From Group Integrals to Extreme QCD

Michael C. Ogilvie
Washington University
September 5th, 2014

CreutzFest 2014
Brookhaven National Laboratory

Mike and group integrals

On invariant integration over SU(N)[a]

Michael Creutz

Physics Department, Brookhaven National Laboratory, Upton, New York 11973

(Received 5 April 1978)

We give a graphical algorithm for evaluation of invariant integrals of polynomials in SU(N) group elements. Such integrals occur in strongly coupled lattice gauge theory. The results are expressed in terms of totally antisymmetric tensors and Kronecker delta symbols.

$$\int_G (dU)\, U_{jk} U^{+}_{lm} = \frac{1}{N} \delta_{jm} \delta_{kl}$$

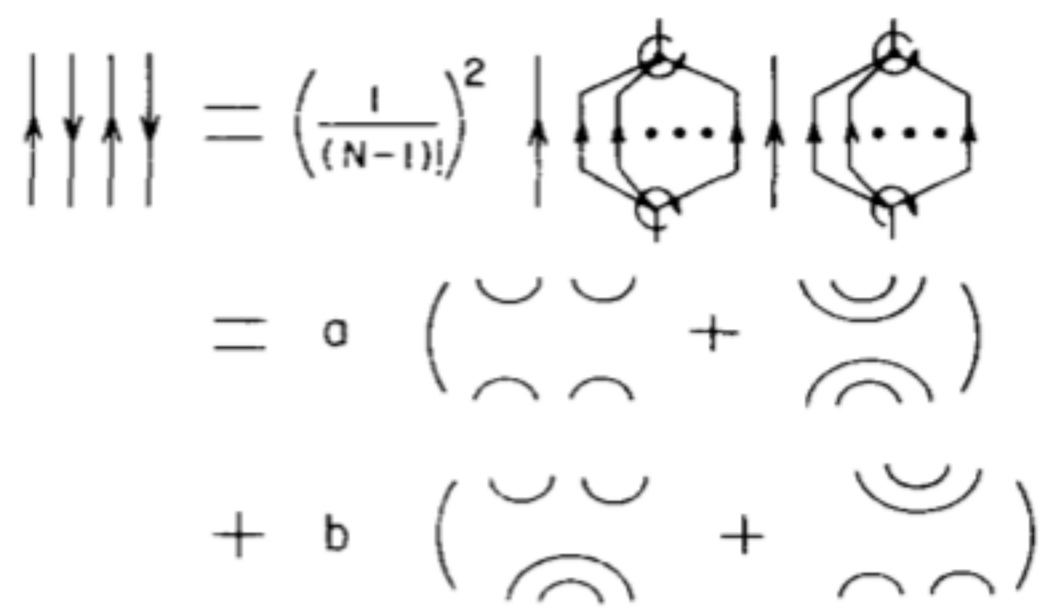


FIG. 9. The integral $\int dg\, g_{i_1 j_1}\, g^{-1}_{k_1 l_1}\, g_{i_2 j_2}\, g^{-1}_{k_2 l_2}$.

Essentially the Feynman rules for strong coupling

Mike and crossover

PHYSICAL REVIEW D VOLUME 21, NUMBER 8 15 APRIL 1980

Monte Carlo study of quantized SU(2) gauge theory

Michael Creutz
Department of Physics, Brookhaven National Laboratory, Upton, New York 11973
(Received 24 October 1979)

Using Monte Carlo techniques, we evaluate path integrals for pure SU(2) gauge fields. Wilson's regularization procedure on a lattice of up to 10^4 sites controls ultraviolet divergences. Our renormalization prescription, based on confinement, is to hold fixed the string tension, the coefficient of the asymptc linear potential between sources in the fundamental representation of the gauge group. Upon reducing cutoff, we observe a logarithmic decrease of the bare coupling constant in a manner consistent with perturbative renormalization-group prediction. This supports the coexistence of confinement and asympt freedom for quantized non-Abelian gauge fields.

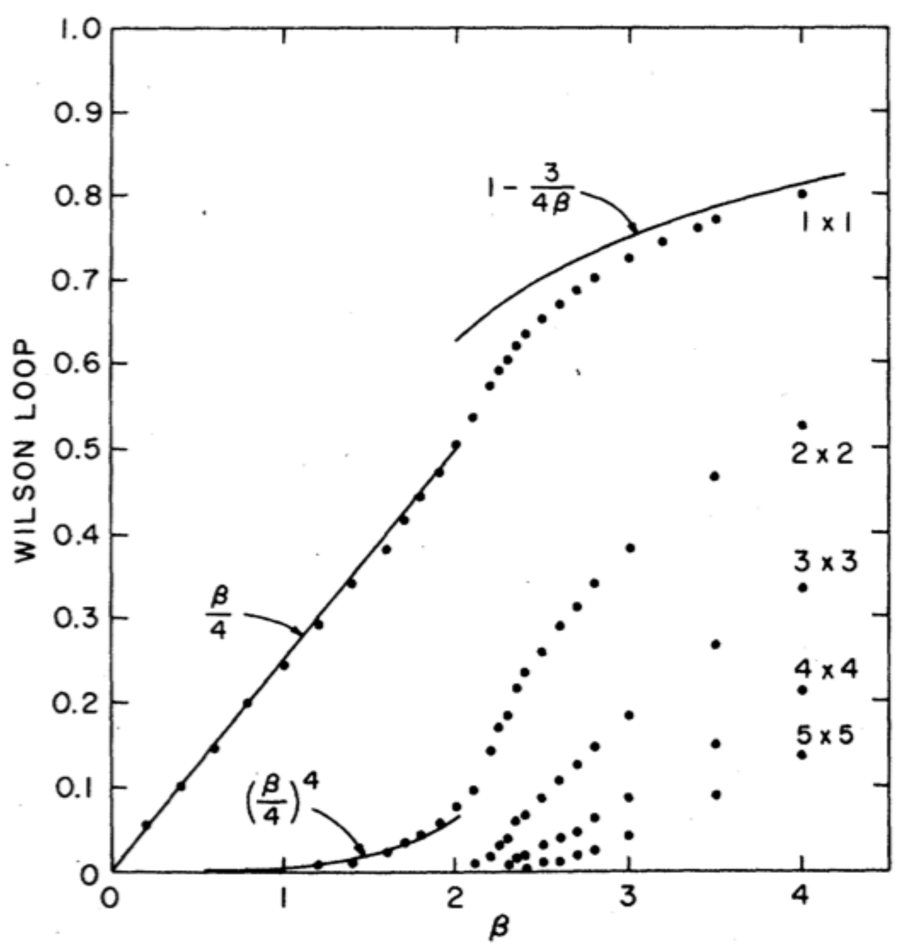


FIG. 4. Wilson loops as a function of β.

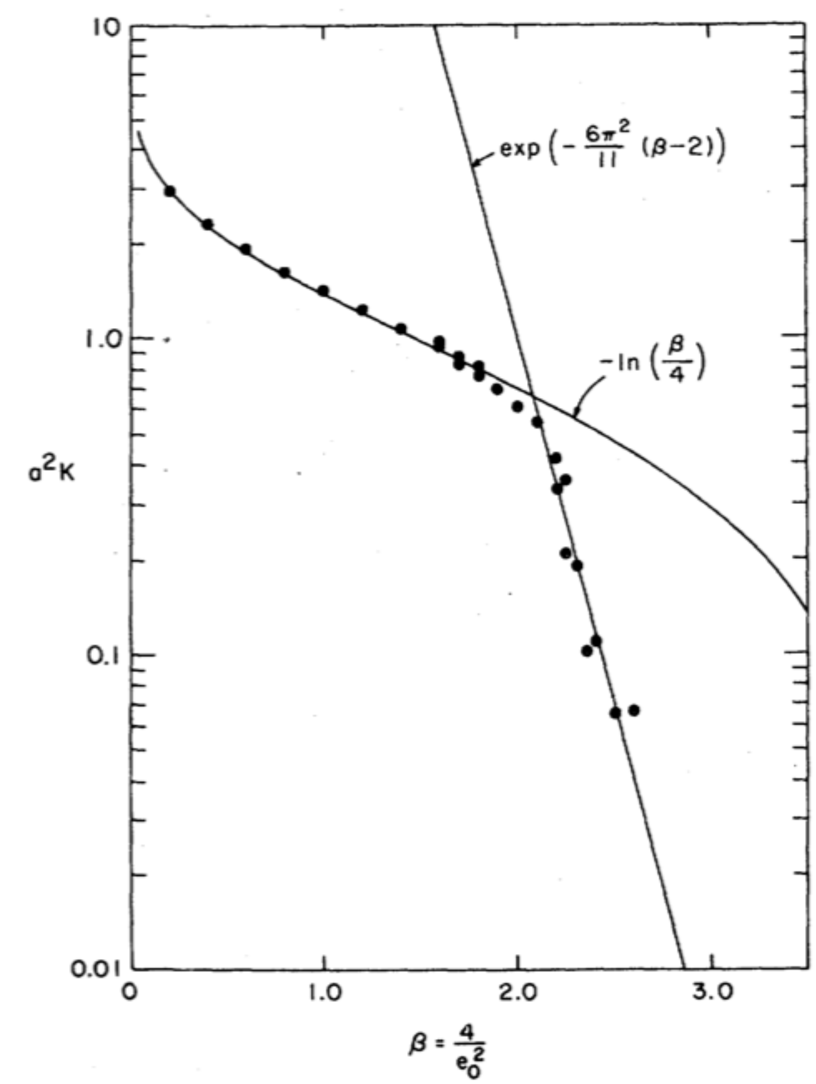


FIG. 6. The cutoff squared times the string tension as a function of β. The solid lines are the strong- and weak-coupling limits.

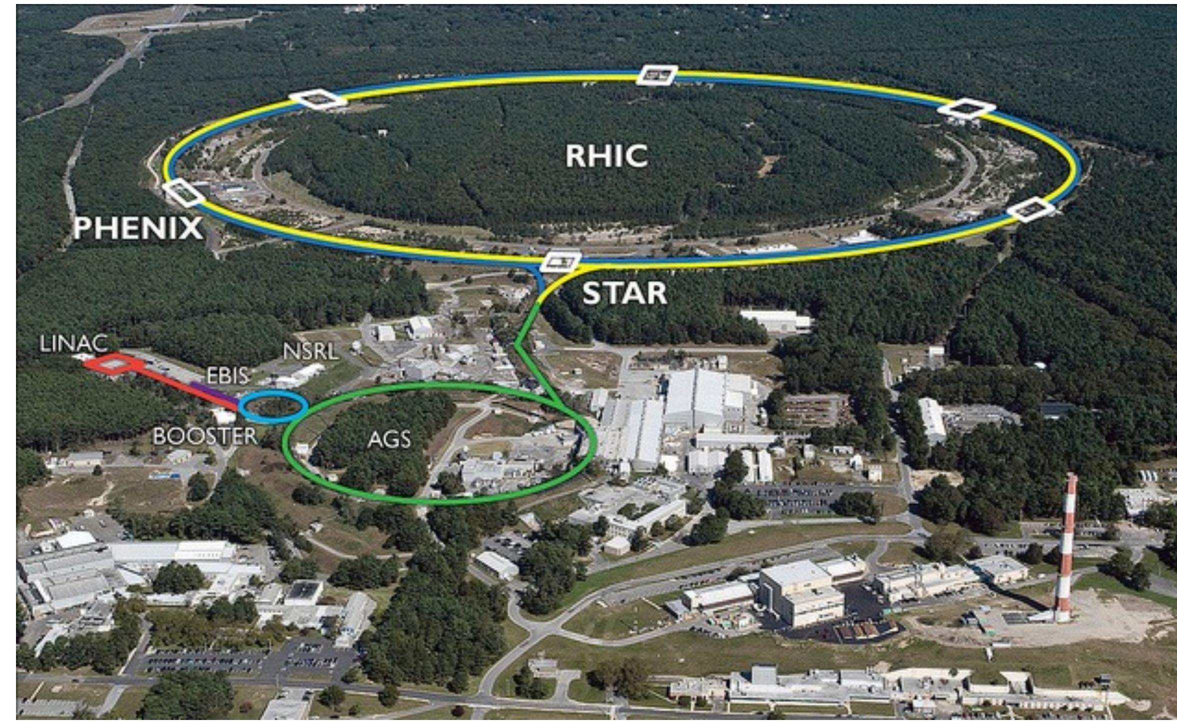
Dark to light: Isabelle to RHIC

The New York Times

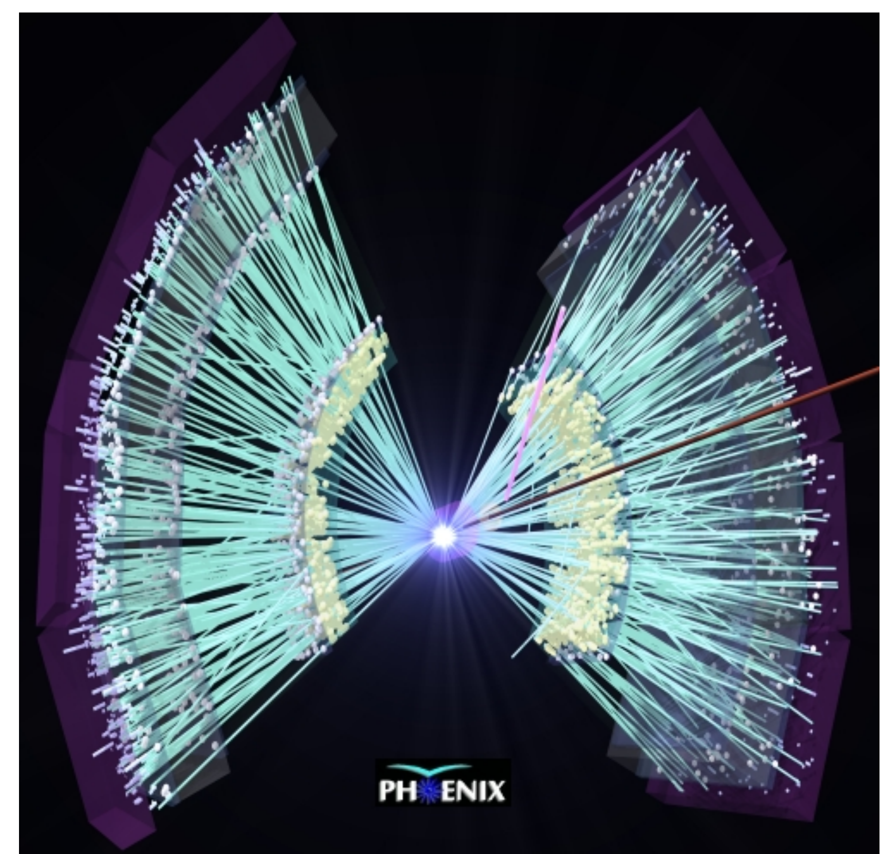
BIG ACCELERATOR ON LONG ISLAND GETS A 'NO' VOTE

By WILLIAM J. BROAD
Published: July 14, 1983

A panel of top physicists who advise the Federal Government recommended yesterday that an incomplete atom smasher at the Brookhaven National Laboratory on Long Island be scrapped.



“There are no second acts
in American lives.”
F. Scott Fitgerald

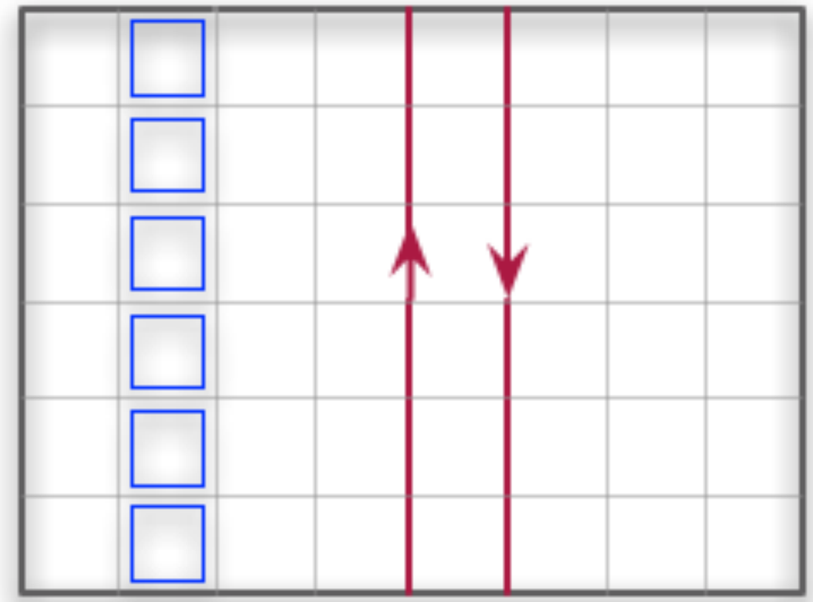


A lattice approach to deconfinement

$$S\left[U_l\right] = \sum_p \frac{\beta}{2N} \mathrm{Tr}\left[U_p + U_p^+\right]$$

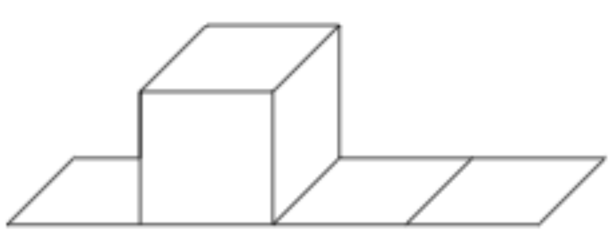
$$S_{eff}\left[P\right] = \sum_{\langle jk \rangle} K \left[\mathrm{Tr}P_j \mathrm{Tr}P_k^+ + \mathrm{Tr}P_j^+ \mathrm{Tr}P_k\right]$$

$$K \simeq \left(\frac{\beta}{2N^2}\right)^{N_t}$$



Polonyi and K. Szlachanyi, 1982; mco, 1984;
Green & Karsch, 1984; Gross & Wheater, 1984

Langelage, Lottini & Philipsen, JHEP 1102 (2011) 057



1st correction

N_τ	$M=1$	$M=3$	$M_1, M_2(\lambda_2) = 3,1$	$M_1, M_2(\lambda_a) = 3,1$	4d YM
4	5.768	5.830	5.813	5.773	5.6925(002)
6	6.139	6.173	6.172	6.164	5.8941(005)
8	6.300	6.324	6.324	6.322	6.0010(250)
10	6.390	6.408	6.408	6.408	6.1600(070)
12	6.448	6.462	6.462	6.462	6.2680(120)
14	6.488	6.500	6.500	6.500	6.3830(100)
16	6.517	6.528	6.528	6.528	6.4500(500)

Table 2. Critical couplings β_c for $SU(3)$ from different effective theories compared to simulations of the 4d theory [33, 35]).

Chiral symmetry breaking the lattice way

Blairon, Brout, Englert and Greensite, 1981

J-M. Blairon et al. / Chiral symmetry breaking 445

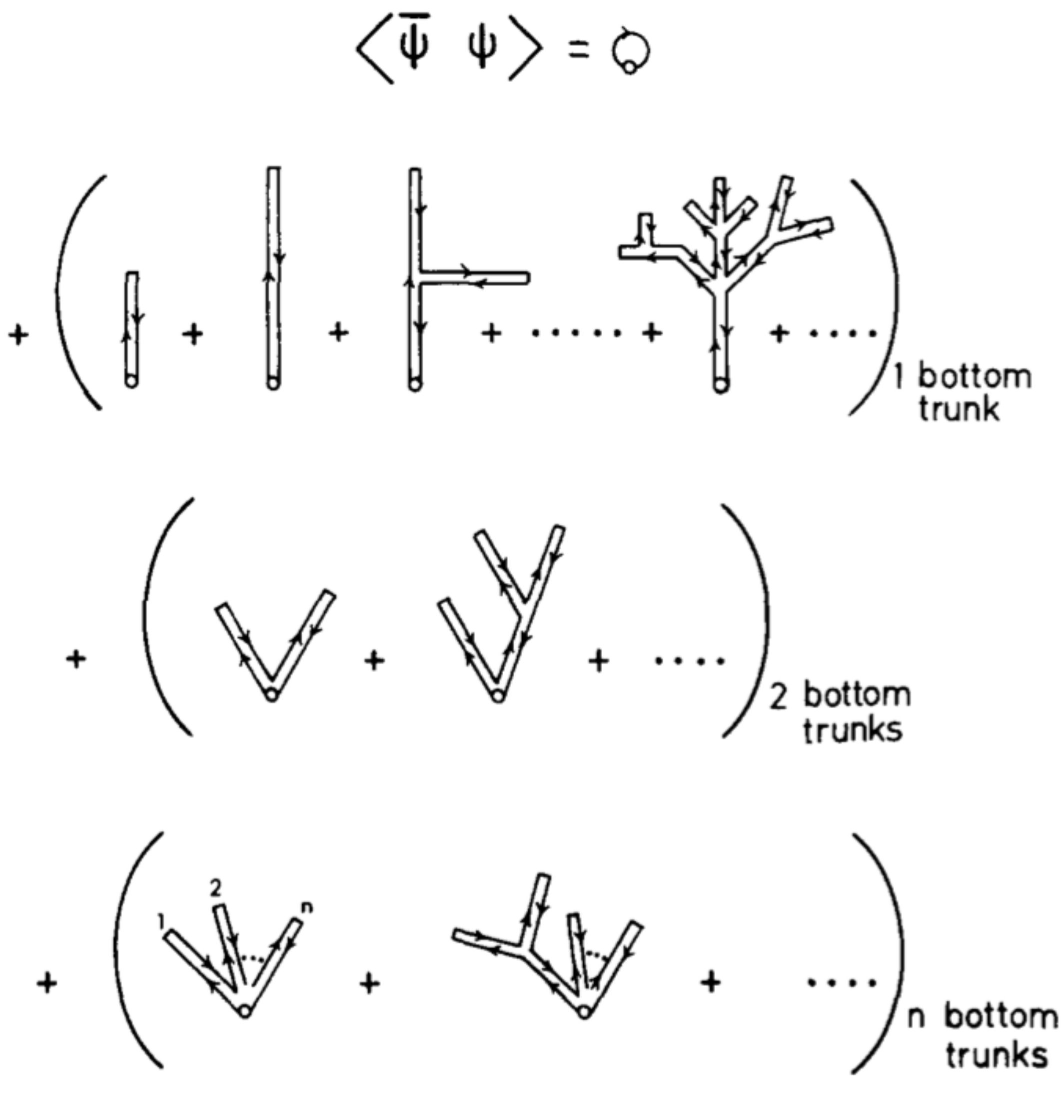


Fig. 4. All tree graphs contributions to $\langle\bar{\psi}\psi\rangle$.

Expansion in non-intersecting polymer branches is a large-d expansion.

Gap equation

$$\langle\bar{\psi}\psi\rangle = \frac{CN}{im}\sum_{n=0}^{\infty}\left[\frac{d}{2im}\,\frac{\langle\bar{\psi}\psi\rangle}{NC}\right]^n$$

$$= \frac{CN}{im - (d/2NC)\langle\bar{\psi}\psi\rangle}\,.$$

Gap equation is nothing but a strong-coupling lattice variant of the gap equation typical in four-fermion interaction models.

Deconfinement and Chiral Symmetry Breaking

PHYSICAL REVIEW D VOLUME 31, NUMBER 4 15 FEBRUARY 1985

Finite-temperature deconfinement and chiral-symmetry restoration at strong coupling

A. Gocksch and Michael Ogilvie
Department of Physics, Brookhaven National Laboratory, Upton, New York 11973
(Received 20 September 1984)

We derive from a d-dimensional lattice gauge theory at finite temperature a $(d-1)$-dimensional effective action in which the dynamical variables are Wilson lines and meson and baryon fields. Analysis of this model shows a first-order deconfinement transition for all values of the bare quark mass, and a second-order chiral transition at a higher temperature for zero bare quark mass. Reasonable values for the two transitions and the hadronic mass spectrum are obtained.

Here we have introduced the quantity E given by

$$\sinh E = m + \epsilon\lambda \ . \tag{2.20}$$

The gauge-dependent formula

$$W^{ab} = e^{in_t\phi_a}\delta^{ab} \tag{2.21}$$

has been used to recover a gauge-invariant result. Putting all this together, we finally arrive at the following effective action:

$$\begin{aligned} S_{\text{eff}} = & \sum_{\text{n.n.}} J[\text{Tr}W(\mathbf{x})\text{Tr}W^\dagger(\mathbf{y}) + \text{H.c.}] \\ & - \tfrac{1}{2} n_t N \sum_{\mathbf{x},\mathbf{y}} \lambda(\mathbf{x}) V^{-1}_{\mathbf{x},\mathbf{y}} \lambda(\mathbf{y}) \\ & + \sum_{\mathbf{x}} \text{Tr}\{n_t E + \ln[1 + e^{-n_t E} W] \\ & \qquad + \ln[1 + e^{-n_t E} W^\dagger]\} \ . \end{aligned} \tag{2.22}$$

- Model includes order parameters for both confinement and chiral symmetry breaking.
- Includes Polyakov loop (W) effects.
- Includes effect of a chiral condensate (λ).
- W and λ are coupled by the effect of quarks moving in a Polyakov loop background.

PNJL: lattice inspires phenomenology

ELSEVIER
Physics Letters B 591 (2004) 277–284
www.elsevier.com/locate/physletb

Chiral effective model with the Polyakov loop

Kenji Fukushima[a,b]

[a] *Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA*
[b] *Department of Physics, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan*

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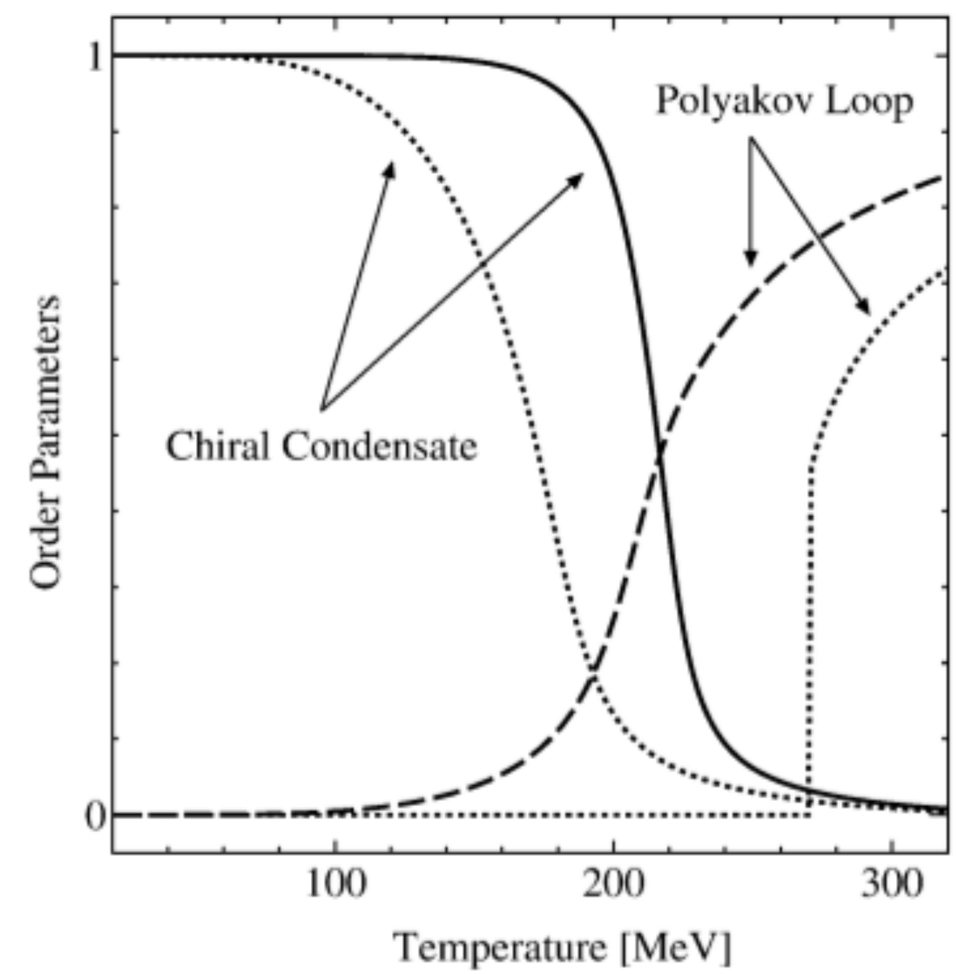
Available online 13 May 2004
Editor: H. Georgi

$$\begin{aligned} \Omega/V = {} & V_{\text{glue}}[L] + \frac{1}{2G}(M - m_q)^2 \\ & - 2N_c N_f \int \frac{d^3 p}{(2\pi)^3} \bigg\{ E_p + T \frac{1}{N_c} \\ & \times \mathrm{Tr}_c \ln\big[1 + L e^{-(E_p - \mu)/T}\big] \\ & + T \frac{1}{N_c} \mathrm{Tr}_c \ln\big[1 + L^\dagger e^{-(E_p + \mu)/T}\big] \bigg\}, \qquad (3) \end{aligned}$$



Unfinished business: large N_f

Conformality in many-flavour lattice QCD at strong coupling

Ph. de Forcrand,[a,b] S. Kim[c] and W. Unger[a,d]

JHEP 1302 (2013) 051

Abstract: It is widely believed that chiral symmetry is spontaneously broken at zero temperature in the strong coupling limit of staggered fermions, for any number of colors and flavors. Using Monte Carlo simulations, we show that this conventional wisdom, based on a mean-field analysis, is wrong. For sufficiently many fundamental flavors, chiral symmetry is restored via a bulk, first-order transition. This chirally symmetric phase appears to be analytically connected with the expected conformal window of many-flavor continuum QCD.

PHYSICAL REVIEW D **87,** 034513 (2013)

Absence of chiral symmetry breaking in multi-flavor strongly coupled lattice gauge theories

E. T. Tomboulis*

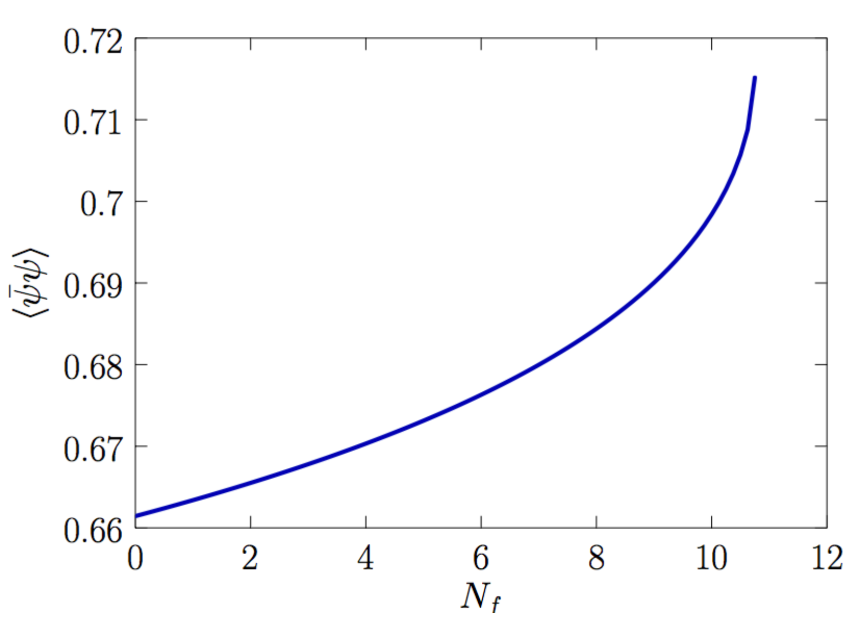
Department of Physics and Astronomy, UCLA, Los Angeles, California 90095-1547, USA

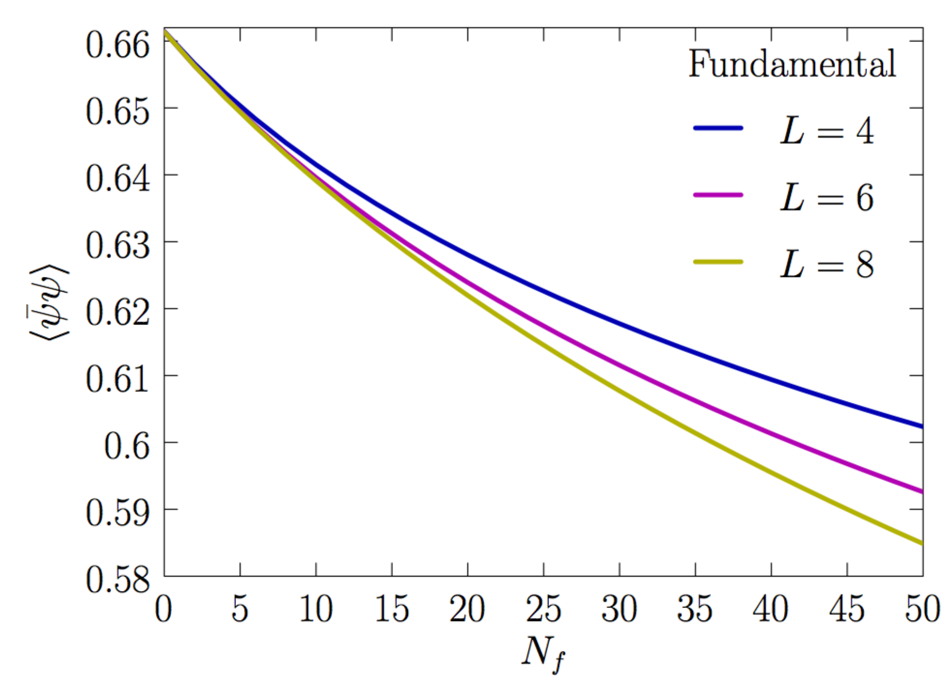
(Received 12 December 2012; revised manuscript received 28 January 2013; published 20 February 2013)

We consider lattice gauge theories at strong coupling with gauge group $U(N_C)$, or $SU(N_C)$ restricted to the meson sector, and coupled to N_F flavors of fundamental representation staggered fermions. We study the formation of a chiral condensate by means of resummation of a hopping expansion. Different classes of graphs become dominant as the parameter (N_F/N_C) is varied. By performing graph resummation we obtain an equation for determining the condensate as a function of (N_F/N_C) and mass m. For values of (N_F/N_C) below a critical value, one reproduces the well-known result of the existence of a nonvanishing condensate solution in the $m = 0$ limit. Above the critical (N_F/N_C) value, however, no such solution exists, its abrupt disappearance indicating a first-order transition to a chirally symmetric phase with composite (colorless) excitation spectrum.

PRELIMINARY Results

Myers et al., XQCD 2014





Unfinished business: finite density

JOURNAL OF MATHEMATICAL PHYSICS **54**, 122301 (2013)

Chiral symmetry restoration at large chemical potential in strongly coupled *SU*(*N*) gauge theories

E. T. Tomboulis[a)]
Department of Physics and Astronomy, UCLA, Los Angeles, California 90095-1547, USA

(Received 23 May 2013; accepted 16 November 2013; published online 9 December 2013; publisher error corrected 11 December 2013)

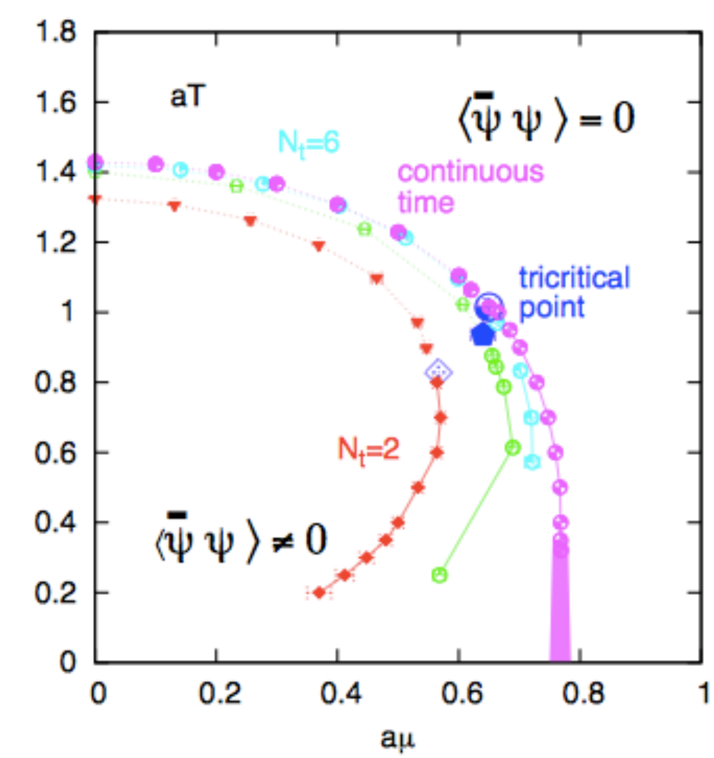
We show that at sufficiently large chemical potential $SU(N)$ lattice gauge theories in the strong coupling limit with staggered fermions are in a chirally symmetric phase. The proof employs a polymer cluster expansion which exploits the anisotropy between timelike and spacelike directions in the presence of a quark chemical potential μ. The expansion is shown to converge in the infinite volume limit at any temperature for sufficiently large μ. All expectations of chirally non-invariant local fermion operators vanish identically, or, equivalently, their correlations cluster exponentially, within the expansion. The expansion itself may serve as a computational tool at large μ and strong coupling. © *2013 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4837115]

- Sign problem
- Color superconductivity problem

The lattice QCD phase diagram in and away from the strong coupling limit

Ph. de Forcrand,[1,2] J. Langelage,[2] O. Philipsen,[3] and W. Unger[3]
[1]*CERN, Physics Department, TH Unit, CH-1211 Geneva 23, Switzerland*
[2]*Institut für Theoretische Physik, ETH Zürich, CH-8093 Zürich, Switzerland*
[3]*Institut für Theoretische Physik, Goethe-Universität Frankfurt, 60438 Frankfurt am Main, Germany*
(Dated: June 18, 2014)

We study lattice QCD with four flavors of staggered quarks. In the limit of infinite gauge coupling, "dual" variables can be introduced, which render the finite-density sign problem mild and allow a full determination of the $\mu - T$ phase diagram by Monte Carlo simulations, also in the chiral limit. However, the continuum limit coincides with the weak coupling limit. We propose a strong-coupling expansion approach towards the continuum limit. We show first results, including the phase diagram and its chiral critical point, from this expansion truncated to next-to-leading order.



arXiv:1406.4397

New/Old business: Resurgence and the lattice

- The behavior of observables are described by a trans-series

$$\langle O \rangle = \sum_{n=0}^{\infty} p_{0,n}\lambda^n + \sum_c e^{-S_c/\lambda} \sum_{n=0}^{\infty} p_{c,n}\lambda^n$$

- Sum includes contributions which are not topologically stable, such as an instanton-anti-instanton contribution.
- Applies even in theories without topologically stable objects.
- Deep connection between perturbative and non-perturbative sector.

- QCD: Argyres and Unsal, 2012.
- CP^{N-1}: Dunne and Unsal, 2012.
- Quantum Mechanics, Dunne and Unsal, 2013.
- Principal Chiral Model: Cherman *et al.*, 2013, 2014.
- Self-dual model: Basar *et al.*, 2013.

- Appears to work best in field theories in a disordered/confined phase, deformed to be effectively Abelian.
- Not clear how to apply in the deconfined phase of QCD, where the perturbative sector is not given as a power series in g^2, and there are severe IR problems at $O(g^6)$.
- Lattice strong-coupling “knows” about the continuum limit. How can we use this?
- Duality works at a deep level on and off the lattice to relate strong-coupling to weak. How can we use this?

My wish for Mike:
More time for play!

"The theorists have only recently realized that modern interactive computer systems allow toying with concepts on a time scale comparable to their attention span.”

Mike Creutz 1983