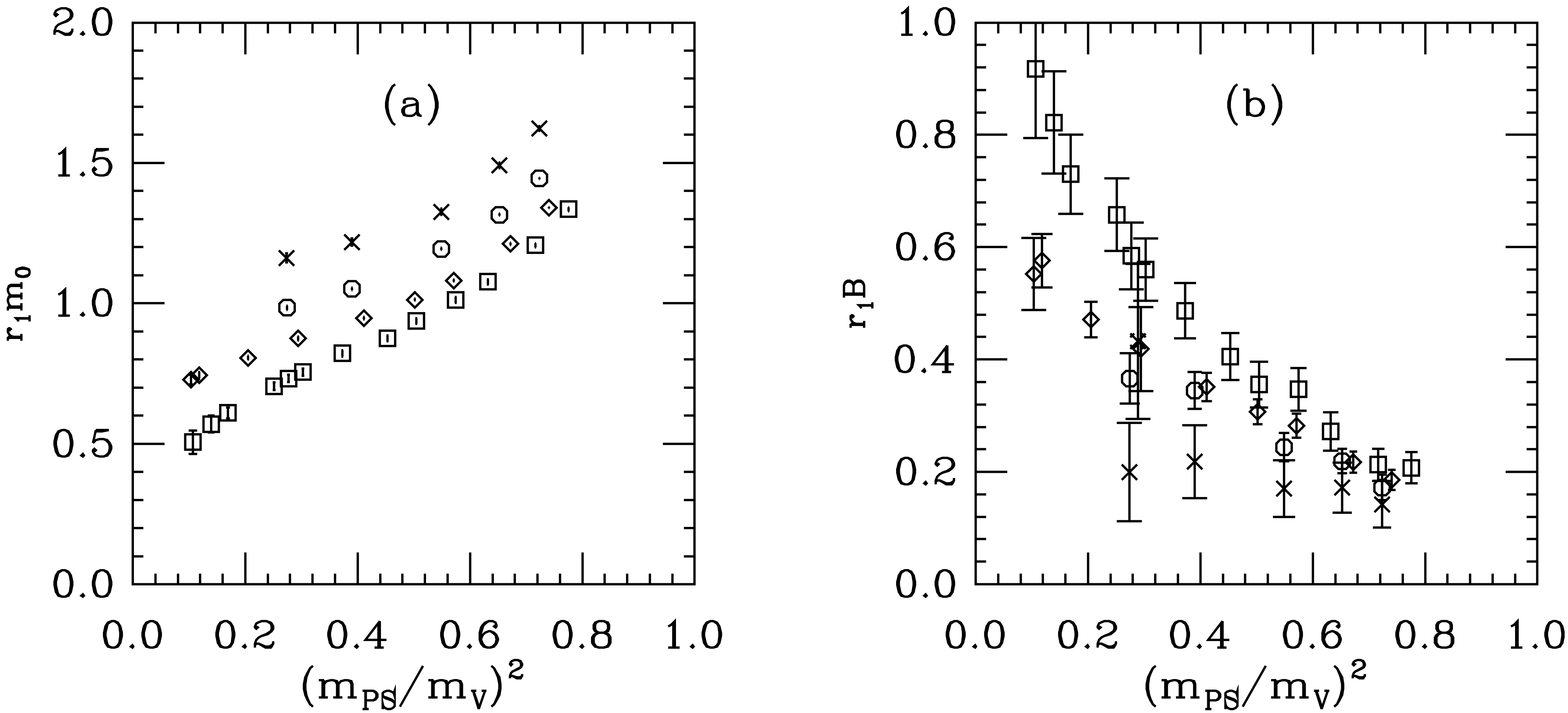
From the top of the multiplet a "rescaled Landé interval rule"

$$M(N, J = N/2) - M(N, J = N/2 - 1) = B \tag{4}$$

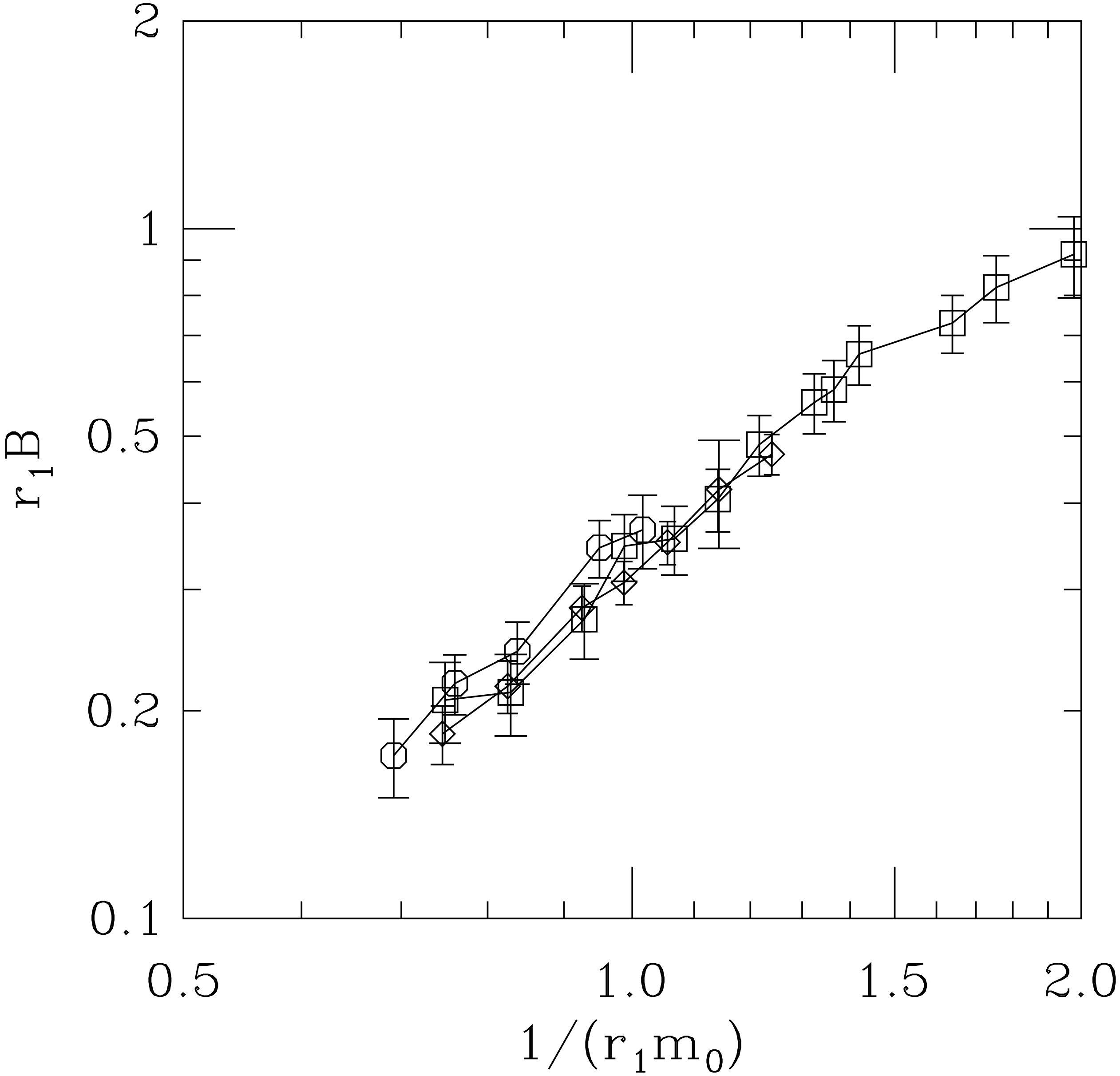
Or, just fit all states' masses to two parameters (m_0 and B), plot vs m_q



Ratios of mass differences for $N = 5$ and $N = 7$. Lines are the analytic ratio (NOT a fit)



m_0 and B from $N = 3$, 5, 7. Crosses extrapolations to $1/N \to 0$. Modest drift with $1/N$.



B vs $1/m_0$ from quenched $SU(N)$ fundamental quark baryons

On to flavor $SU(3)$

The obvious project: flavor $SU(3)$ spectra, with different nonstrange and strange masses

- Lots of mass relations in $1/N$ and $\delta m = m_s - m_{ns}$
- Most published predictions are only for $J = 1/2$ and 3/2 (because those are the only states we have)

There are lots of states, analogs of Δ's, Σ's, Λ's, nucleons at many J's

The project was mostly an issue of organization (and CG coefficients)

A quark model story for baryon spectroscopy

Colorspin! (De Rujula, George, Glashow; also MIT bag model, 1974)

m_i is the constituent quark mass

$$H_0 = \sum_i n_i m_i \tag{5}$$

There is also a color magnetic dipole (hyperfine) interaction

$$H_1 = \sum_{i \neq j} F_{ij} \lambda_i \lambda_j \sigma_i \sigma_j \tag{6}$$

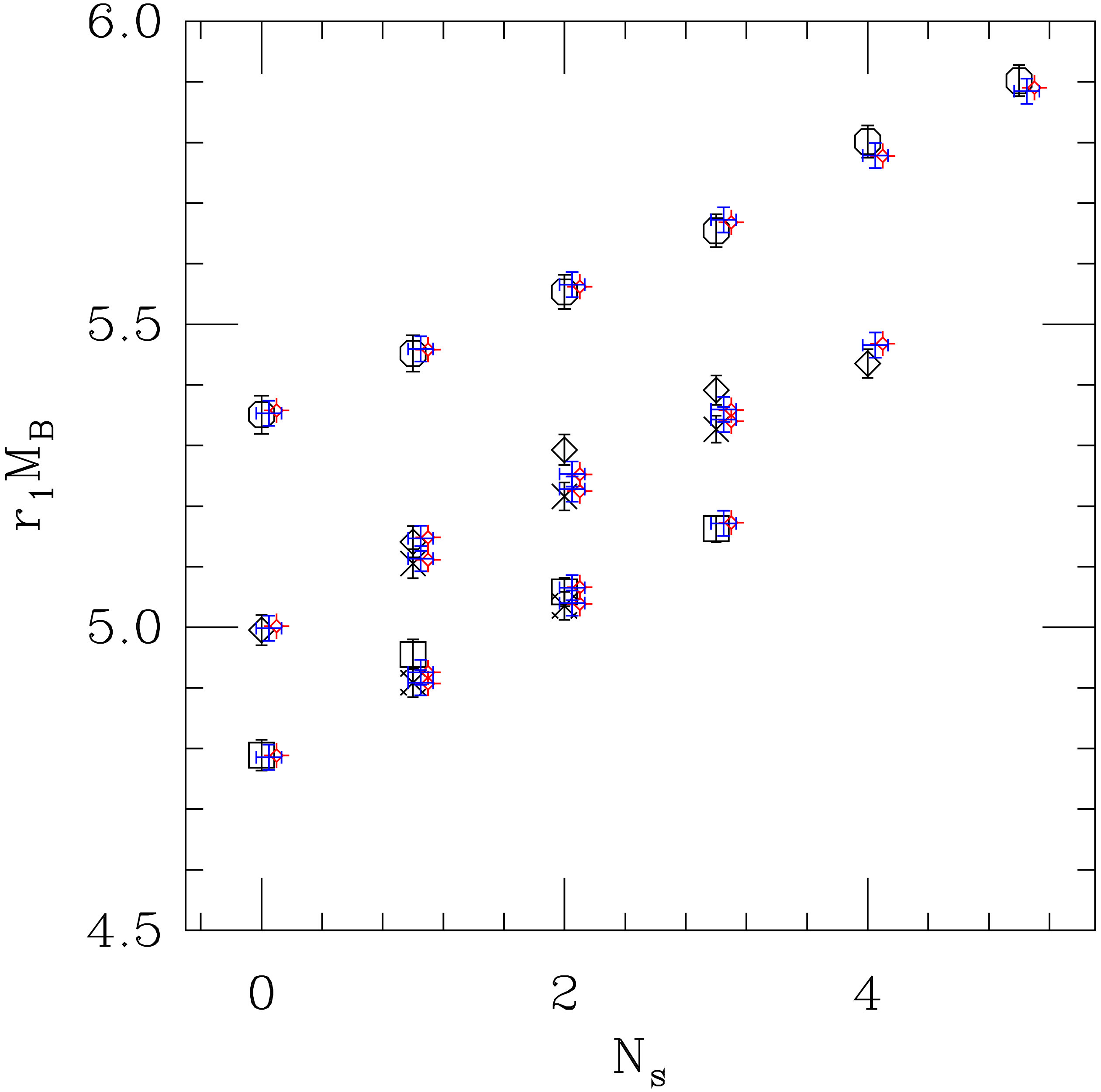
where

$$F_{ij} = \mu_i \mu_j = \frac{g}{m_i} \frac{g}{m_j} \propto g^2 = \frac{\lambda}{N} \tag{7}$$

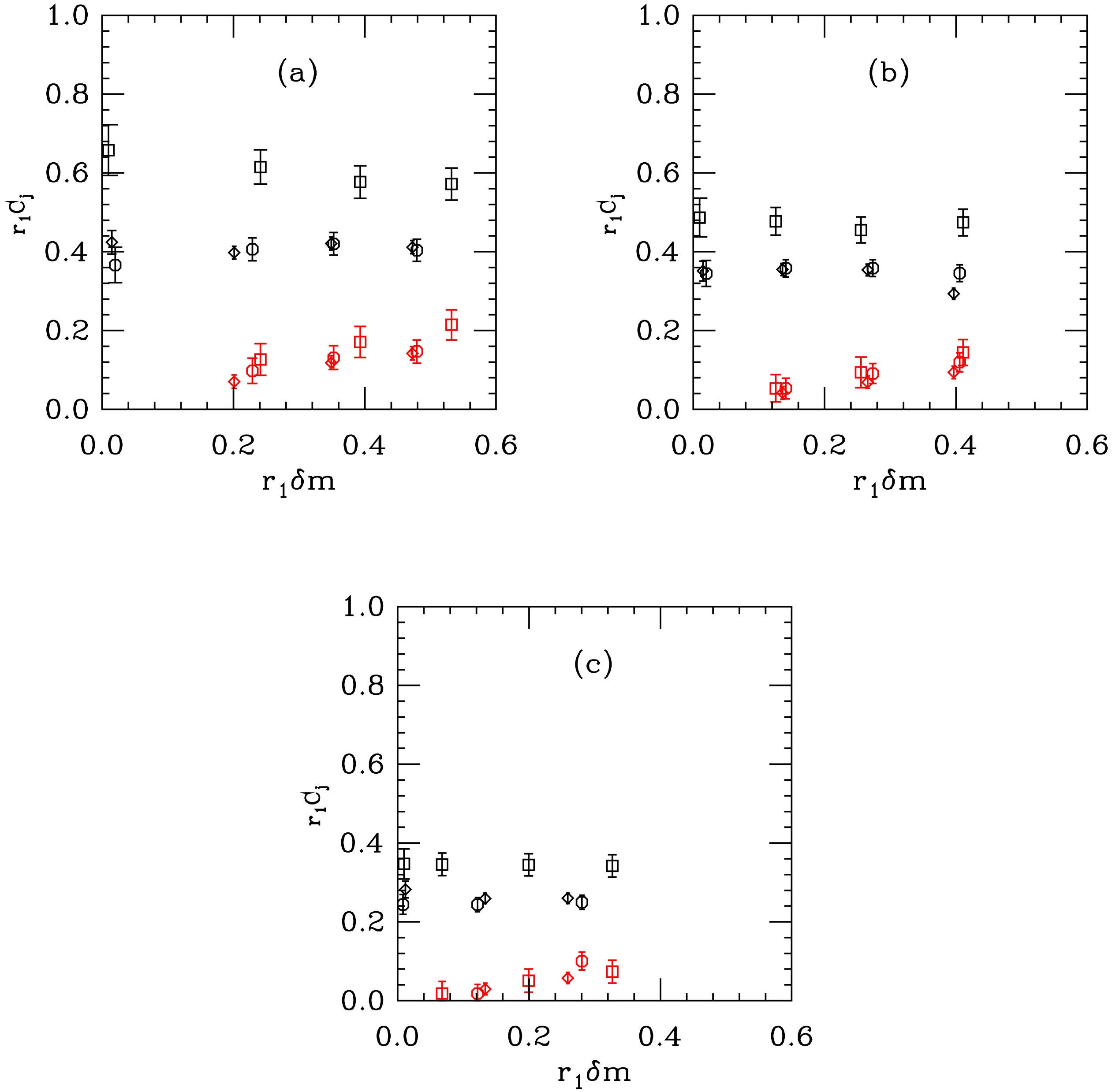
For degenerate mass quarks the rotor spectrum follows, simply,

$$\frac{f}{N}\left(\sum_i \sigma_i \sum_j \sigma_j - \sum_i \sigma_i^2\right) = f\frac{J^2}{N} + C \tag{8}$$

For flavor $SU(3)$, slightly increase $m_s = m_i + \delta m$ and decrease the HFS term between s quarks



$SU(5)$ spectrum at one set of quark masses with two large-N fits overlaid



Color hyperfine interaction model parameters from fits to flavor $SU(3)$ data at matched $(m_{PS}/m_V)^2$ Data are squares for $N = 3$, diamonds for $N = 5$ and octagons for $N = 7$.

Summary

Surprisingly easy to see interesting physics

- Meson masses show expected large-N behavior
- Meson decay constants (mostly) scale appropriately with N
- Rotor spectrum for baryons works well
- Can see corrections to leading order, they seem to be $1/N$ with unsurprising size
- Flavor $SU(3)$ works as expected

Properties of mesons and baryons seem quite generic

Still things to do:

- Smaller m_q, to test chiral PT – this will require dynamical fermions
- Could be a large program of matrix elements vs $1/N$
- Large N will play a big role in the presentation of our (Neil, Svetitsky, Shamir, Jay, Liu) composite Higgs studies

It's a fun project, there is room for more people–

Thanks, Mike, for finding such a nice playground!