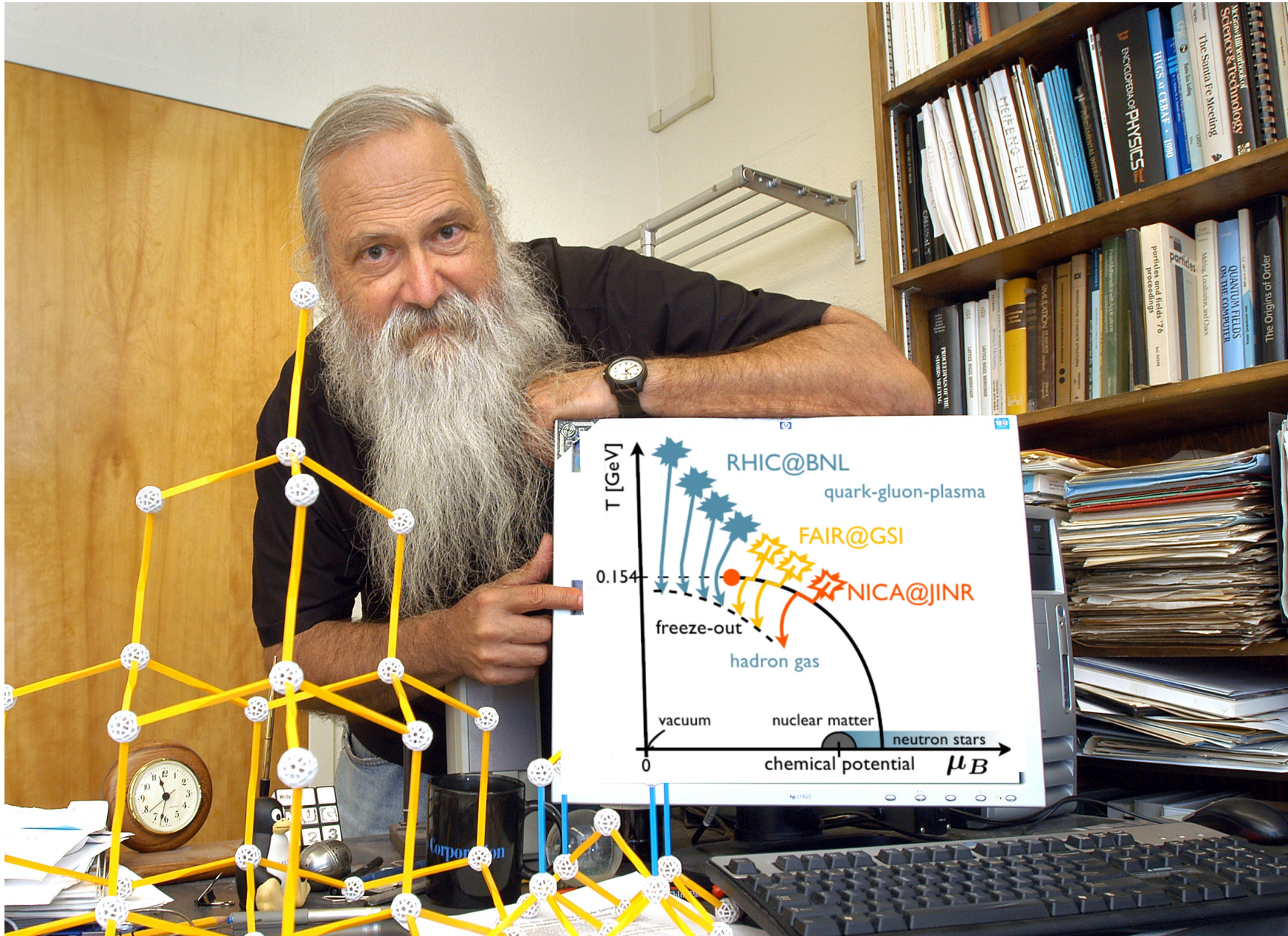


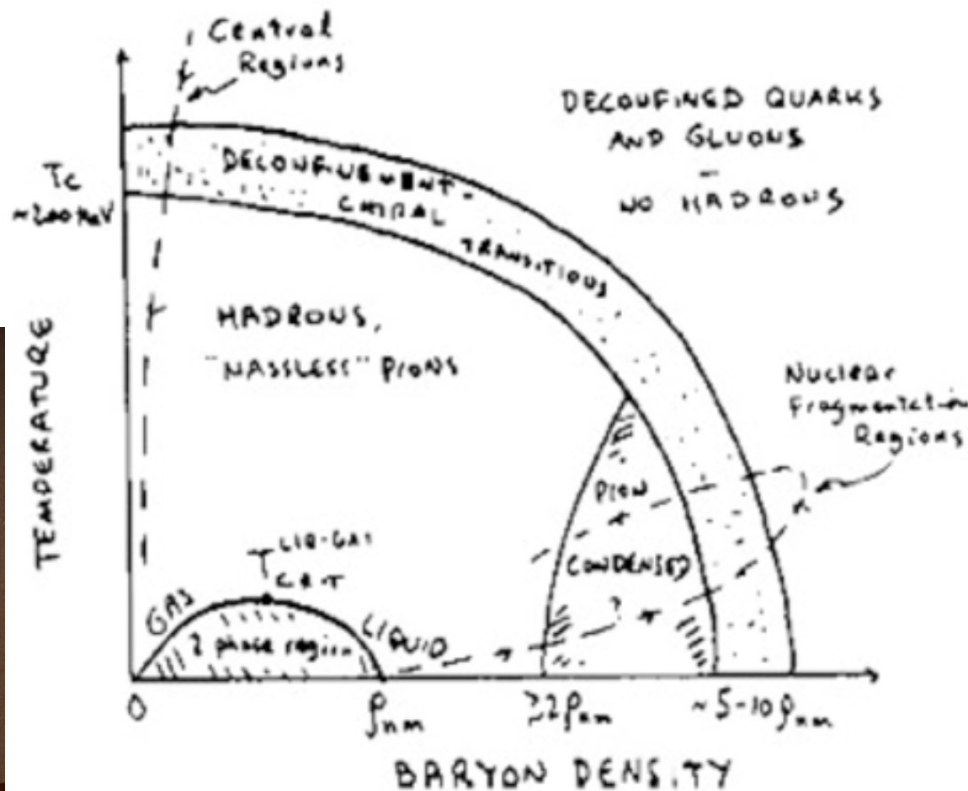
# Conserved charge fluctuations and the phase diagram of strongly interacting matter

Frithjof Karsch, BNL/Bielefeld





# Strongly interacting matter in the '70s



Gordon Baym: Long Range Plan 1983



**Rolf Hagedorn:**  
Hadron resonance gas,  
ultimate temperature?

## Mike Creutz

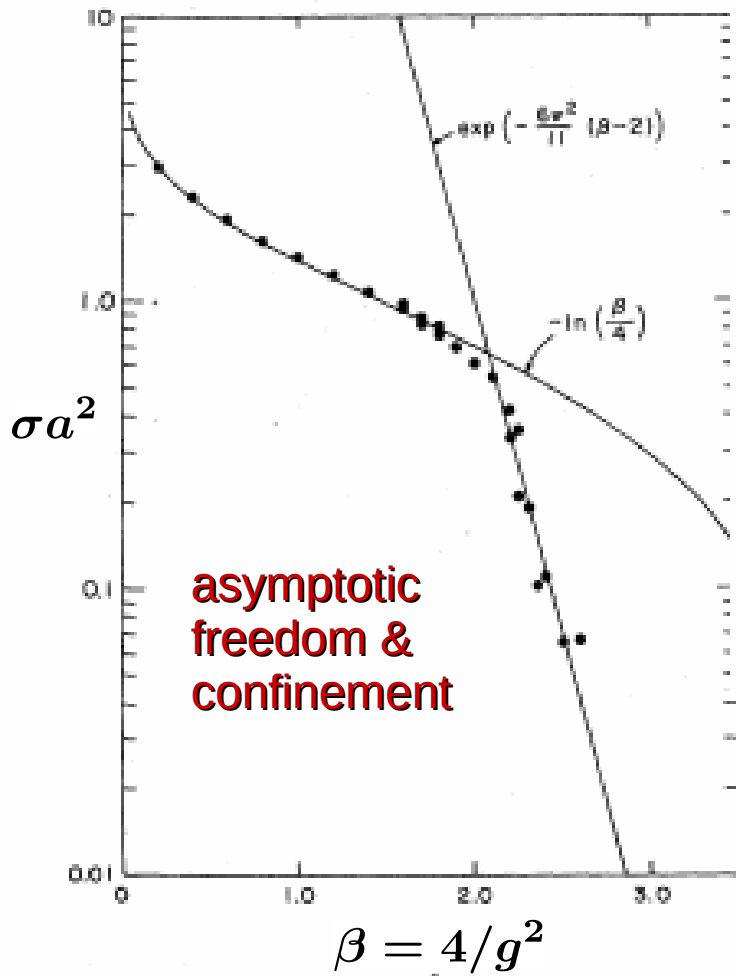
- the physics/**thermodynamics of strong interaction matter** is described by the theory of strong interactions – **Quantum Chromo Dynamics (QCD)**
- understanding highly non-perturbative/collective effects like **phase transitions** requires the application of numerical techniques – **lattice QCD**

# Asymptotic Freedom Equation of state and Hadron Resonance Gas

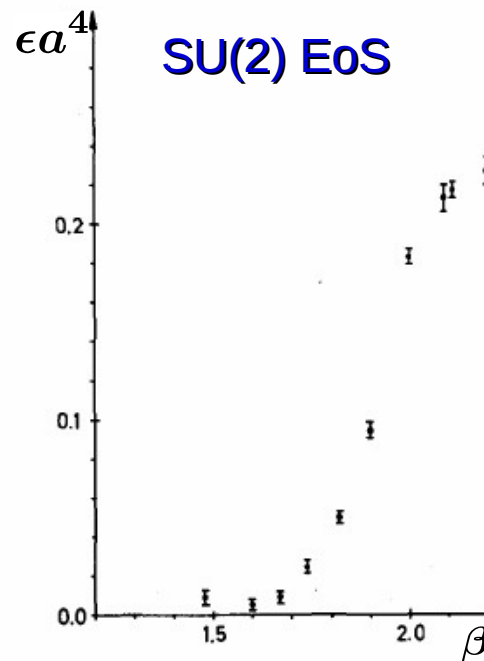
Volume 101B, number 1,2

PHYSICS LETTERS

30 April 1981

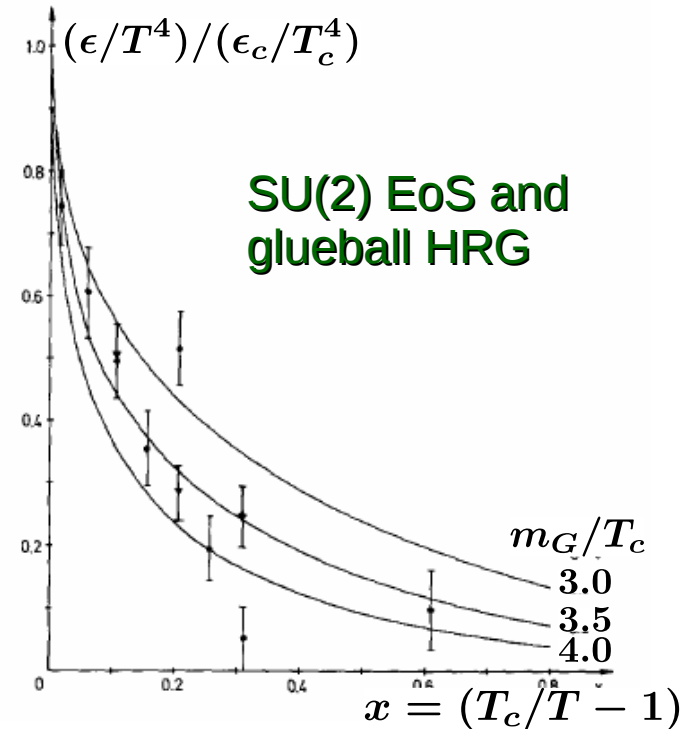


**M. Creutz,**  
**Phys. Rev. D21 (1980) 2308**



**J. Engels et al.,**  
**PL 101B (1981)**

**J. Engels et al.,**  
**PL 102B (1981)**



# QCD-EoS and the Hadron Resonance Gas (HRG)

HRG thermodynamics

Pressure  $\frac{P}{T^4} = \sum_{m \in \text{mesons}} \ln Z_m^b(T, V, \mu) + \sum_{m \in \text{baryons}} \ln Z_m^f(T, V, \mu)$

Trace anomaly

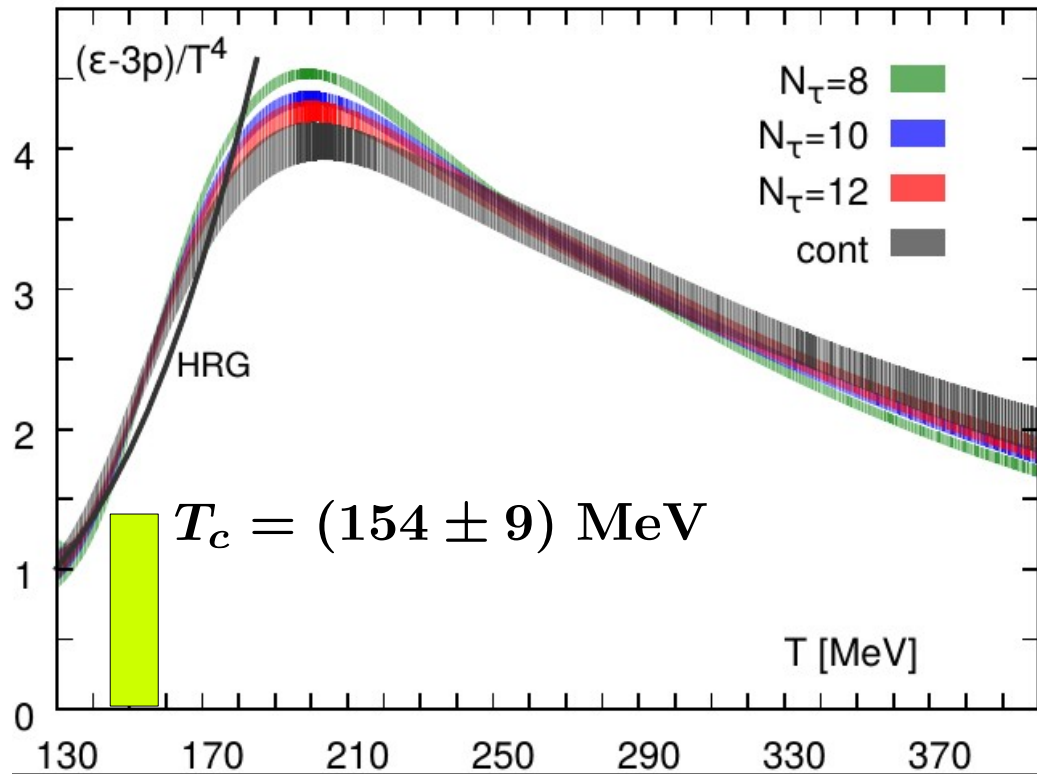
$$\frac{\epsilon - 3P}{T^4} = T \frac{dP/T^4}{dT}$$

$$\left(\frac{\epsilon - 3P}{T^4}\right)_{\text{QCD}} > \left(\frac{\epsilon - 3P}{T^4}\right)_{\text{HRG}}$$



larger pressure, larger trace anomaly

**there is 'room for additional resonances' not accounted for by the HRG model**

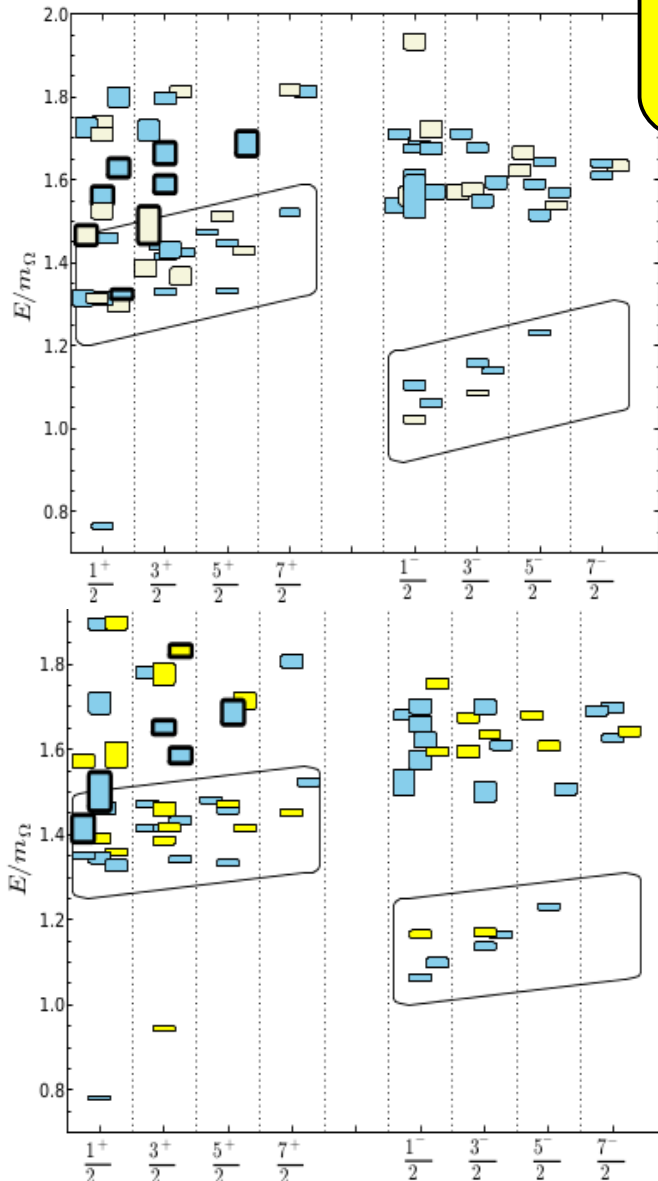


hotQCD , arXiv:1407.6387

# Probing the hadron spectrum using QCD thermodynamics

Lattice QCD

$\Lambda-391$



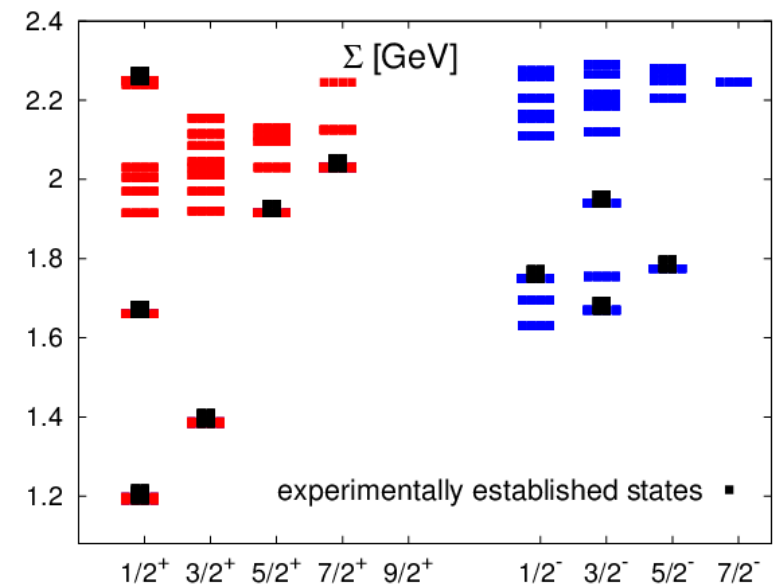
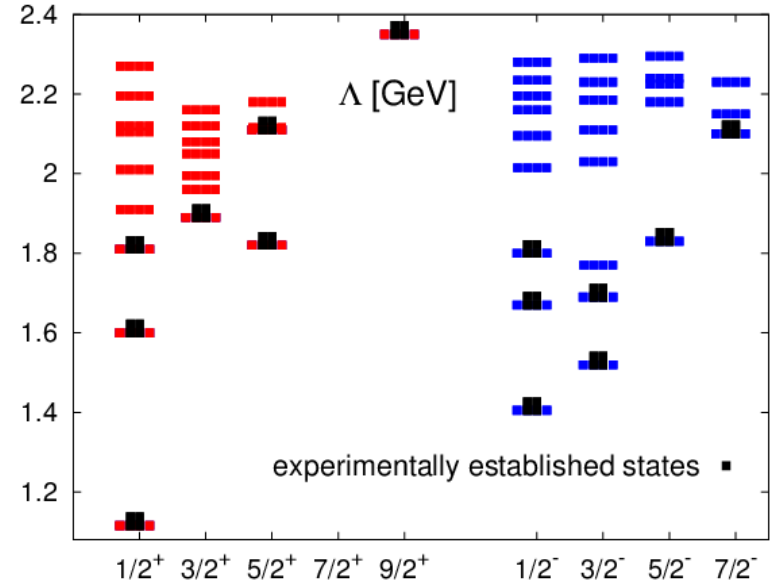
$$P_{tot} = \sum_{h=\text{all hadrons}} P_h$$

strange  
baryons

more  
strangeness

=  
larger fluct.

Quark Model

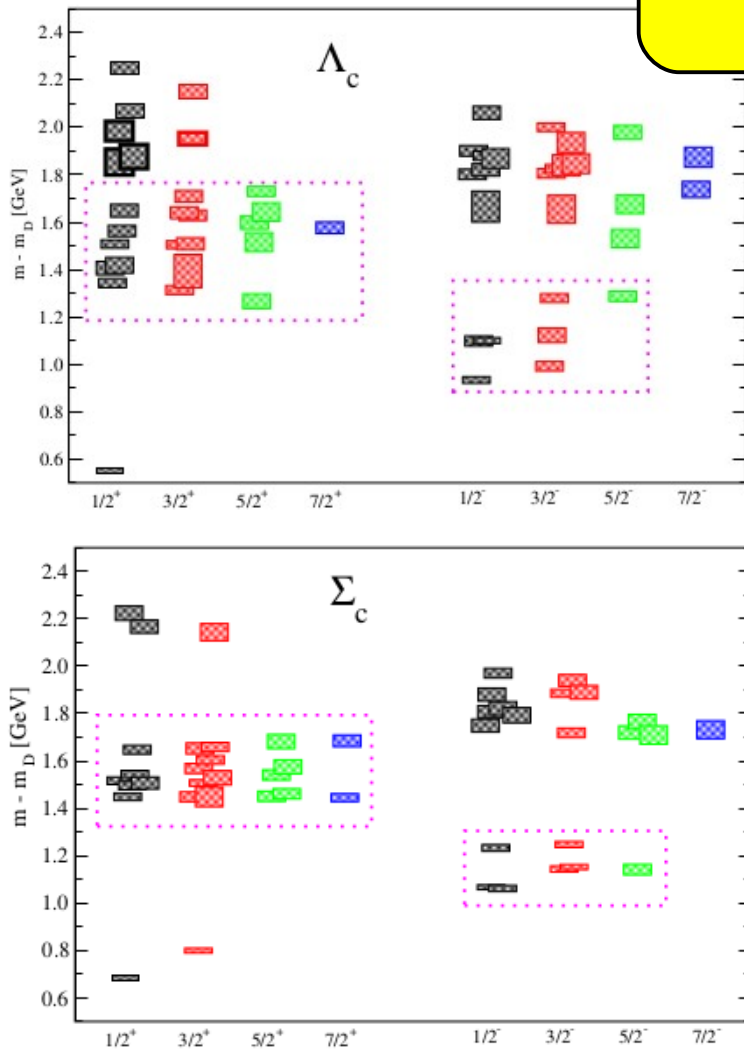


R. Edwards et al., Phys. Rev D87, 054506 (2013)

S. Capstick, N. Isgur, Phys. Rev. D34, 2809 (1986)

# Probing the hadron spectrum using QCD thermodynamics

Lattice QCD

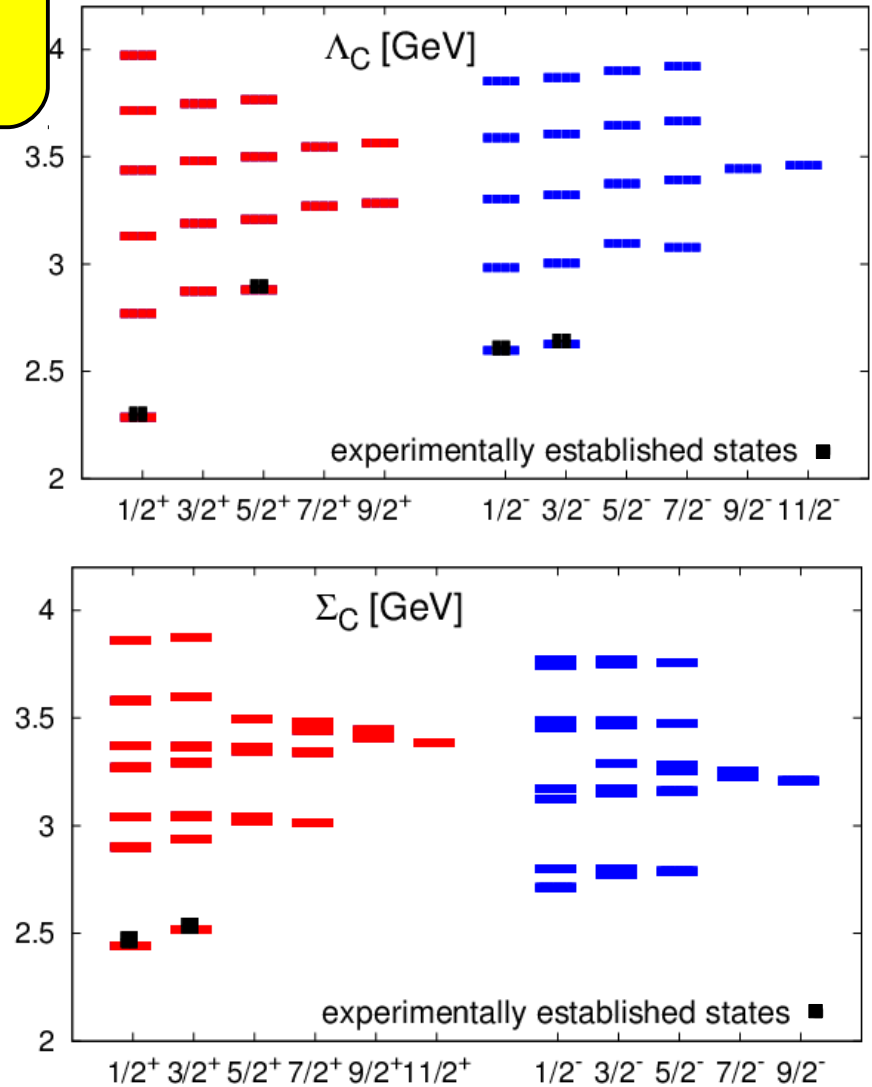


$$P_{tot} = \sum_{h=\text{all hadrons}} P_h$$

charmed  
baryons

more charm  
=  
larger fluct.

Quark Model



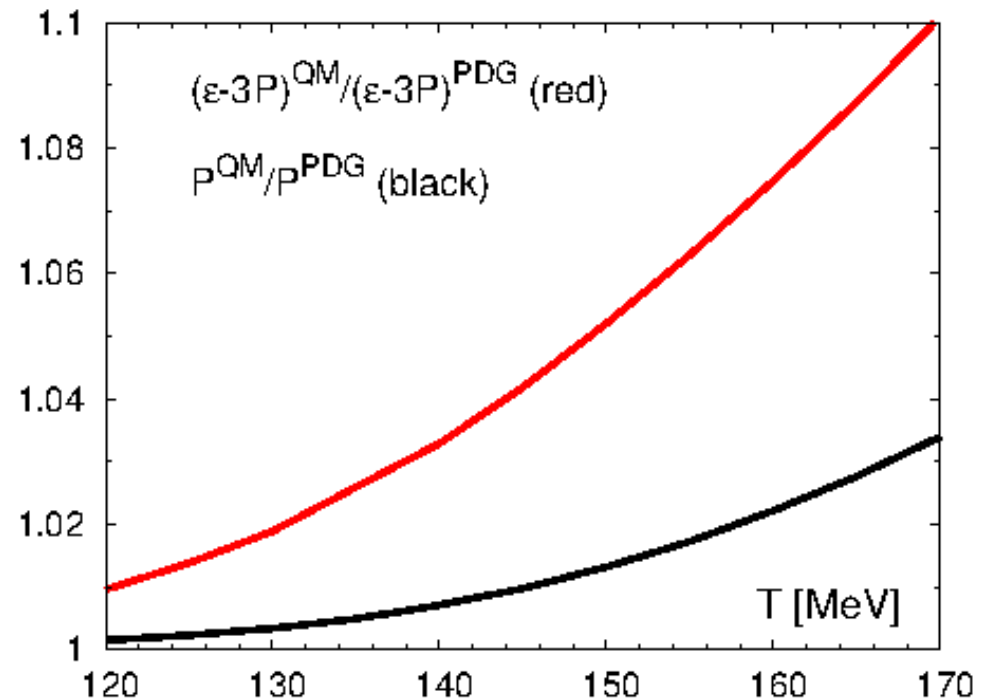
M. Padmanath et al., arXiv:1311.4806

D. Ebert et al., Eur. Phys. J. C66, 197 (2010);  
Phys. Rev. D84, 014025 (2011)

# Probing the hadron spectrum using QCD thermodynamics

- additional resonance in the hadron spectrum increase the pressure, energy density as well as trace anomaly
- charmed baryons are too heavy to have any impact on bulk thermodynamics
- additional strange baryons may increase the pressure by about 3% at  $T=160$  MeV

– need to be more selective to see effects of additional strange and charmed hadronic resonances



# Fluctuations and Correlations: Susceptibilities

- probing the response of a thermal medium to an external field, i.e. variation of one of its external control parameters:  $T$ ,  $\mu$ ,  $m_q$

(generalized) response functions == (generalized) susceptibilities

pressure: 
$$\frac{p}{T^4} \equiv \frac{1}{VT^3} \ln Z(V, T, \mu_{B,Q,S}, m_{u,d,s})$$

net number density

$$\chi_1^q = \frac{1}{VT^3} \frac{\partial \ln Z}{\partial \mu_q/T}$$

(quark) number susceptibility

$$\chi_2^q = \frac{1}{VT^3} \frac{\partial^2 \ln Z}{\partial (\mu_q/T)^2}$$

4<sup>th</sup> order cumulant

$$\chi_4^q = \frac{1}{VT^3} \frac{\partial^4 \ln Z}{\partial (\mu_q/T)^4}$$

mean

variance

kurtosis

generalized quark number susceptibilities:

$$\frac{\partial^{i+j+k} p/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k}$$

evaluated at

$$\hat{\mu}_X \equiv \mu_X/T = 0$$



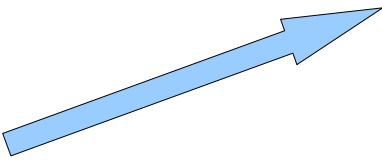
# Correlations and Fluctuations: HRG vs. LQCD

- construct QCD observables that would project onto specific quantum numbers, if QCD = HRG

E.g.: HRG pressure:

$$\frac{P}{T^4} = \sum_{m \in \text{mesons}} \ln Z_m^b(T, V, \mu) + \sum_{m \in \text{baryons}} \ln Z_m^f(T, V, \mu)$$

HRG baryon susceptibilities:

$$\chi_{nmkl}^{BQSC} = \sum_{m \in \text{baryons}} \frac{\partial^{(n+m+k+l)} \ln Z_m^f(T, V, \mu)}{\partial \hat{\mu}_B^n \partial \hat{\mu}_Q^m \partial \hat{\mu}_S^k \partial \hat{\mu}_C^l} \Big|_{\mu=0}$$


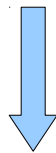
sum "knows" about spectrum

# Correlations and Fluctuations: HRG vs. LQCD

- in a HRG charge fluctuations obey some simple relations because B, Q, S quantum numbers are integer; -- or even restricted to |B|=0, 1
- baryonic part of the pressure:

$$\frac{P^{\text{baryon}}}{T^4} = \sum_{m \in \text{baryons}} f(T, m) \cosh(B\mu_B + S\mu_S + Q\mu_Q)$$

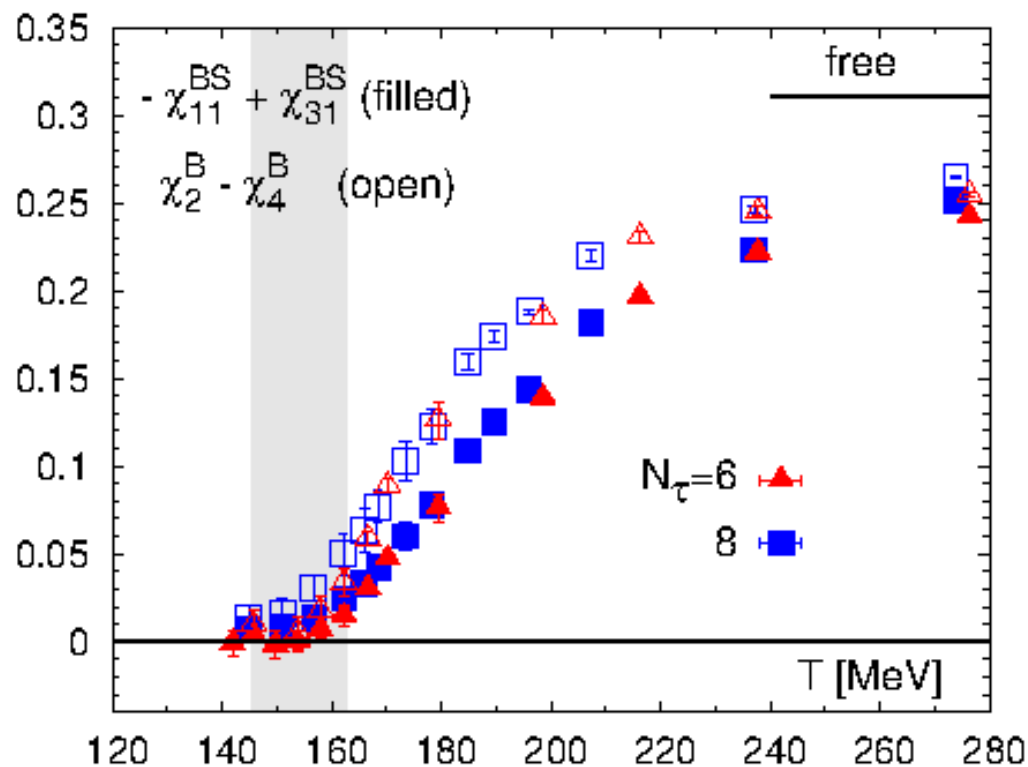
$$\chi_{nmkl}^{BQSC} = \sum_{m \in \text{baryons}} \left. \frac{\partial^{(n+m+k+l)} \ln Z_m^f(T, V, \mu)}{\partial \hat{\mu}_B^n \partial \hat{\mu}_Q^m \partial \hat{\mu}_S^k \partial \hat{\mu}_C^l} \right|_{\mu=0}$$



$$\chi_{amkl}^{BQSC} = \chi_{bmkl}^{BQSC}, \quad a > 0, \quad b > 0, \quad a + b \text{ even}$$

e.g.  $\chi_{11}^{BS} = \chi_{31}^{BS}$ ,  $\chi_2^B = \chi_4^B$  .... valid in any HRG (irresp. of the spectrum)

# Correlations and Fluctuations: HRG vs. LQCD



- HRG model description of fluctuations and correlations breaks down above  $T=160$  MeV
- this also is the case for the strange baryon sector

A. Bazavov et al. (BNL-Bielefeld-CCNU), Phys. Rev. Lett. 111, 082301 (2013)

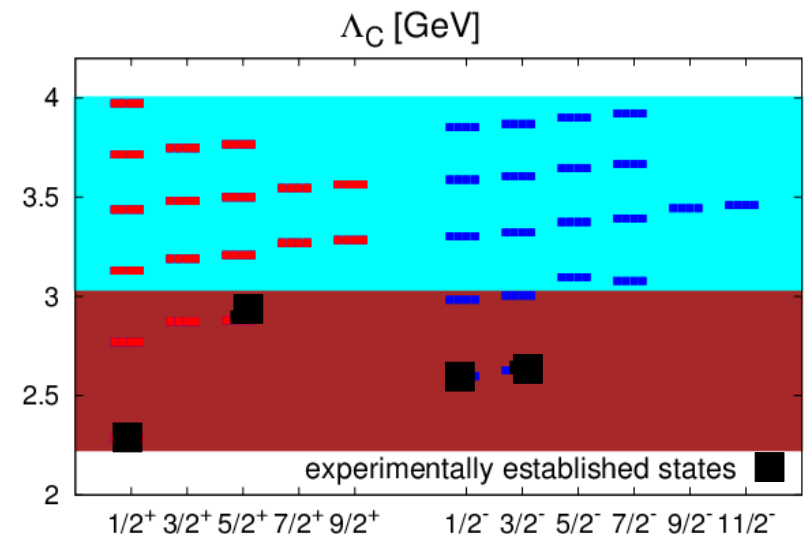
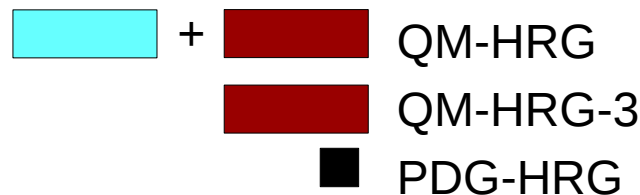
# Evidence for many charmed baryons in thermodynamics

- use charge fluctuations and correlations to probe the hadron spectrum
- HRG pressure of open charmed mesons and baryons particularly simple, because multiple charmed baryons are too heavy to be of thermodynamic relevance; e.g.

$$\chi_{11}^{BC} \simeq \chi_{22}^{BC} \simeq \chi_{13}^{BC}$$

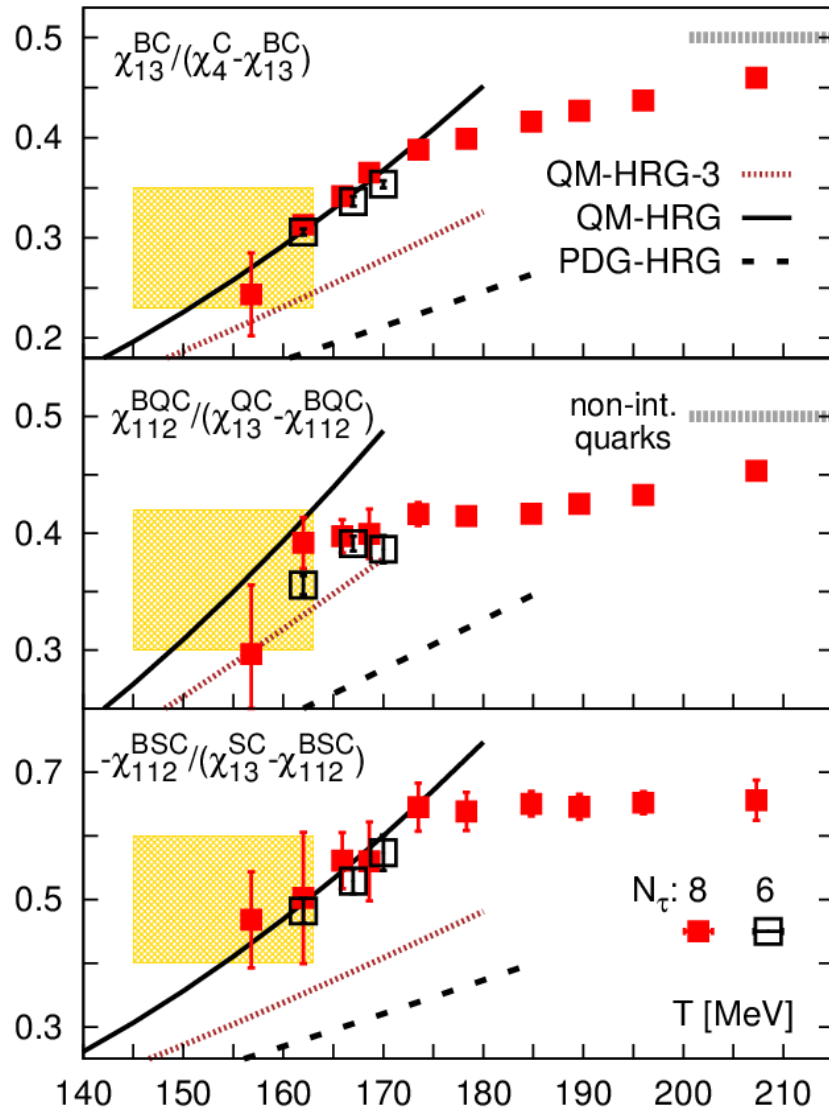
open charm baryon to meson pressure ratio:  $\frac{B_C}{M_C} = \frac{\chi_{13}^{BC}}{\chi_4^C - \chi_{13}^{BC}}$

sum "knows" about spectrum





# Evidence for many **charmed baryons** in thermodynamics



close to  $T_c$  charmed baryon fluctuations are about 50% larger than expected in a HRG based on known charmed baryon resonances (PDG-HRG)

charmed pressure ratios

all charmed baryons/mesons

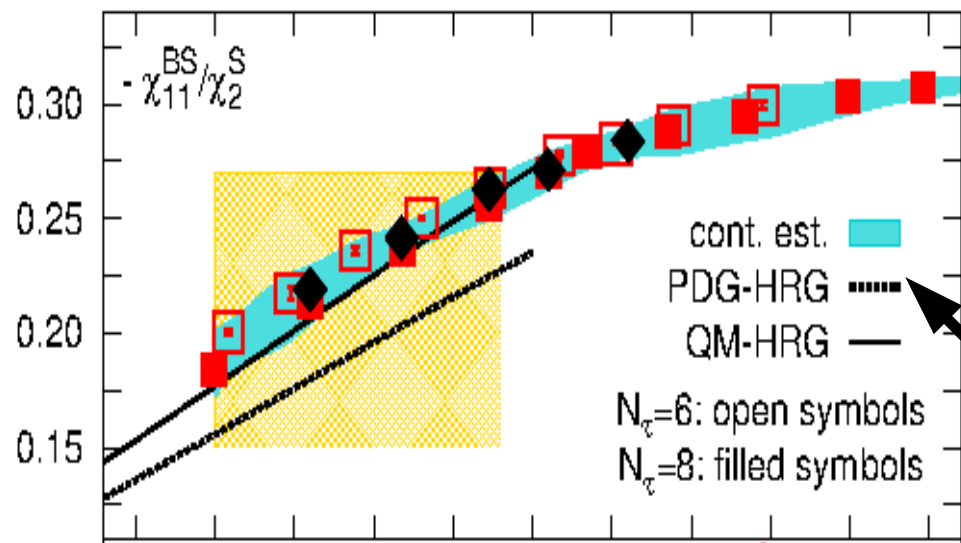
charged charmed baryons/mesons

strange charmed baryons/mesons

including resonance predicted in quark model calculations and observed in lattice QCD calculations allows for a HRG model (QM-HRG) description of lattice QCD results on conserved charge fluctuations and correlations

A. Bazavov et al., arXiv:1404.4043

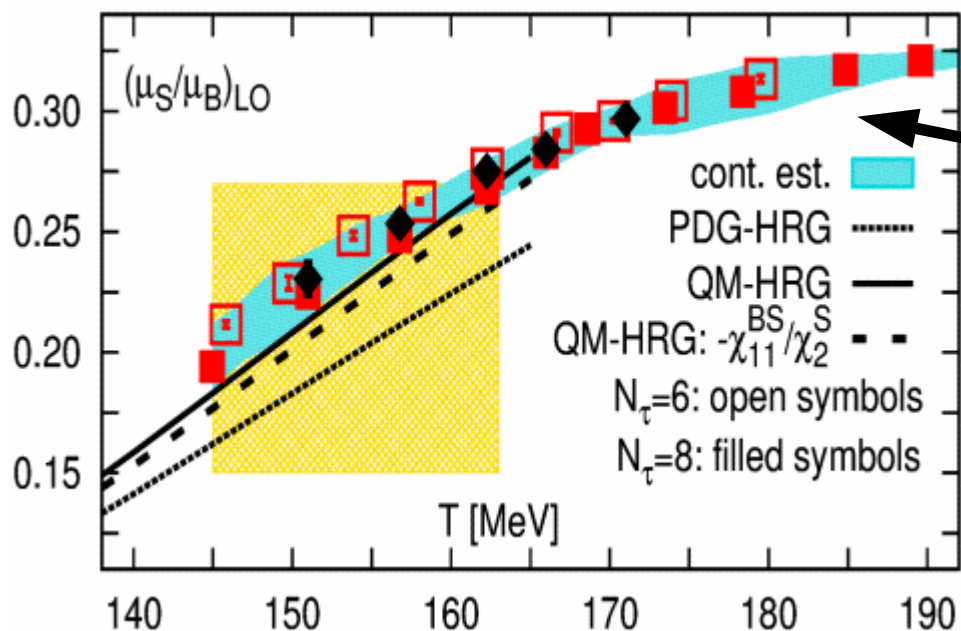
# Evidence for more **strange baryons** in thermodynamics



close to  $T_c$  strange baryon fluctuations are about (10-20)% larger than expected in a HRG based on known strange baryon resonances (PDG-HRG)

QM-HRG model agrees well with lattice QCD

**enhanced** strangeness-baryon correlation over strangeness fluctuations



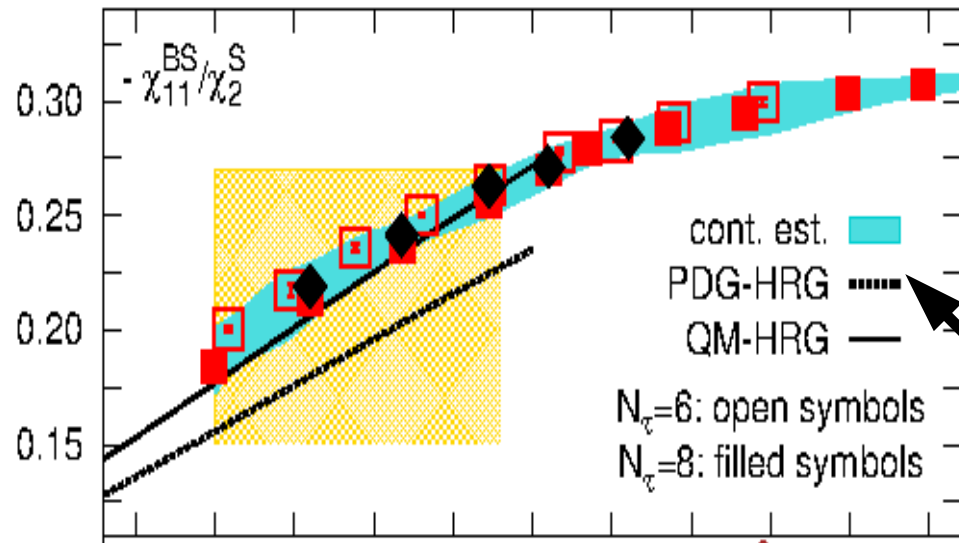
**strangeness neutrality** enforces relation between chemical potentials

$$\begin{aligned} \langle n_S \rangle &= 0 \\ &= \chi_2^S \hat{\mu}_S^2 + \chi_{11}^{BS} \hat{\mu}_S \hat{\mu}_B + \mathcal{O}(\mu^4) \end{aligned}$$

$$\frac{\mu_S}{\mu_B} = -\frac{\chi_{11}^{BS}}{\chi_2^S} + \mathcal{O}(\mu^2)$$

A. Bazavov et al.,  
Phys. Rev. Lett. 113, 072001 (2014), arXiv:1404.6511

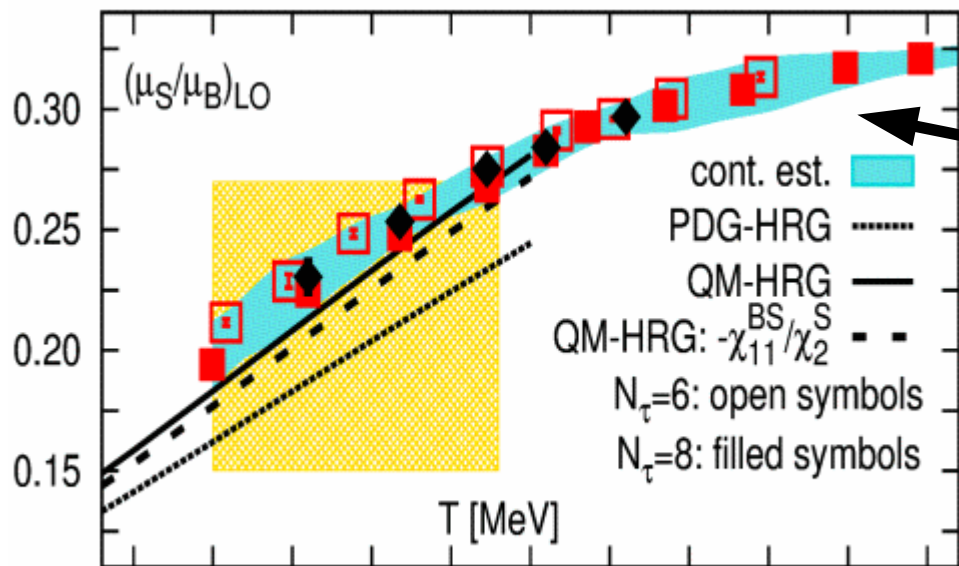
# Evidence for more **strange baryons** in thermodynamics



close to  $T_c$  strange baryon fluctuations are about (10-20)% larger than expected in a HRG based on known strange baryon resonances (PDG-HRG)

QM-HRG model agrees well with lattice QCD

**enhanced** strangeness-baryon correlation over strangeness fluctuations

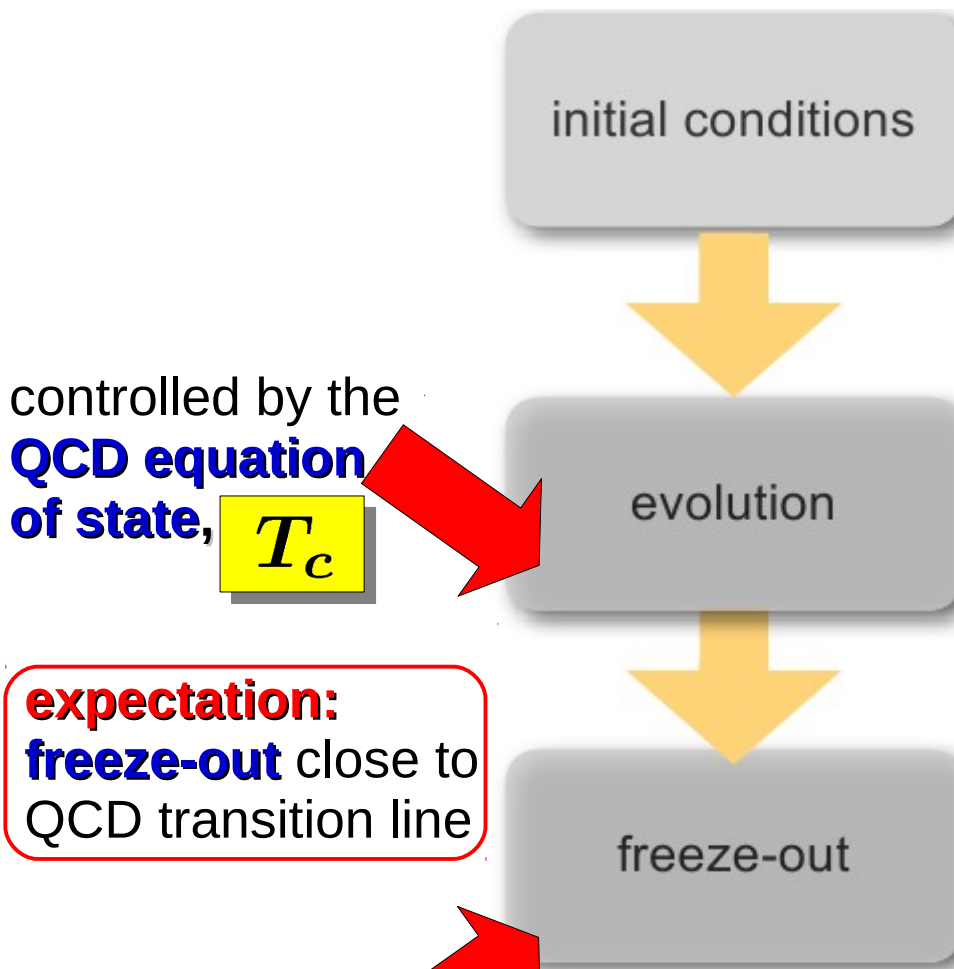


**enhanced** strangeness over baryon chemical potential ratio

leads to the prediction of a **smaller freeze-out temperature**

A. Bazavov et al.,  
Phys. Rev. Lett. 113, 072001 (2014), arXiv:1404.6511

# Exploring the QCD phase diagram

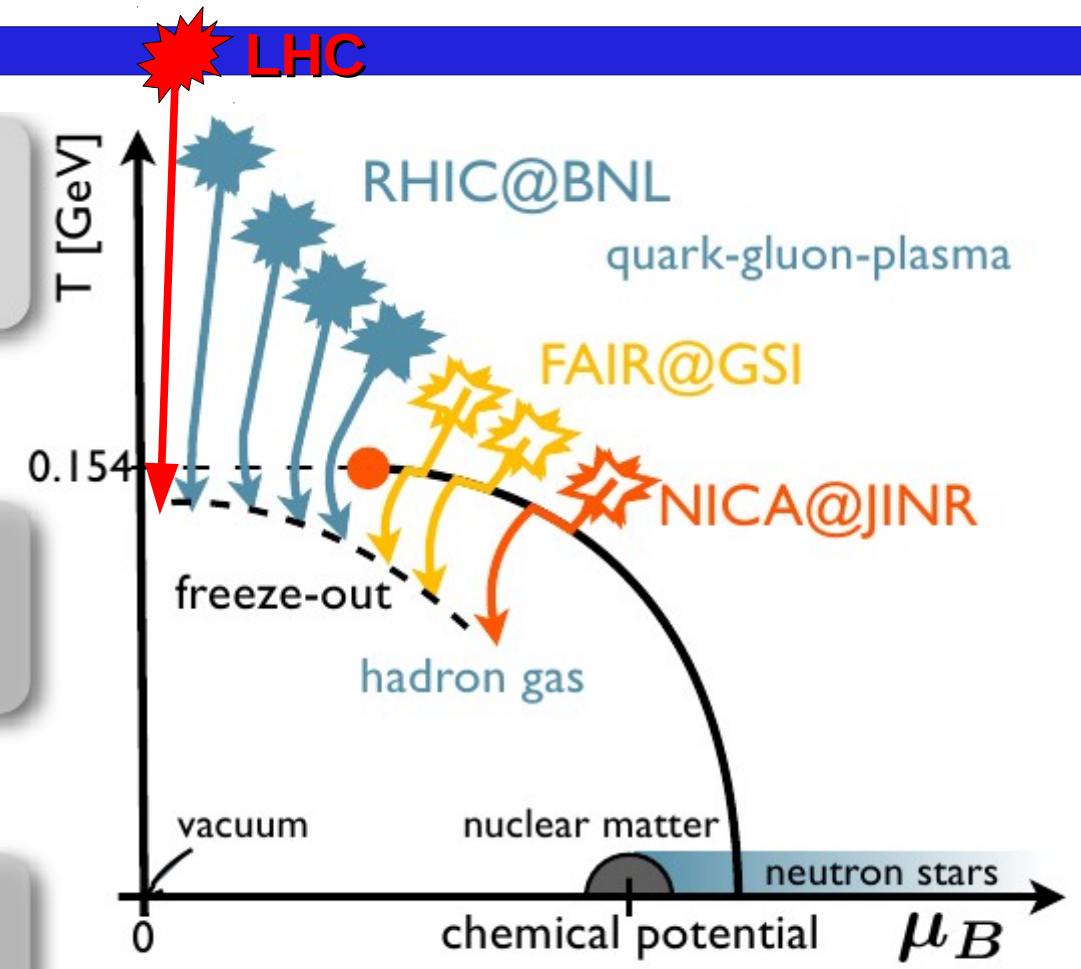


controlled by the QCD equation of state,  $T_c$

**expectation:** freeze-out close to QCD transition line

**observable consequences:** freeze-out/hadronization pattern of mesons and baryons, controlled by

$$T_f, \mu_B, \mu_S$$



- LHC: establish contact with the QCD PHASE transition
- RHIC: locate/provide evidence for the QCD critical point



# HRG model, lattice QCD and critical behavior

- for a wide range of baryon chemical potentials freeze-out happens in or close to the QCD transition region: **predicted** → P. Braun-Munzinger et al., Phys. Lett. B596, 61 (2004)

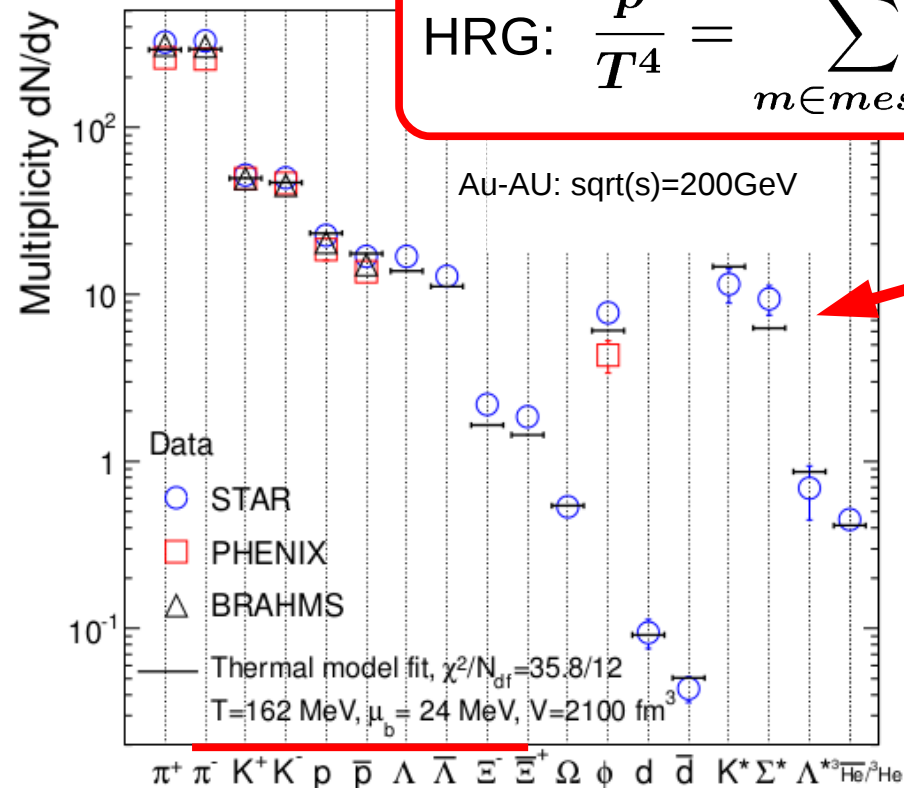
**caveat:** freeze-out parameter extracted from experimental data by comparing to the **Hadron Resonance Gas (HRG)** model, i.e. **not QCD**

$$\text{HRG: } \frac{p}{T^4} = \sum_{m \in \text{meson}} \ln Z_m^b(T, V, \mu) + \sum_{m \in \text{baryon}} \ln Z_m^f(T, V, \mu)$$

$$\sim e^{-m_H/T} e^{(B\mu_B + S\mu_S + Q\mu_Q)/T}$$

**goal:** describe freeze-out in terms of QCD thermodynamics

→ freeze-out parameter from comparison of measured moments of charge fluct. with QCD calculation



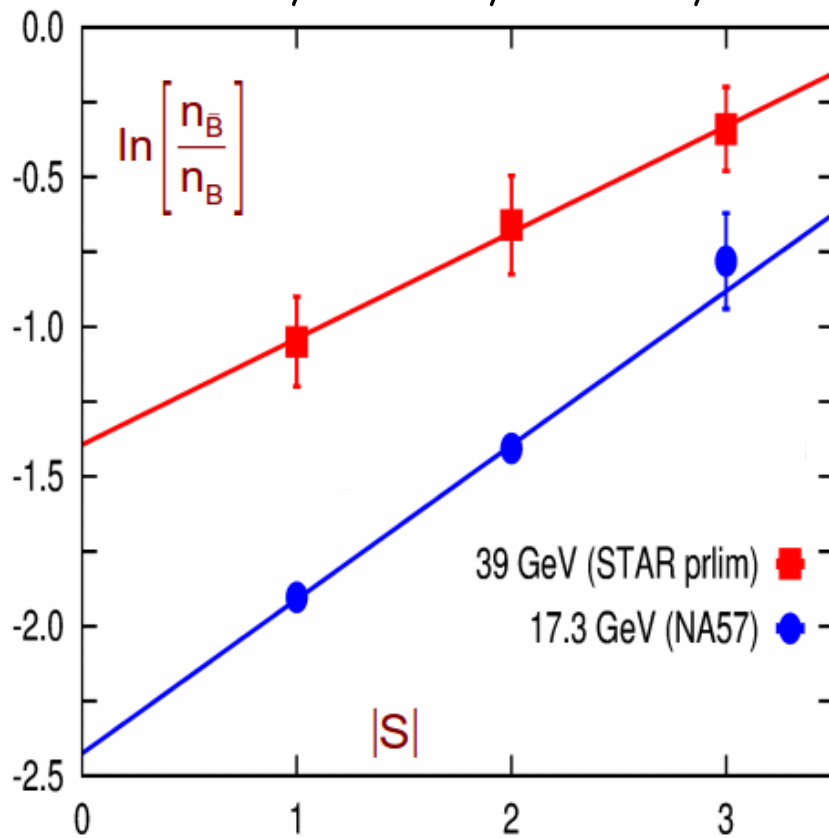
Andornic et al.: Nucl. Phys. A904, 535c (2013)

BNL-Bielefeld, PRL 109, 12302 (2012)

# Strange hadron yields in HIC

$$\frac{n_{\bar{\Lambda}}}{n_{\Lambda}}, \frac{n_{\bar{\Sigma}}}{n_{\Sigma}}, \frac{n_{\bar{\Omega}}}{n_{\Omega}} = \exp \left[ -\frac{2\mu_B^f}{T^f} - \frac{2\mu_S^f}{T^f} |S| \right] = \exp \left[ -\frac{2\mu_B^f}{T^f} \left( 1 - \frac{\mu_S^f}{\mu_B^f} |S| \right) \right]$$

$\Lambda/\bar{\Lambda}$     $\Sigma/\bar{\Sigma}$     $\Omega/\bar{\Omega}$



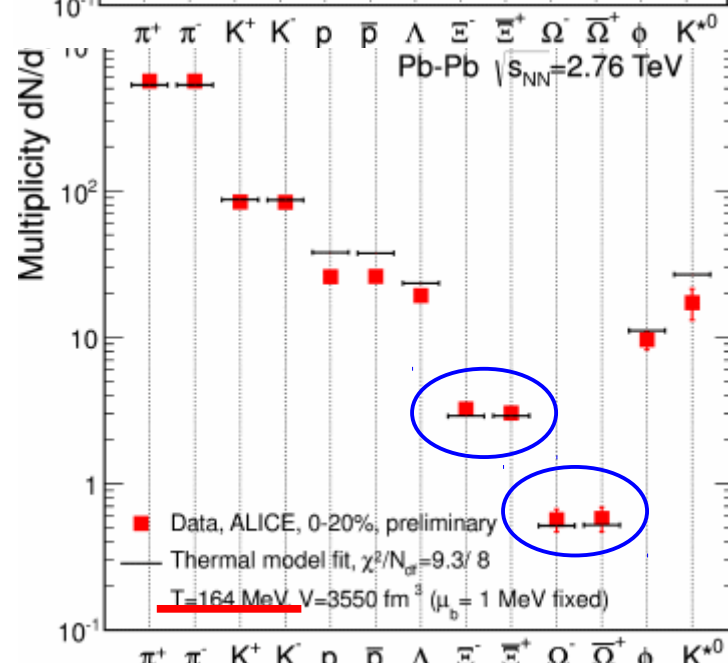
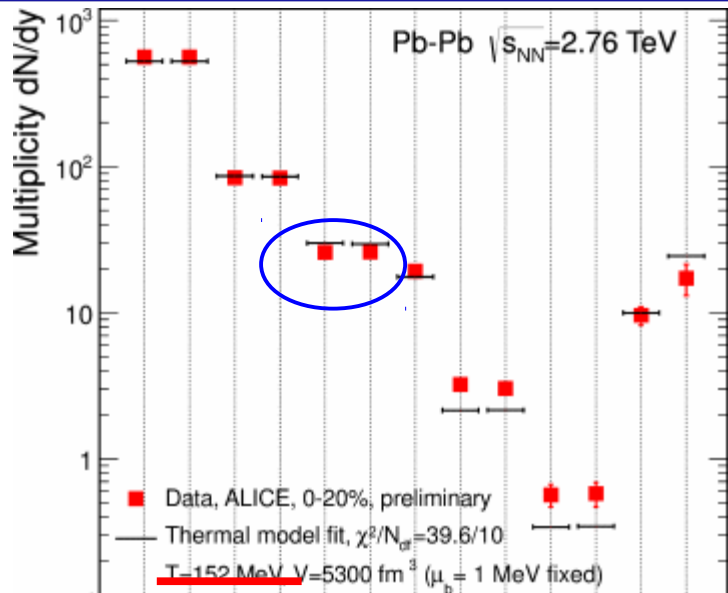
presence of unobserved higher resonances get imprinted in the yields of ground state hadrons

extract chemical potentials from particle/anti-particle ratios of multiple strange baryon yields (eliminates mass dependence)

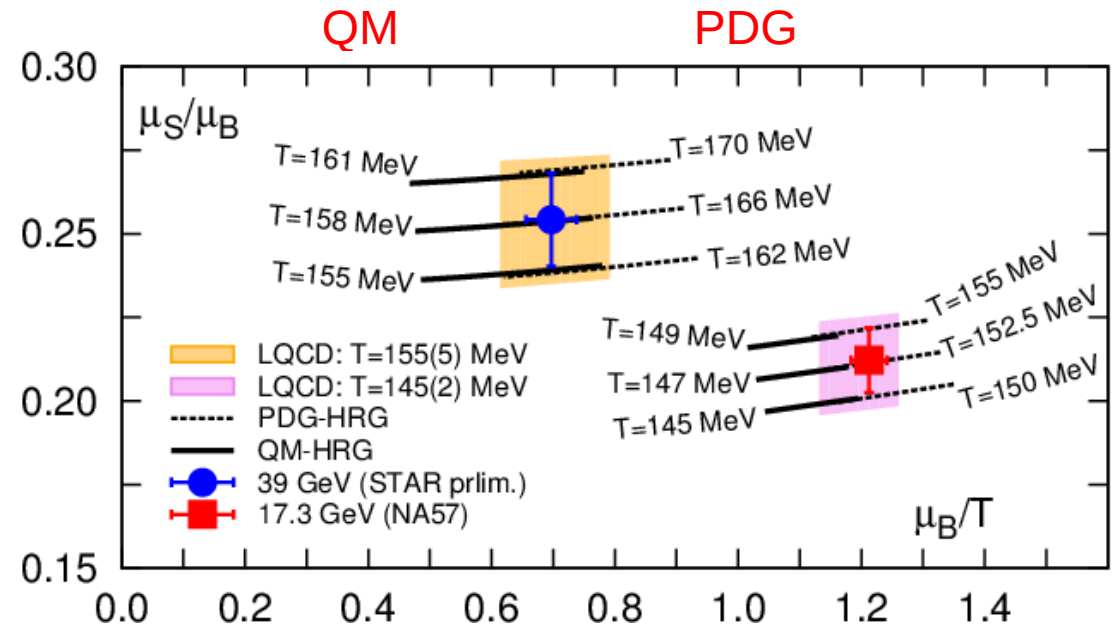
data:

STAR: F. Zhao, PoS CPOD2013 (2013) 036  
 NA57: F. Antinori et al, PLB 595 (2004) 68

# Impact on determination of freeze-out parameter

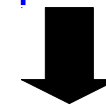


A. Andronic et al.: Nucl. Phys. A904, 535c (2013)



A. Bazavov et al., Phys. Rev. Lett. 113, 072001 (2014), arXiv:1404.6511

including more strange baryons  
will change determination of  
freeze-out parameters



better agreement of strange and  
non-strange particle yields at lower  
freeze-out temperature

# Conclusions

- Thanks to Mike's pioneering work

LGT calculations have achieved two important goals of studies of strong interaction thermodynamics

- determination of transition temperature  $T_c$
- calculation of the equation of state

with physical quark masses in the continuum limit

- Gross features of bulk thermodynamics at low temperatures are compatible with hadron resonance gas thermodynamics;

- deviations from PDG-HRG provide

**evidence for a richer strange and charmed baryon spectrum than so-far known experimentally**