Chiral Symmetry and CP violation

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CreutzFest
Sep 4, 2012

Work done in collaboration with V. Cirigliano, B. Yoon, E. Mereghetti, R. Gupta.

Mass Matrix
Standard Model

Standard model CP violation from

1. Cabibo-Kobayashi-Maskawa mixing

$$\prod(m_i^{(u)} - m_j^{(u)}) \prod(m_i^{(d)} - m_j^{(d)}) \prod \sin 2\theta_{ij} \; (\cos\theta_{13} \sin\delta)$$

2. QCD Θ term

$$\left(\sum \frac{1}{m_i}\right)^{-1} \sin\Theta$$

Both require violation of chiral symmetry:

1. The CKM violation is a mismatch between mass and interaction bases.
2. The Θ term is related to the anomaly.

Mass Matrix

One-flavor theory

A general one flavor QCD

$$\mathcal{L} = \text{Kinetic terms} + \bar{\psi}(\mathcal{R}e\, m + i\gamma_5\, \mathcal{I}m\, m)\psi + \frac{\alpha_s}{8\pi}\Theta G\tilde{G}$$

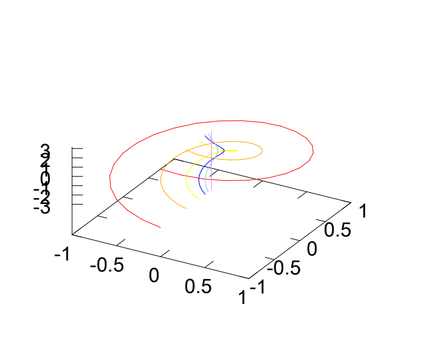
Parameter space three-dimensional: $\mathbb{R}^2 \times \mathbb{S}^1$.

Phase transformation $\psi \to \exp(i\alpha\gamma_5)\psi$ changes

$$m \to m e^{i\alpha}, \qquad \Theta \to \Theta - \alpha\,.$$

This is a *free, smooth, proper* action of $\mathrm{U}(1)$ group.
So, the quotient space $\mathbb{R}^2 \times \mathbb{S}^1/\mathrm{U}(1)$ is a smooth manifold.

The orbits of $\mathrm{U}(1)$ are spirals,
degenerating smoothly into a straight line.



The quotient space is just the $\mathbb{R}^2$ given by $(\mathcal{R}e\, m, \mathcal{I}m\, m)$

Mass Matrix

Two-flavor theory

The mass term in two flavor theory is

$$m_S + i\gamma_5 m_P + m_V \tau_3 + i\gamma_5 m_A \tau_3 \,.$$

Now, there are two chiral transformations:

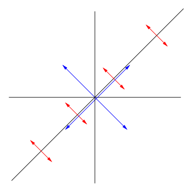
$$\begin{aligned}
(m_S + i m_A) &\rightarrow e^{i\alpha_{NA}}(m_S + i m_A) \\
(m_P + i m_V) &\rightarrow e^{i\alpha_{NA}}(m_P + i m_V) \\
(m_S + i m_P) &\rightarrow e^{i\alpha_A}(m_S + i m_P) \\
(m_A + i m_V) &\rightarrow e^{i\alpha_A}(m_A + i m_V) \\
\Theta &\rightarrow \Theta - 2\alpha_A
\end{aligned}$$

And a vector transformation:

$$m_V \rightarrow -m_V \qquad m_A \rightarrow -m_A$$

The appropriate quotient space is now $\mathbb{R}^4 \times \mathbb{S}^1/\mathrm{U}(1)^2 \times \mathbb{Z}_2$.

The $\mathrm{U}(1)_{NA}$ can be used to remove m_A. Then $\mathrm{U}(1)_A$ can be used to remove Θ. What is then left is $\mathbb{R}^3/\mathbb{Z}_2^2$. There is a coordinate singularity at $m_S = 0$.



Two singularities:

1. The plane $m_V = 0$: Isospin symmetry. (Also $m_S = 0$.)
2. The point $m_S = m_V = m_P = 0$: Nonanomalous chiral symmetry.

Mass Matrix

One zero mass plane

Note that $m_S = m_V,\ m_P = 0$ is not singular. This corresponds to $m_u = 0, \mathcal{I}m\, m_d = 0, \mathcal{R}e\, m_d \neq 0$.

- A generic point has two independent CP-conserving deformations: $\delta m_S, \delta m_V$, and one CP-violating δm_P.
- At the isospin-symmetric plane $\delta m_V \equiv -\delta m_V$.
- At the nonanomalous chiral symmetric point, $(\delta m_V, \delta m_S, \delta m_P) \equiv -(\delta m_V, \delta m_S, \delta m_P)$.
- At most points, $\delta\Theta$ can be chosen instead of δm_P. Not at $m_u = 0$. Not a singularity!

- Why doesn't $\delta\Theta$ violate CP when $m_u = 0$?
- CP interchanges left and right quarks.
- Chiral rotation on left and right quarks unequal.
- CP does not commute with chiral rotation.
- Outer automorphism: $\mathrm{CP}_\chi \equiv \chi^{-1} C P \chi$ is also a "CP."
- $\delta\Theta$ breaks most of the CP_χ, but not all.
- When $m_u = 0$, one CP_χ preserved by $\delta\Theta$.
- At a generic point, a linear combination of $\delta\Theta$ and δm_P does the same.

Standard Model
Vacuum Structure

Which CP_χ are we interested in?

Start with a chirally symmetric theory.
All the CP_χ are good symmetries of the Lagrangian.
Vacuum is degenerate, each vacuum spontaneously breaks almost all CP_χ.

When chiral symmetry is explicitly broken, vacuum mixes with the pseudo Goldstone bosons in degenerate perturbation theory.
To avoid this, start perturbation around the ‘correct’ vacuum.

In particular, if masses are the only chiral symmetry breaking,

$$\langle\Omega \mid m_u^5 \bar\psi_u i\gamma_5 \psi_u + m_d^5 \bar\psi_d i\gamma_5 \psi_d \mid \pi_{uu} - \pi_{dd}\rangle = 0\,.$$

Let

$$\langle\Omega \mid \bar\psi_u i\gamma_5 \psi_u \mid \pi_{uu}\rangle \equiv \delta_S^0 \qquad \langle\Omega \mid \bar\psi_u i\gamma_5 \psi_u \mid \pi_{dd}\rangle \equiv \delta_V^0\,.$$

Then

$$(m_u^5 - m_d^5)(\delta_S^0 - \delta_V^0) = 0\,,$$

which is the reason for our usual phase choice $m_A = 0$. With different choice, vacuum condensate not $\bar\psi\psi$ and $\pi \neq \bar\psi\gamma_5\psi$.

Standard Model
Invariant

Starting from an arbitrary phase choice, we rotate by χ_u and χ_d, such that

$$\begin{aligned} \mathcal{I}m[(m_u + i m_u^5) e^{i\chi_u}] &= \mathcal{I}m(m_d + i m_d^5) e^{i\chi_d}] \\ \Theta + \chi_u + \chi_d &= 0\,. \end{aligned}$$

Let us denote

$$\begin{aligned} k &\equiv \mathcal{I}m[(m_u + i m_u^5) e^{i\chi_u}] \\ |m_{u,d}| e^{i\phi_{u,d}} &\equiv m_{u,d} + i m_{u,d}^5\,. \end{aligned}$$

This can be solved, and only CP violation through

$$k = -\frac{2\bar{m}\sin\frac{\bar{\Theta}}{2}}{\sqrt{1+\epsilon_d^2\tan^2\frac{\bar{\Theta}}{2}}} \approx \bar{m}\bar{\Theta}\,,$$

where

$$\frac{1}{\bar{m}} = \frac{1}{|m_u|} + \frac{1}{|m_d|} \qquad \epsilon_d = \frac{|m_u| - |m_d|}{|m_u| + |m_d|} \qquad \bar{\Theta} = \Theta - \phi_u - \phi_d\,.$$

BSM
Notation

In general, there are many operators that break chiral symmetry. Let

$$\mathcal{L} \supset d_i^\alpha O_i^\alpha$$

where i is flavor index, and α is operator index.
Let

$$\delta_S^\alpha = \langle \Omega \mid \mathcal{I}m\, O_i^\alpha \mid \pi_{ii} \rangle \qquad \delta_V^\alpha = \langle \Omega \mid \mathcal{I}m\, O_i^\alpha \mid \pi_{jj} \rangle \,.$$

Let

$$|d_i| \exp i\phi_i \equiv d_i \equiv \frac{\sum_\alpha d_i^\alpha (\delta_S^\alpha - \delta_V^\alpha)}{\sum_\alpha (\delta_S^\alpha - \delta_V^\alpha)}$$

be the weighted sum of the chiral violating coefficients.
If $|d_i| = 0$ due to cancellation, may need to go beyond this order.

BSM
General Solution

Then to find the CP violating coefficients, we solve the auxiliary equation:

$$\Theta - \sum \phi_i + \sum \sin^{-1} \frac{k}{|d_i|} = 0 \,.$$

In terms of this, the invariant coefficient of $\mathcal{I}m\, O_i^\alpha$ is

$$-k\,\mathcal{R}e\,\frac{d_i^\alpha}{d_i} - \sqrt{|d_i|^2 - k^2}\,\mathcal{I}m\,\frac{d_i^\alpha}{d_i} \approx \bar{d}\bar{\Theta}\,\mathcal{R}e\,\frac{d_i^\alpha}{d_i} - |d_i|\,\mathcal{I}m\,\frac{d_i^\alpha}{d_i}\,,$$

where

$$\frac{1}{\bar{d}} = \sum \frac{1}{|d_i|}$$

BSM
Conclusions

The vacuum chiral condensate picks a direction depending on all the chiral violation in the theory.

If the masses are the major determinants of chiral violation, rest of teh chiral violation not degenerate perturbation. One can read off CP violation in the other terms.

Otherwise, one needs chiral rotations to fix the chiral phase.

Lesson: Calculations need to keep mass.