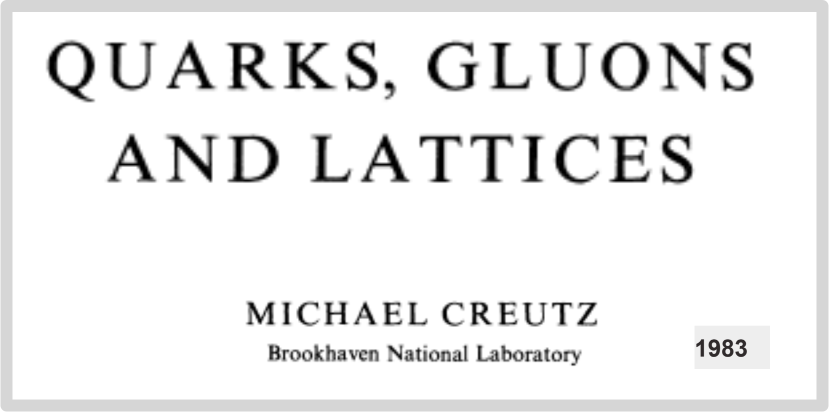


QUARKS, GLUONS
AND LATTICES

MICHAEL CREUTZ
Brookhaven National Laboratory

1983

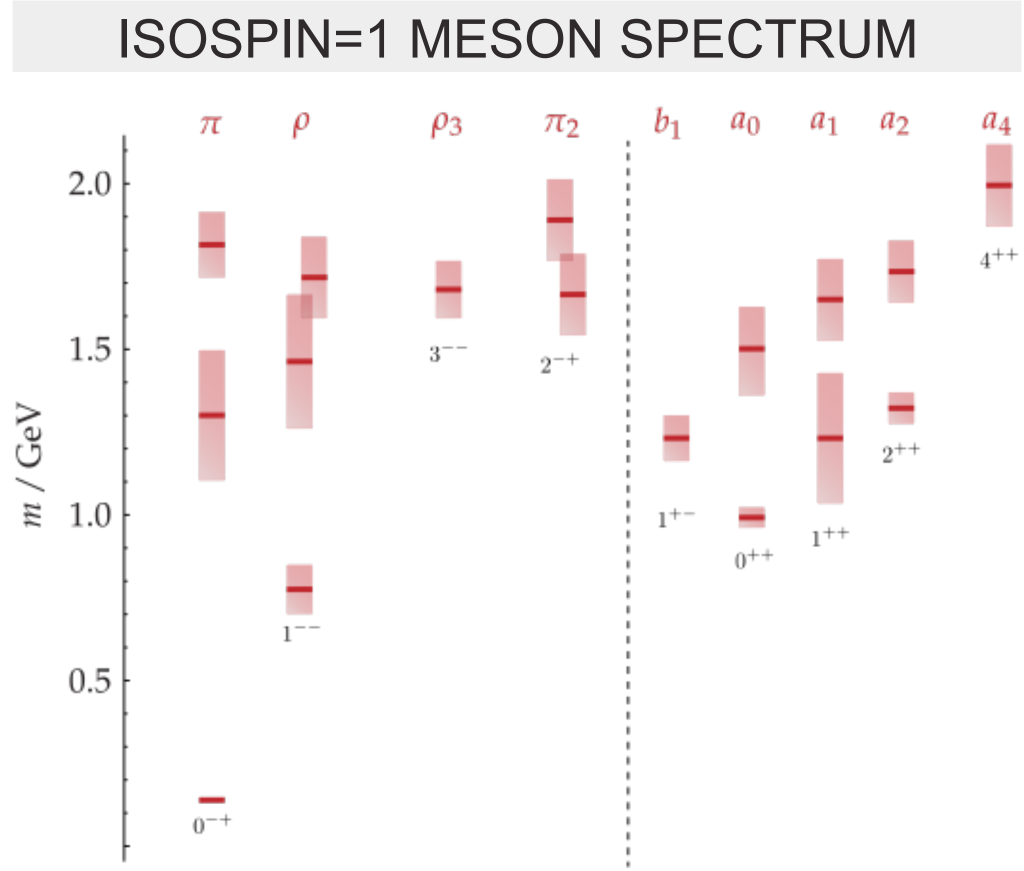
HADRONS

Robert Edwards
Jefferson Lab

Creutz-Fest 2014

Experimental meson spectrum

- Mesons classified by their conserved quantum numbers
  - Spin, isospin, charge-conjugation J^{PC}



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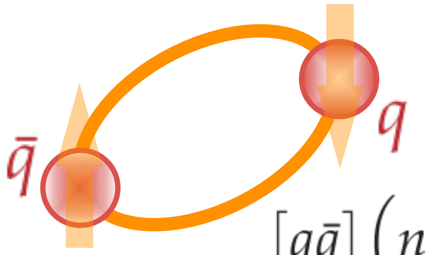
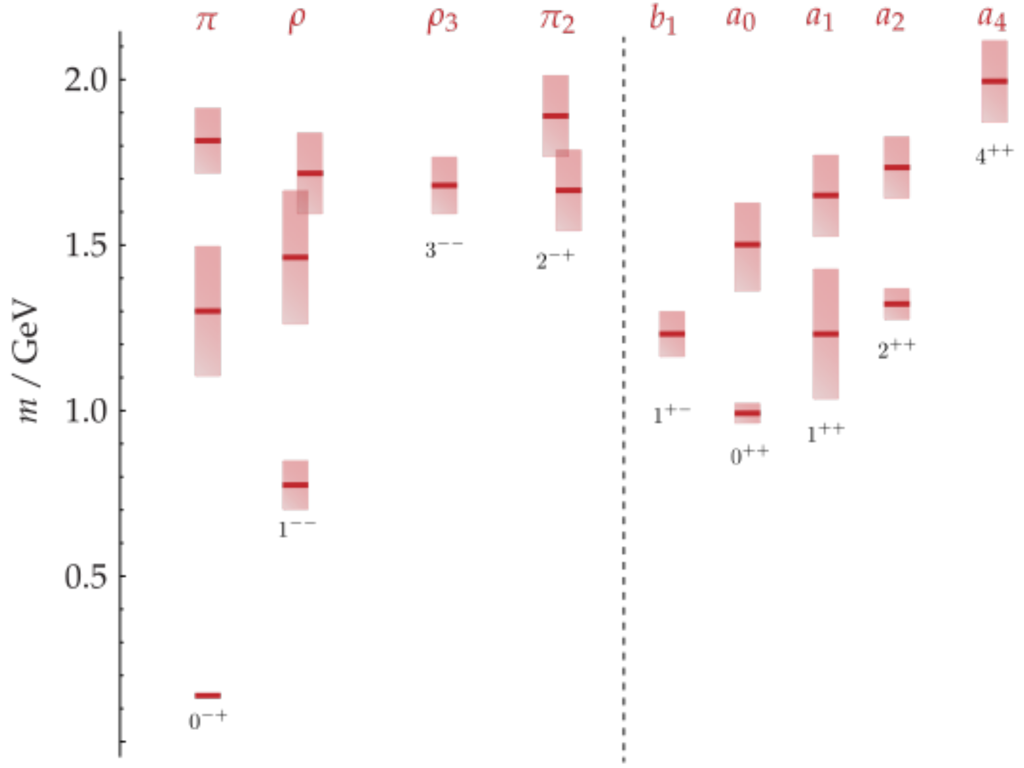


$$[q\bar{q}]\left(n\,^{2S+1}L_J\right)$$

$$L = 0 \,:\, 0^{-+}, 1^{--}$$
$$L = 1 \,:\, 1^{+-}, (0,1,2)^{++}$$
$$L = 2 \,:\, 2^{-+}, (1,2,3)^{--}$$
$$\vdots$$

ISOSPIN=1 MESON SPECTRUM



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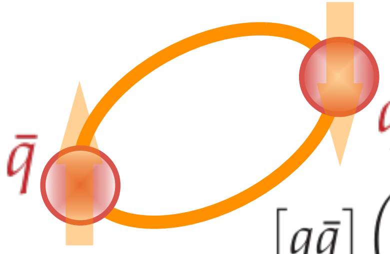

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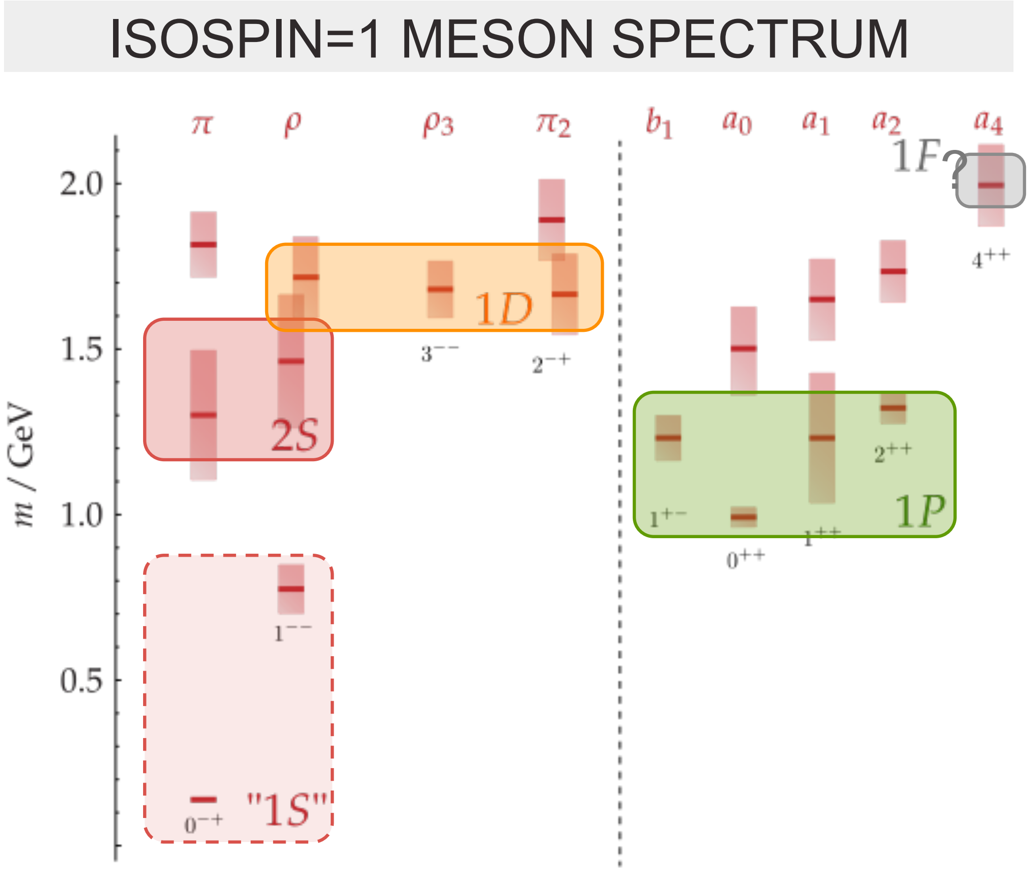
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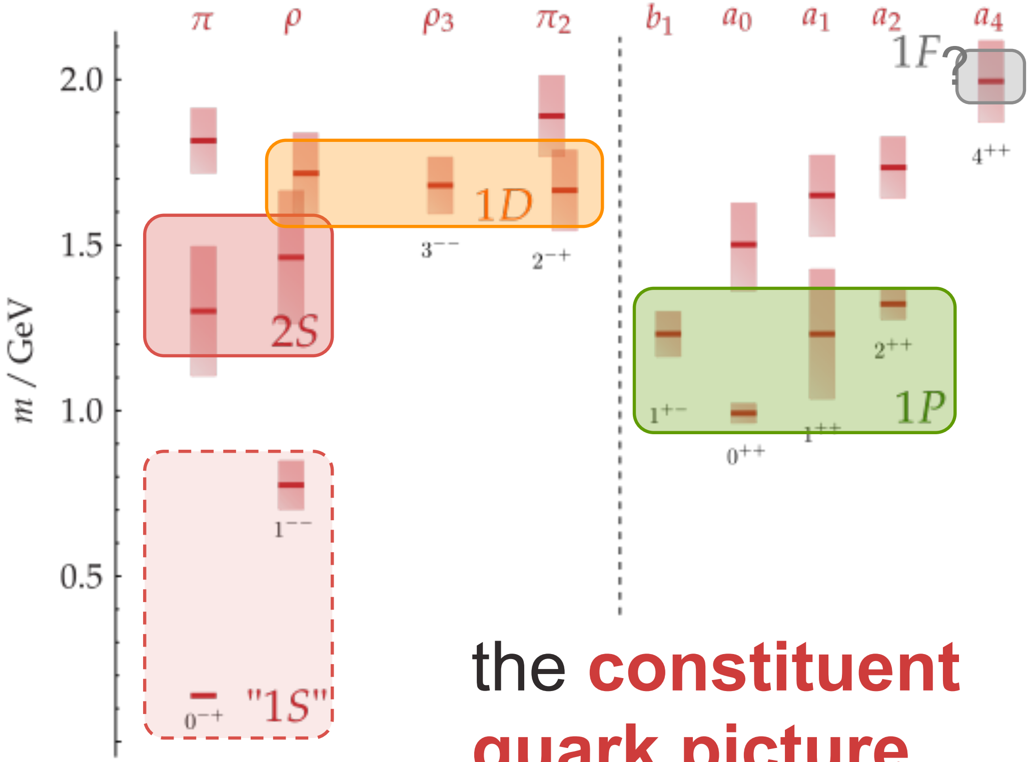
$\vdots$

n.b. absent: $0^{--}, 0^{+-}, 1^{-+}, 2^{+-} \ldots$

ISOSPIN=1 MESON SPECTRUM

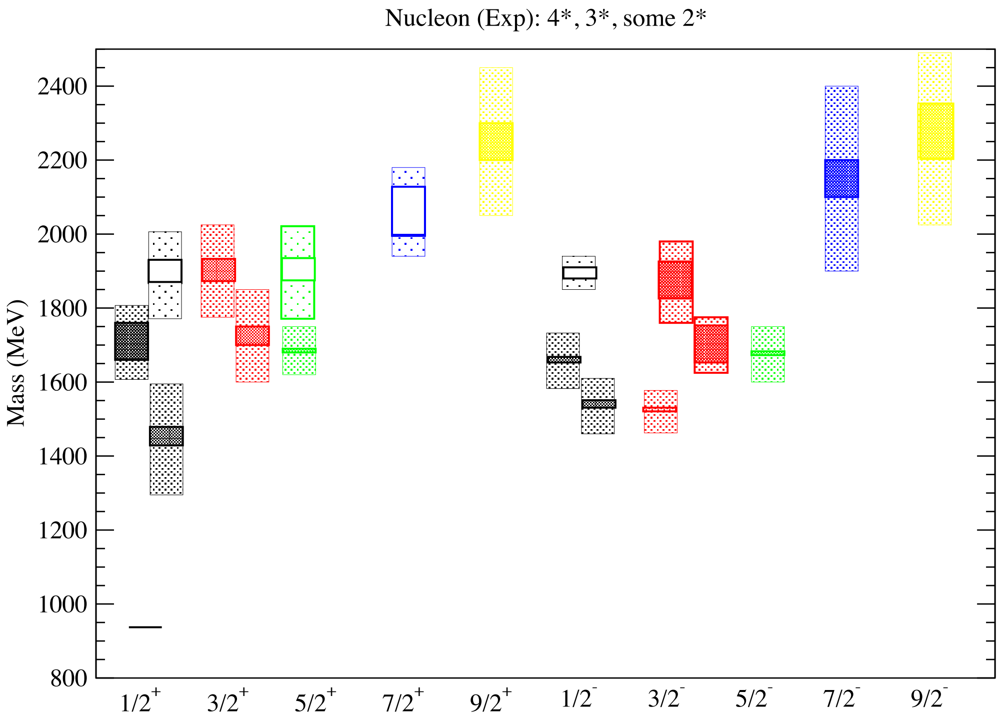


the **constituent quark picture**

Experimental baryon spectrum

- Baryons classified by their conserved quantum numbers
  - Spin, parity, isospin **J^P**

ISOSPIN=1/2 BARYON SPECTRUM



Experimental baryon spectrum

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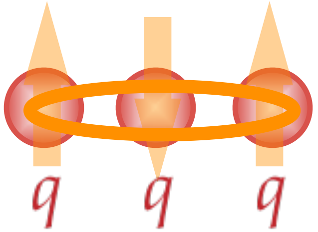

$$[qqq]\left(n^{2S+1}L_{\pi}\right)$$

Antisymmetric under interchange

$$\pi \;=\; \text{permutation of quarks in space}$$

$$L = 0_S \quad : \quad \frac{1}{2}^+$$

$$L = 1_M \quad : \quad \left(\frac{1}{2},\frac{3}{2}\;,\;\frac{1}{2},\frac{3}{2},\frac{5}{2}\right)^-$$

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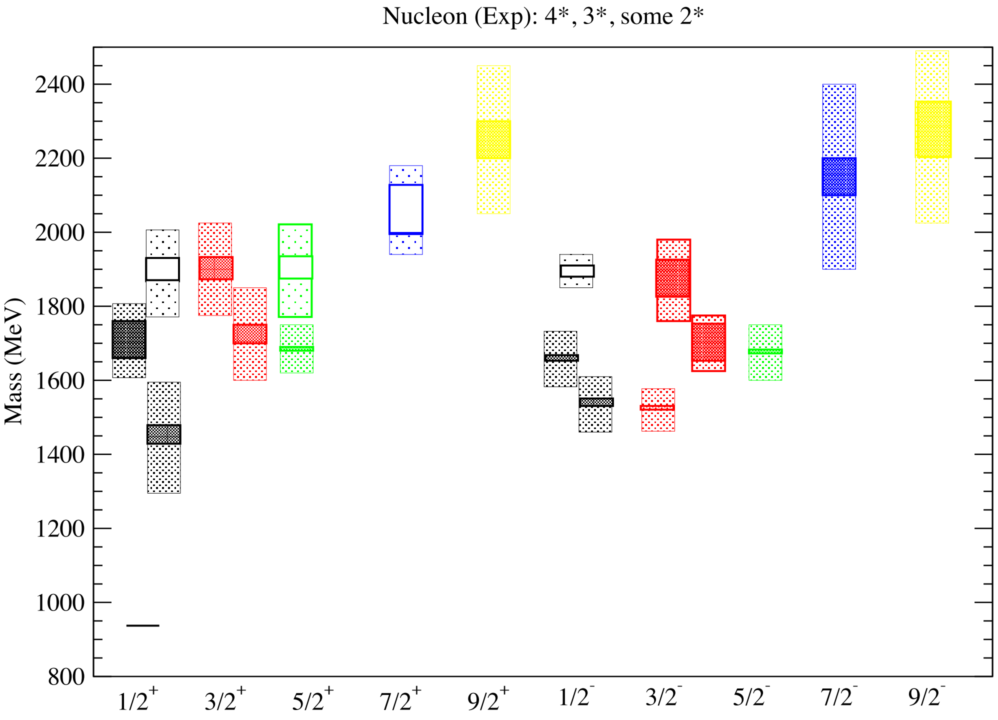

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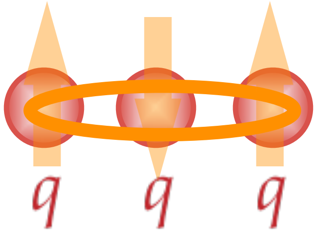

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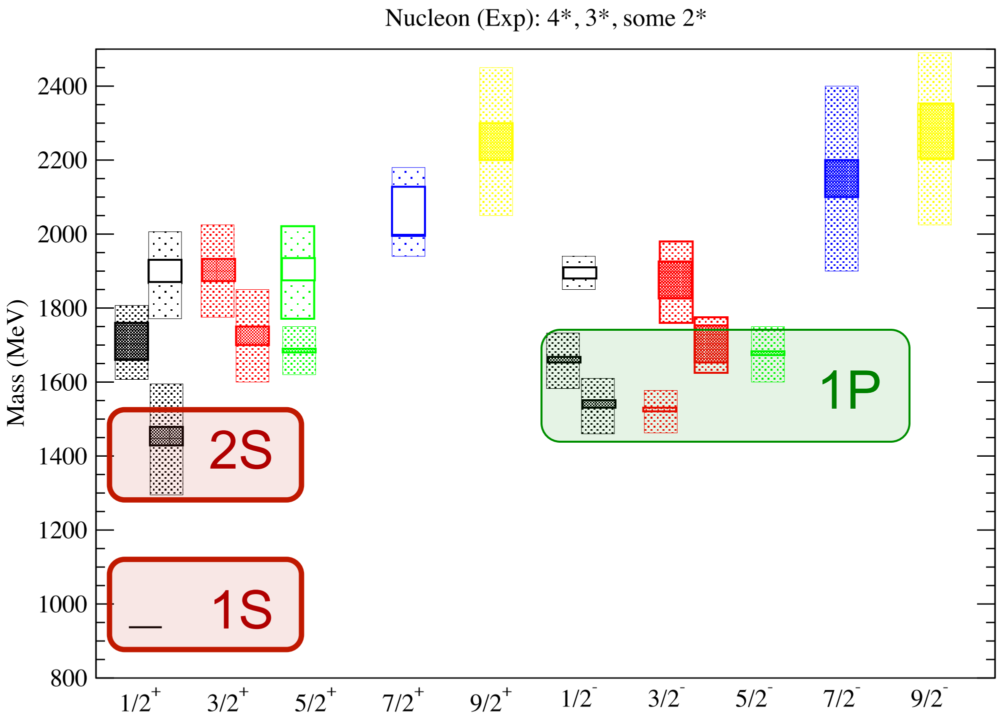
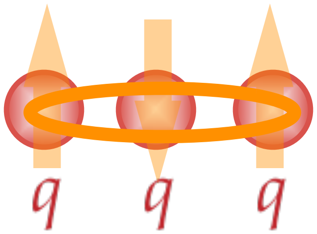


Experimental baryon spectrum

- Some states are “missing” ???



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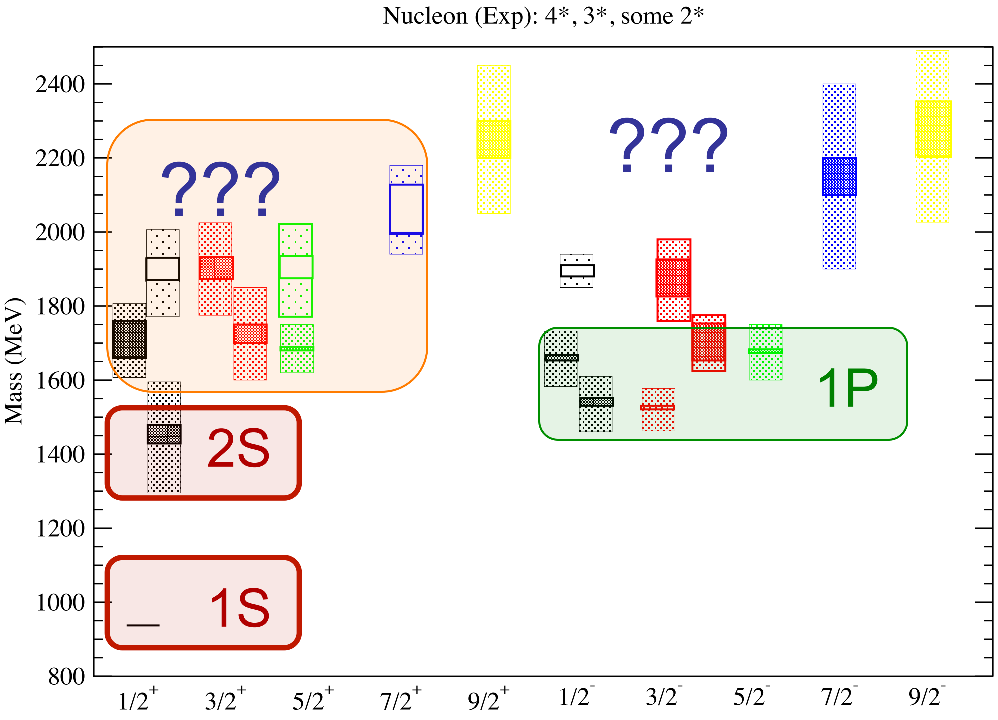
L = 2_S : $\left(\frac{3}{2},\frac{5}{2}\right)^{+}$

L = 2_M : $\left(\frac{3}{2},\frac{5}{2}\right)^{+}\;,\;\left(\frac{1}{2},\frac{3}{2},\frac{5}{2}\right)^{+}$

L = 1_A : $\left(\frac{1}{2},\frac{3}{2}\right)^{+}$

L = 0_M : $\left(\frac{1}{2},\frac{3}{2}\right)^{+}$

ISOSPIN=1/2 BARYON SPECTRUM



Patterns in hadron spectrum

- Observed meson state flavor & J^{PC} systematics: $q\bar{q}$

`constituent quarks'

$$q\bar{q}\,[S,L] \to (J = L \times S)^{PC}, \quad P = (-1)^{L+1},\ C = (-1)^{L+S}$$

$$\boxed{\begin{array}{cccc} \cdot & 0^{-+} & 0^{++} & \cdot \\ 1^{--} & \cdot & 1^{++} & 1^{+-} \\ 2^{--} & 2^{-+} & 2^{++} & \cdot \\ \vdots & & & \end{array}}$$

➢ "Exotic" quantum numbers

$$\boxed{0^{--}, 0^{+-}, 1^{-+}, 2^{+-}}$$

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`constituent quarks'

$\cdot$	0^{-+}	0^{++}	$\cdot$
1^{--}	$\cdot$	1^{++}	1^{+-}
2^{--}	2^{-+}	2^{++}	$\cdot$
$\vdots$			

  - ➢ "Exotic" quantum numbers $\boxed{0^{--}, 0^{+-}, 1^{-+}, 2^{+-}}$

- Could excited gluonic fields play a role – *hybrid* mesons $q\bar{q}G$?
  - ➢ Possibly exotic J^{PC} and extra `non-exotic' states

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- Could excited gluonic fields play a role – *hybrid* mesons $q\bar{q}G$?
  - ➢ Possibly exotic J^{PC} and extra `non-exotic' states

- Constituent quark picture qqq predicts rich baryon spectrum not all experimentally observed
  - ➢ No exotic J^{P} in *hybrid* baryons

However, might lead to extra states

Modeling hybrid mesons

- Long history of suggestions for gluonic excitations coupled to quarks

Modeling hybrid mesons

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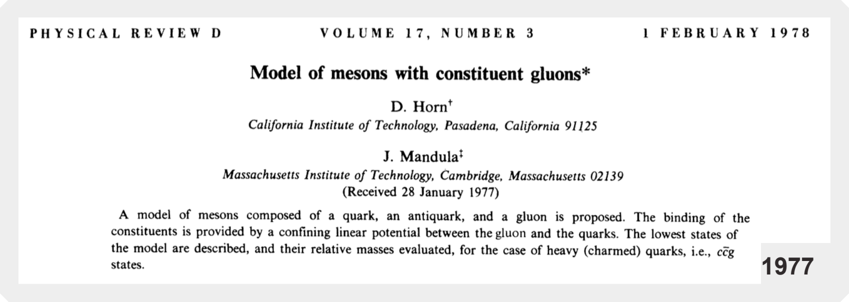


PHYSICAL REVIEW D VOLUME 17, NUMBER 3 1 FEBRUARY 1978

Model of mesons with constituent gluons*

D. Horn[†]
California Institute of Technology, Pasadena, California 91125

J. Mandula[‡]
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 28 January 1977)

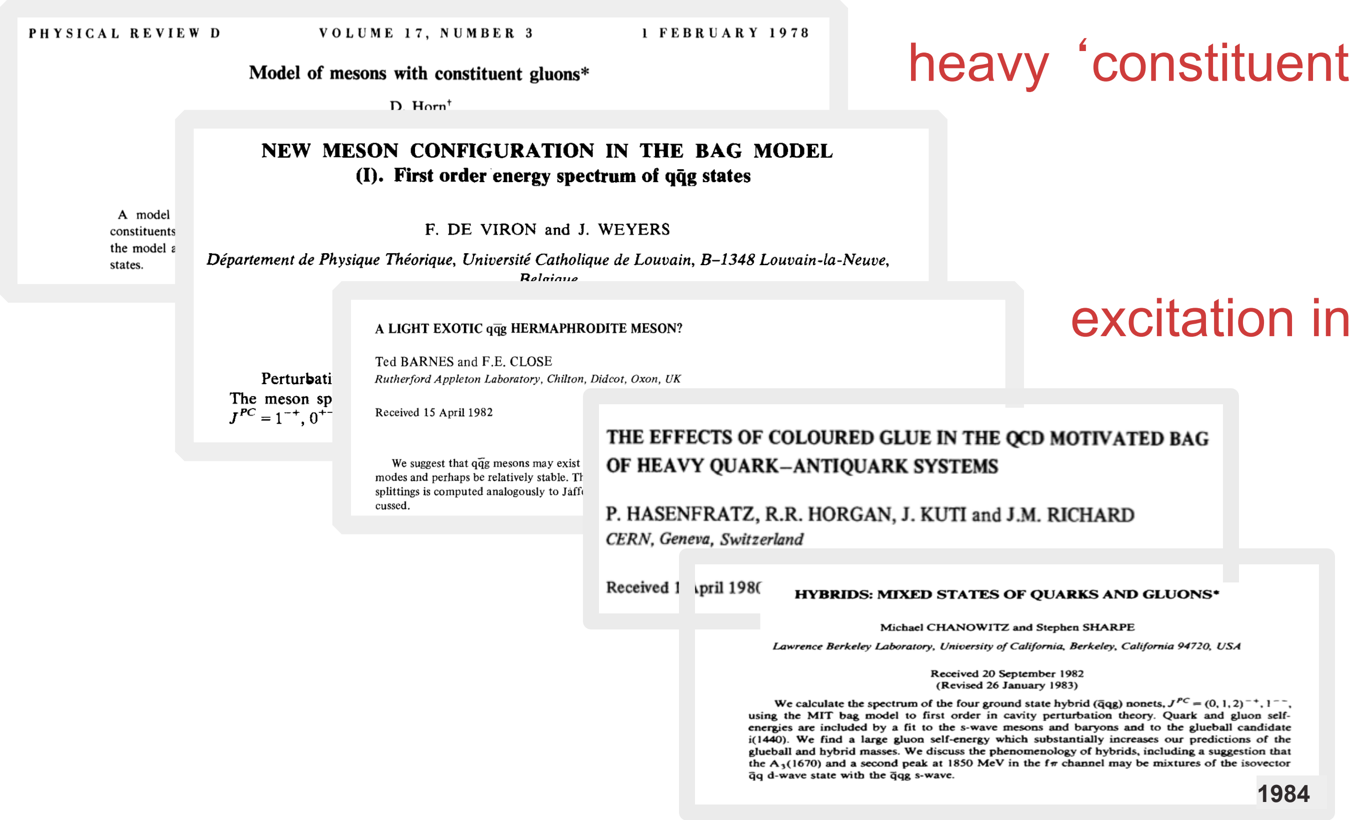
A model of mesons composed of a quark, an antiquark, and a gluon is proposed. The binding of the constituents is provided by a confining linear potential between the gluon and the quarks. The lowest states of the model are described, and their relative masses evaluated, for the case of heavy (charmed) quarks, i.e., $c\bar{c}g$ states.

1977

heavy ‘constituent’ gluon

Modeling hybrid mesons

- Long history of suggestions for gluonic excitations coupled to quarks



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A model
constituents
the model
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NEW MESON CONFIGURATION IN THE BAG MODEL
(I). First order energy spectrum of $q\bar{q}g$ states

F. DE VIRON and J. WEYERS

Département de Physique Théorique, Université Catholique de Louvain, B–1348 Louvain-la-Neuve, Belgique

Perturbati
The meson sp
$J^{PC} = 1^{-+}, 0^{+-}$

A LIGHT EXOTIC $q\bar{q}g$ HERMAPHRODITE MESON?

Ted BARNES and F.E. CLOSE
Rutherford Appleton Laboratory, Chilton, Didcot, Oxon, UK

Received 15 April 1982

We suggest that $q\bar{q}g$ mesons may exist
modes and perhaps be relatively stable. Th
splittings is computed analogously to Jaff
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THE EFFECTS OF COLOURED GLUE IN THE QCD MOTIVATED BAG OF HEAVY QUARK–ANTIQUARK SYSTEMS

P. HASENFRATZ, R.R. HORGAN, J. KUTI and J.M. RICHARD
CERN, Geneva, Switzerland

Received 1 April 198

HYBRIDS: MIXED STATES OF QUARKS AND GLUONS*

Michael CHANOWITZ and Stephen SHARPE
Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720, USA

Received 20 September 1982
(Revised 26 January 1983)

We calculate the spectrum of the four ground state hybrid ($\bar{q}qg$) nonets, $J^{PC} = (0,1,2)^{-+}, 1^{--}$, using the MIT bag model to first order in cavity perturbation theory. Quark and gluon self-energies are included by a fit to the s-wave mesons and baryons and to the glueball candidate i(1440). We find a large gluon self-energy which substantially increases our predictions of the glueball and hybrid masses. We discuss the phenomenology of hybrids, including a suggestion that the $A_3(1670)$ and a second peak at 1850 MeV in the $f\pi$ channel may be mixtures of the isovector $\bar{q}q$ d-wave state with the $\bar{q}qg$ s-wave.

1984

heavy ‘constituent’ gluon

excitation in a confining bag

Modeling hybrid mesons

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VOLUME 54, NUMBER 9 PHYSICAL REVIEW LETTERS 4 MARCH 1985

Gluonic Excitations of Mesons: Why They Are Missing and Where to Find Them

Nathan Isgur and Richard Kokoski
Department of Physics, University of Toronto, Toronto, Ontario M5S 1A7, Canada

and

Jack Paton
Department of Theoretical Physics, University of Oxford, Oxford OX1 3NP, England
(Received 28 November 1984)

We have studied the decays of the low-lying gluonic excitations of mesons (hybrids) predicted by a flux-tube model for chromodynamics. The probable reason for the absence to date of signals for such states is immediately explained: The lowest-lying hybrids decay preferentially to final states with one excited meson [e.g., $B(1235)\pi$, $A_2(1320)\pi$, $K^*(1420)\bar{K}$, $\pi(1300)\pi$, . . .] rather than to two ground-state mesons (e.g., $\pi\pi$, $\rho\pi$, $K^*\bar{K}$, . . .). We make specific predictions of decay channels which will contain J^{PC} exotic hybrid resonance signals and suggest some possibly fruitful production mechanisms.

1984

heavy ‘constituent’ gluon

excitation in a confining bag

flux-tube
oscillation

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PHYSICAL REVIEW D **78,** 056003 (2008)

Heavy quarkonium hybrids from Coulomb gauge QCD

Peng Guo and Adam P. Szczepaniak
Physics Department and Nuclear Theory Center, Indiana University, Bloomington, Indiana 47405, USA

Giuseppe Galatà, Andrea Vassallo, and Elena Santopinto
INFN, Sezione di Genova, via Dodecaneso 33, 16146 Genova, Italy
(Received 23 July 2008; published 12 September 2008)

Using the nonrelativisitc reduction of Coulomb gauge QCD we compute a spectrum of the low mass hybrid mesons containing a heavy quark-antiquark pair. The gluon degrees of freedom are treated in the mean field approximation calibrated to the gluelump spectrum. We discuss the role of the non-Abelian nature of the QCD Coulomb interaction in the ordering of the spin-parity levels.

---

heavy ‘constituent’ gluon

excitation in a confining bag

flux-tube oscillation

Hybrid baryons

- Some suggestions for **hybrid baryons** in QCD

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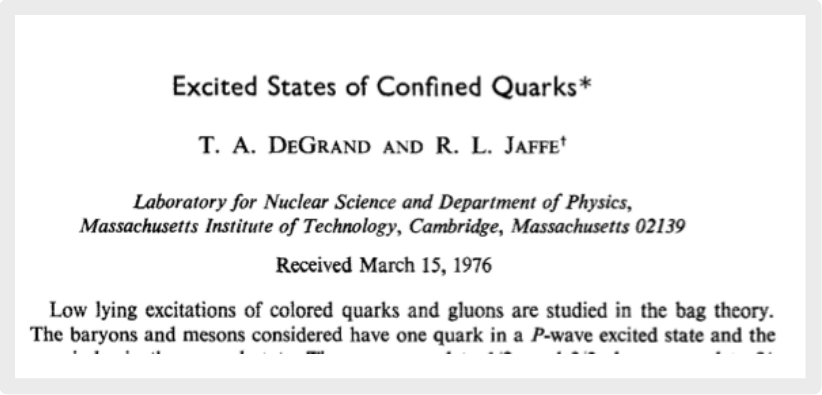


Excited States of Confined Quarks*

T. A. DeGrand and R. L. Jaffe†

Laboratory for Nuclear Science and Department of Physics,
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

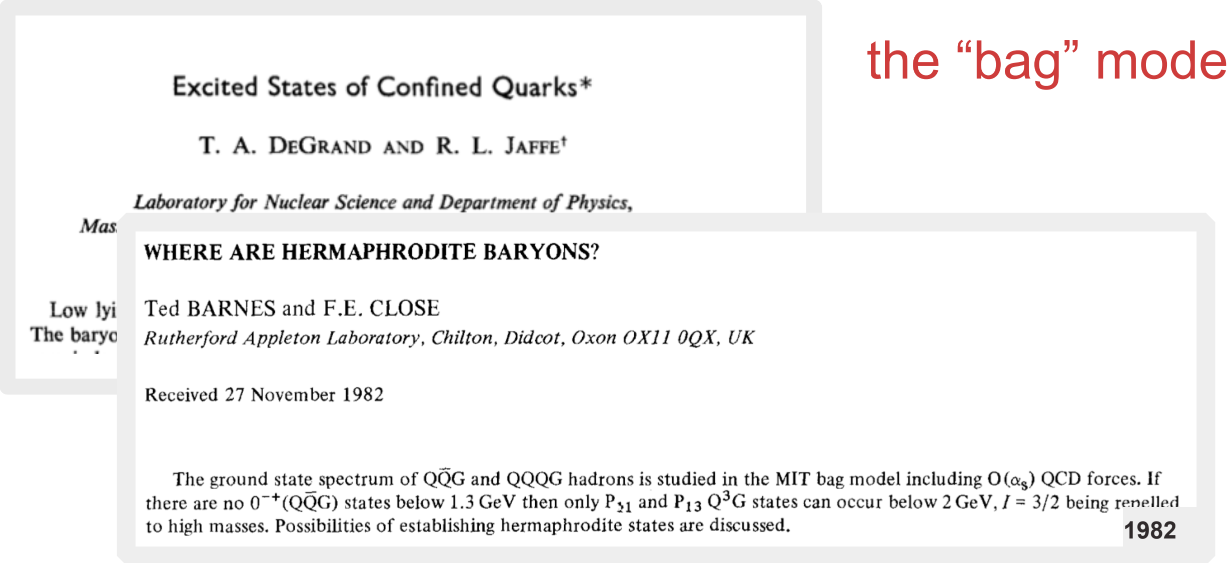
Received March 15, 1976

Low lying excitations of colored quarks and gluons are studied in the bag theory. The baryons and mesons considered have one quark in a *P*-wave excited state and the

the “bag” model

Hybrid baryons

- Some suggestions for **hybrid baryons** in QCD



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T. A. DeGrand and R. L. Jaffe†

Laboratory for Nuclear Science and Department of Physics,

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The baryo

WHERE ARE HERMAPHRODITE BARYONS?

Ted BARNES and F.E. CLOSE

Rutherford Appleton Laboratory, Chilton, Didcot, Oxon OX11 0QX, UK

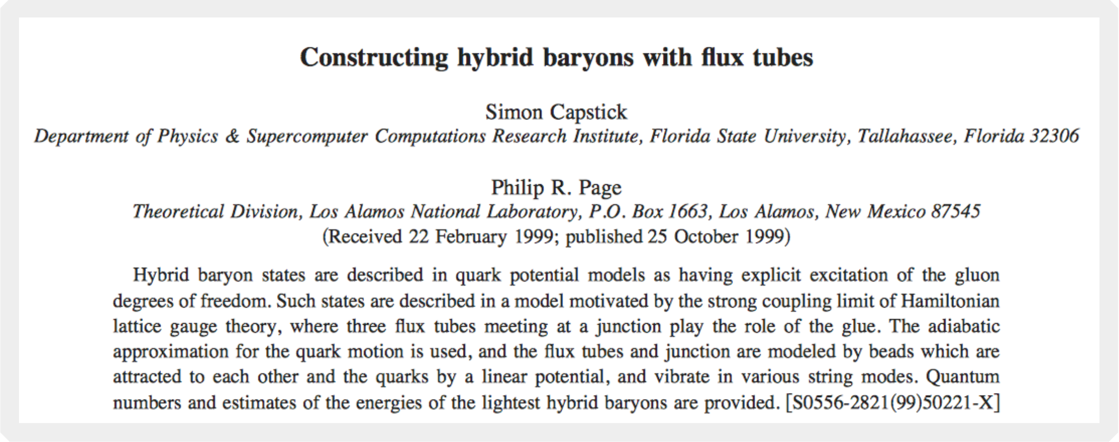
Received 27 November 1982

The ground state spectrum of $Q\bar{Q}G$ and QQQG hadrons is studied in the MIT bag model including $O(\alpha_s)$ QCD forces. If there are no $0^{-+}(Q\bar{Q}G)$ states below 1.3 GeV then only P_{11} and P_{13} Q^3G states can occur below 2 GeV, $I = 3/2$ being repelled to high masses. Possibilities of establishing hermaphrodite states are discussed.

1982

the “bag” model

gluonic excitation in a confining bag



Constructing hybrid baryons with flux tubes

Simon Capstick

Department of Physics & Supercomputer Computations Research Institute, Florida State University, Tallahassee, Florida 32306

Philip R. Page

Theoretical Division, Los Alamos National Laboratory, P.O. Box 1663, Los Alamos, New Mexico 87545

(Received 22 February 1999; published 25 October 1999)

Hybrid baryon states are described in quark potential models as having explicit excitation of the gluon degrees of freedom. Such states are described in a model motivated by the strong coupling limit of Hamiltonian lattice gauge theory, where three flux tubes meeting at a junction play the role of the glue. The adiabatic approximation for the quark motion is used, and the flux tubes and junction are modeled by beads which are attracted to each other and the quarks by a linear potential, and vibrate in various string modes. Quantum numbers and estimates of the energies of the lightest hybrid baryons are provided. [S0556-2821(99)50221-X]

flux-tube oscillation

Lattice QCD

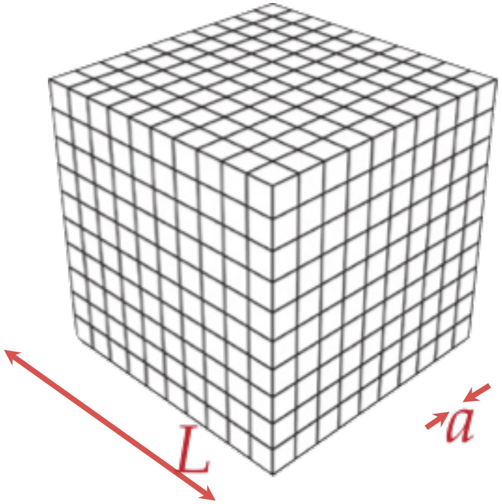
- First-principles numerical approach to the field-theory
- Use lattice as a regulator (UV & IR)

  » in principle recover physical QCD as

$$a \to 0 \qquad L \to \infty \qquad m_q^{\text{calc.}} \to m_q^{\text{phys.}}$$

  » large scale computational problem ...

CUBIC LATTICE



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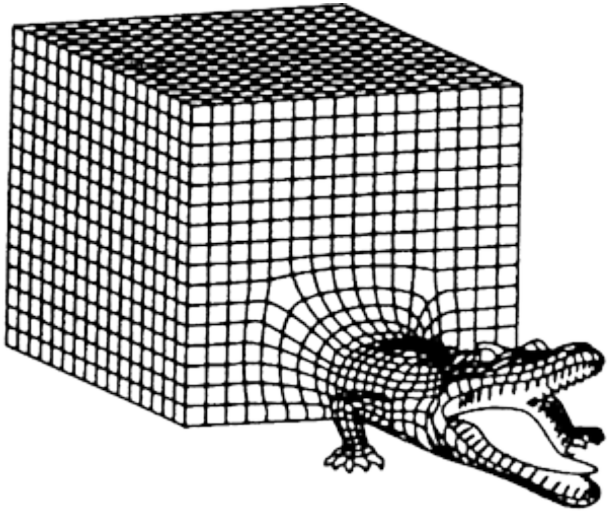
  - » in principle recover physical QCD as

$$a \to 0 \qquad L \to \infty \qquad m_q^{\text{calc.}} \to m_q^{\text{phys.}}$$

  - » which can get really expensive …

CUBIC LATTICE



People wanted to build computers

Some not-so-successful national efforts

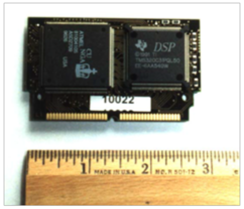
☹ QCD Teraflop 1

☹ QCD Teraflop 2 …

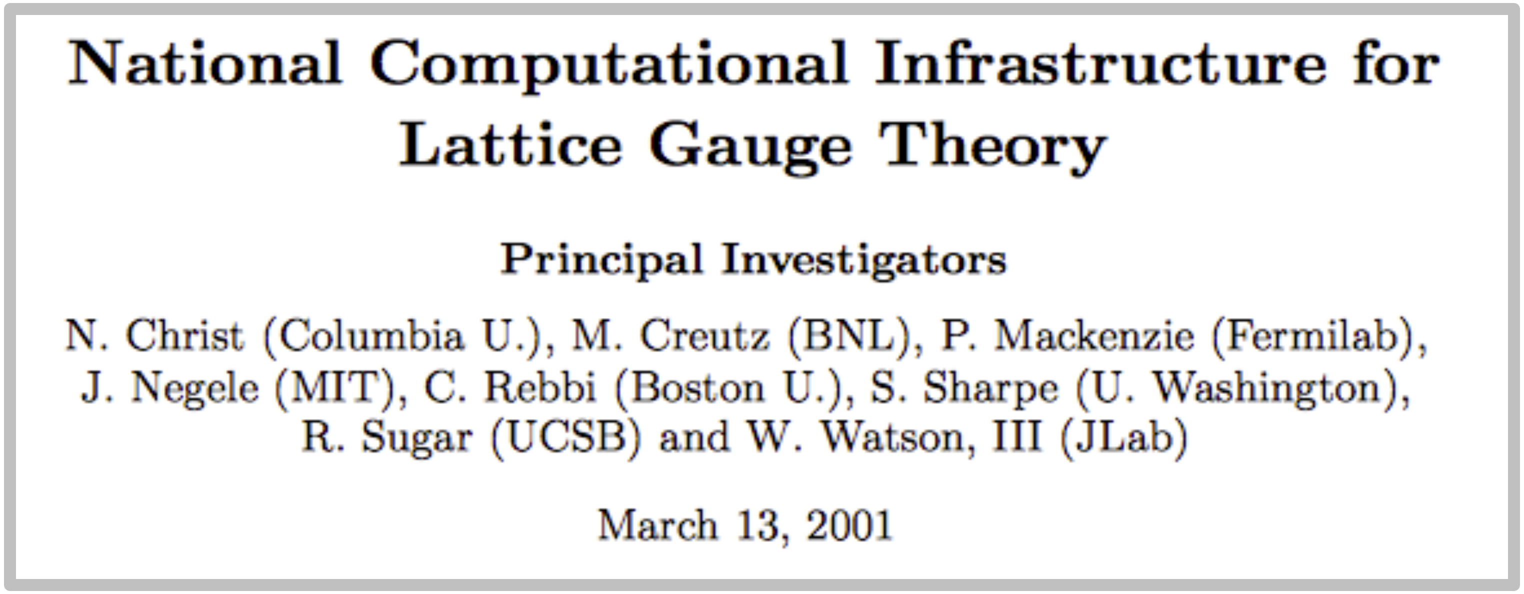
But there were some successes

☺

QCDSP



(Finally) a national proposal was funded…



National Computational Infrastructure for Lattice Gauge Theory

Principal Investigators

N. Christ (Columbia U.), M. Creutz (BNL), P. Mackenzie (Fermilab),
J. Negele (MIT), C. Rebbi (Boston U.), S. Sharpe (U. Washington),
R. Sugar (UCSB) and W. Watson, III (JLab)

March 13, 2001

And it evolved into USQCD

QCDOC

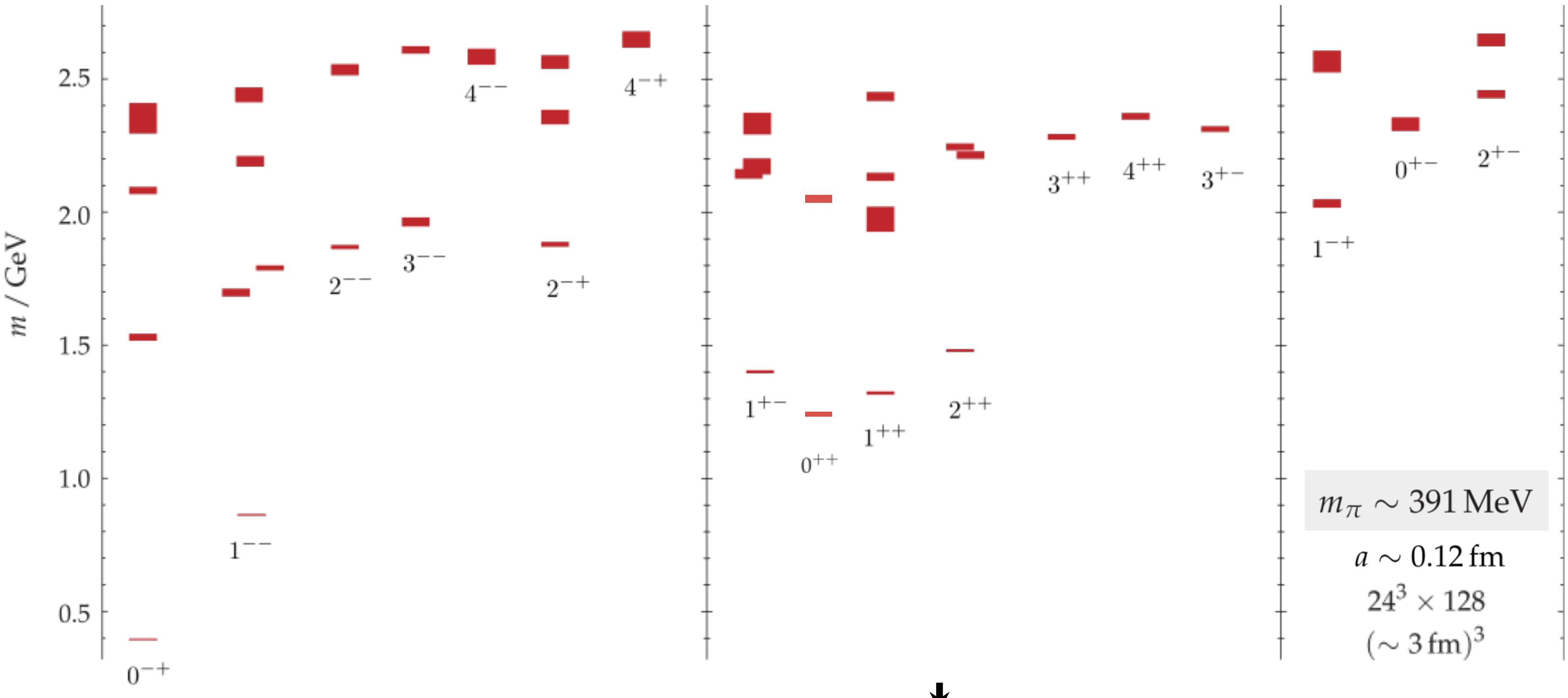
CLUSTERS

GPU

BG/Q

Meson spectrum

Range of hadron interpolators → matrix of correlation functions → variational description



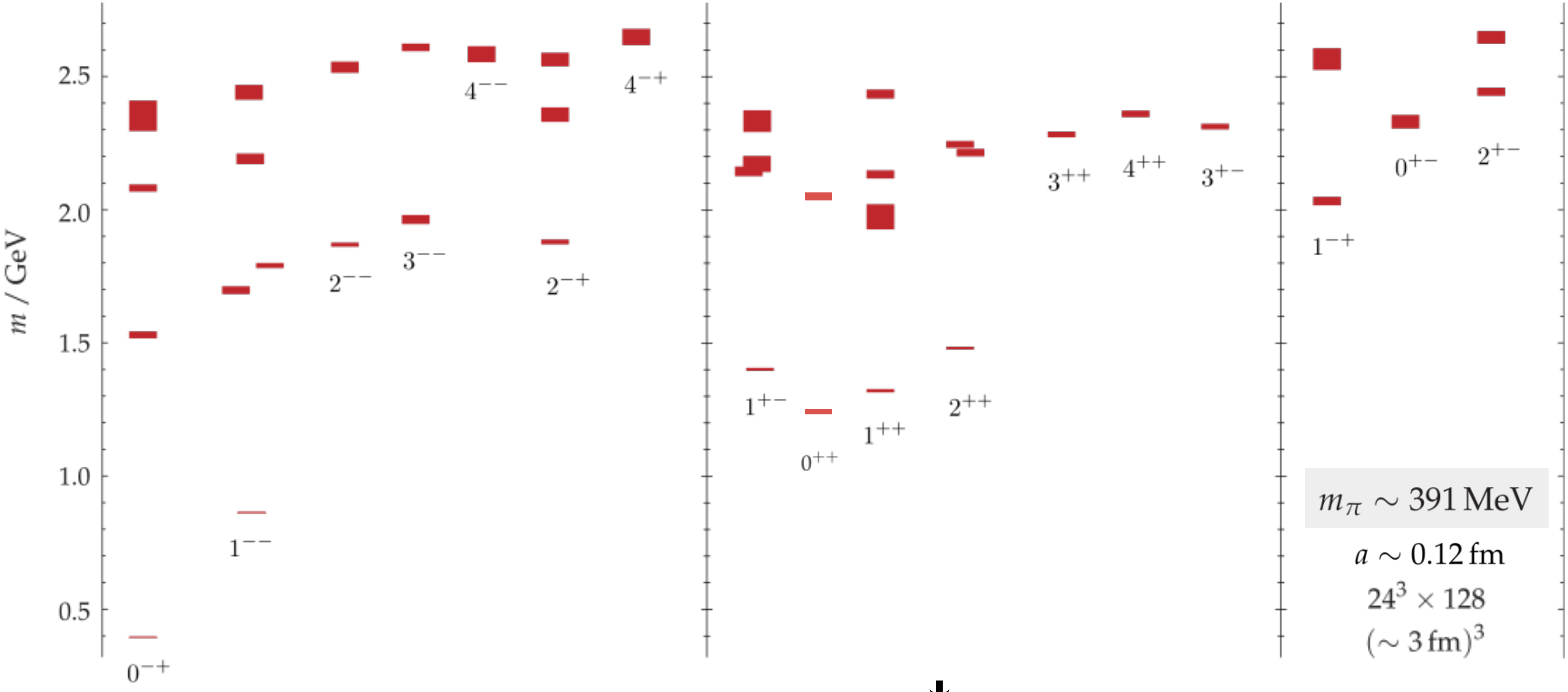
🟥 Monte Carlo stat. uncertainty

arXiv:1004.4930. 1309.2608

Meson spectrum

Range of hadron interpolators → matrix of correlation functions → variational description

- Patterns similar to experiments - even at artificially heavy pion mass



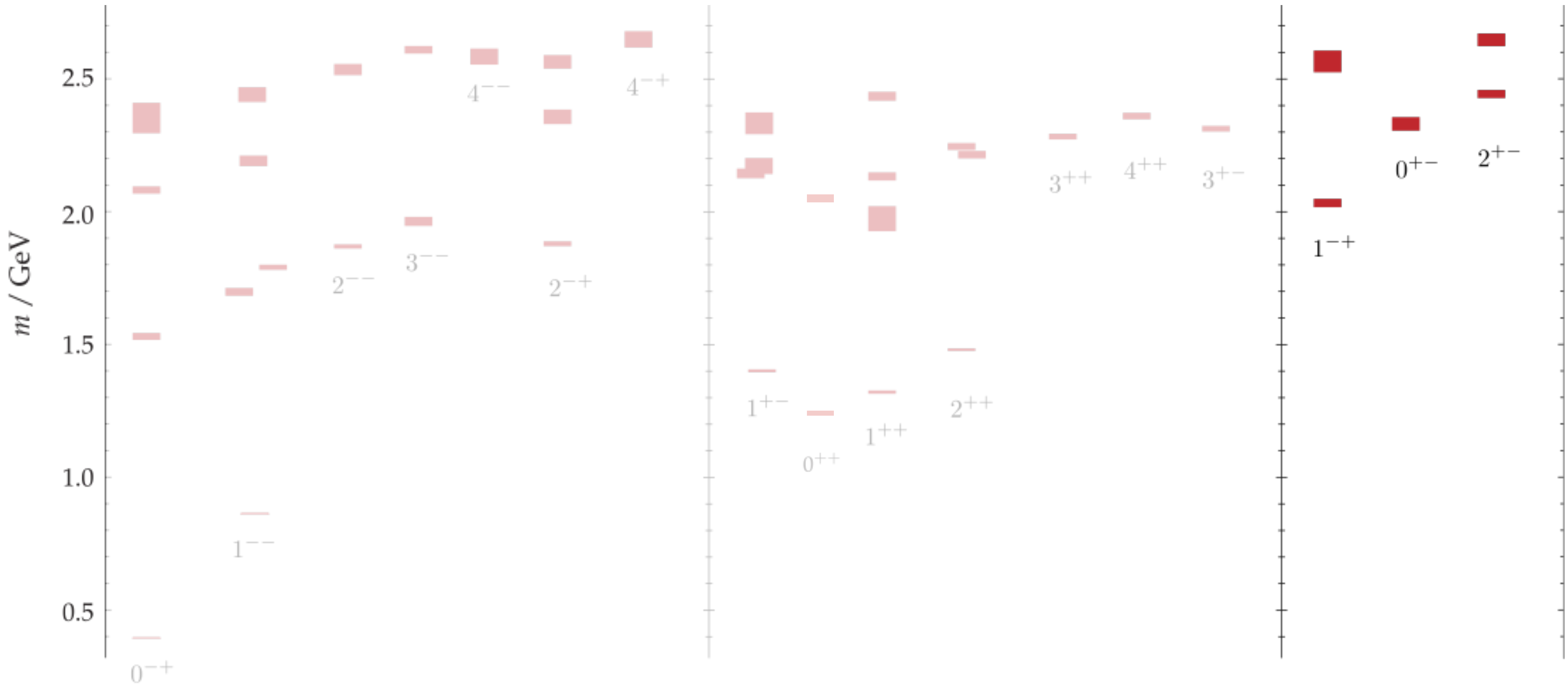
Monte Carlo
stat. uncertainty

arXiv:1004.4930. 1309.2608

Meson spectrum

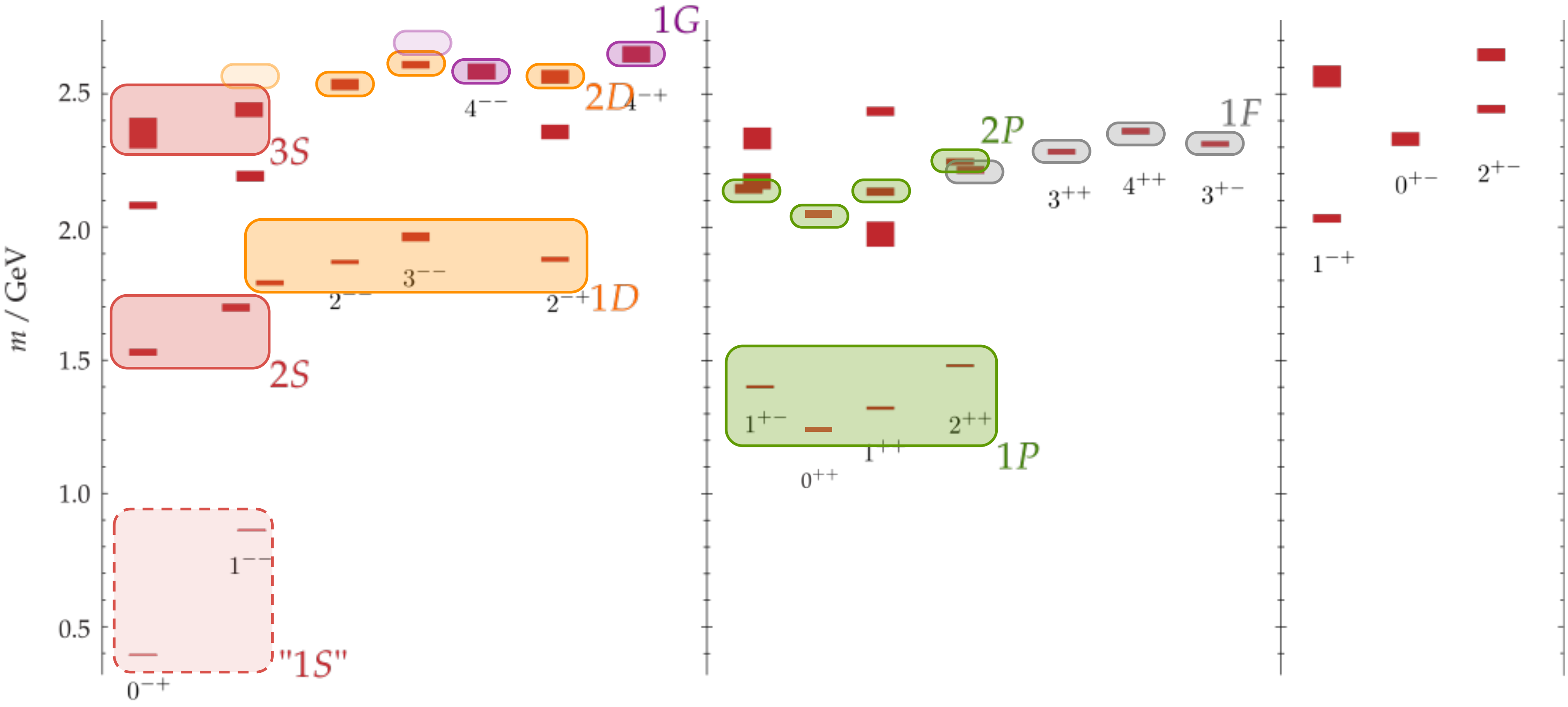
- **Exotic J^{PC}** states are present

EXOTIC J^{PC}



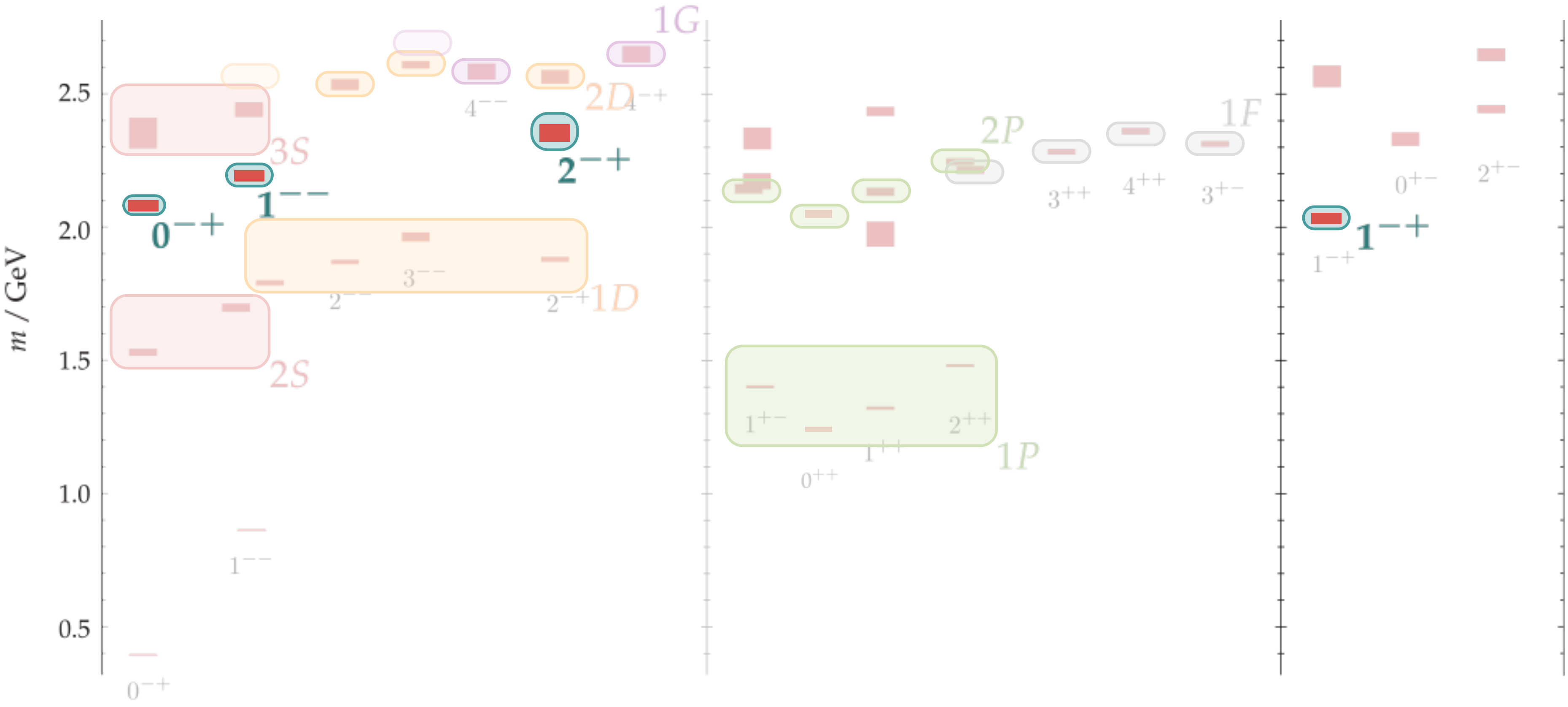
$q\bar{q}$ interpretation?

- Appears to be some $q\bar{q}$-like near-degeneracy patterns



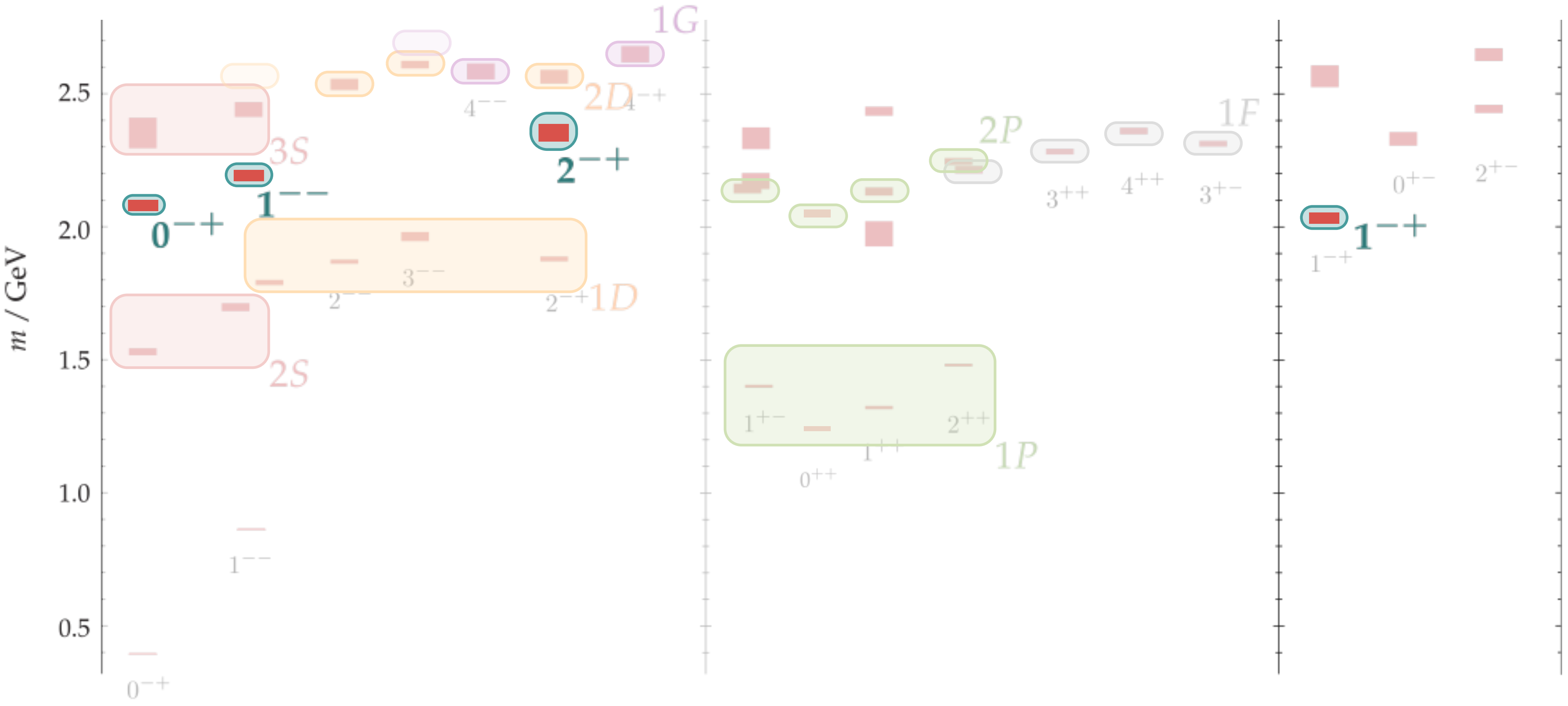
$q\bar{q}$ interpretation?

- “Extra” non-exotic states at same energy scale as lightest exotic?



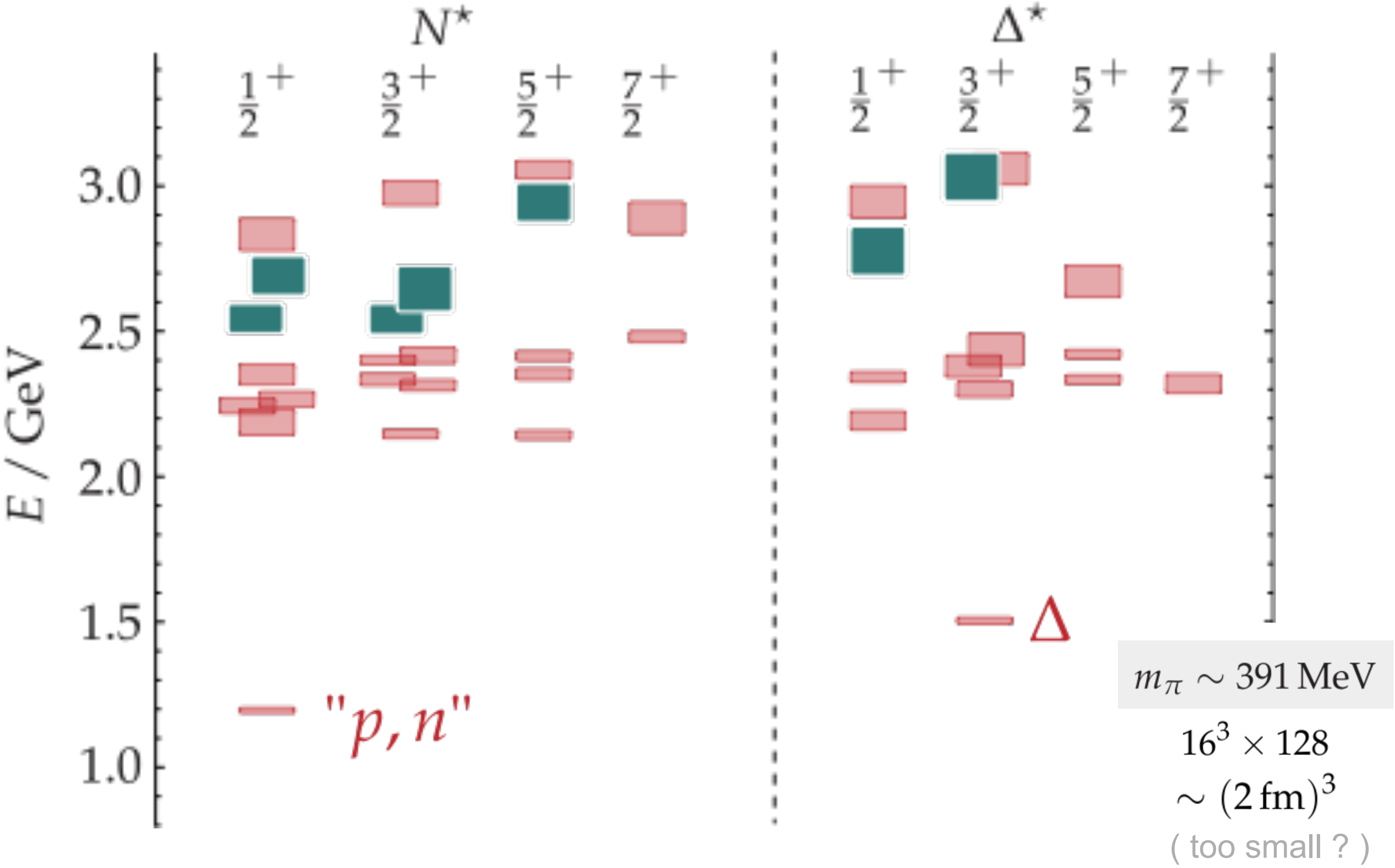
Meson spectrum

- Consider the relative size of operator overlaps $\langle \mathfrak{n} | \mathcal{O}_i^\dagger | \varnothing \rangle$
  - Suggests we have a hybrid meson super-multiplet



Hybrid baryons $qqq\otimes G$

- Lattice QCD spectrum from a large basis of *qqq* operators



arXiv:1104.5152, 1201.2349

Hybrid baryons *qqq*⊗G

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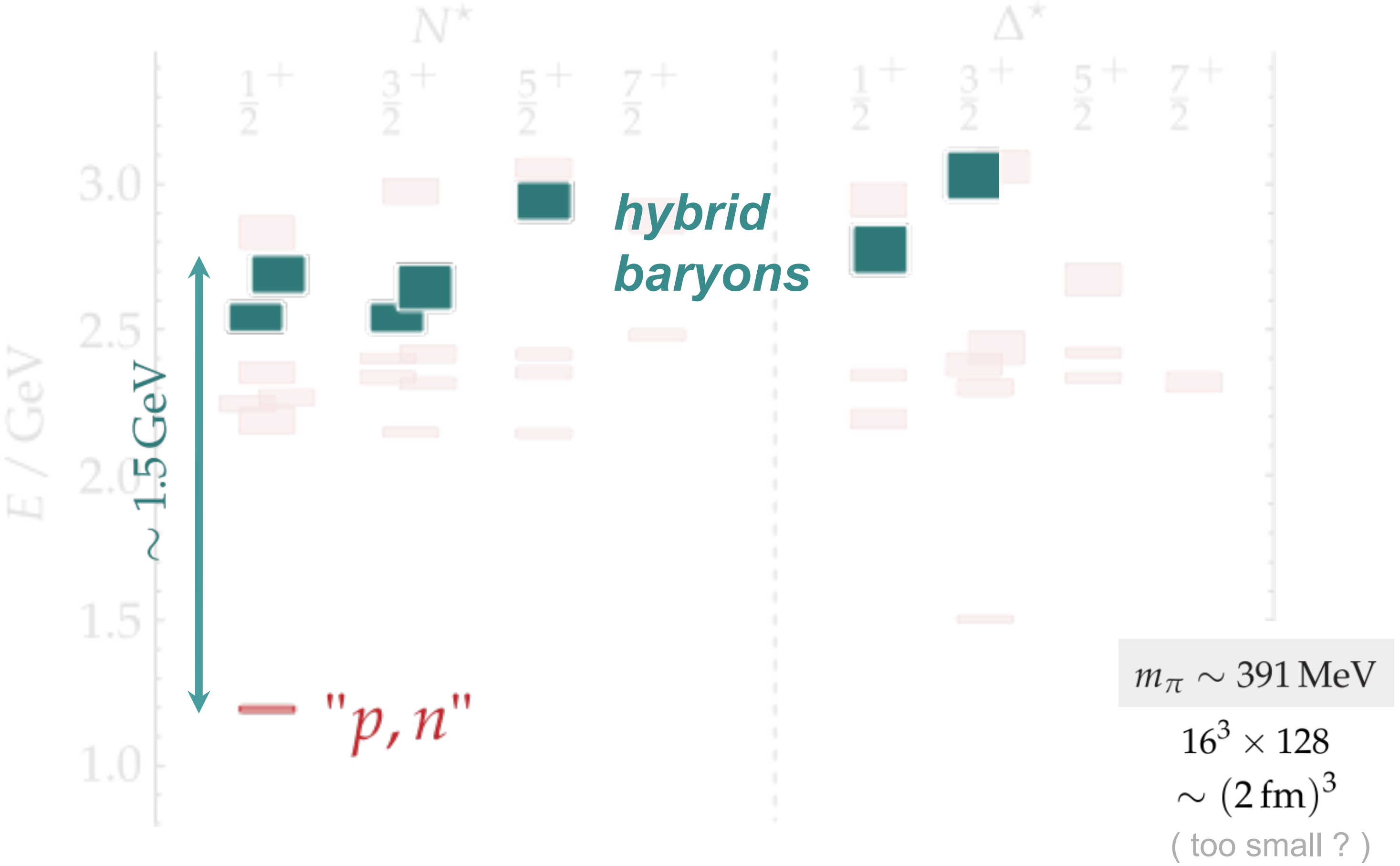
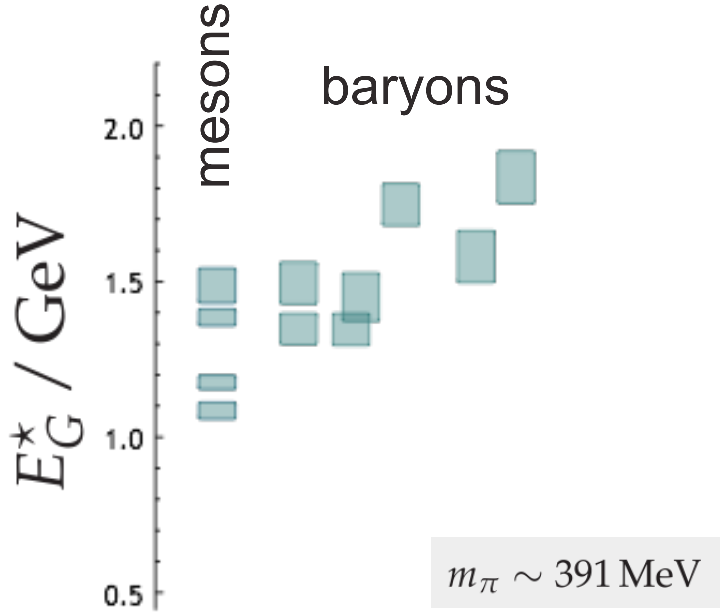


- No exotic quantum numbers for baryons

arXiv:1104.5152, 1201.2349

A common gluonic excitation scale?

- Subtract the ‘quark mass’ contribution



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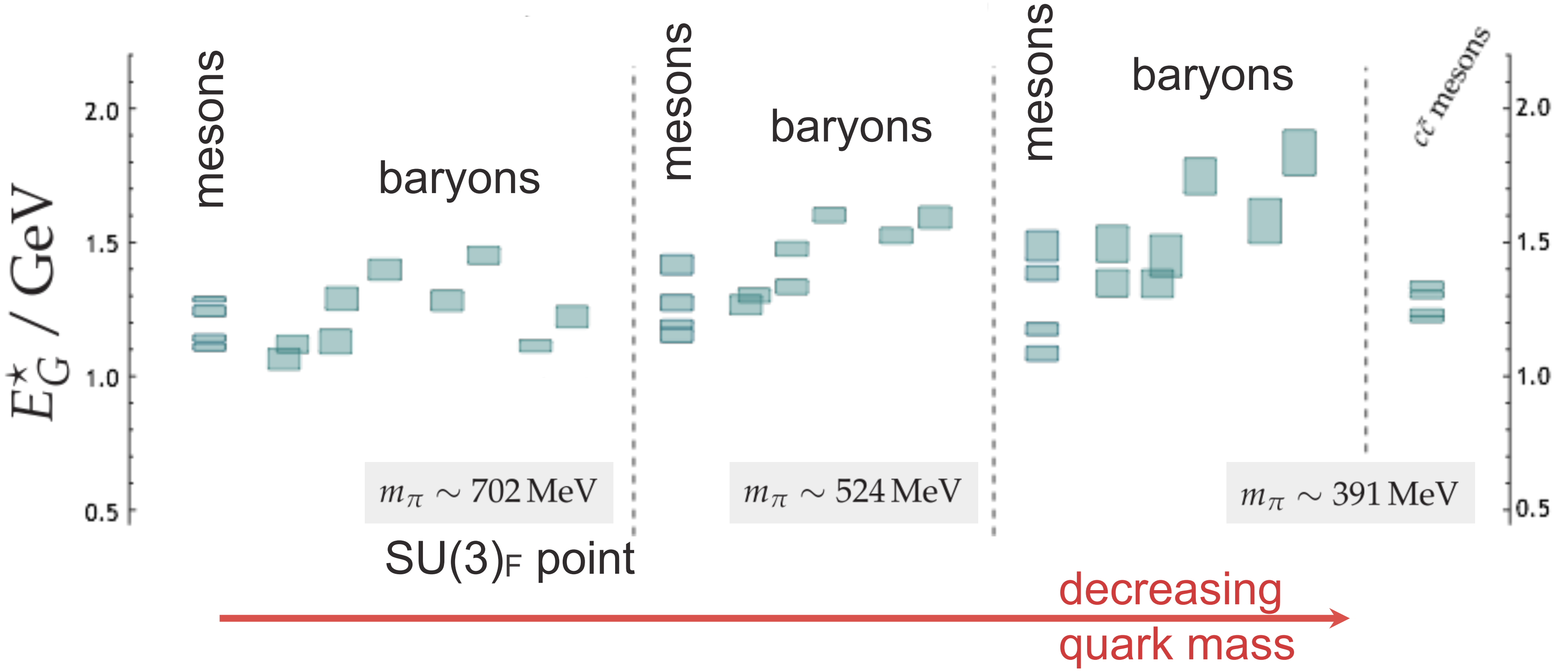

- **Common energy scale of gluonic excitation** $\sim$ 1.3 GeV

What comes next?

- Results so far suggest a rich spectrum of hadrons in QCD
  - Suggests a full baryon spectrum, including hybrid mesons and baryons
  - So far, calculations at artificially heavy quarks
  - And so far don’t resolve the fact they they should decay (*& into what ?*)

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  - But how?
    - Finite volume techniques

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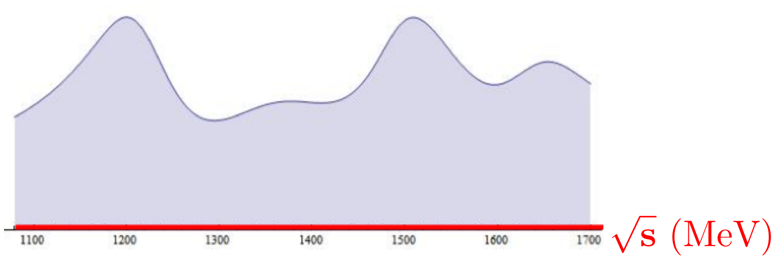
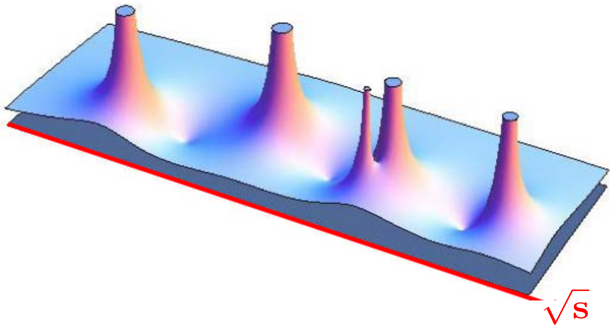
- Provides a direct connection to the S-matrix of QCD
  - Complications: truncation to finite number of partial waves, 3-body decays

Resonances

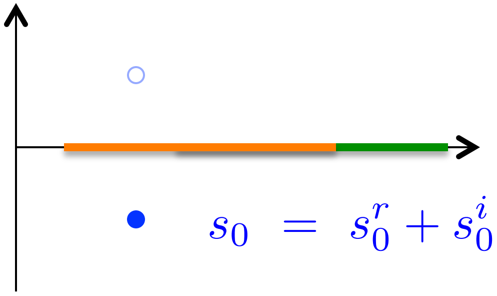
- Most hadrons are resonances





  - Formally defined as a pole in a partial-wave projected scattering amplitude

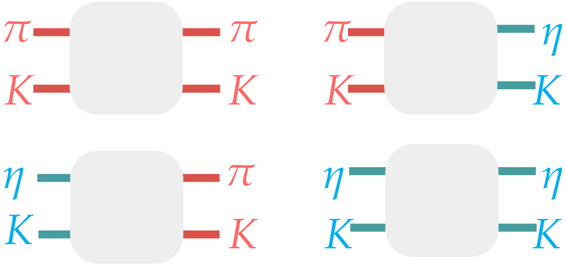
$$t_l(s) \sim \frac{R}{s_0 - s} + \ldots$$



- Can we predict hadron properties from first principles?

Isospin=1/2 πK/ηK scattering

- 73 (real) energies on 3 volumes & momenta
  - Constrain S-matrix in complex plane



channel $i, j = (\pi K, \eta K)$

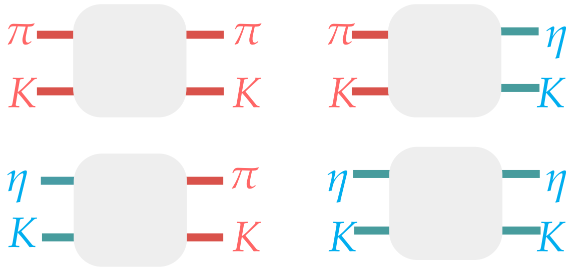
arXiv:1406.4158

Isospin=1/2 πK/ηK scattering

- 73 (real) energies on 3 volumes & momenta
    - Constrain S-matrix in complex plane

- Broad resonance in S-wave πK
- Bound state pole in $J^P = 1^-$
- Narrow resonance in D-wave πK

- all at unphysical quark masses…



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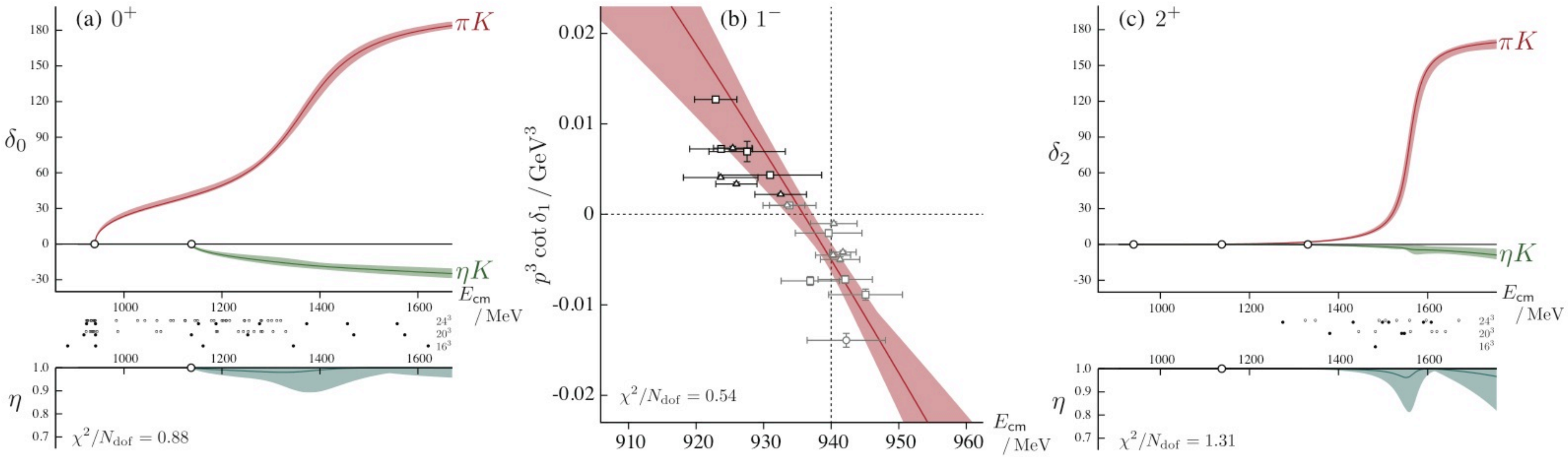

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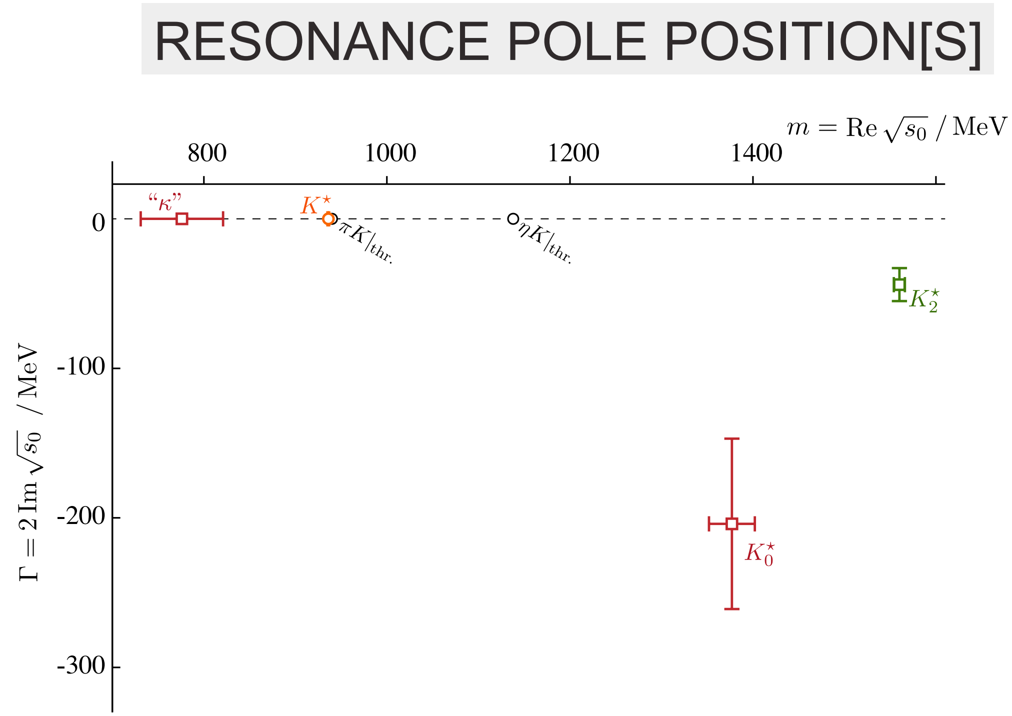
Can even determine pole locations

- Find S & D-wave poles on unphysical sheets
- Also presence of a “virtual” bound-state (pole at small –imag axis of momentum plane)



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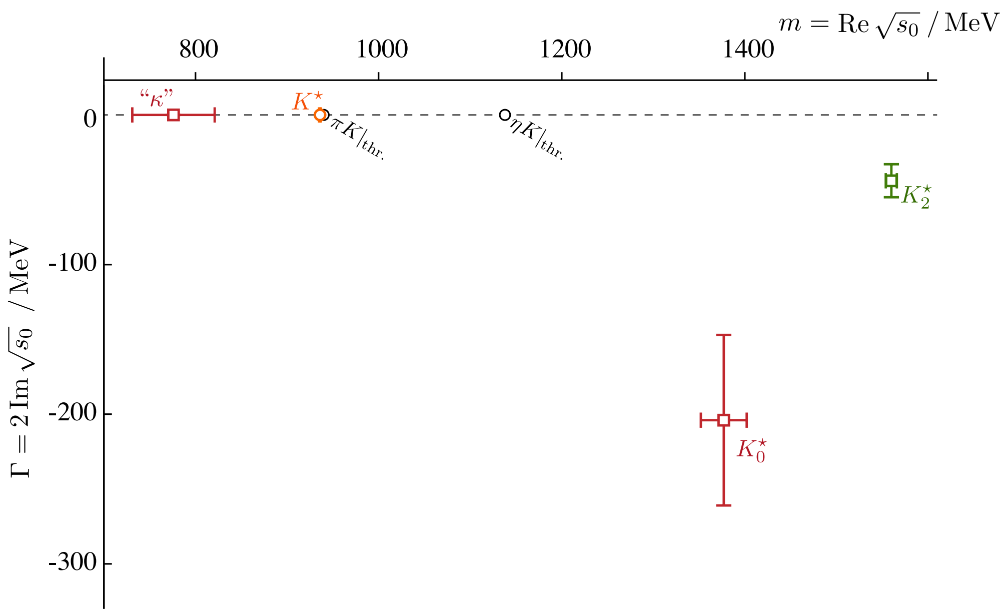
RESONANCE POLE POSITION[S]



Possible because lattice is IR regulator

arXiv:1406.4158

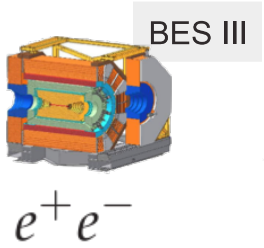
Path forward…

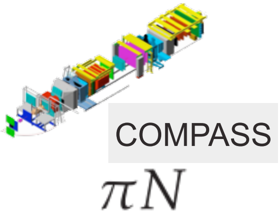
- A first picture of the highly excited spectrum of QCD:
  - Suggests another(?) scale in QCD ~ 1.3 GeV
  - But results are woefully incomplete…

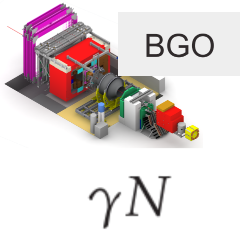
- Next step – determine decays
  - (“Finally”) have a connection between QCD, lattice, and S-matrices
  - Exotics? Hybrids (mesons/baryons)? Scalar sector? Light/charm?

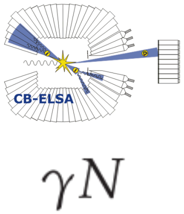
- Future?
  - Glue obviously important in QCD
  - Hard scale should manifest at large Bjorken-x - 12GeV, EIC?

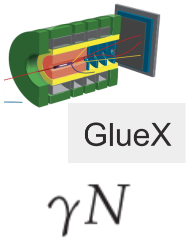
- Made possible by using lattice as a regulator – thank you Mike!

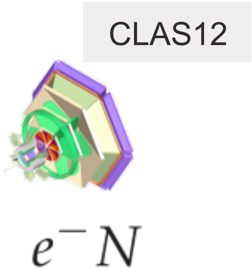






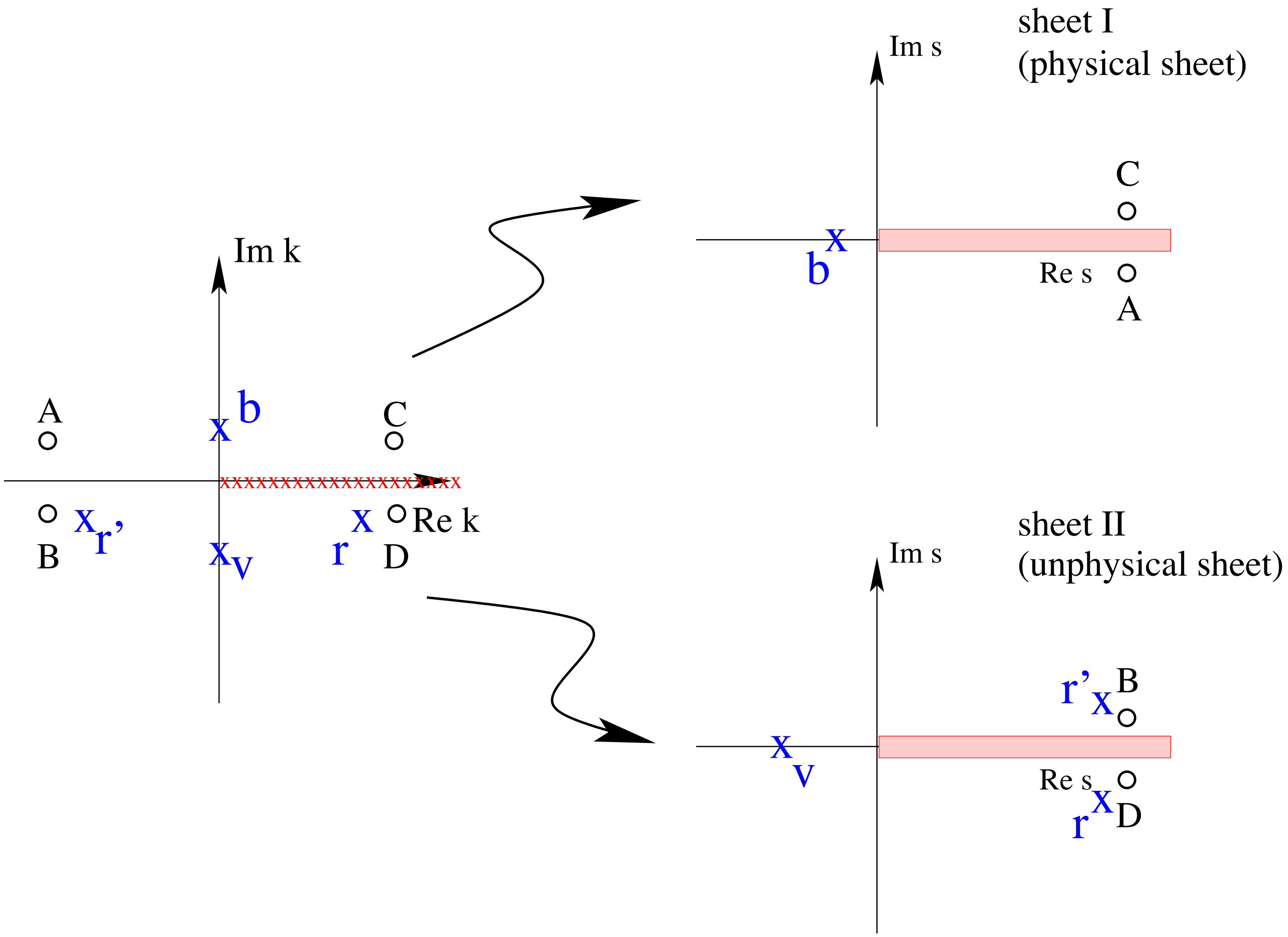






Backup slides

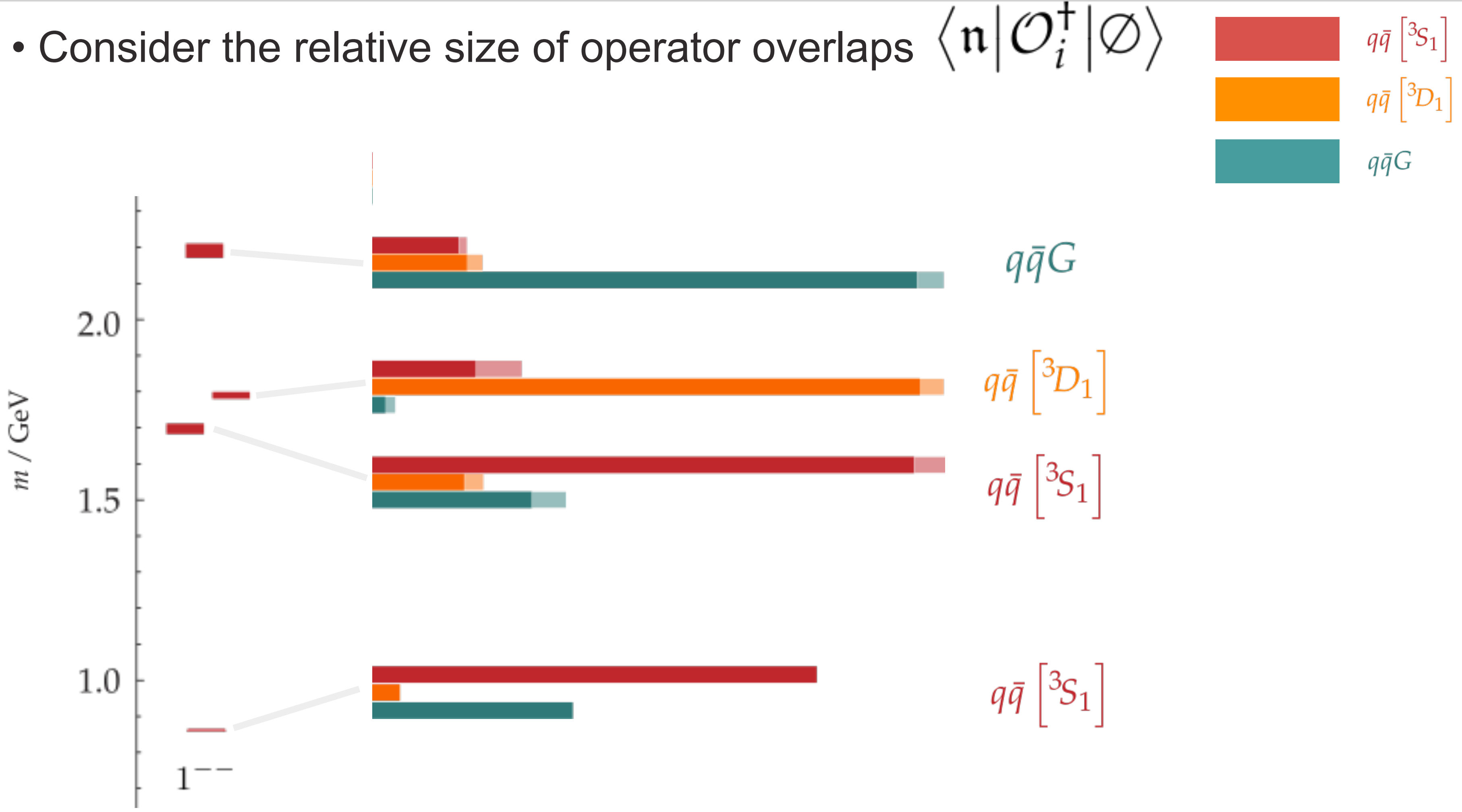
Complex plane



arXiv:1407.7452

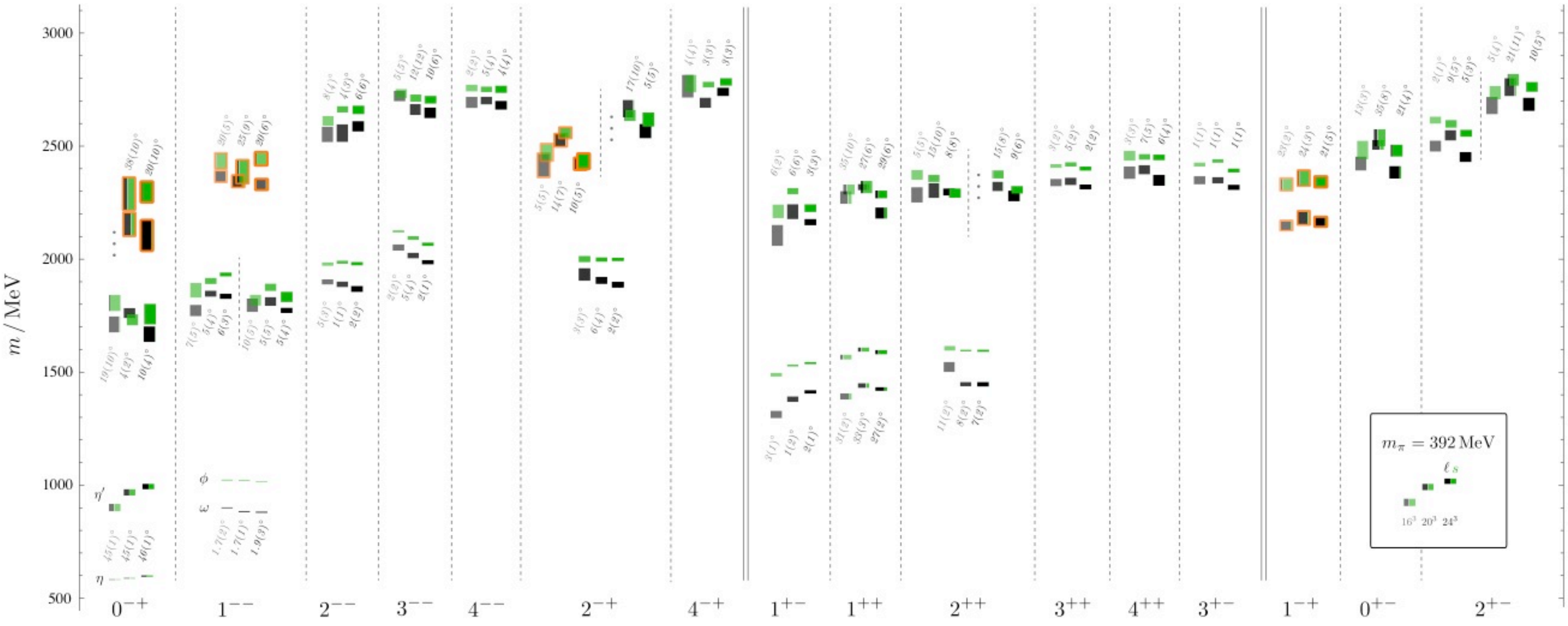
1^{--} operator overlaps

- Consider the relative size of operator overlaps $\langle \mathfrak{n} | \mathcal{O}_i^\dagger | \varnothing \rangle$



Volume dependence: isoscalar mesons

Energies determined from single-particle operators:
Range of J^{PC} - color indicates light-strange flavor mixing



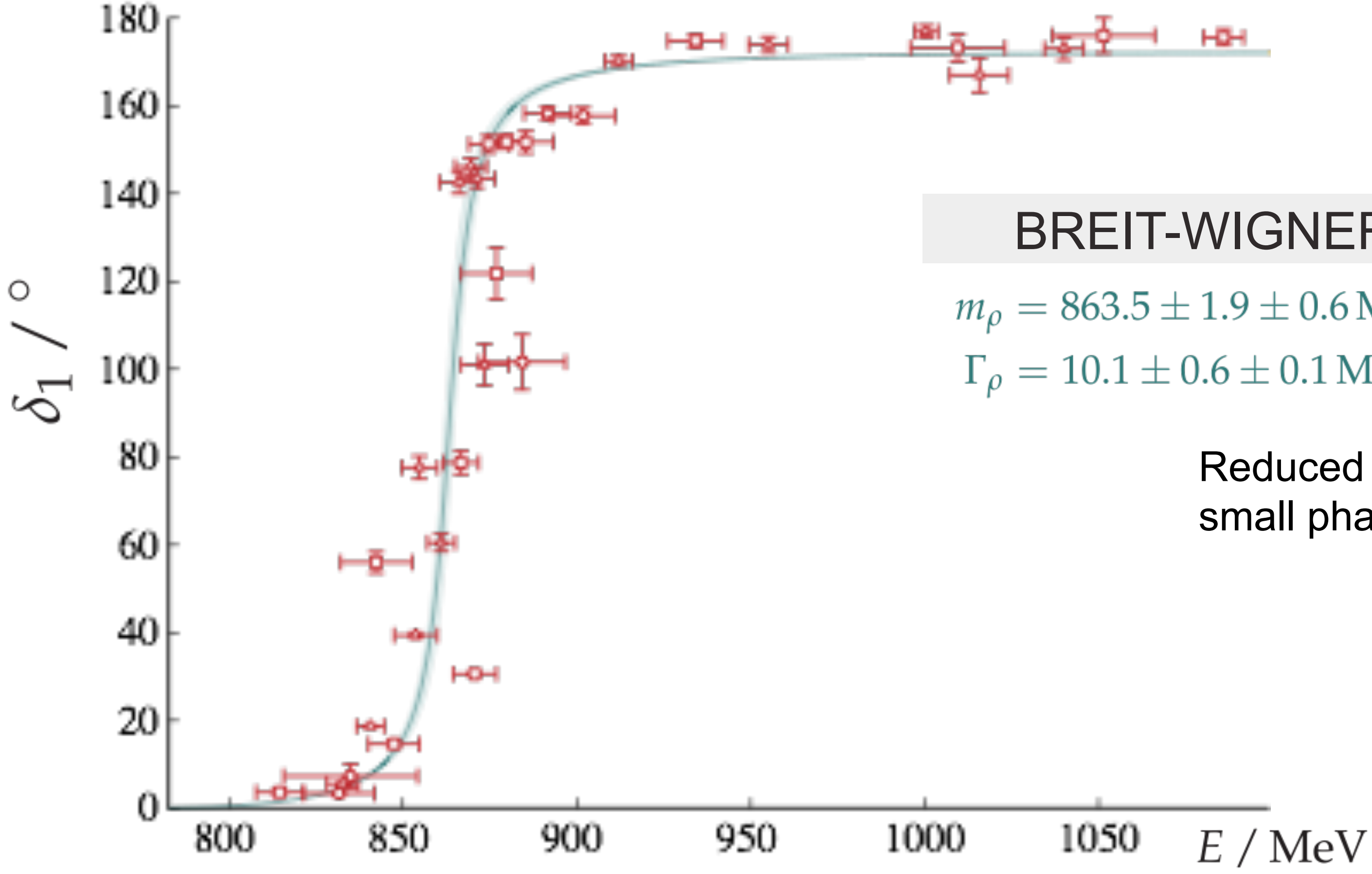
Some volume dependence:

- Interpretation: energies determined up to a hadronic width

arXiv:1309.2608

Isospin=1 $(J^{PC}=1^{--})$ ππ scattering

- Breit-Wigner fit to the energy dependence



BREIT-WIGNER

$$m_\rho = 863.5 \pm 1.9 \pm 0.6\,\text{MeV}$$

$$\Gamma_\rho = 10.1 \pm 0.6 \pm 0.1\,\text{MeV}$$

Reduced width from
small phase-space

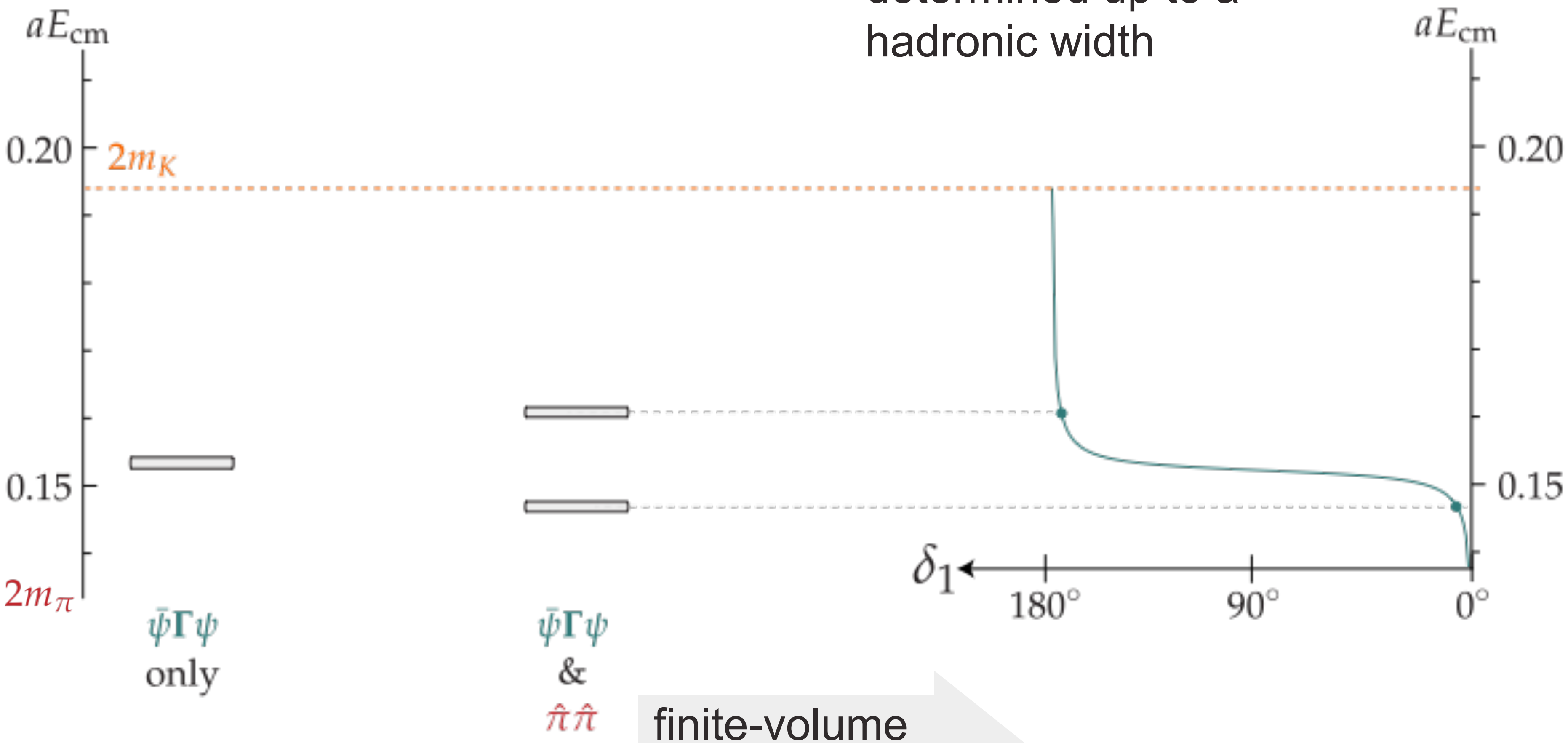
$$g_{\rho\pi\pi} = 4.83 \pm 0.13 \pm 0.02$$

arxiv:1212.0830

$m_\pi \sim 391\,\text{MeV}$

Resolving a resonant ρ meson

Suggests energies determined up to a hadronic width



finite-volume *Lüscher* formalism