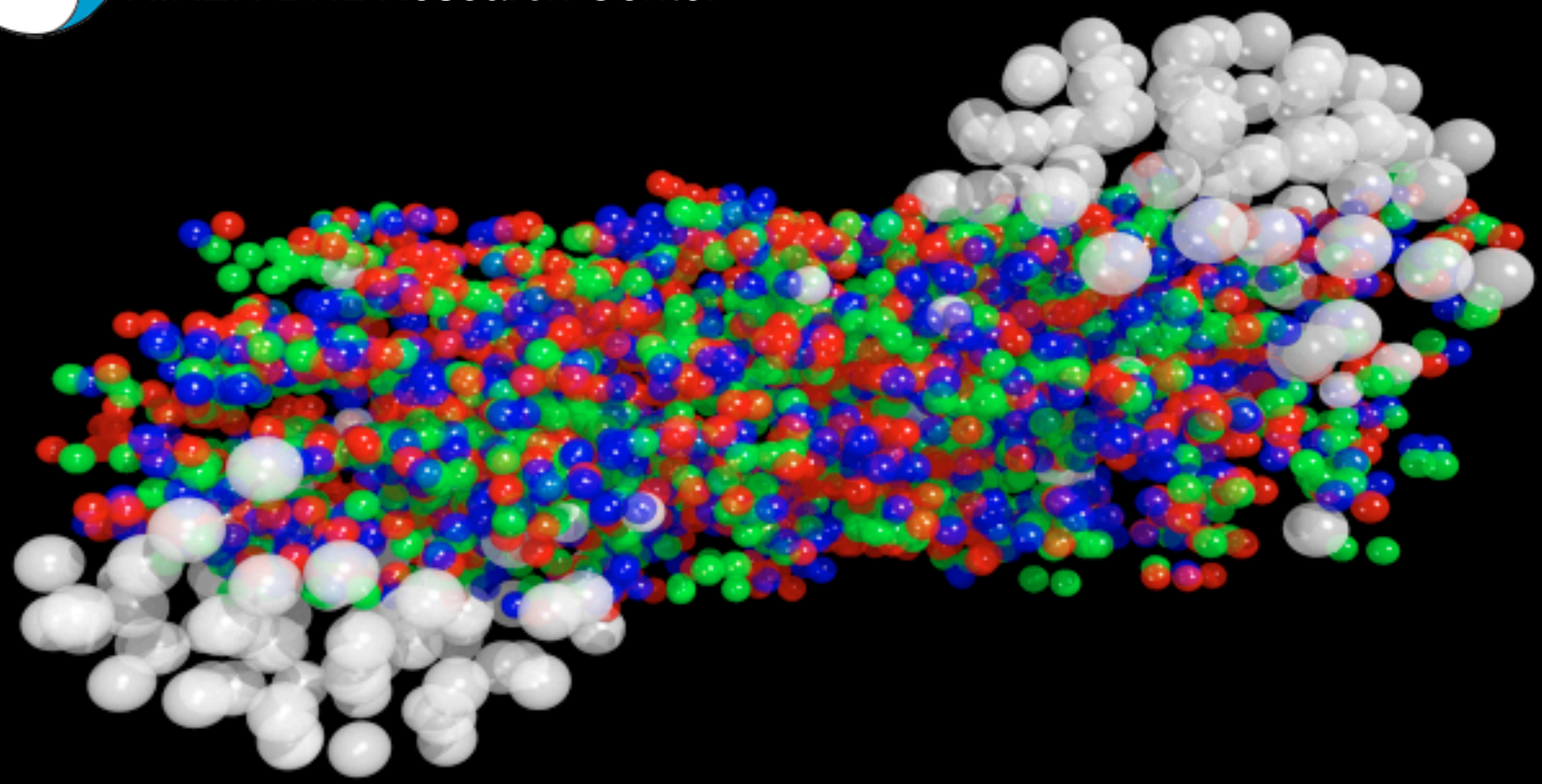
RBRC
RIKEN BNL Research Center

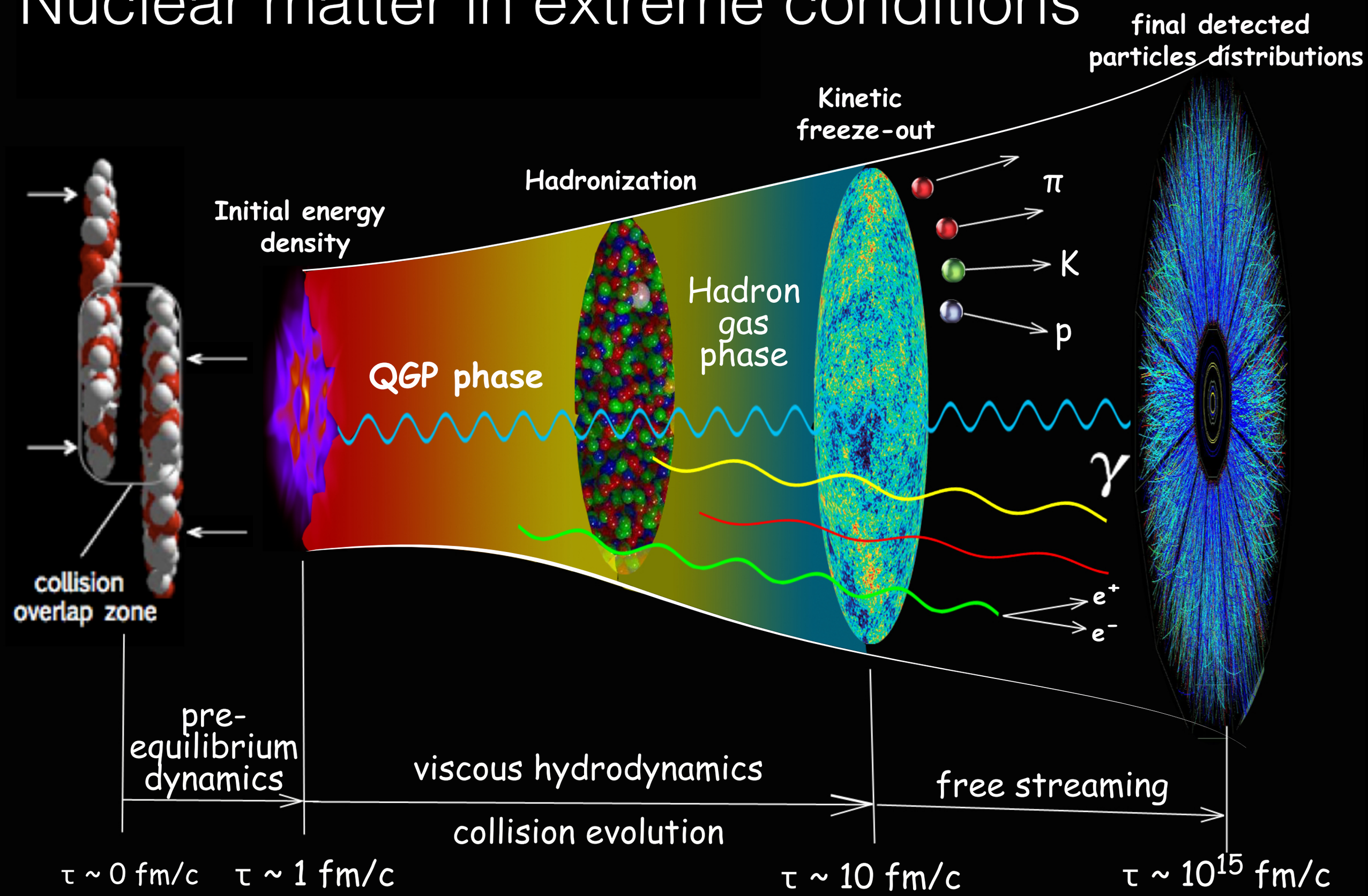


CHUN SHEN

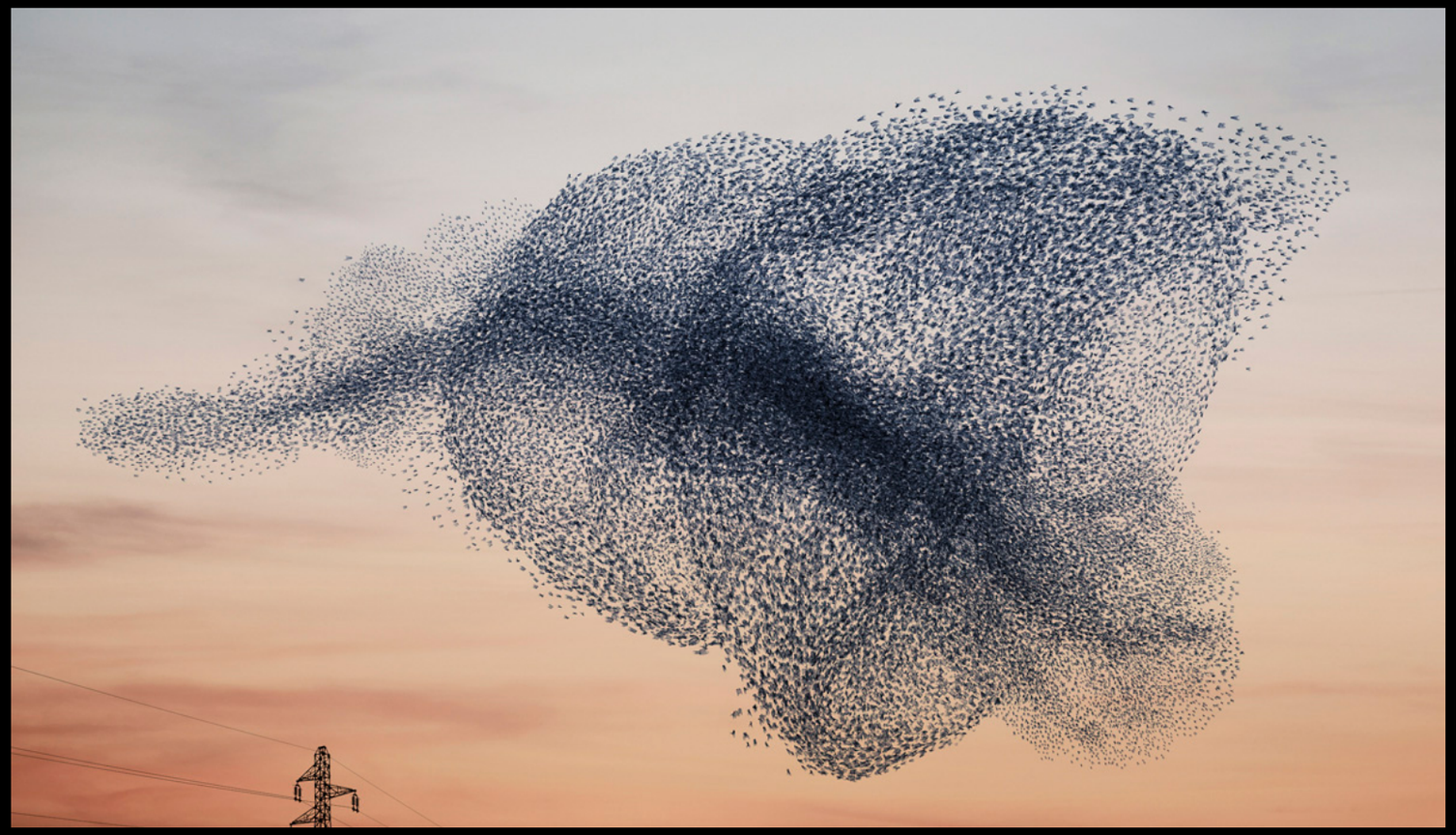
INTRODUCTION TO THE COLLECTIVE QGP

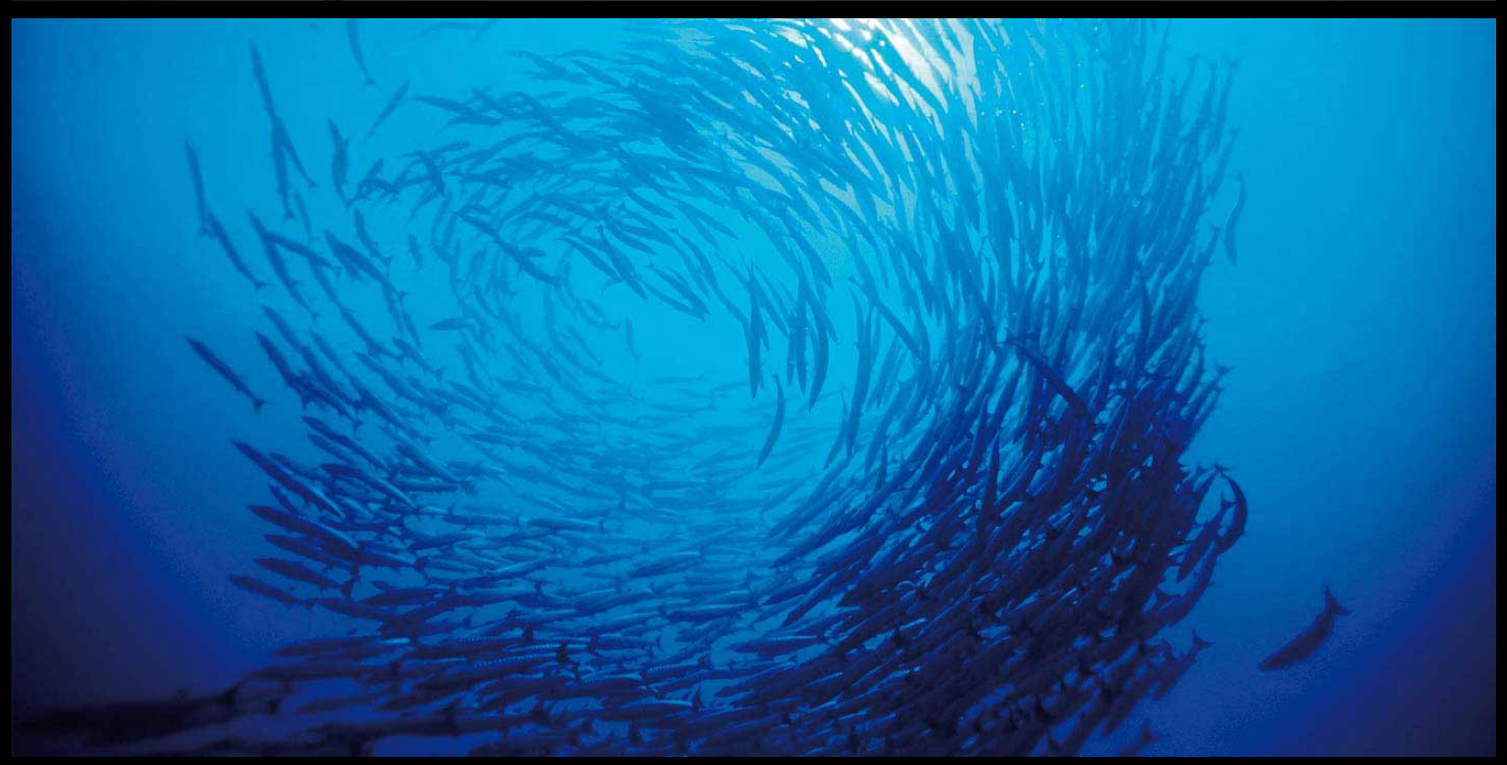
Post your questions in the slack channel: **#bulk-shen**

Nuclear matter in extreme conditions

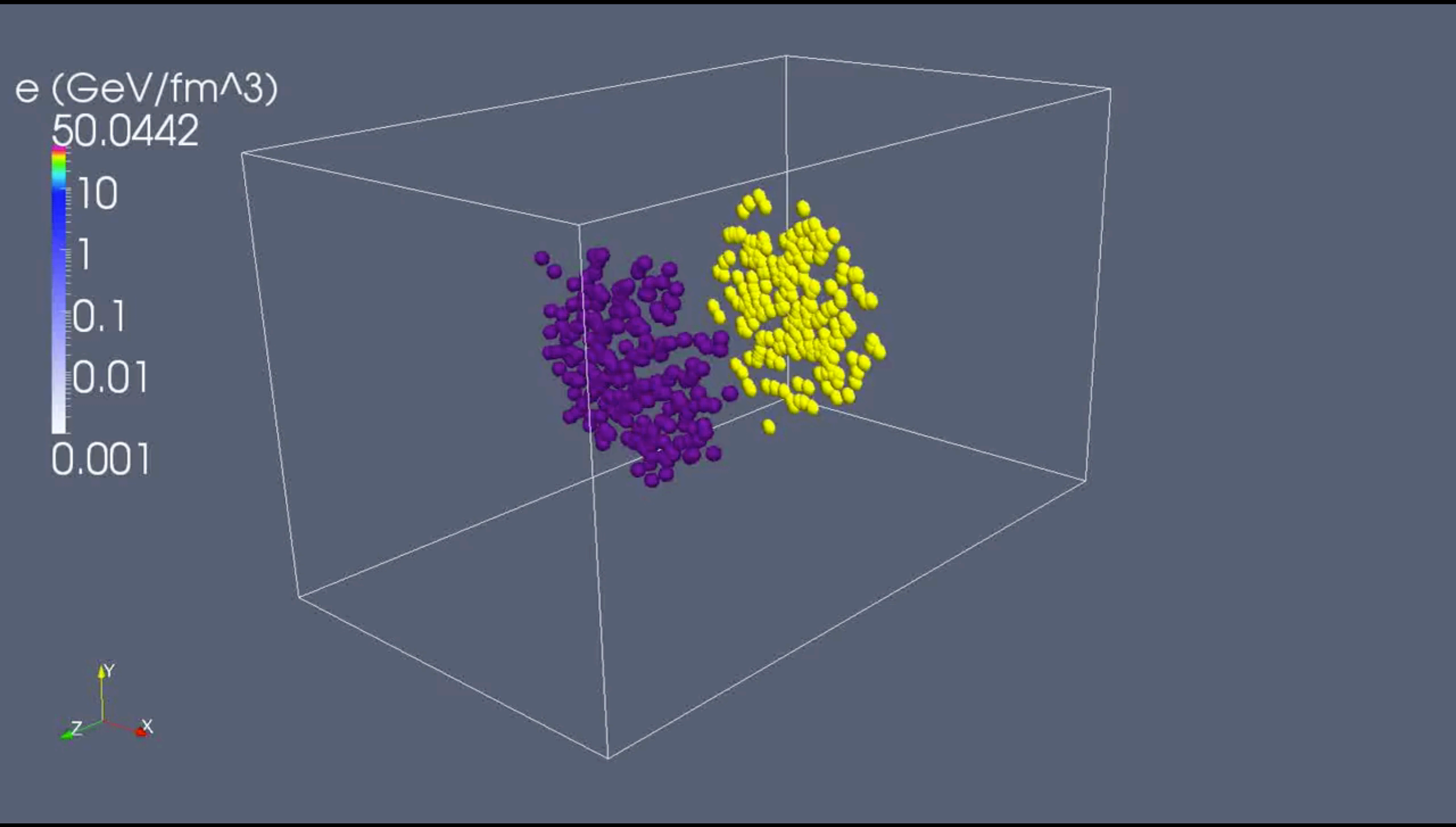


Collective Phenomena



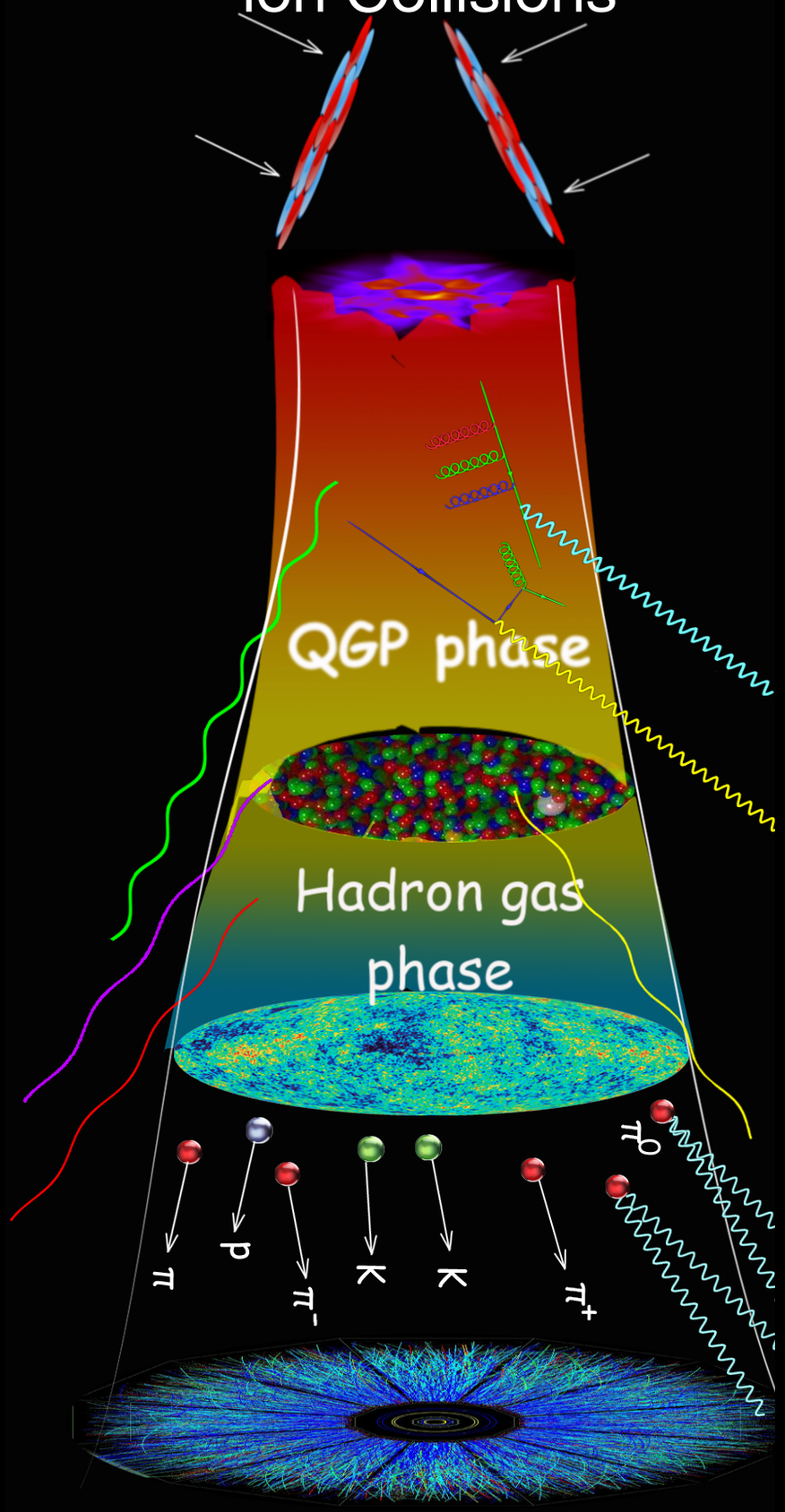


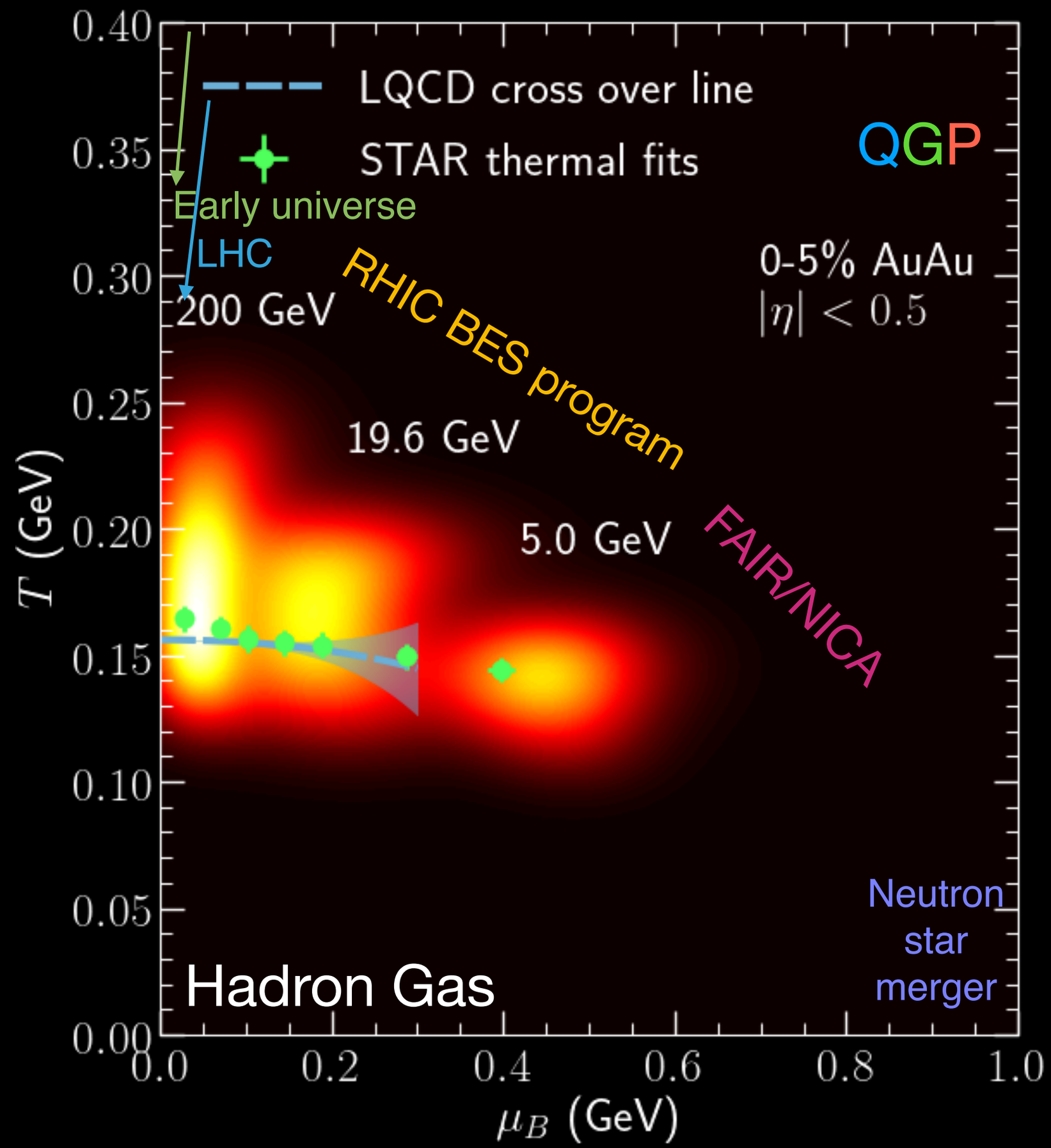
A relativistic QCD many-body system



https://youtu.be/W3h5vQOUJTg

Relativistic Heavy-ion Collisions





Defining the Quark-Gluon Plasma

Which **properties of hot QCD matter** can we determine from relativistic heavy ion data (LHC, RHIC, and FAIR)?

Equation of State $T^{\mu\nu} \Longleftrightarrow e, P, s$ $c_s^2 = \partial P / \partial e\|_{s/n}$	Spectra, collective flow, femtoscopy
Shear and bulk viscosities $\eta/s(T,\mu_B)$, $\zeta/s(T,\mu_B)$	Anisotropic flow v_n
Charge diffusion D_B, D_Q, D_S	Balance functions
Energy-momentum transport $\hat{q}, \hat{e}, \hat{e}_2, \ldots$	Jets and heavy-quarks
Electromagnetic emissivity	Photons and dileptons

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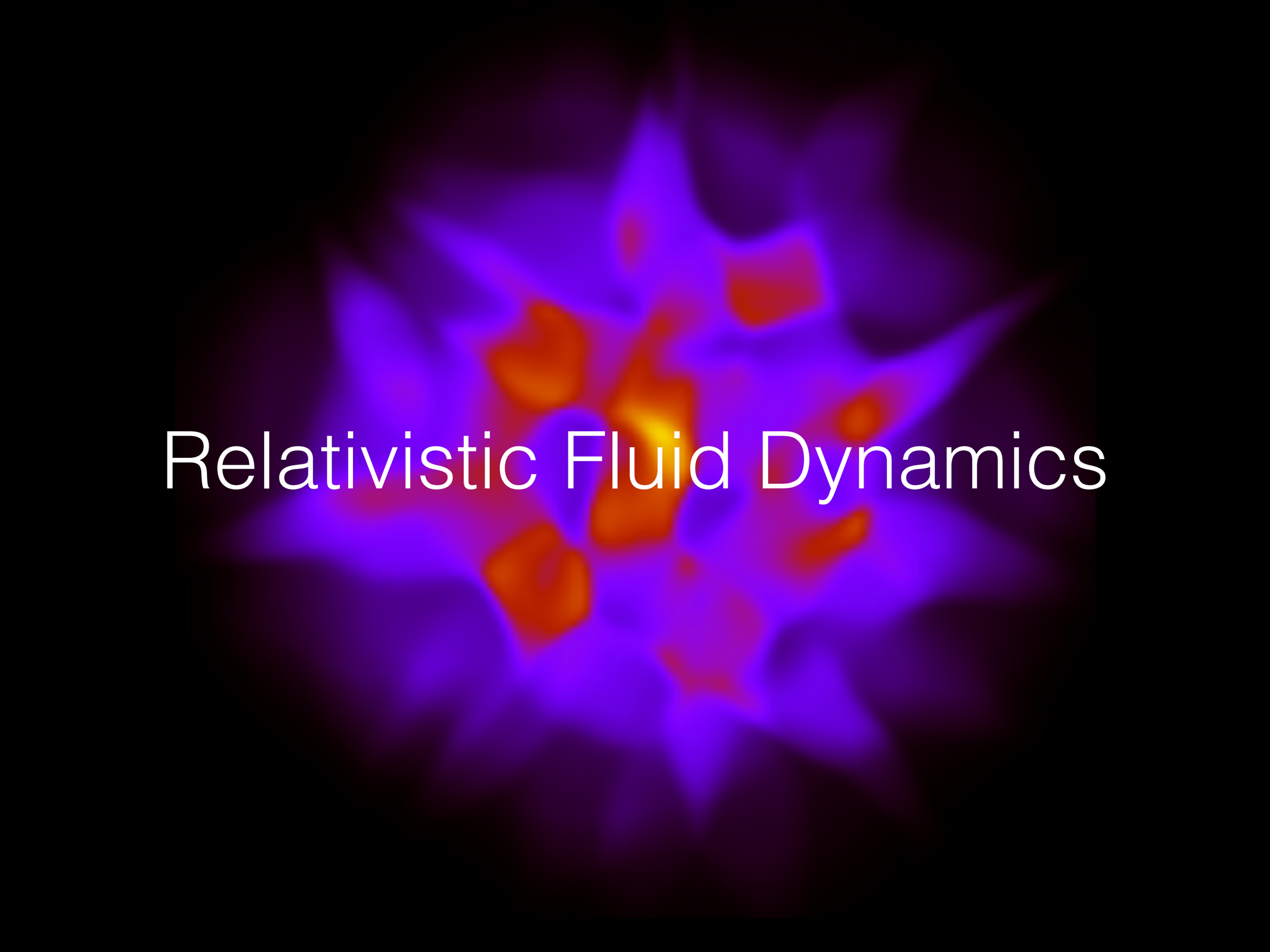
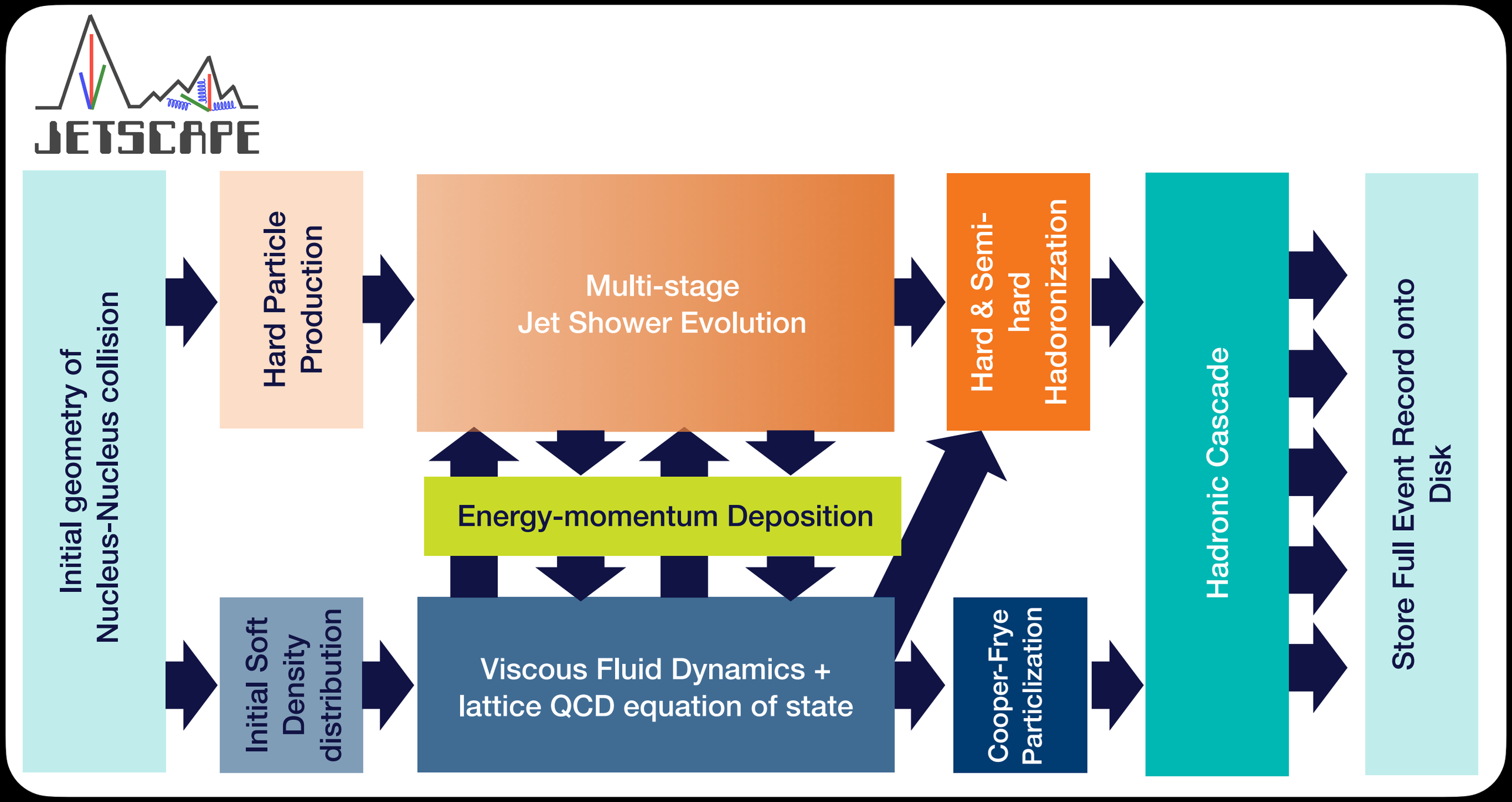


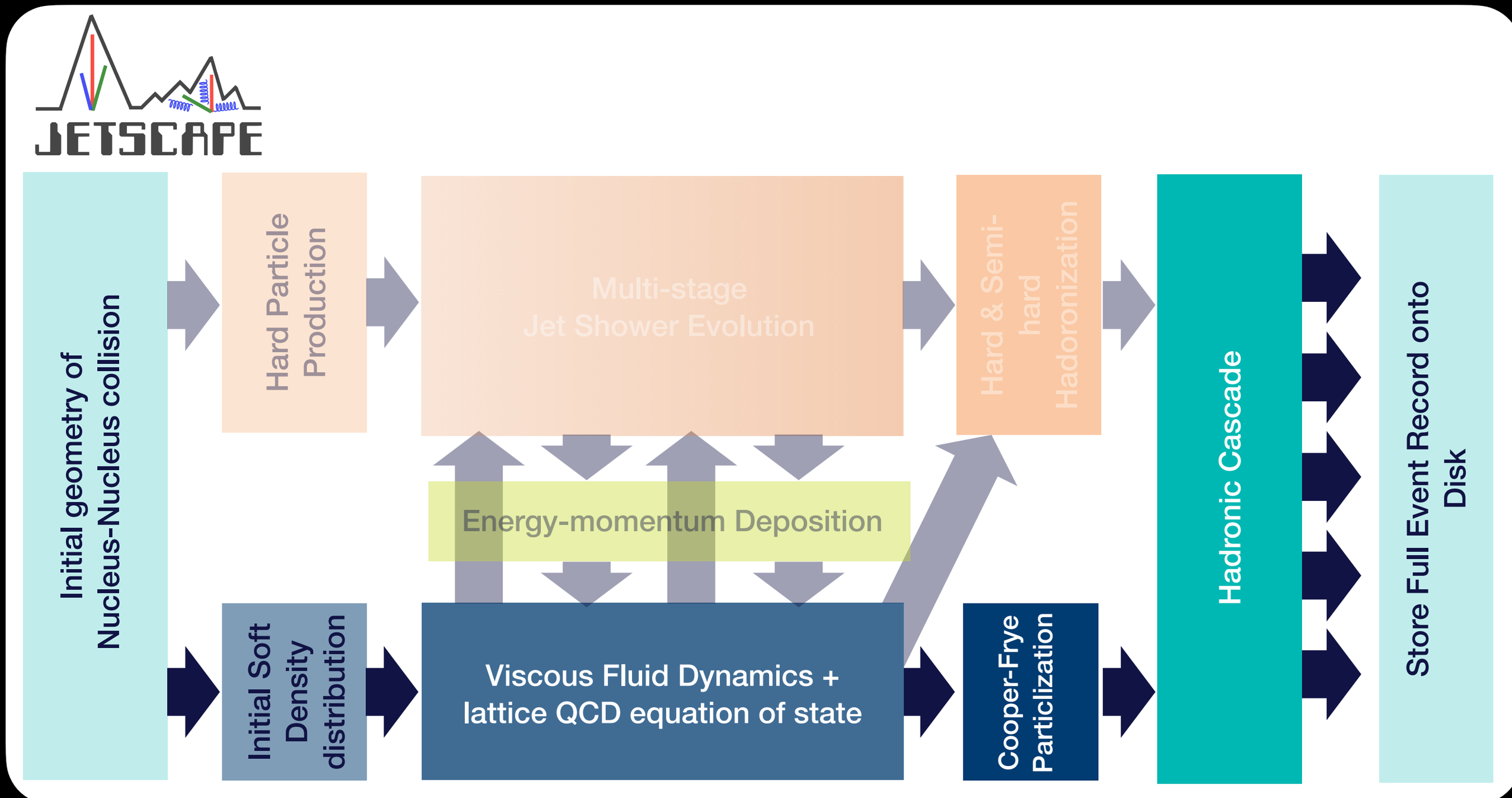
Relativistic Fluid Dynamics

Theoretical framework for heavy-ion collisions



@ Y. Tachibana

Theoretical framework for heavy-ion collisions



@ Y. Tachibana

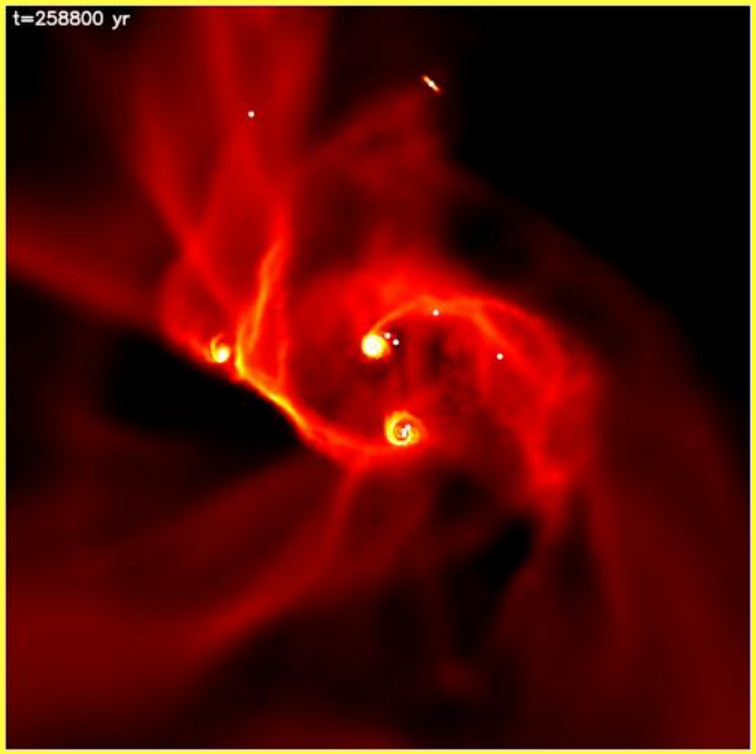
Hydrodynamics

a long wavelength effective description of interacting systems

Conservation laws + Equation of State

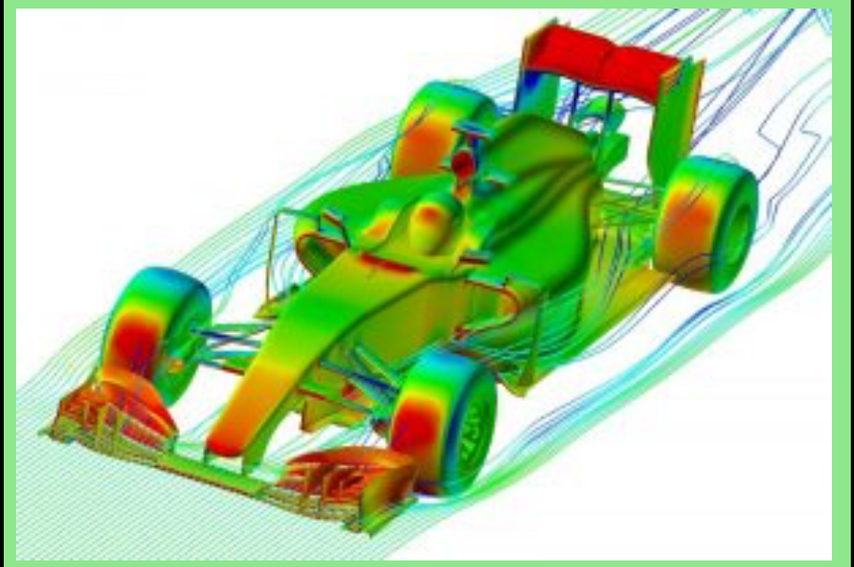
Macroscopic: $L \sim 1/\partial v \gg$ Microscopic: $l \sim l_{\rm mfp} \sim 1/(n\sigma)$

Star formation



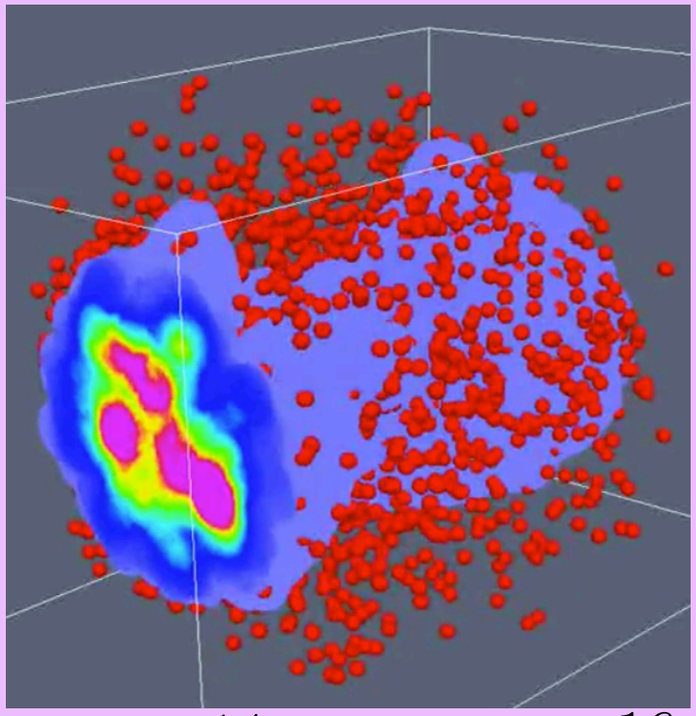
$L \sim 10^{18} m$ $l \sim 10^{10} m$

Air dynamics of race car



$L \sim 1m$ $l \sim 10^{-7} m$

Quark-Gluon Plasma



$L \sim 10^{-14} m$ $l \sim 10^{-16} m$

Conservation laws

Conservation of energy and momentum:

$$\partial_\mu T^{\mu\nu}(x) = 0$$

Conservation of charge:

$$\partial_\mu J^{\mu}(x) = 0$$

Local conservation of particle number and energy-momentum

⟷ **Hydrodynamics!**

This can be generalized to multicomponent systems and systems with several conserved charges:

$$\partial_\mu J_i^{\mu}(x) = 0$$

i = baryon number, strangeness, electric charge ...

Ideal hydrodynamics EoM

In the local rest frame $u^\mu = (1, 0, 0, 0)$

$$D = u^\mu \partial_\mu = \partial_t$$ time derivative

$$\nabla_\mu = \Delta_{\mu\nu} \partial^\nu = \partial_i = \nabla_i$$ spatial gradient

and

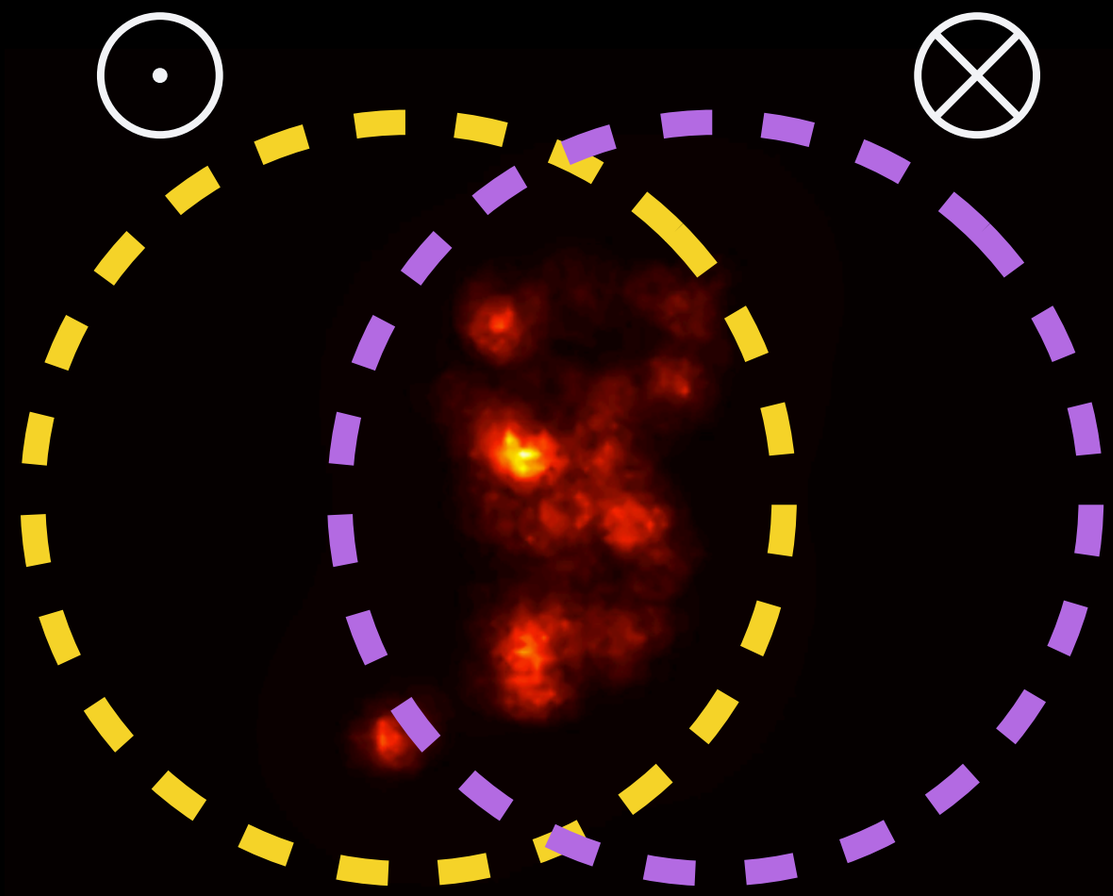
$$De = -(e + P)\theta$$ continuity equation

$$Du^\mu = \frac{\nabla^\mu P}{(e + P)}$$ fluid acceleration

In ideal hydrodynamics, entropy is conserved

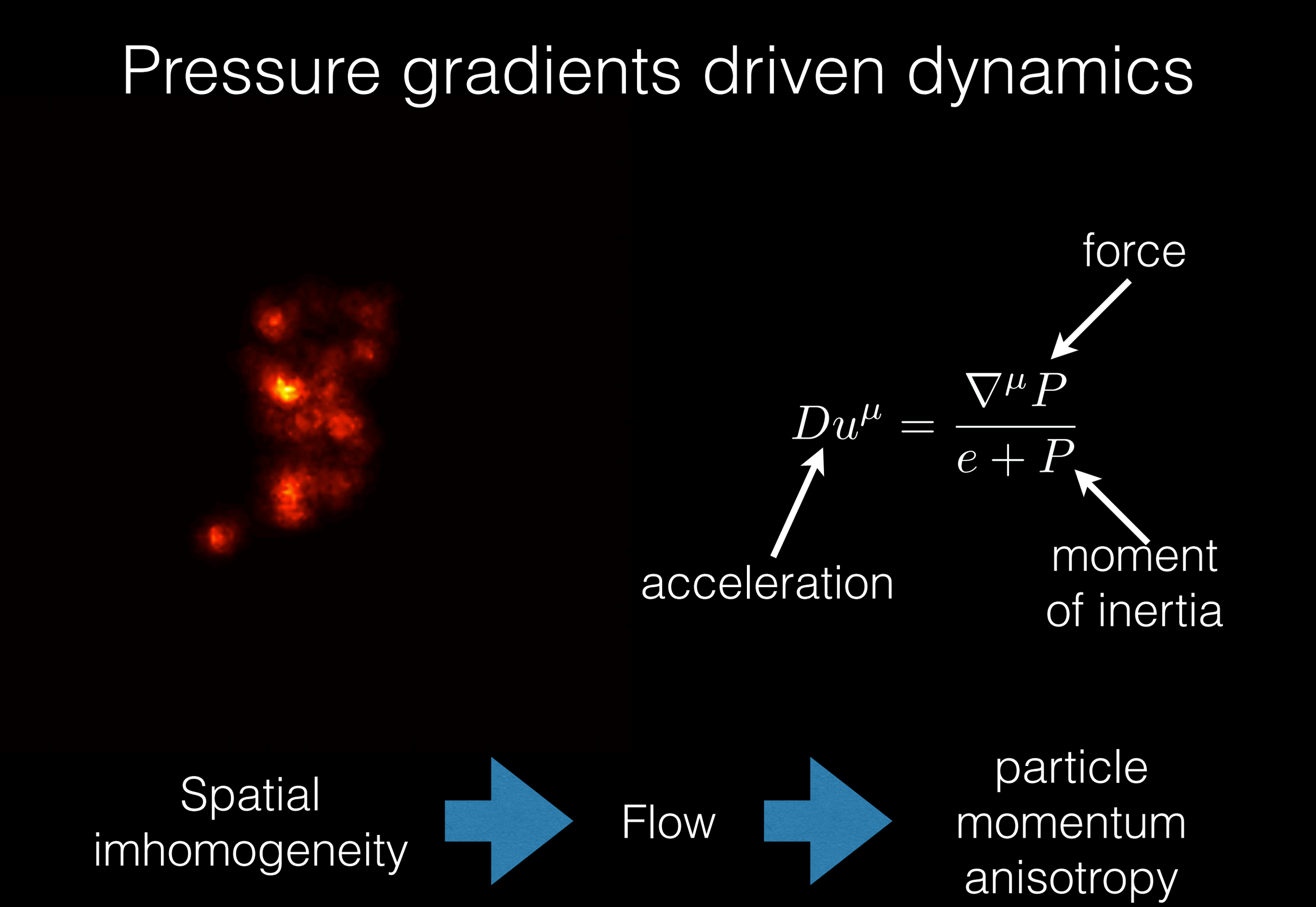
$$\partial_\mu s^\mu = 0 \qquad s^\mu = s u^\mu$$

Pressure gradients driven dynamics



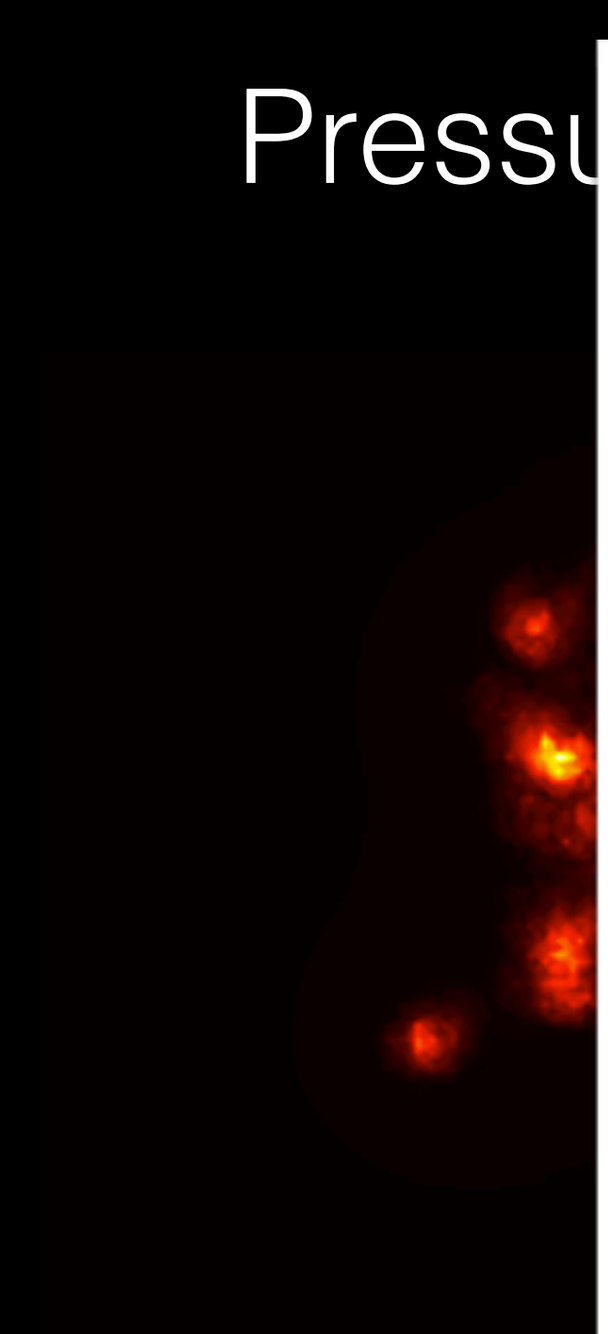
$$Du^\mu = \frac{\nabla^\mu P}{e + P}$$

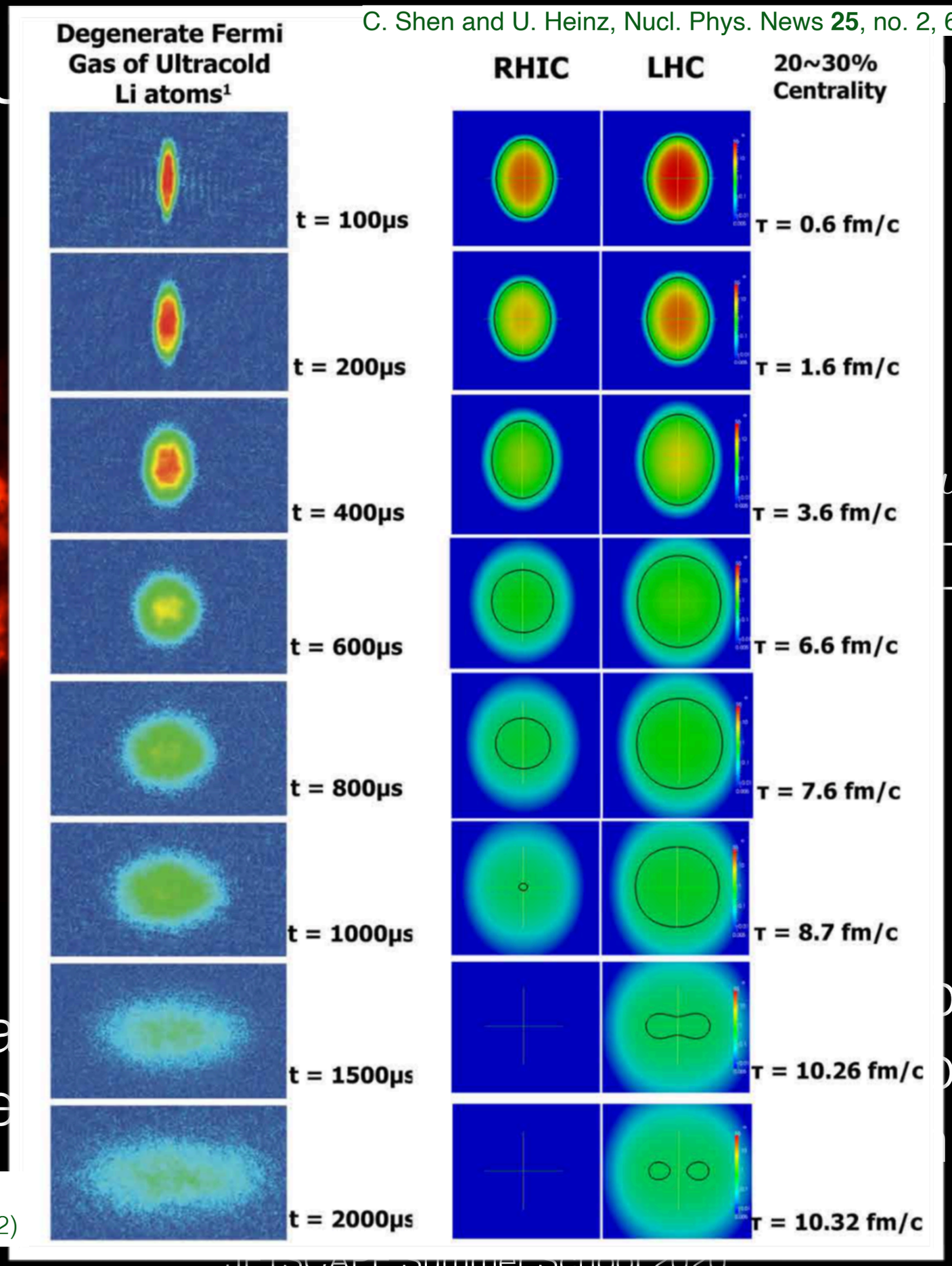
Pressure gradients driven dynamics



Pressu amics

C. Shen and U. Heinz, Nucl. Phys. News **25**, no. 2, 6 (2015)





force

$u\,P$

$-\,P$

moment of inertia

Spatia imhomoge

particle omentum isotropy

K. M O'Hara, et. al, Science 298, 2179 (2002)

Dissipative hydrodynamics

Energy momentum tensor

$$T^{\mu\nu} = eu^\mu u^\nu - (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

Conserved currents

$$J^\mu = nu^\mu + q^\mu$$

Equations of motion

$$\begin{aligned} \partial_\mu T^{\mu\nu} &= 0 \\ \partial_\mu J^\mu &= 0 \end{aligned} \quad + \quad P(e, n)$$

Dissipative quantities are evolved with 2nd order Israel-Stewart type of equations

At Navier-Stokes limit,

$$\pi^{\mu\nu} \sim 2\eta \nabla^{\langle\mu} u^{\nu\rangle} \quad \Pi \sim -\zeta \partial_\mu u^\mu \quad q^\mu \sim \kappa \nabla^\mu \frac{\mu}{T}$$

$$\nabla^\mu = \Delta^{\mu\nu}\partial_\nu$$

QGP transport property

Shear viscosity

Resistance to deformation

$\sim 2\eta\nabla^{\langle\mu}u^{\nu\rangle}$

Bulk viscosity

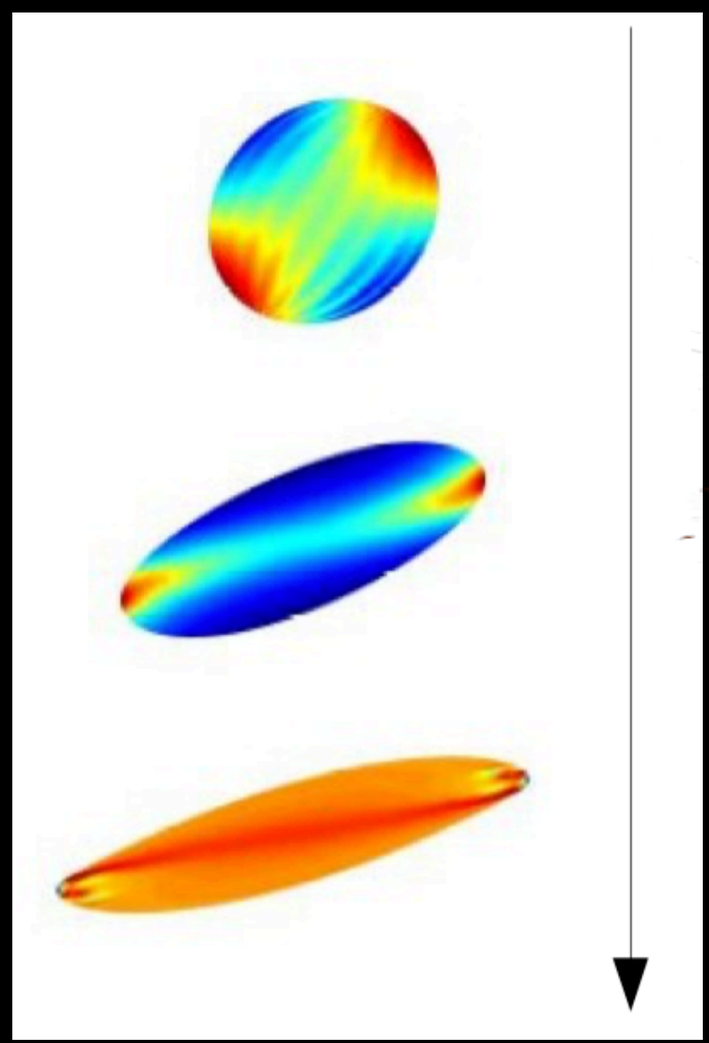
Resistance to expansion

$\sim -\zeta\partial_\mu u^\mu$

Diffusion

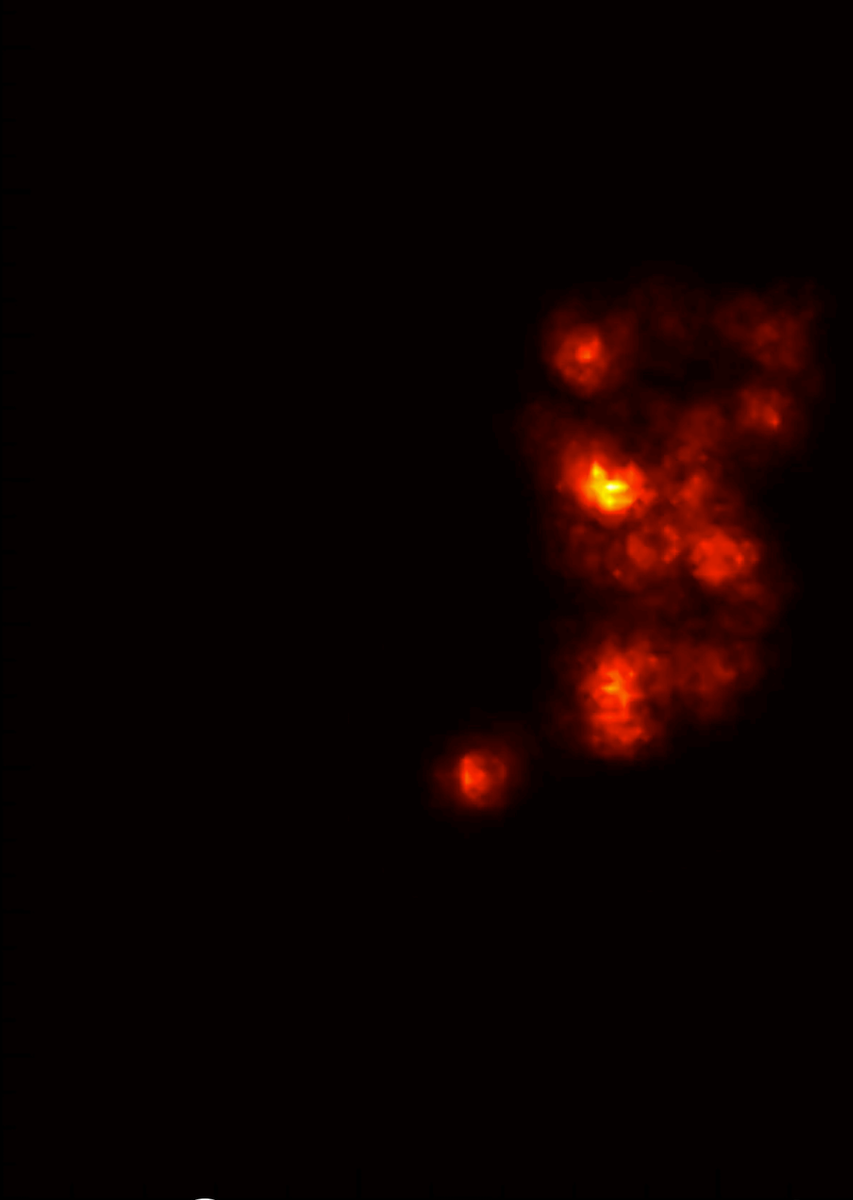
particles diffuse out of fluid cell

$\sim \kappa_B\nabla^\mu(\mu_B/T)$

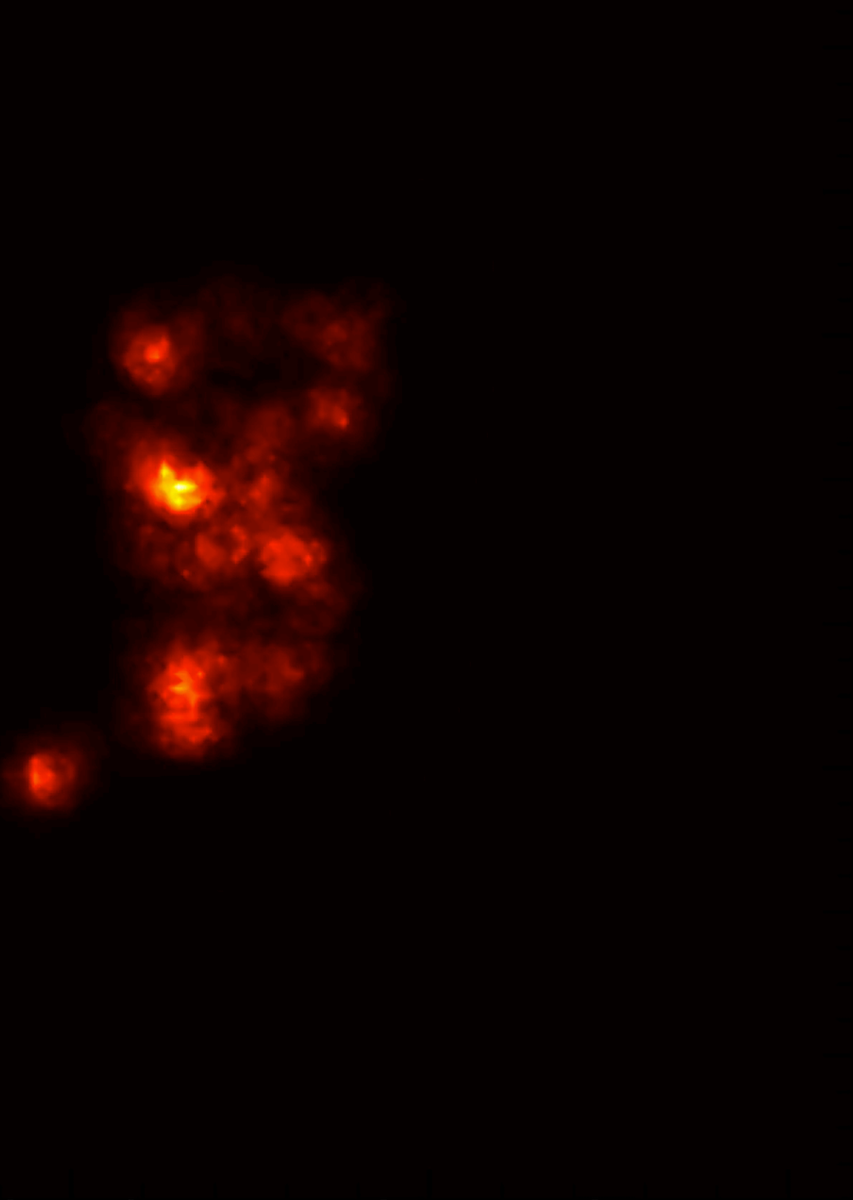


Effects of shear viscosity

no shear viscosity



with shear viscosity



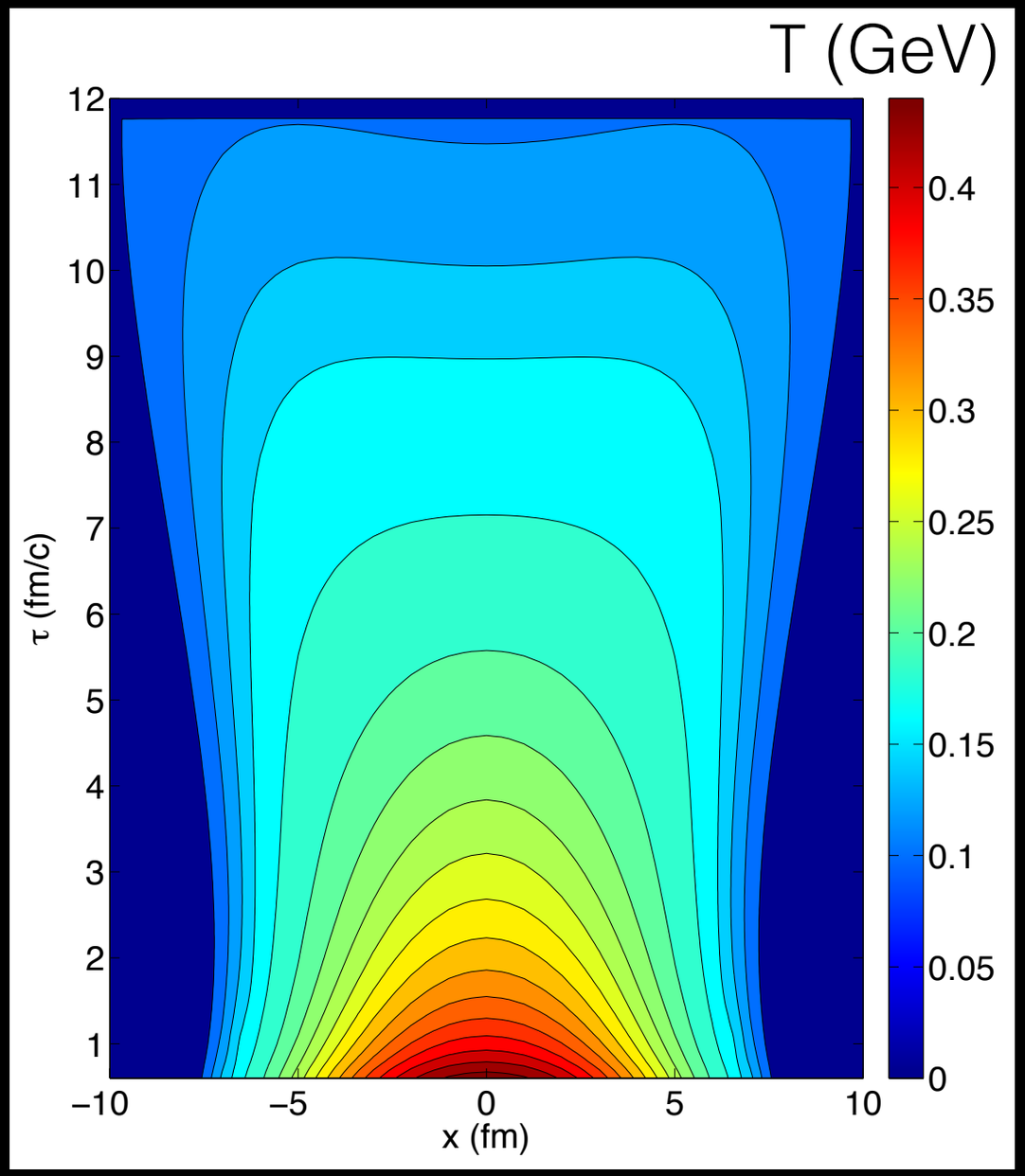
System expands more isotropically with shear viscosity
Shear viscosity smears out the fine flow patterns

A realization of relativistic hydrodynamics

http://www.physics.mcgill.ca/music/

When to end?

- Particle is observed, not fluid
- How and when to convert fluid to particles (evaporation)
- How far is hydrodynamics valid?



Kinetic equilibrium requires

Scattering rate $\gg$ expansion rate

$$\tau_{\rm sc}^{-1} \sim \sigma n \propto \sigma T^3 \qquad \theta = \partial_\mu u^\mu$$

Fluid description breaks down at

$$\tau_{\rm sc}^{-1} = \theta$$

Approximation: decouple takes place at a fixed temperature

$$T = T_{\rm fo} \propto \left(\frac{\theta}{\sigma}\right)^{1/3}$$

Cooper-Frye Particlization

Sample particles on the freeze-out surface (surface of constant (low) energy density) according to

$$E\frac{dN_i}{d^3p} = \frac{g_i}{(2\pi)^3}\int_{\Sigma(T_{\rm fo})} d^3\sigma_\mu p^\mu \left(f^i_{\rm eq}\left(\frac{p^\mu u_\mu}{T}\right) + \delta f^i \right)$$

then feed particles into hadronic cascade (UrQMD/SMASH)

S. A. Bass et al., Prog. Part. Nucl. Phys. 41, 255–369 (1998)
M. Bleicher et al., J. Phys. G25, 1859–1896 (1999)
J. Weil, et al. Rev. C 94, 054905 (2016)

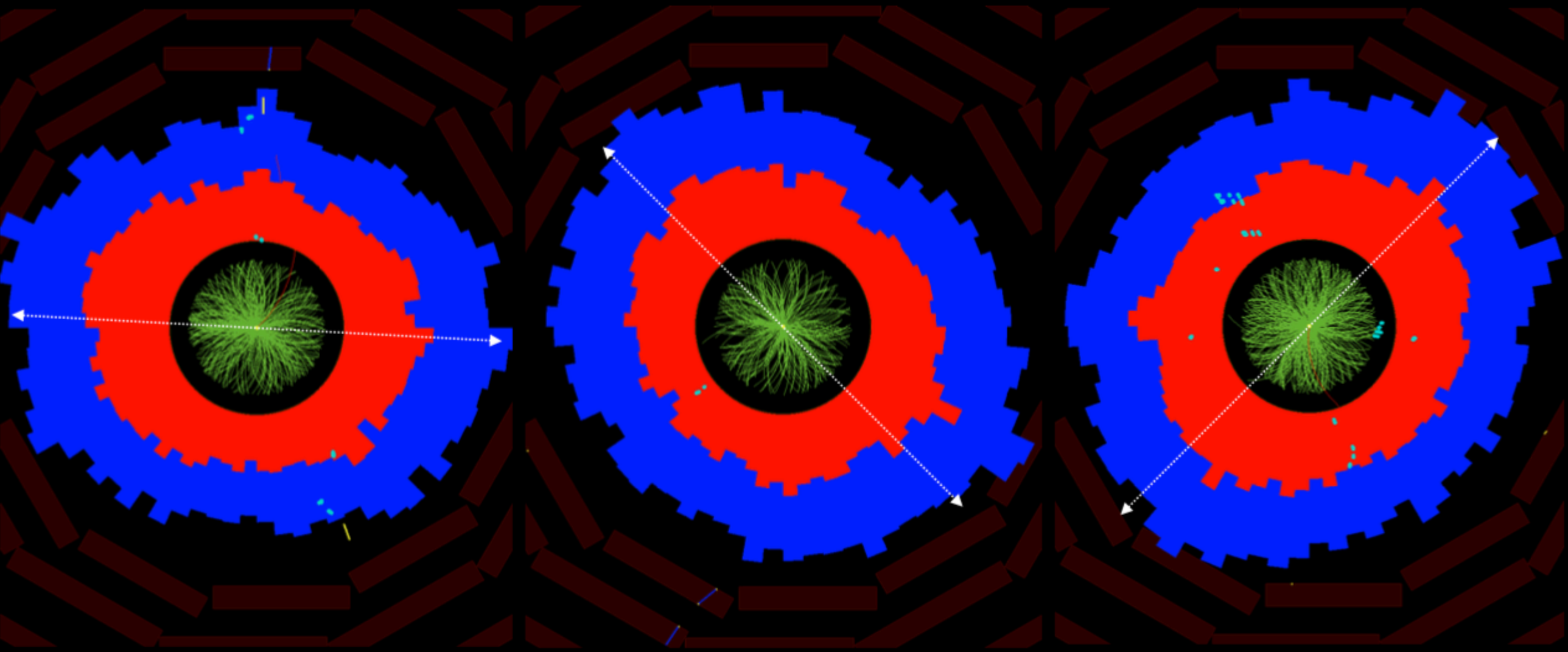
which performs resonance decays and scattering according to hadronic cross sections

Anisotropic Flows

Anisotropic flow

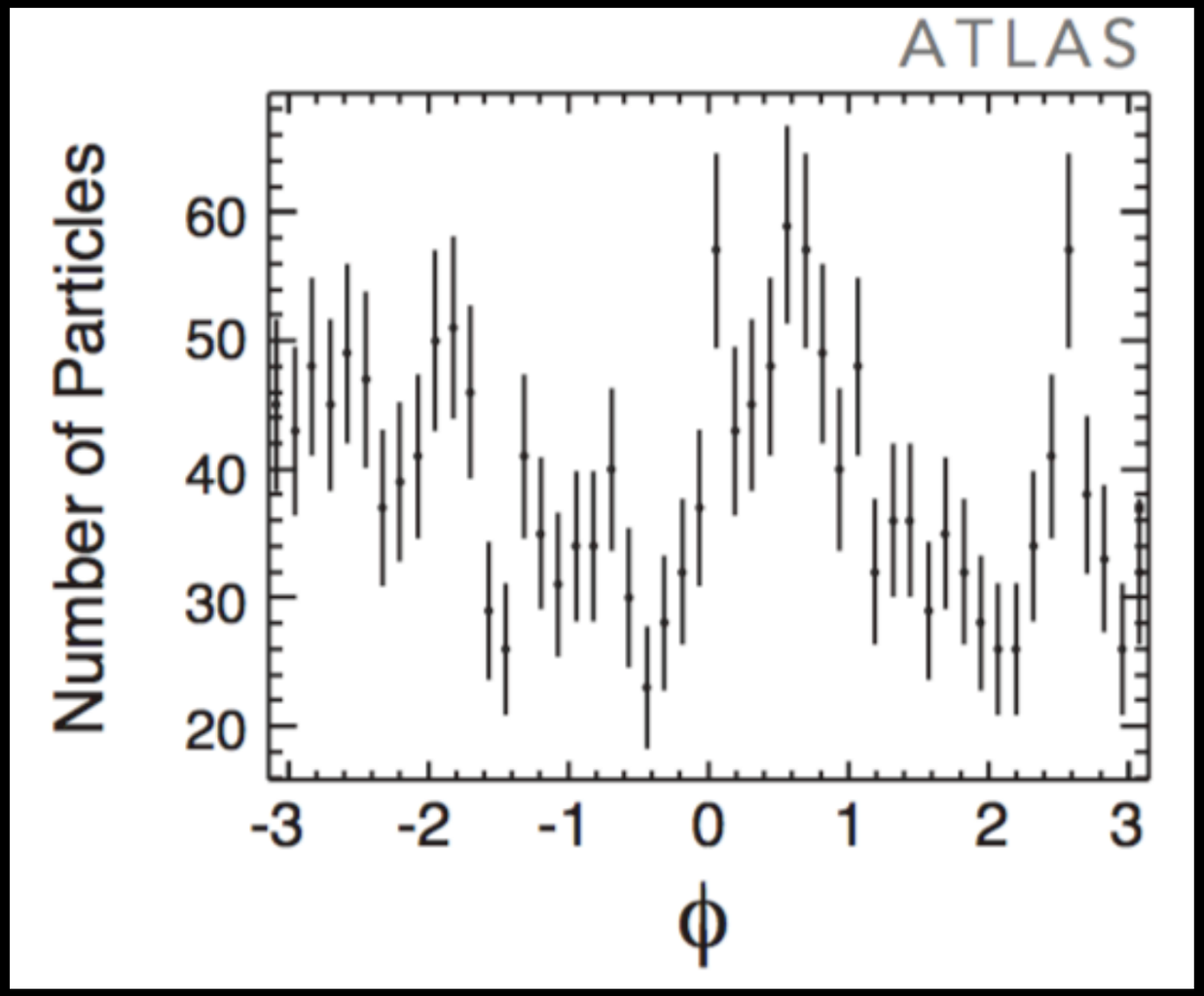
CMS Pb+Pb collisions event display

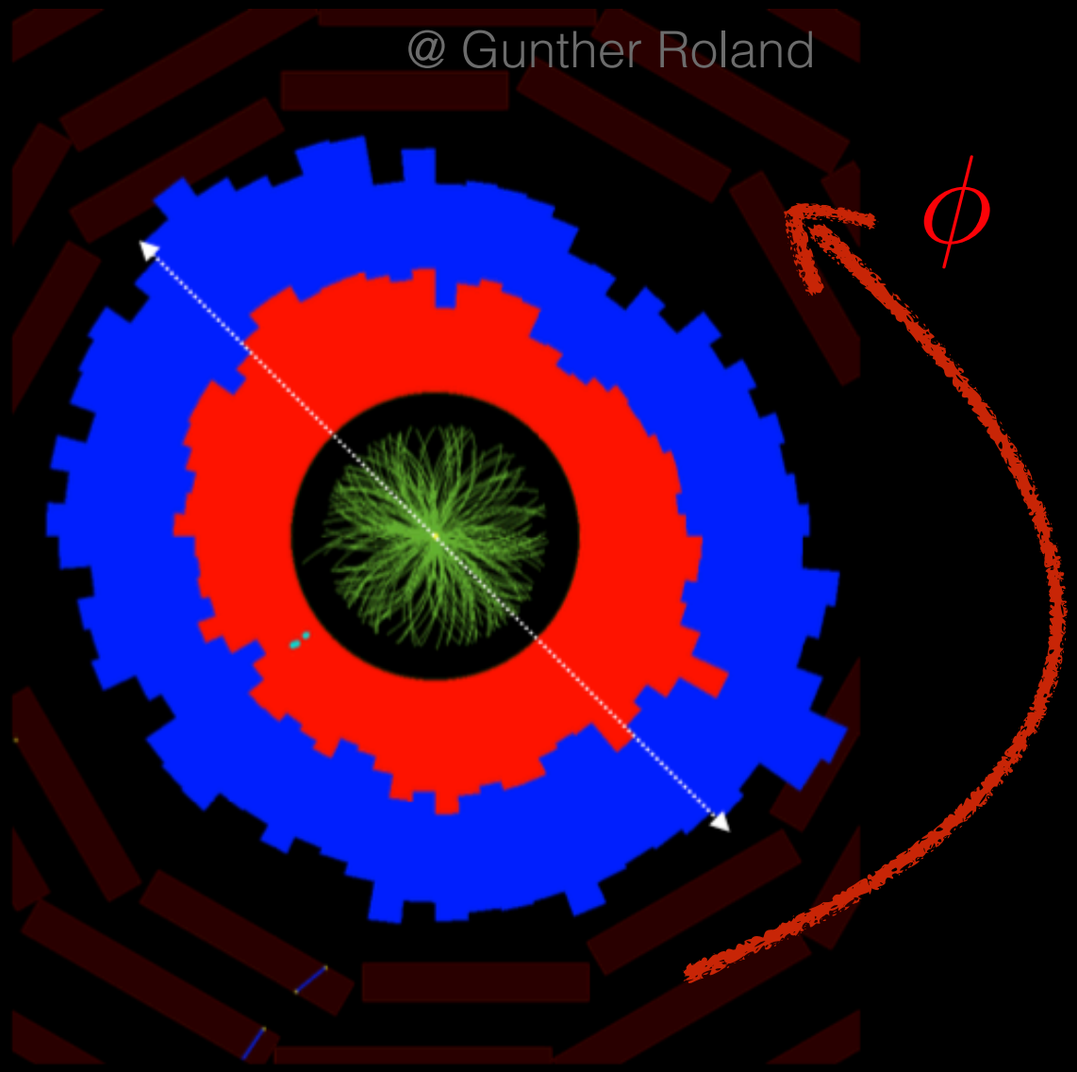
@ Gunther Roland



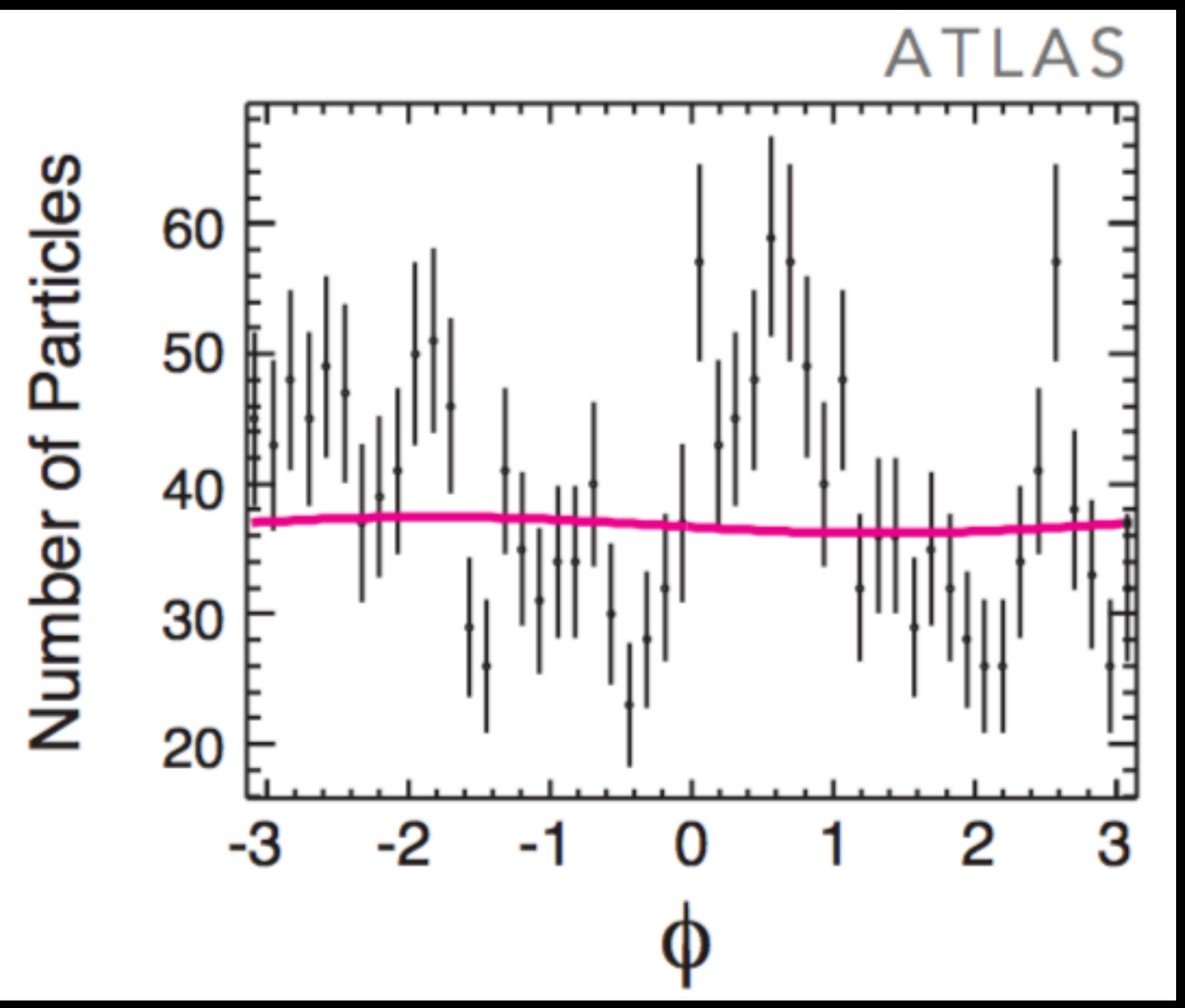
A clear azimuthal anisotropy is seen in the particle momentum distribution

Anisotropic flow

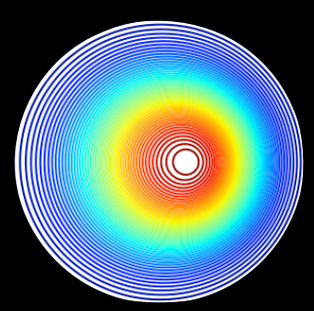




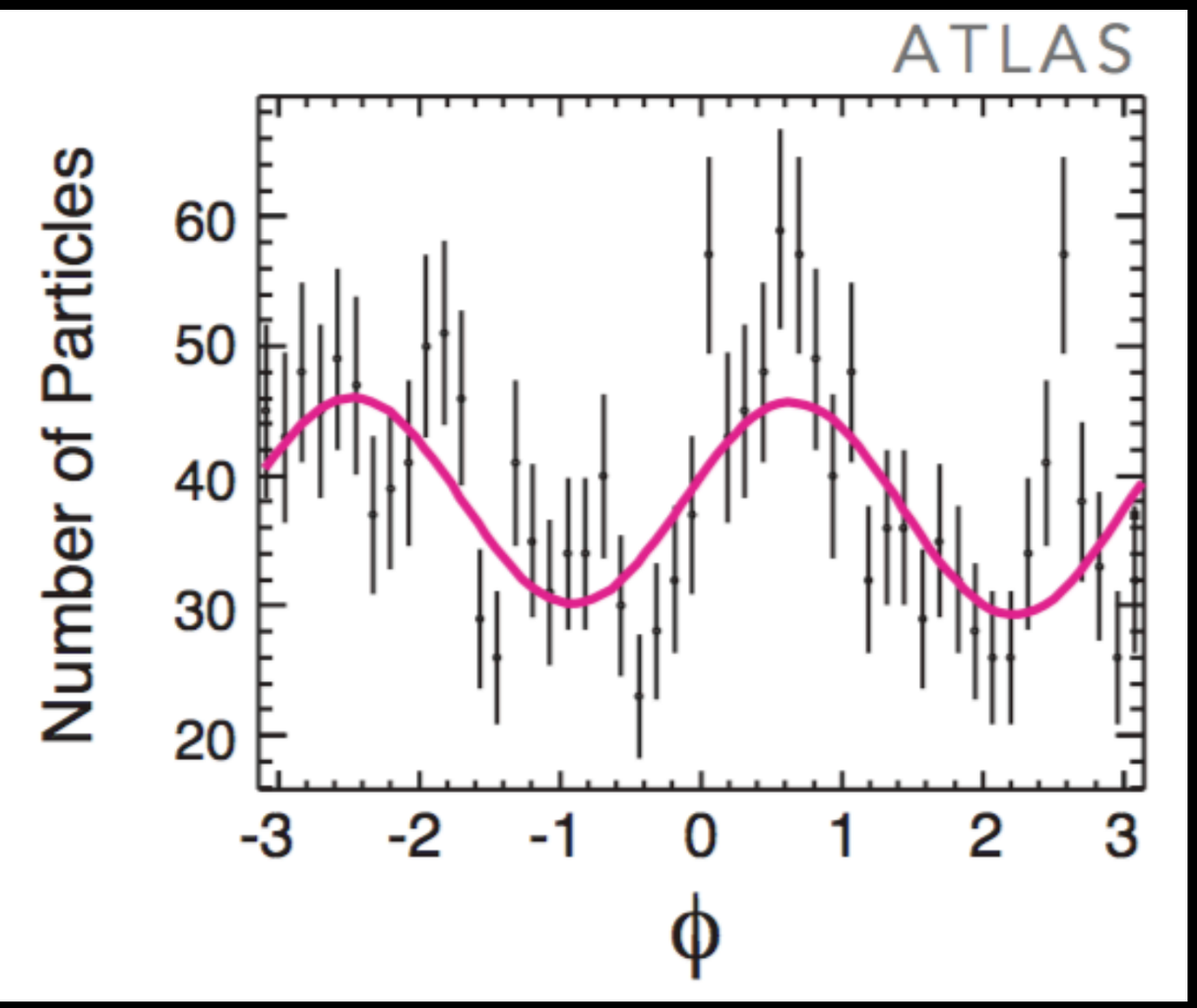
Anisotropic flow



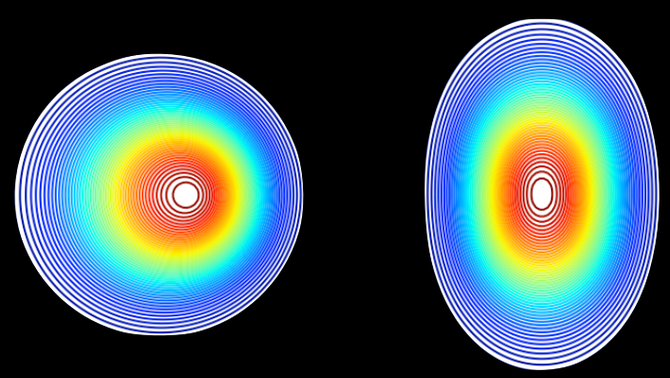
$$\frac{dN}{d\phi} = \frac{N}{2\pi}(1 + 2(v_1 \cos(\phi - \Psi_1))$$



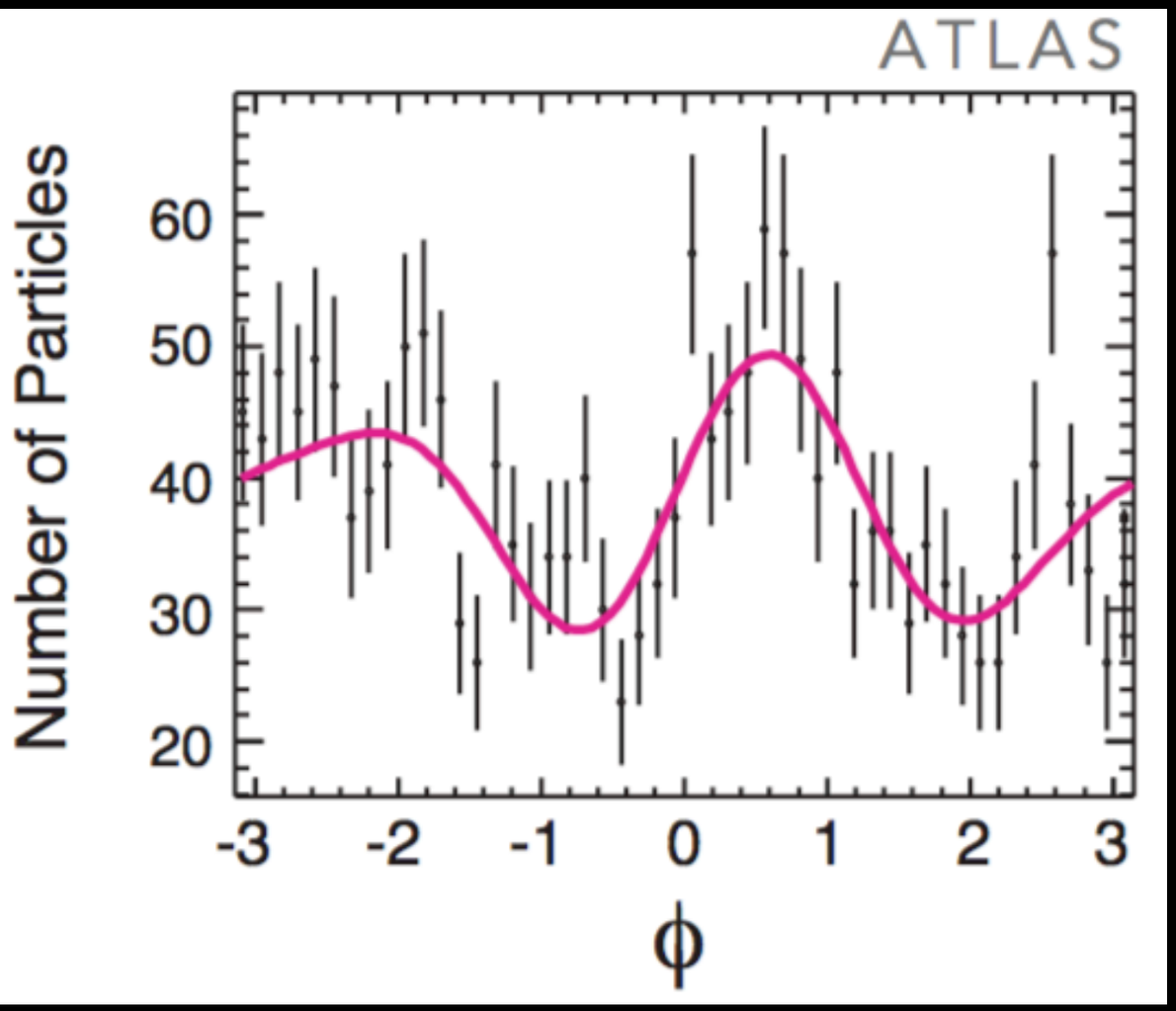
Anisotropic flow



$$\frac{dN}{d\phi} = \frac{N}{2\pi}(1 + 2(v_1 \cos(\phi - \Psi_1) + v_2 \cos(2(\phi - \Psi_2))$$



Anisotropic flow



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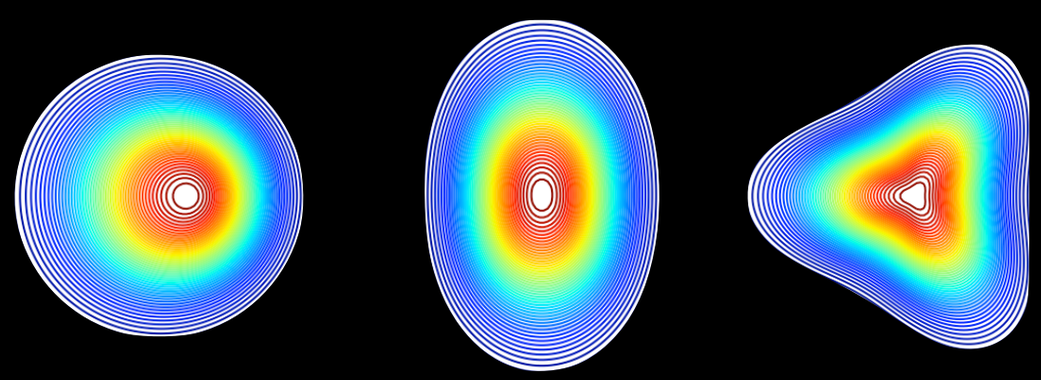
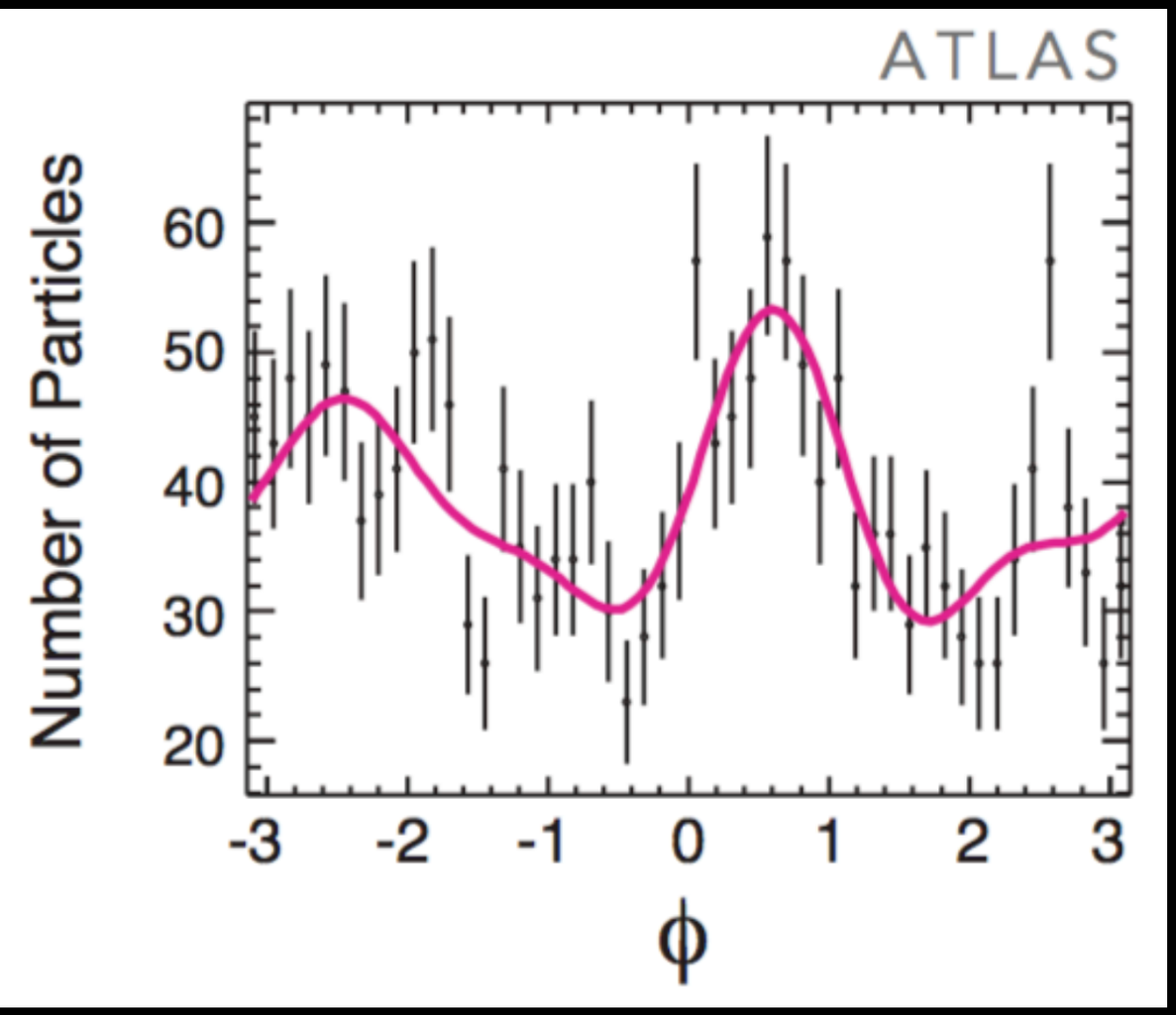


Anisotropic flow



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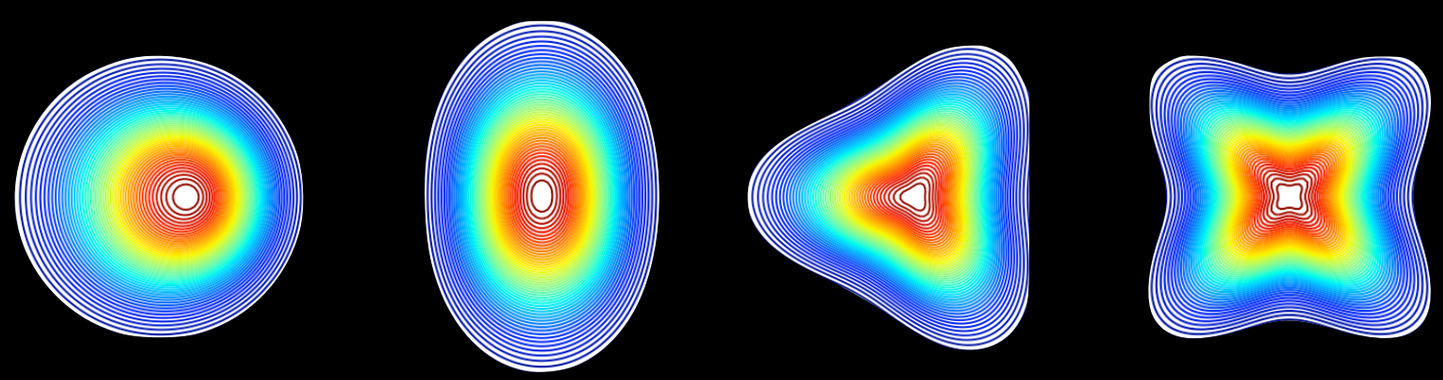
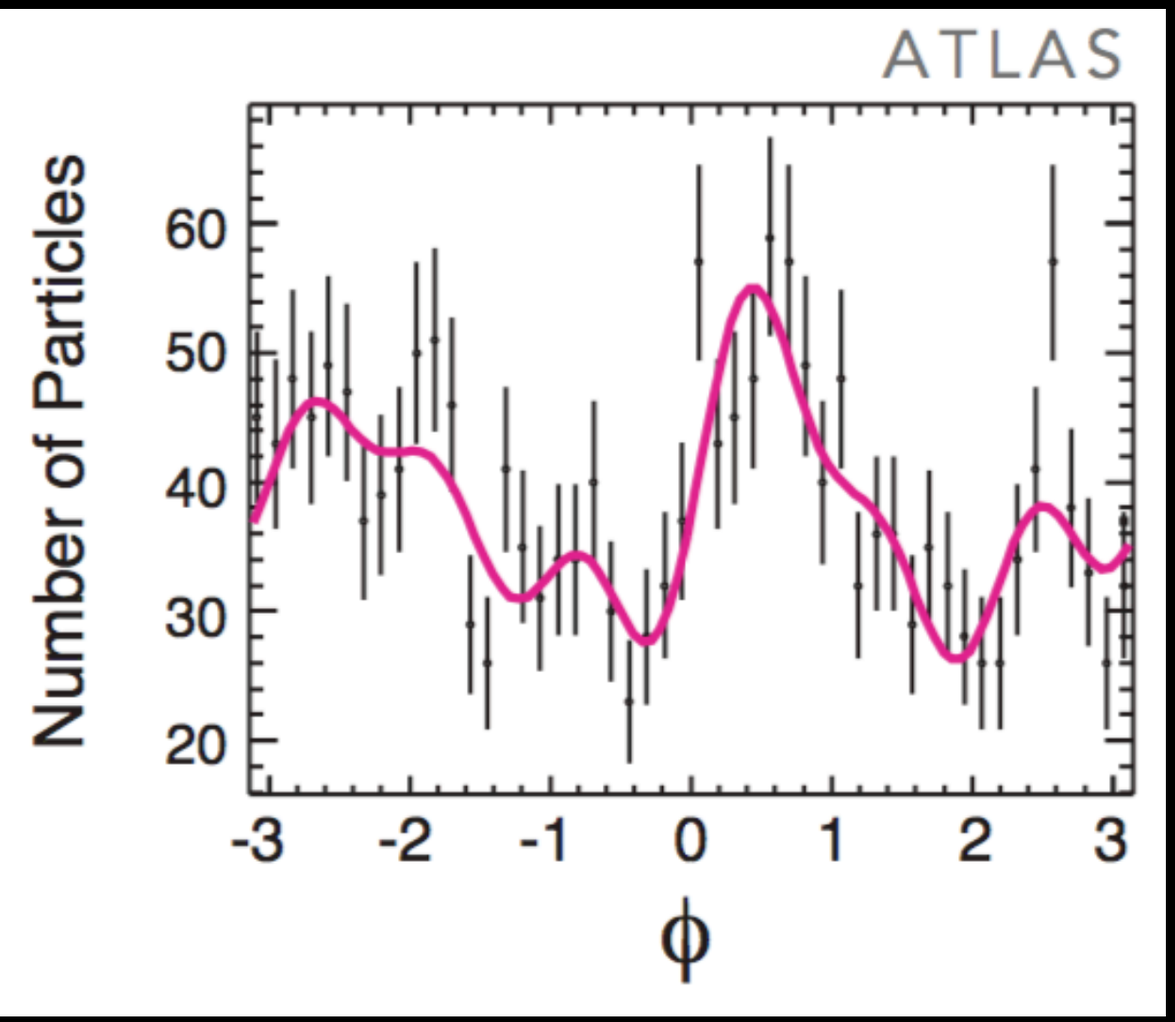


Anisotropic flow



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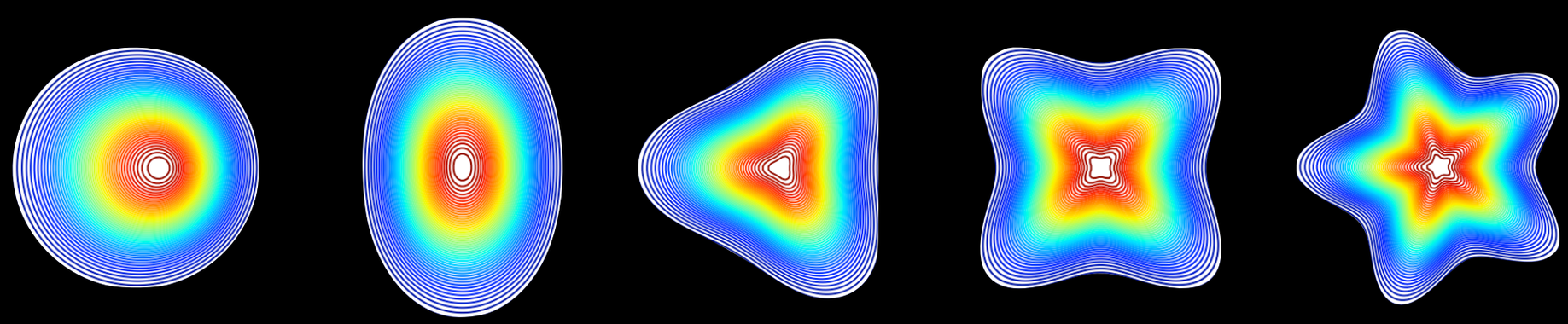
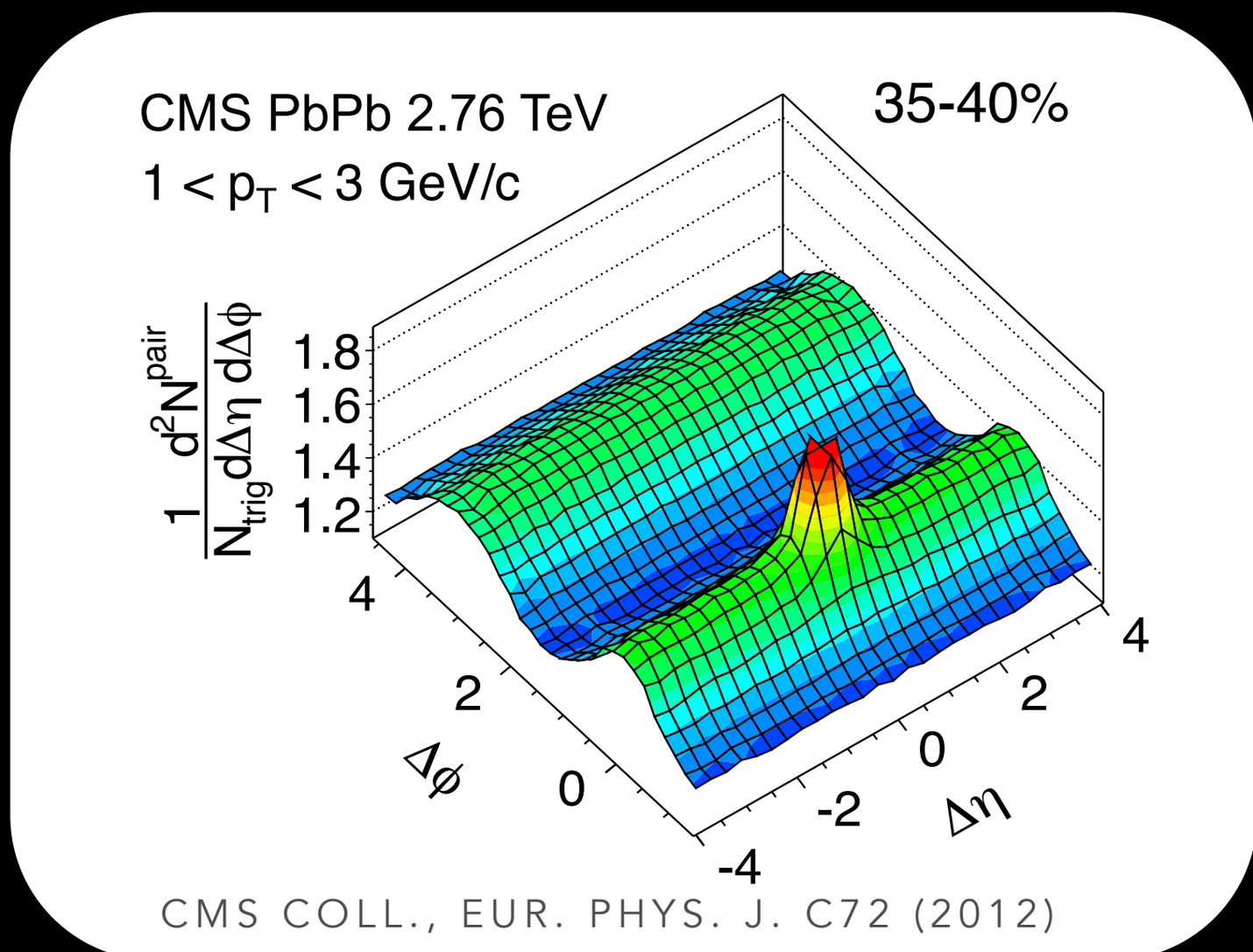


Measure v_n from multi-particle correlation

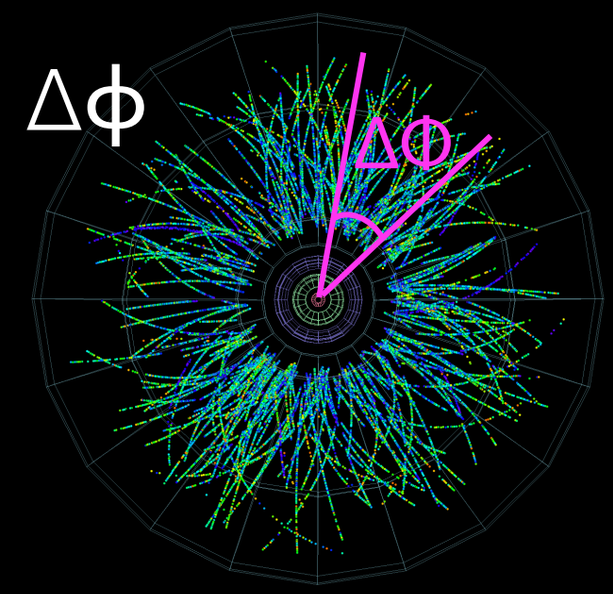
2-particle correlation as a function of $\Delta\eta$ and $\Delta\phi$

$\Delta\eta$: DIFFERENCE IN PSEUDO-RAPIDITY

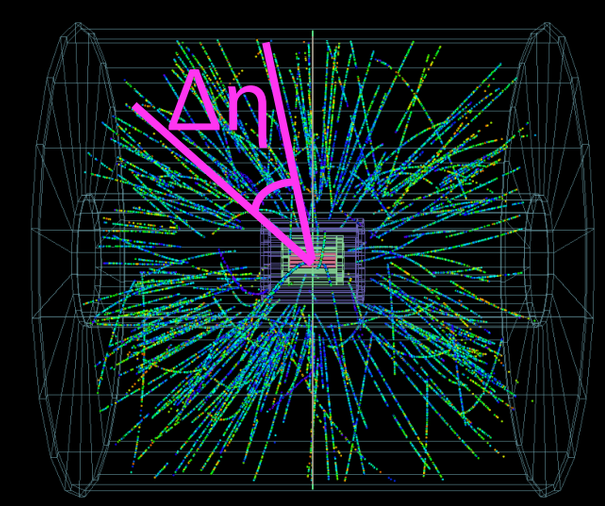
$\Delta\Phi$: DIFFERENCE IN AZIMUTHAL ANGLE



CMS COLL., EUR. PHYS. J. C72 (2012)



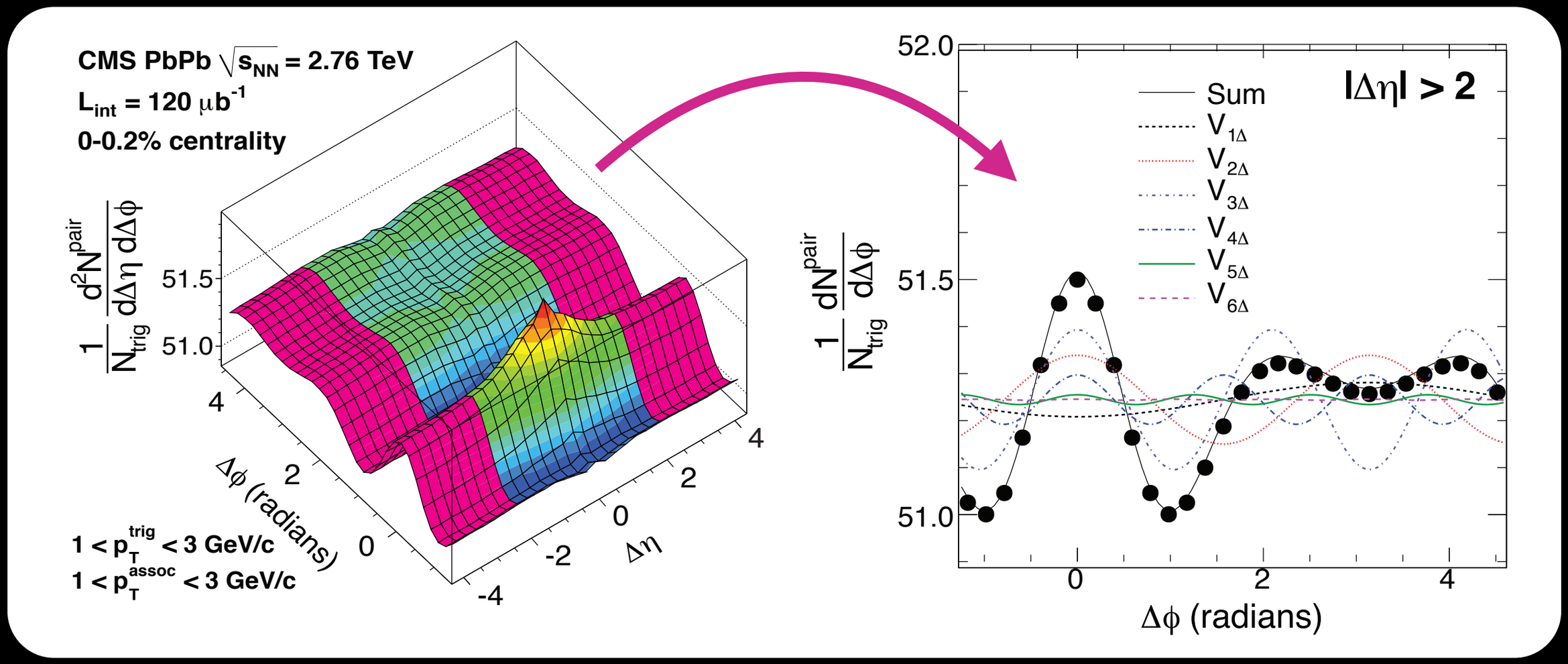
$\Delta\Phi$: DIFFERENCE IN AZIMUTHAL ANGLE



$\Delta\eta$: DIFFERENCE IN PSEUDO-RAPIDITY

Measure v_n from multi-particle correlation

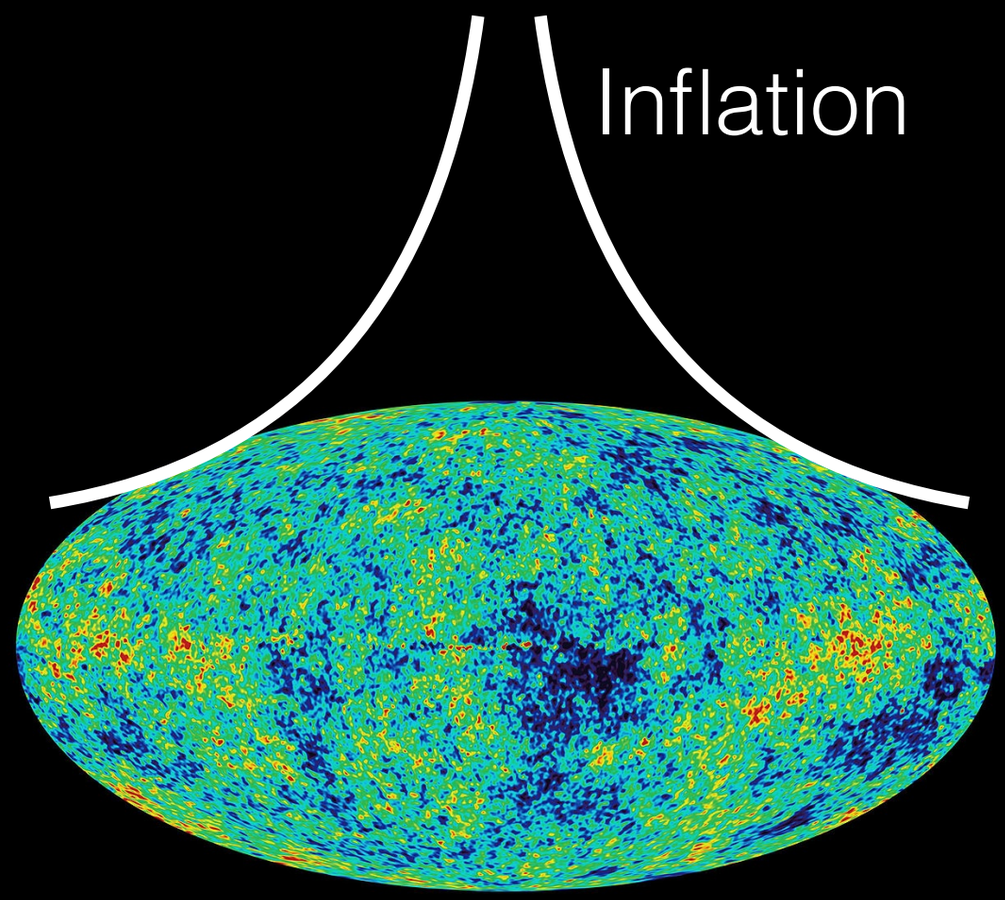
Azimuthal structure quantified using Fourier expansion

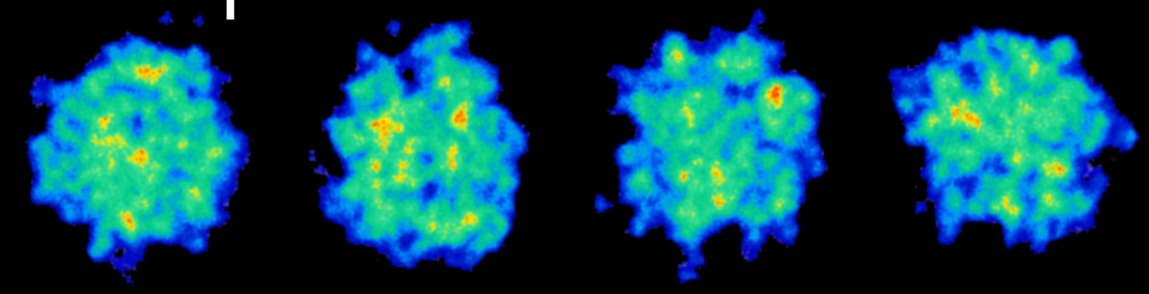


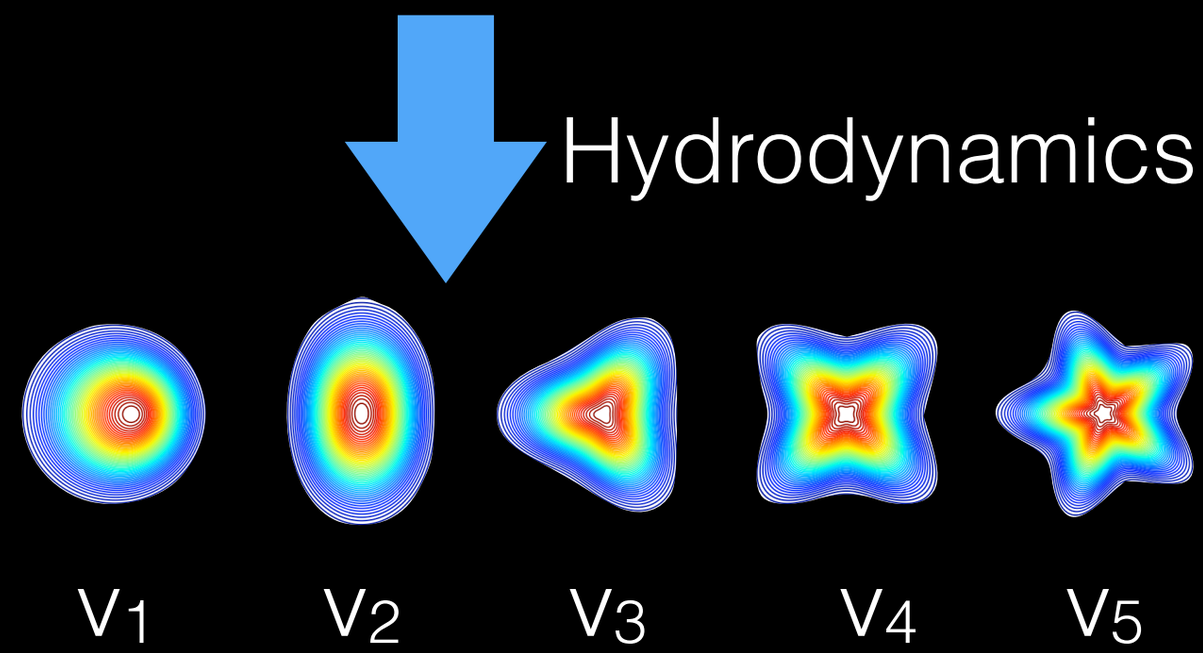
$$\frac{1}{N_{\rm trig}}\frac{dN^{\rm pair}}{d\Delta\phi} \sim 1 + 2\sum_{n=1}^{n=\infty} V_{n\Delta}(p_T^{\rm trig}, p_T^{\rm assoc})\cos(n\Delta\phi) \qquad v_n = \sqrt{V_{n\Delta}}$$

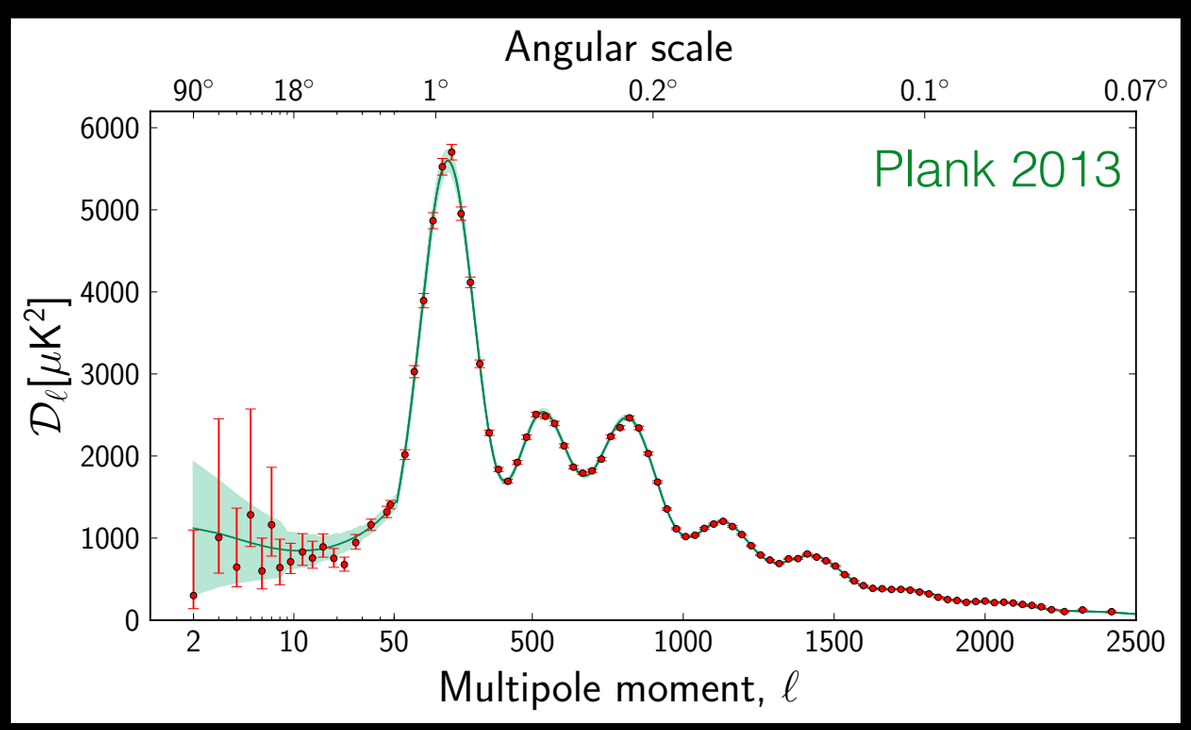
CMS Collaboration, JHEP 02 (2014) 088

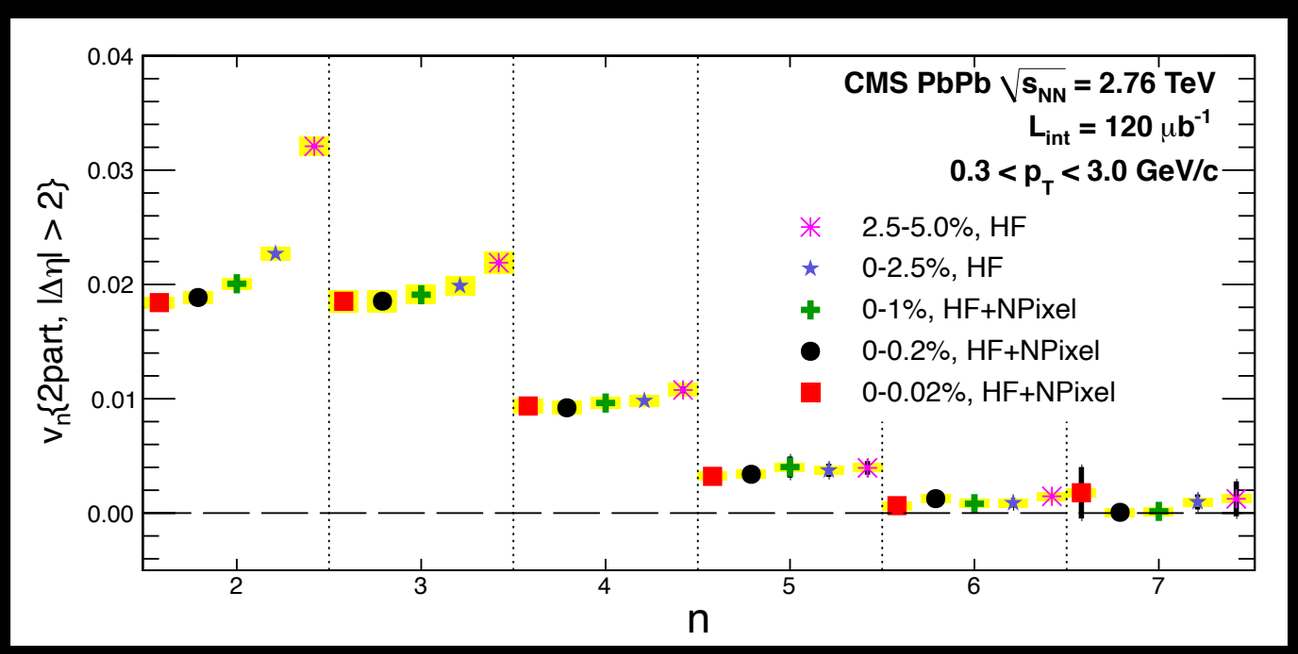
Initial Fluctuation Spectrum









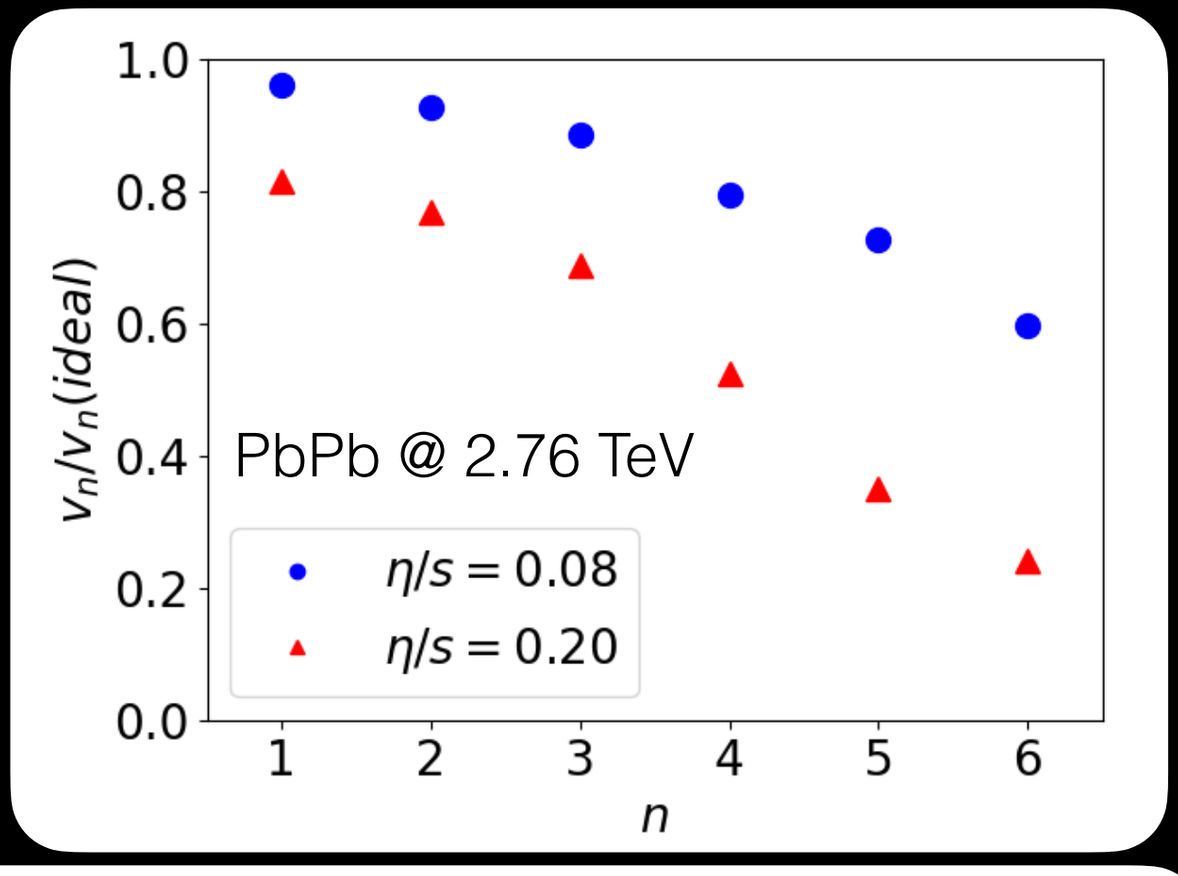


Elucidating QGP Transport Properties with Anisotropic Flows

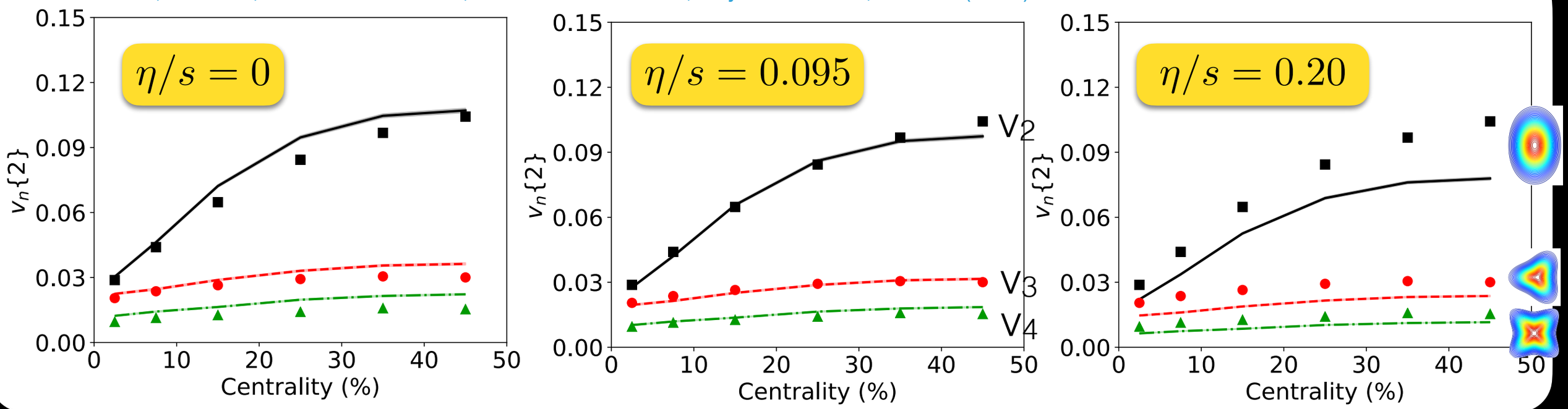
Extract the QGP shear viscosity

C. Shen, Z. Qiu and U. Heinz, Phys. Rev. C 92, 014901 (2015)

- The QGP shear viscosity suppresses particle anisotropic flow; the suppression is larger for higher order v_n

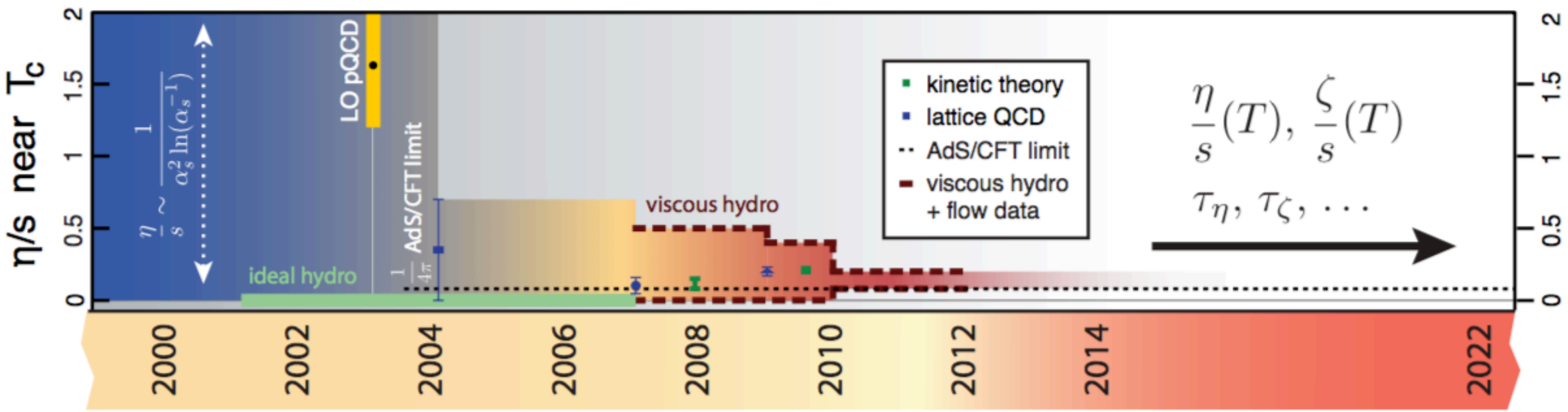


S. McDonald, C. Shen, F. Fillion-Gourdeau, S. Jeon and C. Gale, Phys. Rev. C 95, 064913 (2017)



Quantitative characterization of QGP

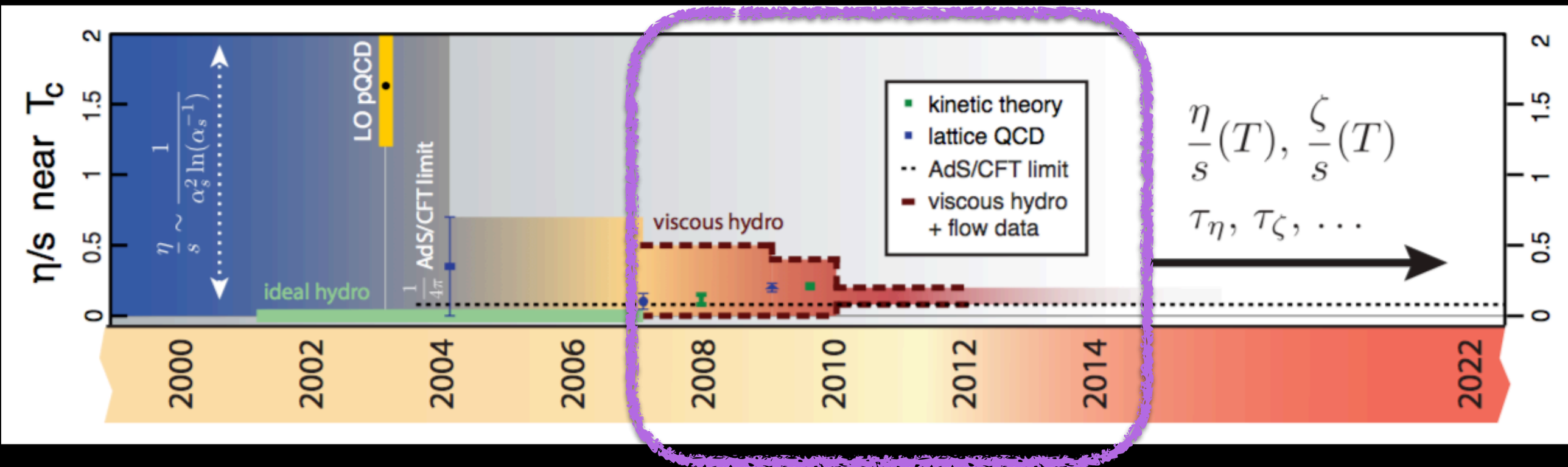
Hot & Dense QCD White Paper



- Broad theoretical efforts and experimental advances lead to precise determination of the QGP transport properties

Quantitative characterization of QGP

Hot & Dense QCD White Paper



- Broad theoretical efforts and experimental advances lead to precise determination of the QGP transport properties
- Phenomenological approaches have set the strongest constraints on the QGP shear viscosity

$$\frac{1}{4\pi} \leq (\eta/s)_{\rm QGP} \leq \frac{2.5}{4\pi}$$

C. Shen and U. Heinz, Nucl. Phys. News **25**, 6 (2015)

Nearly Perfect Fluidity

Specific shear viscosity η/s

P. Danielewicz and M. Gyulassy, Phys. Rev. D 31, 53 (1985)

In kinetic theory, the shear viscosity can be computed as

$$\eta = \frac{1}{3} n \bar{p} l_{\mathrm{mfp}}$$

For a relativistic system, the entropy density $s \sim 4 k_B n$

So the ratio

$$\frac{\eta}{s} \sim \frac{1}{12} \frac{\bar{p} l_{\mathrm{mfp}}}{k_B}$$

~ momentum diffusion per degree of freedom

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$$\frac{\eta}{s} \sim \frac{1}{12} \frac{\bar{p} l_{\rm mfp}}{k_B} \geq \frac{1}{12} \frac{\hbar}{k_B}$$

~ momentum diffusion per degree of freedom

A minimum bound from the uncertainty relation

A universal bound was derived from string theory

Kovtun, Son & Starinets PRL 94 (2005) 111601

$$\frac{\eta}{s} \geq \frac{1}{4\pi} \frac{\hbar}{k_B}$$

Specific shear viscosity η/s

P. Danielewicz and M. Gyulassy, Phys. Rev. D 31, 53 (1985)

T. Schäfer and D. Teaney, Rept. Prog. Phys. **72**, 126001 (2009)

Fluid	$P[\mathrm{Pa}]$	$T[K]$	$\eta[\mathrm{Pa}\cdot s]$	$\eta/s[\hbar/k_B]$
H_2O	10^5	370	2.9×10^{-4}	8.2
^4He	10^5	2.0	1.2×10^{-6}	1.9
$^6Li\ (a=\infty)$	12×10^{-9}	23×10^{-6}	$\leq 1.7\times10^{-15}$	≤ 0.5
QGP	10^{35}	2×10^{12}	5×10^{11}	0.08-0.2

smaller η/s better fluidity

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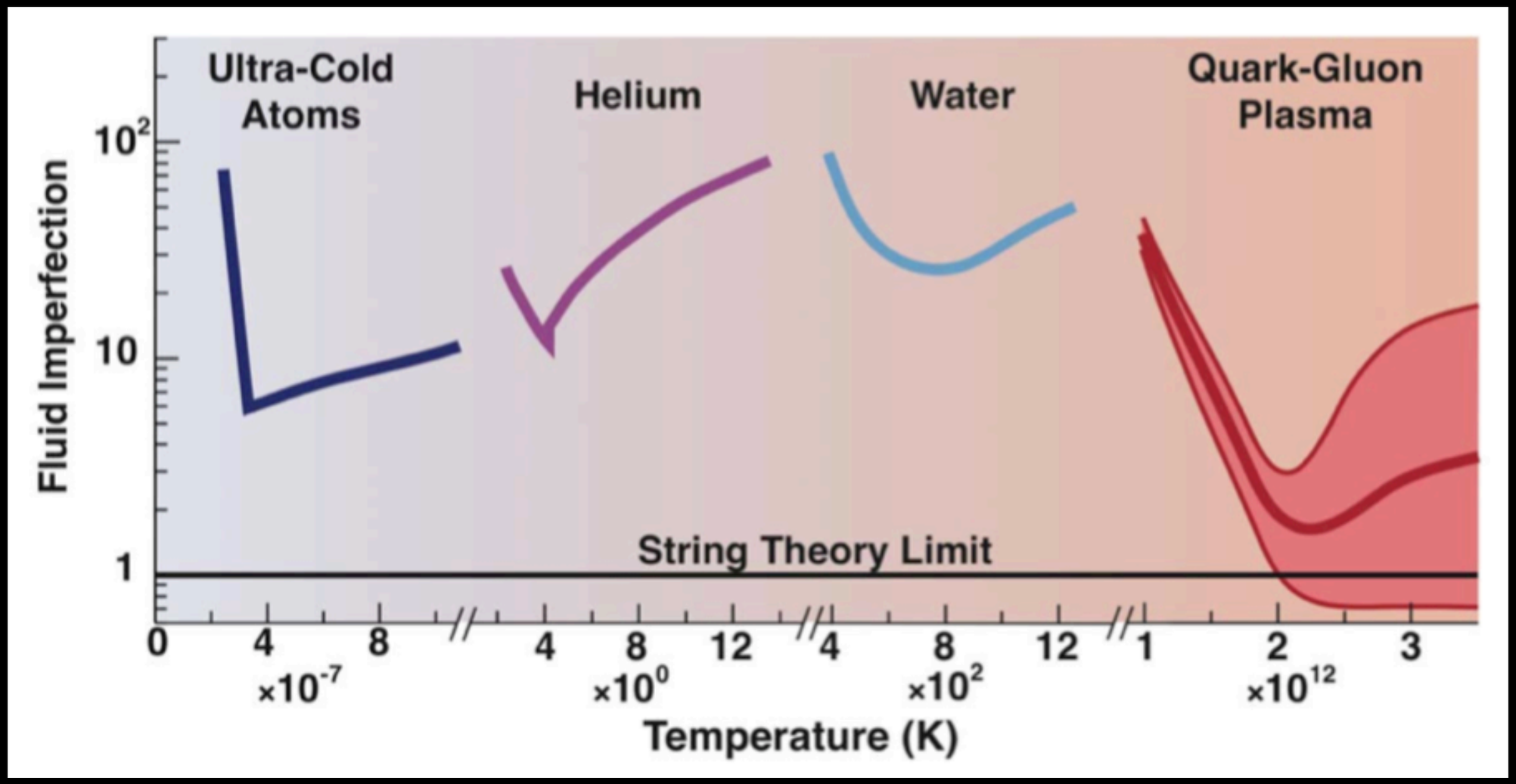
Quantitative characterization of QGP

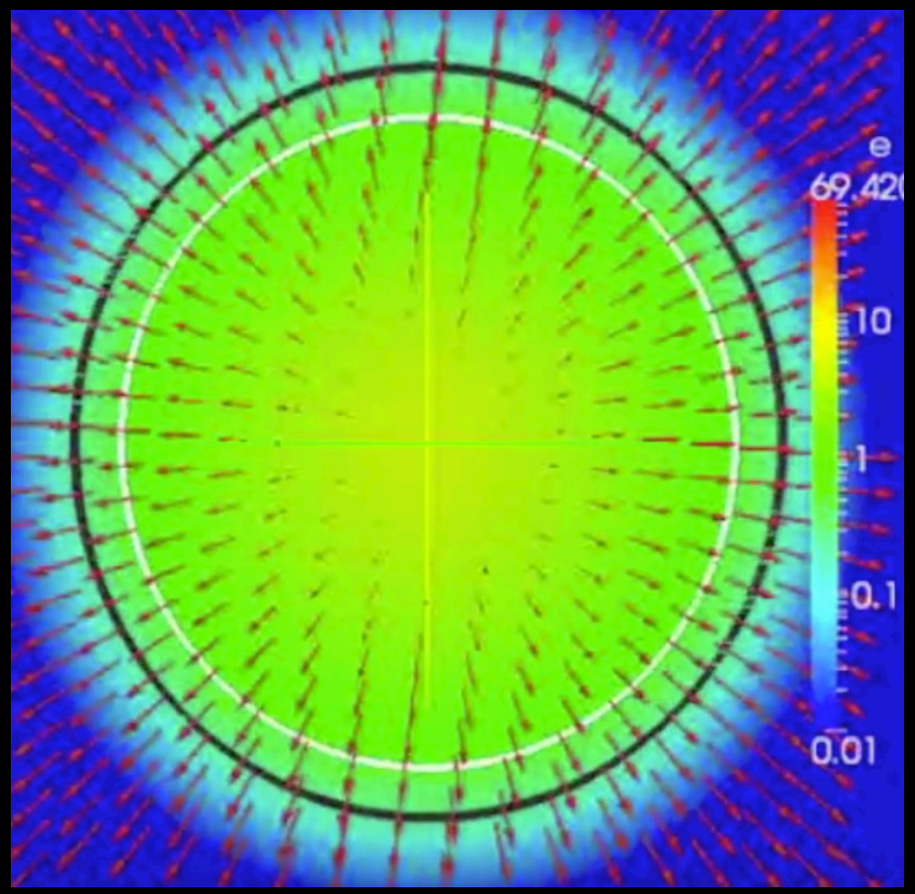
Burrows A et al. 2013 Implementing the 2007 Long Range Plan



- Quark-Gluon Plasma is the hottest, smallest, and the most perfect fluid in nature!

Phenomenological Success of Hydrodynamics Description of QGP

Radial flow and the slopes of particle spectra



- Particles are “surfing” with a relativistic flow

$$E\frac{dN_i}{d^3p} = \frac{g_i}{(2\pi)^3}\int_{\Sigma(T_{\rm fo})} d^3\sigma_\mu p^\mu \frac{1}{e^{p^\mu u_\mu/T} \pm 1}$$

Pions: $T_{\rm slope} \sim T\sqrt{\frac{1+v_\perp}{1-v_\perp}}$

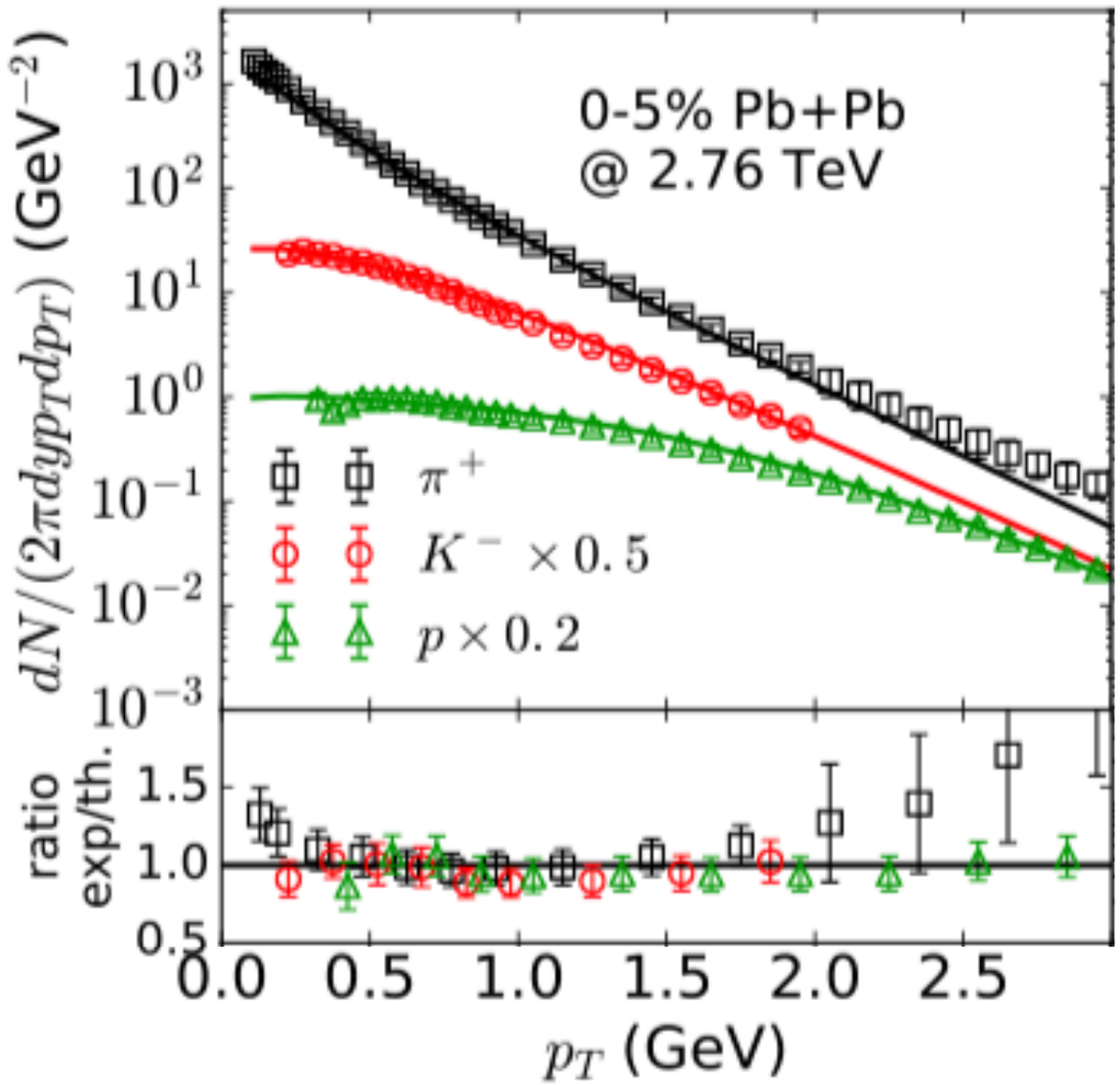
$m \sim T$

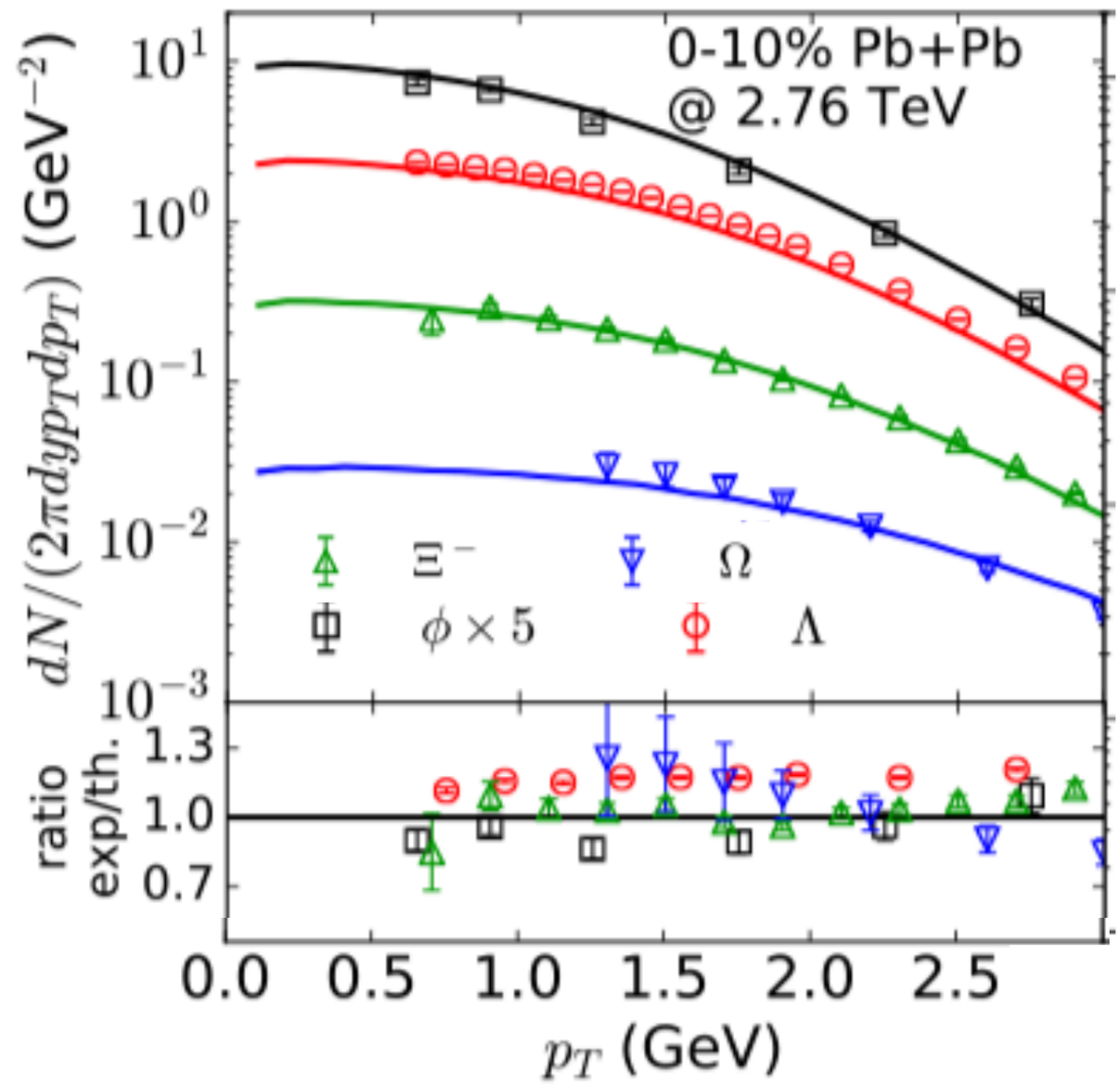
Protons: $T_{\rm slope} \sim T + \frac{1}{2}mv_\perp^2$

$m \gg T$

Identified particle spectra

S. McDonald, C. Shen, F. Fillion-Gourdeau, S. Jeon and C. Gale, Phys. Rev. C 95, 064913 (2017)

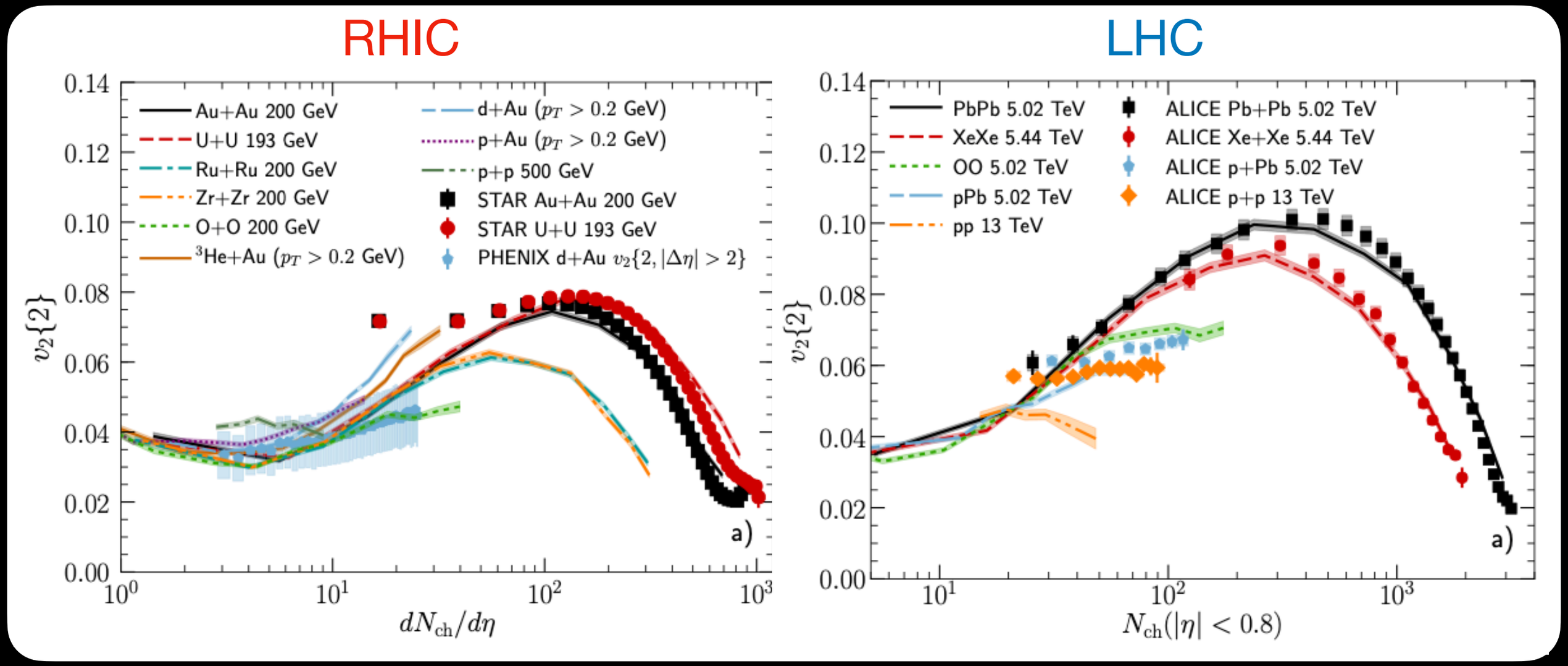




- A common hydrodynamic radial flow can describe a zoo of identified particle spectra within 30% accuracy

Anisotropic flow v_n

B. Schenke, C. Shen and P. Tribedy, arXiv:2005.14682 [nucl-th]

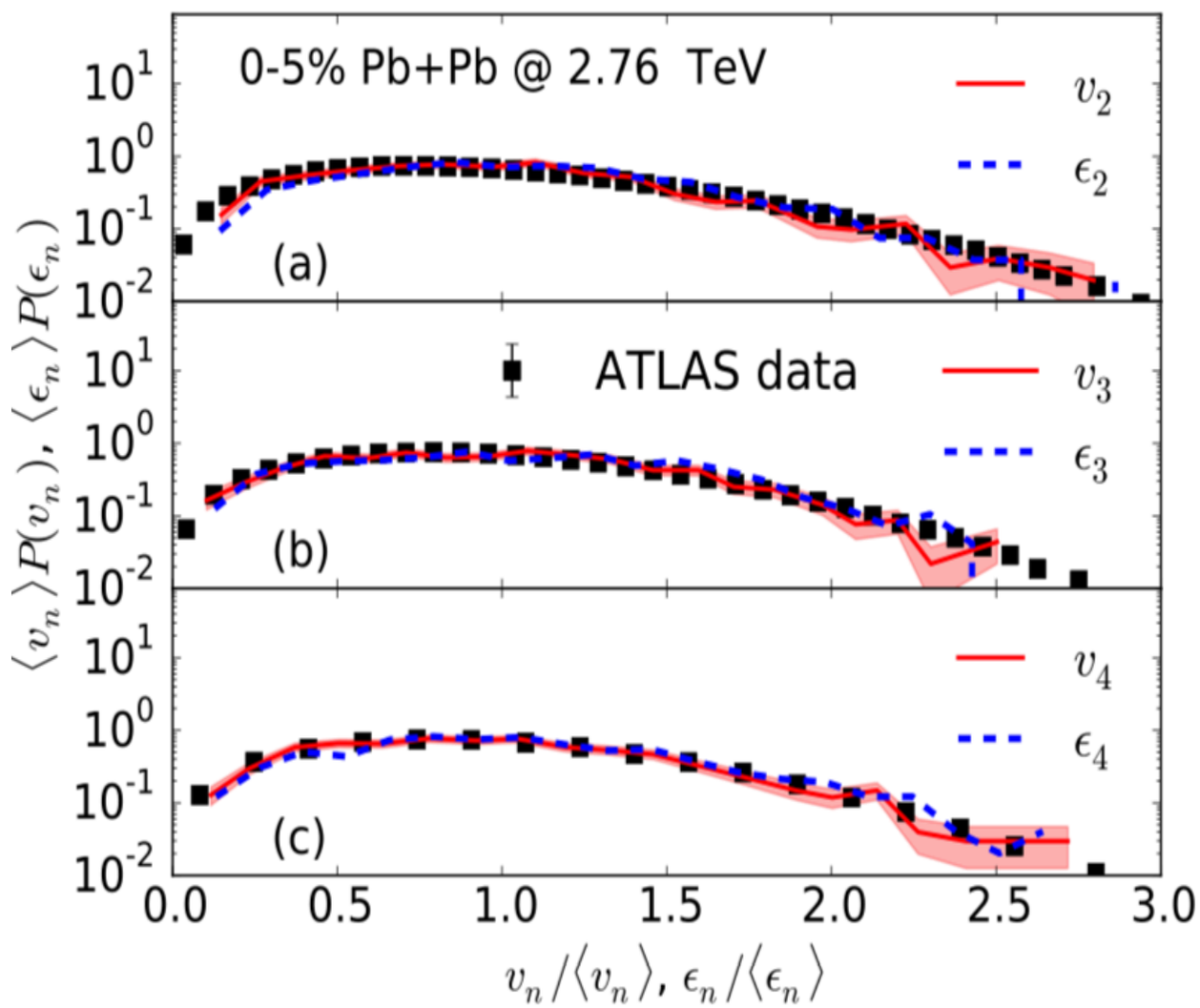


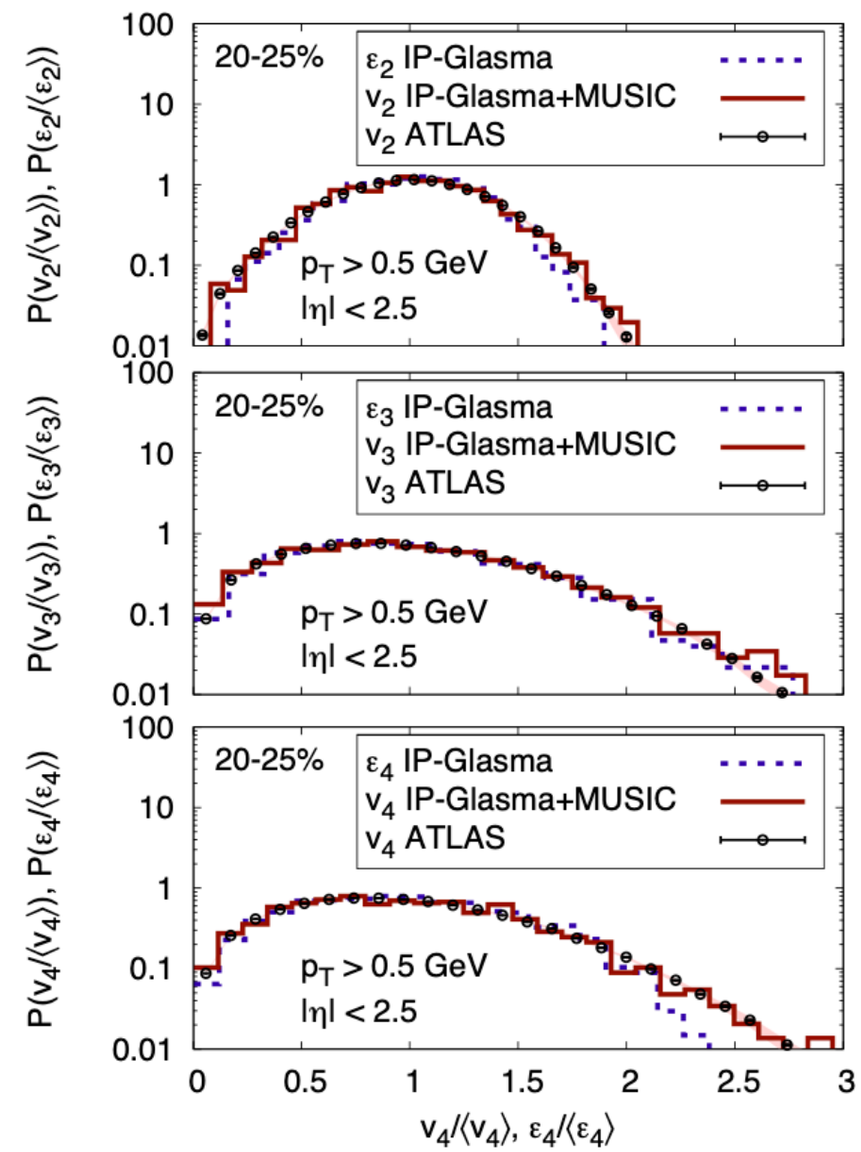
- A framework with viscous hydrodynamics can describe flow coefficients over more than 2 orders of magnitude in charge hadron multiplicity

Event-by-event distribution of v_n

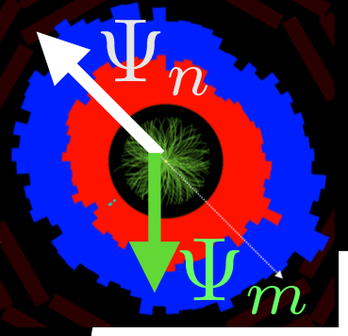
C. Gale, S. Jeon, B. Schenke, P. Tribedy and R. Venugopalan, Phys. Rev. Lett. 10, 012302 (2013)

S. McDonald, C. Shen, F. Fillion-Gourdeau, S. Jeon and C. Gale, Phys. Rev. C 95, 064913 (2017)



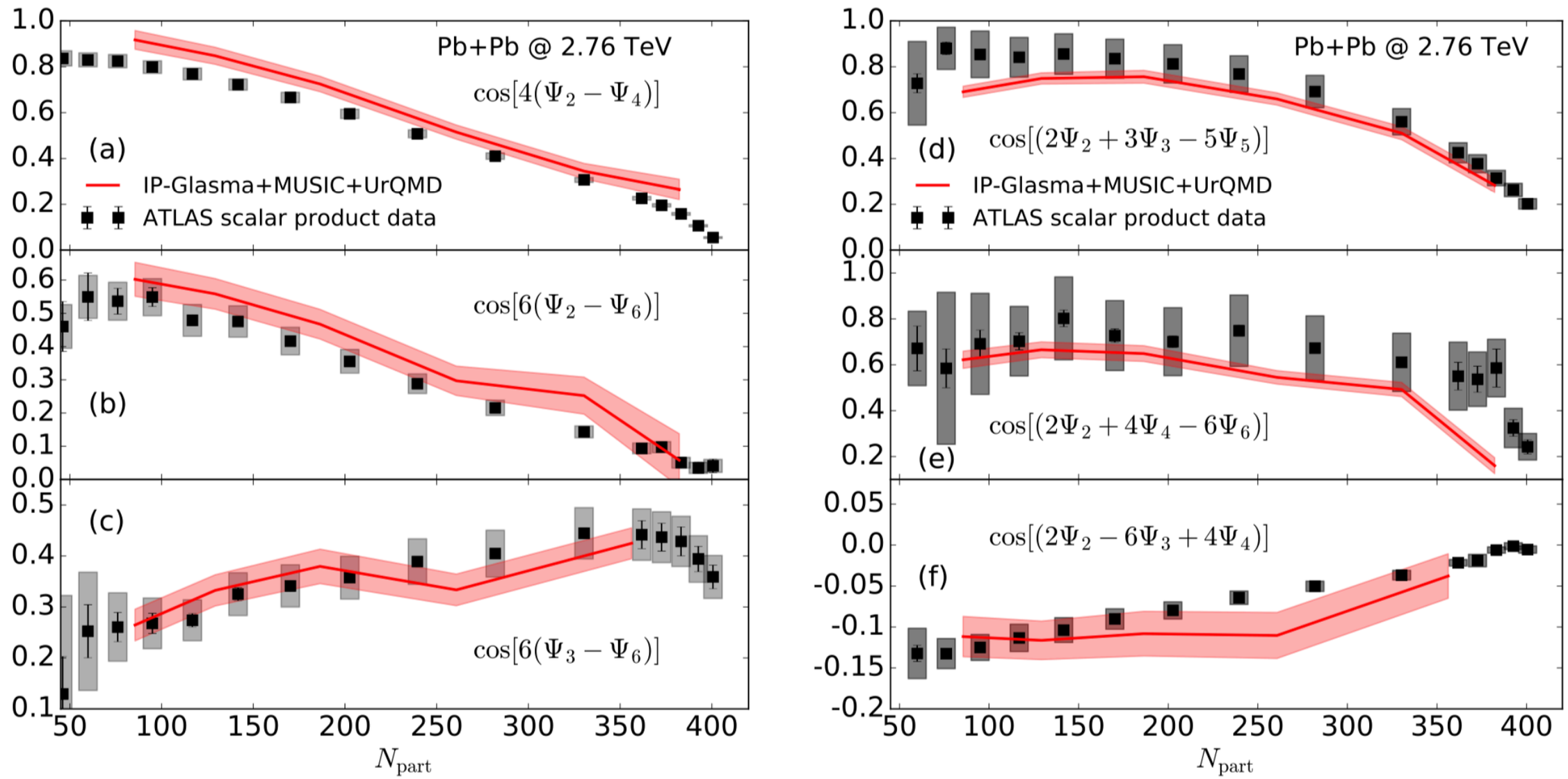


- Hydrodynamic framework can reproduce the charged hadron v_n at LHC on an **event-by-event** basis!



Event plane correlation

Z. Qiu and U. Heinz, Phys. Lett. B 717, 261 (2012)
S. McDonald, C. Shen, F. Fillion-Gourdeau, S. Jeon and C. Gale, Phys. Rev. C 95, 064913 (2017)



- Hydrodynamic framework can reproduce the correlations between different flow harmonics

Summary

- Quark-Gluon Plasma is the hottest, smallest, and the most perfect fluid ever created in the laboratory

  relativistic, strongly coupled, and nonlinear system exhibits universal collective behavior

- Viscous relativistic hydrodynamics provides a **robust**, **reliable**, **efficient**, and **accurate** description of QGP evolution in heavy-ion collisions

  state-of-the-art lattice QCD inputs

  a few remarkable quantitative predictions

  model parameters has direct connection to the physics properties of the strongly-couple QGP