Lattice calculation of hadronic light-by-light scattering contribution to the muon g-2

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Creutz Fest, Brookhaven National Lab, September 5, 2014

Based on arXiv:1407.2923, TB, Saumitra Chowdhury, Masashi Hayakawa, and Taku Izubuchi

For HVP see Mainz work shop mini-proceedings arXiv:1407.4021 and Lattice 2014 talks

BNL circa 1976

Collaborators

Work on g-2 done in collaboration with

HVP	HLbL
Christopher Aubin (Fordham U)	Saumitra Chowdhury (UConn)
Maarten Golterman (SFSU)	Norman Christ (Columbia)
Santiago Peris (Barcelona)	Masashi Hayakawa (Nagoya)
	Taku Izubuchi (BNL/RBRC)
RBC/UKQCD Collaboration	Luchang Jin (Columbia)
	Christoph Lehner (BNL)
	Norikazu Yamada (KEK)

The magnetic moment of the muon

Interaction of particle with static magnetic field

$$V(\vec{x}) \;=\; -\vec{\mu}\cdot\vec{B}_{\rm ext}$$

The magnetic moment $\vec{\mu}$ is proportional to its spin ($c = \hbar = 1$)

$$\vec{\mu} \;=\; g\left(\frac{e}{2m}\right)\vec{S}$$

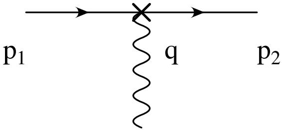
The Landé g-factor is predicted from the free Dirac eq. to be

$$g \;=\; 2$$

for elementary fermions

The magnetic moment of the muon

In interacting quantum (field) theory g gets corrections



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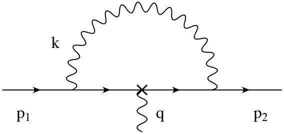


$+ \dots$

$$\gamma^\mu \quad \rightarrow \quad \Gamma^\mu(q) = \left(\gamma^\mu F_1(q^2) + \frac{i\,\sigma^{\mu\nu}\, q_\nu}{2m} F_2(q^2) \right)$$

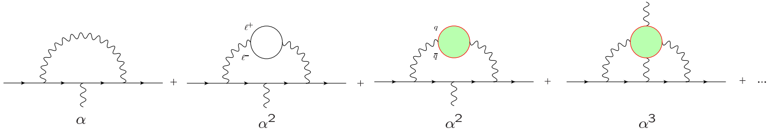
which results from Lorentz and gauge invariance when the muon is on-mass-shell.

$$F_2(0) \quad = \quad \frac{g-2}{2} \equiv a_\mu \qquad\qquad (F_1(0) = 1)$$

(the anomalous magnetic moment, or anomaly)

The magnetic moment of the muon

Compute corrections to g-2 in pert. theory in $\alpha = \dfrac{e^2}{4\pi} = \dfrac{1}{137} + \ldots$



(leading) Schwinger term $= \frac{\alpha}{2\pi} = 0.0011614\ldots$

hadronic contributions $\sim 6 \times 10^{-5}$, 10^{-6} times smaller

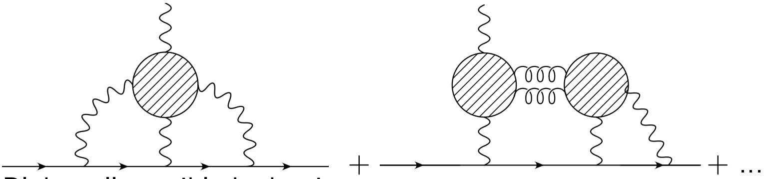
dominate error, ~ 0.4 ppm (exp 0.54 ppm)

QED		116 584 71.8 951 (9)(19)(7)(77)	Aoyama (2012)
EW		15.4 (2)	Czarnecki (2002)
QCD	LO HVP	692.3 (4.2)	Davier (2010)
		694.91 (3.72) (2.10)	Hagiwara (2011)
		701.5 (4.7)	Davier (2010)
	NLO HVP	-9.79 (9)	Hagiwara (2006), Kurz (2014)
	HLbL	10.5 (2.6)	Prades (2009)
	NNLO HVP	1.24 (1)	Kurz (2014)

New experiments + new theory= new physics?

- Fermilab E989 ($\sim$ 2 years away) and J-PARC E34: 0.14 ppm
- a_μ(Expt)-a_μ(SM) $= 287(63)(51)$ ($\times 10^{-11}$), or $\sim$ 3.6σ (or 2.9)
- If both central values stay the same,
  - E989 ($\sim$ 4$\times$ smaller error) $\rightarrow\sim$ 5σ
  - E989+new HLBL theory (models+lattice, 10%) $\rightarrow\sim$ 6σ
  - E989+new HLBL +new HVP (50% reduction) $\rightarrow\sim$ 8σ
- Big discrepancy! (New Physics $\sim$ 2$\times$ Electroweak)
- Lattice calculations crucial
- a_μ good for constraining and explaining BSM physics

The hadronic light-by-light amplitude



Blobs: all possible hadronic states

Model estimates: about $(10-12) \times 10^{-10}$ with a 25-40% uncertainty (difficult to quantify)

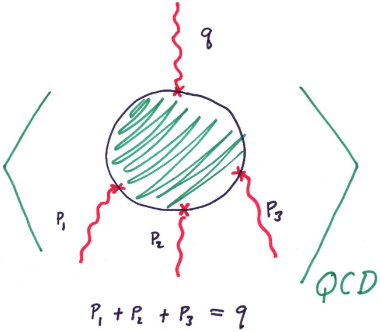
Lattice calculation: model independent, approximations (non-zero a, finite V, ...) systematically improvable

Compute directly on lattice, using QCD and QED

Lattice field theory calculation reminder

- Compute QFT path integrals numerically and stochastically
  - Fields live on finite, discrete, 4d (Euclidean) space-time lattice
  - Generate ensemble of field configurations using monte carlo
  - Average over configurations
- Typically compute correlation function of fields, extract (Minkowski) matrix element or amplitude
- Computation dominated by quark propagators, inverse of large, sparse matrix.
- Extrapolate to continuum, infinite volume, physical quark masses (now directly accessible)

Lattice QCD: conventional approach (*c.f.*, HVP)



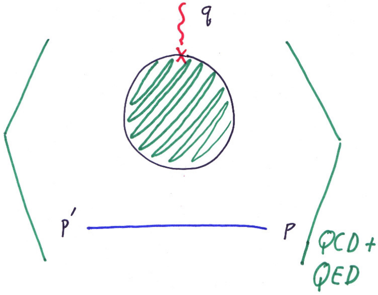
Correlation of 4 EM currents $\Pi^{\mu\nu\rho\sigma}(q, p_1, p_2)$

Two independent momenta $+$external mom q

Compute for all possible values of p_1 and p_2 ($O(V^2)$) four index tensor

several q (extrap $q \to 0$), fit, plug into perturbative QED two-loop integrals

Alternate approach: Lattice QCD+QED



Average over combined gluon *and* photon gauge configurations

Quarks coupled to gluons and photons

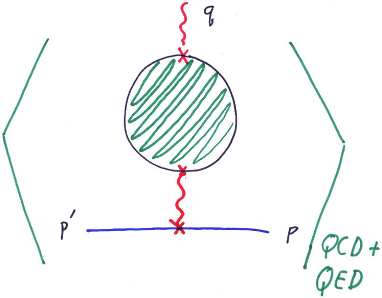
muon coupled to photons

[Hayakawa, *et al.* hep-lat/0509016;

Chowdhury *et al.* (2008);

Chowdhury Ph. D. thesis (2009)]

Alternate approach: Lattice QCD+QED



Attach one photon by hand (see why in a minute)

Correlation of hadronic loop and muon line

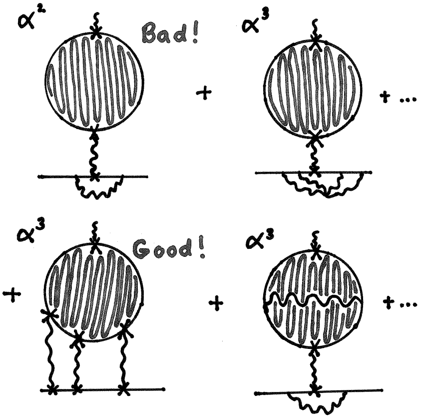
[Hayakawa, *et al.* hep-lat/0509016;

Chowdhury *et al.* (2008);

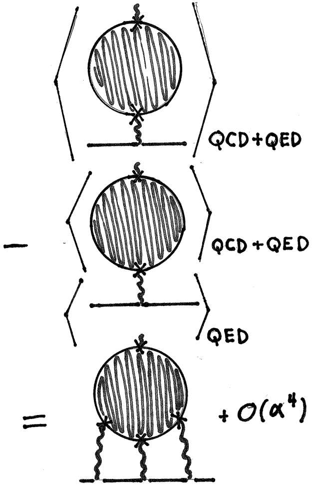
Chowdhury Ph. D. thesis (2009)]

Formally expand in α electromagnetic

The leading and next-to-leading contributions in α to magnetic part of correlation function come from



Subtraction of lowest order piece

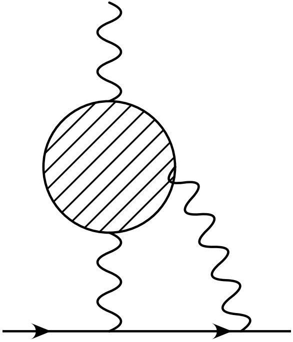


Subtraction term is product of separate averages of the loop and line

Gauge configurations identical in both, so two are highly correlated

In PT, correlation function and subtraction have same contributions except the light-by-light term which is absent in the subtraction

Subtraction of lowest order piece: two photons?

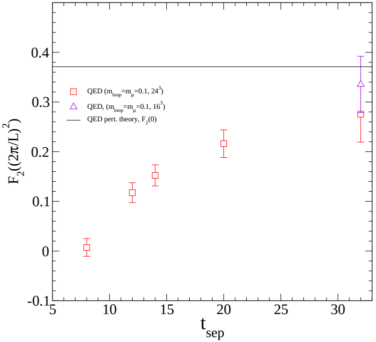


- absent in subtraction term, but vanishes due to Furry's theorem
- Only after averaging over gauge fields, potentially large error ($O(\alpha^2)$ compared to signal of $O(\alpha^3)$)
- Exact symmetry under $\mathbf{p} \rightarrow -\mathbf{p}$
  $e \rightarrow -e$ on muon line only
- If e unchanged, only effect is to flip the sign of all diagrams with two photons, so these cancel on each configuration.
- Observe large reductions in statistical errors after $\pm$ momentum averaging

LbL contribution from lattice QED – a test

- LbL calculation: quenched (no vac. pol.), non-compact QED
  - $m_{\mathrm{lepton}} = 0.1$, loop and line
  - 16^3 and $24^3 \times 64(\times 8)$ Domain Wall Fermions
  - $(L/4)^3 = 64$ and 216 propagators for the lepton loop to enhance statistics
  - incoming muon at rest, $\vec{p} = \pm(2\pi/L, 0, 0)^T$, and permutations, at external vertex
  - several source/sink separations for muon (8-32) to project on to ground states

LbL contribution from lattice QED – a test



- lowest non-trivial momentum only
- Stat. errors only
- Several source/sink separations ($t_{\rm sep}$) for muon (loop is same, only line differs)
- Significant excited state contamination
- $m_\mu = 0.1$ loop and line
- Consistent with expectations from continuum PT

Blum, Chowdhury, Hayakawa, and Izubuchi (arXiv:1407.2923)

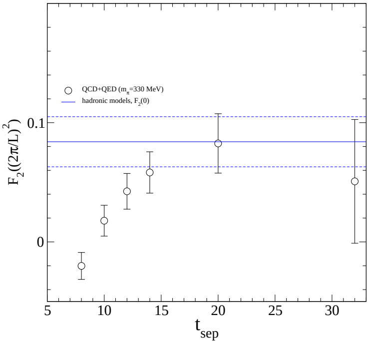
HLbL contribution from lattice QCD+QED

Calculation is almost the same, just take

$U_\mu(x) = U_\mu^{QED}(x) U_\mu(x)^{QCD}$ for the combined gauge field

- HLbL calculation: RBC/UKQCD 2+1f DWF ensemble
  - $m_\pi = 329$ MeV (light quarks heavier than u,d; s is physical)
  - lattice size $24^3 \times 64 (\times 16)$ DWF
  - lattice spacing $a = 0.114$ fm
  - quenched QED for now (sea quarks not charged)
  - $(L/4)^3 = 216$ propagators for the quark loop/configuration.
  - All mode averaging (AMA) technique crucial to make computation tractable

HLbL contribution from lattice QCD+QED

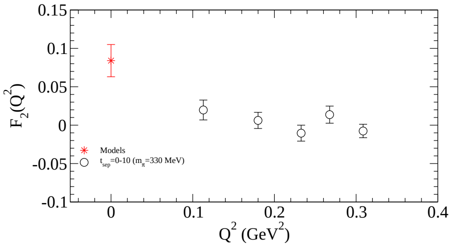


- Stat. errors only, lowest non-trivial momentum
- Several source/sink separations (t_{sep}) for muon
- Significant excited state contamination
- $m_\pi = 329$ MeV
- Model value/error is "Glasgow Consensus"

  (arXiv:0901.0306 [hep-ph], physical masses)
- Subtracted result shows correct e^4 behavior (✓)

Blum, Chowdhury, Hayakawa, and Izubuchi (arXiv:1407.2923)

Momentum dependence

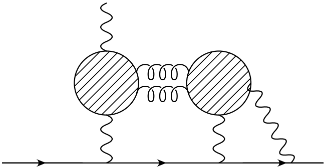


- $t_{\text{sep}} = 10$
- Stat. errors only, lowest non-trivial momentum
- $m_\pi = 329$ MeV
- Model value/error is "Glasgow Consensus"

(arXiv:0901.0306 [hep-ph], physical masses)

Blum, Chowdhury, Hayakawa, and Izubuchi (arXiv:1407.2923)

"Disconnected" diagrams



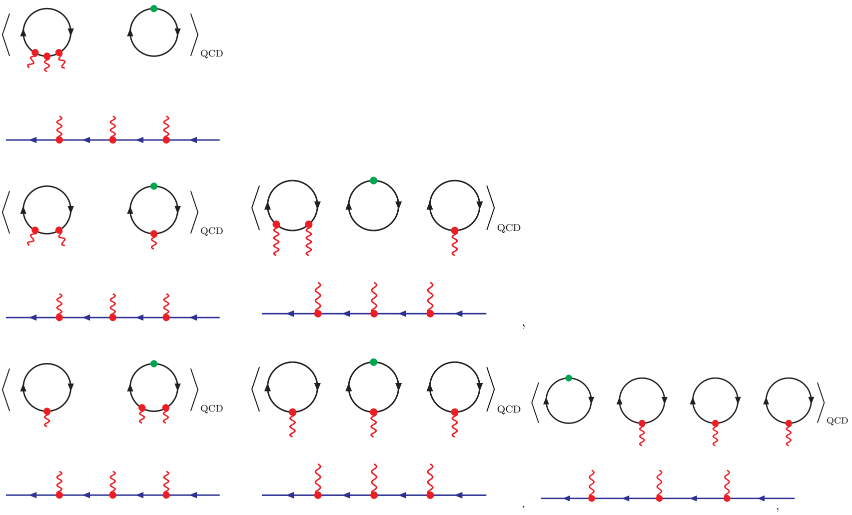
(and similar) not calculated yet

Omission due to use of quenched QED, i.e., sea quarks not electrically charged. Two possibilities,

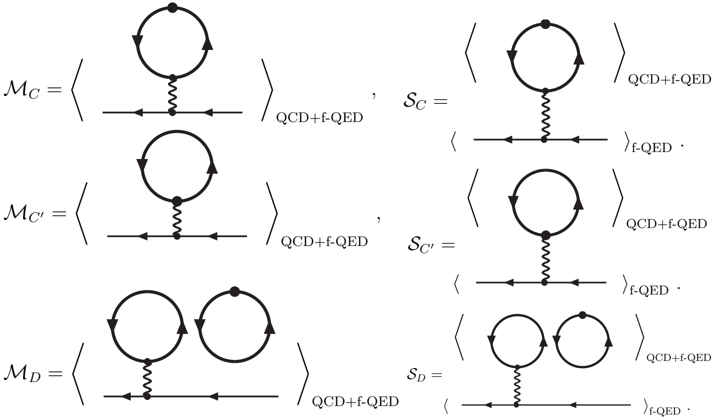
1. Re-weight in α T. Ishikawa, *et al.*, Phys.Rev.Lett. 109 (2012) 072002 or
2. dynamical QED(+QCD) in HMC BMWc arXiv:1406.4088, QCDSF

Use same non-perturbative method as for quenched QED

Disconnected quark loop diagrams



Disconnected quark loop diagrams in non-pert. method



more systematic errors

Need to address

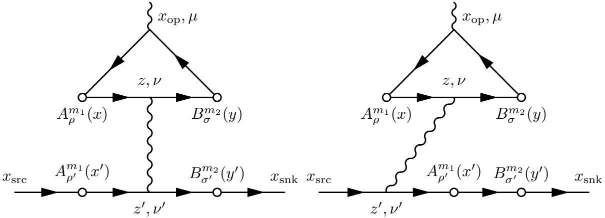
- quark and muon masses (model results: significant dep.)
- quenched QED (reweight QCD ensemble, or dynamical QED)
- Finite volume
- continuum limit $a \to 0$
- $q^2 \to 0$ exptrap
- QED renormalization
- excited states/ "around the world" effects
- $\cdots$

Computationally demanding, but straightforward

Alternative non-pert. method (Luchang Jin)

see talk at Lattice 2014

No subtraction, use 2 independent stochastic photons, one exact



$$
\begin{aligned}
\mathcal{M}^{\rm LbL}_{\mu} &= -(-ie)^6 \frac{1}{M^2} \sum_{m_1,m_2=1}^{M} \frac{1}{V} \sum_k \frac{\delta_{\nu\nu'}}{k^2} \\
&\cdot \sum_z \mathrm{tr}\left\{ \gamma_\mu \left[\sum_x S_q(x_{\rm op};x)\gamma_\rho A^{m_1}_\rho(x) S_q(x;z) \right] \gamma_\nu e^{ik\cdot z} \left[\sum_y S_q(z;y)\gamma_\sigma B^{m_2}_\sigma(y) S_q(y,x_{\rm op}) \right] \right\} \\
&\cdot \sum_{z'} \left\{ \left[\sum_{x'} S(x_{\rm src};x')\gamma_{\rho'} A^{m_1}_{\rho'}(x') S(x';z') \right] \gamma_{\nu'} e^{-ik\cdot z'} \left[\sum_{y'} S(z';y')\gamma_{\sigma'} B^{m_2}_{\sigma'}(y') S(y';x_{\rm snk}) \right] \right. \\
&\quad + S(x_{\rm src};z')\gamma_{\nu'} e^{-ik\cdot z'} \left[\sum_{x'} S(z';x')\gamma_{\rho'} A^{m_1}_{\rho'}(x') \left(\sum_{y'} S(x';y')\gamma_{\sigma'} B^{m_2}_{\sigma'}(y') S(y';x_{\rm snk}) \right) \right] \\
&\quad \left. + \text{other 4 permutations} \right\}
\end{aligned}
$$

Summary and Outlook

- First lattice QCD calculation of HLbL contribution to g-2. Promising, method is feasible.
- Checked QCD+QED method in pure QED (✓)
- Crucial to leverage FNAL E989, J-PARC E34 experiments, search for new physics
- Next HLbL calculations:
  1. RBC/UKQCD 2+1f DWF ensemble

     $m_\pi = 170$ MeV, $32^3 \times 64 \times 32$
  2. 2+1f DWF dynamical or reweighted QCD+QED

     $m_\pi = 300$ MeV, $24^3 \times 64$
  3. RBC/UKQCD 2+1f Möbius-DWF

     $m_\pi = 140$ MeV, $48^3 \times 96 \times 24$

Thanks Mike for sharing your knowledge, wisdom and humor, and for being an inspiration!