# Studying the QCD Phase Diagram via BES

## Fluctuations and the Critical Point

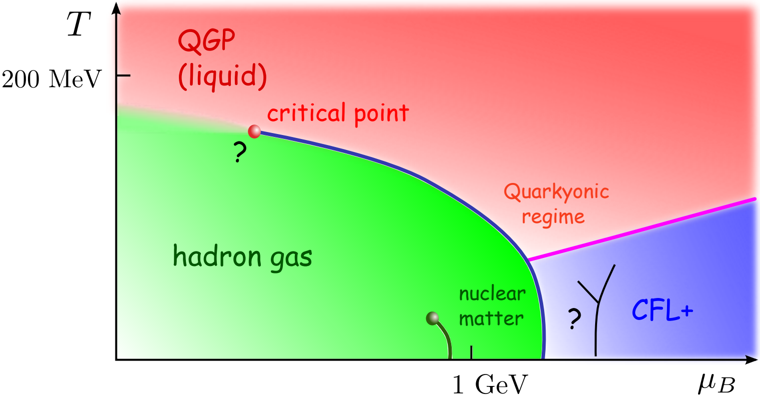
M. Stephanov

U. of Illinois at Chicago

# Outline

1. QCD phase diagram
2. Critical point and fluctuations
    - Higher moments
3. RHIC beam energy scan

# QCD Phase Diagram



Lattice at $\mu_B \lesssim 2T$ (reviewed by S. Mukherjee)

Critical point – a singularity of EOS, anchors the 1st order transition.

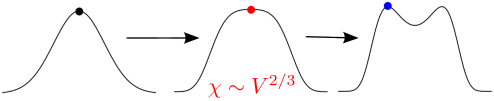
# Critical point and fluctuations

The key equation:

$$P(X) \sim e^{S(X)} \qquad \textit{(Einstein 1910)}$$

At the critical point $S(X)$ has a “flat direction” or “soft-mode”.
Fluctuation measures diverge:

$$\langle X^2 \rangle = -\left(\frac{\partial^2 S}{\partial X^2}\right)^{-1} = VT\chi$$



CLT?

$\xi \to \infty$

# Fluctuations of order parameter and $\xi$

- Fluctuations at CP – conformal field theory.
  Parameter-free → universality. Near CP $\xi = m_\sigma^{-1} < \infty$,

$$P[\sigma] \sim \exp\left\{-\Omega[\sigma]/T\right\},$$

$$\Omega = \int d^3x \left[\frac{1}{2}(\boldsymbol{\nabla}\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \ldots\right].$$

- Moments of order parameter $\sigma_V \equiv \int d^3x\, \sigma(x)$:

  - Each propagator gives $\xi^2$. Thus $\langle\sigma_V^2\rangle = VT\,\xi^2$.
  - As a result higher moments grow faster with $\xi$ with universal exponents

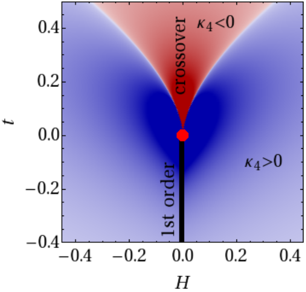
# Sign

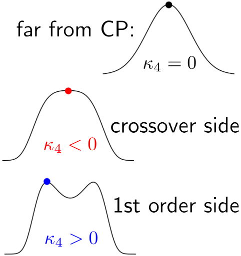
- Higher moments also depend on which side of the CP we are

$$\kappa_3[\sigma_V] = 2VT^{3/2}\,\tilde{\lambda}_3\,\xi^{4.5}\,; \quad \kappa_4[\sigma_V] = 6VT^2\,[\,2(\tilde{\lambda}_3)^2 - \tilde{\lambda}_4\,]\,\xi^7\,.$$

E.g., if symmetry ($\pm\sigma$) constrains $\lambda_3 = 0$ then $\kappa_3 = 0$ and $\kappa_4 < 0$.

- 2 relevant directions. Using universal Ising model variables:




far from CP:

$\kappa_4 = 0$

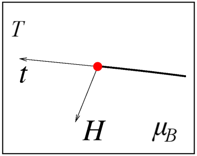
crossover side

$\kappa_4 < 0$

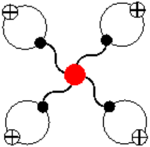
1st order side

$\kappa_4 > 0$

# Mapping to QCD



- In QCD $(t, H) \to (\mu - \mu_{\rm CP}, T - T_{\rm CP})$



- $\kappa_4[N] = \langle N\rangle + \kappa_4[\sigma_V] \times g^4 \Big($  $\Big)^4 + \dots,$

  $\kappa_4[\sigma_V] < 0$ means $\dfrac{\kappa_4[N]}{\langle N\rangle} < 1$

- Lessons: (Athanasiou-Rajagopal-MS 2010)
  - Sensitivity to $g$. Even more to $\mu_B[\mathrm{CP}]$ (exponential).
  - Ratios of cumulants can be used to reduce these uncertanties.
  - At large $\mu_B$ protons are as good as net-protons wrt CP search.

# Why $\xi$ is finite

System expands and is out of equilibrium

Universal scaling law:

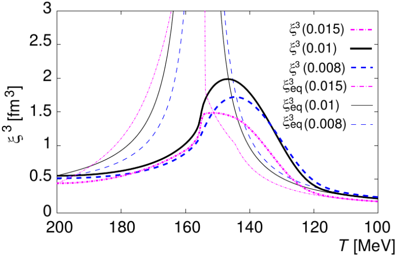
$$\xi \sim \tau^{1/z}, \qquad \text{where } 1/\tau \text{ is expansion rate}$$

and $z \approx 3$ (Son-MS).

Berdnikov-Rajagopal estimate
$\xi \sim 2 - 3$ fm.

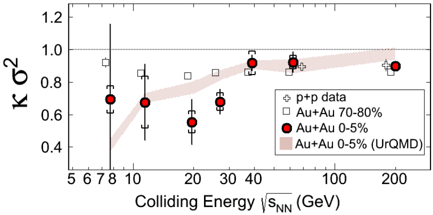
Hydrodynamics with a model EOS by Asakawa-Nonaka:

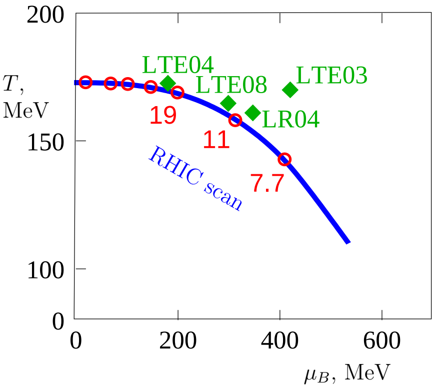
Significant for higher powers of $\xi$.



- Need full critical dynamics to take non-equilibrium into account

# RHIC beam energy scan

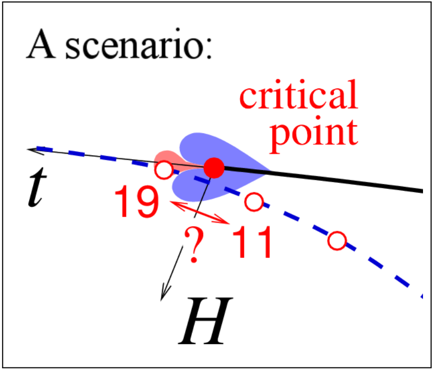


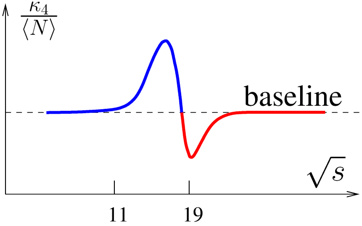


- Negative contribution to $\kappa_4$ around 19 GeV ($\mu_B \sim 200$ MeV).
- O(magnitude) consistent with estimates.

Acceptance effects important (Asakawa-Kitazava 2012
Bzdak-Koch 2012)

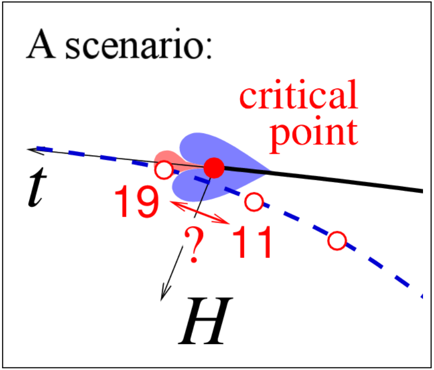
# A scenario/hypothesis

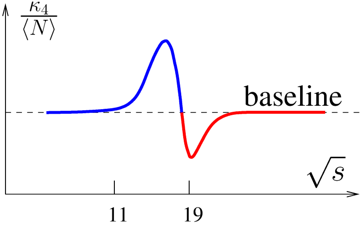




- Assuming critical region $\Delta\mu_B \sim \mathcal{O}(100)$ MeV.
- Critical region fits in the gap between 19 and 11 GeV.

# A scenario/hypothesis

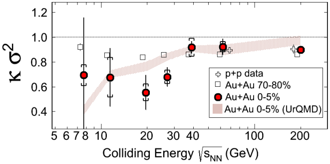




- Assuming critical region $\Delta\mu_B \sim \mathcal{O}(100)$ MeV.
- Critical region fits in the gap between 19 and 11 GeV.
- First order transition signatures at 11 and 7.7 GeV? (Soft EOS)

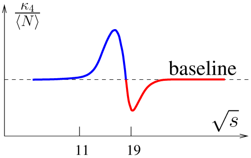
# What have we learned so far

- Consistent with lattice – no signals of the CP at $\mu_B < 200$ MeV.

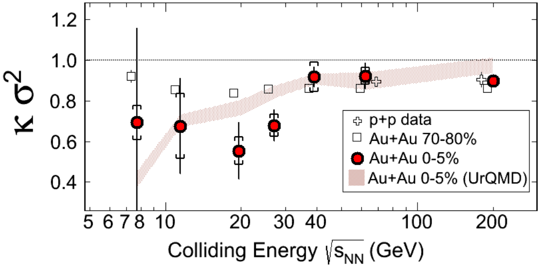


- Signal consistent with the scenario $\mu_B[\text{CP}] \sim 250$ MeV seen in $\kappa_4[N_{\text{protons}}]$.

  Inconclusive without $\kappa_4$ rising above the baseline.



# Questions and Thoughts



- Why in 0-5% but not in 70-80%?
  - Bigger system. Cools *slower*.
    Larger $\xi$ (Berdnikov-Rajagopal) and $\kappa_4 \sim \xi^7$.

- Important to study dynamical evolution of fluctuations.

# What needs to be done: theory

- Non-equilibrium critical dynamics simulations (H. Petersen's talk)

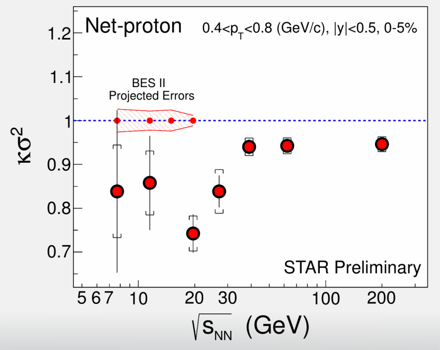
  Determine signal and background (baseline) given EOS.

- Better knowledge of the EOS near the critical point:
  - Critical region: size and shape, mapping $tH \to T\mu_B$

    (Asakawa,Nonaka;Sasaki,Friman,Redlich;Kapusta,Torres-Rincon;Koch,Randrup. . . )
  - Coupling $g$ of critical mode to protons, pions, kaons.

- Prediction of $\mu_B[\text{CP}]$: lattice. (S. Mukherjee's talk)

# What needs to be done: experiment



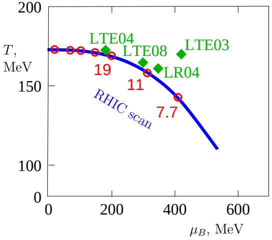
- Data at $\sqrt{s} \in [11 - 19]$ GeV is crucial. $\Rightarrow$ 14.5 GeV data + BESII.

  The rise above the baseline?

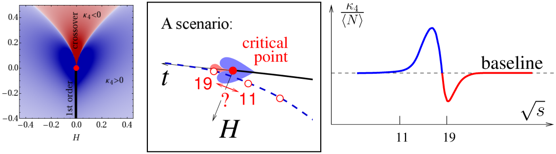
- More statistics at 7.7 and 11 GeV. $\Rightarrow$ BESII. (D. Cebra's talk)

# Summary: Beam Energy Scan and Fluctuations

Lattice and RHIC scan



Universality and 4th moment (kurtosis) near CP:



magnitude and sign strongly depend on $\sqrt{s}$: $\mathcal{O}(\xi^7)$.
Doubly non-monotonous.

- Critical region could fit in the gap between 19 and 11 GeV.
- Data at ∼15 GeV is needed.
- If the scenario above is realized: search for 1st-order transition signatures at 11, 7.7 GeV and lower (+FAIR).