Additive shift in the fermion mass generated by the axial anomaly in one-flavor QCD

Celebrating Mike Creutz

September 4, 2014

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Outline

- Lattice QCD from 1979 to 2014
- Anomalous mass generation
  - The question
  - Numerical exploration
  - Theoretical suggestions

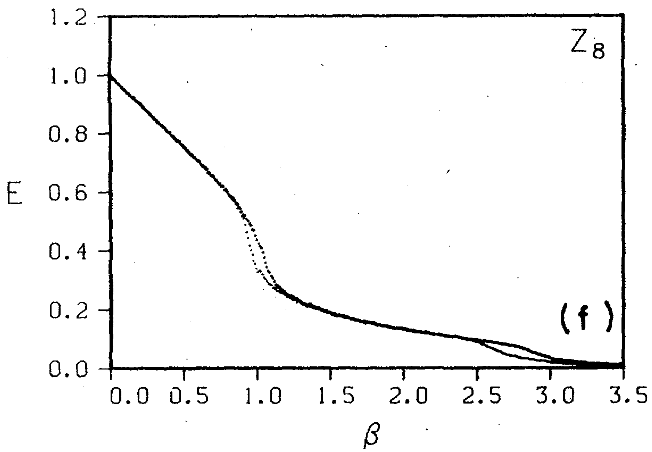
Lattice QCD

1979 – 2014

Lattice QCD 1979 → 2014

QCD “Thermodynamics”

1979

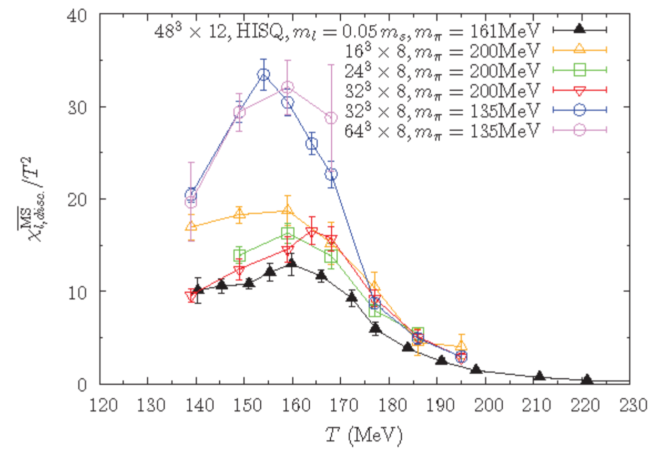


Z_8 gauge theory 8^3 x 20

Phys.Rev. D20 (1979) 1915

Michael Creutz, Laurence Jacobs, Claudio Rebbi

2014



2+1 flavor, SU(3) gauge theory 64^3 x 8

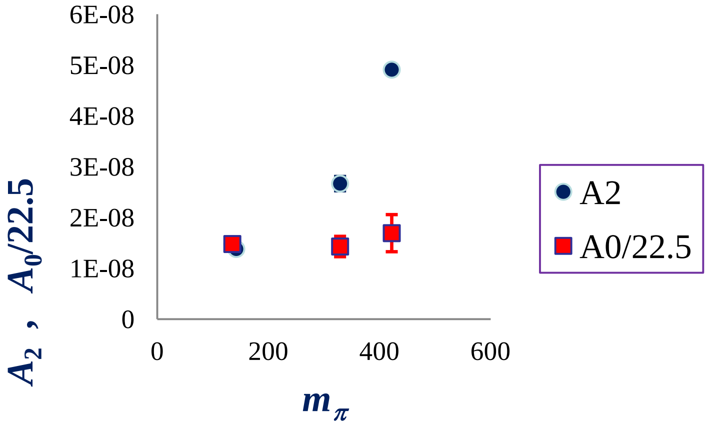
Phys.Rev.Lett. 113 (2014) 082001

T. Bhattacharya, M. Buchoff, N. Christ, H.-T. Ding, R. Gupta, C. Jung, F. Karsch, Z. Lin, R. Mawhinney, G. McGlynn, S. Mukherjee, D. Murphy, P. Petreczky, D. Renfrew, C. Schroeder, R. Soltz, P. Vranas, H. Yin

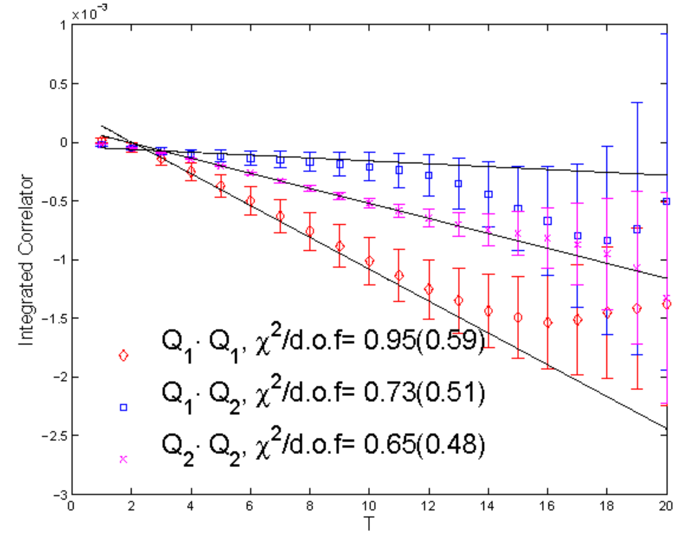
Lattice QCD – 2014

Kaon physics

$\Delta I = 1/2$ rule



M_{K_L} - M_{K_S}



- Continuum results, (preliminary):
  - $\mathrm{Re}(A_2) = (1.583 \pm 0.067_{stat})\times 10^{-8}$ GeV
  - $\mathrm{Im}(A_2) = -(7.51 \pm 27_{stat})\times 10^{-13}$ GeV
- Experiment: $\mathrm{Re}(A_2) = 1.479(4)\ 10^{-8}$ GeV

- Proof of principle:
  - $\Delta M_K = 3.30(34)\ 10^{-12}$ MeV

RBC

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Tobias Tsang (Southampton)

Anomalous Mass Generation

One-Flavor QCD and the Axial Anomaly

- Consider one-flavor QCD:

$$\mathcal{A} = \int d^4x \left\{ \overline{q} \Big(\gamma^\mu (\partial^\mu + igA^\mu) - mq(x) \Big) - \frac{1}{4} F^{\mu\nu} F^{\mu\nu}(x) \right\}$$

- For $N_f = 1$, there is a single axial current $J^{5,\mu(x)} = \overline{q}\gamma^\mu\gamma^5 q(x)$ which obeys an anomalous conservation law:

$$\partial^\mu J^{5,\mu}(x) = 2im\overline{q}\gamma^\mu\gamma^5 q(x) + \frac{g^2}{16\pi^2} F^{\mu\nu}\widetilde{F}^{\mu\nu}$$

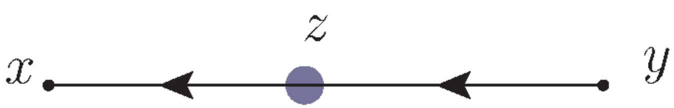
- Is the fermion mass protected when the axial current is not conserved?
- Does *m* receive an additive renormalization?
Mike: "***Yes!***" [Phys. Rev. Lett. 92 (2004) 162003]

Definition of fermion mass

- Meaning of fermion mass obscured by confinement.
- We do not want to confuse:
  - The quark mass in the Lagrangian (current algebra mass)
  - The effective mass generated by vacuum symmetry breaking (constituent mass).
- Easily distinguished by large momentum behavior of the fermion propagator:
  - Input mass: $$\lim_{p\to\infty} \frac{1}{12}\mathrm{tr}\{S_q(p)\} \sim Z_1(p)\frac{m}{p^2}\left(\frac{1}{\ln(p^2)}\right)^{\gamma_1}$$
  - Constituent mass: $$\lim_{p\to\infty} \frac{1}{12}\mathrm{tr}\{S_q(p)\} \sim Z_2(p)\frac{\langle 0|\overline{q}q|0\rangle}{p^4}\left(\frac{1}{\ln(p^2)}\right)^{\gamma_2}$$

Effects of the axial anomaly

- At high momenta, a perturbative expansion about classical Yang-Mills solutions (instantons) will be accurate.
- The leading instanton effects on the quarks are given by 't Hooft's effective Lagrangian.
- For N_f=1 and an instanton of radius R at z



$$
\begin{aligned}
\langle q(x)\overline{q}(y)\rangle &= \rho(R)\frac{1}{\lambda_0}u_0(x-z)\overline{u}_0(z-y) \\
&\propto \rho(R)\frac{R^2}{m_f}\,\frac{\not x-\not z}{\left((x-z)^2\right)^2}\cdot\frac{\not z-\not y}{\left((z-y)^2\right)^2}
\end{aligned}
$$

Effects of the axial anomaly

- `t Hooft's result for the instanton density:

$$\rho(R) \propto \frac{(Rm_f)^{N_f}}{g^8} \frac{1}{R^5} e^{-\frac{8\pi^2}{g^2(\mu)} + \ln(R\mu)\left(11 - \frac{2}{3}N_f\right)} \underset{N_f=1}{\sim} m_f R^{\frac{19}{3}}$$

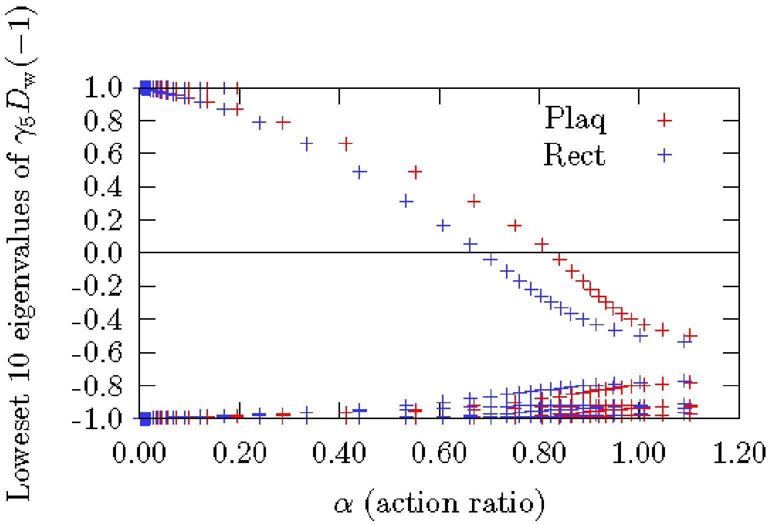
- Gives a momentum dependent mass:

$$M(p) \sim p^{-\frac{28}{3}} \approx p^{-9.33}$$

- However this assumes that `t Hooft's $\rho(R)$ describes lattice-scale instantons!

Lattice instantons

- Follow Luchang Jin's 2013 lattice proceedings
  [PoS LATTICE2013 (2013) 130]
- Start with a localized "instanton" gauge configuration contained in a fixed size box, perhaps 16^4:
  - Gauge action $A_{inst} = \alpha\, 8\pi^2/g_a^2$
  - Lattice Dirac equation has a zero mode $\lambda_0 \sim m$
- Not difficult to construct:
  - Wilson: $\alpha < 0.83$
  - Rectangle: $\alpha < 0.69$



Mass generated by lattice instantons

- Contribution of this lattice instanton to the path integral can be bounded:
  - Not hard to show that for g_a small:

$$\left(\exp -\frac{8\pi^2}{g_a^2}(\alpha+\epsilon)\right) \le \frac{\int_{\Delta\Omega} e^{-A[U]} d[U]}{\int_{\Omega} e^{-A[U]} d[U]} \le \left(\exp -\frac{8\pi^2}{g_a^2}(\alpha-\epsilon)\right)$$

  - Ω is the entire gauge configuration space
  - $\Delta\Omega$ is a sub-volume of configuration space where

$$\left|U_\mu(x) - U_\mu^{\text{inst}}(x)\right| < \epsilon' \quad \text{for all } \mu \text{ and } x$$

Mass generated by lattice instantons

- Appears reasonable to assume:
  - N such configurations widely separately in a large volume would occur with probability:

$$\left(e^{-\alpha\frac{8\pi^2}{g_a^2}}\right)^N$$

  - If N_f flavors of fermions are added this semi-classical probability would be unchanged. At one loop the only important effect would be from the zero modes: $(m_f a)^{N_f}$
- For one-flavor this would result in an anomalous fermion mass:

$$m_{\text{anom}} \sim \left(\frac{1}{a}\right)^{1-\alpha(11-2/3)} = \left(\frac{1}{a}\right)^{1-\alpha\frac{31}{3}}$$

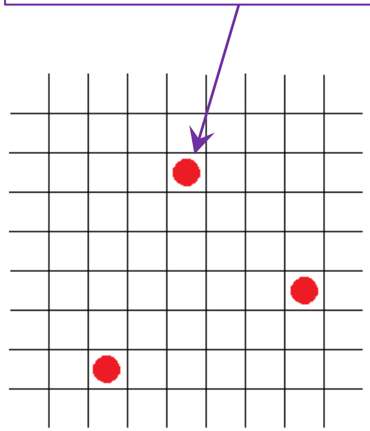
- Require $\alpha \leq 0.097$ and m_{anom} survives the continuum limit!

Engineering the gauge action

- Achieve $\alpha \le 0.097$ by modifying the plaquette action.
- Introduce a coarse-grained lattice of 16^4 sub-blocks.
  - Identify an "artificial" instanton configuration in this sub-block.
  - Change the action to $\alpha\, 8\pi^2/g_a^{\,2}$ if the configuration lies in a volume $\Delta\Omega$ about this configuration:

$$\mathcal{A} = \mathcal{A}_{\mathrm{Wilson}} + N_{\mathrm{inst}}\left(\alpha\frac{8\pi^2}{g_z^2} - \mathcal{A}_{\mathrm{Wilson}}(U_{\mathrm{inst}})\right)$$

Artificial instanton in 16^4 sub-block



- Unconventional but
  - T^{16} gives a positive definite transfer matrix
  - Vacuum structure unaffected if $0 \le \alpha$.
  - Smooth instanton gives fermion zero mode
  - Can choose $\alpha \le 0.097$.

Conclusion

- Reinforce Mike's conclusion that up quark mass may be shifted from a zero input value.
- Recognize another dynamical source of fermion mass for BSM model building.
- Impose a constraint on the gauge action so this cannot happen?
- Look forward to many more years of interesting ideas from Mike!