EVENT GENERATION THEORY OVERVIEW

RAINER J FRIES
TEXAS A&M UNIVERSITY

- Monte Carlo Methods
- Modeling of High Energy Nuclear Collisions
- Summary

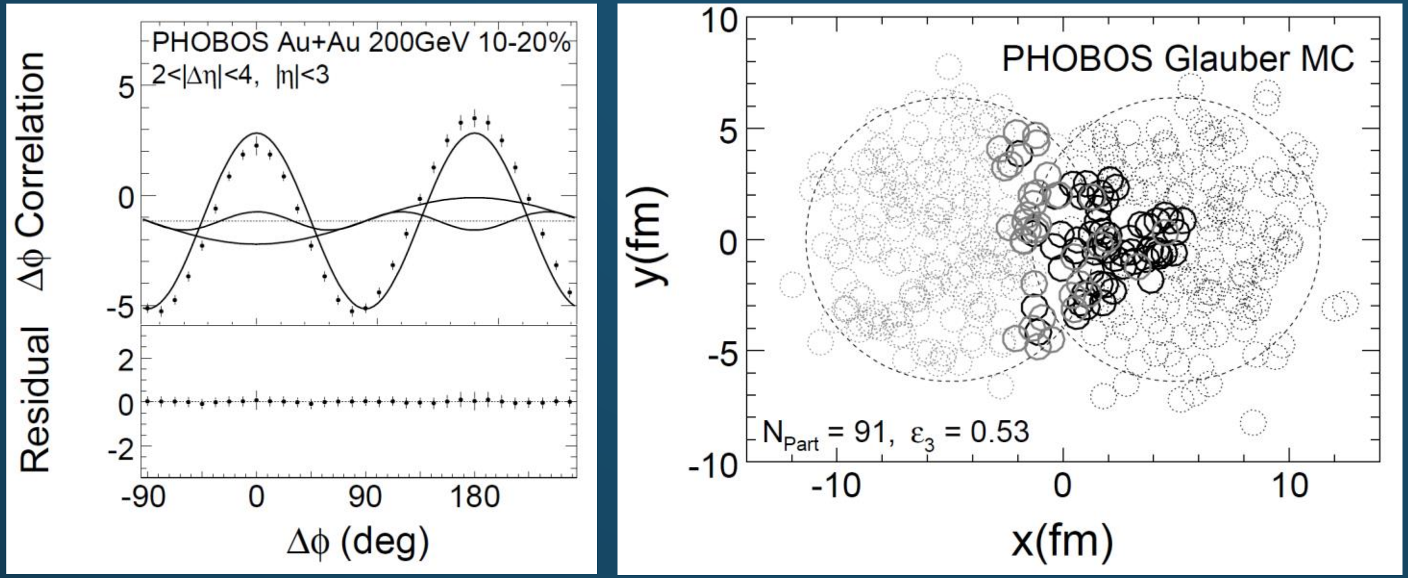
PART I: MONTE CARLO METHODS

WHY MONTE CARLO EVENT GENERATORS?

- In nuclear and particle physics experimentalists have very little control over initial conditions.
- Most experimental analyses are done by adding events to enhance signals and suppress fluctuations.

$$\text{Observable}\left(\sum_i \text{event } i\right) \neq \sum_i \text{Observable}(\text{event } i)$$

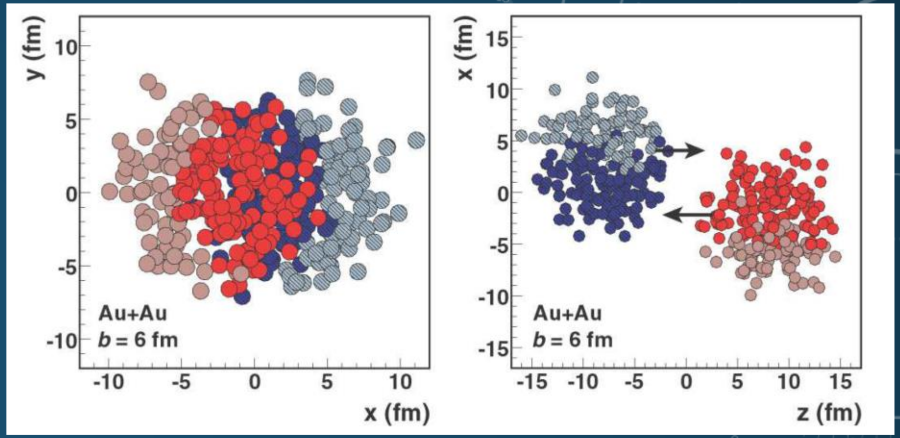
- Historic example: triangular flow v_3



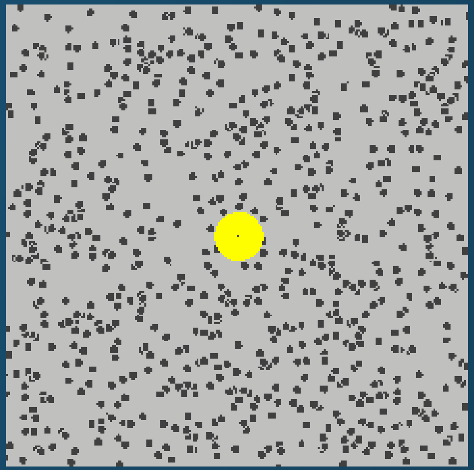
Alver and Roland, PRC 81, 054905 (2010)

WHY MONTE CARLO EVENT GENERATORS?

- Initial state fluctuations
  - Impact parameter, orientation of nuclei (if non-spherical)
  - Positions of nucleons at collision, other details of the wave function frozen at collision
- Fluctuations from classical distributions
  - Brownian motion (e.g. heavy quarks)
  - Thermal distributions (e.g. hadrons at freeze-out)
- Quantum fluctuations
  - E.g. final state gluon radiation



Miller et al., Ann. Rev. Nuc. Part. Sc. 57, 205 (2007)



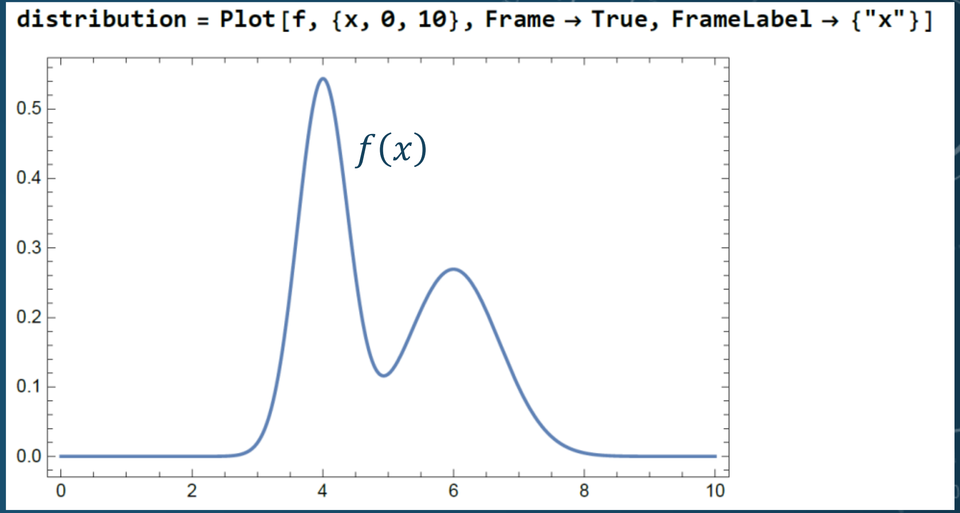
commons.wikimedia.org/wiki/File:Brownian_motion_large.gif

EVENT CREATION

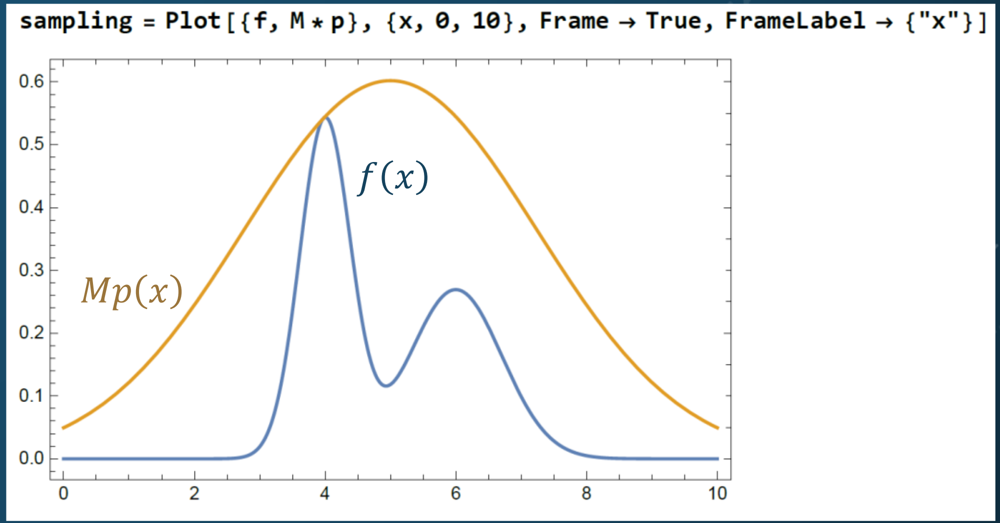
- If the probability distributions of statistical and quantum processes are known, we can sample those to simulate single events → Accumulate large numbers of events.
- Most processes/modules need Monte Carlo sampling as part of their intrinsic dynamics (e.g. parton shower Monte Carlos)
- Some processes/modules are deterministic (e.g. fluid dynamics) but need sampling to determine initial conditions or final particle output
- Powerful Monte Carlo sampling tools are available to do the job.
- Physics needs to determine the probability distributions.

MONTE CARLO SAMPLING: REJECTION SAMPLING

- Rejection Sampling: A simple but effective method for many applications.
- Given a probability distributions $f(x)$ to be sampled in an interval $x \in [a, b]$.
  - Should be positive definite but could have integrable singularities
- Choose a "proposal" function $p(x)$, will be used as an upper bound for $f(x)$
  - Should be easily sampleable, e.g. direct sampling, convenient library; a constant function is often okay (but could be wasteful).
- Scaled proposal function $Mp(x)$ where $M = \sup_x \frac{f}{p}$ is an optimized upper bound for $f(x)$.

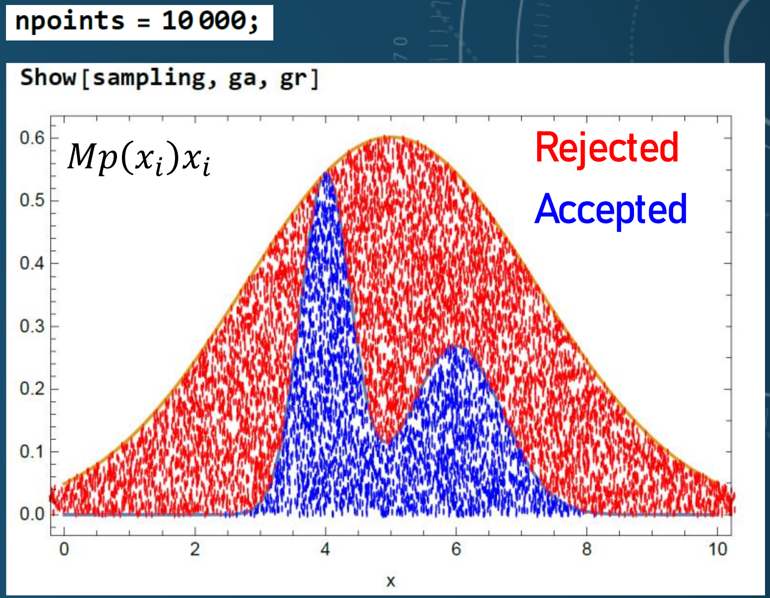


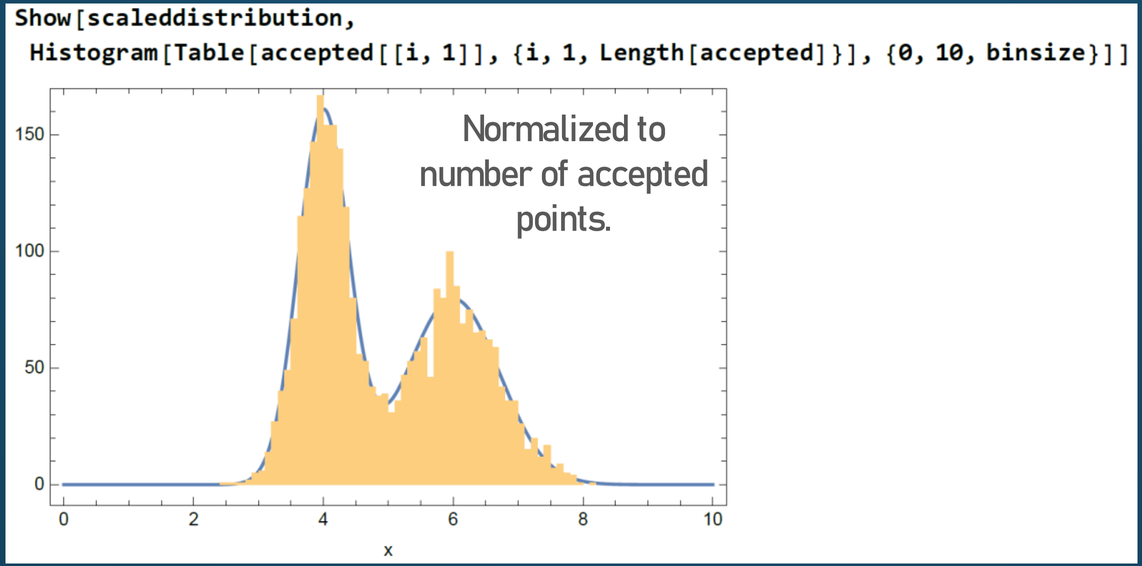

```
M = MaxValue[{f / p, x > 0, x < 10}, x]
0.601856
```



MONTE CARLO SAMPLING: REJECTION SAMPLING

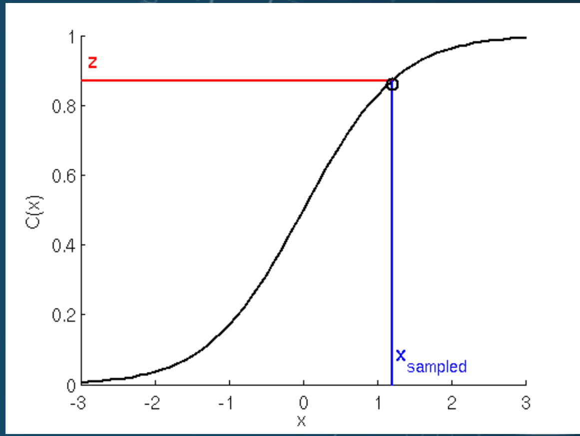
- Throw dice to determine pairs of numbers (x_i sampled from $Mp(x)$, p_i random number sampled from interval $[0,1]$).
- Accept number x_i if $p_i < \frac{f(x_i)}{Mp(x_i)}$.
  - By construction the rhs is always between 0 and 1
- The accepted $\{x_i\}$ represent the original distribution $f(x)$.
- Histogram it:



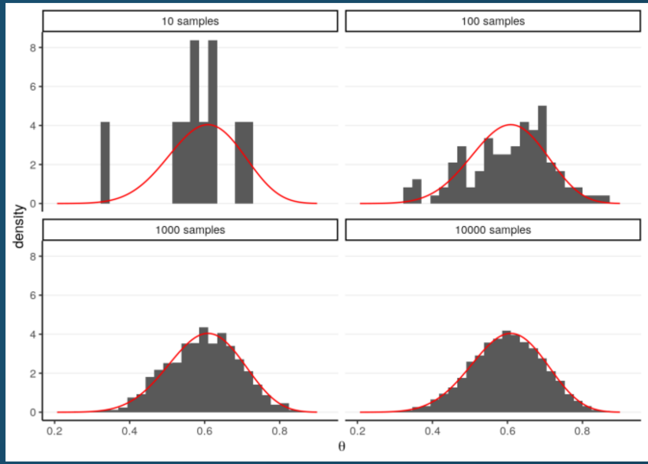


MONTE CARLO SAMPLING: OTHER METHODS

- Direct sampling using transformations of probability distribution functions.



- Sampling of the cumulative distribution (inverse transform sampling).



- Markov Chain (MCMC), e.g. Metropolis algorithm

- Second Example: Thermal blast wave cell, using the former two methods

MONTE CARLO SAMPLING: BOOSTED THERMAL DISTRIBUTION

- Heavy ion collisions at late stages have particles moving collectively and close to local thermal equilibrium.
  - For example: freeze-out from fluid dynamics.
  - The particle distribution is a boosted Boltzmann distribution $f(p) \sim e^{-\frac{p \cdot u}{T}}$ where p is the particle 4-momentum, T is temperature and u is the collective 4-velocity. (we neglect quantum corrections and corrections to perfect kinetic equilibrium for simplicity).
- Step 1: transform into the cell rest frame $\rightarrow$ distribution $f(\tilde{p}) = e^{-\frac{\tilde{E}}{T}}$ acquires spherical symmetry.
  - Undo by applying Lorentz transformation $p^\mu = \Lambda^\mu_\nu(u)\tilde{p}^\nu$ to all particle momenta after rest frame sampling.

MONTE CARLO SAMPLING: BOOSTED THERMAL DISTRIBUTION

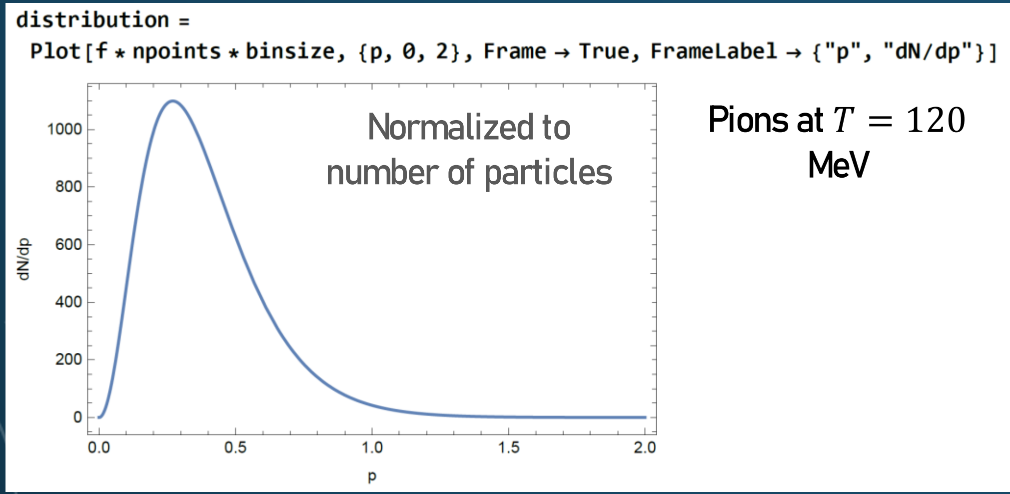
- Step 2: Factorize rest frame distribution in spherical coordinates:

$$f(\tilde{p}) = \frac{dN}{d^3\tilde{p}} = \frac{1}{4\pi\tilde{p}^2}\frac{dN}{d\phi}\frac{dN}{d\cos\theta}\frac{dN}{d\tilde{p}}$$

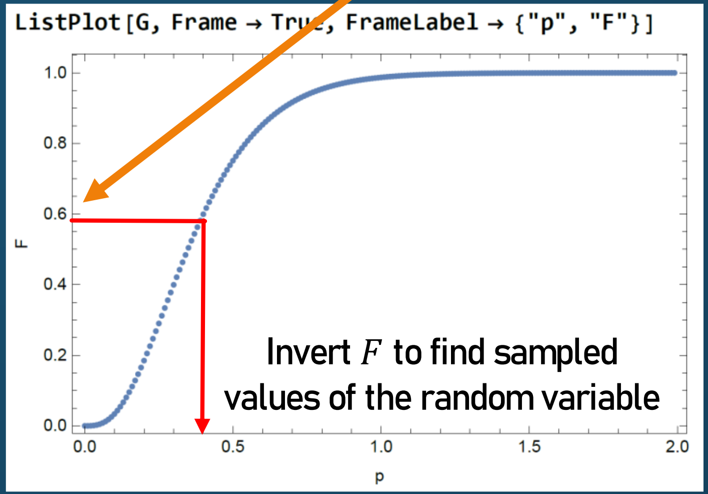
- Step 3: Remaining 1-D distributions are simple to sample

$$\frac{dN}{d\phi}\sim 1 \qquad \frac{dN}{d\cos\theta}\sim 1 \qquad \frac{dN}{d\tilde{p}}\sim \tilde{p}^2 e^{-\frac{\tilde{E}}{T}}$$

  - Compute cumulative distribution $F(P)$ of the probability density associated with $\frac{dN}{d\tilde{p}}$

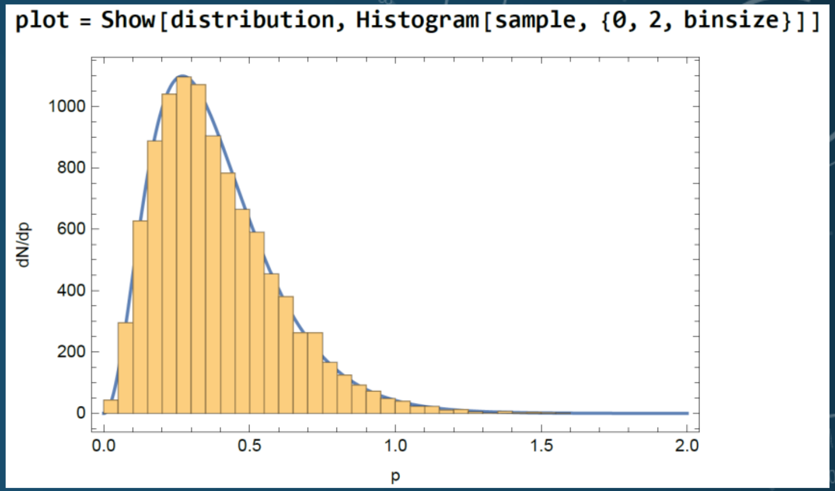


Throw dice in the range [0,1] here

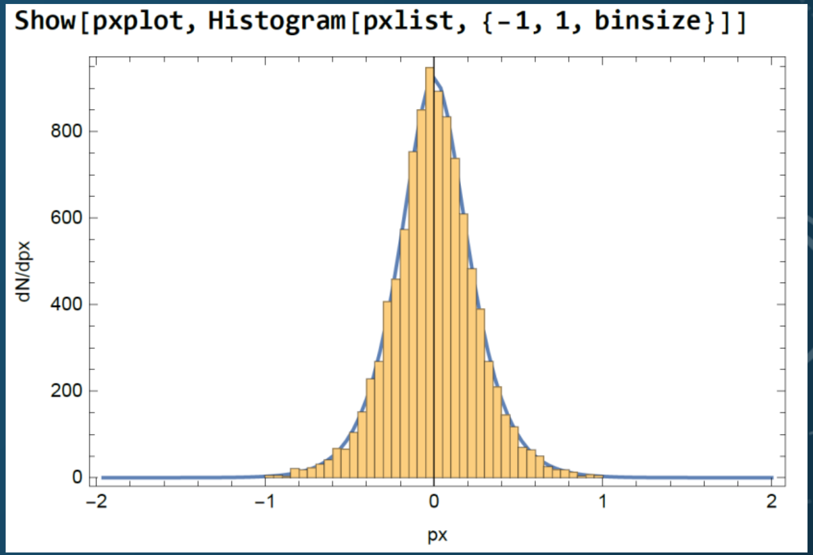


MONTE CARLO SAMPLING: BOOSTED THERMAL DISTRIBUTION

- Inverse transform sampling recovers the original distribution $f(\tilde{p})$.



- Add sampling of the angles to get full momentum vectors.
  - Distribution of the x-momentum, sampled vs analytic.

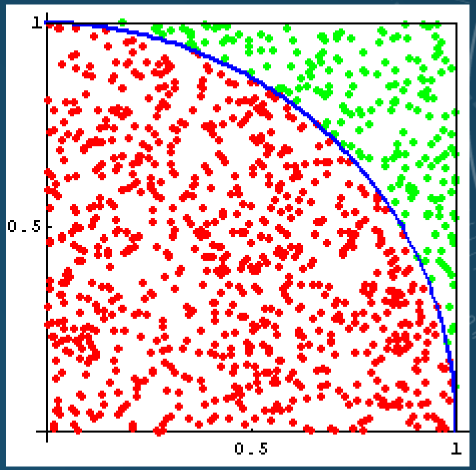


- Step 4: Restore collective motion by boost.

MONTE CARLO SAMPLING

- Suggested homework problems:

- Work out π can be determined by sampling a circle or segment of a circle in 2-D. Write a code using Rejection Sampling to compute π.



- Write a code to sample 100 thermal pions ($m = 140$ MeV) at a temperature of 120 MeV, with collective velocity v=0.7 c in x-direction.

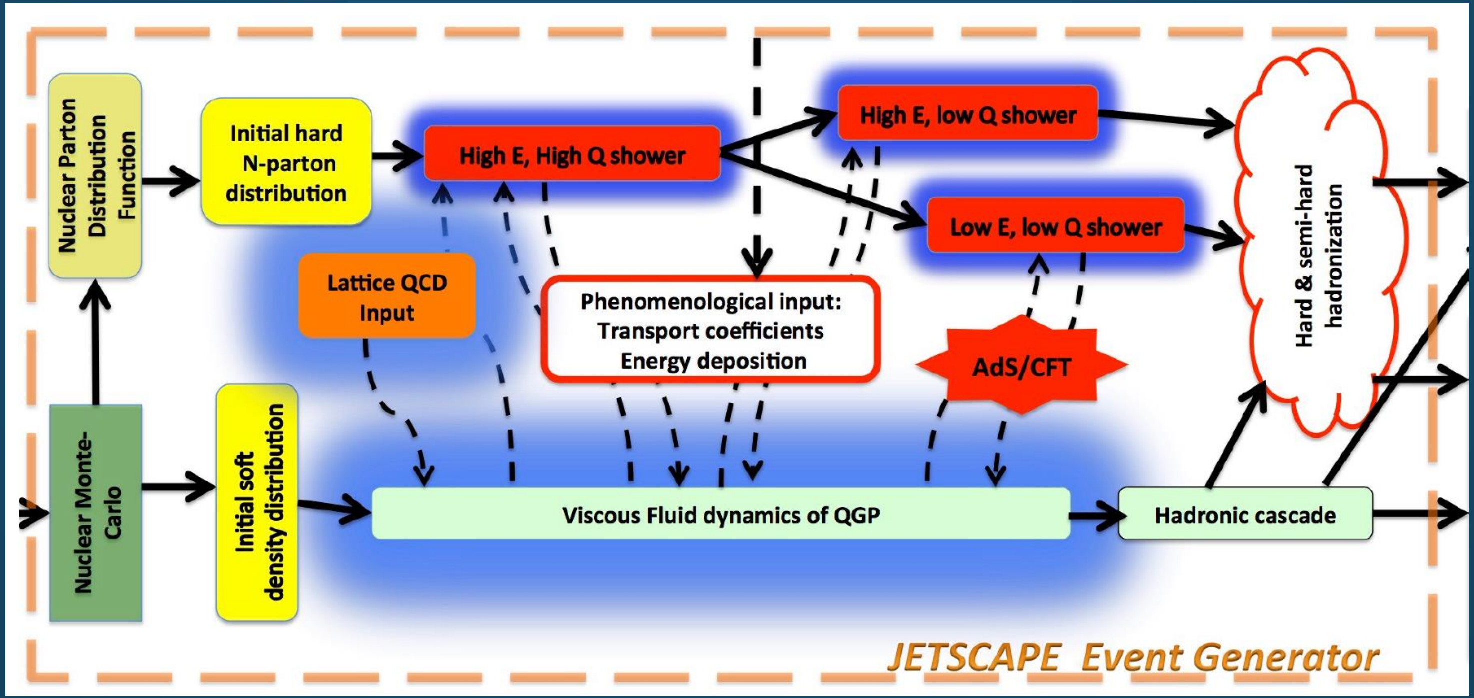
PART II: MODELING OF NUCLEAR COLLISIONS

HIGH ENERGY NUCLEAR COLLISIONS

- There is no comprehensive, self-consistent, QCD based theory for HI collisions

$$amplitude \sim \langle final\ state |\ \ldots QCD \ldots | nuceus\ 1\ \times nucleus\ 2 \rangle$$

- Progress has been made by tackling single aspects of the collision, applying QCD/effective theories/models. JETSCAPE is a tool that allows users to pull these aspects together in unified framework.



HIGH ENERGY NUCLEAR COLLISIONS

- The most important segmentation happens in the initial phase due to the longitudinal momentum of the partons

$$amplitude \sim \langle final\ state|\ \ldots QCD \ldots |nuceus\ 1\ \times nucleus\ 2\rangle$$

$$\sim \langle final\ state| \ldots QCD \ldots | \sum_{partons} |partons(\vec{p}, s, c)\rangle \underbrace{\langle partons(\vec{p}, s, c)|nucleus\ 1\ x\ nucleus\ 2\rangle}$$

Partons with momenta, spins and color in the two nuclear wave functions

Bjorken-x: $x = \frac{p_z}{p_{\text{nucleon}}}$

- Rough classification of parton-parton interactions between the two nuclei:
  - Large-x + large-x $\to$ jets and other hard probes (Rare! Need $p_z \gtrsim 1$ GeV).
  - Large-x + small-x $\to$ single jets at forward/backward rapidities.
  - Small-x + small-x $\to$ soft interactions, full or partial thermalization, bulk!

HIGH ENERGY NUCLEAR COLLISIONS

- Rough classification of parton-parton interactions between the two nuclei:
  - Large-x + large-x $\longrightarrow$ jets and other hard probes (Rare! Need $p_z \gtrsim 1$ GeV).
  - Large-x + small-x $\longrightarrow$ single jets at forward/backward rapidities.
  - Small-x + small-x $\longrightarrow$ soft interactions, full or partial thermalization, bulk!

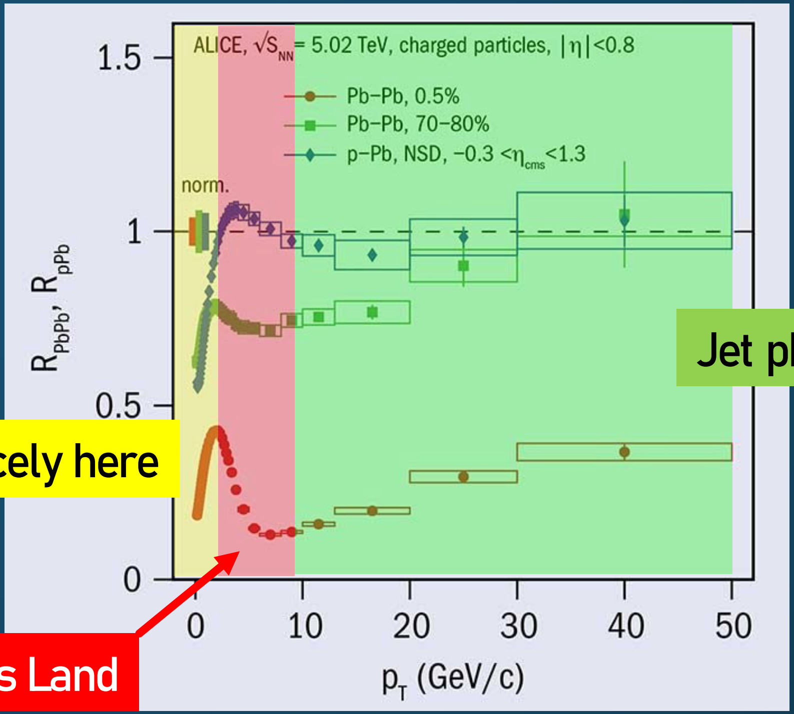
Low-x partons are plentiful $\longrightarrow$ multiple scattering

JETSCAPE bulk physics

JETSCAPE hard probes

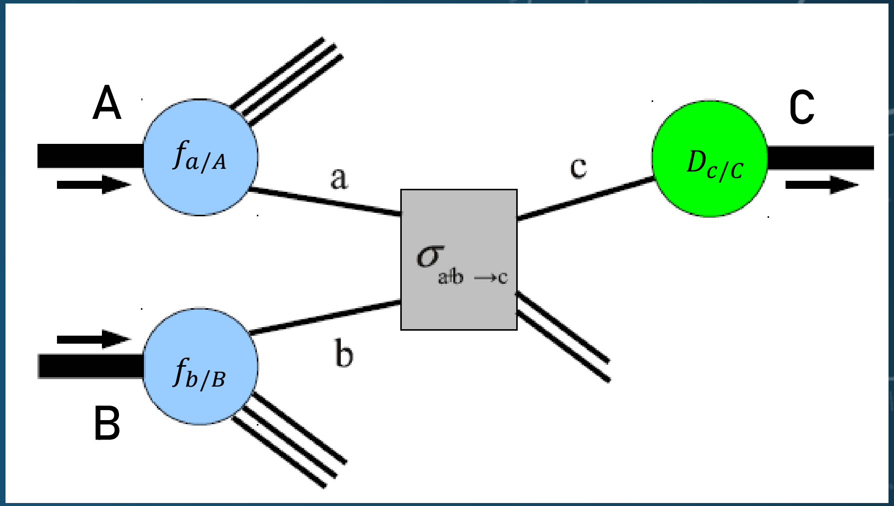
- Of course the different regimes will interact in the subsequent evolution. This is the point of hard "probes".

- However, different domains still clearly visible in data:



FACTORIZATION, PDFS, PQCD

Collins, Soper and Sterman, Adv.Ser.Direct.High Energy Phys.5, 1 (1988)



- High momentum transfer processes:
  - pQCD applicable
  - Factorization theorems to separate universal long-distance behavior from pQCD
- For example, inclusive hadron production in hadron-hadron collisions.

$$d\sigma^{A+B\to C+X} = \sum_{a,b,c} f_{a/A} \otimes d\sigma^{a+b\to c+x} \otimes f_{b/B} \otimes D_{c/C}$$

PDF= parton distribution function → PDF | Parton cross section | PDF | FF ← FF= fragmentation function (parton → hadrons) Universal object

- This is the leading twist contribution, there are corrections of order (Q=large momentum scale) Q/Λ_{QCD}.
- Monte Carlos for jet and hadron production rely on factorization theorems even if they are not proven in the exclusive case.

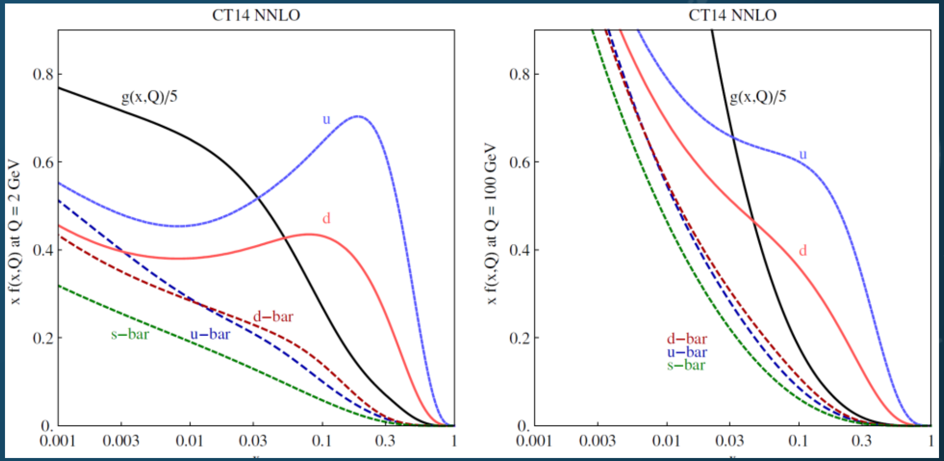
PDFS

- Parton distribution functions and fragmentation functions are universal, i.e. process independent.
- Parton distribution function = probability to find a parton in hadron with a momentum fraction ξ ($0 < \xi < 1$).
- Pdfs rigorously defined in terms of operators (light cone gauge).

$$f_{q/H}(\xi,\mu) = \int \frac{dy^-}{4\pi} e^{-i\xi P^+ y^-} \langle H(P)|\, \bar{q}(y^-)\gamma^+ q(0)\, |H(P)\rangle$$

$$f_{g/H}(\xi,\mu) = \frac{1}{\xi P+} \int \frac{dy^-}{2\pi} e^{-i\xi P^+ y^-} \langle H(P)|\, F_a^{+\nu}(y^-) F_{a\,\nu}{}^{+}(0)\, |H(P)\rangle$$

- Parameterization of pdfs, using observables in e+p and p+p.
- Modifications of pdfs in nuclei compared to free nucleons!



PDFS

- Radiative corrections lead to a scale dependence of pdfs. These are the DGLAP equations with splitting functions $P_{b\to a}$ depending on the momentum fraction in the splitting.

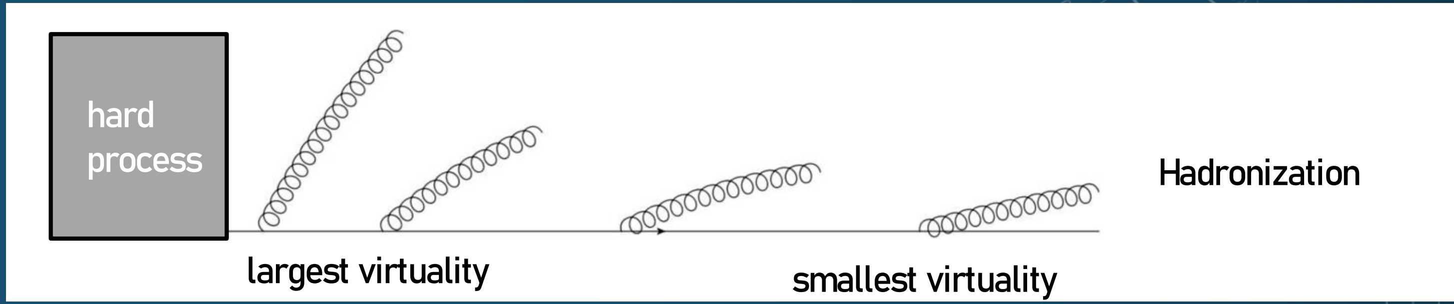
$$\frac{\partial f_a(\xi,\mu)}{\partial \ln \mu} = \frac{\alpha_s}{\pi} \int_x^1 \frac{dy}{y} \sum_b P_{b\to a}(y) f_b\left(\frac{\xi}{y}, \mu\right)$$

- Fragmentation functions have a similar behavior.

- In event generators, typically pdfs and (hard) parton cross sections are sampled. Initial state radiation and additional parton interactions (e.g. MPIs) may be added.
  - JETSCAPE uses PYTHIA 8 as the default module for these tasks.
  - In A+A collisions (transverse) spatial information on hard processes may be added using the Glauber model.
- Fragmentation functions are rarely used in event generators. They are too “inclusive”

PARTON SHOWER MONTE CARLOS

- Fragmentation functions ~ parton shower + hadronization.
  - MC event generators model this process starting with partons emerging from hard processes
- Use DGLAP equations to evolve partons from large virtualities (emerging from the hard process) to small virtualities.

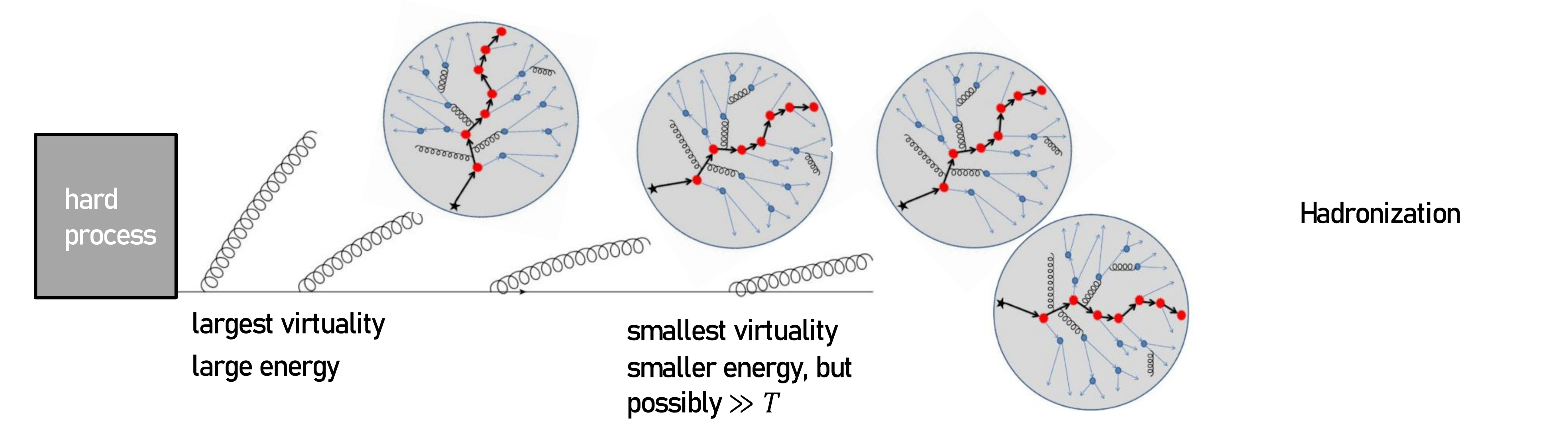


- Need to Monte Carlo the parton splitting.
  - Start with a maximum allowed virtuality $Q_{\max}$ from the hard process.
  - Probability with current virtuality Q_i that the next split will be at Q_{i+1}:
  - probability for no split between Q_i that the next split will be at Q_{i+1} × probability for split at Q_{i+1}

Sudakov form factor $S = \exp\left[-\int_{Q_i^2}^{Q_{i+1}^2} \frac{dt}{t} \frac{\alpha_s}{2\pi} \int_{z_-}^{z_+} dz P(z)\right]$

PARTON SHOWER MONTE CARLOS

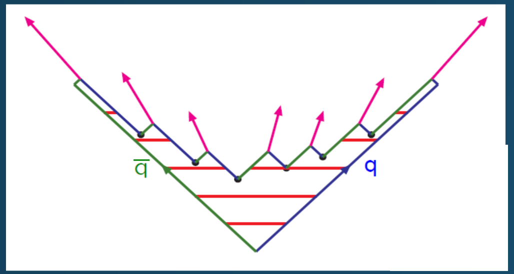
- So far: evolution in the vacuum, driven by virtuality acquired by the hard process. Parton showers in a QCD medium are modified.
- Modifications to virtuality driven shower: e.g. higher twist formalism used in MATTER (standard parton shower MC in JETSCAPE)
- Partons at the end of the virtuality evolution could still have a lot of energy → parton transport in QGP (e.g. LBT or MARTINI).
  - Additional induced radiation and elastic scattering with medium partons



  - See talk by Gojko Vujanovic on Friday.

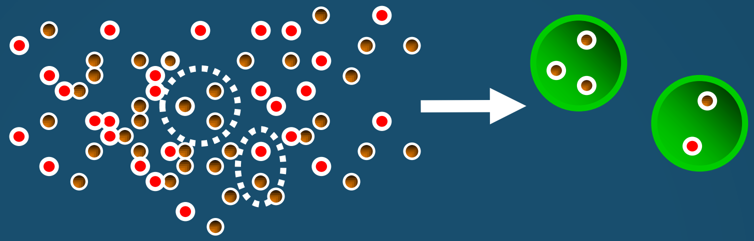
HADRONIZATION

- 3 most relevant models for event generators:
  - Lund string fragmentation



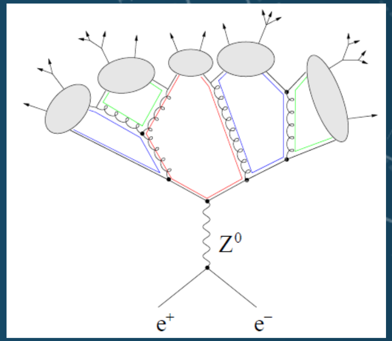
Well tested in e+e- and p+p collisions

  - Quark recombination



Allows jet and medium partons to interact at hadronization

  - Cluster hadronization



- JETSCAPE hadronization:
  - Two different Lund string fragementation schemes (colored and colorless)
  - Hybrid hadronization which interpolates between (colored) string fragmentation in dilute systems and recombination in dense systems by sampling the overlap of quarks with hadron wave functions

BULK PHYSICS IN A+A

- Standard modelling for the bulk of A+A collisions:
  - Colliding color glass condensate $\to$ glasma
  - Equilibration or hydrodynamization
  - Viscous fluid dynamics
  - Sampling of hadrons and subsequent hadronic transport

In JETSCAPE:

Handled by TRENTO initial state parameterizations

State-of-the-art simulations available (MUSIC, SMASH, ...)

- Modelling close to kinetic equilibrium through macroscopic observables (pressure p, energy density e, collective flow u^μ). Energy momentum tensor

$$T^{\mu\nu} = (e + p + \Pi)u^\mu u^\nu - (p + \Pi)g^{\mu\nu} + \pi^{\mu\nu}$$

- Viscous fluid dynamics = conservation laws $\partial_\mu T^{\mu\nu} = 0$ + constitutive equations for shear and bulk stress.
  - Shear and bulk stress are related to deviations of particle distributions from equilibrium: $f = f_{\text{eq}} + \delta f$

BULK PHYSICS IN A+A

- Bulk simulations provide
  - constraints on important parameters of hot nuclear matter (like the shear viscosity)
  - the background for hard probes; shower Monte Carlos read in the local ambient conditions to model the medium
  - Bulk physics can be affected by probes depositing energy and momentum in the medium
- See talks and hands-on sessions by Chun Shen and Yasuki Tachibana

- For SMASH: see talk and session by Dmytro Oliinychenko

HONORABLE MENTION

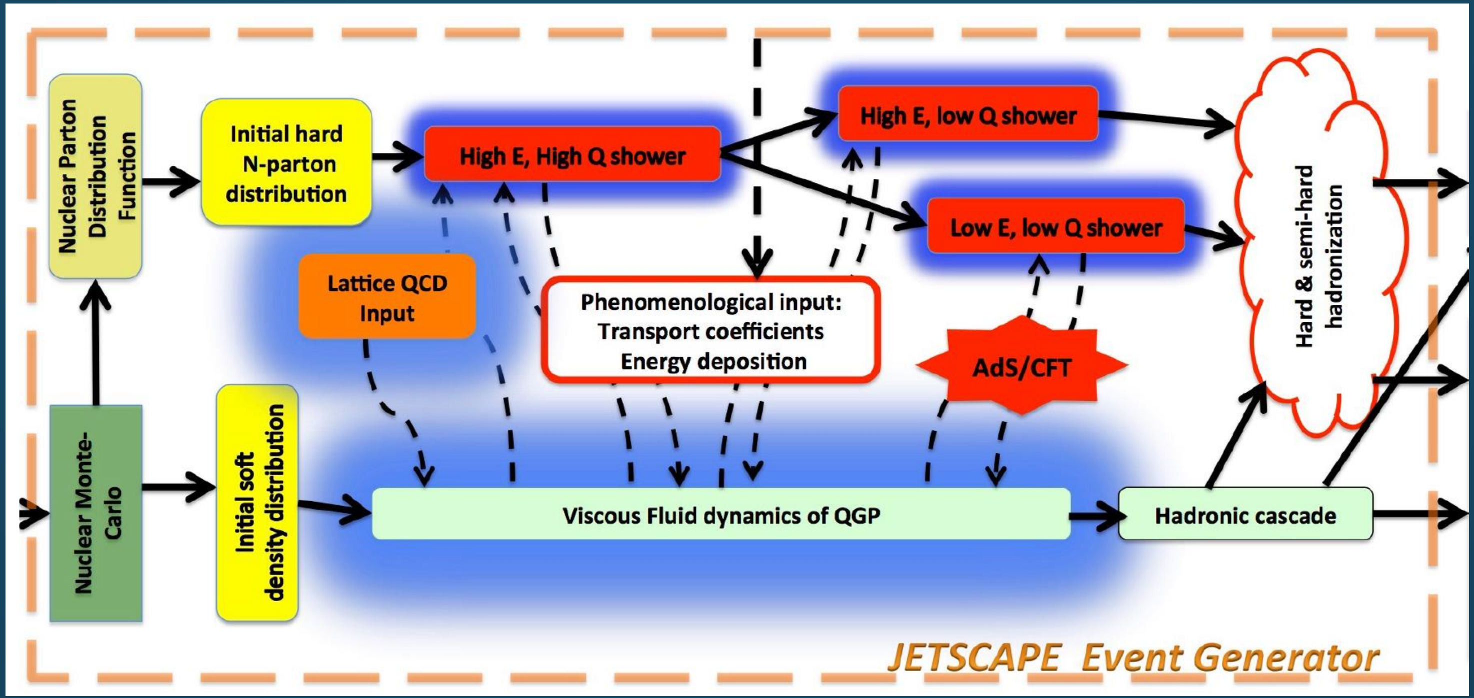
- Photons and heavy quarks are historically very important probes.

- JETSCAPE is adding more capabilities, see JETSCAPE talks at HP2020

PART III: SUMMARY

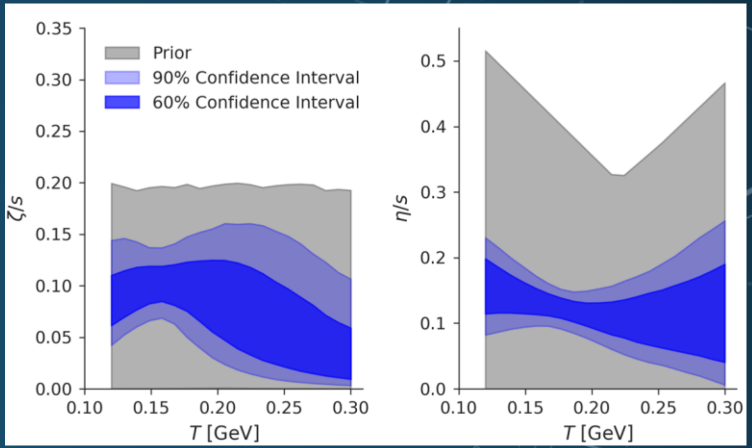
THE JETSCAPE FRAMEWORK

- It should now be easy to follow through the JETSCAPE flow chart.

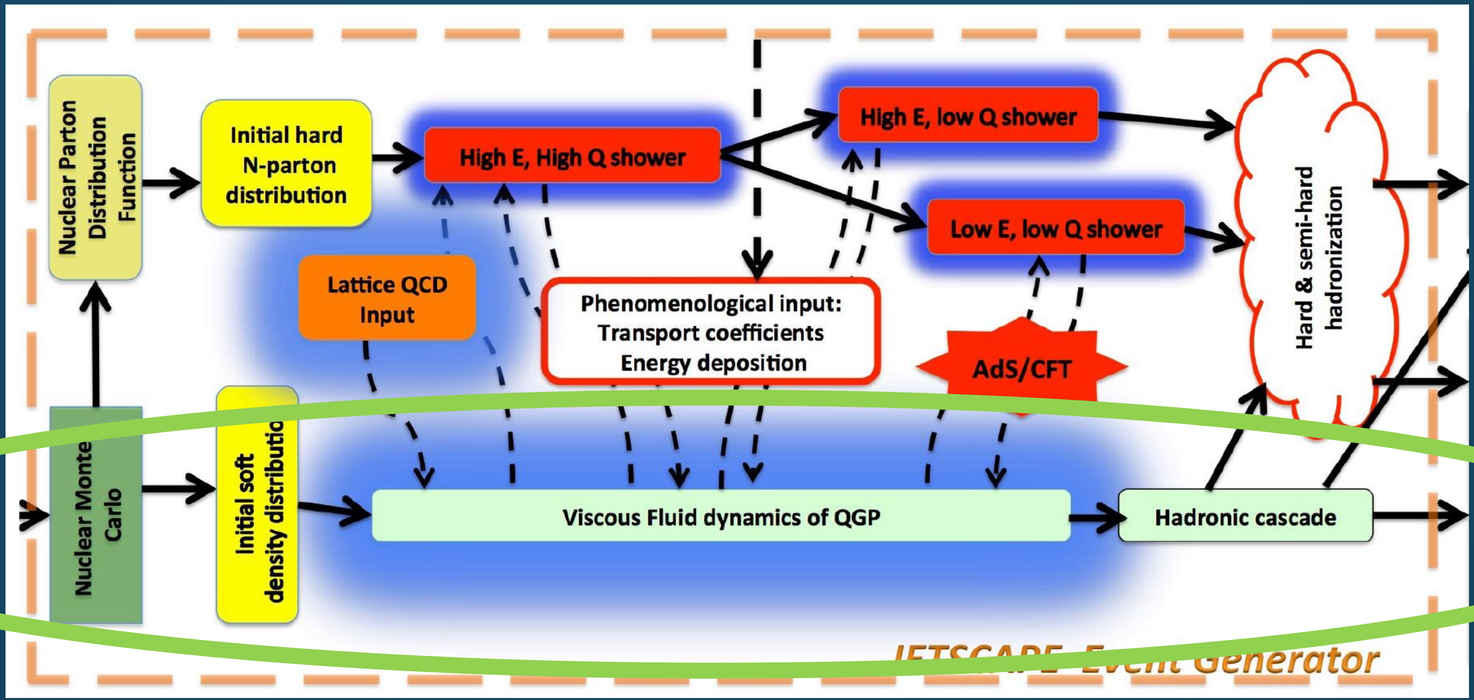


BULK PHYSICS SUMMARY

- State-of-the-art tools
- Comprehensive paper with a bulk analysis in preparation
- Will provide a tuned background for hard probes studies

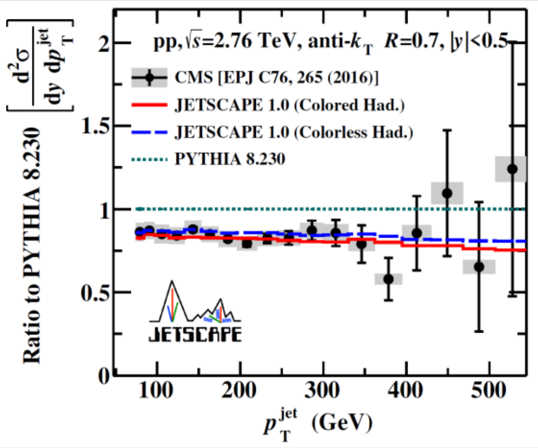


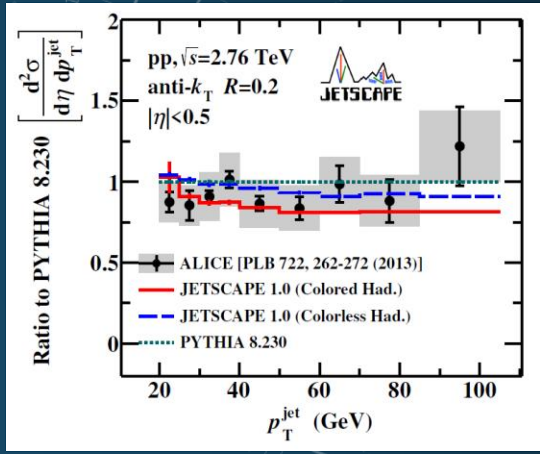
JETSCAPE preliminary @QM 2019



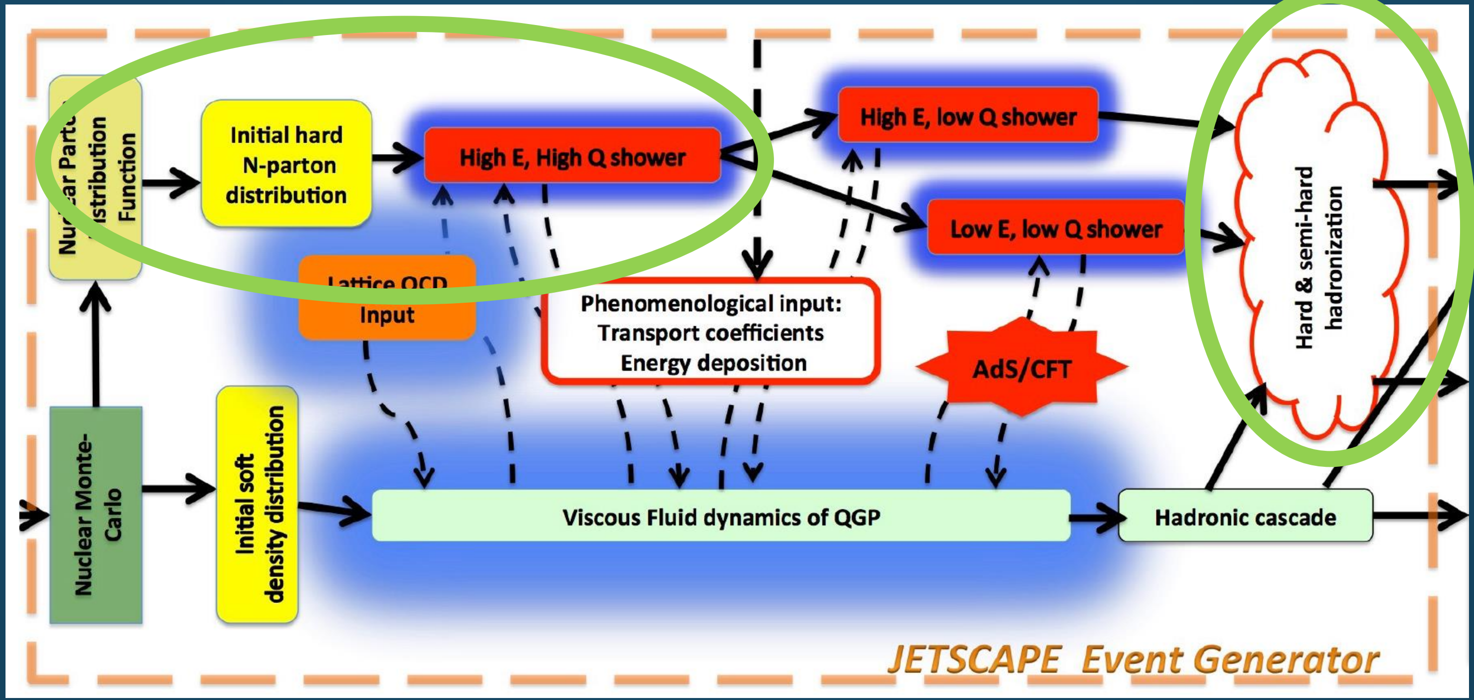
HARD PROBES VACUUM SUMMARY

- Mostly MATTER + string fragmentation
- PP19 tune available, compared to data and PYTHIA 8.



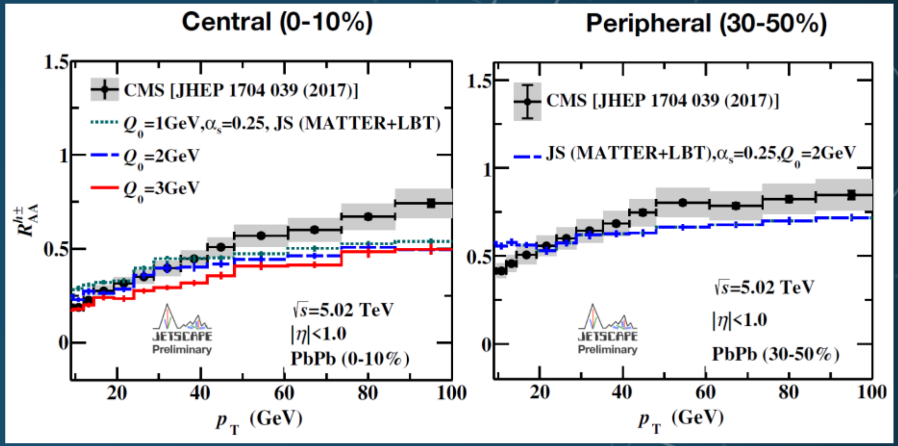


arXiv:1910.05481 [nucl-th]



HARD PROBES A+A SUMMARY

- Several shower MCs/energy loss modules already available, and more can be added.
- Default JETSCAPE configuration: virtuality-driven shower MC MATTER + parton transport LBT or MARTINI
- HYBRID strong coupling module available



JETSCAPE preliminary @HP2020