Transverse Force on Quarks in DIS

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$d_2 \leftrightarrow$ average $\perp$ force on quark in DIS from $\perp$ pol target

polarized DIS:

- $\sigma_{LL} \propto g_1 - \frac{2Mx}{\nu} g_2$
- $\sigma_{LT} \propto g_T \equiv g_1 + g_2$

$\hookrightarrow$ 'clean' separation between g_2 and $\frac{1}{Q^2}$ corrections to g_1

- $g_2 = g_2^{WW} + \bar{g}_2$ with $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$

$$d_2 \equiv 3 \int dx\, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x} \left\langle P,S \left| \bar{q}(0)\gamma^+ gF^{+y}(0)q(0) \right| P,S \right\rangle$$

color Lorentz Force on ejected quark (MB, PRD 88 (2013) 114502)

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v} \times \vec{B}\right)^y \text{ for } \vec{v} = (0,0,-1)$$

matrix element defining d_2 $\leftrightarrow$ 1^{st} integration point in QS-integral
$\hookrightarrow$ Sivers

$\int x^2 e(x)$ (scalar twist-3 PDF) $\leftrightarrow$ 'Boer-Mulders force':
$\perp$ pol. quarks; unpol. target

OAM from Wigner Functions

Wigner Functions (Belitsky, Ji, Yuan; Metz et al.)

$$W(x,\vec{b}_\perp,\vec{k}_\perp) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} \int \frac{d^2\xi_\perp d\xi^-}{(2\pi)^3} e^{ik\cdot\xi} e^{-i\vec{q}_\perp\cdot\vec{b}_\perp} \langle P'S'|\bar{q}(0)\gamma^+ q(\xi)|PS\rangle.$$

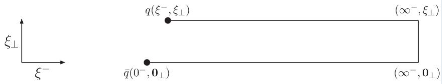
- TMDs: $f(x,\mathbf{k}_\perp) = \int d^2\mathbf{b}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)$
- GPDs: $q(x,\mathbf{b}_\perp) = \int d^2\mathbf{k}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)$
- $L_z = \int dx \int d^2\mathbf{b}_\perp \int d^2\mathbf{k}_\perp W(x,\vec{b}_\perp,\vec{k}_\perp)(b_x k_y - b_y k_x)$
- need to include Wilson-line gauge link $\mathcal{U}_{\prime\xi}$ to connect 0 and ξ (Ji, Yuan; Hatta; Lorcé;...)

straight line (Ji et al.)

straigth Wilson line from 0 to ξ yields Ji-OAM:

$$L^q = \int d^3x \langle P,S| q^\dagger(\vec{x}) \left(\vec{x}\times i\vec{D}\right)^z q(\vec{x})|P,S\rangle$$

Light-Cone Staple for $\mathcal{U}^{\pm LC}_{0\xi}$ (Hatta)



'light-cone staple' yields $\mathcal{L}_{Jaffe-Manohar}$

Quark OAM from Wigner Distributions

straight line (→Ji)

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + L_q + J_g$$

$$L_q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{D}\right)^z q(\vec{x})|P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

light-cone staple (→ Jaffe-Manohar)

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

$$\mathcal{L}^q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{\mathcal{D}}\right)^z q(\vec{x})|P,S\rangle$$

$$i\mathcal{D}^j = i\partial^j - gA^j(x^-,\mathbf{x}_\perp) - g\int_{x^-}^\infty dr^- F^{+j}$$

difference $\mathcal{L}^q - L^q$

$$\mathcal{L}^q - L^q = -g\int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left[\vec{x}\times\int_{x^-}^\infty dr^- F^{+\perp}(r^-,\mathbf{x}_\perp)\right]^z q(\vec{x})|P,S\rangle$$

color Lorentz Force on ejected quark (MB, PRD 88 (2013) 014014)

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v}\times\vec{B}\right)^y \quad \text{for } \vec{v} = (0,0,-1)$$

Torque along the trajectory of q

$$T^z = \left[\vec{x}\times\left(\vec{E} - \hat{\vec{z}}\times\vec{B}\right)\right]^z$$

Change in OAM

$$\Delta L^z = \int_{x^-}^\infty dr^- \left[\vec{x}\times\left(\vec{E} - \hat{\vec{z}}\times\vec{B}\right)\right]^z$$

straight line (→Ji)

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + L_q + J_g$$

$$L_q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x} \times i\vec{D}\right)^z q(\vec{x})|P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}(\vec{x})$

light-cone staple (→ Jaffe-Manohar)

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

$$\mathcal{L}^q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x} \times i\vec{\mathcal{D}}\right)^z q(\vec{x})|P,S\rangle$$

- $i\vec{\mathcal{D}} = i\vec{\partial} - g\vec{A}(x^- = \infty, \mathbf{x}_\perp)$

difference $\mathcal{L}^q - L^q$ (MB, PRD 88 (2013) 014014)

$\mathcal{L}^q - L^q = \Delta L^q_{FSI}$ = change in OAM as quark leaves nucleon

example: torque in magnetic dipole field