The neutron-proton mass difference

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Outline

Introduction

Isospin symmetry:
'up' and 'down' quarks have identical properties (mass,charge)
$M_n = M_p, \quad M_{\Sigma^+} = M_{\Sigma^0} = M_{\Sigma^-}, \quad$ etc.

The symmetry is explicitly broken by
- up, down quark mass difference
- up, down quark electric charge difference

The breaking is large on the level of quarks ($m_d/m_u \approx 2$)
but small (typically sub-percent) compared to hadronic scales.

These two competing effects provide the tiny M_n-M_p mass difference $\approx 0.14\%$ is required to explain the universe as we observe it

Big bang nucleosynthesys and nuclei chart

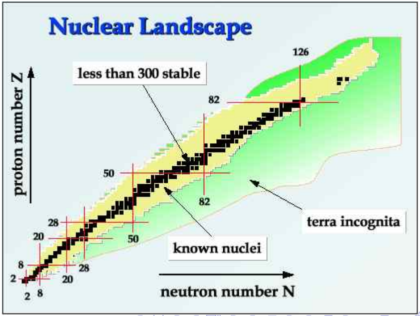
if $\Delta m_N < 0.05\%$ $\rightarrow$ inverse β decay leaving (predominantly) neutrons
$\Delta m_N \gtrsim 0.05\%$ would already lead to much more *He* and much less *H*
$\rightarrow$ stars would not have ignited as they did

if $\Delta m_N > 0.14\%$ $\rightarrow$ much faster beta decay, less neutrons after BBN
burinng of *H* in stars and synthesis of heavy elements difficult

The whole nuclei chart is based on precise value of Δm_N

Could things have been different?

Jaffe, Jenkins, Kimchi, PRD 79 065014 (2009)



The challenge of computing $M_n - M_p$ (on the 5σ level)

Unprecedented precision is required

$\Delta M_N/M_N = 0.14\% \rightarrow$ sub-permil precision is needed to get a high significance on ΔM_N

$m_u \neq m_d$ $\rightarrow$ 1+1+1+1 flavor lattice calculations are needed $\rightarrow$ algorithmic challenge

(Previous QCD calculations were typically 2+1 or 2+1+1 flavors)

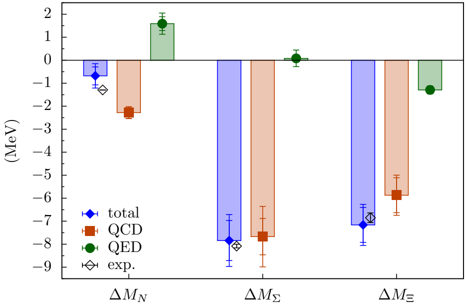
Inclusion of QED: no mass gap

$\rightarrow$ power-like finite volume corrections expected

$\rightarrow$ long range photon field may cause large autocorrelations

Electroquenched results

Isospin breaking effects can be included in the quenched approximation (only in the measurements)



Budapest-Marseille-Wuppertal, PRL 111 (2013) 252001

much higher precision/accuracy (we aim for 5σ) is needed: hard
usually similar systematic/statistical errors (no use improving on one)
reduce systematics by a factor of 5, increase statistics by $\times 25$

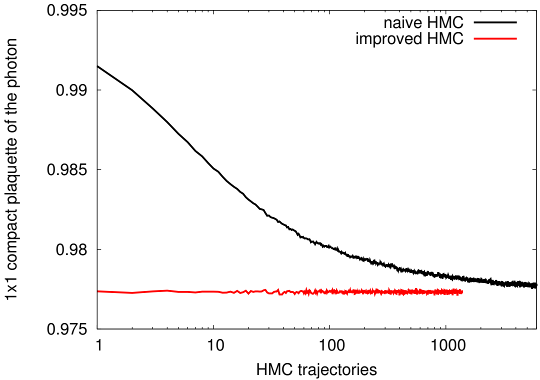
extension steps for a fully realistic theory

1. include dynamical charm:
usually easy since existing codes can include many fermions
since m_c is quite heavy it is computationally cheap
one needs small lattice spacings to have am_c small enough

2. include QED:
difficult, since the action/algorithmic setup must be changed
conceptual difficulties for finite V, since QED is not screened
additional computational costs are almost negligable

3. include $m_u \neq m_d$ (similarly large effect as QED):
usually easy since existing codes can include many fermions
$m_u \approx m_d/2$: more CPU-demanding than 2+1 flavors
since m_u is small larger V needed to stabilize the algorithm:
more CPU but large V (upto 8 fm) is good for other purposes

Autocorrelation of the photon field



Standard HMC has $\mathcal{O}(1000)$ autocorrelation

Improved HMC has none

Small coupling to quarks introduces a small autocorrelation

Ensambles

strategy to tune to the physical point: 3+1 flavor simulations
pseudoscalar masses: $M_{\bar{q}q} = 410$ MeV and $M_{\bar{c}c} = 2980$ MeV
lattice spacings was determined by using $w_0 = 0.1755$ fm (fast)
for the final result a spectral quantity, M_Ω was used

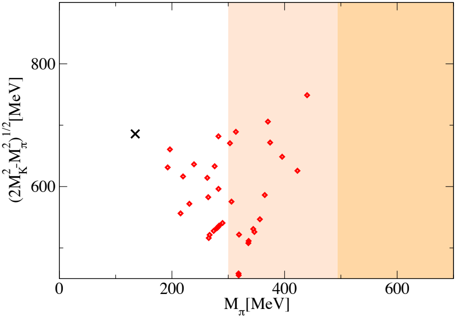
series of $n_f = 1 + 1 + 1 + 1$ runs: QCDSF strategy
decreasing $m_{u/d}$ & increasing m_s by keeping the sum constant
small splitting in the mass of the up and down quarks
$\Longrightarrow$ 27 neutral ensembles with no QED interaction: e=0

turning on electromagnetism with $e = \sqrt{4\pi/137}, 0.71, 1$ and 1.41
significant change in the spectrum $\Rightarrow$ we compensate for it
additive mass: connected $M_{\bar{q}q}$ same as in the neutral ensemble
$\Longrightarrow$ 14 charged ensembles with various L and e
four ensembles for a large volume scan: L=2.4 ... 8.2 fm
five ensembles for a large electric charge scan: e=0 ... 1.41

Lattice spacings and pion masses

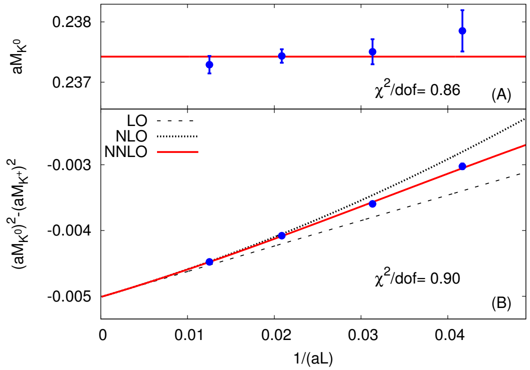
final result is quite independent of the lattice spacing
$\Longrightarrow$ four lattice spacings with a=0.102, 0.089, 0.077 and 0.064 fm

even the pion mass dependence is –surprisingly– quite weak
41 ensembles with M_π=195–440 MeV (various cuts)



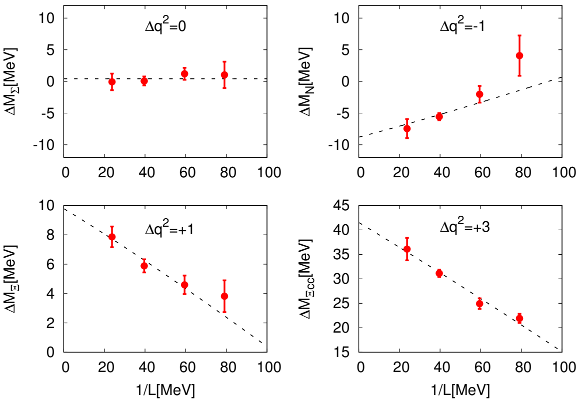
large parameter space: helps in the Kolmogorov-Smirnov analysis

Finite V dependence of the kaon mass



Neutral kaon shows no volume dependence
Volume dependence of the K splitting is perfectly described
$1/L^3$ order is significant

Finite V dependence of baryon masses



Σ splitting (identical charges) shows no volume dependence
V dependence of all baryons is well described by the universal part
$1/L^3$ order is insignificant for the volumes we use

Electric charge: signal/noise problem

symmetric operators under charge conjugation: depends on e^2
on a given gauge configuration (or on the level of the action):
no such symmetry, linear contribution in e
signal is proportional to e^2, whereas the noise is of $O(e)$

on electro-quenched configurations there is an elegant solution:
use a charge +e and a charge -e for the measurements
in the sum $O(e)$ parts drop out and only the quadratic remains
(the QED field generation has the +e versus -e symmetry)

for electro-unquenched configurations: no +e versus -e symmetry
dynamical configurations do feel the difference between up/down
due to their different charges they feel the QED field differently
small but important effect (we look for sub permil predictions)

Take couplings larger than 1/137

simulate at couplings that are larger than the physical one:
in such a case the signal outweighs the noise
precise mass and mass difference determination is possible

for e=0 and $m_u = m_d$ we know the isospin splittings exactly
$\Longrightarrow$ they vanish, because isospin symmetry is restored
$\alpha = e^2/4\pi \gg 1/137$ and e=0 can be used for interpolation

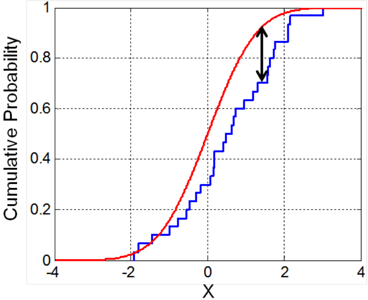
this setup will be enough to determine the isospin splittings
leading order finite volume corrections: proportional to α
leading order QED mass-splittings: proportional to α
no harm in increasing α, only gain (renormalization)

(perturbative Landau-pole is still at a much higher scale:
hundred-million times higher scale than our cutoff/hadron mass)

Kolmogorov-Smirnov analysis

select a good fit range: correlated χ^2/dof should be about one(?)
not really: χ^2/dof should follow instead the χ^2 distribution
probability that from t_{min} the χ^2/dof follow the distribution
(equivalently: goodnesses of the fits are uniformly distributed)

Kolmogorov-Smirnov: difference D (max. between the 2 distributions)



significance:

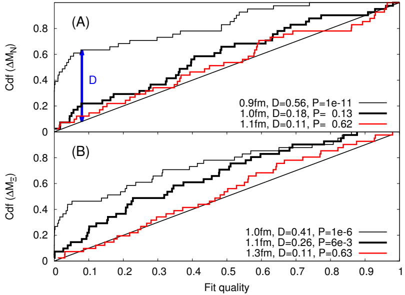
$$Q_{KS}(x) = 2\sum_j (-1)^{j-1} e^{-2j^2x^2}$$

with $Q_{KS}(0) = 1$ and $Q_{KS}(\infty) = 0$

Probablility(D>observed)
$=Q_{KS}([\sqrt{N} + 0.12 + 0.11/\sqrt{N}] \cdot D)$

Different fit intervals for the hadronic chanels

for each hadronic chanel: use the Kolmogorov-Smirnov test P>0.3



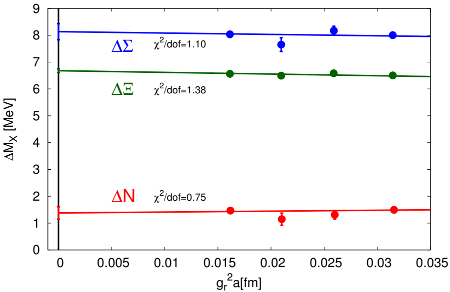
ΔM_N & ΔM_Ξ isospin mass differences with 41 ensembles
(with even more ensembles one can make it mass dependent)
the three t_{min} values give very different probabilities

ΔM_N: 1.1 fm; ΔM_Σ 1.1 fm; ΔM_Ξ 1.3 fm; ΔM_D 1.2 fm; $\Delta M_{\Xi cc}$: 1.2 fm

Getting the final results

extra- and interpolations to the physical point
a. mass-independent or ratio method; b. form for ΔM_X
c. two different fitting ranges d. $(8\tau)^{-1/2} = 280/525$ MeV for α

$\mathcal{O}(500)$ fits, for which we use AIC/goodness/no weights



essentially no lattice spacing dependence (also small for M_π)

Systematic uncertainties/blind analysis

various fits go into BMW Collaboration's hystogram method
its mean: central value with the central 68%: systematic error
use AIC/goodness/no: same result within 0.2σ (except Ξ_{cc}: 0.7σ)
2000 bootstrap samples: statistical uncertainty

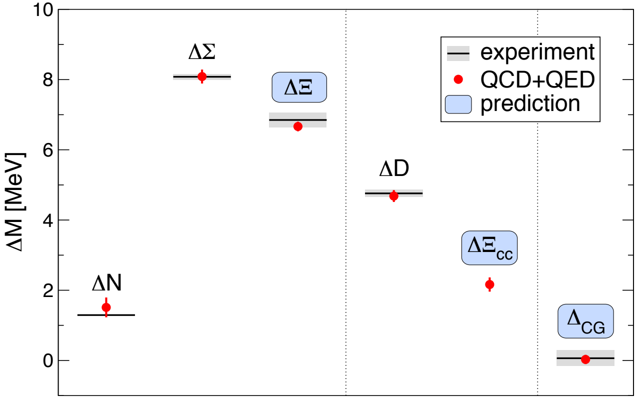
ΔM_X has tiny errors, it is down on the 0.1 permil level
many of them are known $\Longrightarrow$ possible bias $\Longrightarrow$ blind analysis

medical research: double-blind randomized clinical trial (Hill, 1948)
both clinicians and patients are not aware of the treatement
physics: e/m of the electron with angle shift (Dunnington 1933)

we extracted M_X & multiplied by a random number between 0.7–1.3
the person analysing the data did not know the value $\Longrightarrow$
reintroduce the random number $\Longrightarrow$ physical result (agreement)

Isospin splittings

splittings in channels that are stable under QCD and QED:



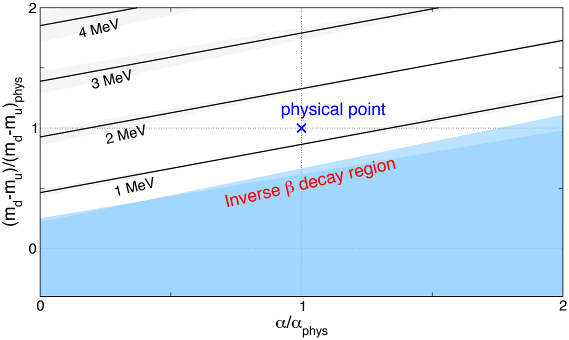
ΔM_N, ΔM_Σ and ΔM_D splittings: post-dictions

ΔM_Ξ, $\Delta M_{\Xi_{cc}}$ splittings and Δ_{CG}: predicitions

Quantitative anthropics

Precise scientific version of the great question:
Could things have been different (string landscape)?

eg. big bang nucleosynthesis & today's stars need $\Delta M_N \approx$ 1.3 MeV



(lattice message: too large or small α would shift the mass)

Summary

Motivations:

- neutrons are more massive than protons ΔM_N=1.3 MeV
- existence/stability of atoms (as we know them) relies on this fact
- splitting: significant astrophysical and cosmological implications
- genuine cancellation between QCD and QED effects: new level

Computational setup:

- 1+1+1+1 flavor full dynamical QCD+QED simulations
- four lattice spacings in the range of 0.064 to 0.10 fm
- pion masses down to 195 MeV
- lattice volumes up to 8.2 fm (large finite L corrections)

Technical novelties (missing any of them would kill the result):

- dynamical QED_L: zero modes are removed on each time slice
- analytic control over finite L effects (larger than the effect)
- high precision numerics for finite L corrections
- large autocorrelation for photon fileds $\Rightarrow$ new algorithm
- improved Wilson flow for electromagnetic renormalization
- Kolmogorov-Smirnov analysis for correlators
- Akakike information criterion for extrapolation/interpolation
- fully blind analysis to extract the final results

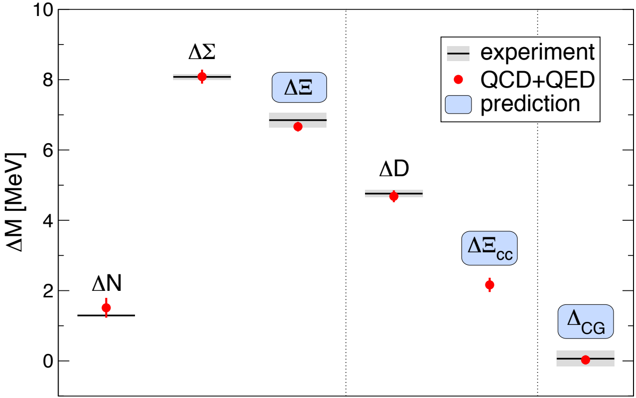
$\Rightarrow$ all extrapolated to the continuum and physical mass limits

Results:

- ΔM_N is greater than zero by five standard deviations
- ΔM_N, ΔM_Σ and ΔM_D splittings: post-dictions
- ΔM_Ξ, $\Delta M_{\Xi_{cc}}$ splittings and $\Delta_{\rm CG}$: predicitions
- quantitative anthropics possible (fairly large region is OK)

Isospin splittings

splittings in channels that are stable under QCD and QED:



ΔM_N, ΔM_Σ and ΔM_D splittings: post-dictions

ΔM_Ξ, $\Delta M_{\Xi_{cc}}$ splittings and Δ_{CG}: predicitions