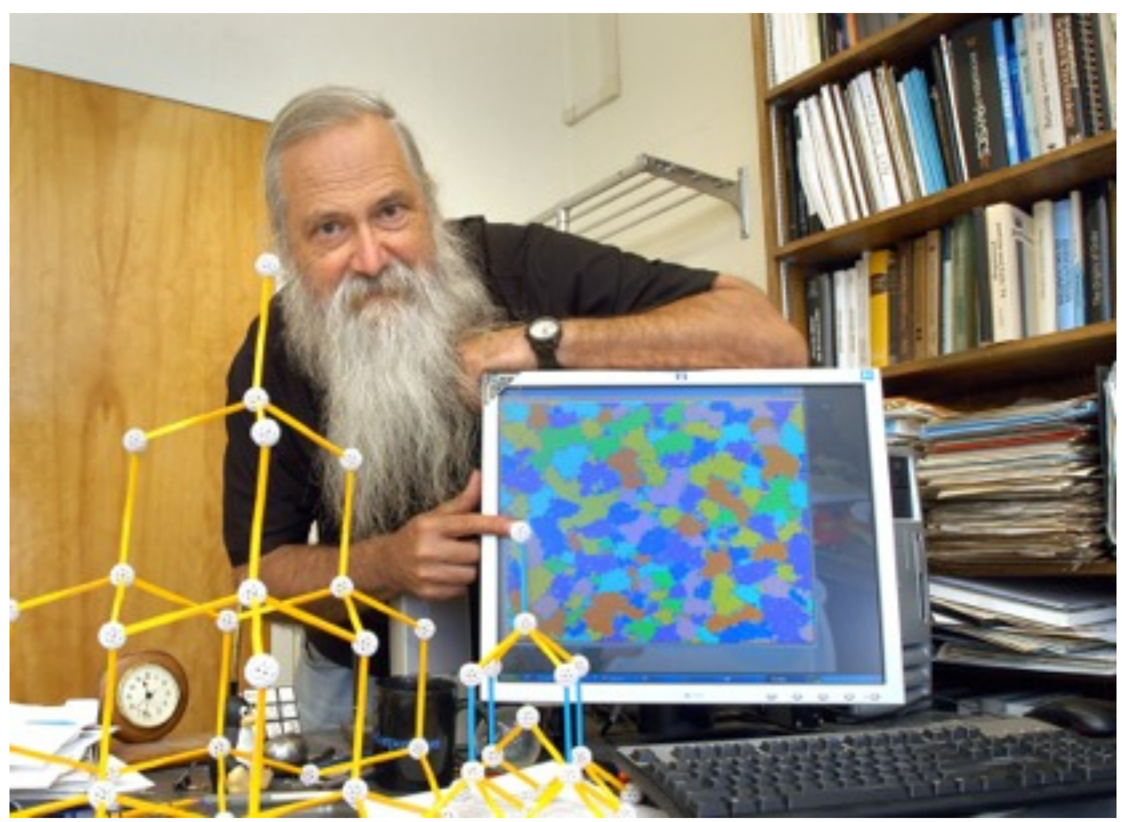
Applying chiral perturbation theory to LQCD: successes and challenges

Steve Sharpe
University of Washington



Outline

- Inspired by Mike
- Playing catch-up with Mike, round 1
- Playing catch-up with Mike, round 2
- Some open questions

Inspired by Mike

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  - There was no tex or email (or PCs)!
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  - This was fun, but there was a very unsatisfying model dependence
  - I wanted to **really** calculate nonperturbative properties in QCD
- So, I started reading preprints....
  - ... and was excited & inspired by the flood of papers on LQCD

Inspired by Mike

1979

BNL-26307

Monte Carlo Study of Abelian Lattice Gauge Theories

Michael Creutz, Laurence Jacobs* and Claudio Rebbi

Physics Department

Brookhaven National Laboratory

Upton, New York 11973

Abstract

Using Monte Carlo techniques, we study the thermodynamics of four-dimensional Euclidian lattice gauge theories, with gauge groups Z_N and U(1). For $N \leq 4$ the models exhibit a single first order phase transition while for $N \geq 5$ we observe two transitions of higher order. As N increases, one of these transitions moves toward zero temperature, whereas the other remains at finite temperature and survives in the U(1) limit. The behavior of the Wilson loop factor is also analyzed for the Z_2 and Z_6 models.

Inspired by Mike

1980

BNL 27752

Asymptotic Freedom Scales

Michael Creutz
Physics Department
Brookhaven National Laboratory
Upton, New York 11974

Abstract

Using Monte Carlo methods with Wilson's lattice cutoff, I calculate the asymptotic freedom scales of SU(2) and SU(3) gauge theories without quarks.

Inspired by Mike

1981

BNL 29840

Numerical Studies of Gauge Field Theories*

Michael Creutz
Physics Department
Brookhaven National Laboratory
Upton, Upton 11973

INTRODUCTION

Monte Carlo simulation of statistical systems is a well established technique of the condensed matter physicist. In the last few years, particle theorists have rediscovered this method and are having a marvelous time applying it to quantized gauge field theories. The main result has been strong numerical evidence that the standard SU(3) non-Abelian gauge theory of the strong interaction is capable of simultaneously confinging quarks into the physical hadrons and exhibiting asymptotic freedom, the phenomenon of quark interactions being small at short distances.

Inspired by Mike

1982

BNL 31488

Numerical Studies of Wilson Loops in
SU(3) Gauge Theory in Four Dimensions

Michael Creutz

Brookhaven National Laboratory
Upton, New York 11973

and

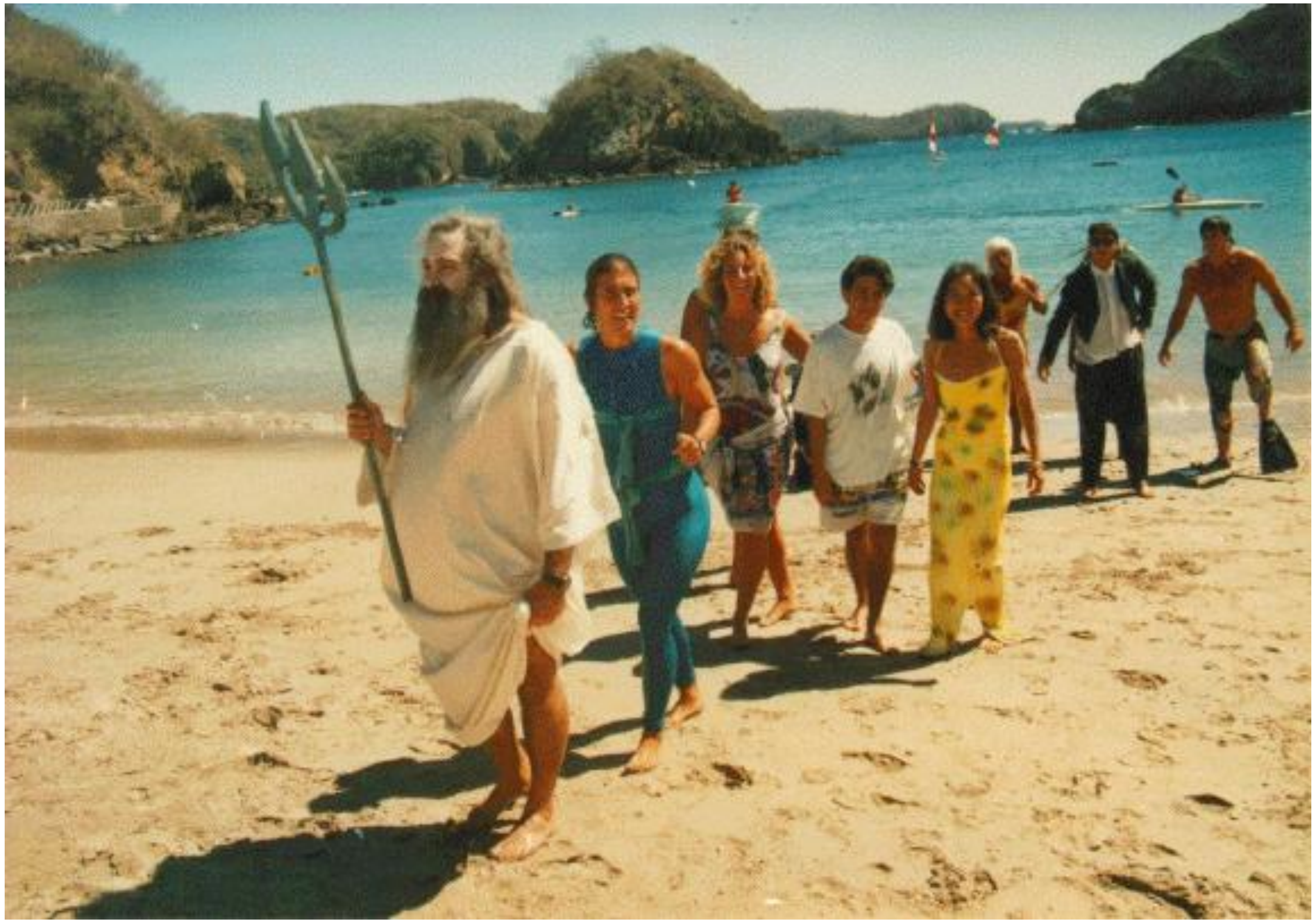
K. J. M. Moriarty

Department of Mathematics
Royal Holloway College
Englefield Green, Surrey, TW20 OEX, U.K.

Abstract

Monte Carlo simulations are used to calculate Wilson loops for pure SU(3) gauge theory on a 6^4 lattice. Previous measurements of the scale parameter Λ_o are improved.

Inspired by Mike



Inspired by Mike

[Creutz & Moriarty, 82]

Creutz ratios show confinement!

$\chi(I,J)$

$$\chi(I,J) \xrightarrow[I,J\to\infty]{} a^2\sigma$$

$$a^2 \approx e^{-1/(\beta_0 g^2)}$$

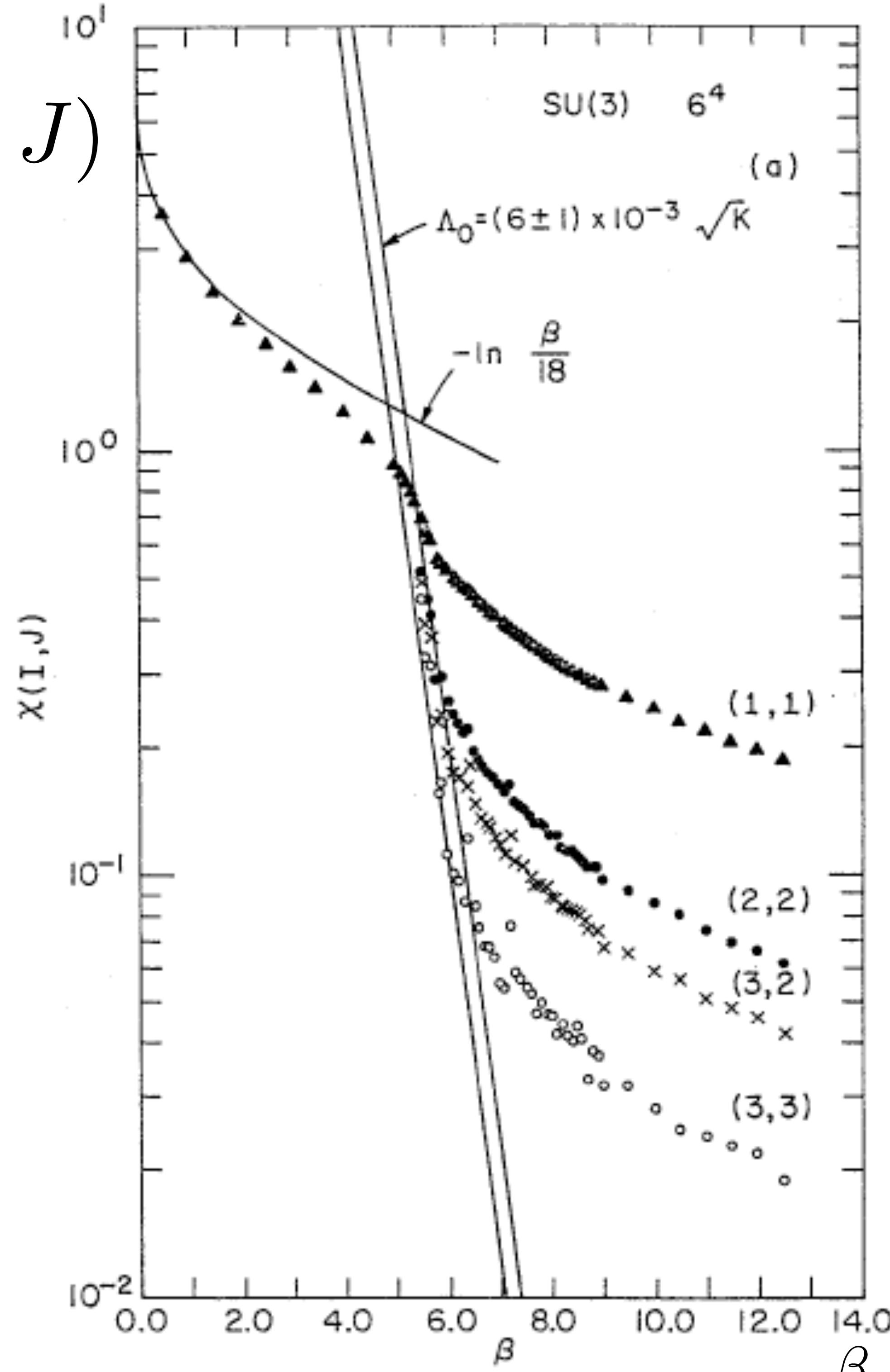


Fig. 4

$$\beta = \frac{6}{g^2}$$

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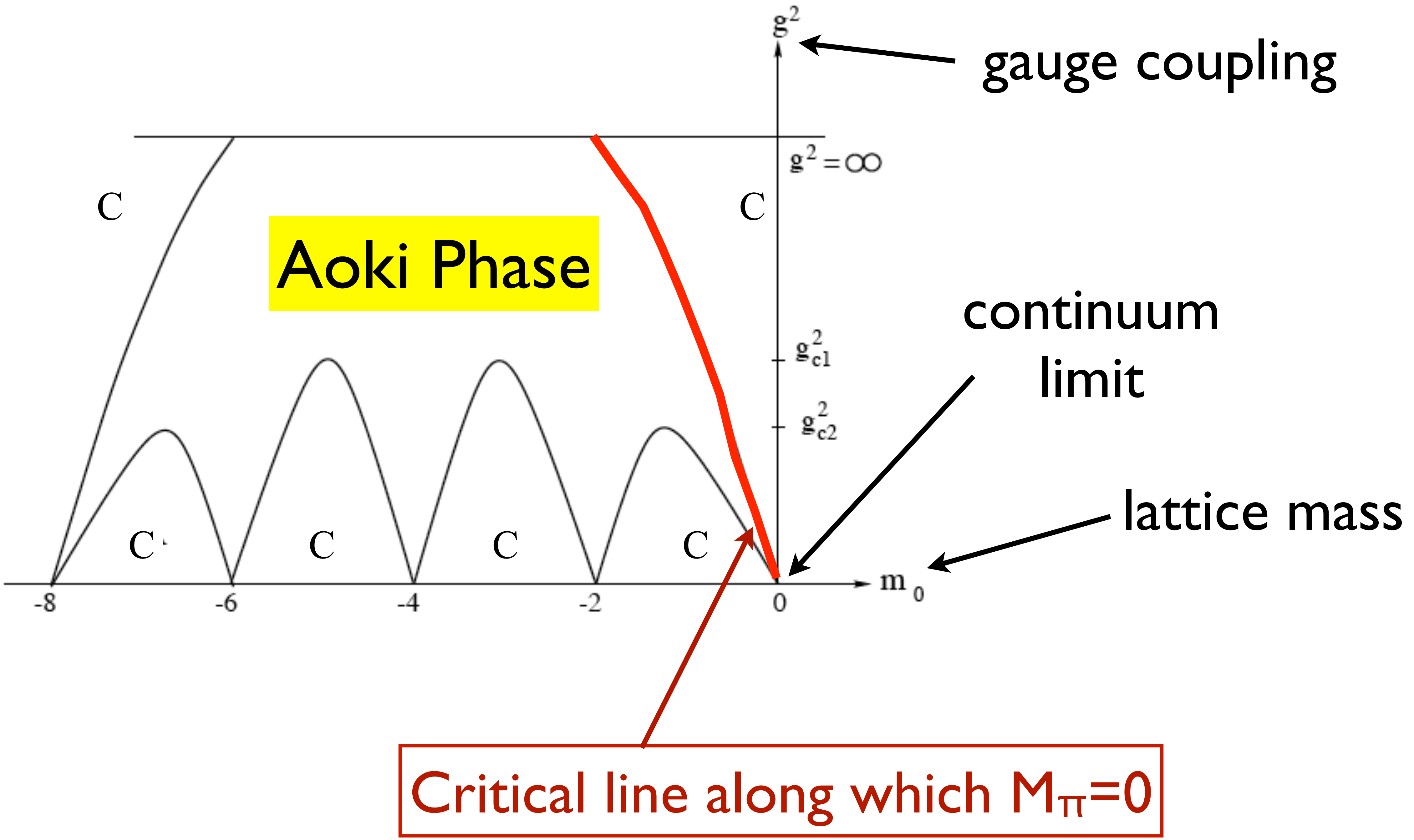
Playing catch-up, round 1

- Mike has long been interested in interconnections between lattice fermions, chiral symmetry, discretization errors, topology and the θ-term
- One example: how can one obtain a massless triplet of pions in Nf=2 lattice QCD with Wilson fermions, given the absence of chiral symmetry?
  - [Aoki 1989] proposed a solution: the Aoki phase (π_0 condensation)
  - Spontaneous flavor & parity (& CP) breaking on the lattice leads to exactly massless Goldstone bosons
  - Supported by arguments from strong coupling and by simulations

LQCD phase diagram (Wilson fermions)



Aoki vs 1st-order scenario

- [SS + Singleton, 98] analyzed the vacuum structure using "Wilson Chiral Perturbation Theory"
  - Low-energy effective theory of LATTICE QCD (with Wilson fermions)

$$\mathcal{L}_\chi^{(LO)} = \frac{f^2}{4}\langle\partial_\mu\Sigma\partial_\mu\Sigma^\dagger\rangle - \frac{f^2}{4}\chi\langle\Sigma+\Sigma^\dagger\rangle + \frac{c_2}{16}\langle\Sigma+\Sigma^\dagger\rangle^2$$

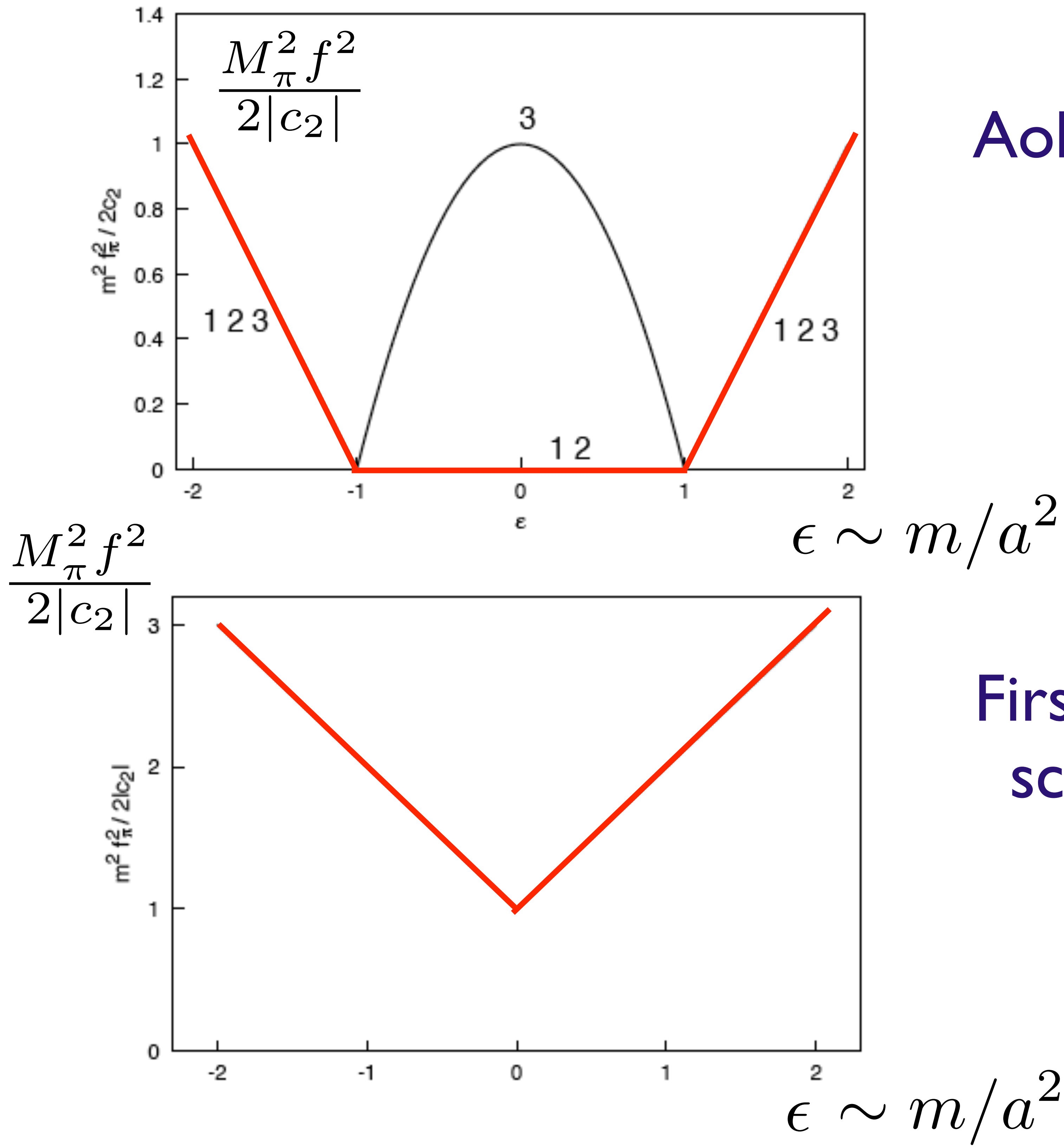
$\Sigma = \exp(i\vec{\pi}\cdot\vec{\tau}/f)$ $\quad 2\mathcal{B}_0\mathcal{M}$ $\quad \sim a^2\Lambda^6$

  - Sign of low-energy coefficient c_2 determines phase structure

Aoki vs 1st-order scenario

$c_2>0$ — Aoki-phase

$c_2<0$ — First-order scenario



1st-order scenario occurs!

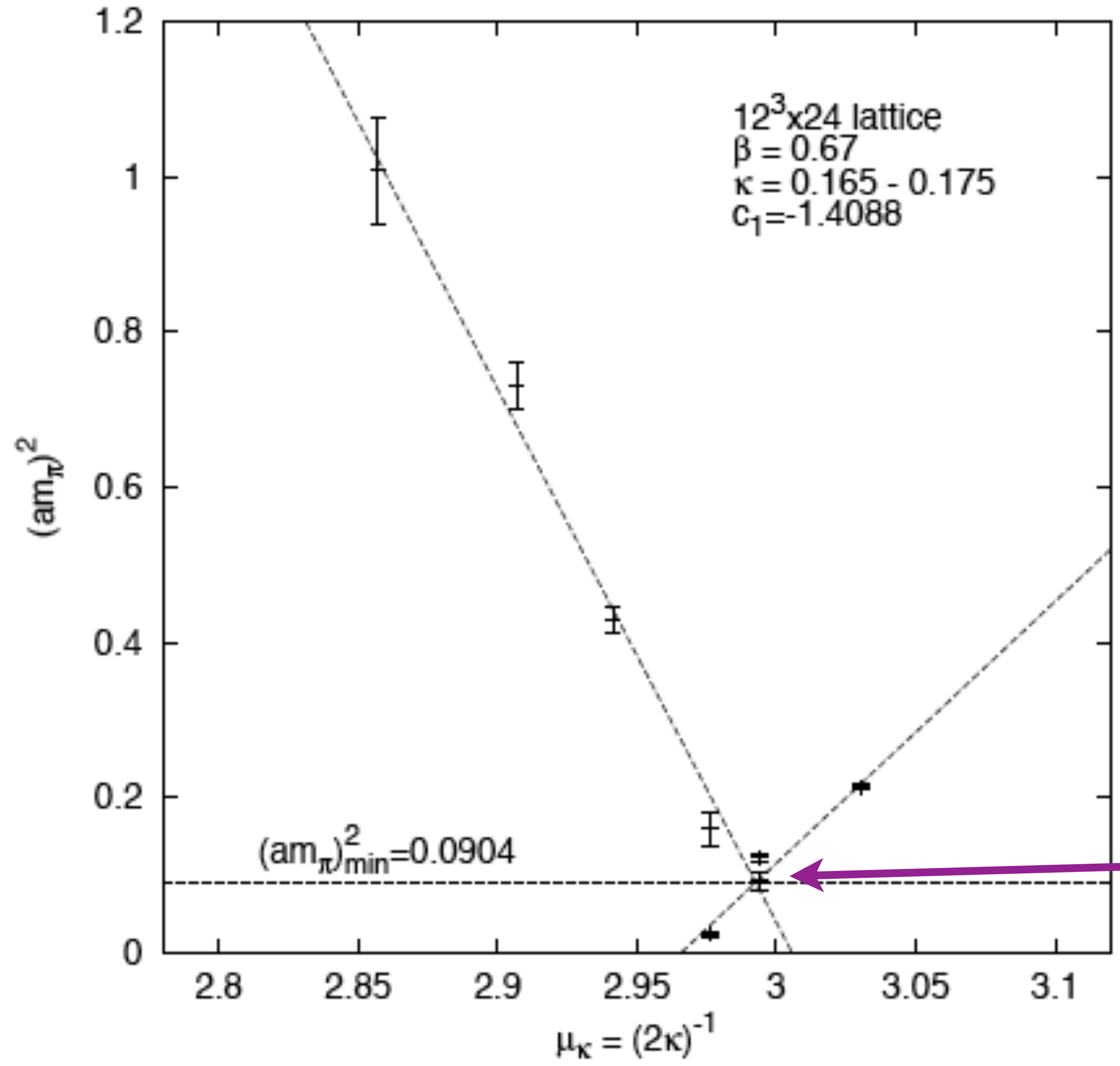


Figure 9. Unquenched results for $(am_\pi)^2$ as a function of $(2\kappa)^{-1} = m_0 + 4$ for $\mu = 0$ and with $a^{-1} \approx 0.2$ fm[60]. Straight lines are to guide the eye.

[Farchioni et al., 05]

a≈0.2fm

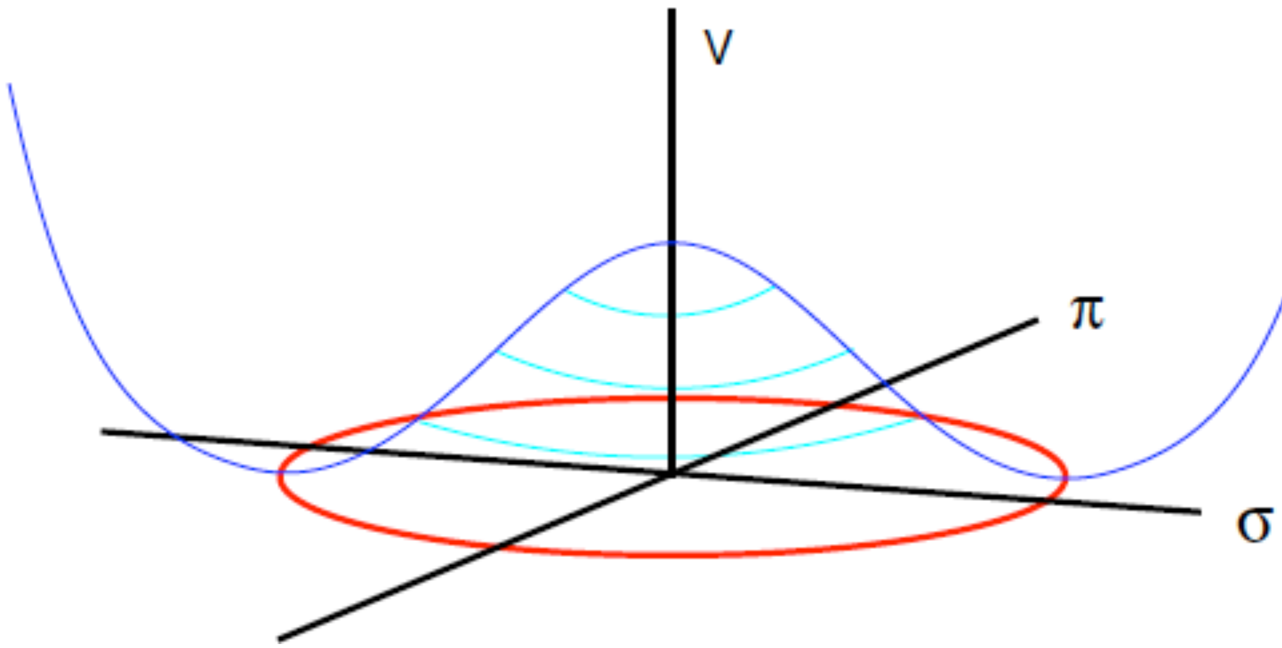
First-order scenario with minimum pion mass

Mike's '96 sigma-model analysis

[M. Creutz, hep/lat 9608024, "Wilson fermions at finite temperature"]

- Linear sigma model analysis, assuming correction $a^2\sigma^2$

$$V \sim \lambda(\sigma^2 + \vec{\pi}^2 - v^2)^2$$



Banana diagrams!

- Two possibilities from "quadratic warping" correspond exactly to two choices of sign of c_2

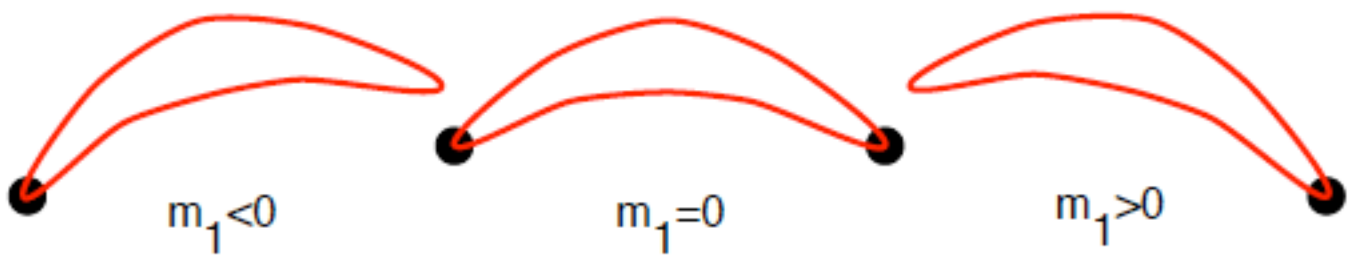


Fig. (2) The effect of a downward warping of the effective potential. The curve represents the warped bottom of the sombrero potential. Here m_1 represents the distance from the critical point. The solid circles represent possible states of the æther. The phase transition now occurs without a diverging correlation length.

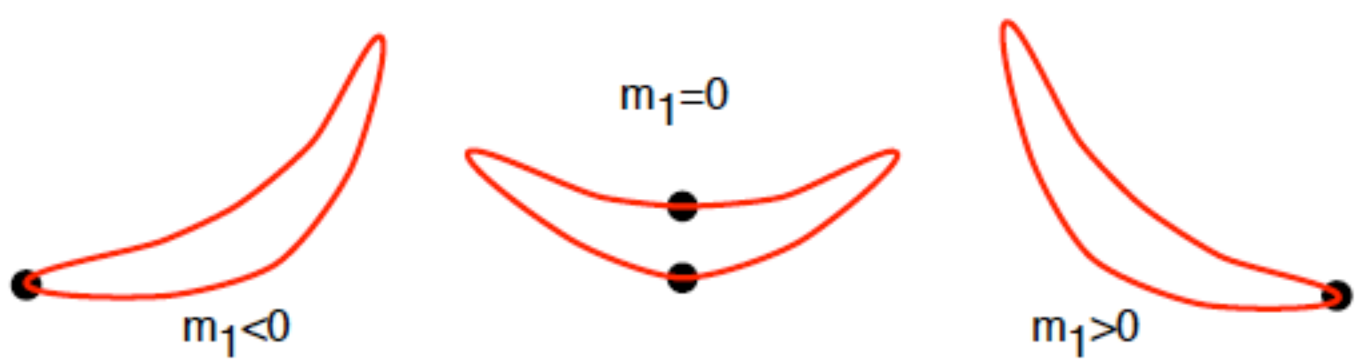


Fig. (3) The effect of an upward warping of the effective potential. Here m_1 represents the distance from $K_c(\beta)$. Now there are two phase transitions, with the intermediate phase having an expectation for the pion field.

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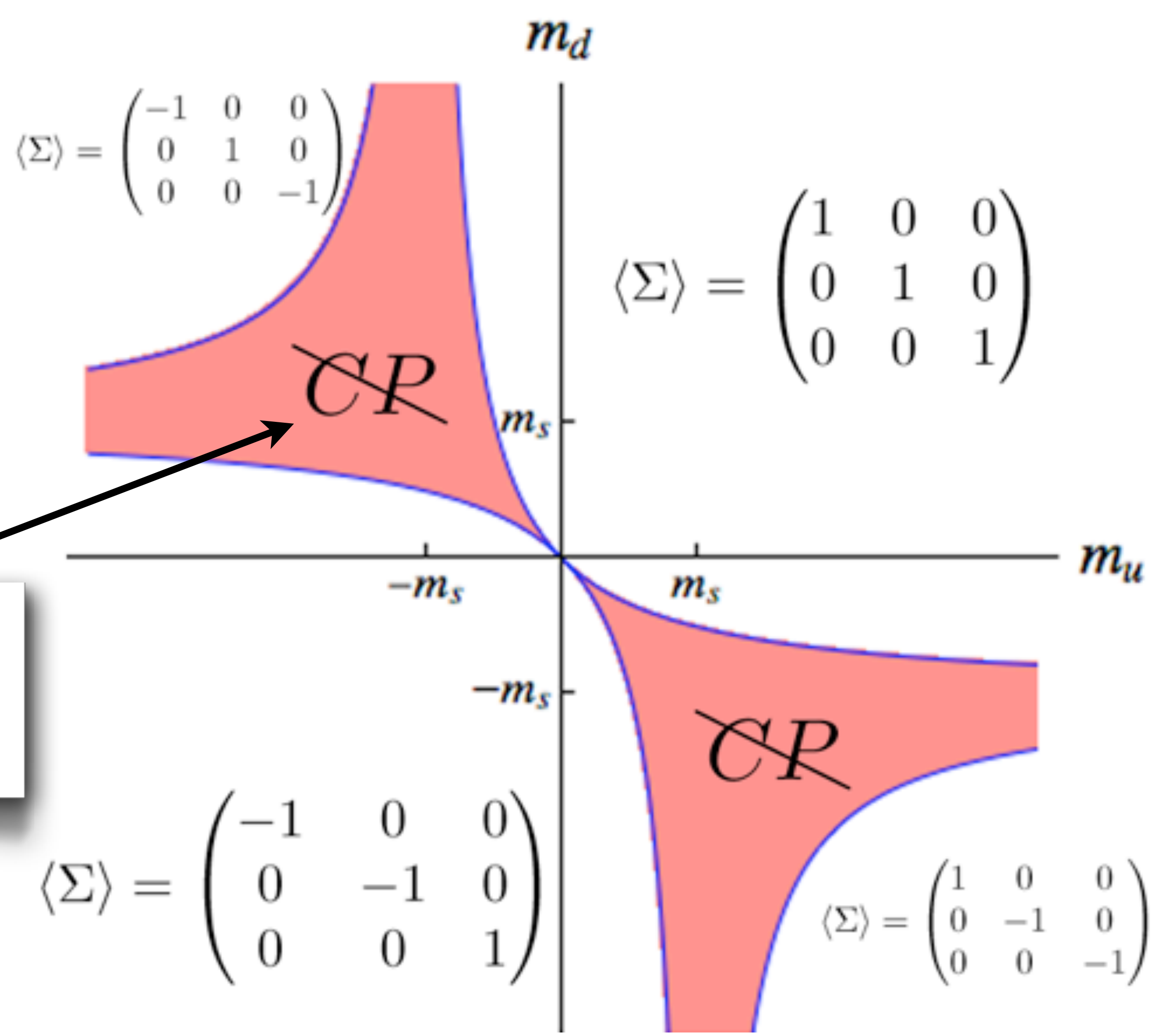
Playing catch-up, round 2

- $m_u \neq m_d$ leads to Dashen's CP-violating phase

[Creutz, 2004]

Prediction from leading-order SU(3) Chiral PT

$$\langle\Sigma\rangle = \begin{pmatrix} \exp i\phi & 0 & 0 \\ 0 & \exp i\psi & 0 \\ 0 & 0 & \exp -i(\phi+\psi) \end{pmatrix}$$



Playing catch-up, round 2

- Mike's question: What happens to the CP-violating phase in the presence of discretization errors?

Playing catch-up, round 2

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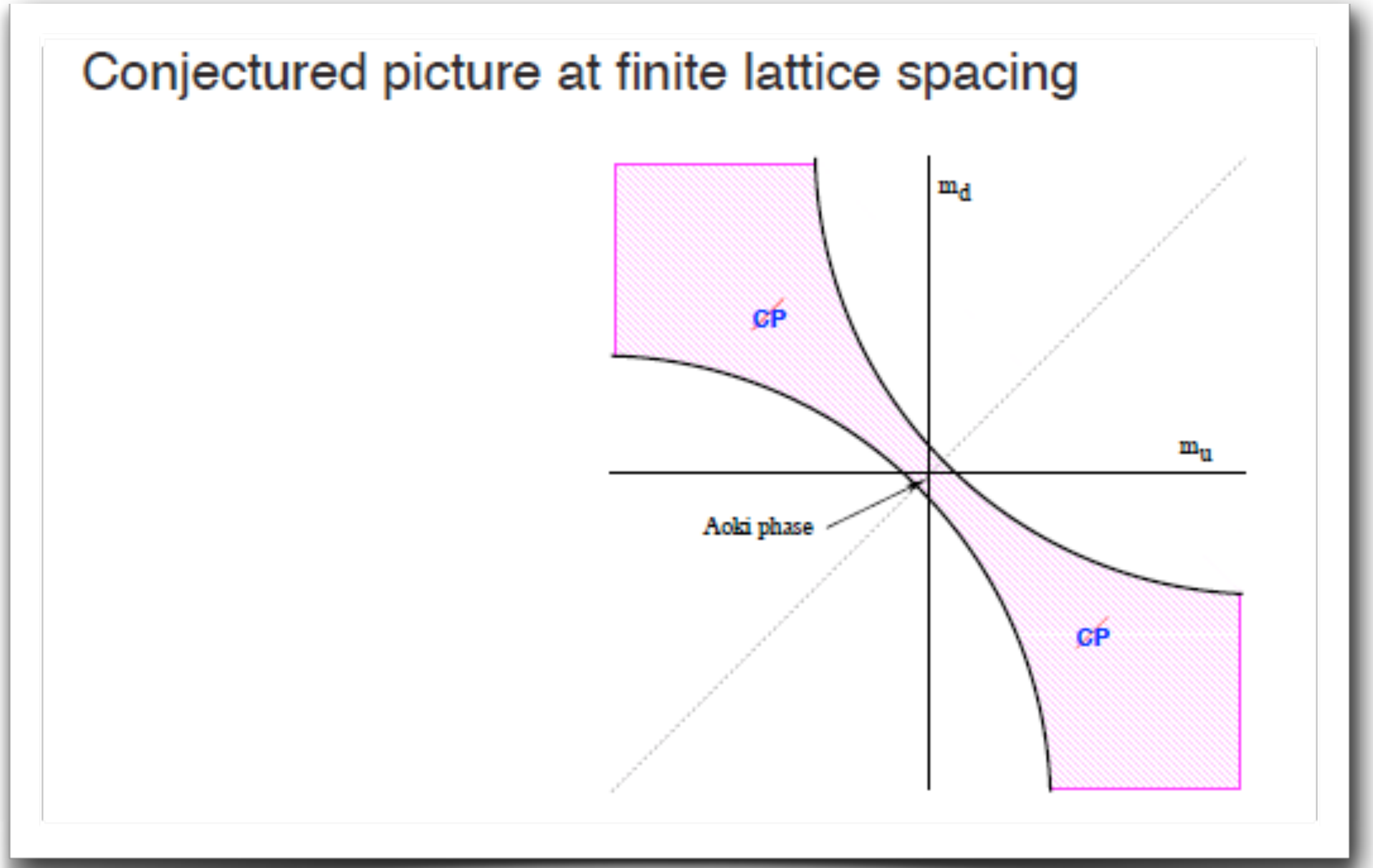
- Mike's conjectured answers

[Creutz, talks "Isospin breaking & the Aoki phase," 2013]



Playing catch-up, round 2

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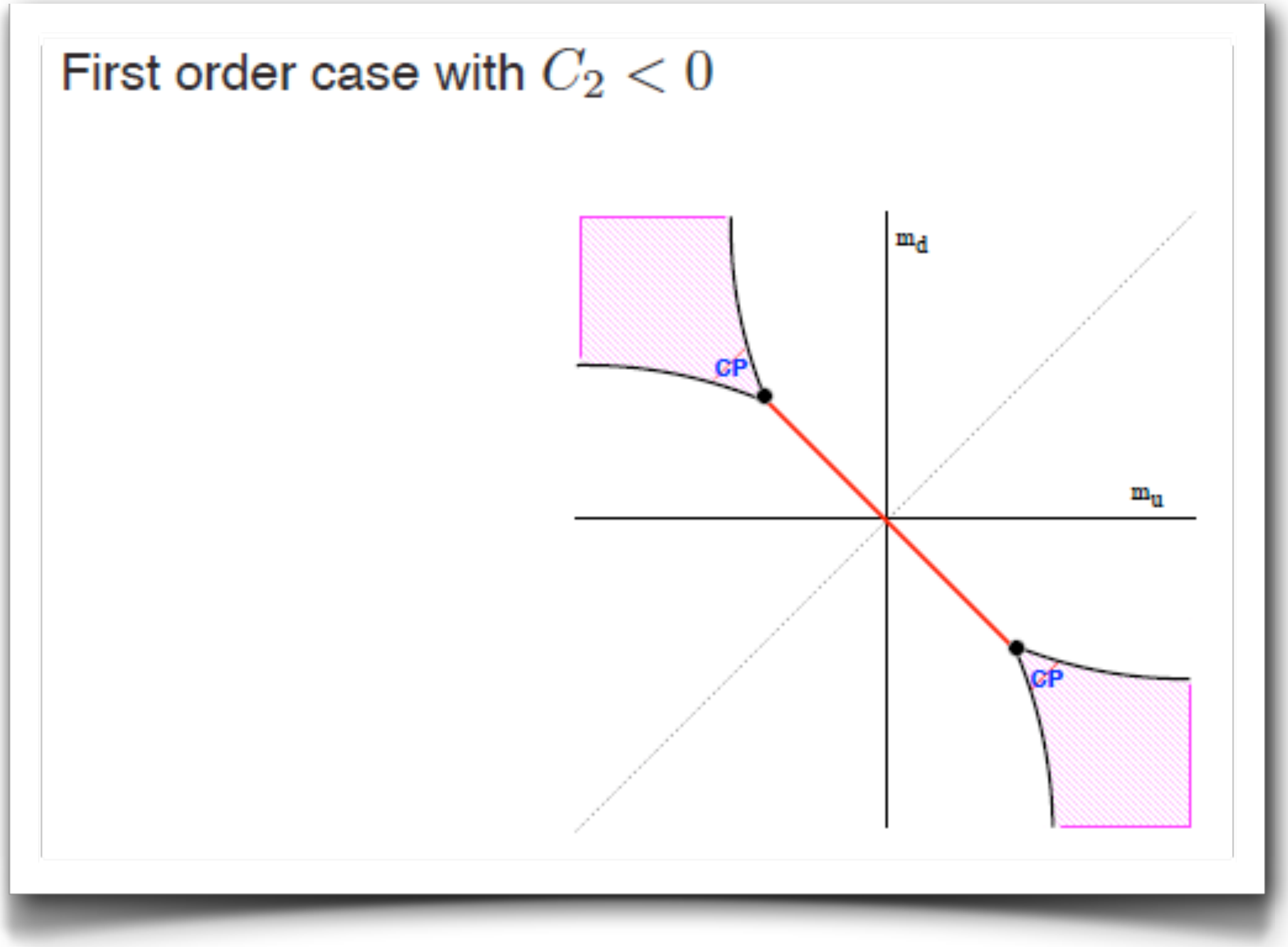


Confirming Mike's conjecture

[Derek Horkel, talk at Lattice 2014; Horkel & SS, arXiv:1409.xxxx]

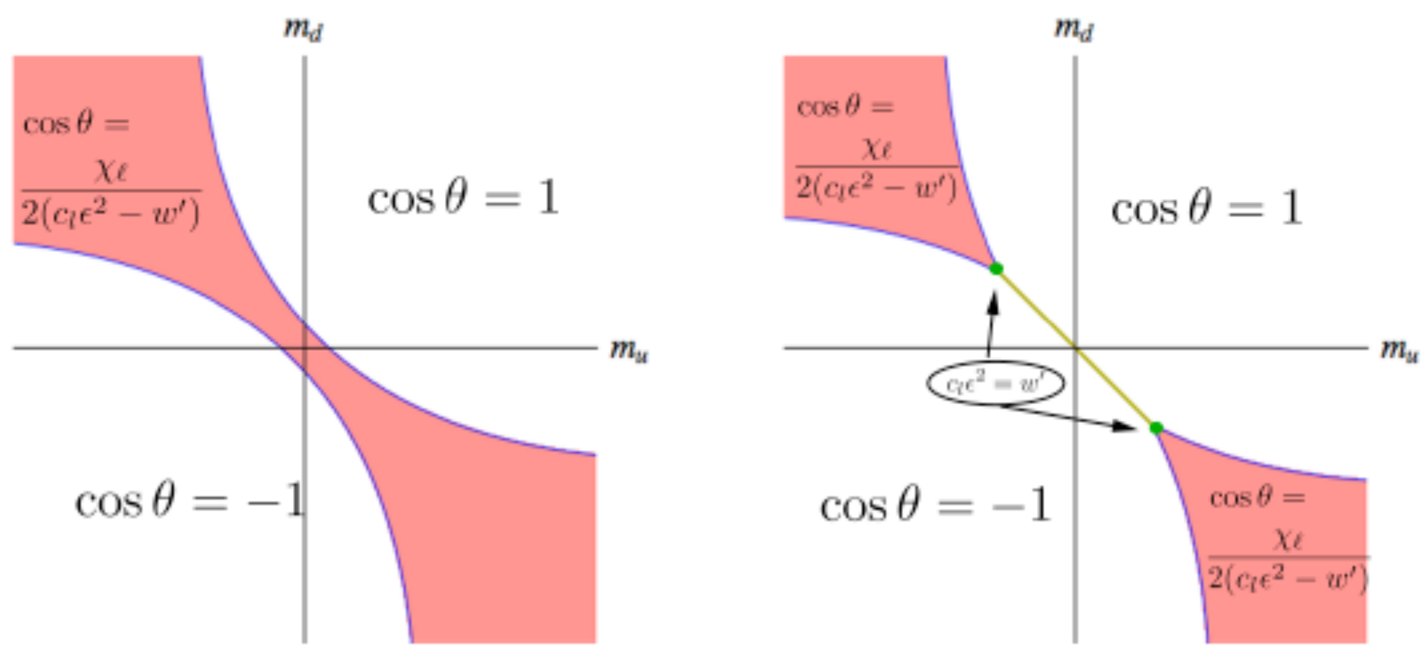
- Use SU(2) Wilson ChPT with NLO term obtained by matching with SU(3) LO ChPT (integrating out η)

$$\mathcal{V}_{a^2,\ell_7} = -\frac{f^2}{4}\mathrm{tr}(\chi^\dagger\Sigma + \Sigma^\dagger\chi) - W'[\mathrm{tr}(A^\dagger\Sigma + \Sigma^\dagger A)]^2 + \frac{\ell_7}{16}[\mathrm{tr}(\chi^\dagger\Sigma - \Sigma^\dagger\chi)]^2$$

$W' \propto -c_2$

$\frac{\ell_7}{16}[\mathrm{tr}(\chi^\dagger\Sigma - \Sigma^\dagger\chi)]^2 \propto (m_u - m_d)^2/m_s$

$$\langle\Sigma\rangle = e^{i\theta\tau_3}$$

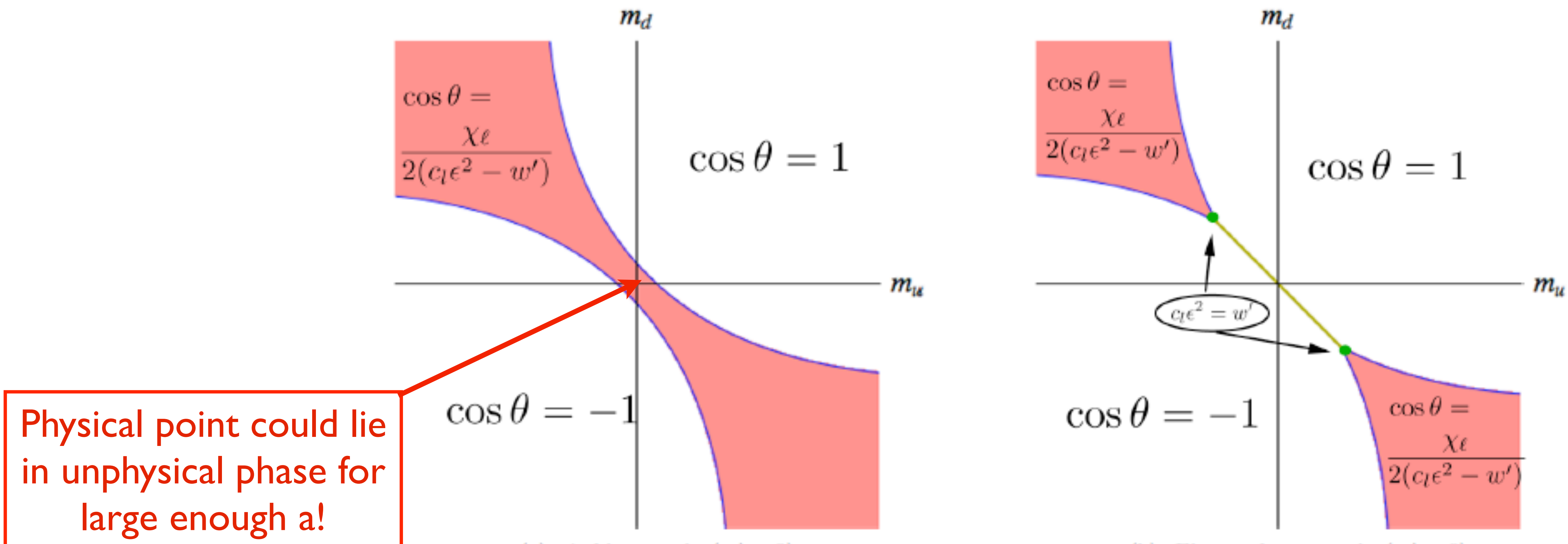


(a) Aoki scenario ($w' < 0$). (b) First-order scenario ($w' > 0$).

Relevant for simulations?

- Simulation frontier: physical quark masses including $m_u \neq m_d$ (& EM)
- How are these impacted by discretization errors ($1/a \approx 3$ GeV)?

$$m_u \approx 2.5\,\mathrm{MeV}, \quad m_d \approx 5\,\mathrm{MeV}, \quad a^2\Lambda^3_{\mathrm{QCD}} \approx 3\,\mathrm{MeV}$$



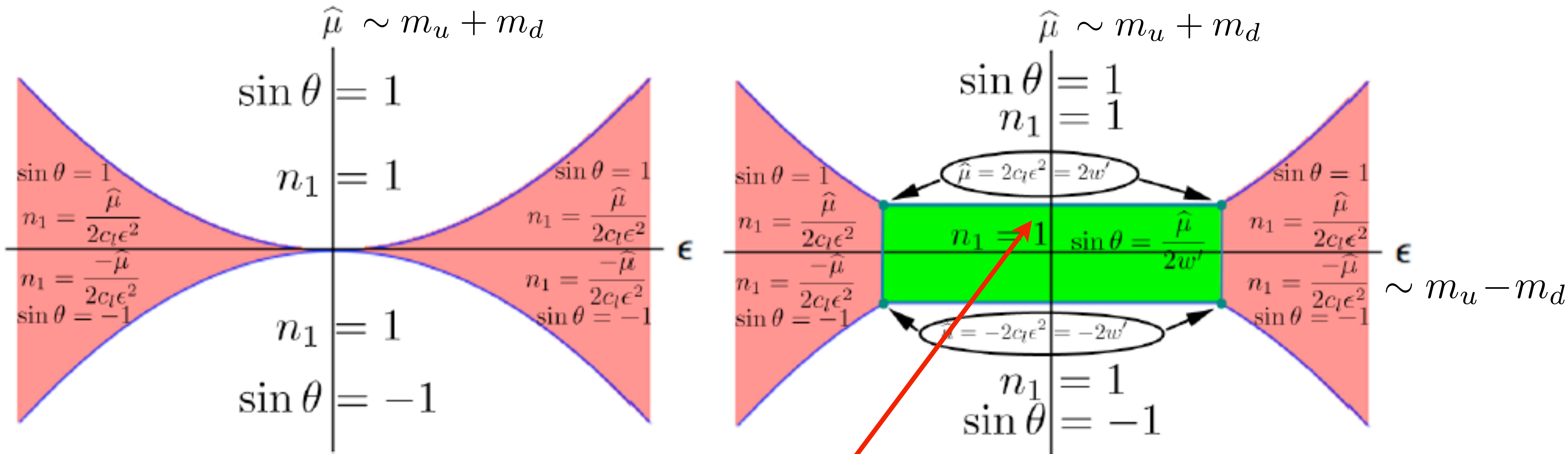
(a) Aoki scenario ($w' < 0$). (b) First-order scenario ($w' > 0$).

Wilson fermions

Maximally twisted fermions

- Role of two scenarios is interchanged

[Horkel & SS, 2014]



(a) Aoki scenario or continuum ($w' \leq 0$)

(b) First-order scenario ($w' > 0$)

Physical point could lie in unphysical phase for large enough a!

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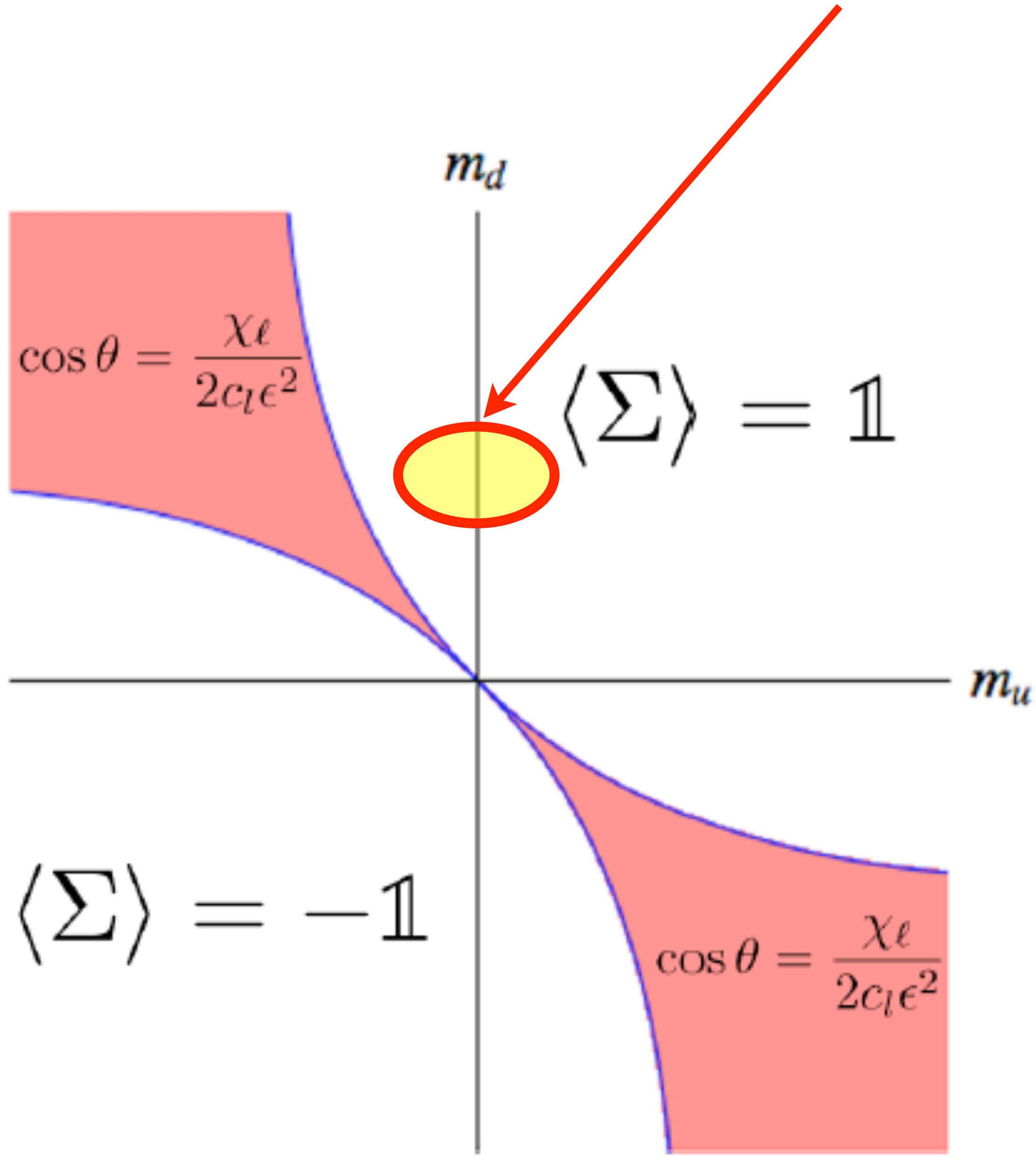
Resolvable open questions

- Focus on $m_u \approx 0$ with fixed $m_d > 0$



- No symmetry enhancement at m_u=0
- All physical quantities (m_π, condensate) are smooth at m_u=0
- For low-energy properties (scale $\ll m_\pi$) equivalent to N_f=1 QCD at $m \approx 0$

Resolvable open questions

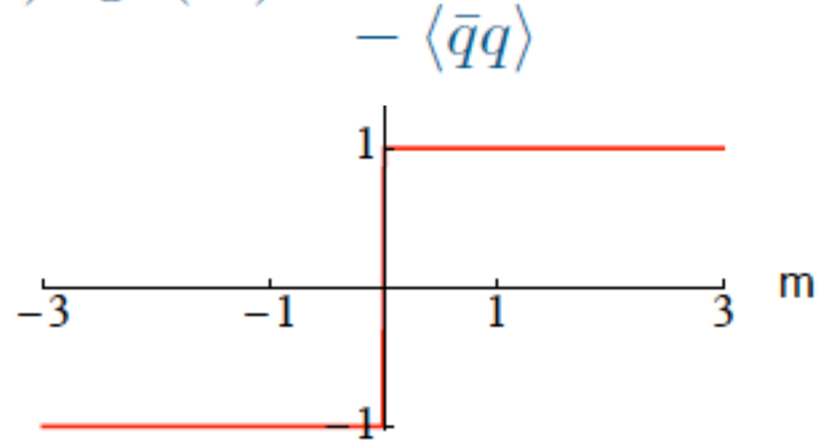
- Is m=0 in N_f=1 QCD universal in the continuum limit?
  - To resolve, calculate m_η/m_Δ using overlap fermions with two different kernels and take $a \to 0$ holding m=0
  - For a>0, there will be differences in m_η/m_Δ due to discretization errors (proportional to a^2)
  - Do differences remain when $a \to 0$?
  - Note that there are no massless particles so no IR issues
  - Can formulate question similarly for N_f=2 with m_u=0

Resolvable open questions

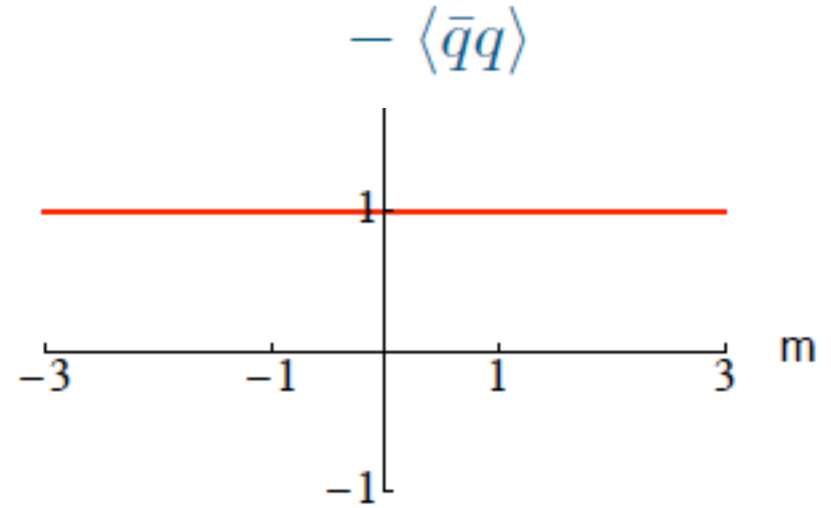
- Does the density of (slightly non-zero) eigenvalues of the Dirac operator vanish when m→0 in N_f=1 QCD?
  - Might expect to vanish from Banks-Casher, since [Creutz, 2006]

$$-\langle \bar{q}q \rangle \underset{m\to 0}{=} \frac{\pi}{V}\rho(0)\mathrm{sign}(m)$$

apparently implies



which is inconsistent with prediction from ChPT



Resolvable open questions

- Does the density of (slightly non-zero) eigenvalues of the Dirac operator vanish when $m \to 0$ in $N_f=1$ QCD?
  - But Banks-Casher fails for $m<0$ (ρ not positive) [Leutwyler & Smilga, Verbaarschot, ...]
  - [Verbaarschot, Lat14] $\rho(0) \neq 0$ is consistent with ChPT, and indeed can be calculated in ε-regime
  - [Degrand et al., 2006] Small eigenvalues agree with ε-regime predictions which assume $\rho(0) \neq 0$

Resolvable open questions

- Does the density of (slightly non-zero) eigenvalues of the Dirac operator vanish when $m \to 0$ in $N_f=1$ QCD?
    - Why do we care?
    - Because if $\rho(0)=0$ then partially-quenched ChPT fails

- Thanks to Mike for inspiration and many lively and stimulating (and ongoing) discussions!