

HADRON TOMOGRAPHY

(A. Metz, Temple University)

What is it about ?

What did we learn ?

Looking ahead

Overlap with talks by Qiu, Peng, Seidl, and various other long and short contributions

Acknowledge discussions with Aschenauer, Avakian, Chen, Diehl, Hyde, Liu, Lorcé, Munoz-Camacho, Roche

Nucleon Structure: the Nobel Prizes

- Protons' anomalous magnetic moment (Estermann, Frisch, Stern, 1933)

$$\mu_p \sim 3 \times \mu_{\text{Dirac}}$$

→ proton cannot be pointlike



- Elastic electron-proton scattering (Hofstadter, McAllister, 1955)

$$r_{p,\text{RMS}}^{\text{charge}} = (0.74 \pm 0.24) \text{ fm}$$

→ first rather precise idea about size of the proton



- Deep-inelastic electron-proton scattering (Friedman, Kendall, Taylor et al, 1968)

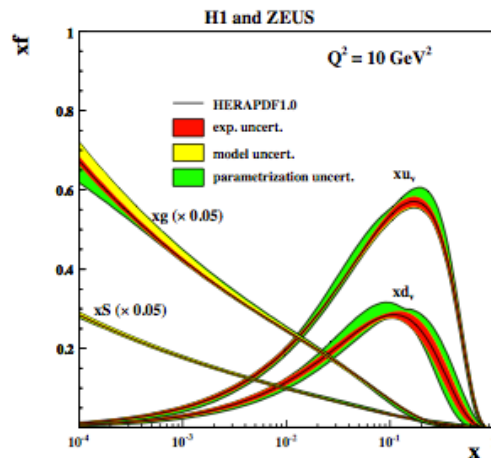
→ proton has partonic substructure

→ experiments paved ground for discovery of QCD

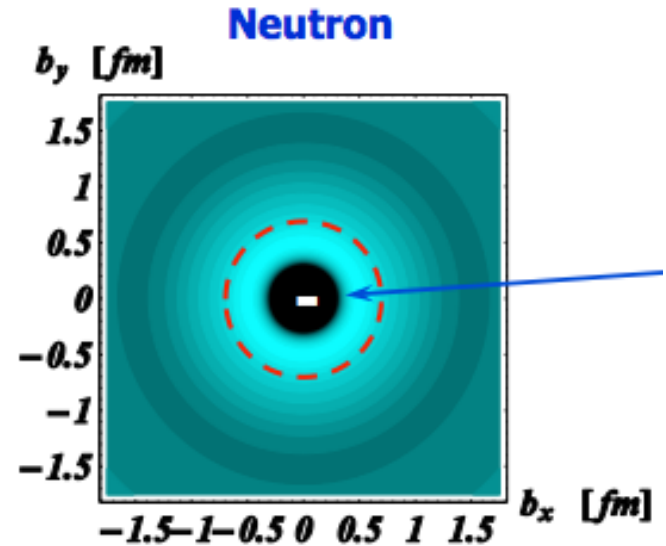


New Era: 3-D Imaging

- Information from DIS experiments and from form factors



1-D long. momentum densities $f^a(x)$

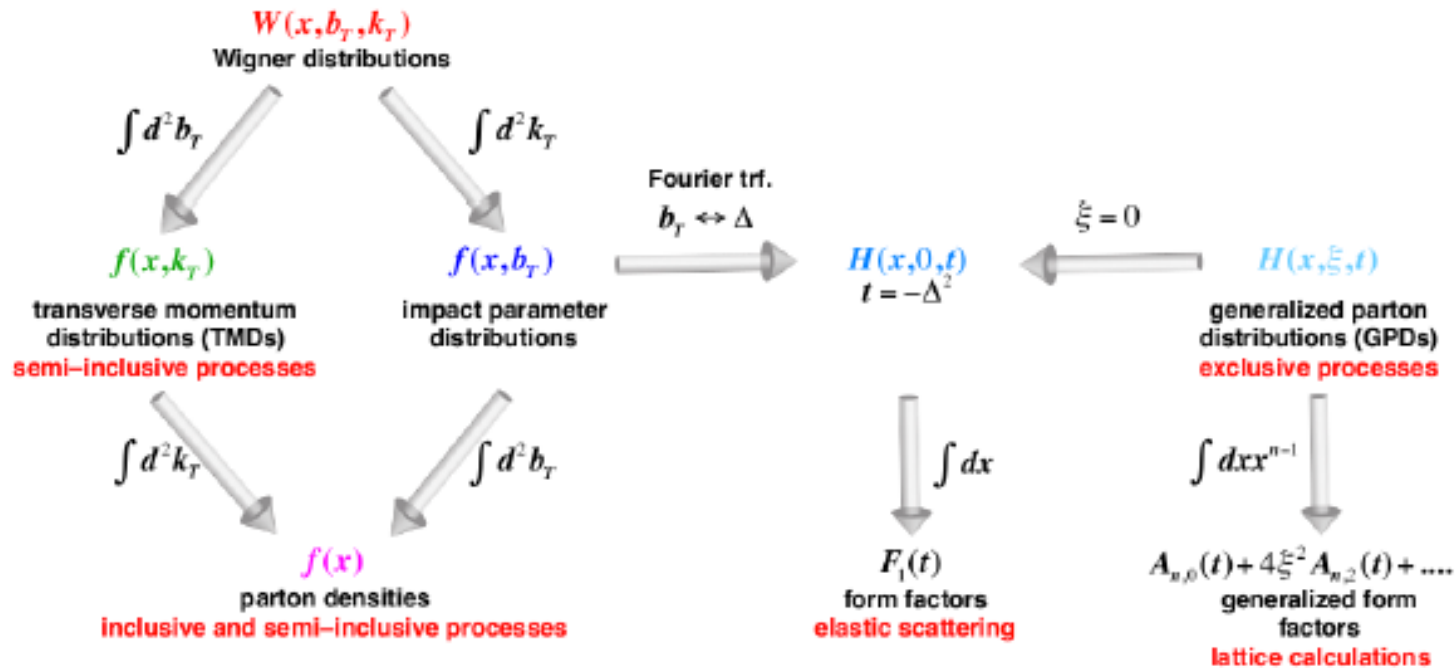


2-D trans. position densities $f^a(\vec{b}_T)$

(Miller, 2007 / courtesy of Lorcé)

- Several pieces are missing
 - helicity distribution $g_1(x)$ (spin sum rule), transversity distribution $h_1(x)$, many open questions about form factors
 - $f^a(\vec{b}_T) = \int dx f^a(x, \vec{b}_T)$ GPDs: 3-D “spatial” densities
 - $f^a(x) = \int d^2\vec{k}_T f^a(x, \vec{k}_T)$ TMDs: 3-D momentum densities (confined motion)

3-D Imaging: Overview of Tools



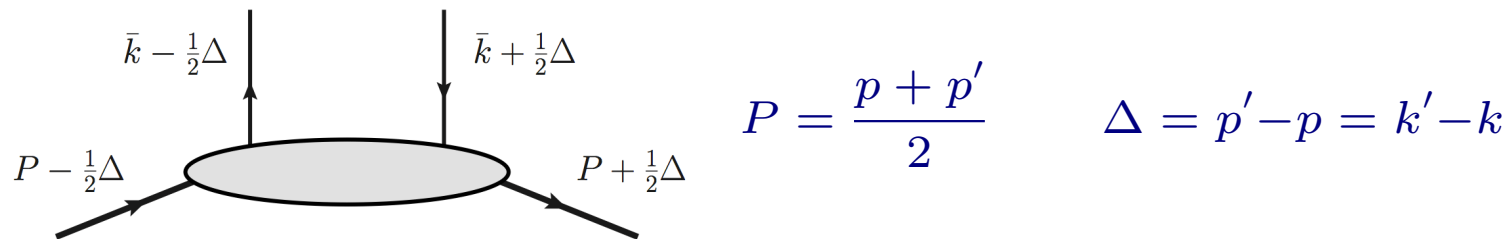
(from arXiv:1212.1701)

Objects of main interest for 3-D imaging

1. $f(x, \vec{b}_T)$ GPDs (in impact parameter space)
2. $f(x, \vec{k}_T)$ TMDs
3. $W(x, \vec{b}_T, \vec{k}_T)$ Wigner distributions (5-D quasi-probability distribution)
4. all those parton correlation functions expressible through light-cone wave functions (if one ignores some “subtleties”)

GPDs and Spatial Imaging of the Nucleon

- Appear in QCD-description of hard exclusive reactions (DVCS, HEMP)
- Kinematics (symmetric frame)



- GPD-correlator (for unpolarized quarks)

$$\begin{aligned}
 F^q[\gamma^+] &= \frac{1}{2} \int \frac{dz^-}{2\pi} e^{i\bar{k}\cdot z} \langle p' | \bar{\psi}^q\left(-\frac{z}{2}\right) \gamma^+ \mathcal{W}_{GPD} \psi^q\left(\frac{z}{2}\right) | p \rangle \Big|_{z^+=z_\perp=0} \\
 &= \frac{1}{2P^+} \bar{u}(p') \left(\gamma^+ H^q(x, \xi, t) + \frac{i\sigma^{+\mu} \Delta_\mu}{2M} E^q(x, \xi, t) \right) u(p) \\
 x &= \frac{\bar{k}^+}{P^+} \quad \xi = -\frac{\Delta^+}{2P^+} \quad t = \Delta^2
 \end{aligned}$$

- Eight leading twist GPDs for quarks and gluons

- Relation to forward PDFs and form factors (crucial for modeling)

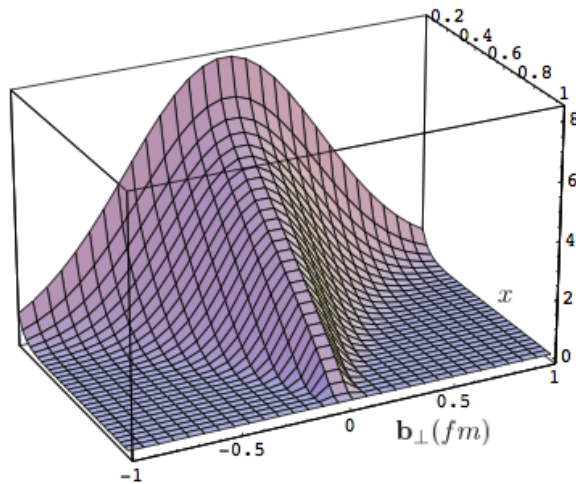
$$H^q(x, 0, 0) = f_1^q(x) \quad \int_{-1}^1 dx H^q(x, \xi, t) = F_1^q(t)$$

- Impact parameter representation ($\xi = 0$) → density interpretation (Burkardt, 2000, 2002)

$$\begin{aligned} \mathcal{F}^q(x, \vec{b}_T; \uparrow) &= \int \frac{d^2 \vec{\Delta}_T}{(2\pi)^2} e^{-i\vec{\Delta}_T \cdot \vec{b}_T} F^q(x, \vec{\Delta}_T; \uparrow) \\ &= \mathcal{H}^q(x, \vec{b}_T^2) + \frac{(\vec{S}_T \times \vec{b}_T)^z}{M} \frac{\partial}{\partial \vec{b}_T^2} \mathcal{E}^q(x, \vec{b}_T^2) \end{aligned}$$

- 3-D structure in (x, \vec{b}_T) -space (“spatial” imaging)
- \vec{b}_T relative to transverse center of longitudinal momentum $\sum_i p_i^+ \vec{b}_{Ti} / \sum_i p_i^+$
- term containing \mathcal{E}^q generates dipole pattern
→ (numerically large) distortion of $\mathcal{F}^q(x, \vec{b}_T; \uparrow)$

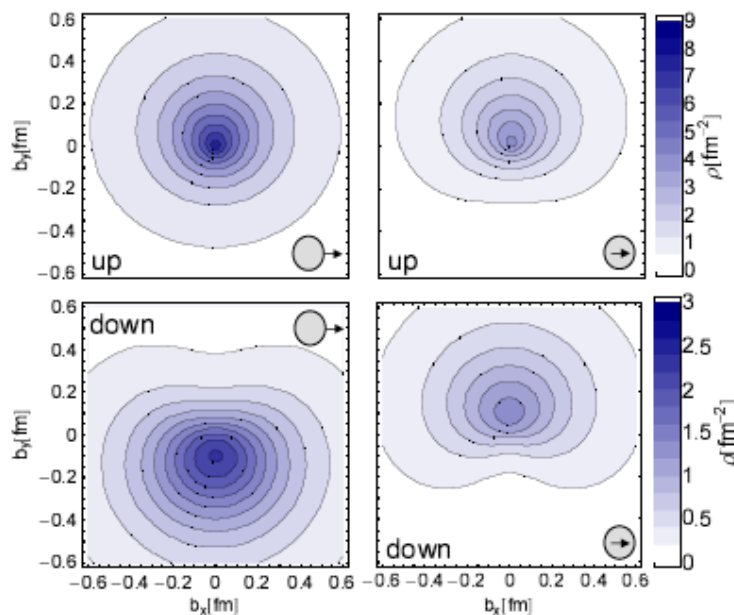
- GPDs in impact parameter space: sample plots
 - no polarization



toy model for GPD (Burkardt, 2002)

- * b_T distribution gets narrow at large x
- * general pattern agrees with phenomenology

- transverse polarization included (nucleon and quark) (QCDSF Collaboration, 2006)



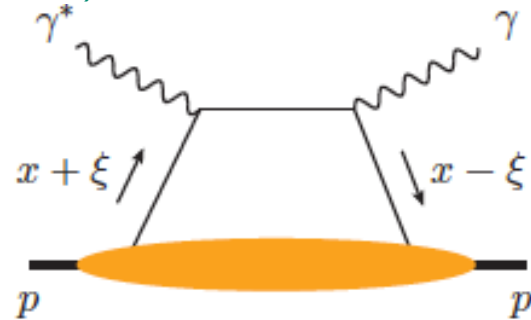
left: unpolarized quarks in transversely polarized target

right: transversely polarized quarks in unpolarized target

- * distortion stronger for down quarks
- * distortion stronger for transv. pol. quarks in unpol. nucleon
- * similar results in models and GPD parameterizations

GPDs in Experiment

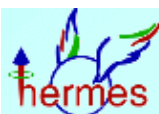
- Deep-virtual Compton scattering (DVCS) $\ell N \rightarrow \ell N \gamma$
 - handbag diagram (leading order)



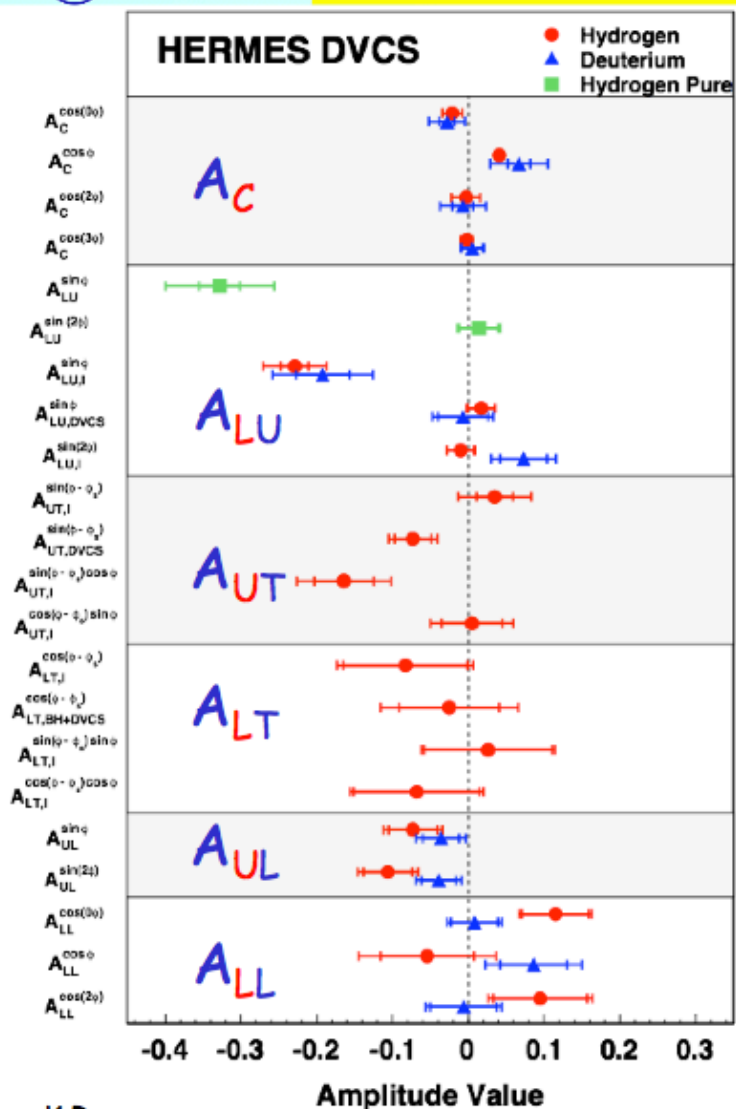
- Observables depend on four (complex-valued) Compton form factors

$$CFF(\xi, t) = \int_{-1}^1 dx \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) H(x, \xi, t)$$

- **1st step:** extract imaginary and real part of CFFs
- **2nd step:** extract GPDs from CFFs (so far with some model-dependence)
- data from HERA (H1, ZEUS), HERMES, JLab (\rightarrow talk by Girod)
- Hard exclusive meson production (HEMP) $\ell N \rightarrow \ell N M$ (\rightarrow talk by Kim)
- **Future plan:** gluon GPD E^g at RHIC in $p^\uparrow Au$ collisions (\rightarrow talk by Aschenauer)



DVCS asymmetries measured @ HERMES



K.R.

DIS2014

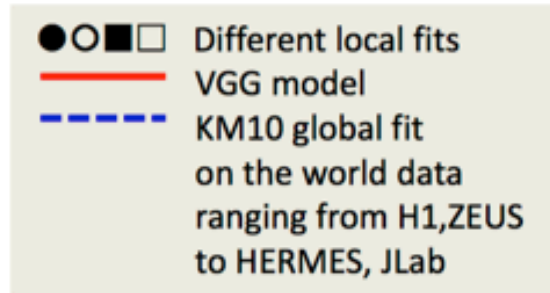
- Beam charge asymmetry
GPD H
H: PRL 87 (2001) 182001
PR D 75 (2007) 011103
JHEP 11 (2009) 083
JHEP 07 (2012) 032
JHEP 10 (2012) 042 (recoil det.)
JHEP 01 (2014) 077 (recoil det.)
D: Nucl. Phys. B 829 (2010) 1
nuclei: PR C 81 (2010) 035202
- Beam helicity asymmetry
GPD H
- Transverse target-spin asymmetries
GPD E
H: JHEP 06 (2008) 066
H: PLB 704 (2011) 15
- Longitudinal target spin asymmetries
GPD H
H: JHEP 06 (2010) 019
D: Nucl. Phys. B 842 (2011) 265

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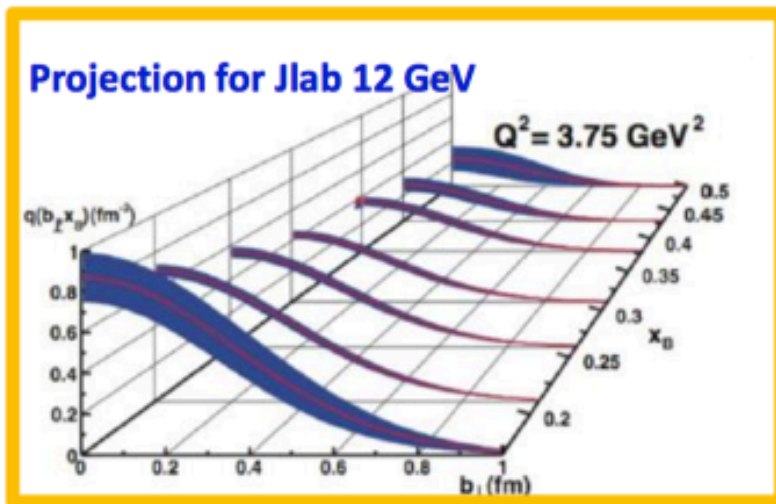
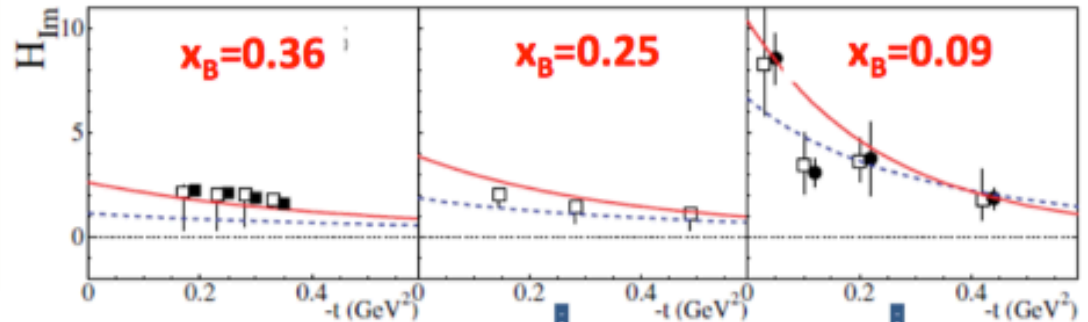
(slide from talk of Rith at DIS 2014)

Towards the 3D Structure of the Proton (past 7 years)

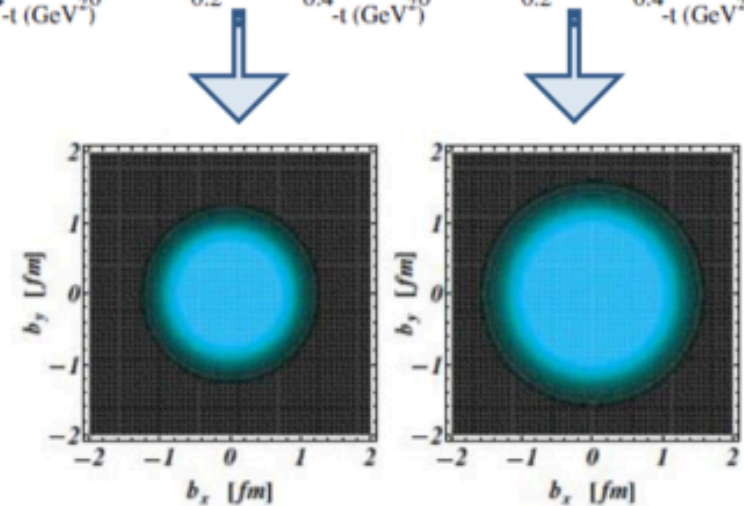
the GPD H in Im DVCS



	e^- 6 GeV	e^\pm 27 GeV
Jlab Hall A	Jlab CLAS	HERMES
Beam Spin Diff Beam Spin Sum	Beam Spin Asym Long Pol targ Asym	Beam Spin Asym Beam Charge Asym



Dudek et al., EPJA48 (2012)



Guidal, Moutarde, Vanderhaeghen, Rept. Prog. Phys. 76 (2013)

Slide from N. [d'Hose](#) (CEA-France)

(slide from Roche and JLab GPD-community)

Towards the 3D Structure of the Proton (next 7 years)

Simplest process: $e + p \rightarrow e' + p + \gamma$ (DVCS)

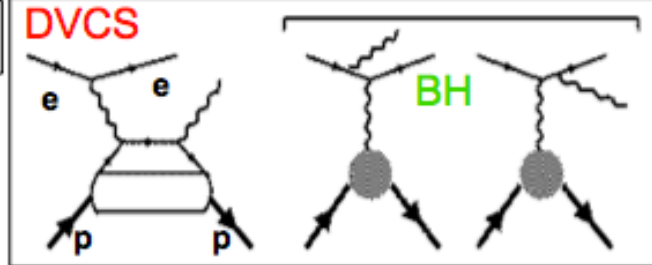
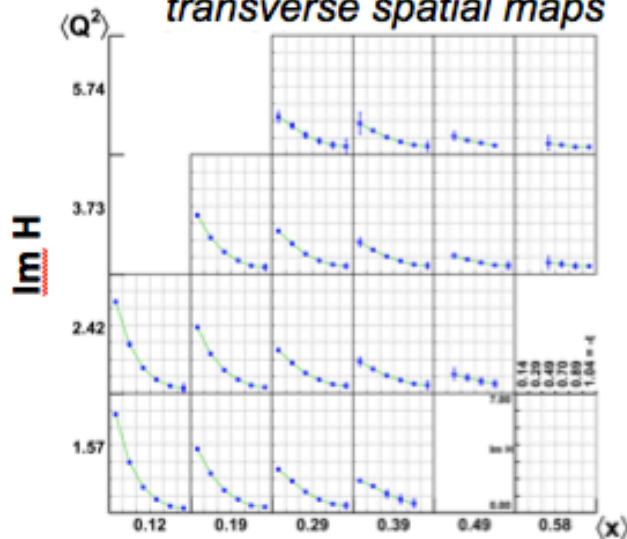
Well defined formalism relating cross-sections to GPDs observables up to Twist 3

Belitsky and Mueller, Phys. Rev. D82 (2010) 074010

6 GeV data:

Hall B beam-spin asymmetry data show potential for imaging studies from analysis in x , Q^2 and t .

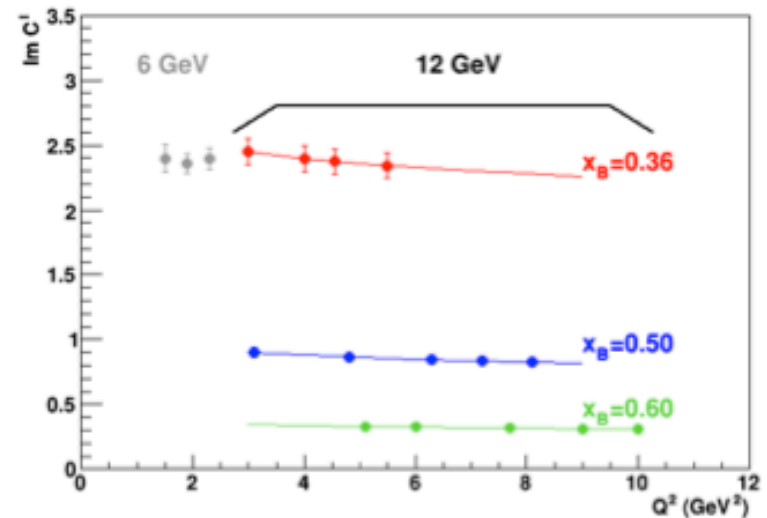
12 GeV projections for Hall B:
(beam-spin and target-spin asymmetries)
transverse spatial maps



6 GeV data:

Hall A data for Compton form factor (over limited Q^2 range) agree with hard-scattering

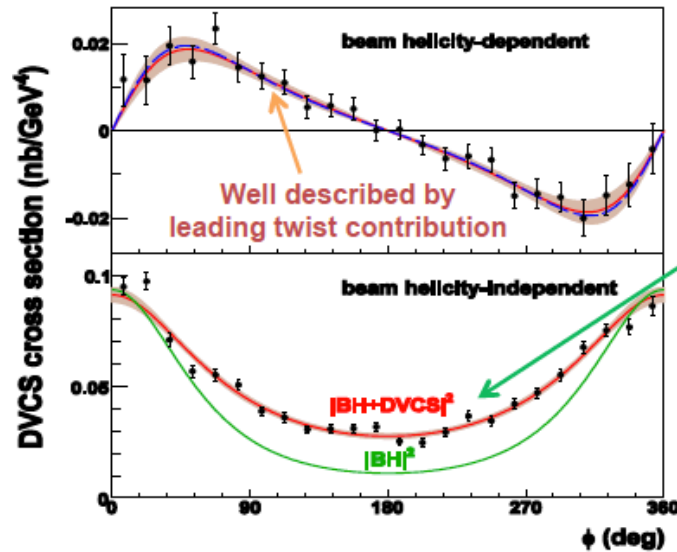
12 GeV projections for Hall A/C:
confirm formalism



(slide from Roche and JLab GPD-community)

Accessing the Real part of the DVCS amplitude

(next 7 years)



Results from Hall A E00-110:

Significant deviation from Bethe-Heitler:

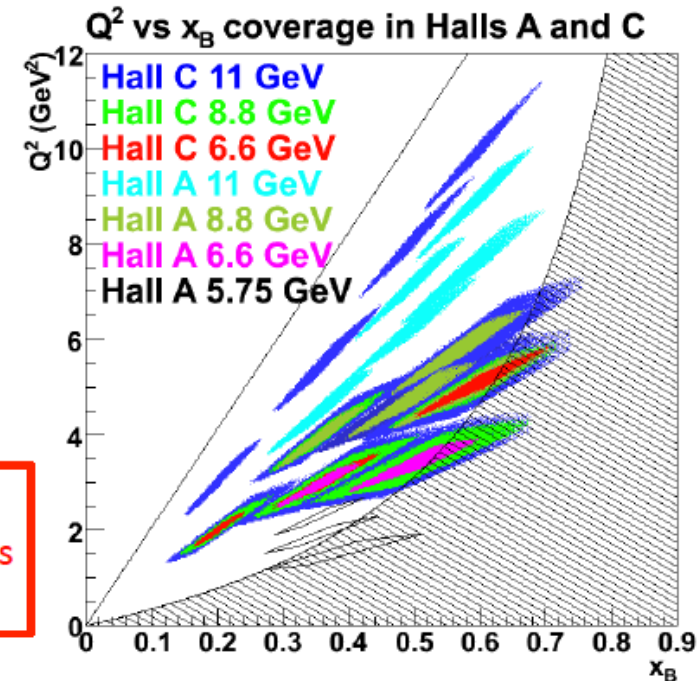
Both $Inter(BH \cdot DVCS)$ and $|DVCS|^2$ contribute to the cross-section

Rosenbluth-like technique: beam energy separation at constant Q^2 , x_B and t

$$\sigma = |BH|^2 + Re[DVCS^* BH] + |DVCS|^2$$

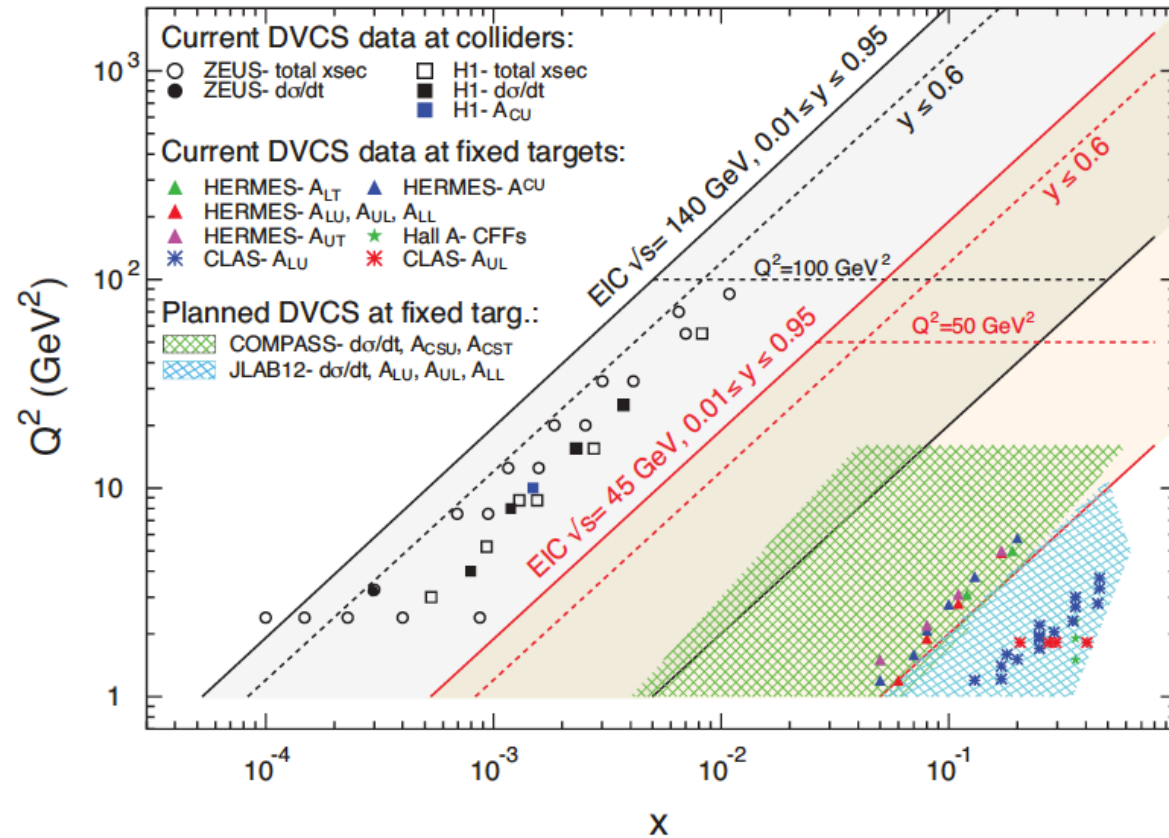
$$\sim E_{Beam}^2 \quad \sim E_{Beam}^3$$

Extracting the real part of CFFs from DVCS requires measuring the cross section at multiple beam energies (DVCS²-Interference separation)



(slide from Roche and JLab GPD-community)

- Future DVCS experiments (beyond JLab 12)

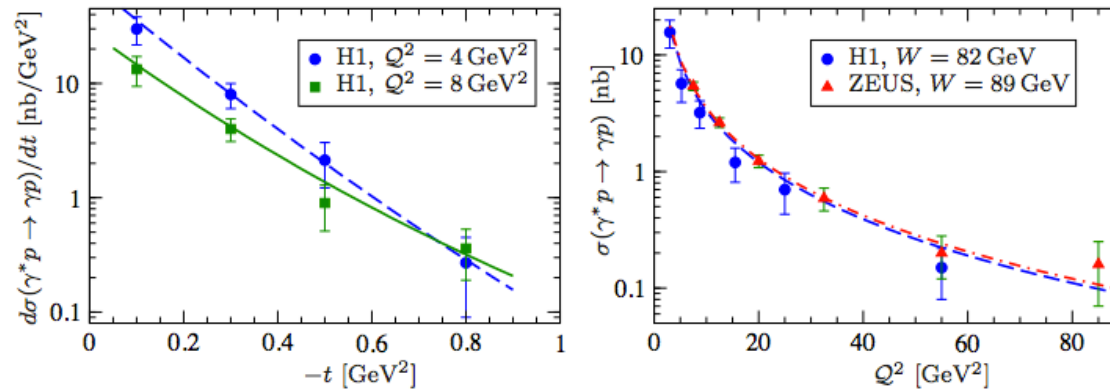


(from arXiv:1212.1701)

- COMPASS: starts getting into region of sea quarks and gluons
- EIC: high luminosity, precision imaging all the way into the small x region, very high Q^2 at large x (exploit scaling violations for GPD extraction), polarization (→ dedicated EIC talks by Mueller, McKeown, Mezziani, Prokudin, ...)

Further Progress in GPD Theory (selection)

- Flexible parameterizations of GPDs allow one to fit wide range of DVCS data (Kumericki, Müller, 2009, ... / other groups)
 - example



(Kumericki, Müller, 2009)

- Neural Network analysis applied for the first time to DVCS (Kumericki, Müller, 2011)
- Kinematical higher twist corrections for DVCS clarified up to twist-4 (Braun, Manashov, 2011 / ...)
- Conceptual progress in computing parton correlators on the lattice (\rightarrow talk by Ji) (Ji, 2013 / ...)
 - new method should allow one to compute at smaller x (no need for moments)

TMDs and confined motion

- Appear in QCD-description of many hard semi-inclusive reactions
 $\ell N \rightarrow \ell h X$, $\ell N \rightarrow \text{jet jet } X$, etc
 $pp \rightarrow (\gamma^*, Z, W)$, $pp \rightarrow \gamma \gamma X$, $pp \rightarrow \text{Higgs } X$, $pp \rightarrow (h \text{ jet}) X$, etc
 $e^+ e^- \rightarrow h_1 h_2 X$, etc
 (→ talks by Seidl, Liu, Aschenauer, ...)

- TMD-correlator (for unpolarized quarks)

$$\begin{aligned} \Phi^{q[\gamma^+]}(x, \vec{k}_T; \uparrow) &= \frac{1}{2} \int \frac{dz^-}{2\pi} \frac{d^2 \vec{z}_\perp}{(2\pi)^2} e^{ik \cdot z} \langle p | \bar{\psi}^q \left(-\frac{z}{2} \right) \gamma^+ \mathcal{W}_{TMD} \psi^q \left(\frac{z}{2} \right) | p \rangle \Big|_{z^+=0} \\ &= f_1^q(x, \vec{k}_\perp^2) + \frac{(\vec{S}_T \times \vec{k}_T)^z}{M} f_{1T}^{q\perp}(x, \vec{k}_\perp^2) \end{aligned}$$

- 3-D structure in (x, \vec{k}_T) -space (momentum imaging)
- new correlation due to confined motion (k_T -dependence): f_{1T}^\perp (Sivers, 1989)

- Eight leading twist TMDs for quarks and gluons

- Overview of leading twist quark TMDs

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 = \odot$		$h_1^\perp = \uparrow - \downarrow$ Boer-Mulders
	L		$g_{1L} = \rightarrow - \leftarrow$ Helicity	$h_{1L}^\perp = \rightarrow - \leftarrow$
	T	$f_{1T}^\perp = \uparrow - \downarrow$ Sivers	$g_{1T}^\perp = \rightarrow - \leftarrow$	$h_1 = \uparrow - \downarrow$ Transversity $h_{1T}^\perp = \rightarrow - \leftarrow$

(from arXiv:1212.1701)

- New physics aspects due to confined motion
 - transverse momentum dependence of f_1 , g_1 , h_1
 - new correlation between \vec{S}_T , \vec{s}_T , \vec{k}_T (h_{1T}^\perp)
 - new correlation between \vec{S}_T , \vec{k}_T (f_{1T}^\perp), and between \vec{s}_T , \vec{k}_T (h_1^\perp)
 - new correlation between \vec{S}_T , λ , \vec{k}_T (g_{1T}^\perp), and between Λ , \vec{s}_T , \vec{k}_T (h_{1L}^\perp)
 - connection to single-spin asymmetries and quark-gluon-quark correlations
 - ideal playground for pQCD: factorization, universality, resummation
 - allow one to directly study impact of local color gauge invariance of QCD
 - etc

→ “new structures, new physics/phenomena” (applies also to GPDs)
(quote from X. Ji at recent JLab pretown meeting)

- “Stamp collection”? ... maybe ... but we are in good company
 - periodic table of elements

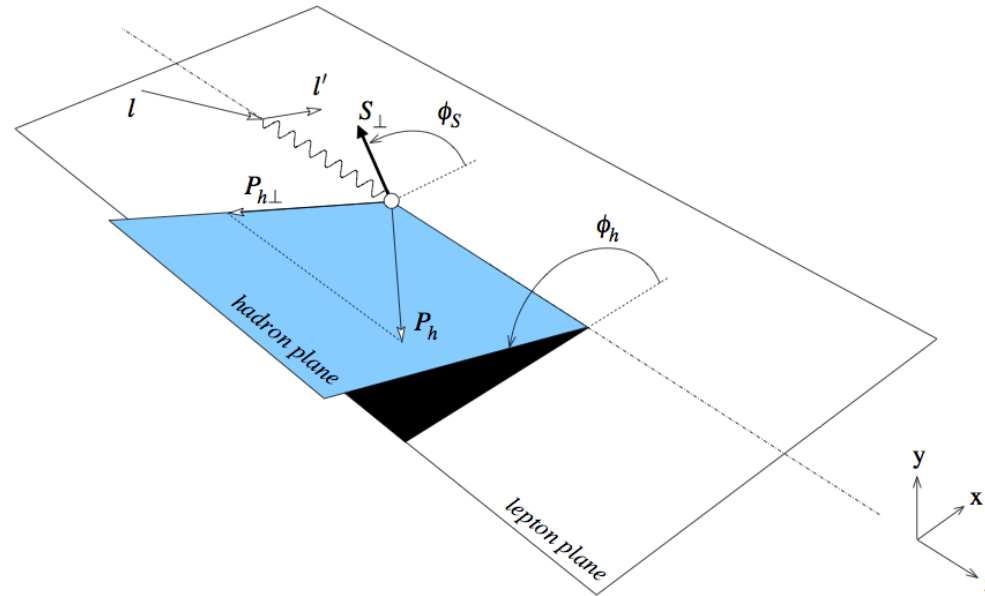
1																	2										
H																	He										
3	4											5	6	7	8	9	10										
Li	Be											B	C	N	O	F	Ne										
11	12											13	14	15	16	17	18										
Na	Mg											Al	Si	P	S	Cl	Ar										
19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36										
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr										
37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54										
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe										
55	56											72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	
Cs	Ba											Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn	
87	88											104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	
Fr	Ra											Rf	Db	Sg	Bh	Hs	Mt	Ds	Rg	Cn	Uut	Fl	Uup	Lv	Uus	Uuo	
57	58	59	60	61	62	63	64	65	66	67	68	69	70	71													
La	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu													
89	90	91	92	93	94	95	96	97	98	99	100	101	102	103													
Ac	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr													

don't forget the isotopes ...

- (supersymmetric) extensions of the Standard Model
- materials science
- **major driving forces:** (1) curiosity, (2) search for new physics/phenomena, (3) new insight into existing puzzles, (4) applications

TMDs in Semi-Inclusive DIS: $\ell N \rightarrow \ell h X$

- 6 independent kinematical variables: $x \quad Q^2 \quad \phi_S \quad z \quad P_{h\perp} \quad \phi_h$

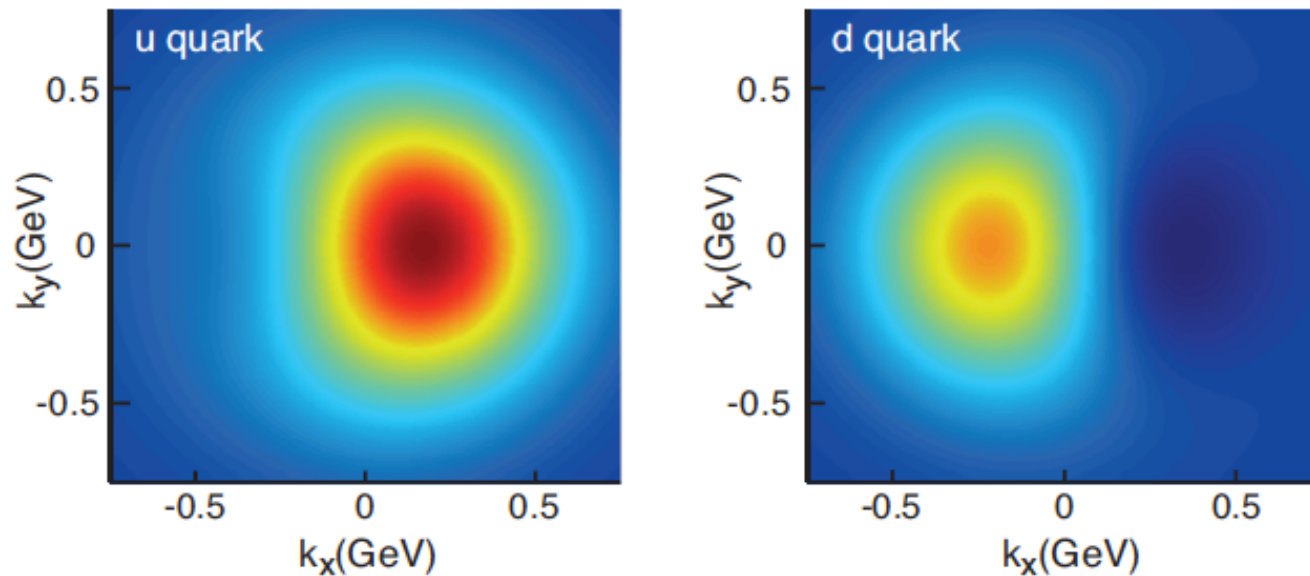


- At low $P_{h\perp}$, 8 structure functions are related to 8 leading twist quark TMDs
→ complete experiment possible
- Data from COMPASS, HERMES, JLab (→ talk by Seidl)
- Transverse target polarization: Sivers component, Collins component, etc

$$d\sigma^\uparrow \sim \sin(\phi_h - \phi_S) f_{1T}^\perp \otimes D_1 + \sin(\phi_h + \phi_S) h_1 \otimes H_1^\perp + \dots$$

- Distortion of k_T distribution due to Sivers effect

$$\Phi^{q[\gamma^+]}(x, \vec{k}_T; \uparrow) = f_1^q(x, \vec{k}_\perp^2) + \frac{(\vec{S}_T \times \vec{k}_T)^z}{M} f_{1T}^{q\perp}(x, \vec{k}_\perp^2) \quad (x = 0.1)$$

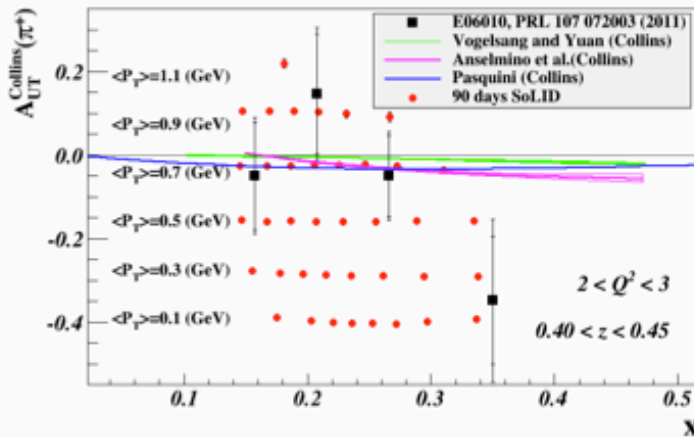


(from arXiv:1212.1701, based on Anselmino et al, 2011)

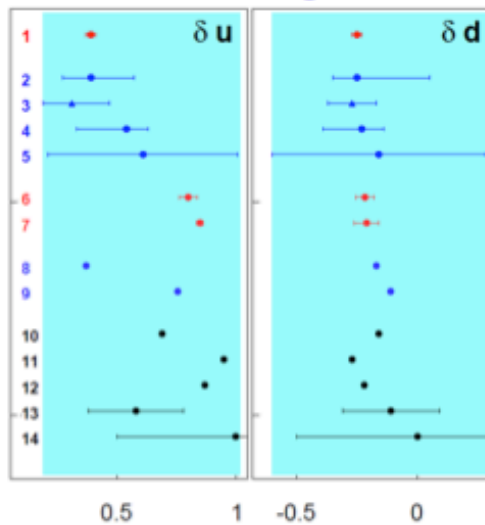
- Sivers effect generates distorted distribution of unpolarized quarks
- plots based on data
- distortion in k_T space and in b_T space have (model-dependent) relation (applies to some other correlations too)
(Burkardt, 2002, ... / Burkardt, Hwang, 2002 / Meißner, AM, Goeke, 2007 / ...)

Nucleon Structure with SoLID-SIDIS

Collins Asymmetry *Total > 1400 points*



Tensor Charges



SoLID projections

Extractions from existing data

LQCD

DSE

Models

Semi-inclusive Deep Inelastic Scattering program:

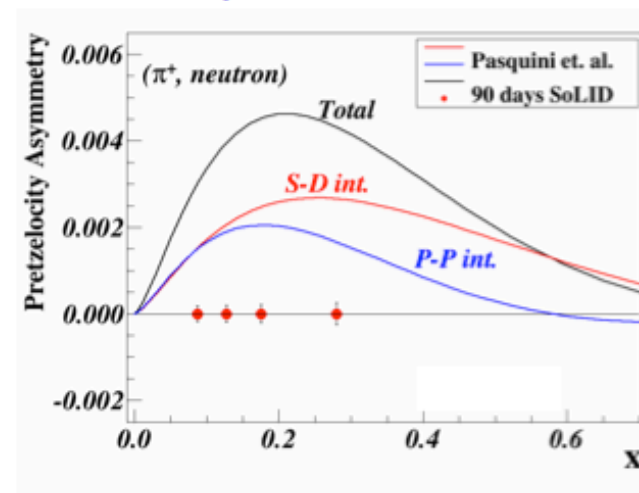
Large Acceptance + High Luminosity
+ Polarized targets

→ 4-D mapping of Collins, Sivers, and pretzelocity asymmetries,...

→ Tensor charge of quarks, transversity distributions, TMDs...

→ Benchmark test of Lattice QCD, probe QCD Dynamics and quark orbital motion

Pretzelocity → information on OAM



(slide from Chen and SoLID Collaboration)

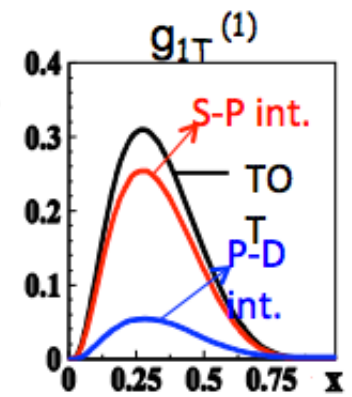
→ talks by Chen, Souder, ...

Worm-gear Functions

- Dominated by **real** part of interference between **L=0 (S)** and **L=1 (P)** states
- **No** GPD correspondence
- Exploratory lattice QCD calculation:
Ph. Hägler et al, EPL 88, 61001 (2009)

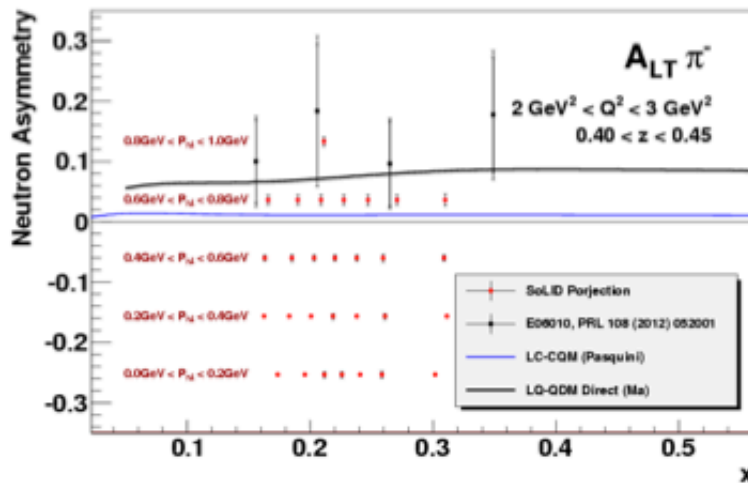
$$g_{1T} = \begin{array}{c} \uparrow \\ \circ \leftarrow \\ \circ \leftarrow \end{array} - \begin{array}{c} \uparrow \\ \circ \leftarrow \\ \circ \leftarrow \end{array}$$

$$h_{1L}^\perp = \begin{array}{c} \uparrow \\ \circ \leftarrow \\ \circ \leftarrow \end{array} - \begin{array}{c} \uparrow \\ \circ \leftarrow \\ \circ \leftarrow \end{array}$$

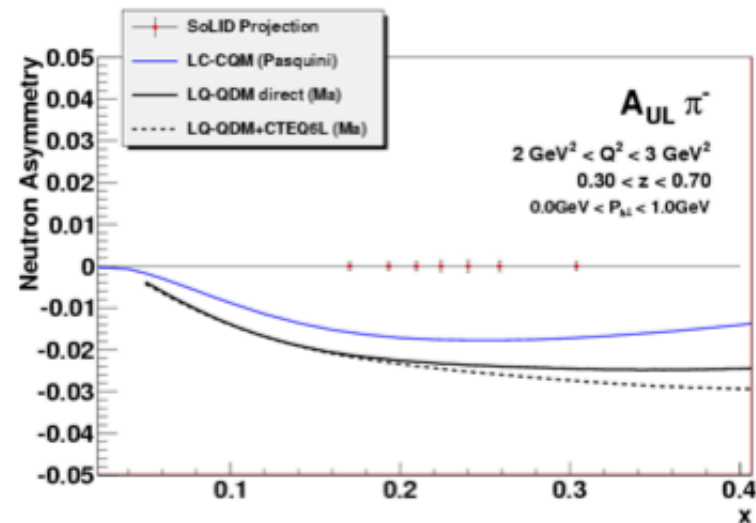


Light-Cone CQM by B. Pasquini
B.P., Cazzaniga, Boffi, PRD78, 2008

Neutron Projections,



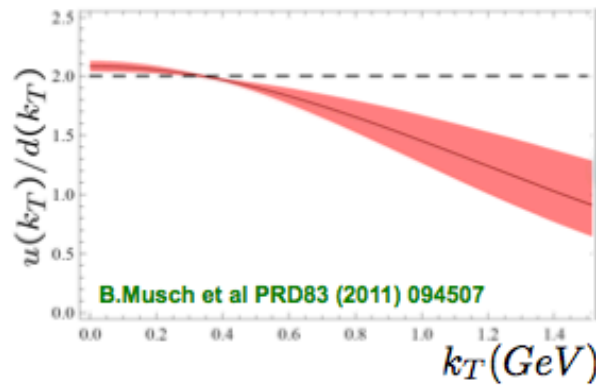
$$A_{LT} \sim g_{1T}(x)D_1(z)$$



$$A_{UL} \sim h_{1L}^\perp(x) \otimes H_1^\perp(z)$$

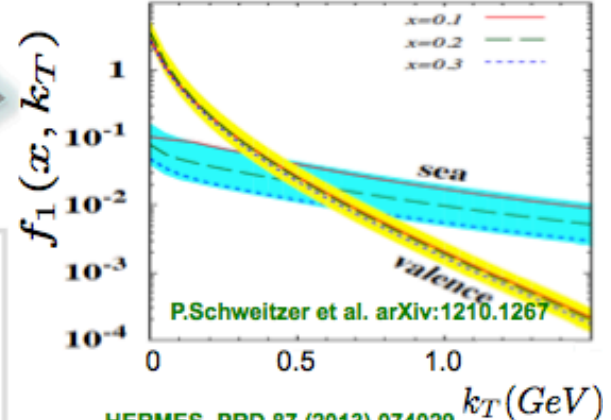
(slide from Chen and SoLID Collaboration)

Probing the flavor-dependence of k_T -distributions

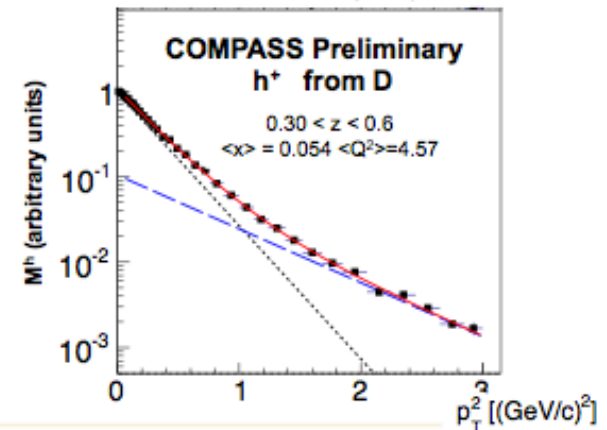
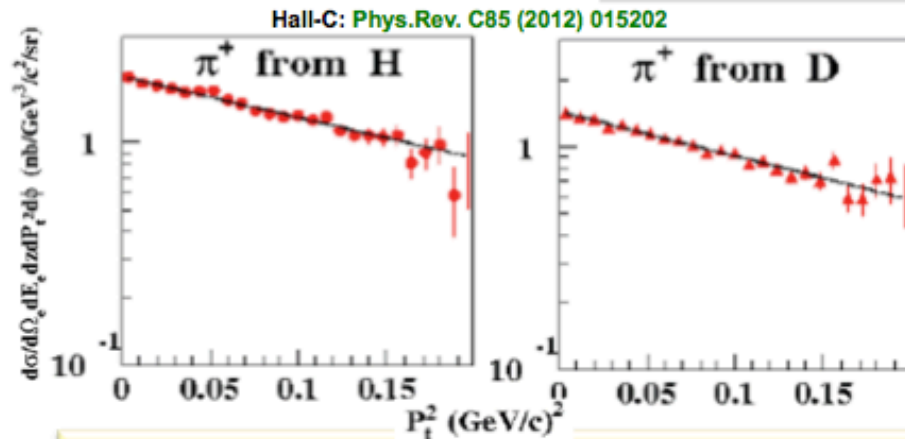


Higher probability to find more sea & d-quarks at large k_T

Measurements of hadronic multiplicities provide a crucial input for studies of k_T dependence of spin independent distributions

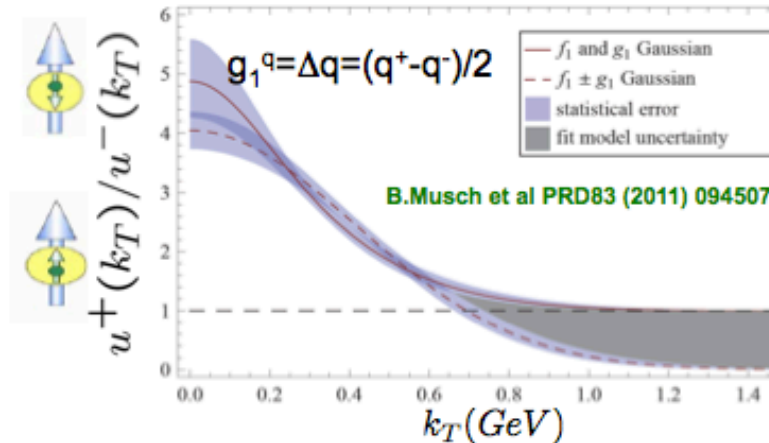


HERMES, PRD 87 (2013) 074029
 COMPASS, EPJC 73 (2013) 2531



There are indications from both theory (lattice, chiral constituent quark model) and experimental data of the k_T dependence of quark flavor distribution.

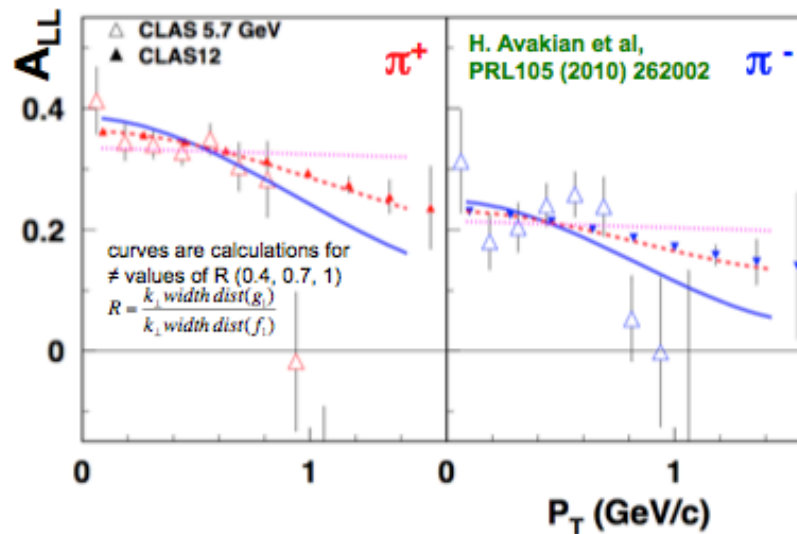
Probing the helicity-dependence of k_T -distributions



- Higher probability to find a quark anti-aligned with proton spin at **large k_T**
- Important to have q^+ and q^- k_T dependent distribution separately
- q^- sensitive to orbital motion:

$$q_{L=1}^- \sim (1-x)^5 \log^2(1-x)$$

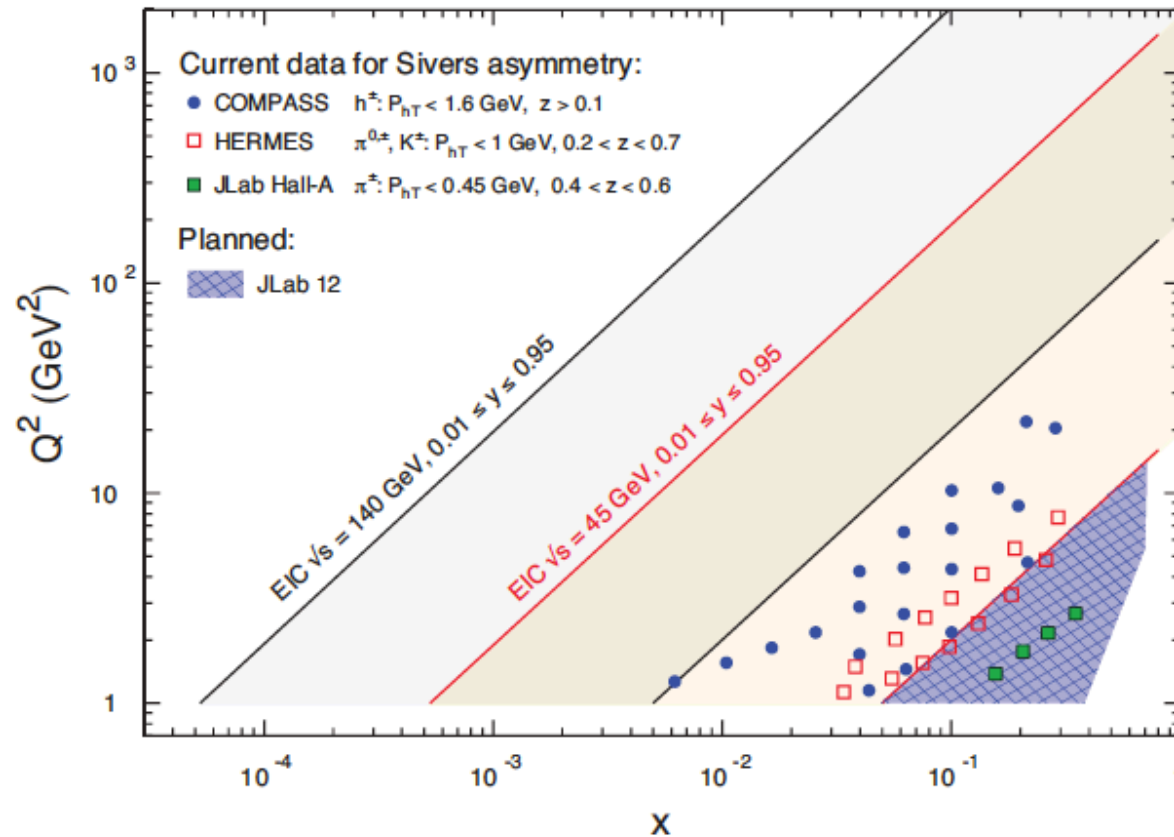
H. Avakian et al. PRL 99 (2007) 082001



- Double spin asymmetries from CLAS@JLab consistent with wider k_T distributions for f_1 than for g_1
- **Wider range in P_T from CLAS12 is crucial !**

Measurements of the P_T -dependence of A_{LL} ($\propto g_1/f_1$) provide access to transverse momentum distributions of quarks anti-aligned with the proton spin.

- TMDs at an EIC



(from arXiv:1212.1701)

- high luminosity, (precision) imaging in the small x region (sea quarks and gluons), suppression of higher twist at large x , polarization, systematic studies of pQCD techniques (evolution, resummation), studies of parton saturation (→ dedicated EIC talks by Mueller, McKeown, Meiziani, Prokudin, ...)

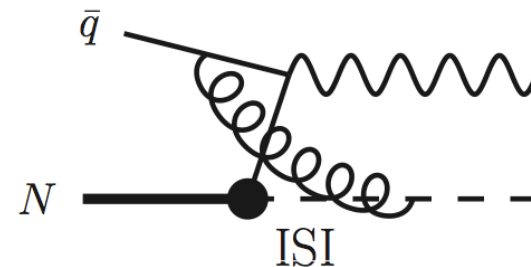
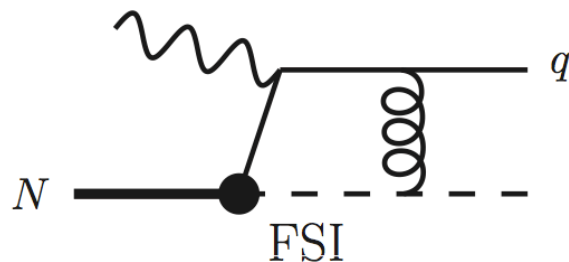
Universality Properties: TMDs in SIDIS vs DY

(→ talk by Peng)

- Prediction based on operator definition in quantum field theory (Collins, 2002)
(follows from TMD factorization)

$$f_{1T}^\perp|_{DY} = -f_{1T}^\perp|_{SIDIS} \quad h_1^\perp|_{DY} = -h_1^\perp|_{SIDIS}$$

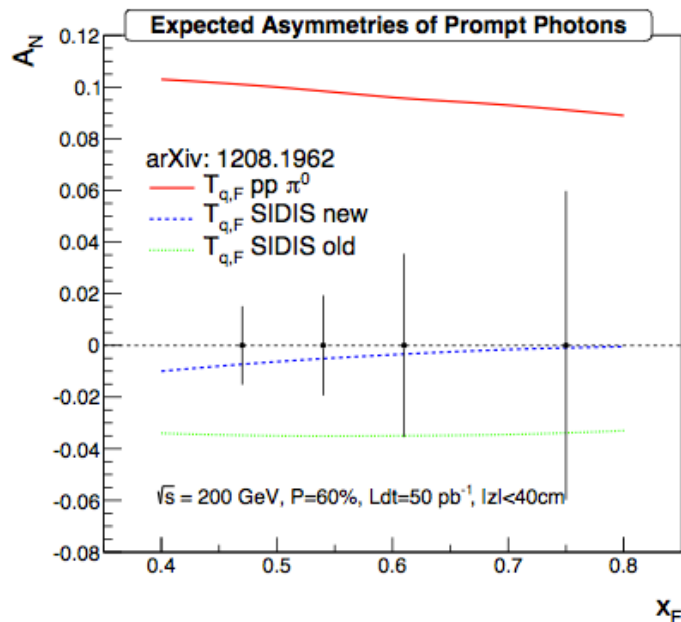
- Underlying physics: re-scattering of active partons with hadron remnants:
Final state interaction in semi-inclusive DIS vs **Initial state interaction** in Drell-Yan
(Brodsky, Hwang, Schmidt, 2002)



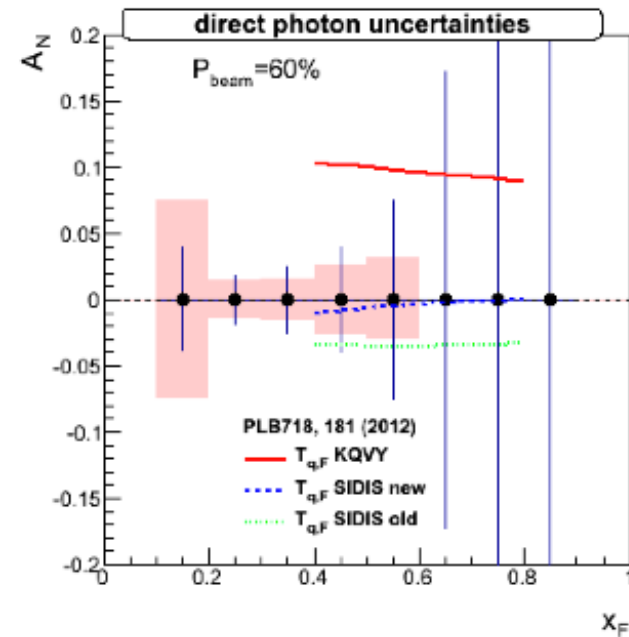
- Several labs worldwide aim at measurement of Sivers effect in Drell-Yan:
BNL, CERN, FermiLab, FAIR, IHEP, JINR, J-PARC
(→ talks by Peng, Seidl, Lorenzon, Aschenauer, ...)
- **Experimental verification of sign reversal is pending** (DOE milestone HP13!)

- First indications of process dependence of Sivers function from phenomenology
 - combined study of Sivers effect in SIDIS and transverse SSA A_N in inclusive DIS (AM, Pitonyak, Schäfer, Schlegel, Vogelsang, Zhou, 2012)
 - data on A_N for $p^\uparrow p \rightarrow \text{jet } X$ from AnDY compatible with process dependence (Gamberg, Kang, Prokudin, 2013)
 - first RHIC results on Sivers asymmetry in $p^\uparrow p \rightarrow (W, Z) X$ (\rightarrow talks by Seidl, Aschenauer, ...)
- Promising observable for studying process dependence: A_N in $p^\uparrow p \rightarrow \gamma X$

(PHENIX, scheduled)



(STAR, requested)

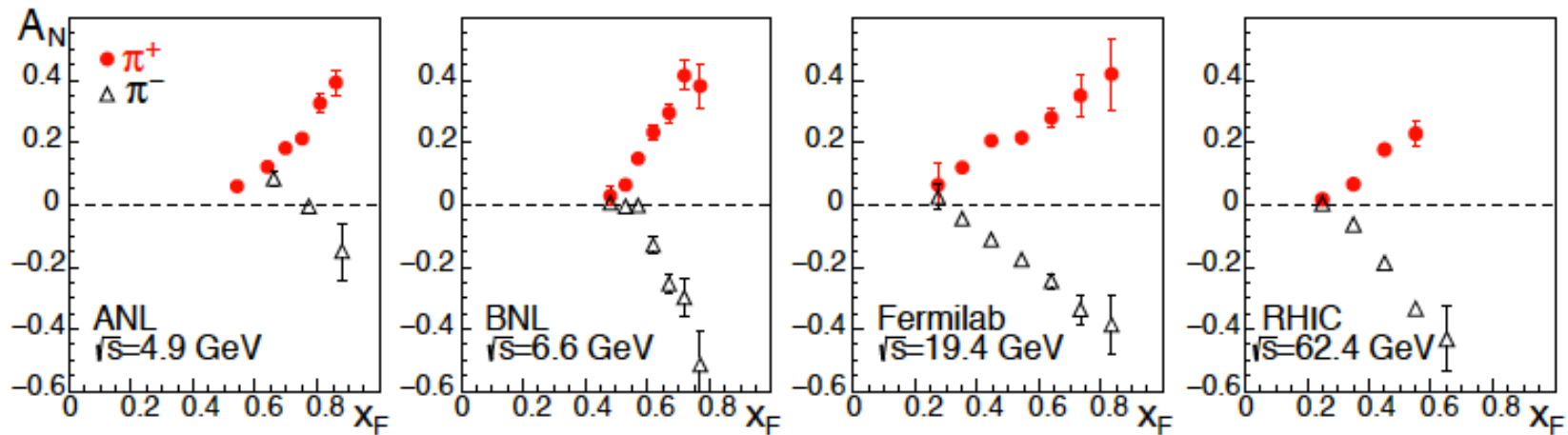


Transverse SSA in $p^\uparrow p \rightarrow h X$ and TMDs

(\rightarrow talks by Seidl, Liu, Aschenauer, ...)

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

$$x_F = \frac{2P_{hL}}{\sqrt{s}}$$



(from arXiv:1209.2803)

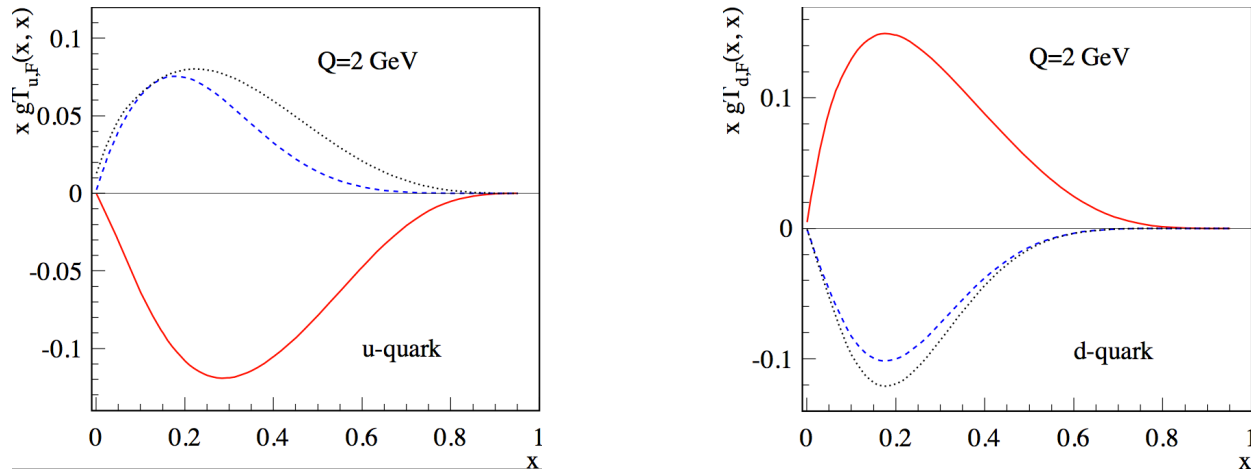
- Very striking effects
- Plenty of recent data from RHIC (BRAHMS, PHENIX, STAR)
- Such data are challenge for QCD calculations since about four decades

- A_N is twist-3 observable \rightarrow quark-gluon-quark correlations (T_F)
- Relation to Sivers function (Boer, Mulders, Pijlman, 2003)

$$g T_F(x, x) = - \int d^2 \vec{k}_T \frac{\vec{k}_T^2}{M} f_{1T}^\perp(x, \vec{k}_T^2) \Big|_{SIDIS}$$

\rightarrow two independent ways to determine T_F

- Sign-mismatch problem (Kang, Qiu, Vogelsang, Yuan, 2011)

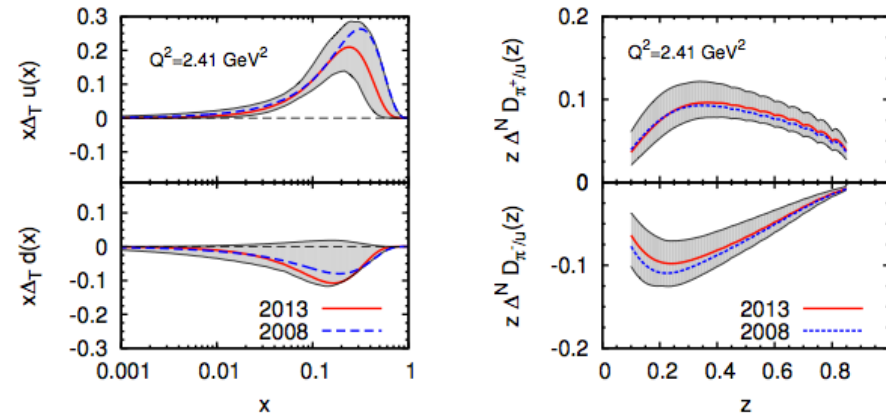


\rightarrow progress due to info from both proton-proton and lepton-proton collisions

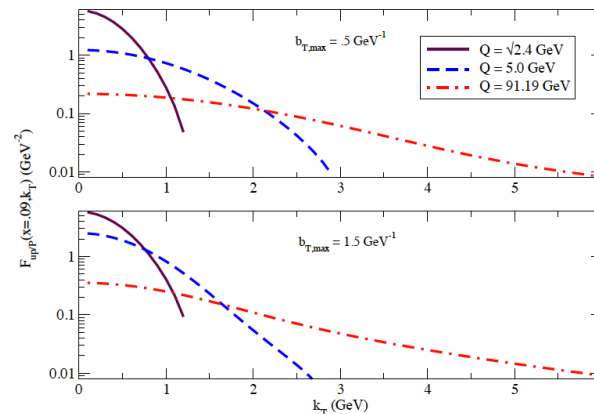
- (Twist-3) Sivers effect (T_F) most likely not main cause of A_N (AM, Pitonyak, Schäfer, Schlegel, Vogelsang, Zhou, 2012)
- Twist-3 fragmentation contribution could be main cause of A_N (AM, Pitonyak, 2012 / Kanazawa, Koike, AM, Pitonyak, 2014)

Further Progress in TMD Theory (selection)

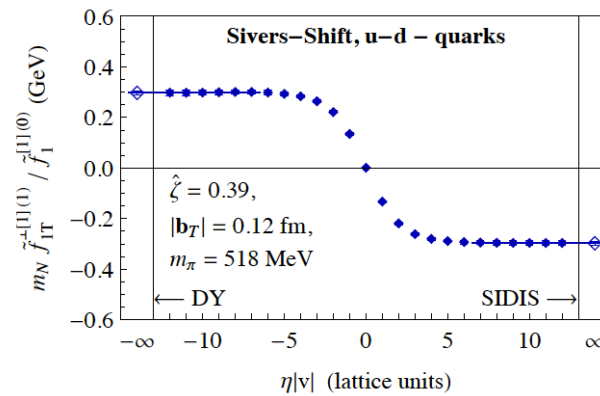
- Extraction of f_{1T}^\perp , Collins function H_{1T}^\perp , transversity h_1 , etc from recent data (Anselmino, Boglione, D'Alesio, Melis, Murgia, Prokudin, ... 2008 ... / other groups)
 - example: transversity and Collins function



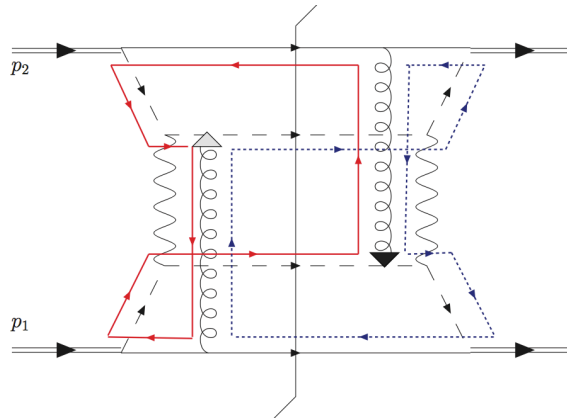
- Evolution of TMDs (Aybat, Rogers, 2011 / other groups)
 - example: unpolarized parton distribution $f_1^u(x, \vec{k}_T^2)$ at $x = 0.09$



- Pioneering results on TMDs in Lattice QCD (Hägler, Musch, Negele, Schäfer, 2009 / ...)
 - example: Sivers effect (from Musch, Hägler, Engelhardt, Negele, Schäfer, 2011)



- TMD factorization broken for processes like $pp \rightarrow \text{jet jet } X$ (Rogers, Mulders, 2010)



- Gluon TMDs at small x (regime of parton saturation)
 - Relation between TMD factorization and Color Glass Condensate approach (Dominguez, Marquet, Xiao, Yuan, 2010, 2011 ...)
 - Gluons at small x largely linearly polarized \rightarrow exploit to study parton saturation? (AM, Zhou, 2011 / Domingez, Qiu, Xiao, Yuan, 2011)

Wigner functions

- Wigner quasi-probability distribution in QM (calculable from wave function)

$$|\psi(x)|^2 = \int dp W(x, p)$$

$$|\psi(p)|^2 = \int dx W(x, p)$$

$$\langle O(x, p) \rangle = \int dx dp O(x, p) W(x, p)$$

- Analogy: Wigner distributions for 3-D imaging of hadrons

(Belitsky, Ji, Yuan, 2003 / Lorcé, Pasquini, Vanderhaeghen 2011)

$$f(x, \vec{b}_T) = \int d^2 \vec{k}_T W(x, \vec{b}_T, \vec{k}_T)$$

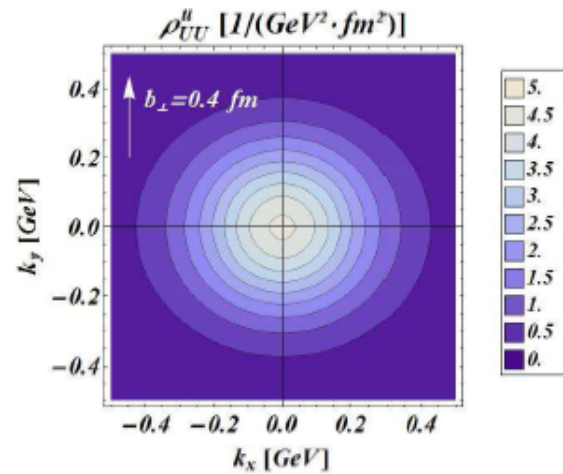
$$f(x, \vec{k}_T) = \int d^2 \vec{b}_T W(x, \vec{b}_T, \vec{k}_T)$$

$$\langle O(x, \vec{b}_T, \vec{k}_T) \rangle = \int dx d^2 \vec{b}_T d^2 \vec{k}_T O(x, \vec{b}_T, \vec{k}_T) W(x, \vec{b}_T, \vec{k}_T)$$

→ contain more information than $f(x, \vec{b}_T)$ and $f(x, \vec{k}_T)$, but generally less information than the wave function

- Full classification of Wigner functions exists for quarks and gluons (Meißner, AM, Schlegel, 2009 / Lorcé, Pasquini, 2013)

- Example of (quasi)-probability interpretation (Lorcé, Pasquini, 2011)



- distortion of $\int dx W_{UU}(x, \vec{b}_T, \vec{k}_T)$
- intuitive understanding from confinement
- many more examples exist

- Further application (Lorcé, Pasquini, 2011 / Hatta, 2011 / Ji, Xiong, Yuan, 2012)

$$L_z = \int dx d^2\vec{b}_T d^2\vec{k}_T (\vec{b}_T \times \vec{k}_T)_z W_{LU}(x, \vec{b}_T, \vec{k}_T)$$

→ in particular, L_z^{JM} may be calculable in Lattice QCD (Hatta, 2011)

- Parameterize/compute Wigner functions first, and then project onto GPDs, TMDs
- Open question: (how) can Wigner functions be extracted from experiment?

Expect the Unexpected

- **Concept of GPDs**
Müller et al, 1994 / Ji, 1996 / Radyushkin 1996
 - **Density interpretation of GPDs**
Burkardt, 2000, 2002 / Ralston, Pire, 2001 / Diehl, 2002
 - **Sivers function and its re-discovery**
Sivers, 1989 / Brodsky, Hwang, Schmidt, 2002 / Collins, 2002
 - **Wigner functions and their applications**
Belitsky, Ji, Yuan, 2003 / Meißner, AM, Schlegel, 2009 / Lorcé, Pasquini, Vanderhaeghen, 2011
 - **Sign-mismatch puzzle**
Kang, Qiu, Vogelsang, Yuan, 2011
 - **Electric form factor of the proton**
Jones et al, 1999
 - **Proton radius puzzle**
Pohl et al, 2010
 - **etc.**
- None of those crucial developments was (major) part of a long range plan
- Though we need plans for the future, scientific progress can hardly be planned

Summary

- Hadron structure studies have a very rich and successful history
- Hadron tomography (3-D imaging) can be considered a new era in this field
- The tools are GPDs, TMDs, and Wigner functions
- Tremendous progress in the last decade (experiment and theory)
- Plenty of open questions and challenges (experiment and theory)
- Future experimental facilities can advance the field to the next level (EIC would be crucial !)
- Wigner functions might be the ultimate tools to use
- Surprises can safely be expected