RECONSTRUCTION AND REWEIGHTING PROPOSAL

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1. Cross-sections

Our goal is to re-weight vertex level Monte Carlo Event Generators (MCEG) to reflect a given NLO theoretical cross-section σ_{nlo}. As an example lets look at the neutral current (NC) case. For a given kinematic bin in x and Q^2 the differential NC cross-section may be written as :

$$\frac{d\sigma^{NC}_{Born}(i)}{dxdQ^2} = \frac{N^{NC}_{Born}(i)}{\Delta x \Delta Q^2}\frac{1}{\mathcal{L}} \tag{1}$$

The bin i denotes the (x, Q^2) measured at the vertex and $\mathcal{L}$ is the integrated luminosity. N^{NC}_{Born} is the number of reconstructed NC events, fully corrected for background, smearing, acceptance and radiative effects. N^{NC}_{Born} is extracted from the raw number of identified NC events, N^{NC}_{Raw}:

$$N^{NC}_{Raw}(j) = N^{NC}_{Born}(j)(1+\delta_v) + N^{NC}_{Elastic}(j) + N^{NC}_{Inelastic}(j) + N^{\pi^-}_{Bkg}(j) + N^{PS}_{Bkg}(j) \tag{2}$$

The bin j denotes the (x, Q^2) as reconstructed in the detector where radiative, resolution and acceptance effects have been included. Note that for deuteron/ion an additional term for the radiative quasi-elastic tail should be included. The contributions are defined as:

$N^{NC}_{Born}\delta_v$	contribution from internal (vertex, vacuum) radiative effects
$N^{NC}_{Elastic}$	counts from NC radiative elastic tail
$N^{NC}_{Inelastic}$	counts from NC radiative inelastic tail
$N^{\pi^-}_{Bkg}$	counts from mis-identified charged pions
N^{PS}_{Bkg}	counts from pair symmetric backgrounds ($\pi^0 \to e^+e^-\gamma$, $\gamma \to e^+e^-$)

Next we calculate N^{NC}_{Corr} by subtracting the background contributions from N^{NC}_{Raw} and correcting for all detector effects:

$$N^{NC}_{Corr}(k) = [N^{NC}_{Raw}(j) - N^{\pi^-}_{Bkg}(j) - N^{PS}_{Bkg}(j)] \times D(j,k)|_{k=j} \tag{3}$$

Here D(j,k) is defined as the ratio of the number of NC events in a given kinematic bin k before detector smearing $N^{NC}_{true}(k)$ to the number of NC events in a given kinematic bin j after smearing $N^{NC}_{det}(j)$. Both terms include radiative effects so the D(j,k) factor only corrects for detector effects.

Next we correct for radiative effects by multiplying N^{NC}_{Corr} by R(k,i), the ratio of the number of counts without radiative effects in bin i N^{NC}_{noRC} to the number of counts with radiative effects included: N^{NC}_{RC}.

$$N^{NC}_{Born}(i) = N^{NC}_{Corr}(k) \times R(k,i)|_{i=k} \tag{4}$$

Assuming we use the multiplicative factor then $N^{NC}_{Born}(i)$ is the number of inelastic NC current events binned at the vertex level and the fully corrected NC born cross-section is now:

$$\frac{d\sigma^{NC}_{Born}(i)}{dxdQ^2} = \frac{N^{NC}_{Raw}(j) - N^{\pi^-}_{Bkg}(j) - N^{PS}_{Bkg}(j)}{\Delta x \Delta Q^2} \times D(j,k) \times R(k,i) \times \frac{1}{\mathcal{L}} \tag{5}$$

In practice, because $N^{NC}_{true}(k) == N^{NC}_{RC}(k)$, the detector and radiative effects corrections can be combined into one factor $N^{NC}_{noRC}(i)/N^{NC}_{det}(j)$. Equation 5 is a trivial equivalency since we are using simulated data, but it should allow us to calculate errors that reflect our corrections and therefore our detector effects.

Next, lets consider how to modify each term for the re-weighting. The weights are defined as the ratio of the NLO theory and LO MCEG cross-sections at the vertex level, $w(i) = \frac{\sigma_{nlo}(i)}{\sigma_{mceg}(i)}$, and should be applied to the $N^{NC}_{noRAD}(i)$ term in the numerator. Technically $w(i)$ should also be applied to N^{NC}_{Corr} and N^{NC}_{det} counts as well, but they will simply cancel as they factor into the numerator and the denominator in the same way. Note that we should be careful to make sure $\mathcal{L}$ is held constant with the re-weighting procedure - that is $N_{thrown} = \mathcal{L} \times \sigma_{NLO}$ and not $\mathcal{L} \times \sigma_{MCEG}$.

1.1. **Questions.**

- Experimentally there is some variation in how radiative corrections are implemented. SLAC used multiplicative and additive factors but I used only additive at JLAB and this will change the error bars. How do we want to implement?
- How do we construct N^{NC}_{RAW}? How do we handle two electrons in an event? Do we only apply the Pair-symmetric correction if it is the only electron in the event?

2. Asymmetries

Our goal is to construct a MCEG sample that reflects a given NLO theoretical partonic asymmetry $\hat{a}_{nlo}$. Lets use the case of $A_{||}$, necessary for the extraction of g_1 and g_2, as a working example. For a given kinematic bin in x and Q^2 the inclusive asymmetry $A_{||}$ may be written as :

$$A^{Born}_{||}(i) = \frac{1}{P_e P_p} \times \frac{\frac{N^{\uparrow\uparrow+\downarrow\downarrow}_{Born}(i)}{L^{\uparrow\uparrow+\downarrow\downarrow}} - \frac{N^{\uparrow\downarrow+\downarrow\uparrow}_{Born}(i)}{L^{\uparrow\downarrow+\downarrow\uparrow}}}{\frac{N^{\uparrow\uparrow+\downarrow\downarrow}_{Born}(i)}{L^{\uparrow\uparrow+\downarrow\downarrow}} + \frac{N^{\uparrow\downarrow+\downarrow\uparrow}_{Born}(i)}{L^{\uparrow\downarrow+\downarrow\uparrow}}} = \frac{1}{P_e P_p} \times \frac{N^{+}_{Born}(i) - RN^{-}_{Born}(i)}{N^{+}_{Born}(i) + RN^{-}_{Born}(i)} \tag{6}$$

The bin i denotes the (x, Q^2) measured at the vertex, while $L^{\uparrow\uparrow+\downarrow\downarrow}$ and $N_{Born}^{\uparrow\uparrow+\downarrow\downarrow}(i)$ are the luminosity and the number of reconstructed NC events, respectively, when the helicity of the electron and proton beams are the same. $N_{Born}^{+}(i)$ is fully corrected for background, smearing, acceptance and radiative effects. Following the same procedure as for the cross-sections above $N_{Born}^{+}(i)$ is extracted from the raw number of counts N_{Raw}^{+}:

$$N_{Raw}^{+}(j) = N_{Born}^{+}(1+\delta_v) + N_{Elastic}^{+}(j) + N_{Inelastic}^{+}(j) + N_{\pi^-}^{+}(j) + N_{PS}^{+}(j) \tag{7}$$

The bin j denotes the (x, Q^2) as reconstructed in the detector where radiative, resolution and acceptance effects have been included. The NC contributions are defined as:

$N^{+}\delta_v$	contributions from internal (vertex, vacuum) radiative effects
$N_{Elastic}^{+}(j)$	counts from NC radiative elastic tail
$N_{Inelastic}^{+}(j)$	counts from NC inelastic radiative elastic tail
$N_{\pi^-}^{+}(j)$	counts from mis-identified charged pions
$N_{PS}^{+}(j)$	counts from pair symmetric backgrounds ($\pi^0 \to e^+e^-\gamma$, $\gamma \to e^+e^-$)

If the analyzer is using Djangoh then it is possible to generate two data samples, one for σ^+ and one for σ^-, follow the correction procedure described in Section 1 for $N_{Raw}^{+}(j)$ and $N_{Raw}^{-}(j)$ and then construct the asymmetry $A_{ll}^{Born}(i)$ from the fully corrected $N_{Born}^{+}(i)$ and $N_{Born}^{-}(i)$. If the analyzer is using an unpolarized generator it is still possibly to simply weight two samples with the appropriate cross-sections and change the weight accordingly to reflect the unpolarized PDFs.

3. Binning

The bin widths for a given measurement are typically determined by the detector resolution. For each detector configuration implemented in EICsmear the purity for the detected electron should be calculated and the binning re-optimized if necessary. Purity is defined as the fraction of electrons reconstructed in the same bin they were generated. Purities above 30% are acceptable.