2-Flavor QCD with Non-degenerate Quark Masses

Sinya AOKI

Yukawa Institute for Theoretical Physics, Kyoto University

Creutz Fest 2014

Physics Department, BNL, USA, September 4-5, 2014

My overlap with Mike Creutz

In 1986, I was in the last year of graduate course@U. of Tokyo.

I attended Lat'86@BNL at my own expense. During the meeting, I handed my preprint to Mike and others.

No arXiv existed. Tex was just appearing but not used.

Later I applied several postdoc positions including BNL.

No email, of course.

In January, 1987, I got a call in my apartment from Mike, offering a pos-doc position@BNL. I accepted it without hesitation. I started my 1st postdoc, in October, 1987 at BNL.

On the first day@BNL, I asked Mike "What should I do as your post-doc ? "

He answered "Whatever you would like to do."

Even though I have appreciated Mike's generosity, I had regretted for a long time that I had never collaborated with him in research.

Recently, this situation changes.

This work has been done in collaboration with Mike Creutz.

base on

S.A and M. Creutz, PRL 112(2014) 141603 (arXiv:1402.1837[hep-lat])

1. Introduction

θ term in QCD $$i\theta\frac{1}{32\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}(x)F_{\alpha\beta}(x)\equiv i\theta q(x)$$ CP odd

↓

Neutron Electric Dipole Moment(NEDM)

Experimental bound

$$|\vec{d_n}|\leq 6.3\times 10^{-26}e\cdot cm$$

Model estimate

$$|\vec{d_n}|/\theta\simeq 10^{-15}\sim 10^{-17}e\cdot cm$$

⇒ $$\theta=\theta_{\rm QCD}+\theta_{\rm EW}\leq O(10^{-8})$$

Strong CP problem !

One possible "solution" $m_u=0$ massless up quark

(Lattice QCD already ruled out this ?)

chiral rotation

$$u\longrightarrow e^{i\alpha\gamma_5}u,\quad \bar{u}\longrightarrow \bar{u}e^{i\alpha\gamma_5},$$

$$m_u\,\bar{u}u\to m_u\,\bar{u}e^{i2\alpha\gamma_5}u$$

$$\theta\longrightarrow\theta'=\theta+2\alpha N_f$$ chiral anomaly

if $m_u=0$, we can make

$$\theta'=0$$

by $\alpha=-\frac{\theta}{2N_f}$

Mike Creutz, “Quark masses, the Dashen phase, and gauge field topology”
arXiv:1306.1245[hep-lat]

Mike’s Oracles

$m_d > 0$ fixed, then

1. Nothing special happens at $m_u = 0$.

2. Massless neutral pion: $m_{\pi^0} = 0$ at $m_u =^{\exists} m_c < 0$.
   critical quark mass

3. Pion condensation (Dashen phase): $\langle \pi^0 \rangle \neq 0$ at $m_u < m_c < 0$.

4. $\chi = \infty$ at $m_u = m_c$.

$$\chi = \frac{1}{V}\langle Q^2 \rangle$$ topological susceptibility

5. $\chi = 0$ at $m_u = 0$. ?

In this talk, I show the above properties by ChPT including the anomaly effect.
In addition, we discuss an interesting prediction related to these in 2-flavor QCD.

ChPT with "anomaly"

$$\mathcal{L} \;=\; \frac{f^2}{2}\mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) - \frac{1}{2}\mathrm{tr}\left(M^\dagger U + U^\dagger M\right) - \underline{\frac{\Delta}{2}\left(\det U + \det U^\dagger\right)}$$

effect of anomaly

Note: large N argument by Witten (fundamental rep. for quarks)

$\frac{\Delta}{2}(\det U + \det U^\dagger)$ → $\frac{c}{N}(\log\det U)^2$

N=3 quark → fundamental ?

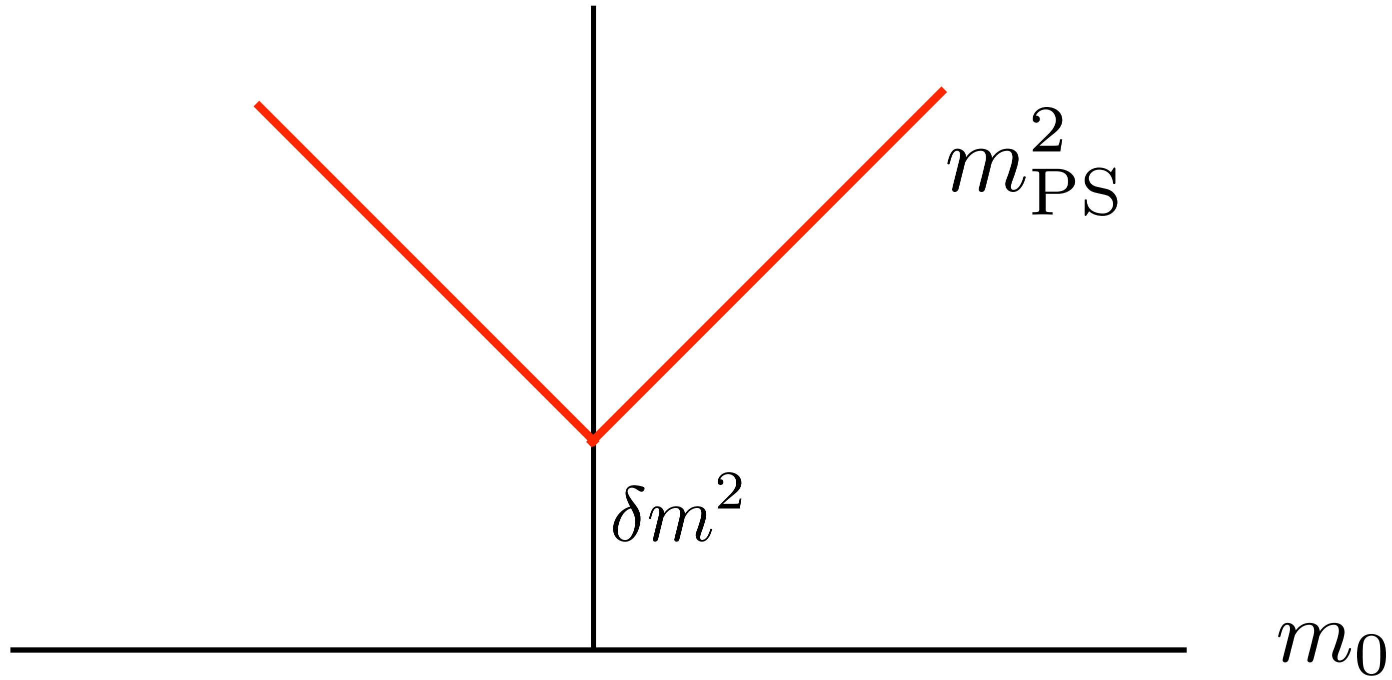
→ 2-index anti-symmetric ?

in the large N limit

For simplicity, we use $\frac{\Delta}{2}(\det U + \det U^\dagger)$ but check results with $\frac{c}{N}(\log\det U)^2$

Warm-up: $N_f = 1$ case

naive guess $$m_{\rm PS}^2 = \frac{2B}{f^2}|m_0| + \delta m^2$$ No massless "pion"(eta)



correct behavior $U = U_0 = e^{i\varphi_0}$ vacuum ansatz $m = 2Bm_0$

$$V(\varphi_0) = -(m+\Delta)\cos\varphi_0$$

potential

$$\varphi_0 = \begin{cases} 0 & m+\Delta > 0 \\ \pi & m+\Delta < 0 \end{cases}$$

minimum

$U(x) = U_0 e^{i\pi(x)/f}$

PS meson field

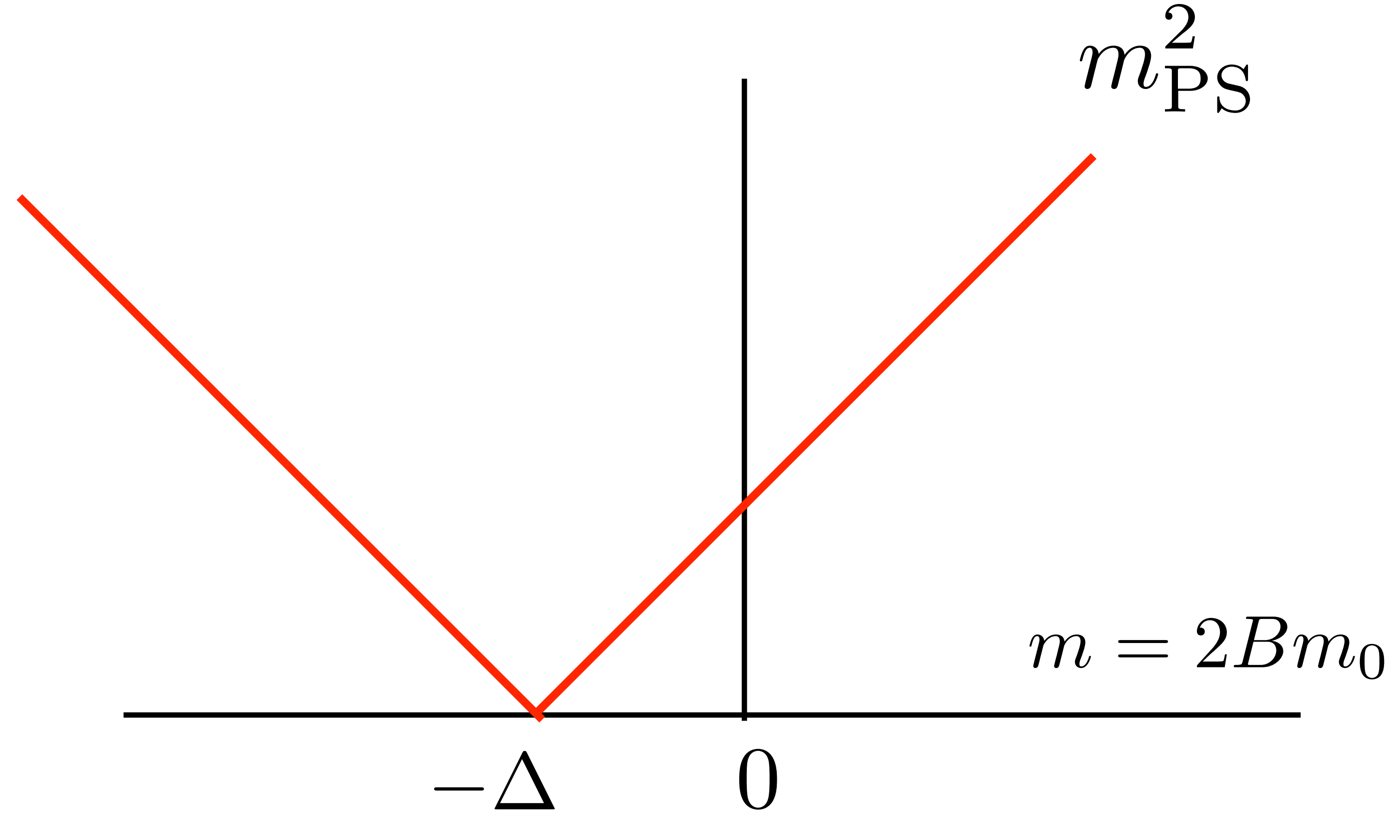
$$\begin{aligned} \mathcal{L} &= \frac{1}{2}\partial_\mu\pi(x)\partial^\mu\pi(x) - (m+\Delta)U_0\cos(\pi(x)/f) \\ &= \frac{1}{2}\left[(\partial_\mu\pi(x))^2 + \frac{|m+\Delta|}{f^2}\pi(x)^2\right] + O(\pi^4) \end{aligned}$$

$$m_{\rm PS}^2 = \frac{|m+\Delta|}{f^2}$$

$m = 0$ is note special

non-symmetric under $m \to -m$

massless PS meson at $m = -\Delta$



2. Phase structure and pion masses at N_f=2

$$M = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} \equiv 2B \begin{pmatrix} m_{0u} & 0 \\ 0 & m_{0d} \end{pmatrix}$$

mass term

$$U = U_0 = e^{i\varphi_0} \begin{pmatrix} e^{i\varphi_3} & 0 \\ 0 & e^{-i\varphi_3} \end{pmatrix}$$

vacuum

VEV

$$\langle \bar\psi \psi \rangle \equiv \frac{1}{2}\mathrm{tr}\,(U_0 + U_0^\dagger) = 2\cos(\varphi_0)\cos(\varphi_3),$$

$$\langle \bar\psi \tau^3 \psi \rangle \equiv \frac{1}{2}\mathrm{tr}\,\tau^3(U_0 + U_0^\dagger) = -2\sin(\varphi_0)\sin(\varphi_3),$$

$$\langle \bar\psi i\gamma_5 \psi \rangle \equiv \frac{1}{2i}\mathrm{tr}\,(U_0 - U_0^\dagger) = 2\sin(\varphi_0)\cos(\varphi_3),$$

$$\langle \bar\psi i\gamma_5 \tau^3 \psi \rangle \equiv \frac{1}{2i}\mathrm{tr}\,\tau^3(U_0 - U_0^\dagger) = 2\cos(\varphi_0)\sin(\varphi_3).$$

potential

$$V(\varphi_0,\varphi_3) = -m_u\cos(\varphi_0+\varphi_3) - m_d\cos(\varphi_0-\varphi_3) - \Delta\cos(2\varphi_0).$$

$$\frac{\partial V}{\partial \varphi_0} = m_u\sin(\varphi_0+\varphi_3) + m_d\sin(\varphi_0-\varphi_3) + 2\Delta\sin(2\varphi_0) = 0$$

$$\frac{\partial V}{\partial \varphi_3} = m_u\sin(\varphi_0+\varphi_3) - m_d\sin(\varphi_0-\varphi_3) = 0.$$

gap equations

$\sin\varphi_0 = \sin\varphi_3 = 0$ is a trivial solution

Non-trivial Solutions

$$0 < m_d < \Delta$$

$$\sin^2(\varphi_3) = \frac{(m_d - m_u)^2\{(m_u + m_d)^2\Delta^2 - m_u^2 m_d^2\}}{4 m_u^3 m_d^3}$$
$$\sin^2(\varphi_0) = \frac{(m_u + m_d)^2\Delta^2 - m_u^2 m_d^2}{4 m_u m_d \Delta^2},$$

$$m_c^- < m_u < m_c^+$$

Dashen phase

$$\Delta < m_d$$

$$\sin^2(\varphi_3) = \frac{(m_d - m_u)^2\{(m_u + m_d)^2\Delta^2 - m_u^2 m_d^2\}}{4 m_u^3 m_d^3}$$
$$\sin^2(\varphi_0) = \frac{(m_u + m_d)^2\Delta^2 - m_u^2 m_d^2}{4 m_u m_d \Delta^2},$$

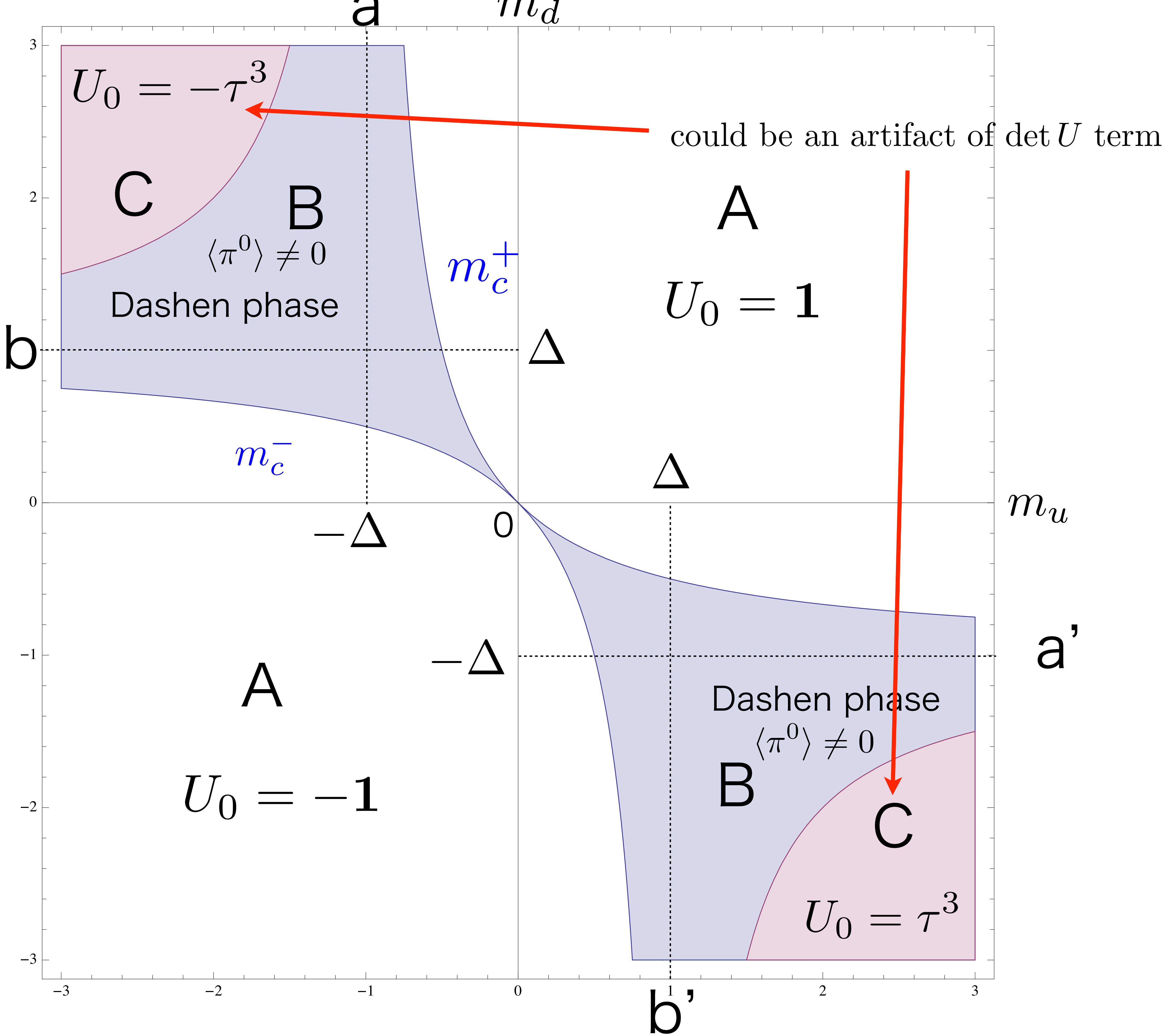
$$-m_c^- < m_u < m_c^+$$

Dashen phase

$$U_0 = \pm \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad (\sin^2(\varphi_3) = \sin^2(\varphi_0) = 1).$$

$$m_u < -m_c^-$$

$$m_c^\pm = -\frac{m_d \Delta}{\Delta \pm m_d} < 0,$$



a
m_d
$U_0 = -\tau^3$
could be an artifact of $\det U$ term
C
B
A
$\langle \pi^0 \rangle \neq 0$
m_c^+
Dashen phase
$U_0 = \mathbf{1}$
b
Δ
m_c^-
Δ
m_u
$-\Delta$
0
$-\Delta$
a'
A
Dashen phase
$\langle \pi^0 \rangle \neq 0$
B
$U_0 = -\mathbf{1}$
C
$U_0 = \tau^3$
b'

PS meson masses

$$U(x) = U_0 e^{i\Pi(x)/f}, \qquad \Pi(x) = \begin{pmatrix} \frac{\eta(x)+\pi_0(x)}{\sqrt{2}} & \pi_-(x) \\ \pi_+(x) & \frac{\eta(x)-\pi_0(x)}{\sqrt{2}} \end{pmatrix}$$

$$\mathcal{L}^{(2)} = \frac{1}{2}\{(\partial_\mu \pi_0(x))^2 + (\partial_\mu \eta(x))^2 + 2\partial_\mu \pi_+(x)\partial^\mu \pi_-(x)\} + \frac{\delta m}{2f^2}\eta^2(x)$$
$$+ \frac{m_+(\vec{\varphi})}{4f^2}\{\eta^2(x) + \pi_0^2(x) + 2\pi_+(x)\pi_-(x)\} - \frac{m_-(\vec{\varphi})}{2f^2}\underline{\eta(x)\pi_0(x)}, (26)$$

$\pi_0 - \eta$ mixing

$$m_{\pi_\pm}^2 = \frac{m_+(\vec{\varphi})}{2f^2}$$ charged pion

$$m_{\tilde{\pi}_0}^2 = \frac{1}{2f^2}[m_+(\vec{\varphi}) + \delta m - X]$$ neutral pion

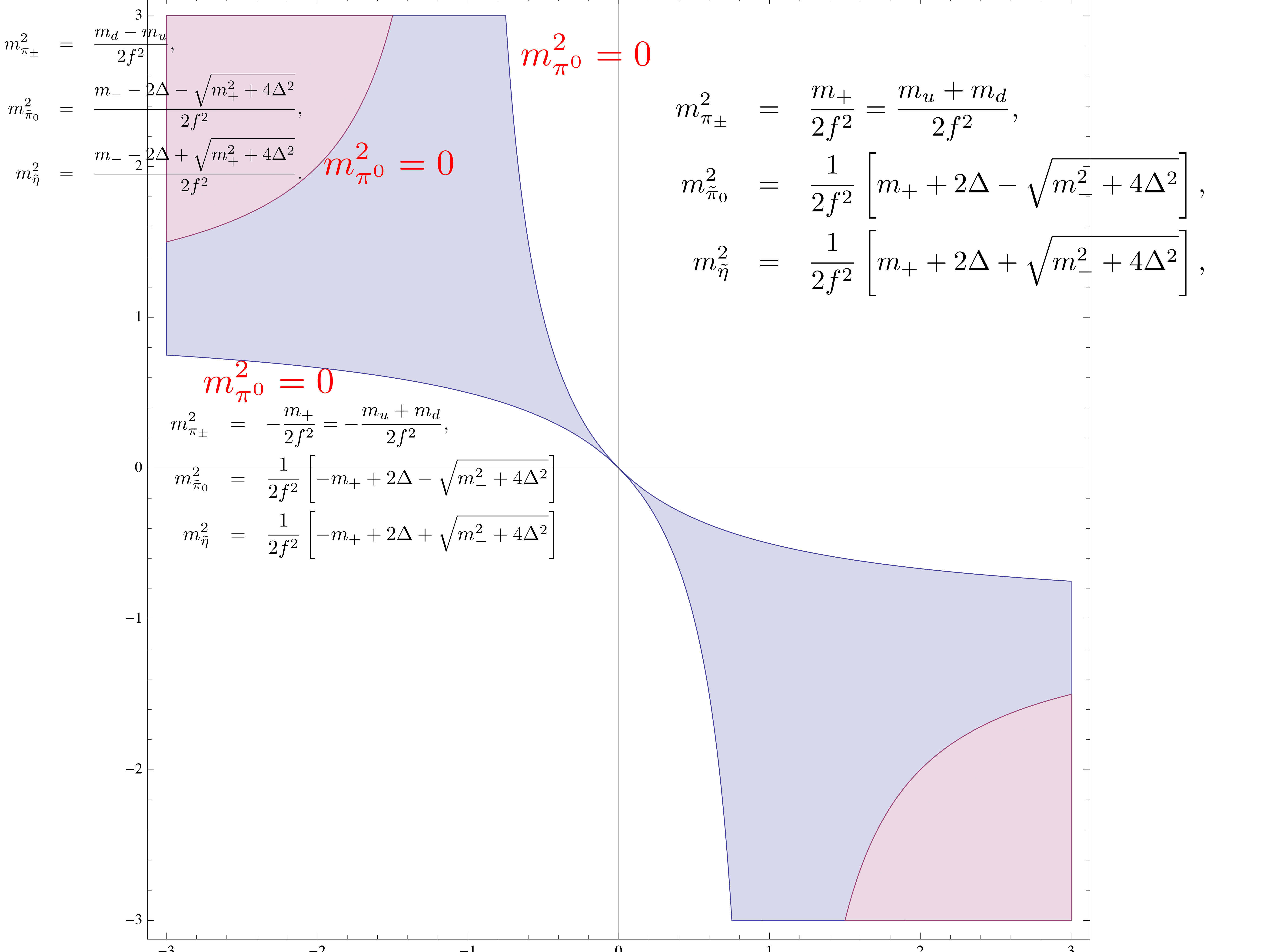
$$m_{\tilde{\eta}}^2 = \frac{1}{2f^2}[m_+(\vec{\varphi}) + \delta m + X]$$ eta meson

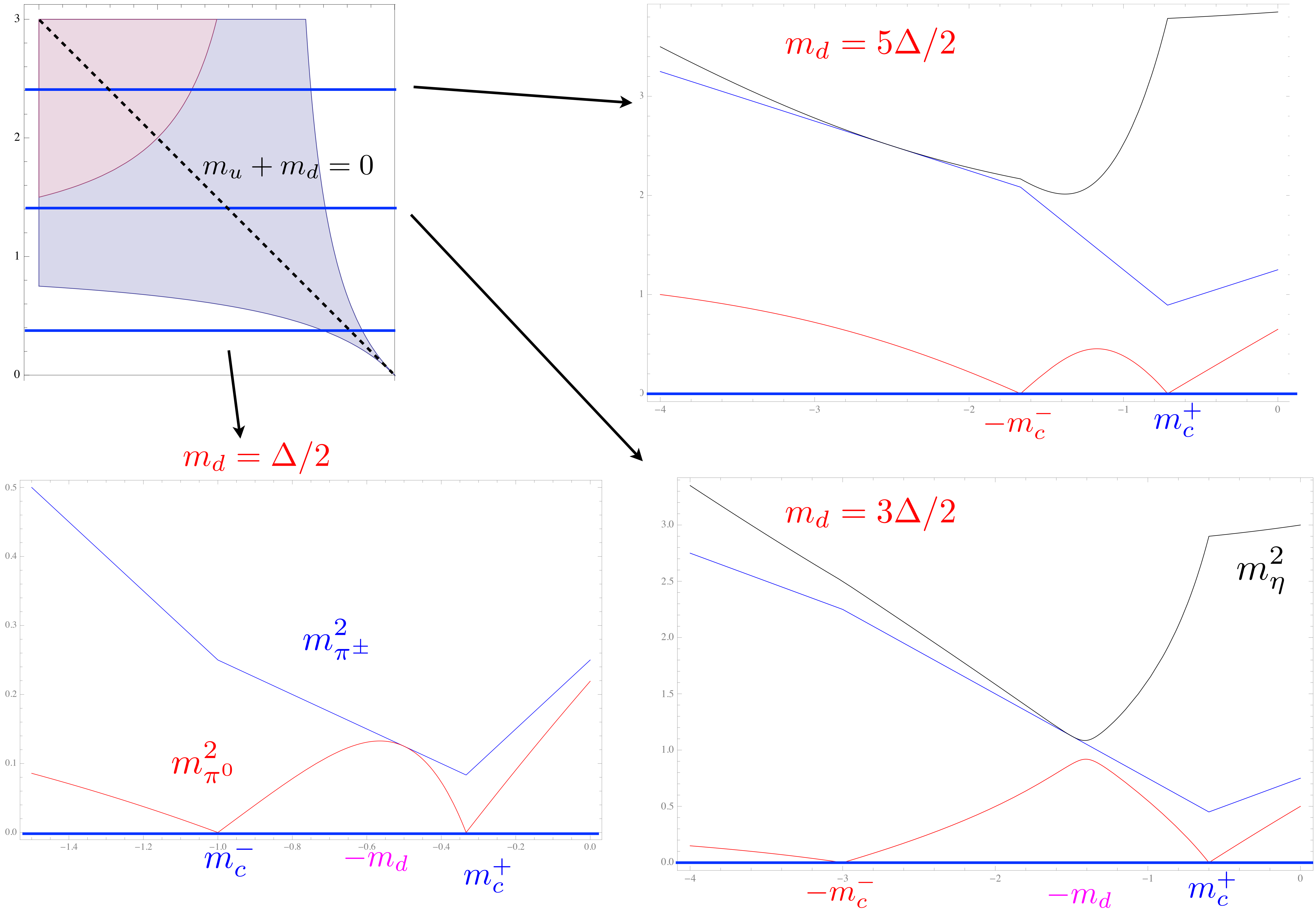
$\Rightarrow$ $m_{\tilde{\pi}_0} < m_{\pi_\pm}$

$$X = \sqrt{m_-(\vec{\varphi})^2 + \delta m^2},$$

$$m_\pm(\vec{\varphi}) = m_d \cos(\varphi_0 - \varphi_3) \pm m_u \cos(\varphi_0 + \varphi_3)$$
$$= m_\pm \cos(\varphi_0)\cos(\varphi_3) + m_\mp \sin(\varphi_0)\sin(\varphi_3),$$
$$\delta m = 2\Delta \cos(2\varphi_0).$$

$$m_\pm = m_d \pm m_u.$$


$m_{\pi\pm}^2 = \frac{m_d - m_u}{2f^2},$
$m_{\tilde{\pi}0}^2 = \frac{m_- - 2\Delta - \sqrt{m_+^2 + 4\Delta^2}}{2f^2},$
$m_{\tilde{\eta}}^2 = \frac{m_- - 2\Delta + \sqrt{m_+^2 + 4\Delta^2}}{2f^2}.$
$m_{\pi^0}^2 = 0$
$m_{\pi^0}^2 = 0$
$m_{\pi\pm}^2 = \frac{m_+}{2f^2} = \frac{m_u + m_d}{2f^2},$
$m_{\tilde{\pi}0}^2 = \frac{1}{2f^2}\left[m_+ + 2\Delta - \sqrt{m_-^2 + 4\Delta^2}\right],$
$m_{\tilde{\eta}}^2 = \frac{1}{2f^2}\left[m_+ + 2\Delta + \sqrt{m_-^2 + 4\Delta^2}\right],$
$m_{\pi^0}^2 = 0$
$m_{\pi\pm}^2 = -\frac{m_+}{2f^2} = -\frac{m_u + m_d}{2f^2},$
$m_{\tilde{\pi}0}^2 = \frac{1}{2f^2}\left[-m_+ + 2\Delta - \sqrt{m_-^2 + 4\Delta^2}\right]$
$m_{\tilde{\eta}}^2 = \frac{1}{2f^2}\left[-m_+ + 2\Delta + \sqrt{m_-^2 + 4\Delta^2}\right]$
3
2
1
0
−1
−2
−3
−3
−2
−1
0
1
2
3



$m_u + m_d = 0$
$m_d = 5\Delta/2$
$-m_c^-$
m_c^+
$m_d = \Delta/2$
$m_{\pi^\pm}^2$
$m_{\pi^0}^2$
m_c^-
$-m_d$
m_c^+
$m_d = 3\Delta/2$
m_η^2
$-m_c^-$
$-m_d$
m_c^+

3. Topological susceptibility and massless up quark

(anomalous) WT identities

$$\chi \equiv \int d^4x \, \langle q(x) q(y) \rangle = \frac{2m_u}{N_f} \int d^4x \; \langle \bar{u}\gamma_5 u(x) q(y) \rangle \, .$$

$$q(x) = \frac{g^2}{16\pi^2} \varepsilon^{\mu\nu\alpha\beta} G_{\mu\nu}(x) G_{\alpha\beta}(x),$$

topological charge density

?

$$\chi = \infty \text{ at } m_{\pi^0} = 0 \text{ and } m_u \neq 0$$

$$\chi = 0 \text{ at } m_{\pi^0} \neq 0 \text{ and } m_u = 0$$

Anomalous WT identities in N_f=2 ChPT

WT identities

$$\langle \delta_x S \mathcal{O}(y)\rangle = \delta^{(4)}(x-y)\,\langle \delta\mathcal{O}(y)\rangle$$

$$\delta_x S = i\theta(x)\left[\partial^\mu A_\mu(x) + \mathrm{tr}\left\{MU^\dagger(x) - M^\dagger U(x)\right\} - \Delta\{\det U(x) - \det U^\dagger(x)\}\right],$$

$2N_f q(x)$: topological charge density

$$A_\mu(x) = f^2 \mathrm{tr}\left\{U^\dagger(x)\partial_\mu U(x) - U\partial_\mu U^\dagger(x)\right\}$$

$$2N_f\chi = \frac{\Delta^2}{4}\int d^4x\,\langle\{\det U(x) - \det U^\dagger(x)\}\{\det U(y) - \det U^\dagger(y)\}\rangle$$

$$+ \frac{\Delta}{2}\langle \det U(y) + \det U^\dagger(y)\rangle,$$

$$-\frac{2\Delta^2}{f^2}\int d^4x\,\langle\eta(x)\eta(y)\rangle$$

effect of contact term $= \Delta$

$$2N_f\chi = -\frac{4\Delta^2 m_+(\vec\varphi)}{m_+^2(\vec\varphi) - m_-^2(\vec\varphi) + 2m_+(\vec\varphi)\delta m} + \Delta.$$

$m_u = 0$ $\Rightarrow$ $m_+(\vec{\varphi}) = m_-(\vec{\varphi}) = m_d$ and $\delta m = 2\Delta$

$\Rightarrow$

$$2N_f\chi \;=\; -\frac{4\Delta^2 m_d}{4m_d\Delta} + \Delta = 0,$$

$m^2_{\pi^0} = 0$ $\Rightarrow$

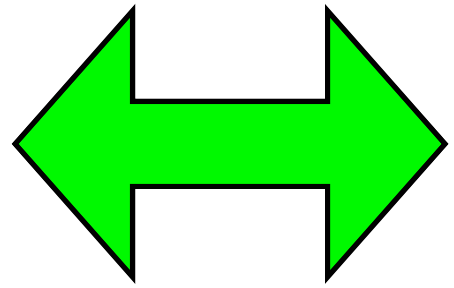
$$\int d^4x \, \langle \eta(x)\eta(y) \rangle \;=\; \frac{1}{2X}\left(\frac{X_-}{m^2_{\tilde{\pi}_0}} + \frac{X_+}{m^2_{\tilde{\eta}}} \right) \quad \longrightarrow \infty$$

$\Rightarrow$

$$2N_f\chi \;\to\; -\infty, \quad m_{\tilde{\pi}_0} \to 0,$$

4. An interesting application

$$m_u = -m_d = -m$$
$$\theta = 0$$



$$m_u = m_d = m$$
$$\theta = \pi$$

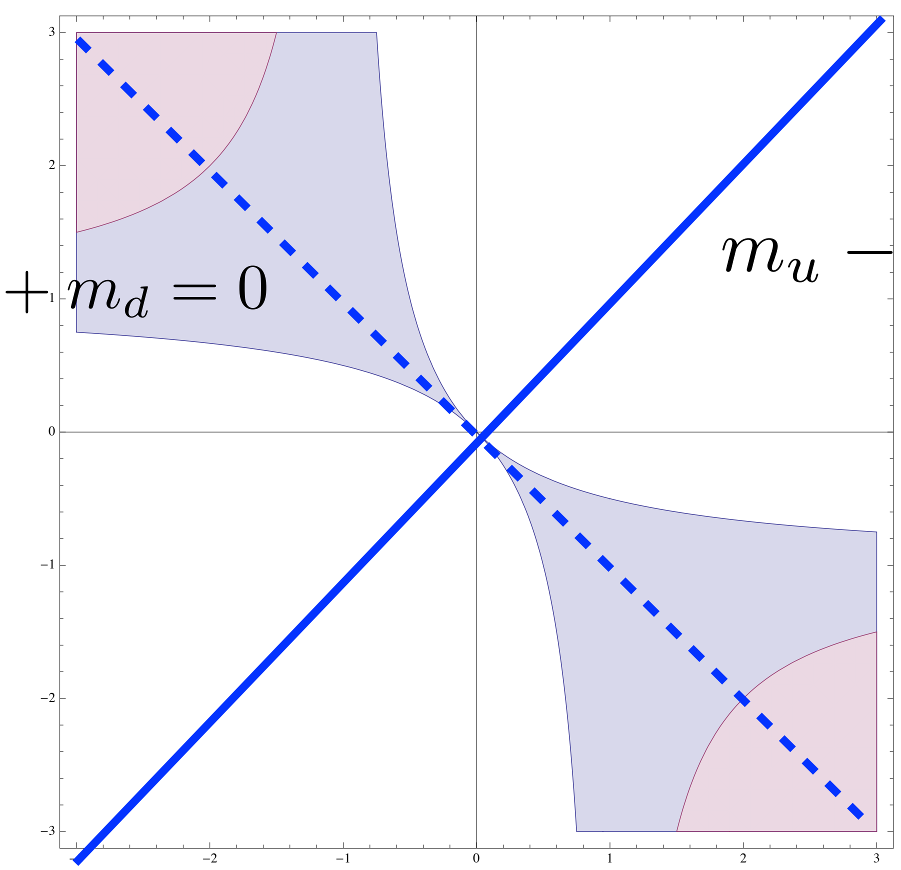
chiral rotations

$$M = \begin{pmatrix} -m & 0 \\ 0 & m \end{pmatrix}$$

$$V_R = e^{i(\theta_0+\theta_3\tau^3)} = V_L^\dagger, \quad M' = V_L^\dagger M V_R = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$$

$$\theta_0 = \theta_3 = \pi/4,$$

$$\Delta \qquad\qquad -\Delta$$



ChPT analysis

VEV

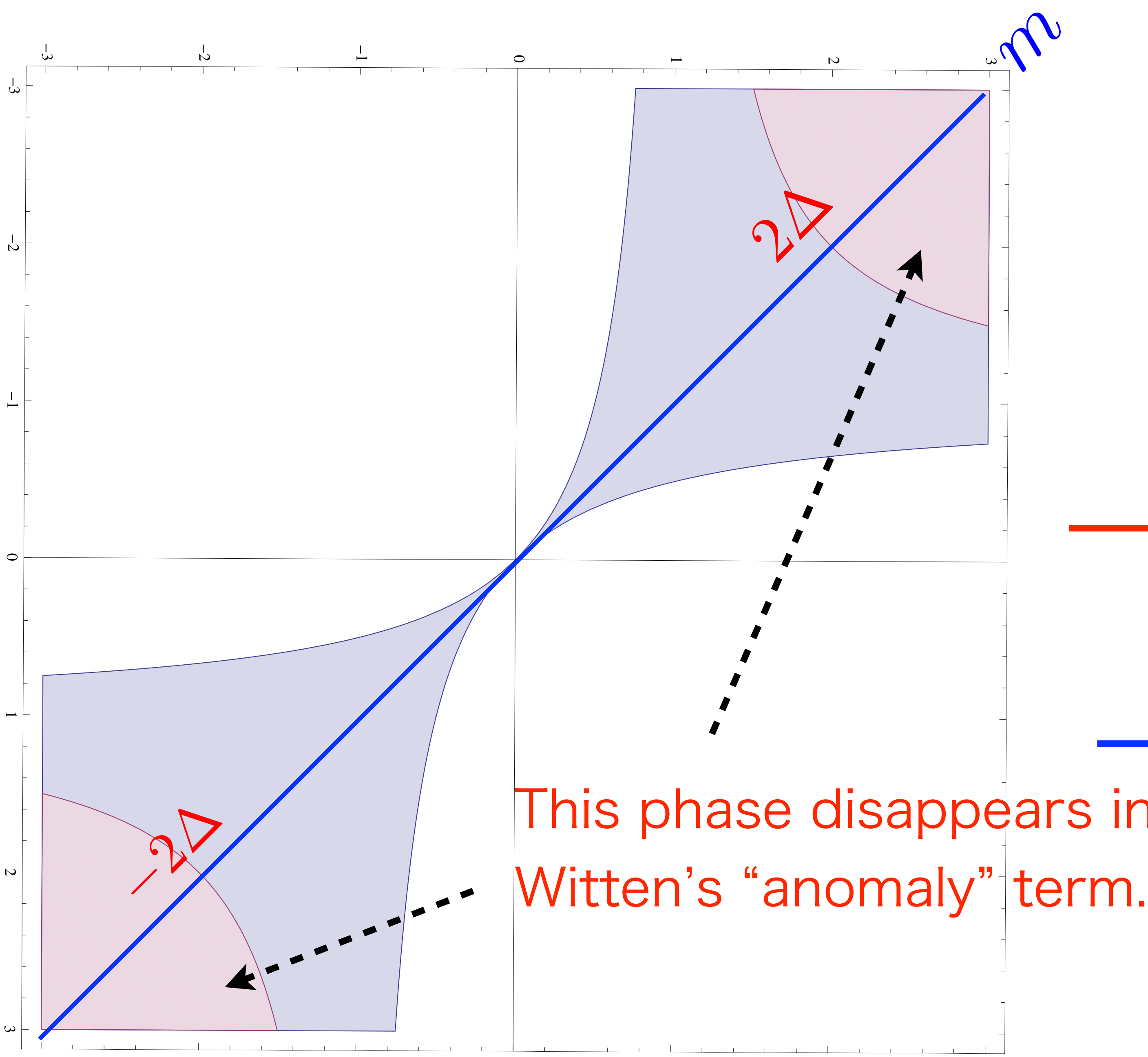
$$\langle\bar{\psi}\psi\rangle^2 + \langle\bar{\psi}i\gamma_5\psi\rangle^2 = 4$$

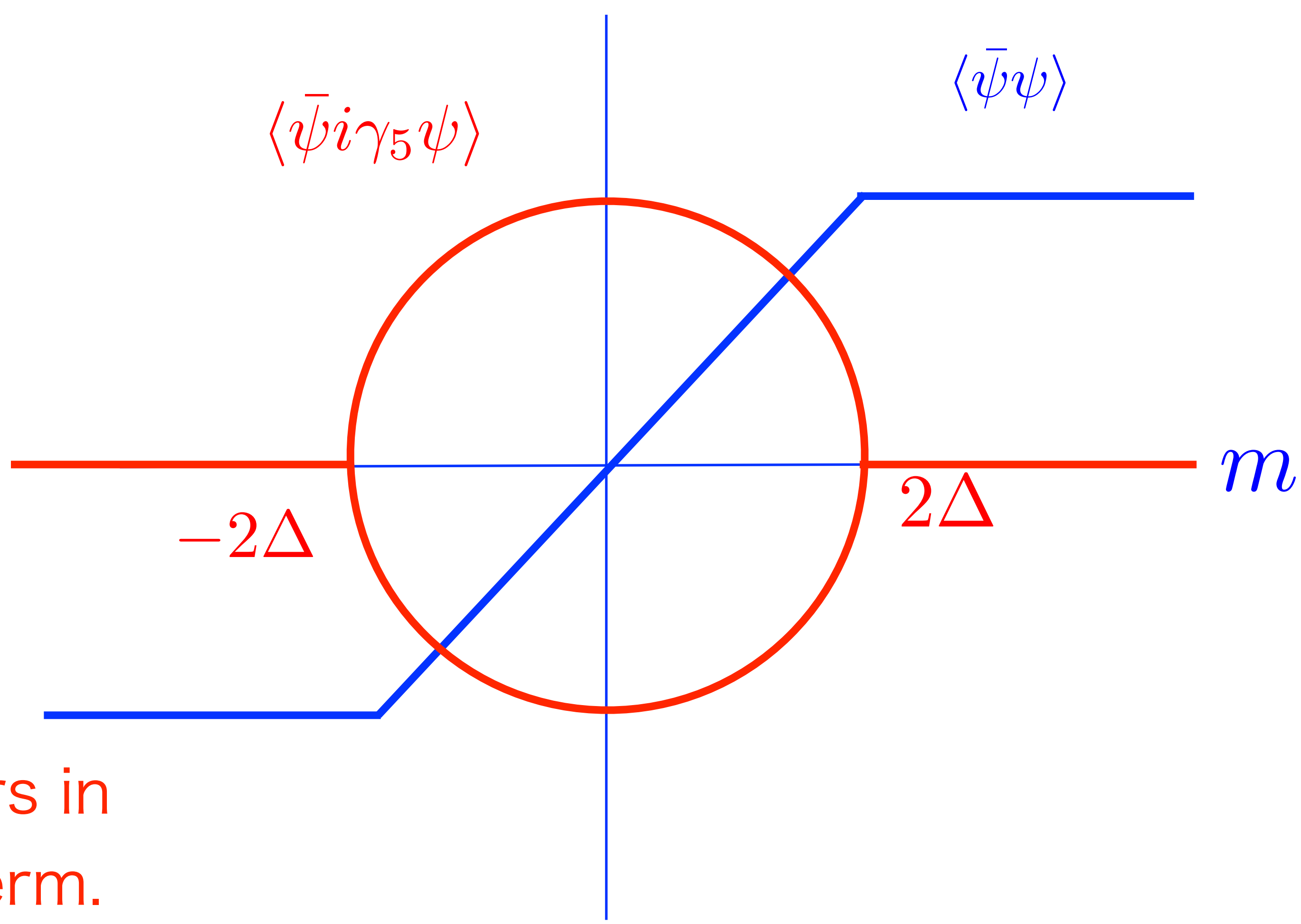
$$\cos\varphi_3 = 1$$

$$\cos\varphi_0 = \begin{cases} 1, & 2\Delta \le m \\ \dfrac{m}{2\Delta}, & -2\Delta < m < 2\Delta \\ -1, & m \le -2\Delta \end{cases}$$

$$\langle\bar{\psi}i\gamma_5\psi\rangle = 2\sin\varphi_0\cos\varphi_3 = \begin{cases} 0, & m^2 \ge 4\Delta^2 \\ \pm 2\sqrt{1-\dfrac{m^2}{4\Delta^2}}, & m^2 < 4\Delta^2 \end{cases}$$

$$\langle\bar{\psi}\psi\rangle = 2\cos\varphi_0\cos\varphi_3 = \begin{cases} 2, & 2\Delta \le m \\ \dfrac{m}{\Delta}, & -2\Delta < m < 2\Delta \\ -2, & m \le -2\Delta \end{cases},$$



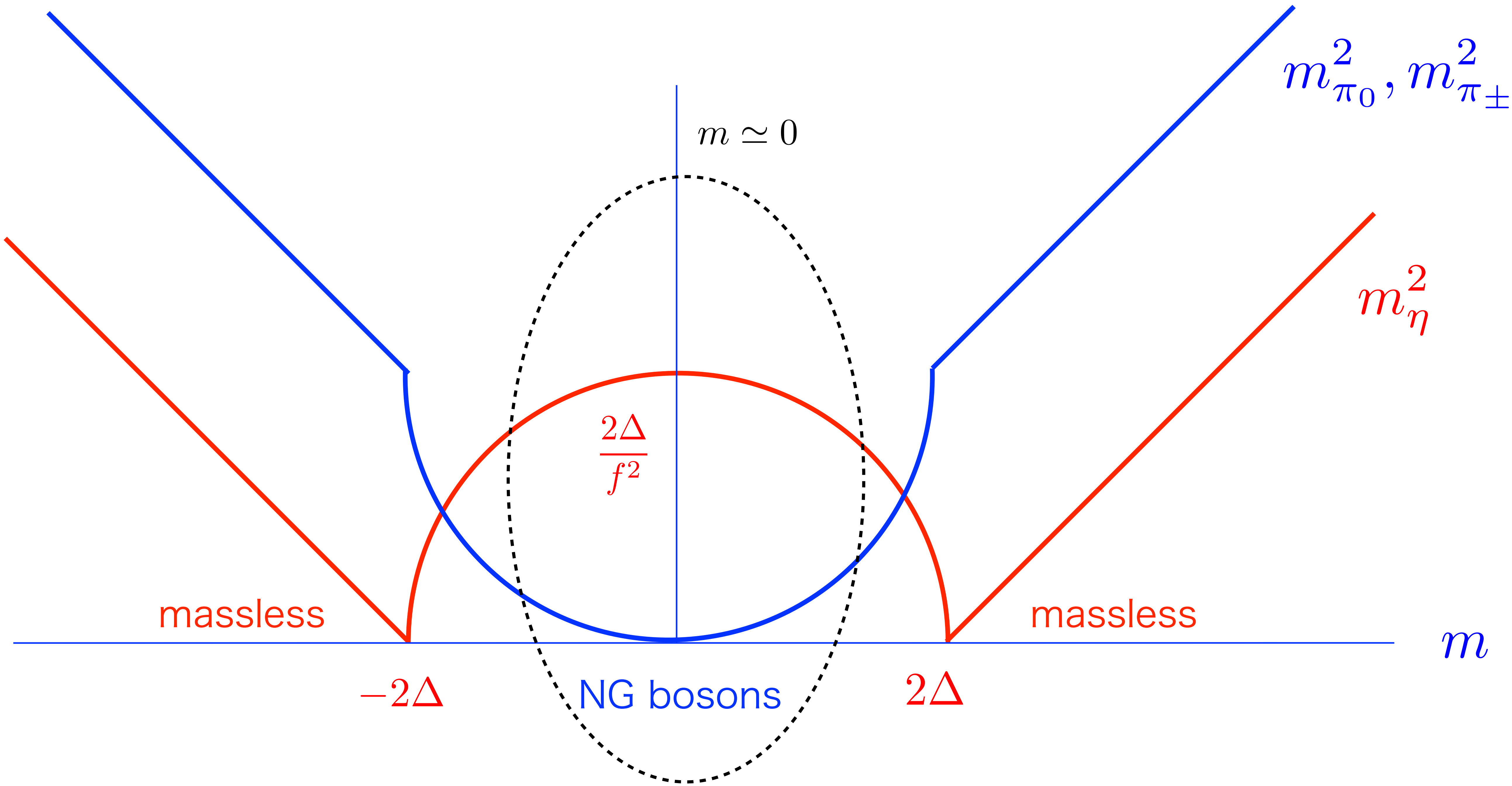


Spontaneous CP violation !
(eta condensation)

PS meson masses

$$m_{\pi_\pm}^2 = m_{\pi_0}^2 = \begin{cases} \frac{1}{2f^2} 2|m|, & m^2 \geq 4\Delta^2 \\ \frac{1}{2f^2}\frac{m^2}{\Delta}, & m^2 < 4\Delta^2 \end{cases} \qquad m_\eta^2 = \begin{cases} \frac{1}{2f^2}\left[2|m| - 4\Delta\right], & m^2 \geq 4\Delta^2 \\ \frac{1}{2f^2}\frac{4\Delta^2 - m^2}{\Delta}, & m^2 < 4\Delta^2 \end{cases},$$

non-standard PCAC relation !



$$m_\pi^2 = \frac{1}{2f^2}\frac{m^2}{\Delta}$$

How can we get this from WT-identities ?

$$\langle\{\partial^\mu A_\mu^3 + m\,\mathrm{tr}\tau^3(U^\dagger - U)\}(x)\mathcal{O}(y)\rangle = \langle\delta^x\mathcal{O}(y)\rangle$$

taking $\mathcal{O} = \mathrm{tr}\,\tau^3(U^\dagger - U)$ and integrating over x

$$\Rightarrow \quad m\int d^4x\,\langle \underline{\mathrm{tr}\,\tau^3(U^\dagger - U)(x)}\;\mathrm{tr}\,\tau^3(U^\dagger - U)(y)\rangle = -2\langle\underline{\mathrm{tr}(U + U^\dagger)(y)}\rangle$$

$$\mathrm{tr}\,\tau^3(U^\dagger - U)(x) = -i\frac{2\sqrt{2}}{f}\cos\varphi_0\,\pi_0(x) \qquad \langle\mathrm{tr}(U + U^\dagger)(y)\rangle = 4\cos\varphi_0$$

$$\Rightarrow \quad m\frac{\cos^2\varphi_0}{f^2}\underline{\int d^4x\langle\pi_0(x)\pi_0(y)\rangle} = \cos\varphi_0 \quad \Rightarrow \quad m_{\pi_0}^2 = \frac{m}{f^2}\underline{\cos\varphi_0}$$

$$\int d^4x\langle\pi_0(x)\pi_0(y)\rangle = \frac{1}{m_{\pi_0}^2} \qquad \cos\varphi_0 = \frac{m}{2\Delta}$$

$$\Rightarrow \quad m_{\pi_0}^2 = \frac{m}{f^2}\frac{m}{2\Delta}$$

one m form WTI, the other m from VEV.

5. Conclusions

Using ChPT with anomaly effect, we show

1. $m_u = 0$ is nothing special if $m_d \neq 0$. (no symmetry)
2. At $m_u = m_c^{\pm}, -m_c^{-} \neq 0$, $m_{\pi^0} = 0$.
3. $\langle \pi^0 \rangle \neq 0$ at $m_c^{-}(-m_c^{-}) < m_u < m_c^{+}$. Dashen phase

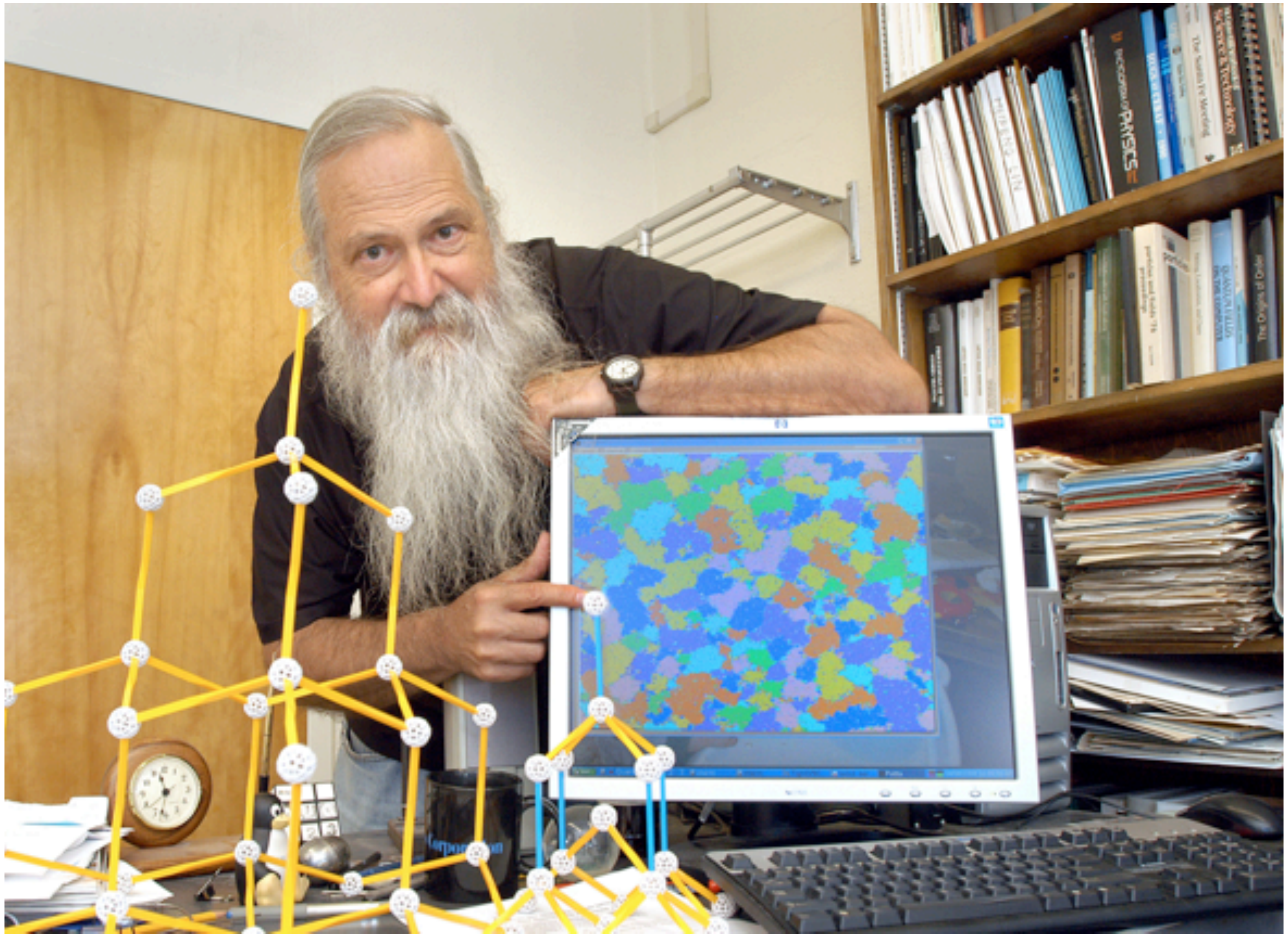
   rooted Staggered quark can not reproduce this.
4. $\chi = \infty$ at $m_u = m_c$.
5. $\chi = 0$ at $m_u = 0$. ➡ a solution to strong CP problem

Mike's Oracles are confirmed by ChPT.

New predictions for 2-flavor QCD with $m_u = m_d$ and $\theta = \pi$

1. Spontaneous CP violation : $\langle \eta \rangle \neq 0$
2. Non-standard PCAC relation: $m_\pi^2 \propto m_q^2$

I thank Mike a lot for his many contributions to our lattice community.



I wish Mike has fruitful and enjoyable life,
and sometimes visit me in Kyoto !