Creutz ratio in the continuum limit

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Creutzfest

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Recently, Gonzalez-Arroyo and I propose a new method
to calculate Creutz ration in the continuum limit
from 4 dimensionally smeared Wilson loops.

We demonstrate the practical advantage of our method
by applying it to the calculation of the string tension both in

SU(N) lattice gauge theory

and

Twisted Eguchi-Kawai (TEK) model for large N LGT.

- I was postdoc for three years in Long Island.
  - ◎ Sep. 1981 – Aug.1983 : BNL
  - ◎ Sep. 1983 – May.1984 : Stony Brook

- I wrote two papers with Mike
  - ★ Generalized actions in Z(P) lattice gauge theory,
    M. Creutz and M. O. , Nucl. Phys. B220 (1983) 149.
  - ★ Microcanonical renormalization group,
    M. Creutz, A. Gocksch, M. Ogilvie and M. O. ,
    Phys. Rev. Lett. 53 (1984) 875.

- I also have *"Quarks, Gluons, and Lattices", but* ・・・・

- I also wrote several papers on twisted space-time reduced model with A. Gonzalez-Arroyo at BNL.

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Twisted-Eguchi-Kawai model: A reduced model for large-N lattice gauge theory

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(Received 15 December 1982)

We study the large-N reduced model recently proposed by the present authors. This model is a modified version of the Eguchi-Kawai model incorporating twisted boundary conditions. It is shown that the Schwinger-Dyson equations of our model are the same as in the infinite-lattice theory provided $[U(1)]^4$ symmetry is not spontaneously broken. We study the model at strong coupling, weak coupling, and intermediate coupling using analytical and Monte Carlo techniques. At weak coupling, it is shown that for a particular choice of twist, $[U(1)]^4$ symmetry is not broken and we prove how one recovers usual planar perturbation theory. Monte Carlo data for χ ratios show striking agreement with Wilson-theory results.

I. INTRODUCTION

The $N\to\infty$ limit[1] of lattice gauge theories[2] has been the subject of many interesting studies in recent years. The ultimate hope is to be able to solve the theory in this limit. However, this aim has not been achieved at present, despite the remarkable simplifications already discovered. It is known that only planar diagrams survive in this limit,[1] and that the was known even before the work of Eguchi and Kawai.[14] Since then Monte Carlo simulations have shown that this in fact takes place.[13,15]

To save the idea of the reduction of degrees of freedom Bhanot, Heller, and Neuberger[13] proposed the quenched-Eguchi-Kawai (QEK) model. In this model the eigenvalues of the link matrices are quenched, i.e., treated as classical variables. Physical quantities are averaged over all values of the

● Eguchi-Kawai（EK) model (1982)

Obtained from the usual SU(N) lattice gauge theory

$$Z_W = \int \prod_{x,\mu} dU_{x,\mu} \exp\left\{ bN \sum_x \sum_{\mu\neq\nu=1}^{d} Tr\left(U_{x,\mu} U_{x+\mu,\nu} U^+_{x+\nu,\mu} U^+_{x,\nu}\right) \right\}$$

by neglecting the space-time dependence of the link variables

$$U_{x,\mu} \to U_\mu$$

$$Z_{EK} = \int \prod_{\mu} dU_\mu \exp\left\{ bN \sum_{\mu\neq\nu=1}^{d} Tr\left(U_\mu U_\nu U^+_\mu U^+_\nu\right) \right\}, \qquad b = \frac{1}{g^2 N}$$

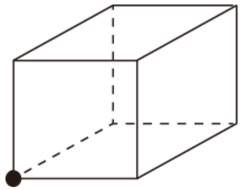
Both theories are equivalent in the large N limit, if $Z(N)^d$ symmetry is not broken.

$$U_\mu \to e^{i\theta_\mu} U_\mu, \qquad e^{i\theta_\mu} \in Z(N)$$

However, this symmetry is spontaneously broken in the weak coupling region.

Bhanot, Heller and Neuberger (1982)

● Twisted Eguchi-Kawai (TEK) model   Gonzalez-Arroyo, M. O. (1983)



EK model can be viewed as the usual Wilson gauge theory having only one site with the periodic boundary conditions.

We introduce twisted boundary conditions in $SU(N)$, $N = L^2$ theory

$$S_{TEK} = bN \sum_{\mu \neq \nu = 1}^{d} Tr\left(Z_{\mu\nu} U_{\mu} U_{\nu} U_{\mu}^{+} U_{\nu}^{+} \right)$$

$$Z_{\mu\nu} = \exp\left(k \frac{2\pi i}{L} \right), \quad Z_{\nu\mu} = -Z_{\mu\nu}, \qquad \mu > \nu$$

In 2003, Tomomi Ishikawa and I found that $Z(N)^d$ symmetry is broken for N>100 if k=1.

see also Teper, Vairinhos (2007)

Azeyanagi, Hanada, Hirata, Ishikawa (2008)

Ishikawa and I also conjectured that by keeping $k / \sqrt{N}$ finite as we take large N limit, this problem should be circumvented.

In 2010, Gonzalez-Arroyo and I confirmed this conjecture up to N =1521.

If the TEK model is correct nonperturbatively, we should be able to calculate the string tension in the large N limit.

It is well know that Wilson loops $W(R,T)$ with large R,T are quite noisy.

We tried to calculate string tension from $q\bar{q}$ potential evaluated from 3-dmensionally smeared spatial Wilson loops.

However, for TEK model, noise of $q\bar{q}$ potential is quite large.

We then try alternative method.

● Creutz ratio from 4-dimensional smearing method

Our proposal is to calculate Creutz ratio with 4-dimensional Ape smearing

$$U_{n,\mu}^{smeared} = \mathrm{Proj}_{SU(N)}\left[(1-f)U_{n,\mu} + \frac{f}{6}\sum_{\nu\neq\mu=\pm1}^{\pm4} U_{n,\nu}U_{n+\nu,\mu}U_{n+\mu,\nu}^{+} \right]$$

Let $t = f * n_s / 6$ with n_s the number of smearing steps.

In the following, we explain our method using the data of SU(3) LGT on a 32^4 lattice at $\beta = 6.17$ with $f = 0.1$.

Wilson loop and potential have huge t dependences, which makes 4-d Ape smearing almost useless for these quantities

It is crucial to consider Creutz ratio

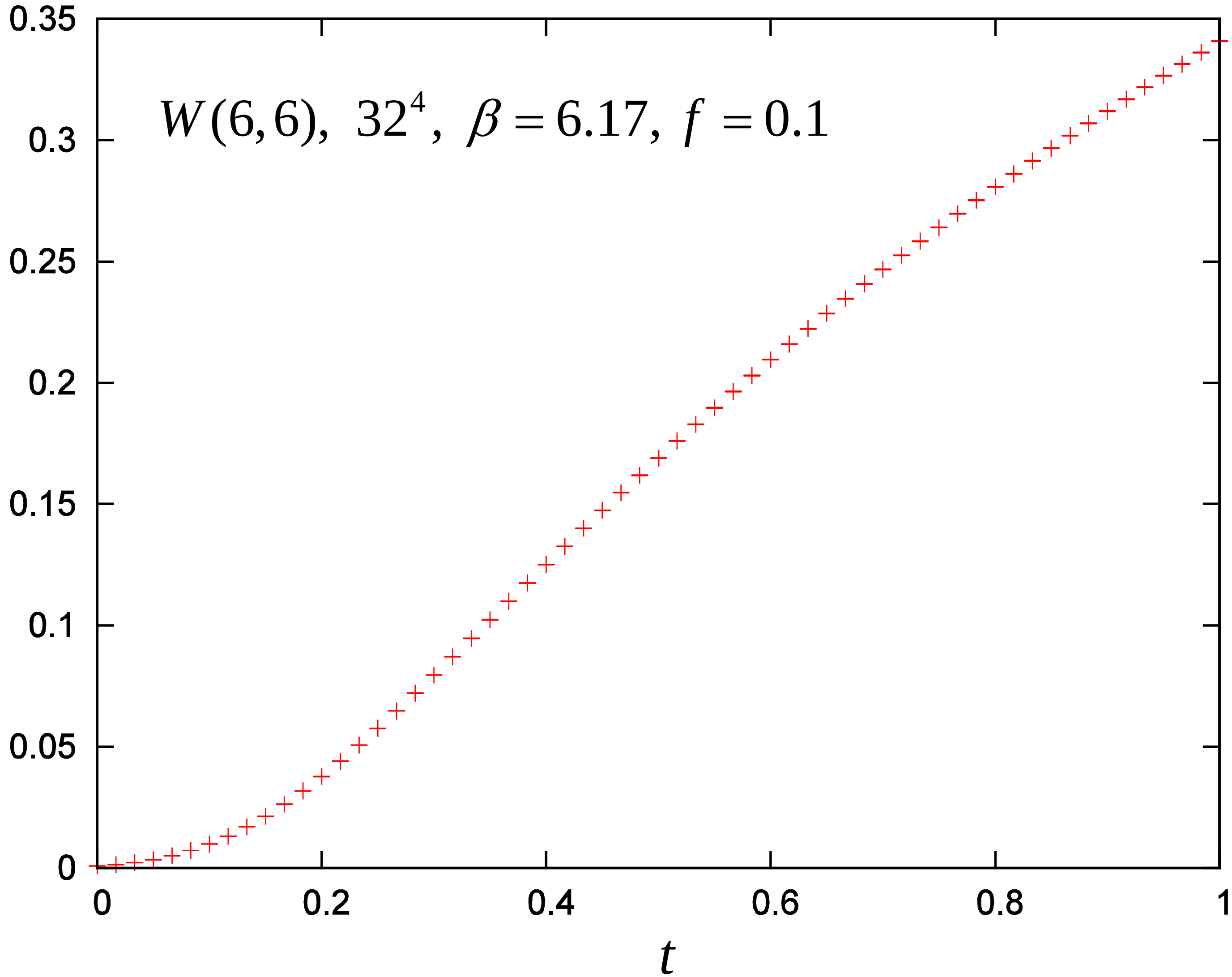
$$\chi(R,T) = -\log \frac{W(R+1/2, T+1/2)W(R-1/2, T-1/2)}{W(R+1/2, T-1/2)W(R-1/2, T-1/2)}$$

which is free from ultraviolet divergences and its t dependence is quite well fitted by

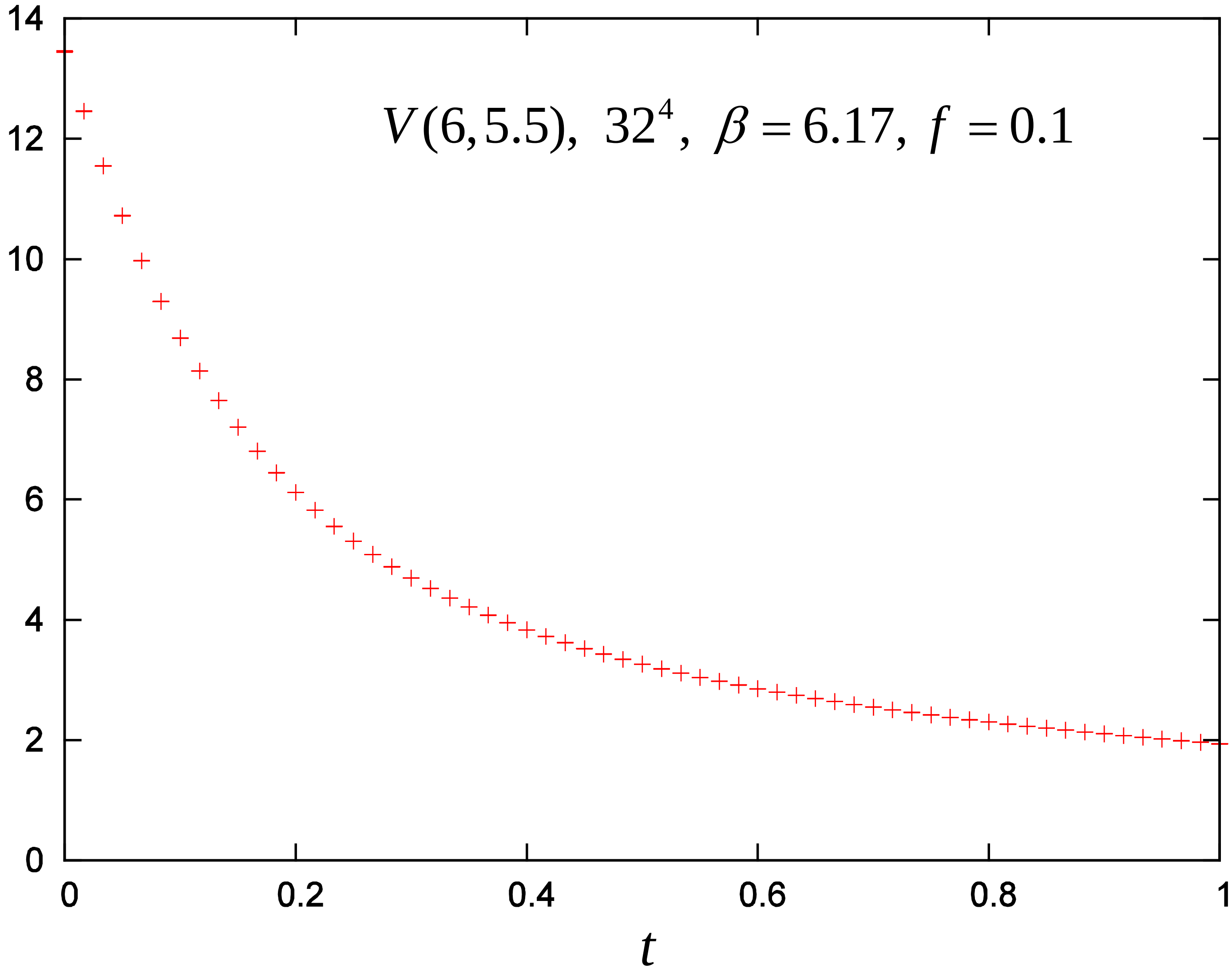
$$\chi(t) = a\left(1 - \exp\left(\frac{b}{t+c}\right)\right)$$

This is the method proposed in Phys. Lett. B718 (2013) 1524.

$t = f * n_s / 6$ dependence of Wilson loop $W(R,T)$

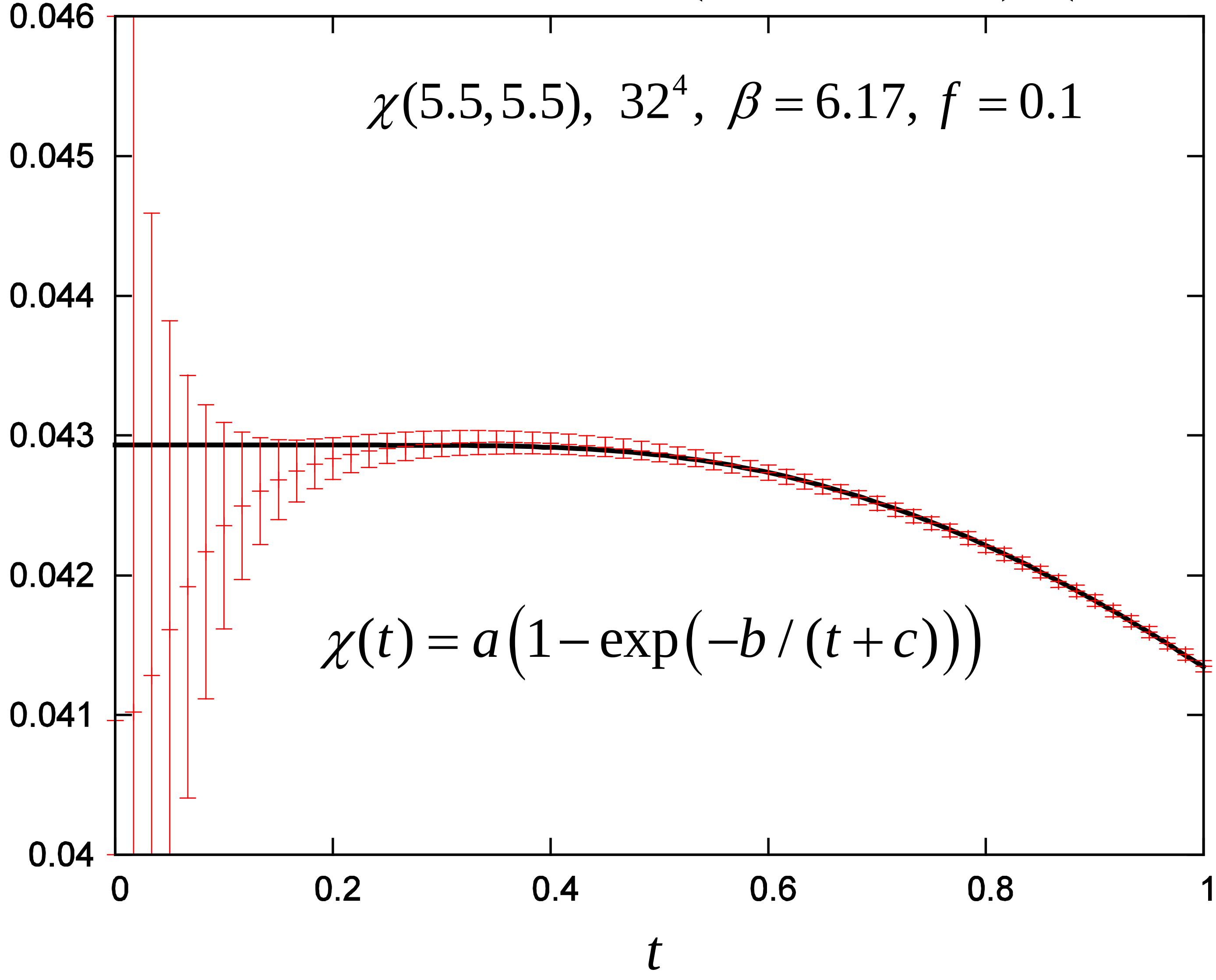


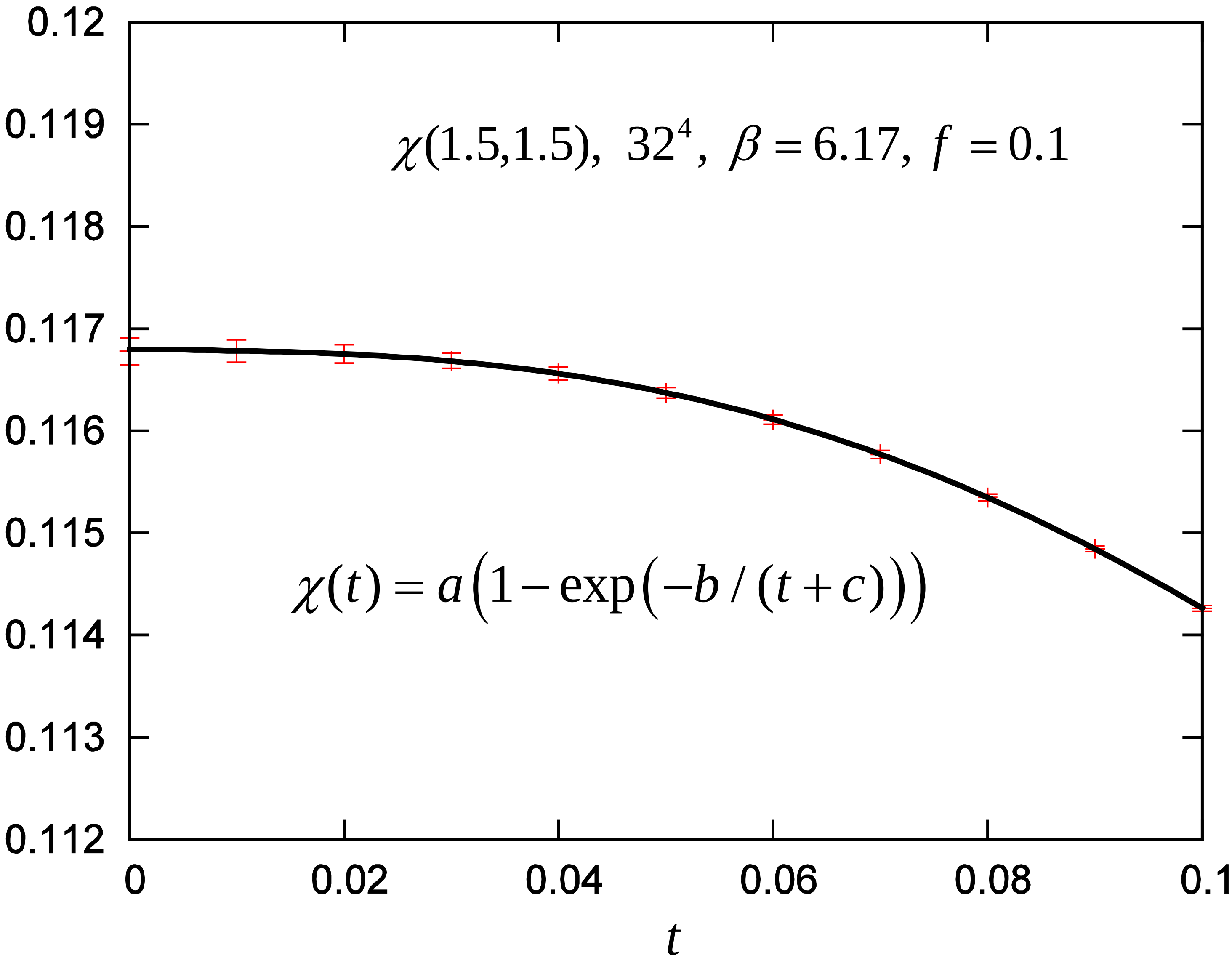
$t = f * n_s / 6$ dependence of Potential $V(R,T) = -\log \frac{W(R,T+1/2)}{W(R,T-1/2)}$



$t = f * n_s / 6$ dependence of Creutz ratio

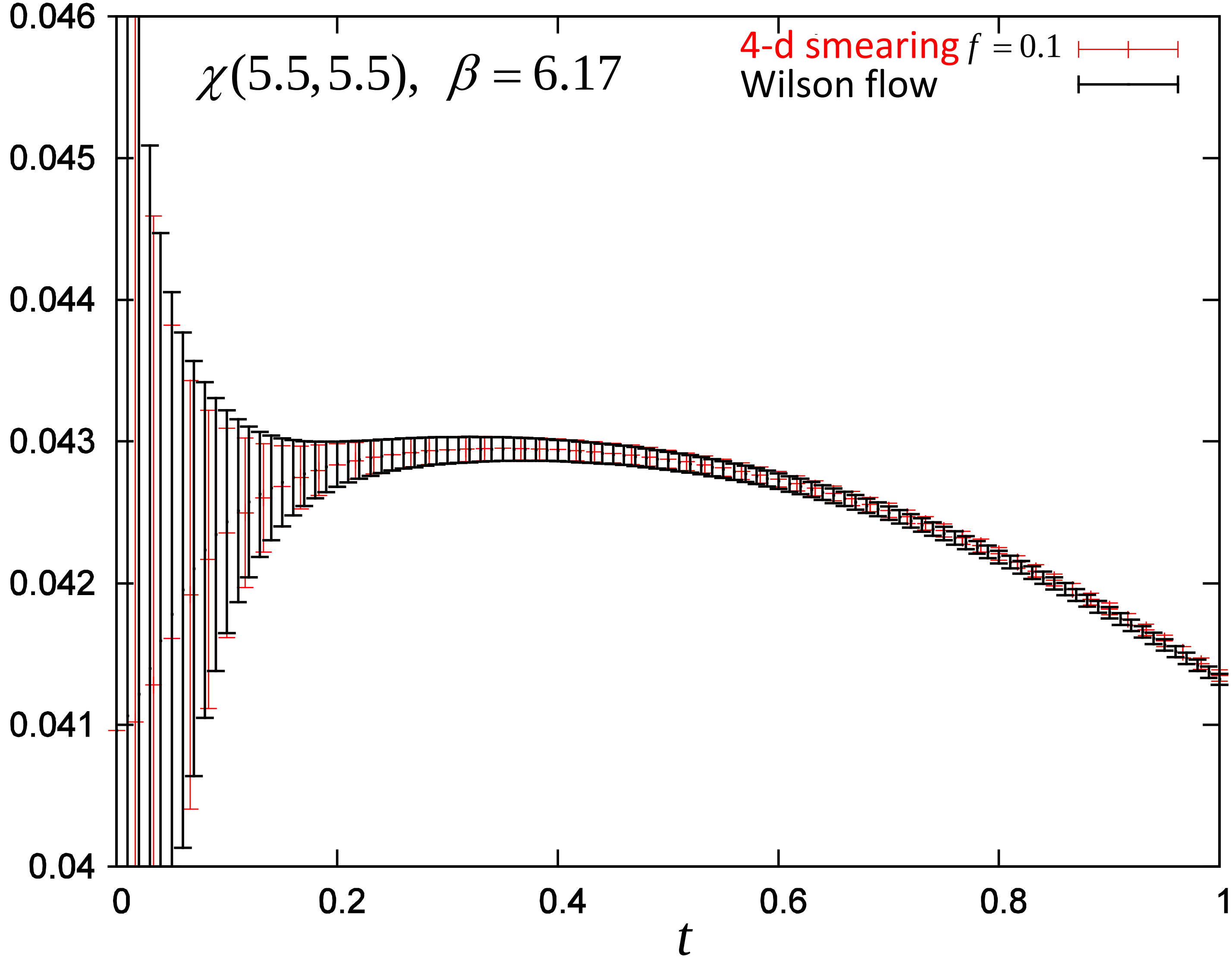
$$\chi(R,T) = -\log \frac{W(R+1/2, T+1/2)W(R-1/2, T-1/2)}{W(R+1/2, T-1/2)W(R-1/2, T+1/2)}$$





0.12
0.119
0.118
0.117
0.116
0.115
0.114
0.113
0.112
$\chi(1.5,1.5)$, 32^4, $\beta = 6.17$, $f = 0.1$
$\chi(t) = a\left(1 - \exp\left(-b/(t+c)\right)\right)$
0
0.02
0.04
0.06
0.08
0.1
t

For small t, 4-d smearing and Wilson flow results are essentially identical.



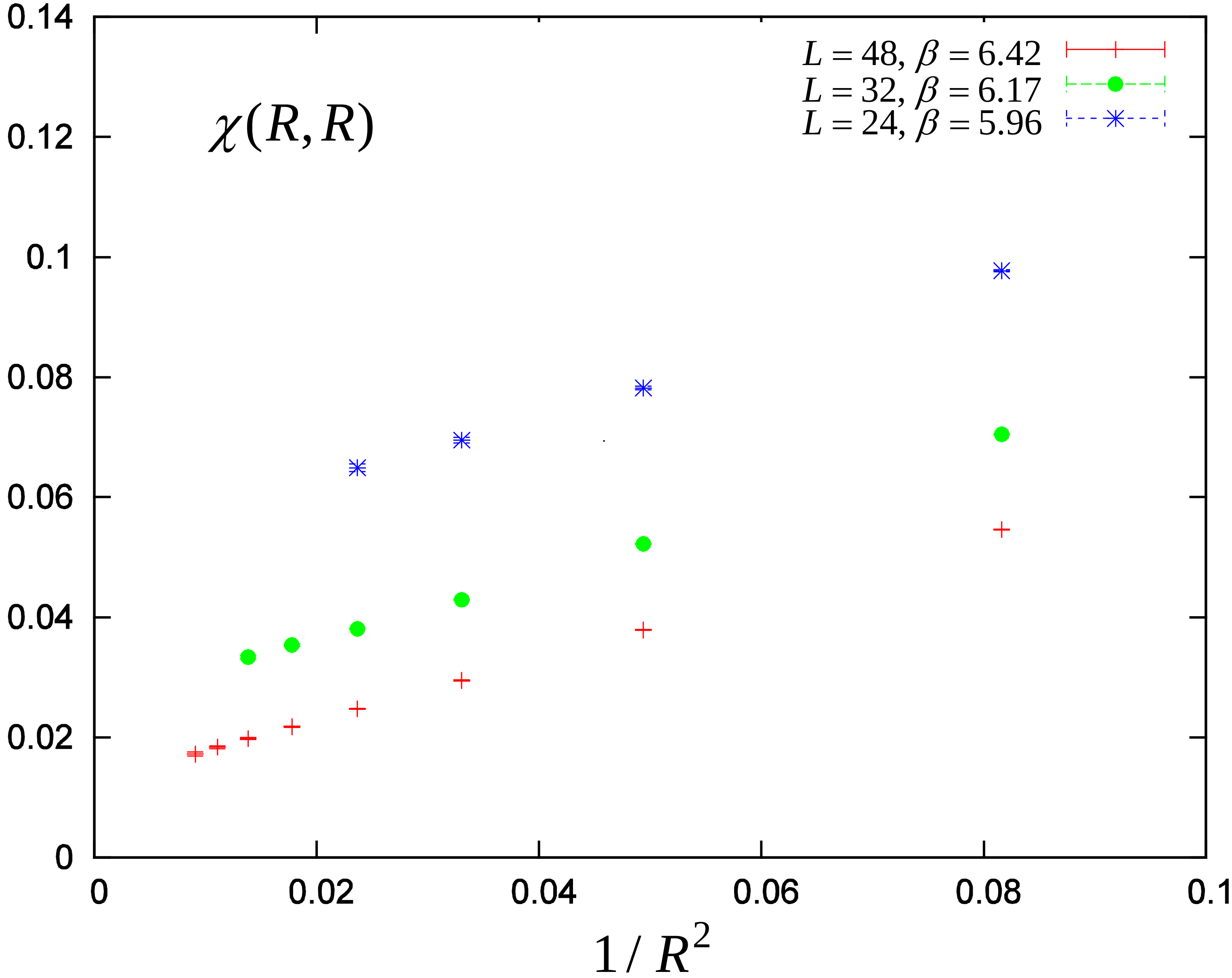
- Creutz ratio in the continuum limit

We made SU(3) simulations at three lattices having almost same physical volume $(La)^4$

$Lattice$	β	N_{cnfg}
24^4	5.96	1200
32^4	6.17	600
48^4	6.42	100

In this talk, we concentrate on the diagonal $\chi(R,R)$, although

there shoud be a lot of interesting physics in off-diagonal $\chi(R,T)$



χ(R,R)
L = 48, β = 6.42
L = 32, β = 6.17
L = 24, β = 5.96
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
0.02
0.04
0.06
0.08
0.1
1 / R²

Introducing a scale $\bar{r}$ and writing $r = Ra$, dimensional analysis implies that there should be $O(a^2)$ lattice artifact in $1/r^4$ term.

$$\left(\frac{\bar{r}}{a}\right)^2 \chi(R,R) = \sigma \bar{r}^2 + 2\gamma \left(\frac{\bar{r}}{r}\right)^2 + 4\left(\frac{\bar{r}}{r}\right)^4 \left(c + d\left(\frac{a}{\bar{r}}\right)^2\right)$$

$\tilde{F}(r)$ defined by

$$\bar{r}^2 \tilde{F}(r) \equiv \sigma \bar{r}^2 + 2\gamma \left(\frac{\bar{r}}{r}\right)^2 + 4c\left(\frac{\bar{r}}{r}\right)^4$$

is the Creutz ratio in the continuum limit.

We can fix the scale a la Sommer as

$$\bar{r}^2 \tilde{F}(\bar{r}) = \sigma \bar{r}^2 + 2\gamma + 4c = 1.65$$

Eliminating c from the expression and replacing r to Ra , we finally find a fitting function

$$\left(\frac{\bar{r}}{a}\right)^2 \chi(R,R) = \sigma\bar{r}^2 + 2\gamma\left(\frac{\bar{r}}{a}\right)^2 \frac{1}{R^2} + 4\left(\frac{\bar{r}}{a}\right)^4 \frac{1}{R^4}\left(c + d\left(\frac{a}{\bar{r}}\right)^2\right)$$

$$4c = 1.65 - \sigma\bar{r}^2 - 2\gamma$$

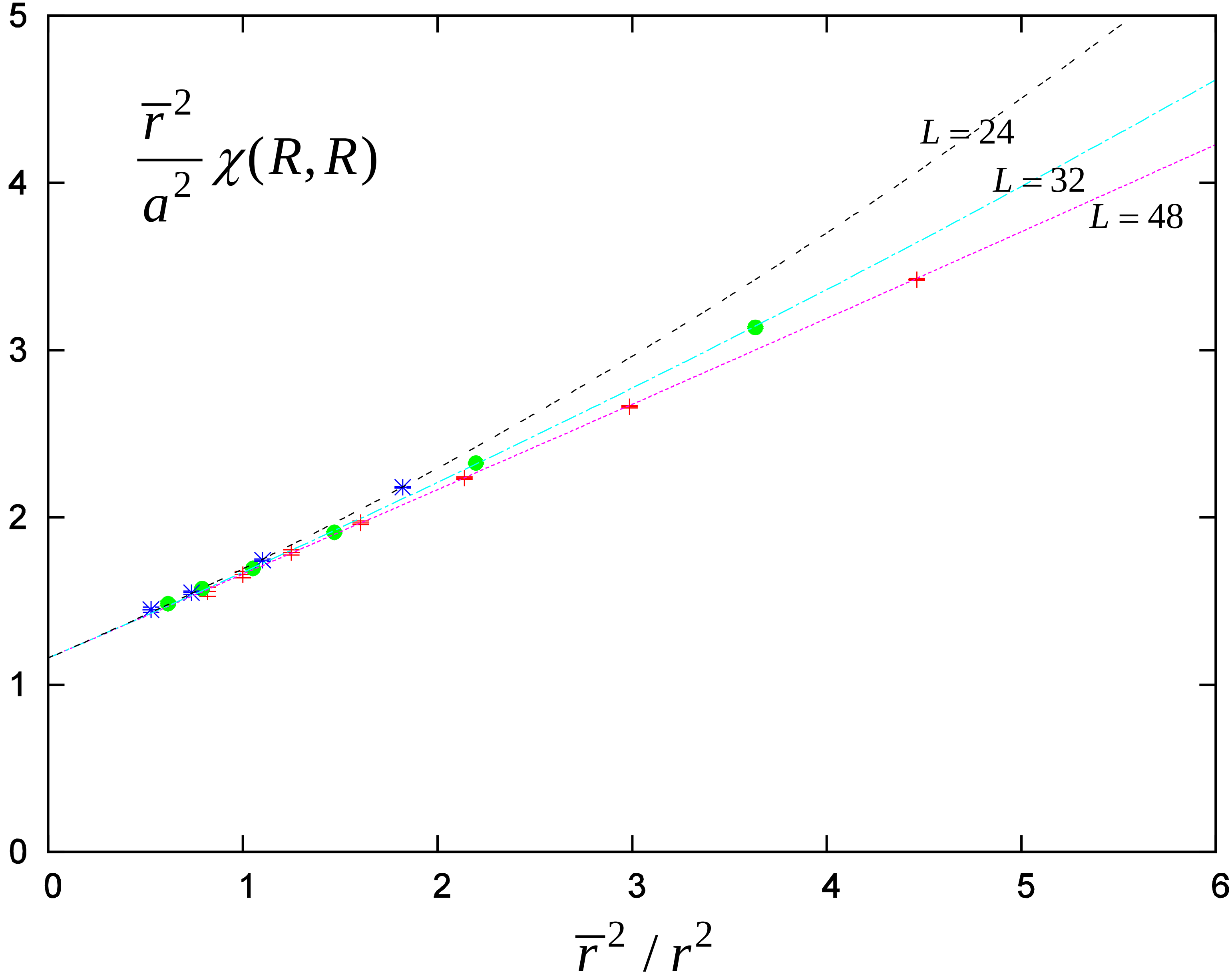
with 6 fitting parameters

$$\sigma\bar{r}^2, \gamma, d, \frac{a(\beta=5.96)}{\bar{r}}, \frac{a(\beta=6.17)}{\bar{r}}, \frac{a(\beta=6.42)}{\bar{r}}$$

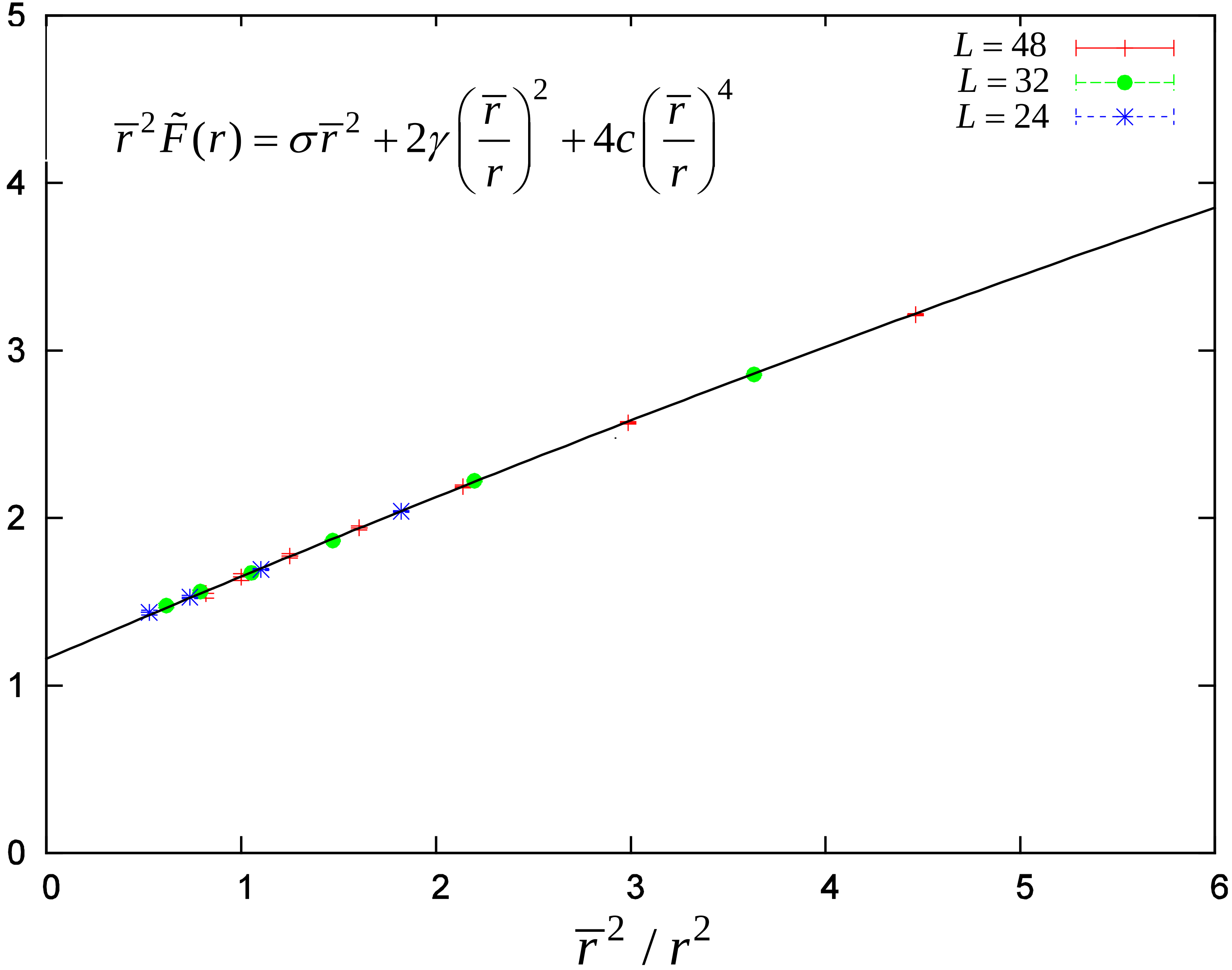
The resultant fit has $\chi^2 / (\#\, of \;\; freedom) = 1.05$ with

$$\sigma\bar{r}^2 = 1.159(6) \qquad \gamma = 0.250(3), \; d = 0.24(2)$$

$$a(\beta = 5.96)/\bar{r} = 0.2117(5),$$
$$a(\beta = 6.17)/\bar{r} = 0.1499(2),$$
$$a(\beta = 6.42)/\bar{r} = 0.1052(3)$$



$\frac{\overline{r}^2}{a^2}\chi(R,R)$
$L = 24$
$L = 32$
$L = 48$
0
1
2
3
4
5
6
$\overline{r}^2 / r^2$

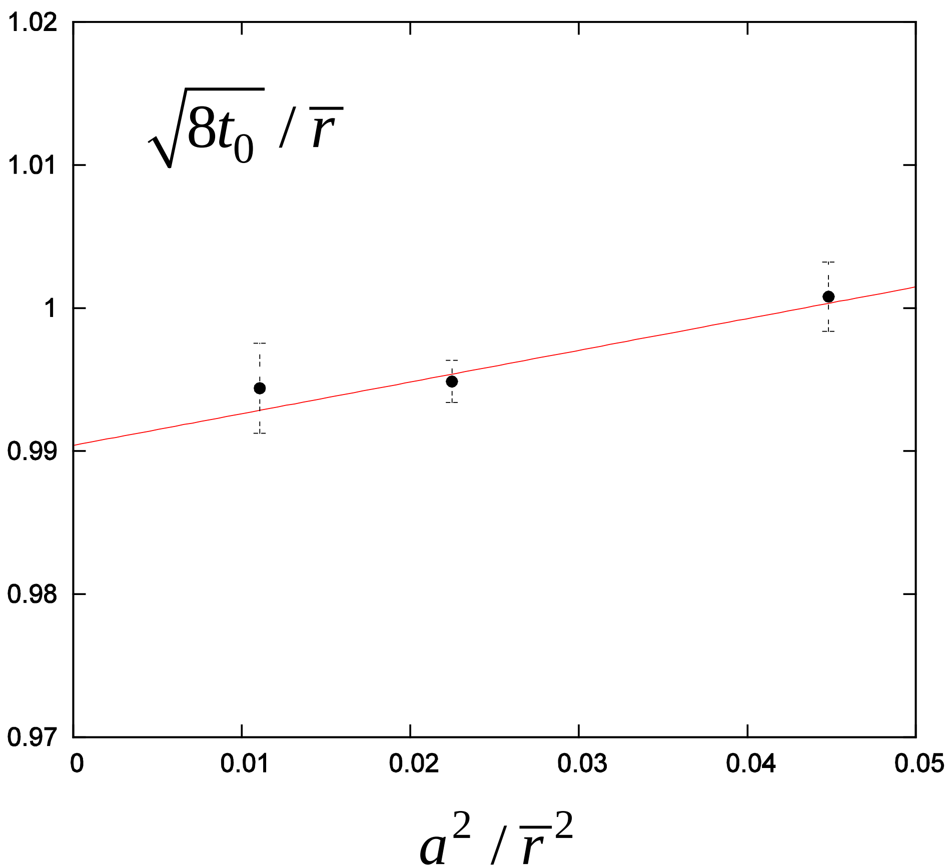


$\bar{r}^2\tilde{F}(r) = \sigma\bar{r}^2 + 2\gamma\left(\frac{\bar{r}}{r}\right)^2 + 4c\left(\frac{\bar{r}}{r}\right)^4$
$L = 48$
$L = 32$
$L = 24$
0
1
2
3
4
5
0
1
2
3
4
5
6
$\bar{r}^2 / r^2$

- relation between

$\overline{r}$ and t_0

β	$a/\overline{r}$	t_0/a^2	$\sqrt{8t_0}/\overline{r}$
5.96	0.2117(5)	2.794(3)	1.001(2)
6.17	0.1499(2)	5.506(7)	0.995(2)
6.42	0.1052(3)	11.17(3)	0.994(3)



In the continuum limit

$$\sqrt{8t_0}/\overline{r} = 0.990(3)$$

$$\sigma\overline{r}^2 = 1.159(6)$$

$$\therefore\ \sqrt{8t_0\sigma} = 1.066(4)$$

● comparison to 3-d smearing methods

In the continuum limit, $\sqrt{8t_0}\,/\,r_0 = 0.948(6)$. Lüscher JHEP08(2010) 071.

Then, our result is converted to $r_0\sqrt{\sigma} = 1.124(8)$

Previous results of $r_0\sqrt{\sigma}$ derived from 3-d smeared Potential are almost consistent with the value $r_0\sqrt{\sigma} = \sqrt{1.65 - \pi/12} = 1.178$

It is not clear how to interpret this 5% difference, however,

they are obtained from quite different geometries of Wilson loops !

3-d smearing, $W(r,t)$ with finite r and $t = \infty$

$$r_0^2 F(r) \;=\; r_0^2\sigma + \frac{\pi}{12}\frac{r_0^2}{r^2} \;\simeq\; r_0^2\sigma + 0.2612\frac{r_0^2}{r^2}$$

4-d smearing, $W(r,t)$ with finite $r \approx t$

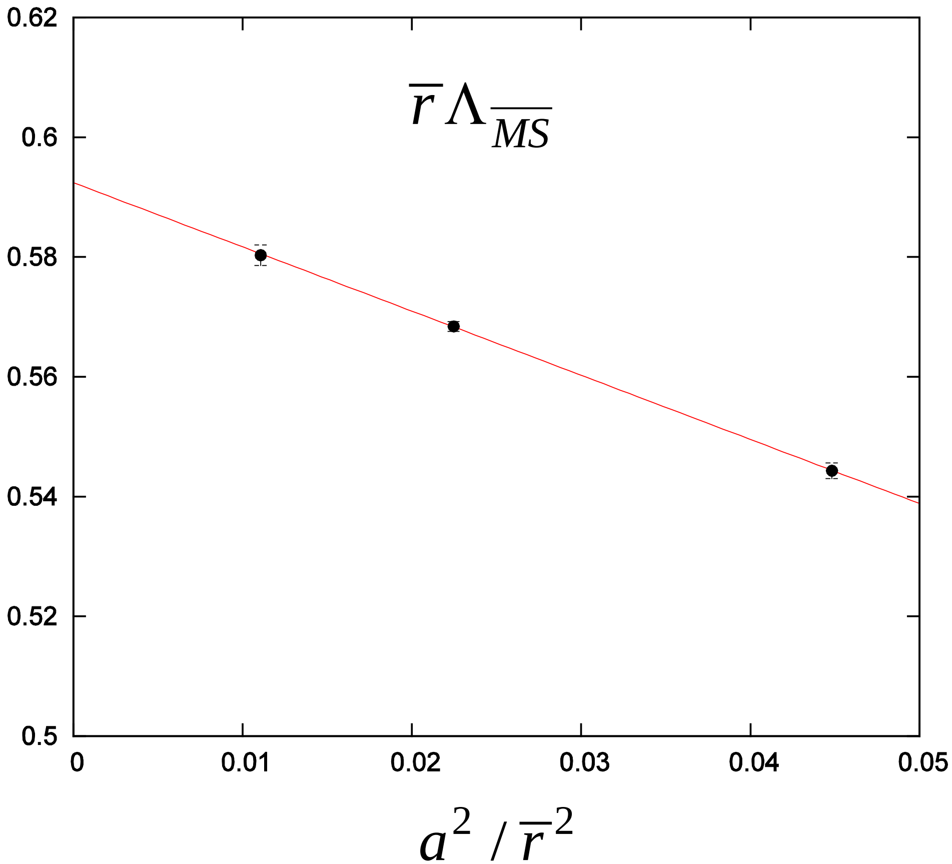
$$\bar{r}^2\tilde{F}(r) \;=\; \bar{r}^2\sigma + 2\gamma\frac{\bar{r}^2}{r^2} + 4c\frac{\bar{r}^4}{r^4} \;\simeq \bar{r}^2\sigma + 0.499\frac{\bar{r}^2}{r^2} - 0.008\frac{\bar{r}^4}{r^4}$$

- relation between $\bar{r}$ and $\Lambda_{\overline{MS}}$

We use $g_E^2 = 3(1-u_P)$ for

$a\Lambda_{\overline{MS}} = a\Lambda_E(\Lambda_{\overline{MS}} / \Lambda_E)$

β	$a/\bar{r}$	$a\Lambda_{\overline{MS}}$	$\bar{r}\Lambda_{\overline{MS}}$
5.96	0.2117(5)	0.11523	0.5443(13)
6.17	0.1499(2)	0.08520	0.5684(8)
6.42	0.1052(3)	0.06105	0.5803(17)



In the continuum limit

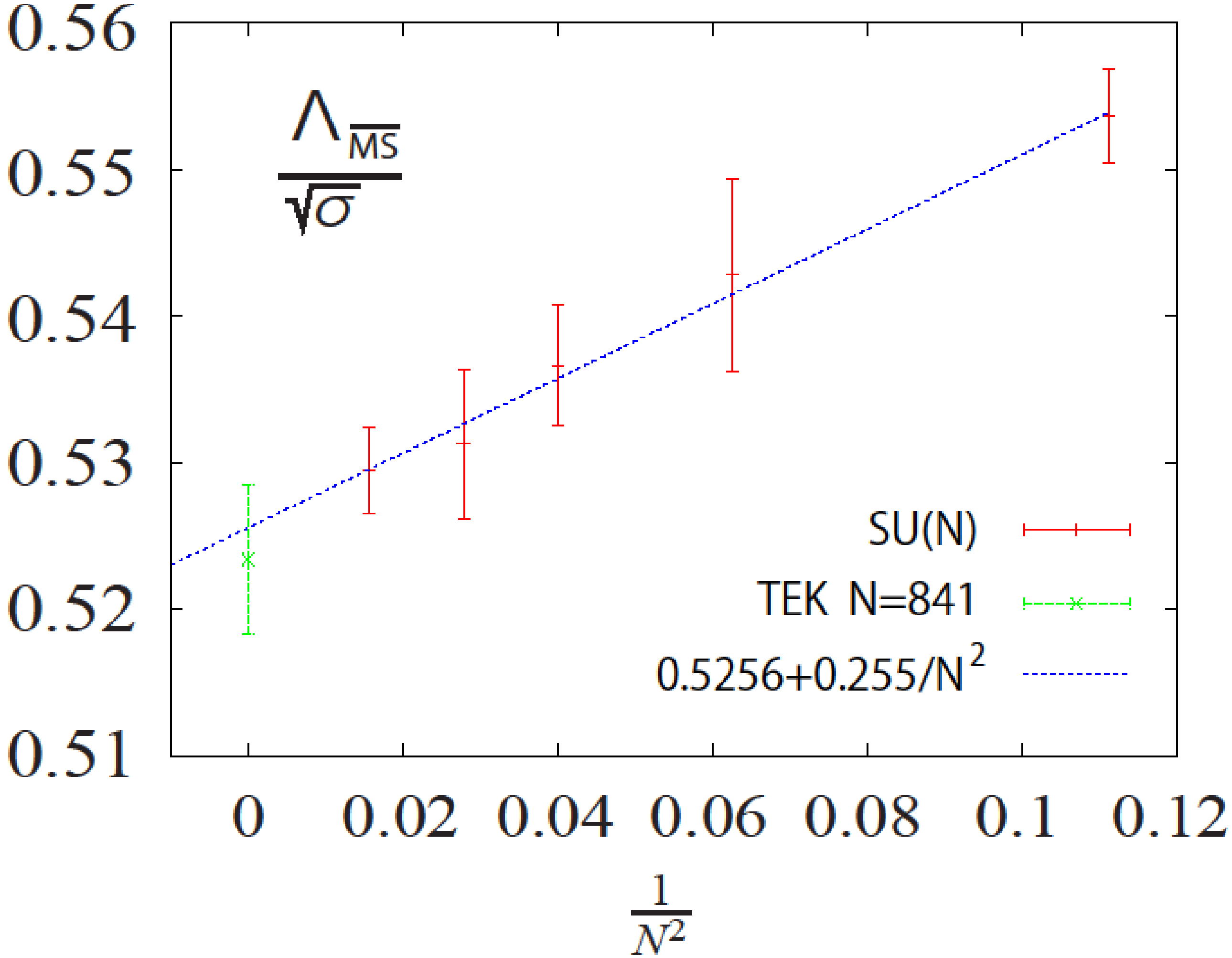
$$\bar{r}\Lambda_{\overline{MS}} = 0.5924(16)$$

$$\sigma\bar{r}^2 = 1.159(6)$$

$$\therefore \frac{\Lambda_{\overline{MS}}}{\sqrt{\sigma}} = 0.550(2)$$

Comparison of the continuum string tension $\Lambda_{\overline{MS}} / \sqrt{\sigma}$

TEK model with $N = 841 = 29^2$ and LGT with $N = 3, 4, 5, 6, 8$



● Conclusion

Creutz ratios in the continuum limit can be evaluated precisely by 4-d Ape smearing, giving rather reliable determination of the string tension.

Wilson flow technique gives the same physical results.

We show that the TEK model is able to calculate the string tension in the large N limit, thus demonstrating the correctness of TEK model nonperturbatively.

It is worth calculating Wilson loops during Wilson flow updating.

They should give fruitful physics !