Modification of Hard Jets in a Dense Medium

Gojko Vujanovic

Wayne State University

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Conseil de recherches en sciences naturelles et en génie du Canada

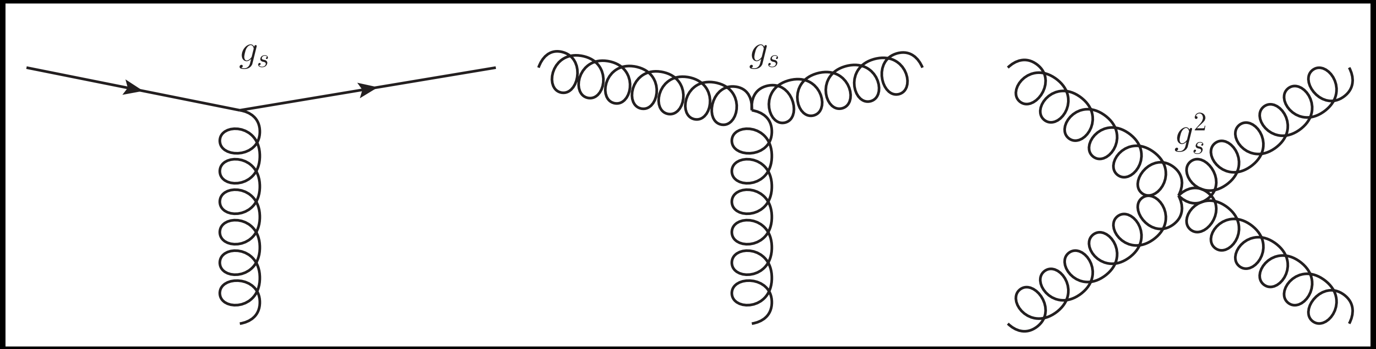
Preparing for the hands-on

- % = host machine
- $ = docker container
- % mkdir ~/MATTER_LBT_result
- Download and run the docker container
  - % docker run -it -v ~/MATTER_LBT_result:/home/jetscape-user/JETSCAPE/MATTER_LBT_results -p 8888:8888 gvujan/jetscape-school:latest

Learning about QCD

- Two regimes of JETSCAPE calculations :
  - Non-perturbative: Hydrodynamics (lattice EoS, etc) ⇒ Chun's lecture
  - Perturbative regime: parton propagation in the QGP (no hadronization) ⇒ this talk

- QCD is an SU(3) gauge theory
  - Quark-gluon interaction reminiscent of QED
  - Gluon-gluon couplings new to QCD



- The coupling in the QCD Lagrangian is g_s.
  - More often we think in terms of $\alpha_s = \frac{g_s^2}{4\pi}$

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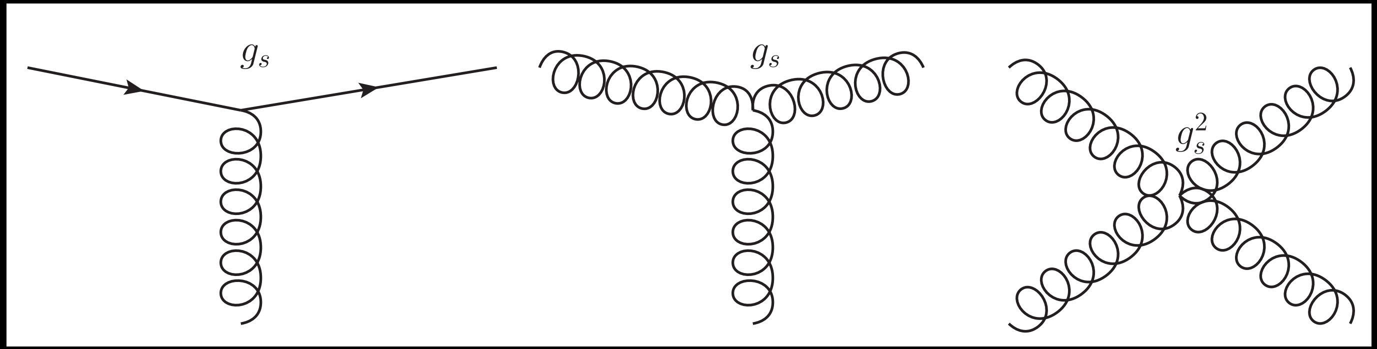
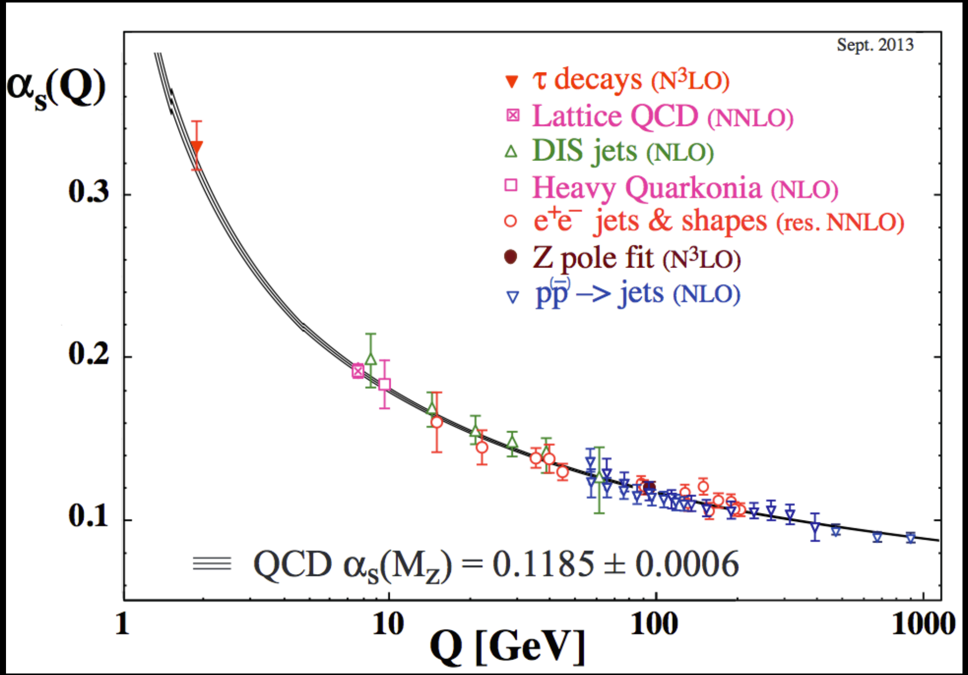
- These extra interactions in QCD make it asymptotically free. Perturbative QCD (pQCD) can only be performed at scale, e.g. energy (E) or more commonly virtuality $Q \gg 1$ GeV^2, where $g_s(Q) \ll 1$ *or* $\alpha_s(Q) \lll 1$.



$$|Q^2| = E^2 - \boldsymbol{p}^2 - m^2$$

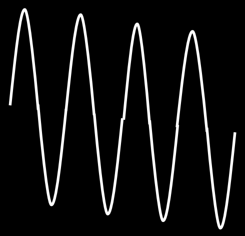
$$\frac{g_s^2}{4\pi} = \alpha_s = \frac{12\pi}{(11N_c - 2N_f)\log\left(\frac{Q^2}{\Lambda_{QCD}^2}\right)}$$



Learning about QCD

- Factorization:
  - Are pQCD processes even related to observables? Yes, as there's a separation of scales (factorization)

A,B: Long wavelength, hadrons $E \lesssim 1$ GeV, $\mathrm{Q} \lesssim 1$ GeV



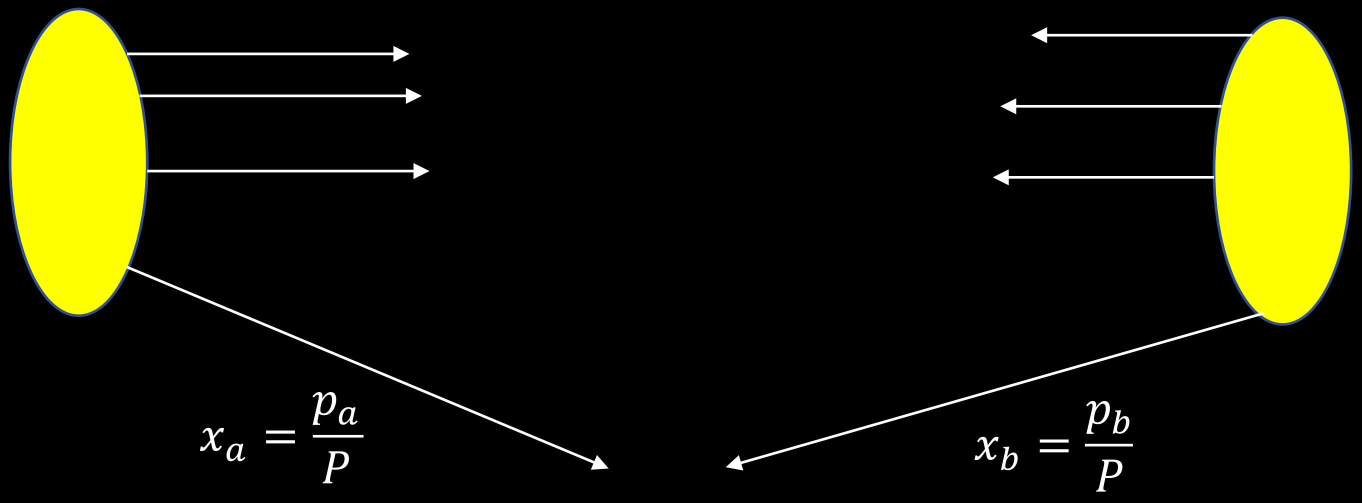
C,D: Short wavelength, partons $E \gtrsim 10$ GeV, $Q \gtrsim 10$GeV

  - As long as there is a large separation between scales, "interference terms" between the pair (A,B) and the pair (C,D) are negligible. More formally,

$$\mathcal{M} \propto (A + B)(C + D)$$

$|\mathcal{M}|^2 \propto |(A + B)(C + D)|^2 \Rightarrow |A + B|^2 |C + D|^2$ under factorization, i.e. there are no cross-terms (AC, BD)

Factorization at work: Leading order diagram



- Parton Distribution Function (PDF) G: Prob. of finding a parton from the hardon
  - a non-perturbative process, most easily measured in $e + p$ experiment (e.g. HERA)

$$\frac{d\sigma_1^h}{dy dp_{T_1}} \sim \int dx_a dx_b \boxed{G(x_a)G(x_b)} \left[\frac{d\hat{\sigma}}{d\hat{t}}\right] D(z_1)$$

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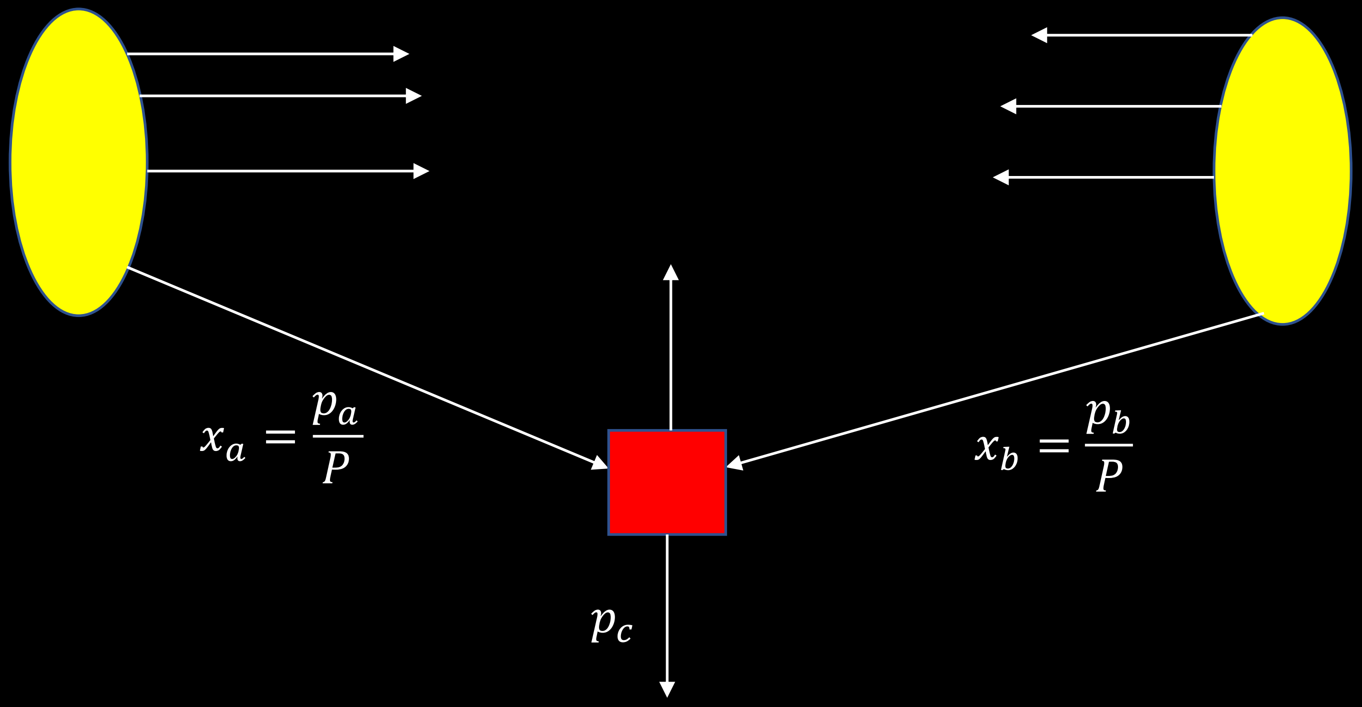

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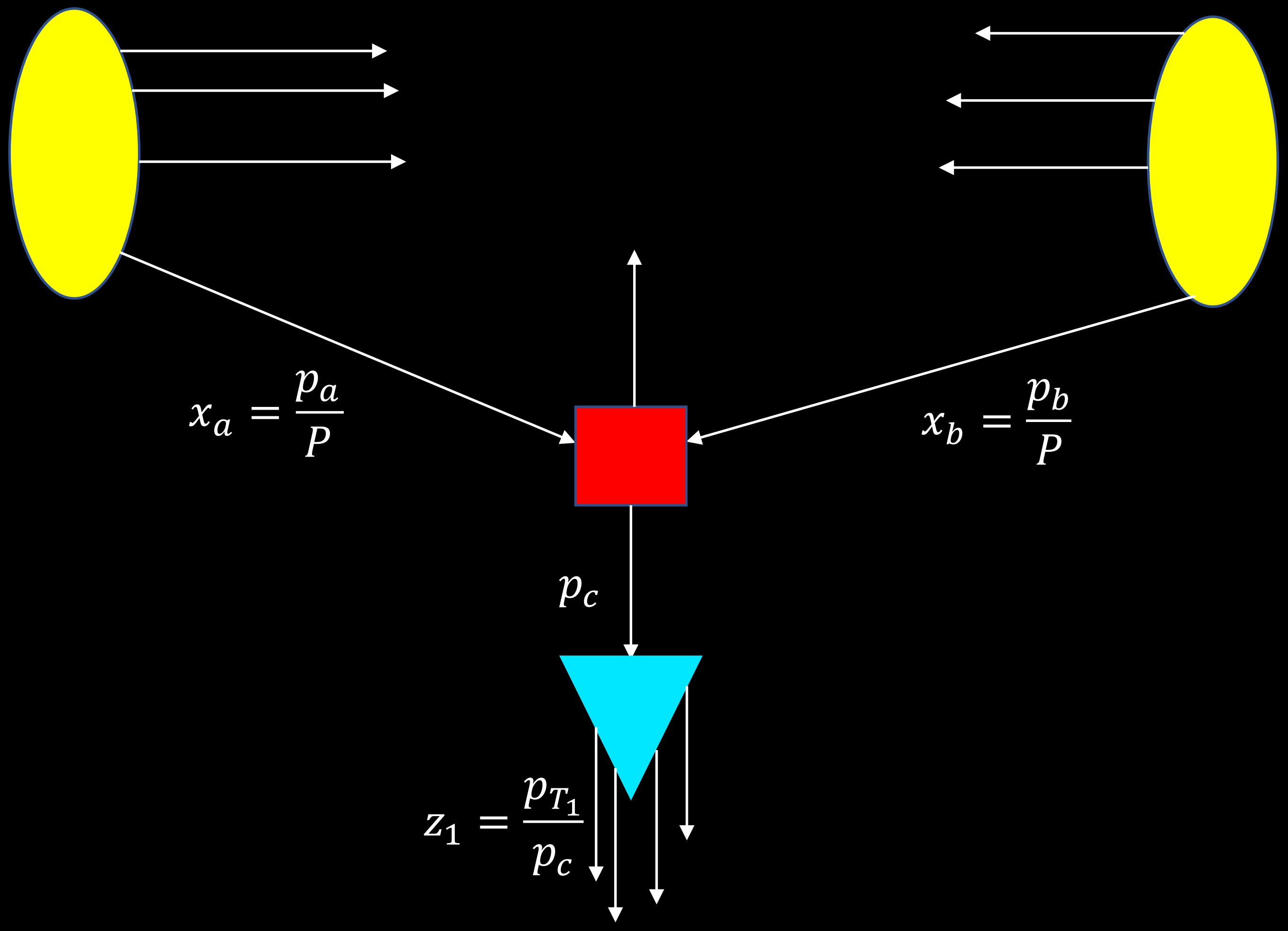

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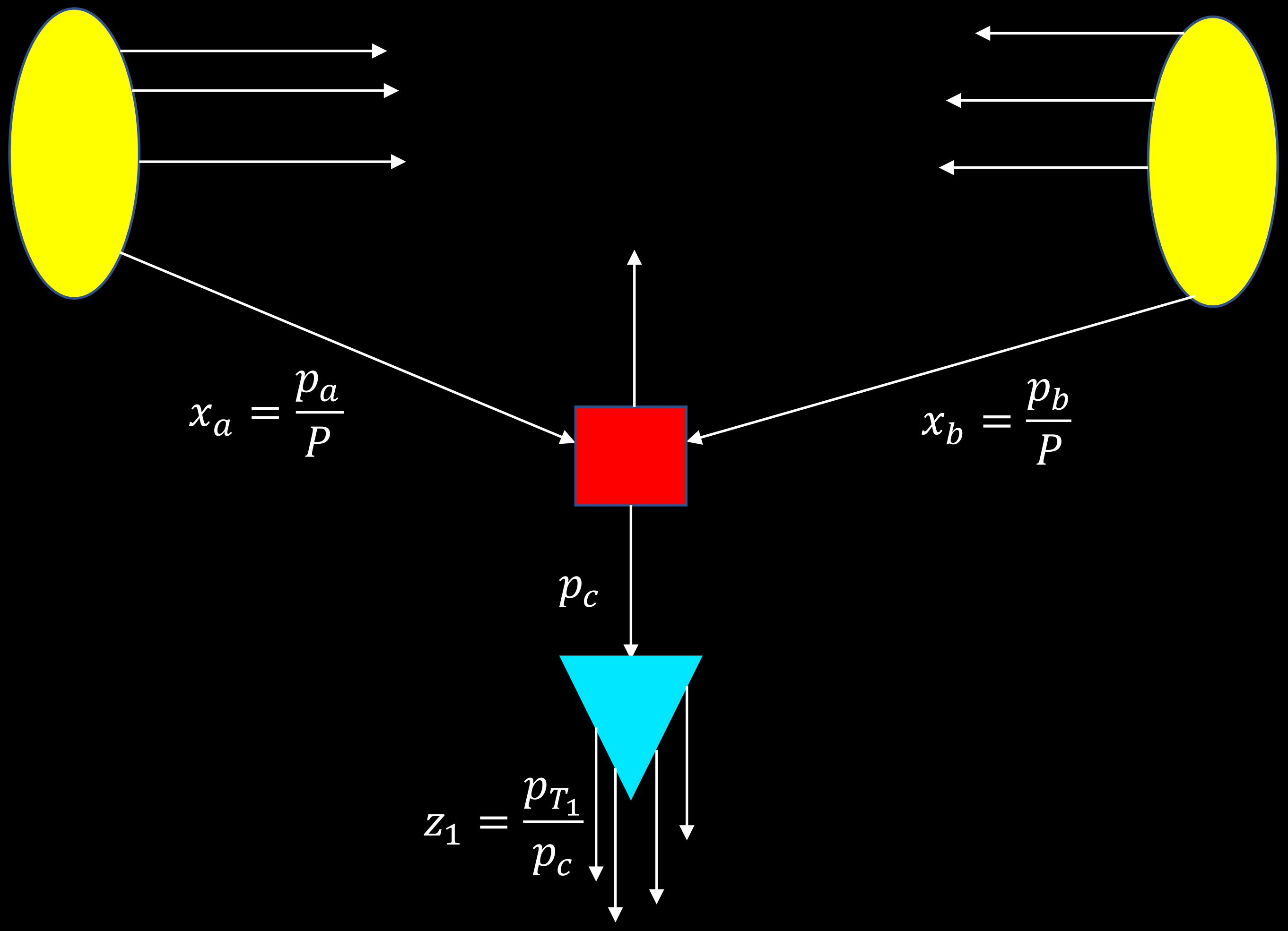

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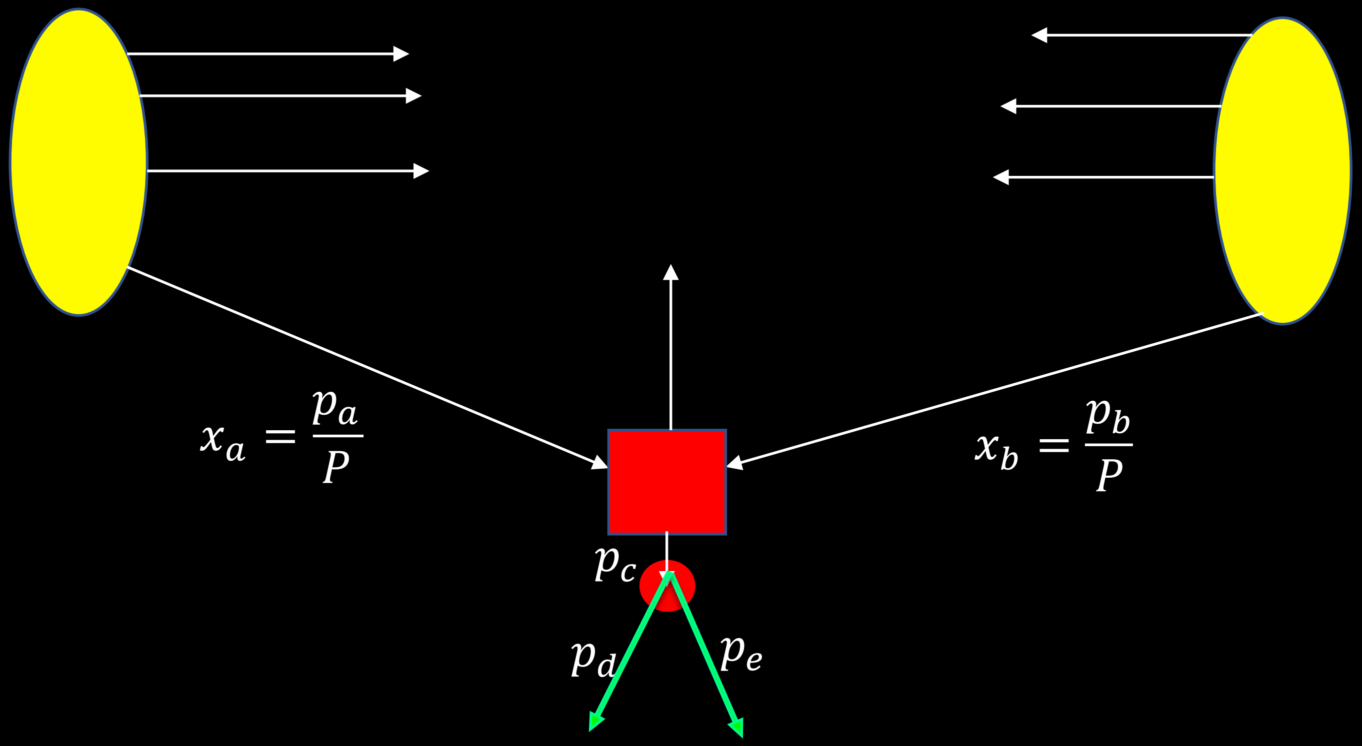
- The separation of scale depends on the process. The formula on the left only works for $2 \rightarrow 2$

Sketch of possible corrections



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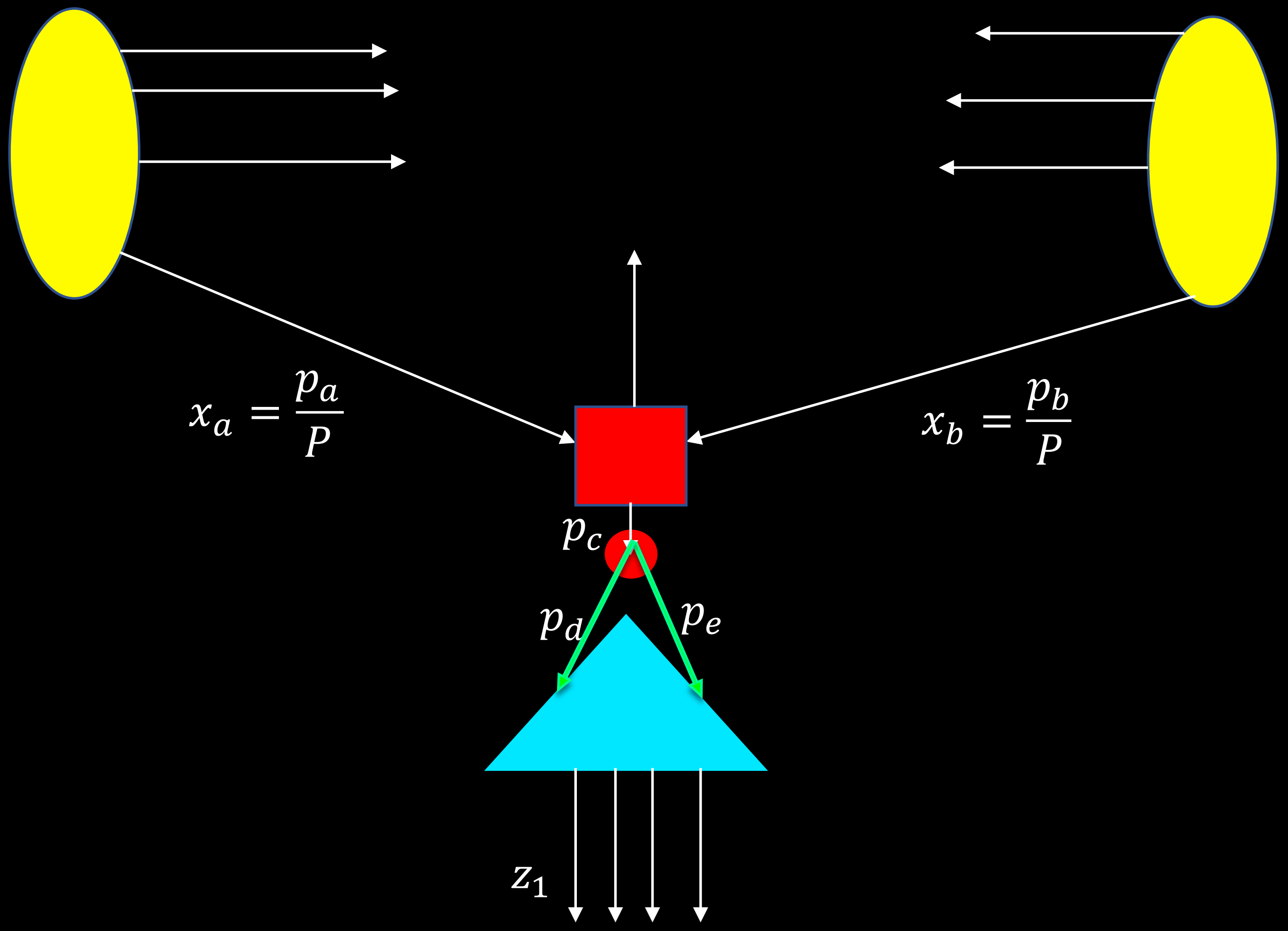

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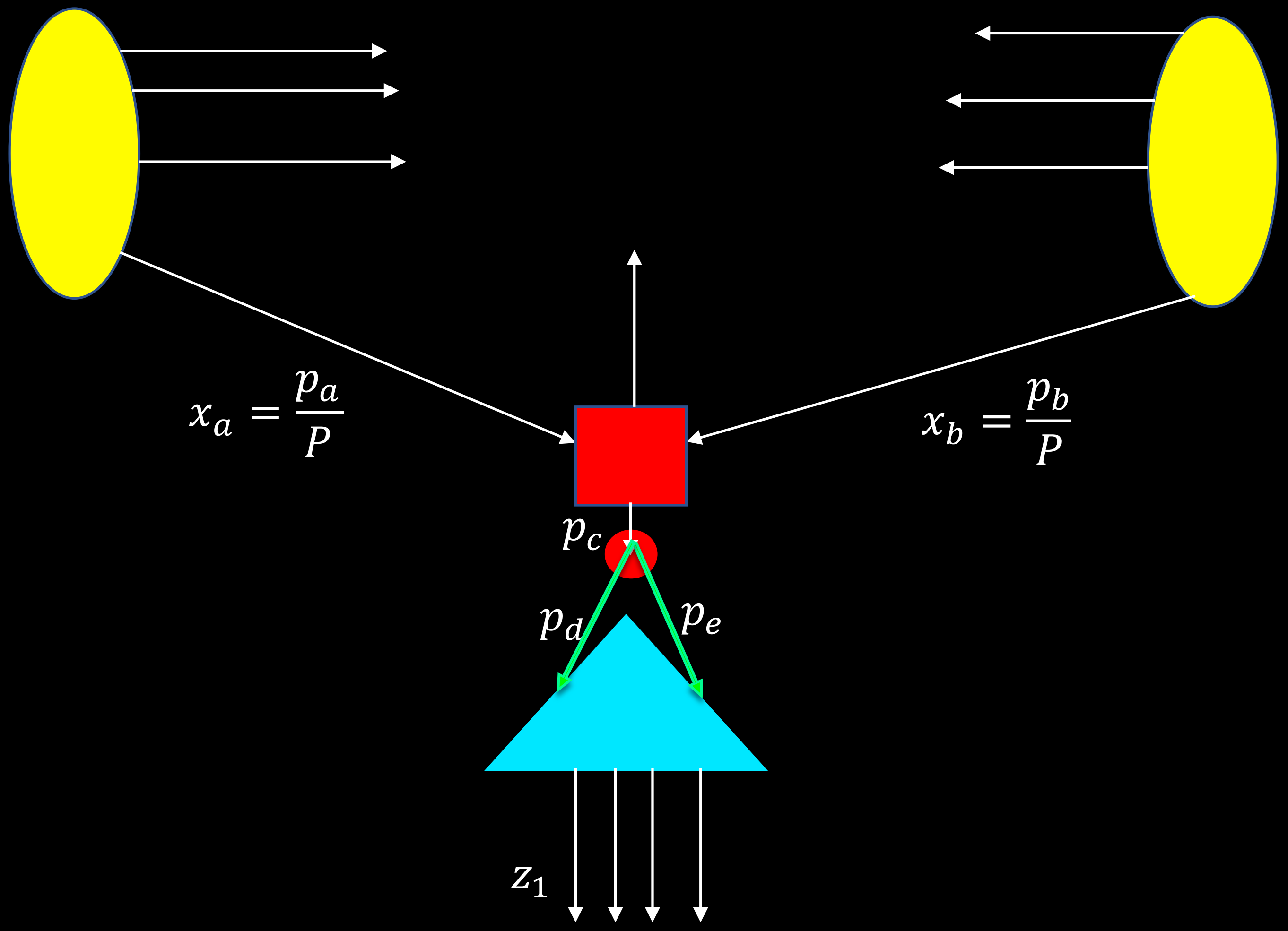

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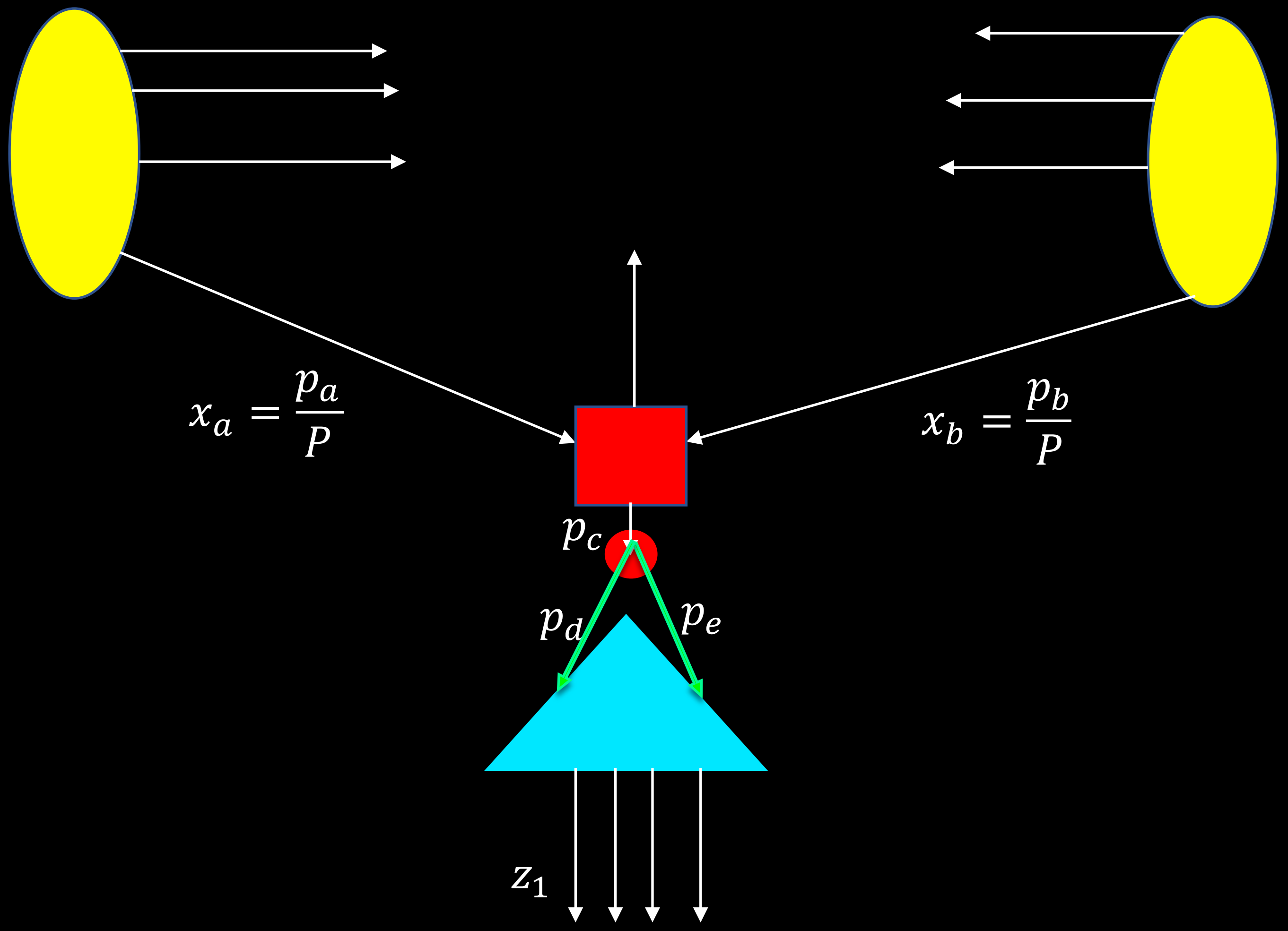

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- Need to use scale dependent PDFs and FFs

$$G(x_a) \rightarrow G(Q, x_a); \qquad D(z_1) \rightarrow D(Q, z_1)$$

- What's the meaning of $G(Q\ )$?

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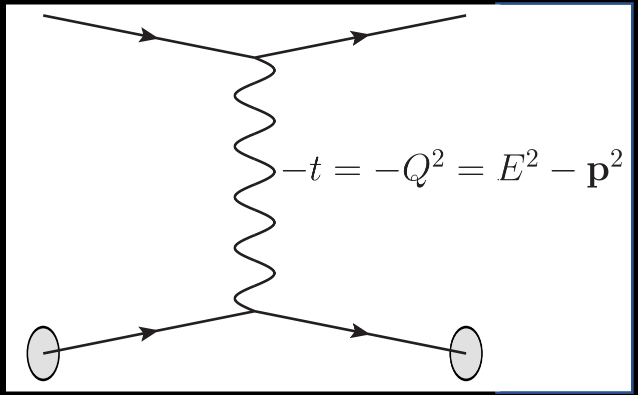
A system to study PDFs: $e^- + p$

- $|t| = |Q^2| \sim 0.1$ GeV², no proton substructure



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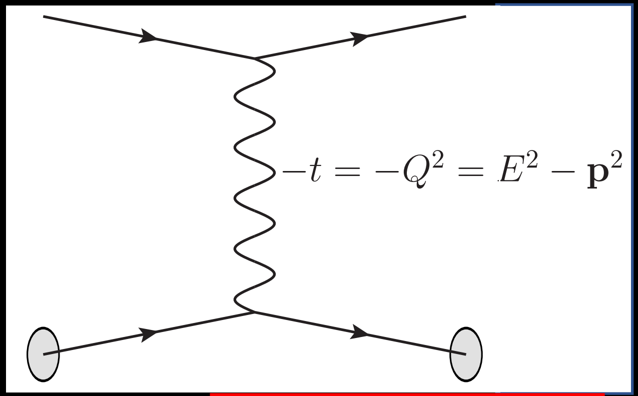

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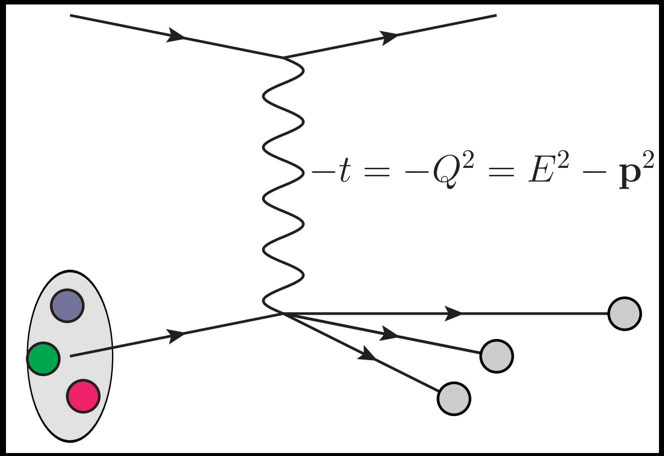
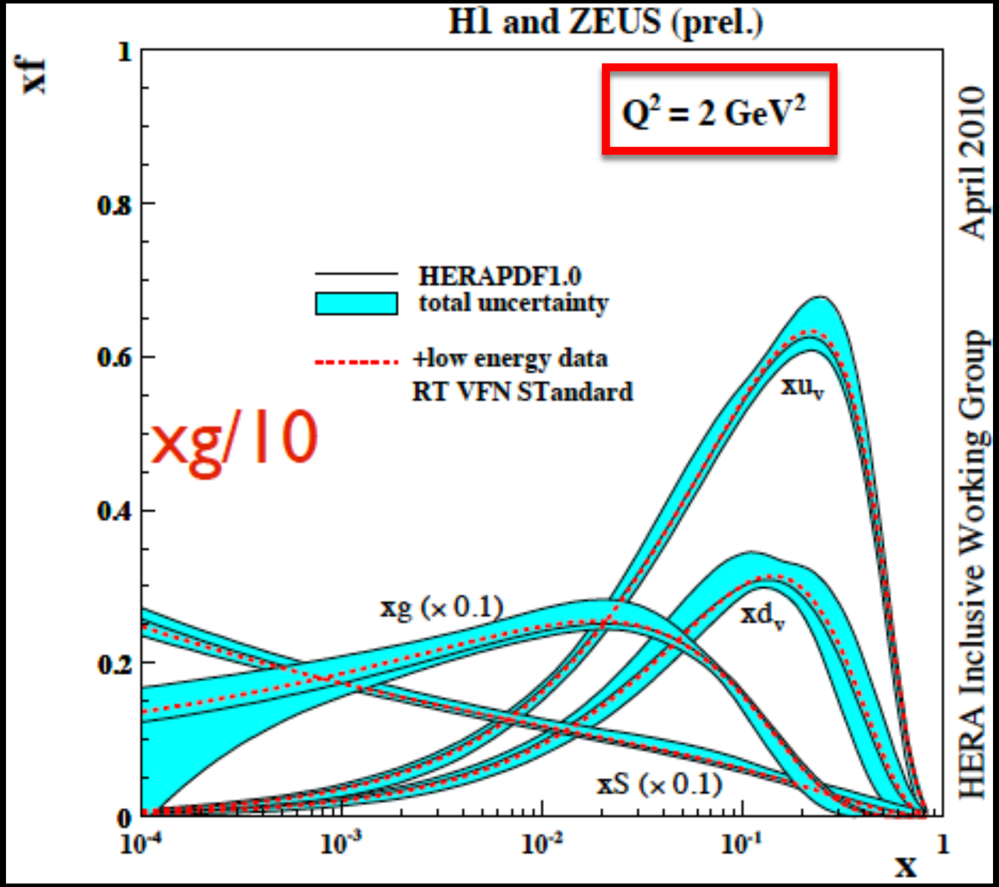
- $|t| = |Q^2| \sim 2$ GeV²

Larger $Q^2 \Rightarrow$ smaller features seen in p





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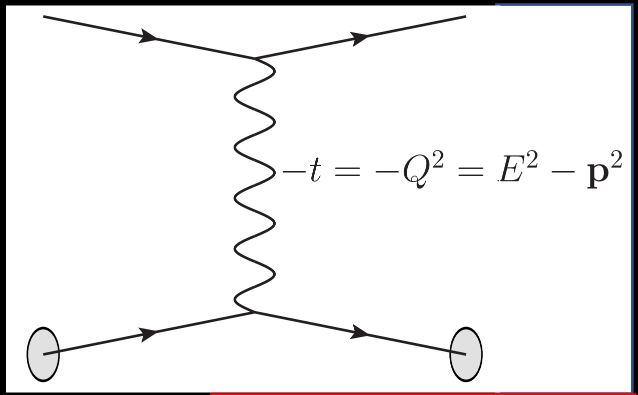
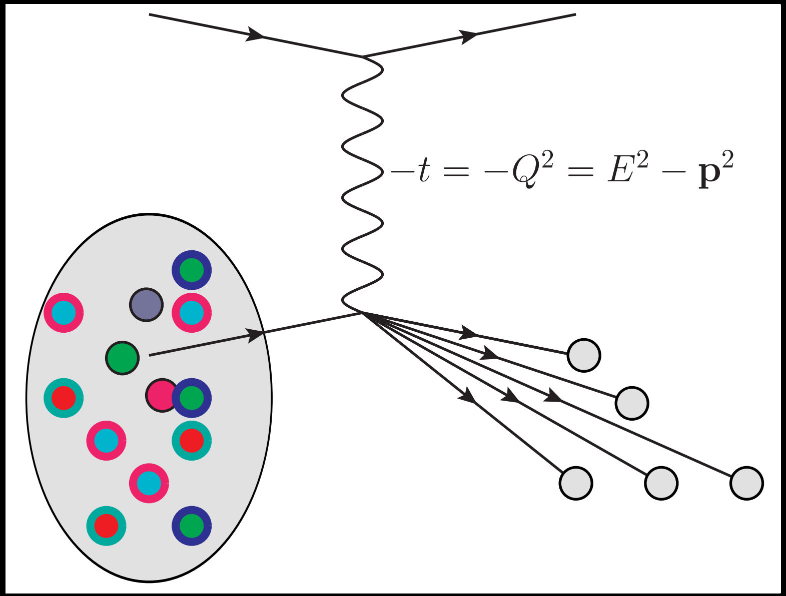


- $|t| = |Q^2| \sim 10$ GeV²



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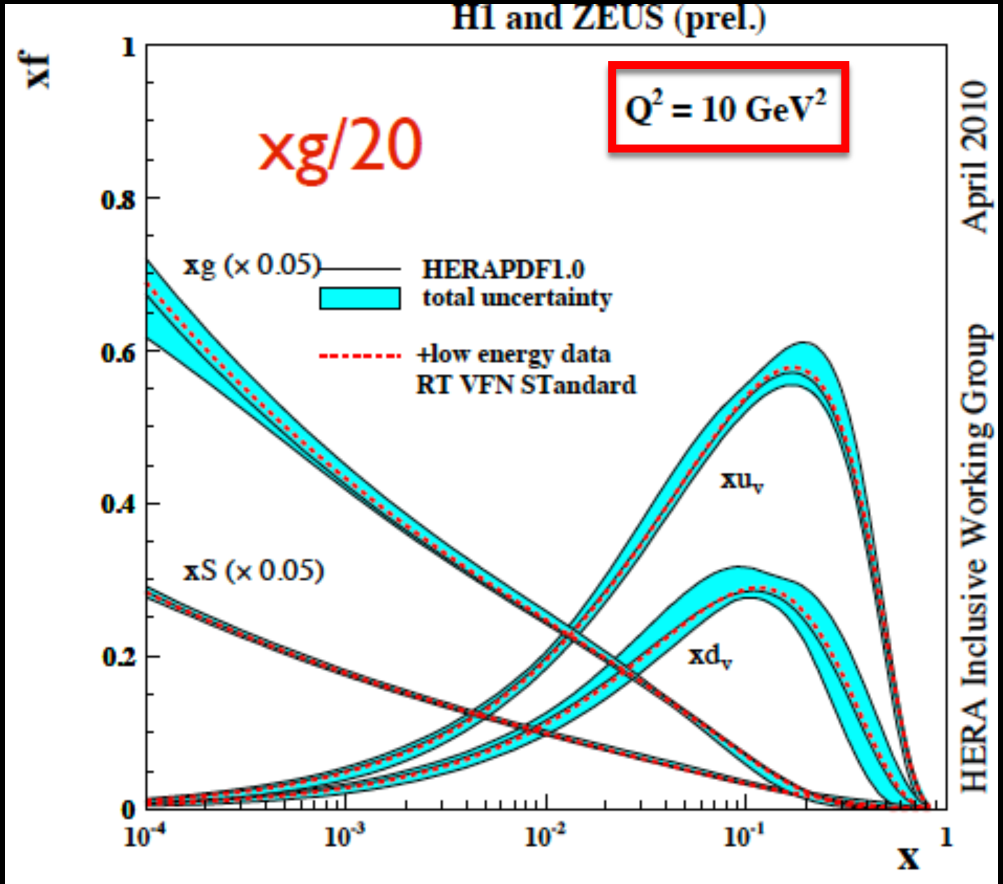

Parton showering

The basic ideas

Parton splitting & Hadronization

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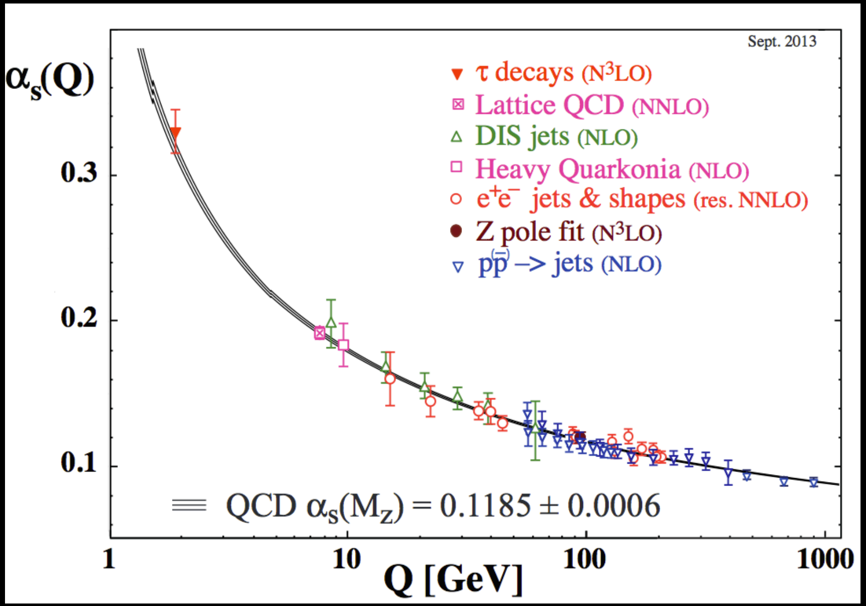

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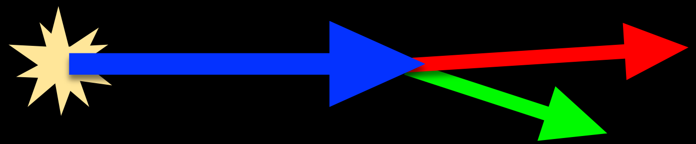

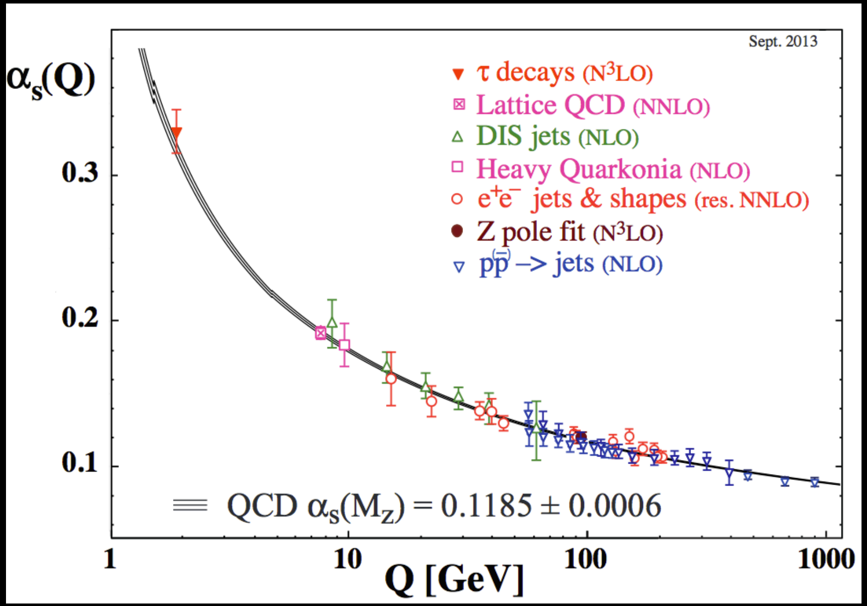

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- Thick/thin arrow ⇒ High/low Virtuality

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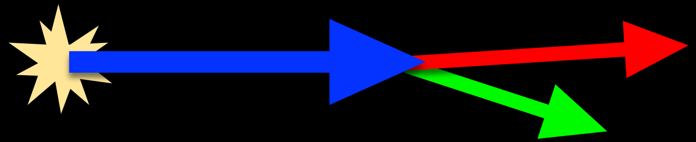

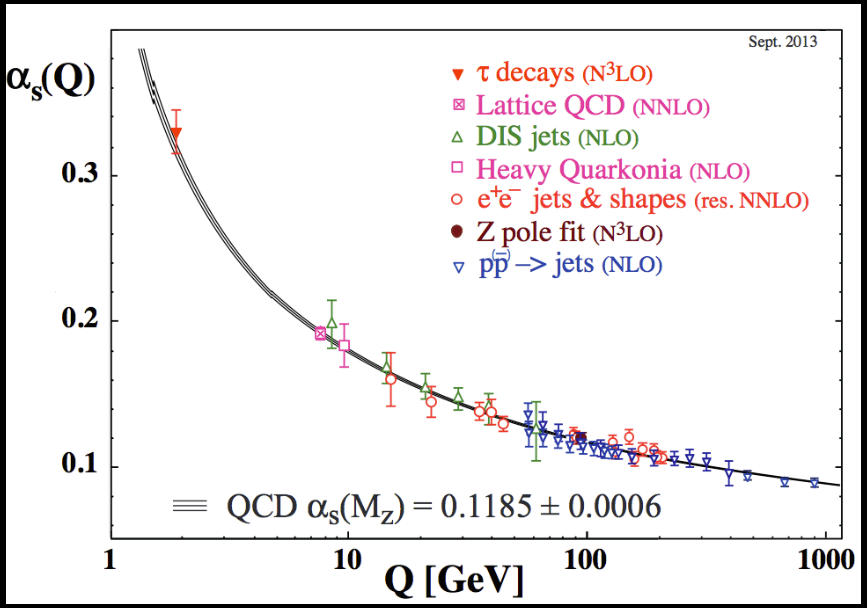

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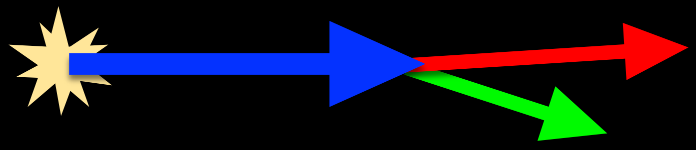

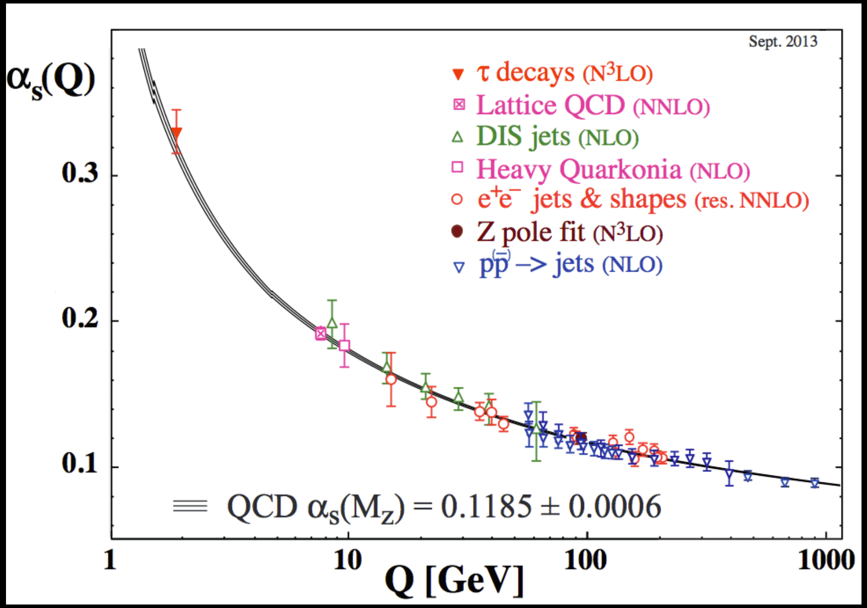

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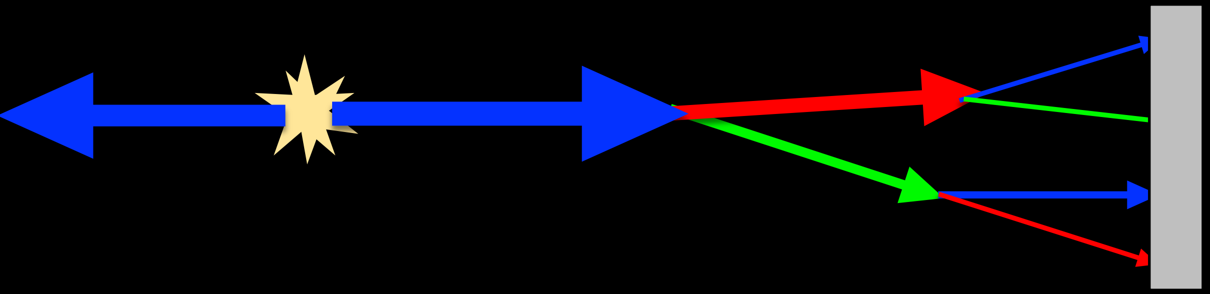
  - Interaction between daughter partons has negligible effect on shower evolution at high Q^2 (as α_s is small and the lifetime is short).

Parton splitting & Hadronization

- As multiple splits happen, less virtual (lower Q^2) partons (thinner arrows) are created, which are longer lived.



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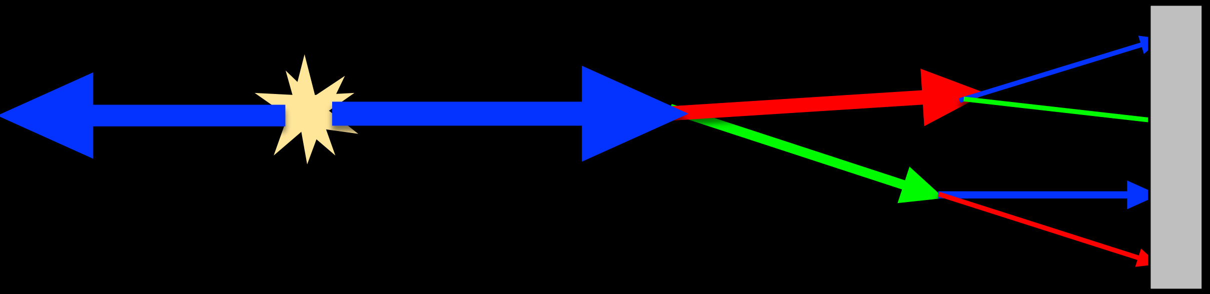

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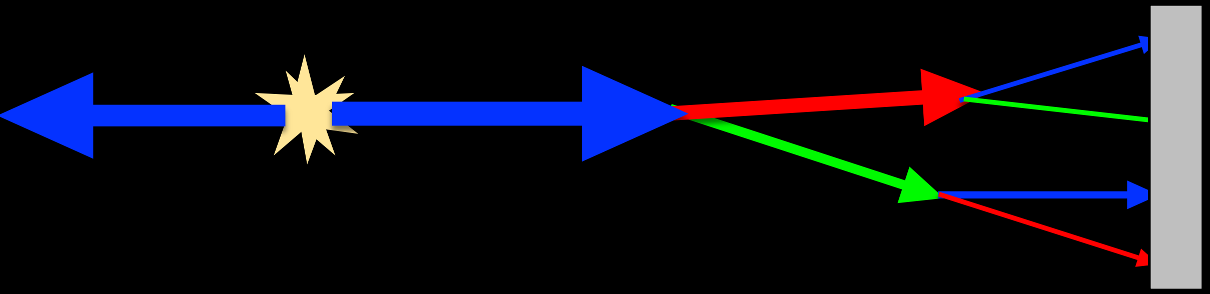

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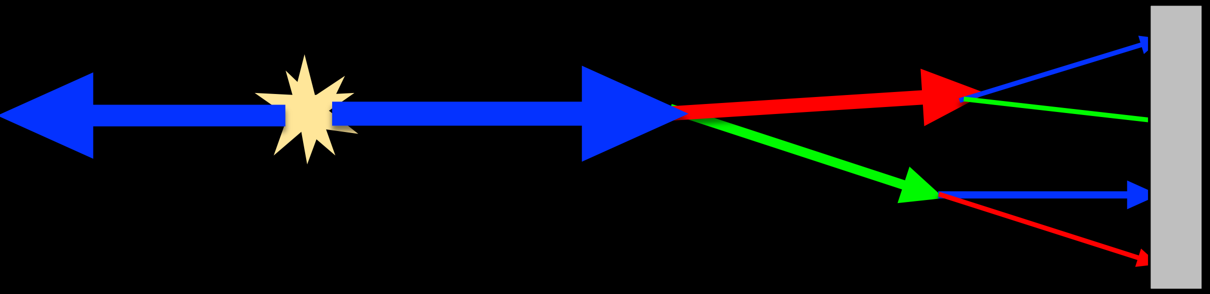

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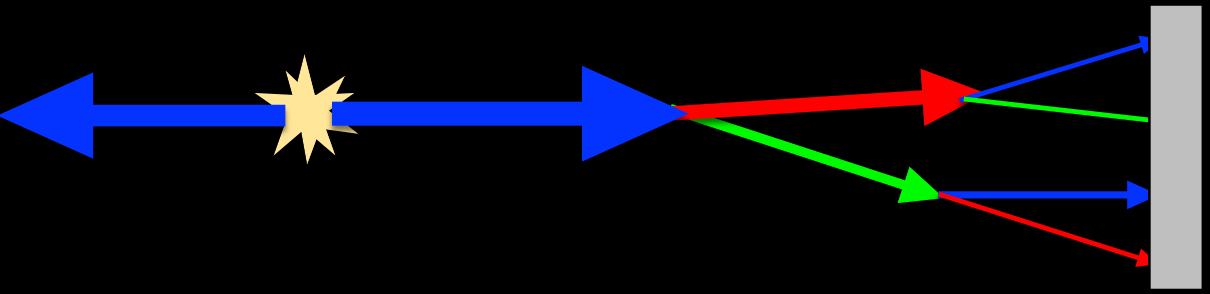

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- In the QGP, the interaction between partons is even more complicated as the showering partons can interact with partons in the QGP.
  - The parton picture needs to be broken down in terms of regions of phase space:
    - High Q, high E
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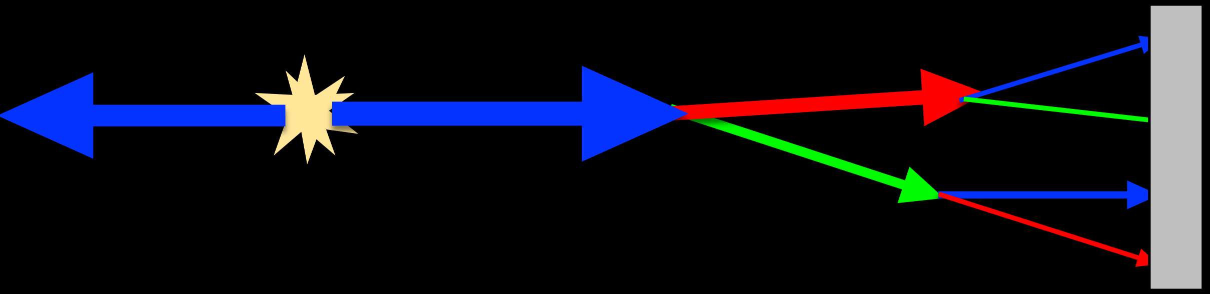

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    - Low Q, High E }
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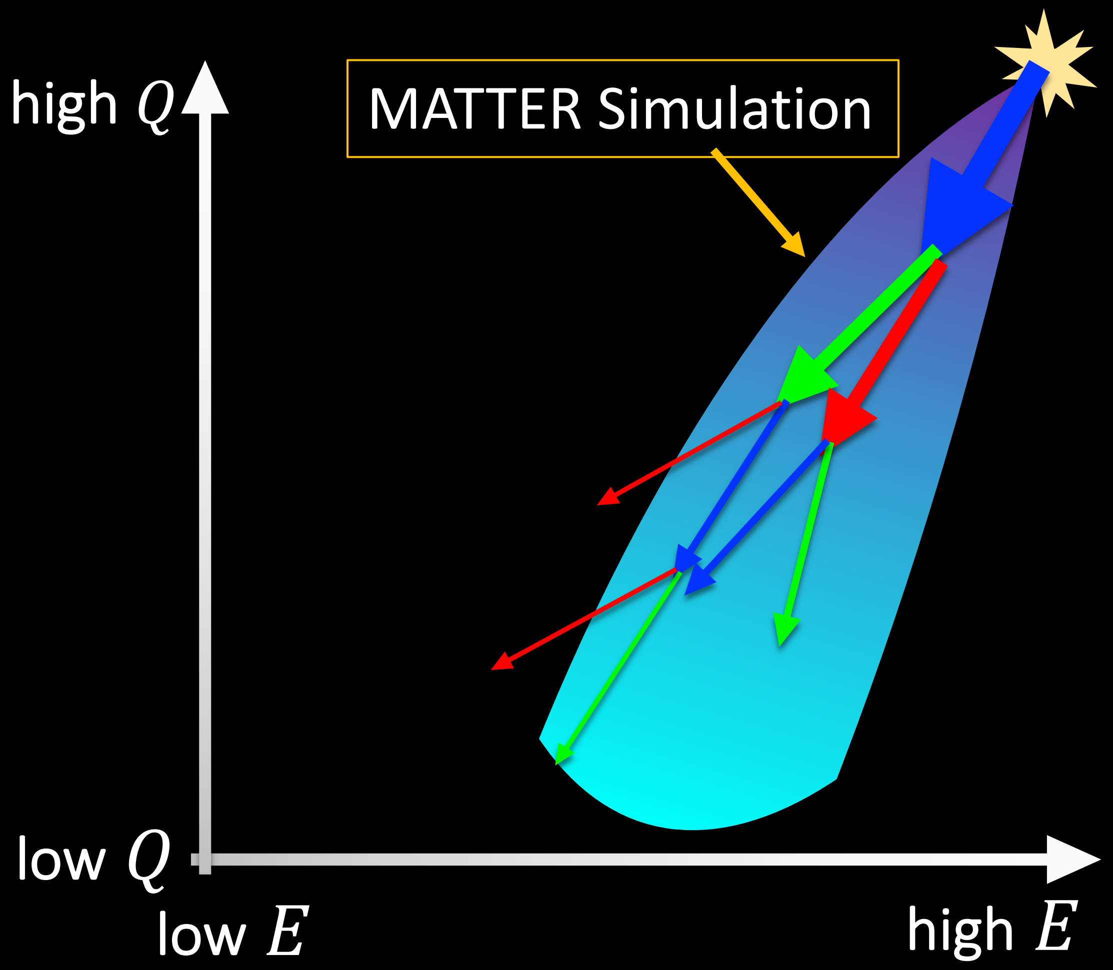
Parton Energy Loss

The JETSCAPE Paradigm

Multi-stage parton evolution in JETSCAPE



- High→Lower Q, High E: Rapid virtuality loss through radiation (i.e. splitting)

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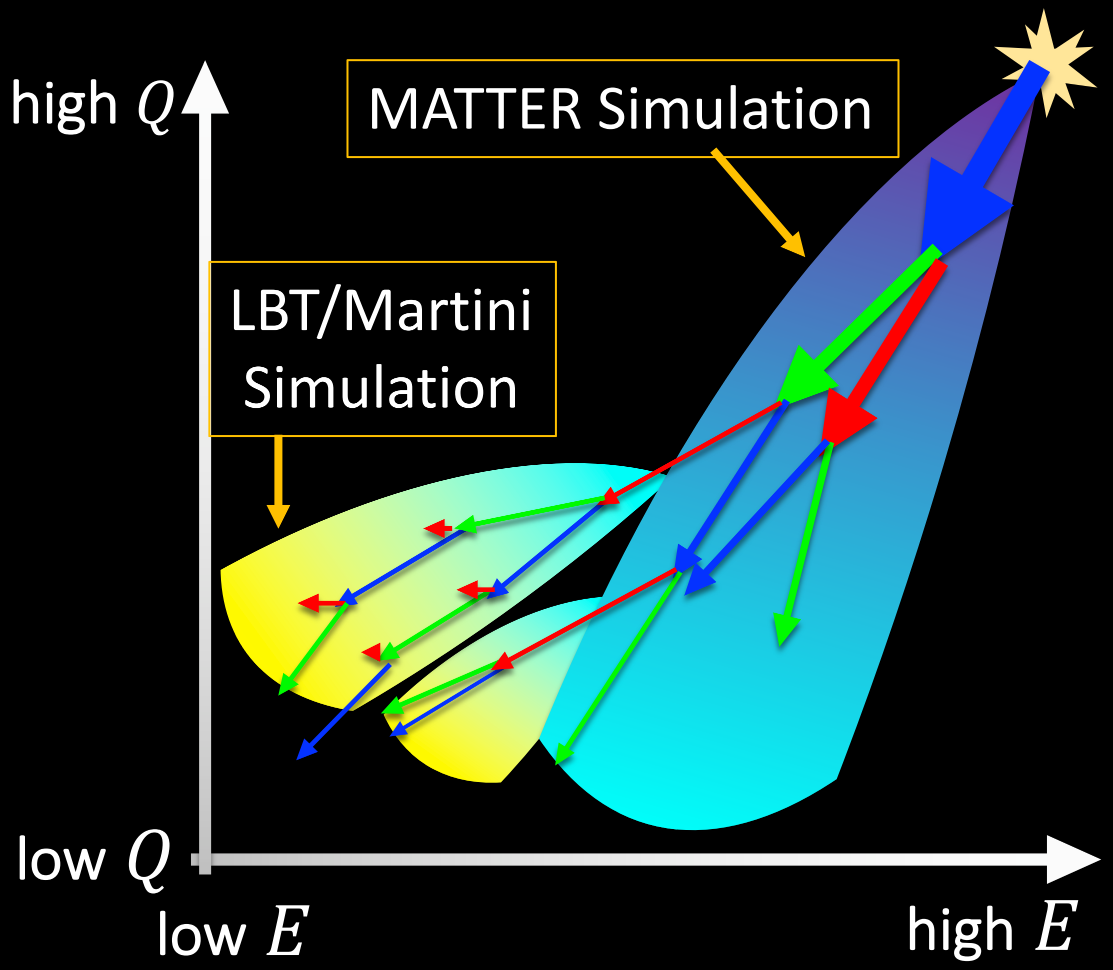

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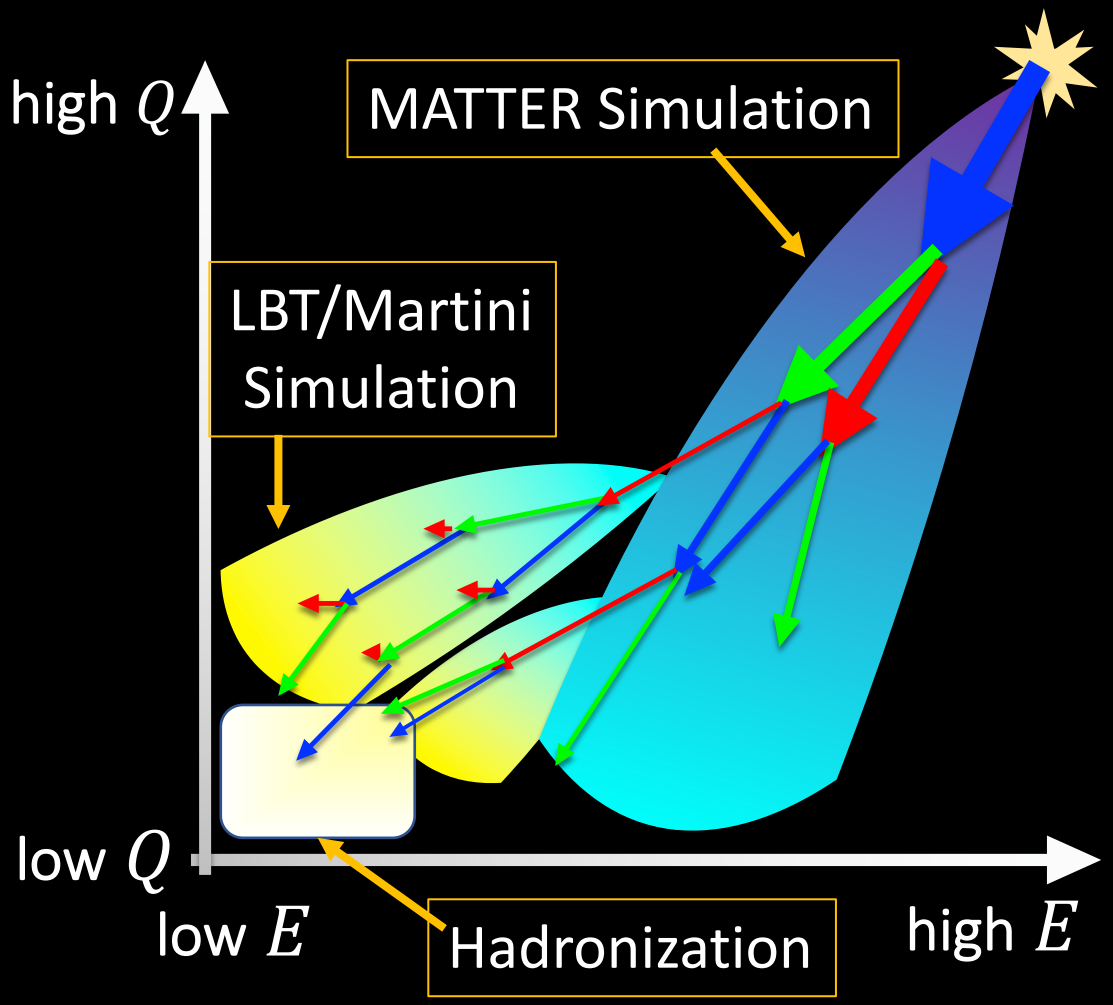

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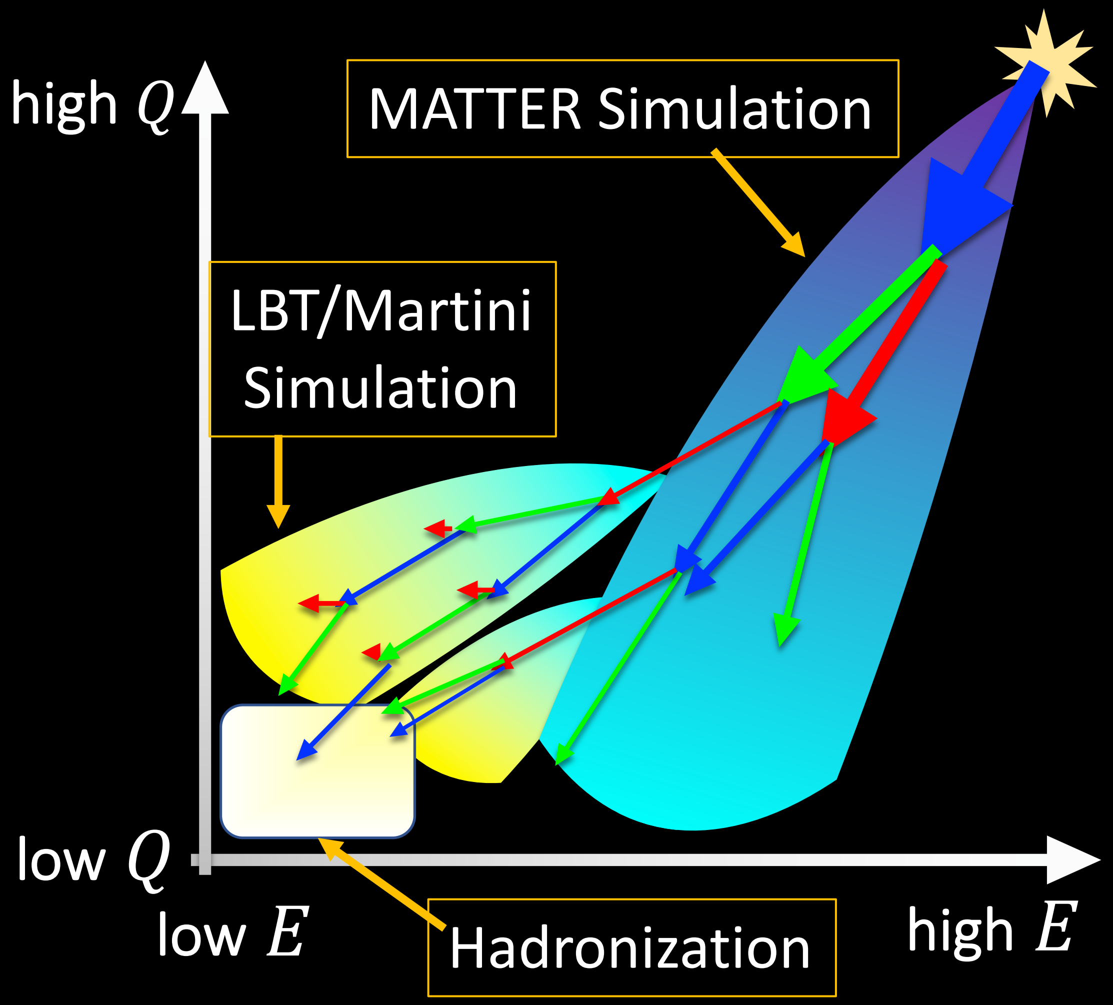

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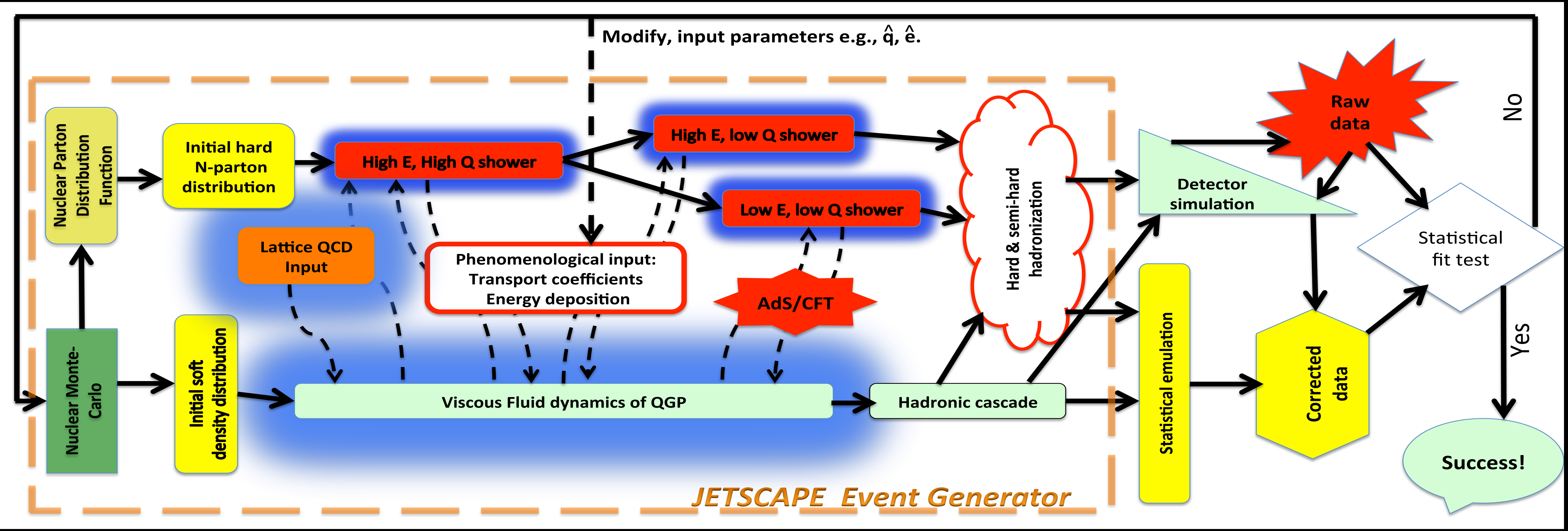
Different physics mechanisms for in-medium energy loss in different kinematic regimes ⇒ a *per parton* multi-stage approach is needed for an accurate description

The JETSCAPE Framework

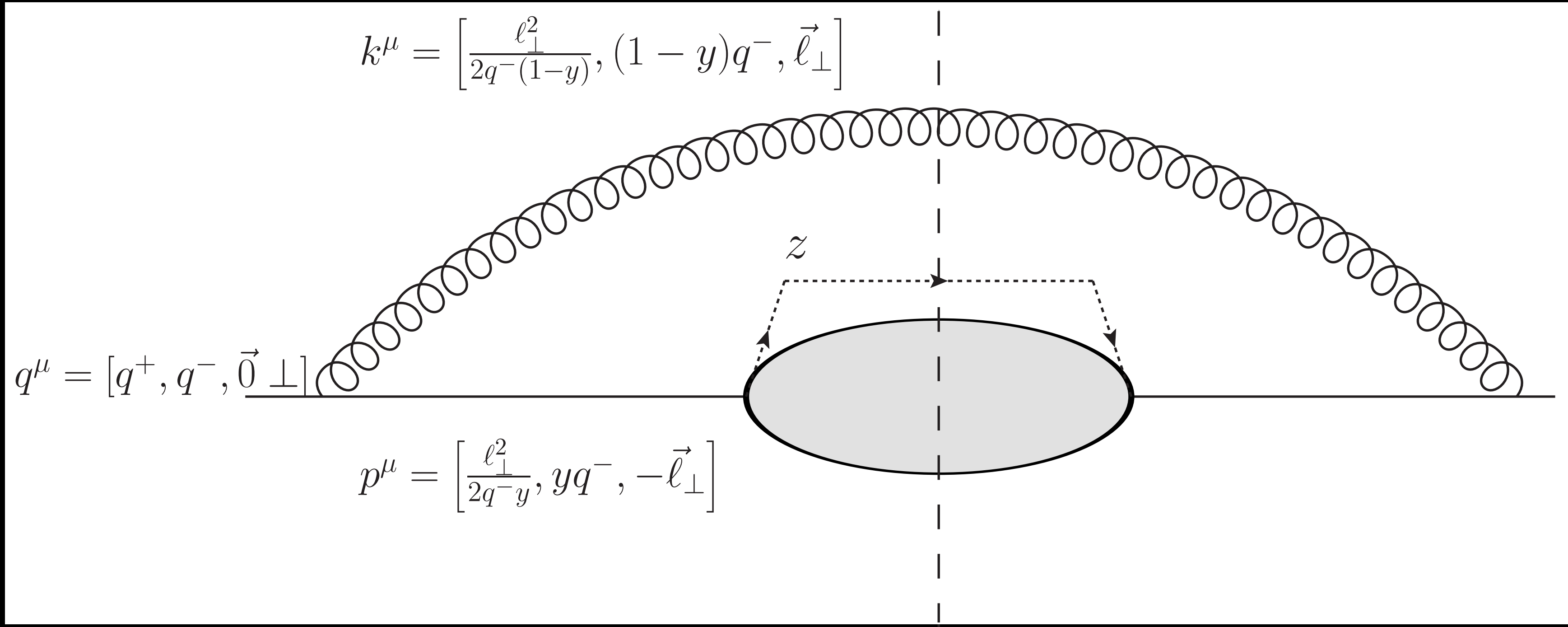


- JETSCAPE framework is such that parton energy-momentum exchange with the medium is happening on a parton by parton basis.
  - Initial hard partons are given by Pythia (not discussed here)
  - High E, High Q shower is given by MATTER ✓
  - High E, low Q shower is given by LBT, MARTINI ✓
  - Parton energy-momentum gain/loss is associated with a source/sink in the fluid simulation, see Yasuki's talk up next.

Parton Energy “Loss”

The story of high energy and high virtuality partons

Kinematics of a split



- Work in light-cone coordinates:

$$q^+ = \frac{(q^0 + q^z)}{\sqrt{2}}, \quad q^- = \frac{(q^0 - q^z)}{\sqrt{2}}$$

$$\begin{aligned} Q^2 = q^\mu q_\mu &= (q^0)^2 - \vec{q}\cdot\vec{q} \\ &= 2q^+q^- - \vec{q}_\perp \cdot \vec{q}_\perp \\ &= 2q^+q^- \end{aligned}$$

- Energy-momentum conservation

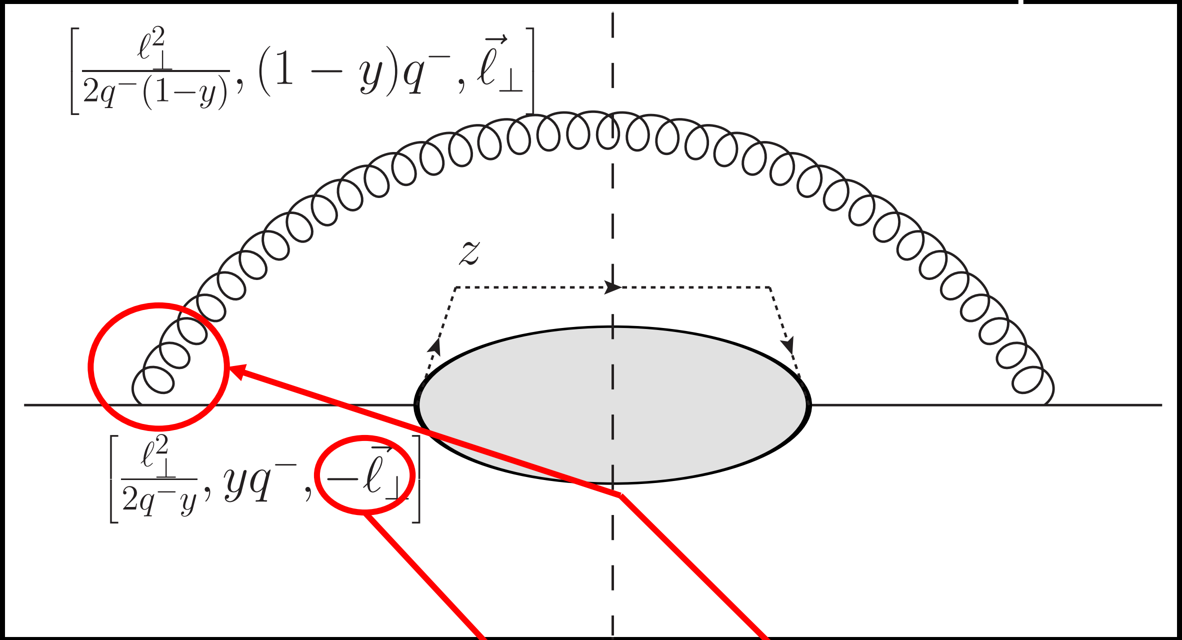
$$\begin{aligned} q^\mu &= k^\mu + p^\mu \\ k^\mu k_\mu &= 0 \\ p^\mu p_\mu &= 0 \\ k^- &= (1-y)q^- \\ p^- &= yq^- \end{aligned}$$

$$\Rightarrow k^+ = \frac{\ell_\perp^2}{2q^-(1-y)}$$

$$\Rightarrow p^+ = \frac{\ell_\perp^2}{2q^- y}$$

$$\Rightarrow Q^2 = \frac{\ell_\perp^2}{y(1-y)}$$

In an inclusive process: probability of a split



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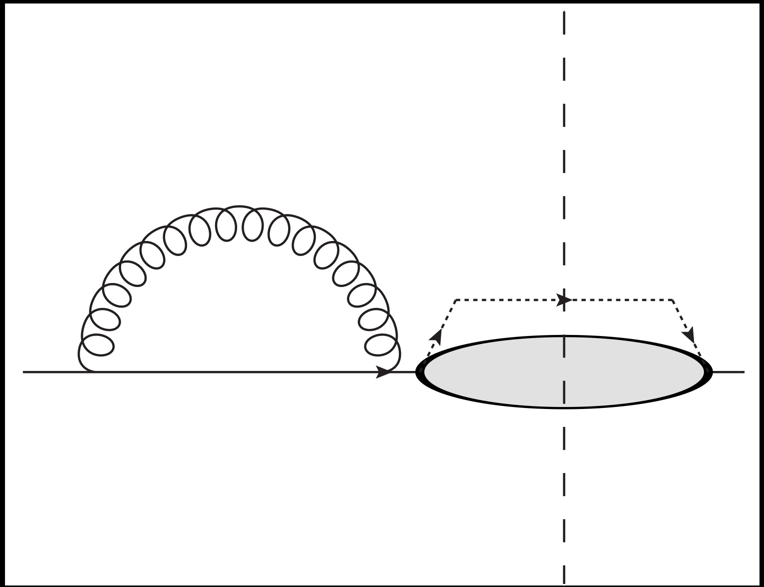


$$Q^2 y(1-y) = \ell_\perp^2$$

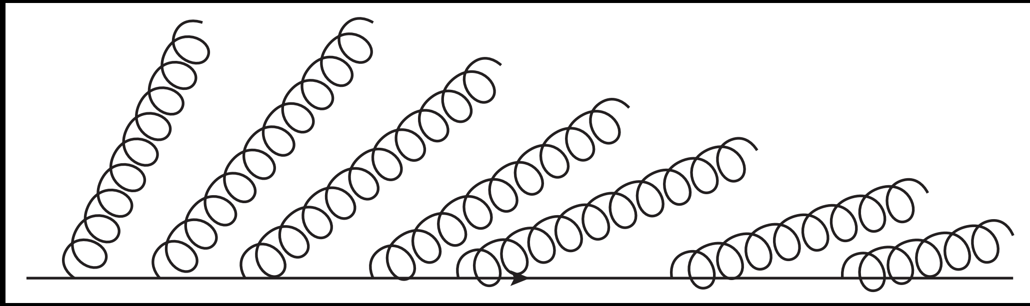
$$\frac{d\sigma}{\sigma_0} = \int_{Q_0^2}^{Q_{max}^2} \frac{d\ell_\perp^2}{\ell_\perp^2} \frac{\alpha_s(\ell_\perp^2)}{2\pi} \int_z^1 \frac{dy}{y} P(y) D\left(\frac{z}{y}\right) - \int_{Q_0^2}^{Q_{max}^2} \frac{d\ell_\perp^2}{\ell_\perp^2} \frac{\alpha_s(\ell_\perp^2)}{2\pi} D(z) \int_0^1 dy\, P(y)$$

$$\frac{d\sigma}{\sigma_0} \equiv \int_{Q_0^2}^{Q_{max}^2} \frac{d\ell_\perp^2}{\ell_\perp^2} \frac{\alpha_s(\ell_\perp^2)}{2\pi} \int_z^1 \frac{dy}{y} P_+(y) D\left(\frac{z}{y}\right)$$

- Note that individual contributions are singular at $y = 1$, but together the result is finite.

Probability of multiple splits

- At leading order, we have ordered radiations, such that $\ell_{\perp,0}^2 > \ell_{\perp,1}^2 > \ell_{\perp,2}^2 > \cdots > \ell_{\perp,n}^2$



$$\frac{d\sigma/\sigma_0}{\Pi_i dy_i d\ell_{\perp,i}^2} = \left[\prod_{i=0}^{n} \frac{\alpha_s\left(\ell_{\perp,i}^2\right)}{2\pi\ell_{\perp,i}^2} \frac{P_+(y_i)}{y_i}\right] D\left(\frac{z}{\Pi_i y_i}\right)$$

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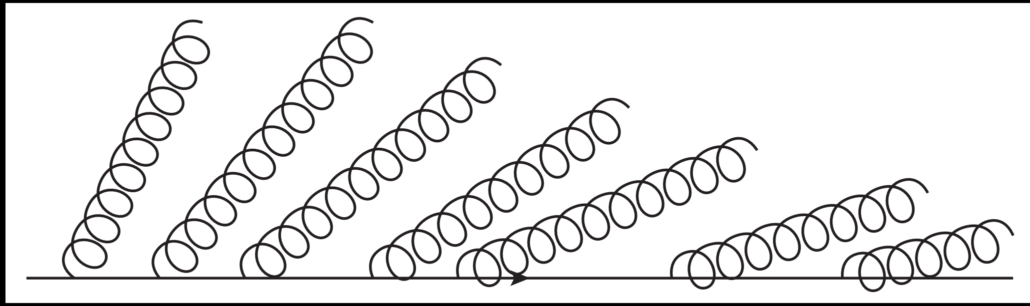

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- An ordering in $\ell^2_{\perp,i}$ also implies an ordering in angle θ_i between the gluons and the quark.
- Thus, interferences between subsequent radiations can be neglected.

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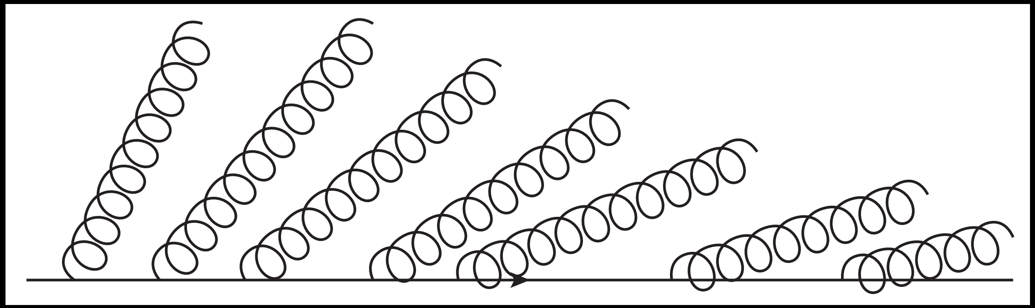

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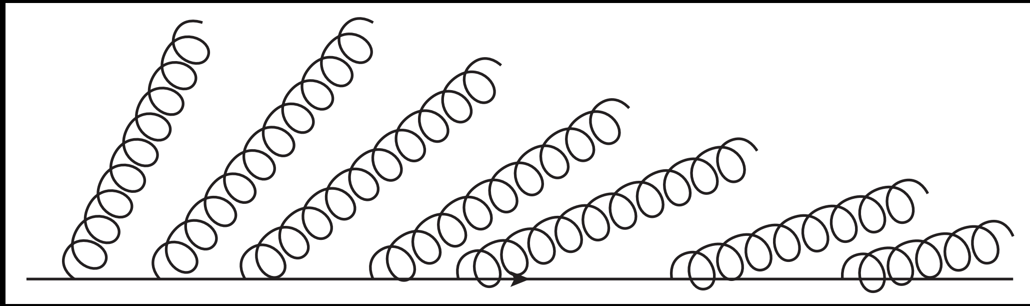

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- Thus, interferences between subsequent radiations can be neglected.
- This allows to resum radiations with 1,2,… multiple gluons giving [see e.g. R.D. Field *Applications of pQCD*, chap. 3.4]:

$$D(z,Q^2) = \exp[\kappa P_+ *]\, D(z,Q_0) = \sum_{n=0}^{\infty} \frac{[\kappa P_+ *]^n}{n!} D(z,Q_0) \qquad P_+ * D = \int_z^1 \frac{dy}{y} P_+(y) D\left(\frac{z}{y}\right)$$

$$\kappa = \int_{Q_C^2}^{1} dQ^2 \frac{\alpha_s(Q^2)}{2\pi Q^2} \qquad \frac{\kappa^n}{n!} = \int_{Q_C^2}^{1} dQ_1^2 \frac{\alpha_s(Q_1^2)}{2\pi Q_1^2} \int_{Q_c^2}^{Q_1^2} dQ_2^2 \frac{\alpha_s(Q_1^2)}{2\pi Q_1^2} \int_{Q_c^2}^{Q_2^2} dQ_3^2 \frac{\alpha_s(Q_3^2)}{2\pi Q_3^2} \cdots \int_{Q_c^2}^{Q_{n-1}^2} dQ_n^2 \frac{\alpha_s(Q_n^2)}{2\pi Q_n^2}$$

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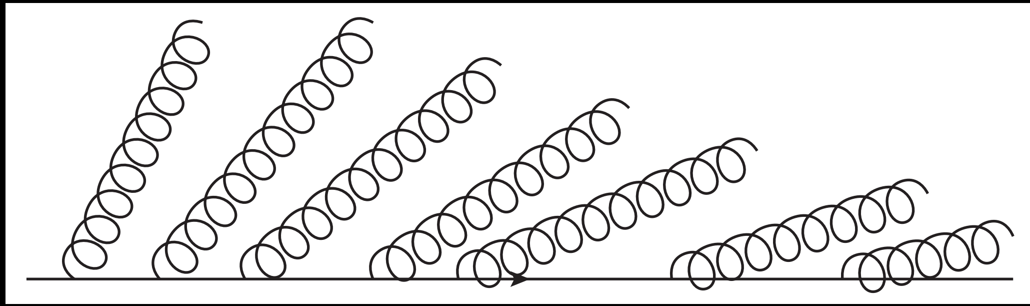

$$\frac{d\sigma/\sigma_0}{\Pi_i dy_i d\ell^2_{\perp,i}} = \left[\prod_{i=0}^{n} \frac{\alpha_s\left(\ell^2_{\perp,i}\right)}{2\pi\ell^2_{\perp,i}} \frac{P_+(y_i)}{y_i}\right] D\left(\frac{z}{\Pi_i y_i}\right)$$

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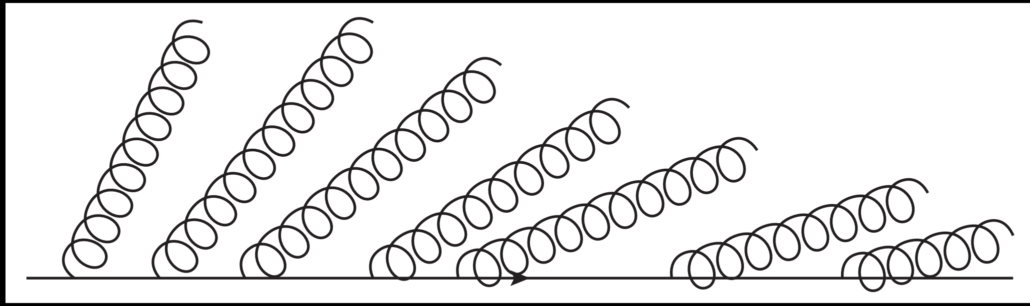

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$$\frac{dD(z,Q^2)}{d\log Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_z^1 \frac{dy}{y}\, P_+(y) D\left(\frac{z}{y}, Q^2\right)$$

- This is DGLAP evolution of the fragmentation function with scale Q^2 $\left[Q^2 = \frac{\ell^2_\perp}{y(1-y)}\right]$.

Probability of multiple splits

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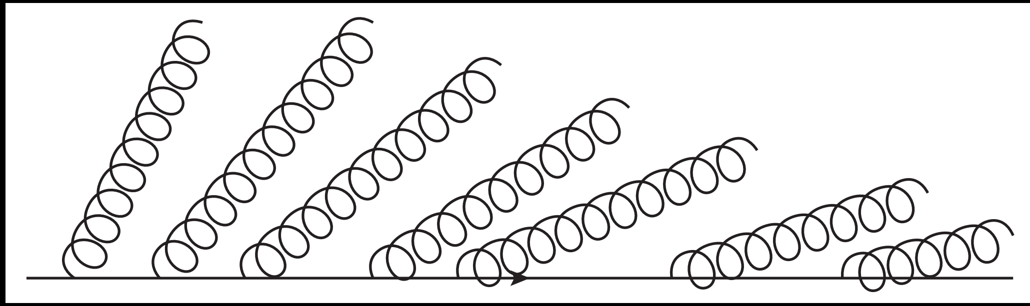

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- In literature, the + in the *distribution* $P_+(y)$ is often implied, while parton species are added as $P_{q\leftarrow q}(y)$ or $P_{g\leftarrow q}(y)$ where y is the momentum fraction of the quark/gluon, respectively.
    - $P_{q\leftarrow q}(y)$ or $P_{g\leftarrow q}(y)$ are related, i.e. if q takes fraction y, then g must take $(1-y)$, and vice versa.

Modified splitting inside the QGP

- The in-medium contributions comes through $P(y) \rightarrow \mathcal{P}(y)$

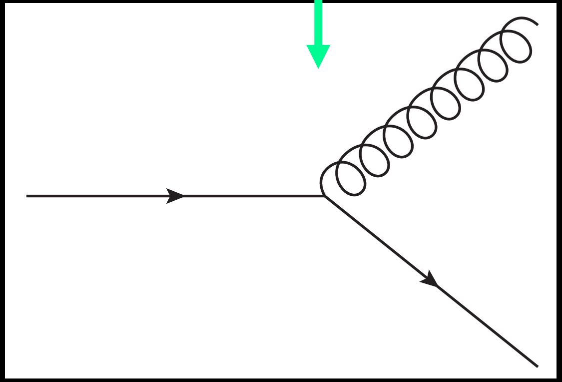
$$\mathcal{P}(y) = P(y) + \tilde{\mathcal{P}}(y)$$

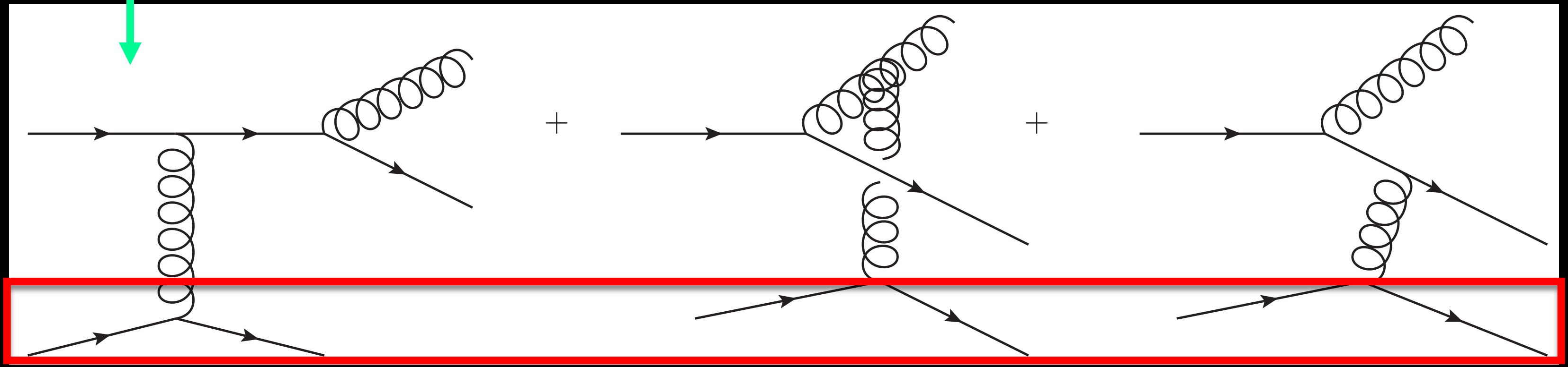
$$\mathcal{P}(y) = P(y) + \frac{P(y)\left\{\int_{\tau_i}^{\tau_f} dt\, \hat{q}(t)\left[4\sin^2\left[\frac{t-\tau_i}{2\tau_f}\right]\right]\right\}}{y(1-y)Q^2}$$

$$\tau_f = \frac{2q^+}{Q^2}$$

$$\hat{q} = \frac{\langle \boldsymbol{p}_T^2 \rangle}{\boldsymbol{L}}$$

τ_i time at which enters the QGP or is created in the QGP





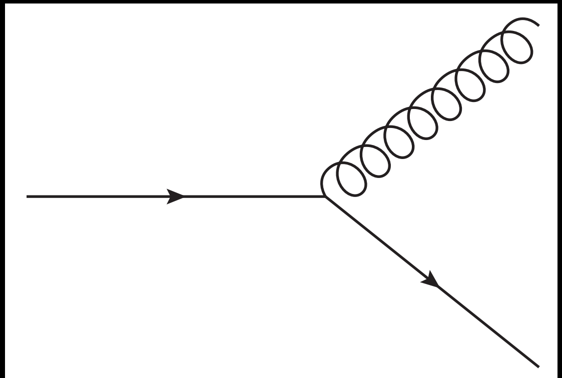
Scattering partners are coming from the QGP. Also (not shown) diagrams with $q \rightarrow g$ are included in the calculation of $\hat{q}$.

- The largest medium contribution stems from $\hat{q}$ (the average p_T^2 gained per unit length) owing to the parton interacting with the QGP.

Monte Carlo simulation at high Q, high E

- With $\mathcal{P}(y)$ at hand, one proceeds to construct a Monte Carlo algorithm as follows:
  - Generate original partons via Pythia
  - Assign virtuality Q to parent partons via the Sudakov form factor [**Adv.Ser.Direct.HEP, 573 (1989); NPA 696, 788 (2001)**] which gives the probability not to split

$$\Delta(Q^2, Q_0^2) = \exp\left[-\int_{Q_0^2}^{Q^2} \frac{d\hat{Q}^2}{\hat{Q}^2} \frac{\alpha_s(\hat{Q}^2)}{2\pi} \int_{y_{\min}}^{y_{\max}} dy\, \mathcal{P}(y)\right]$$

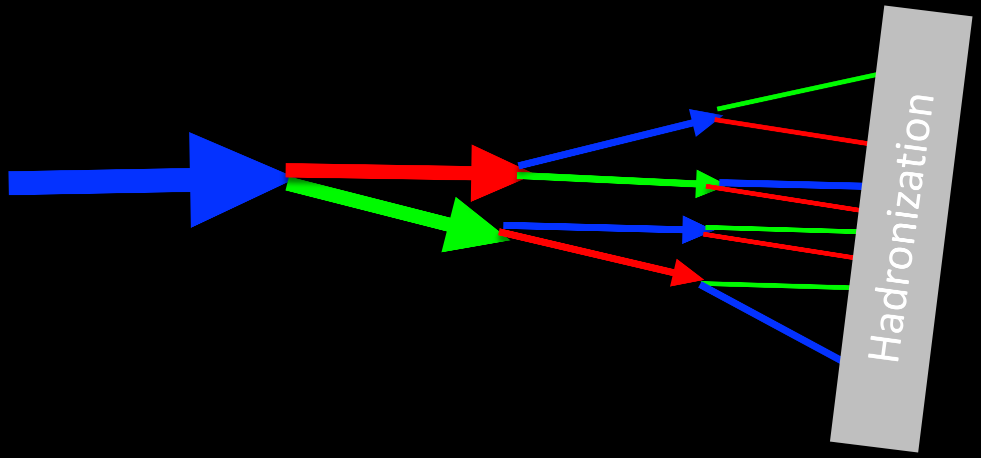


  - Calculate the momentum fraction of the daughter partons, after the split using $\mathcal{P}(y)$
  - Estimate the daughter parton's virtuality using the Sudakov and kinematic
- This Monte Carlo showering at high Q and high E in JETSCAPE is done via **MATTER** (**M**odular **A**ll **T**wist **T**ransverse-scattering **E**lastic-drag and **R**adiation)
  - MATTER also slightly modifies the E or $\vec{p}$ (leaving Q unchanged) using $2 \to 2$ scattering rates with partons from the QGP.

Shower in the vacuum vs QGP

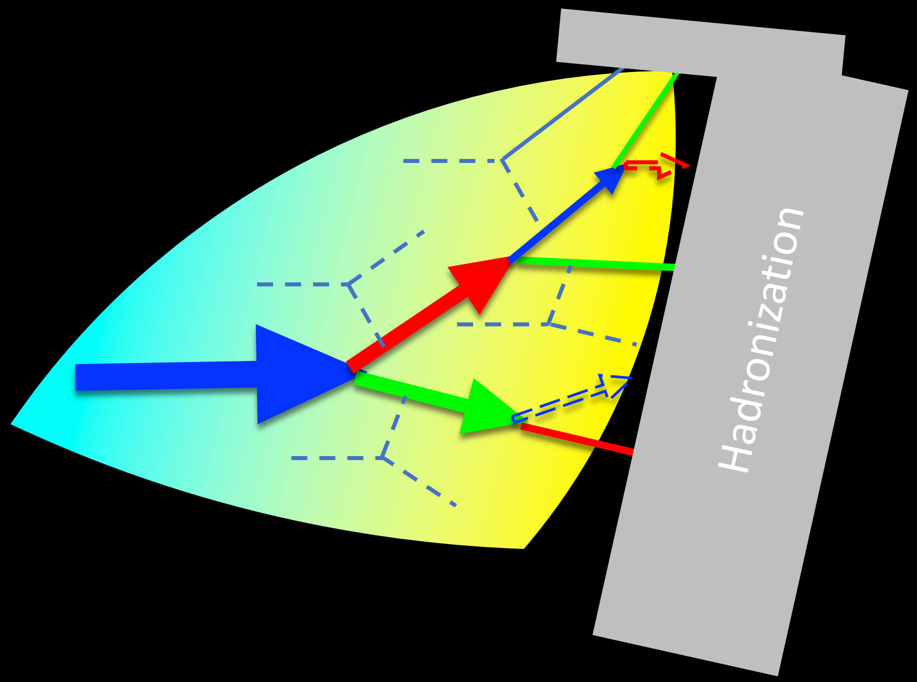
What do these showers look like?

Jet shower in vacuum



- The particles in a jet after hadronization occupy a narrow cone.

Modified splitting inside the QGP



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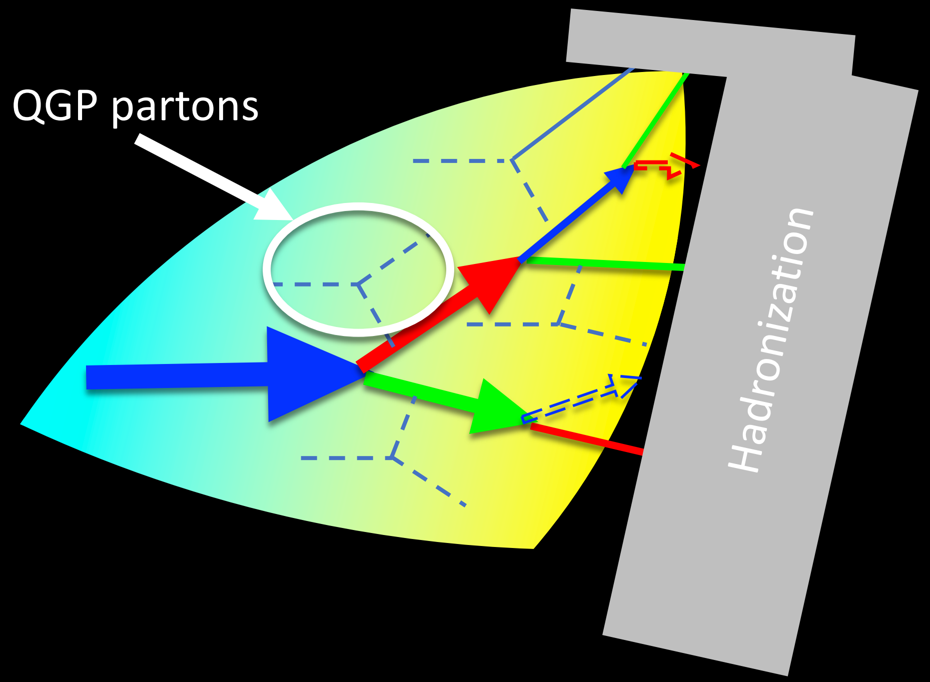
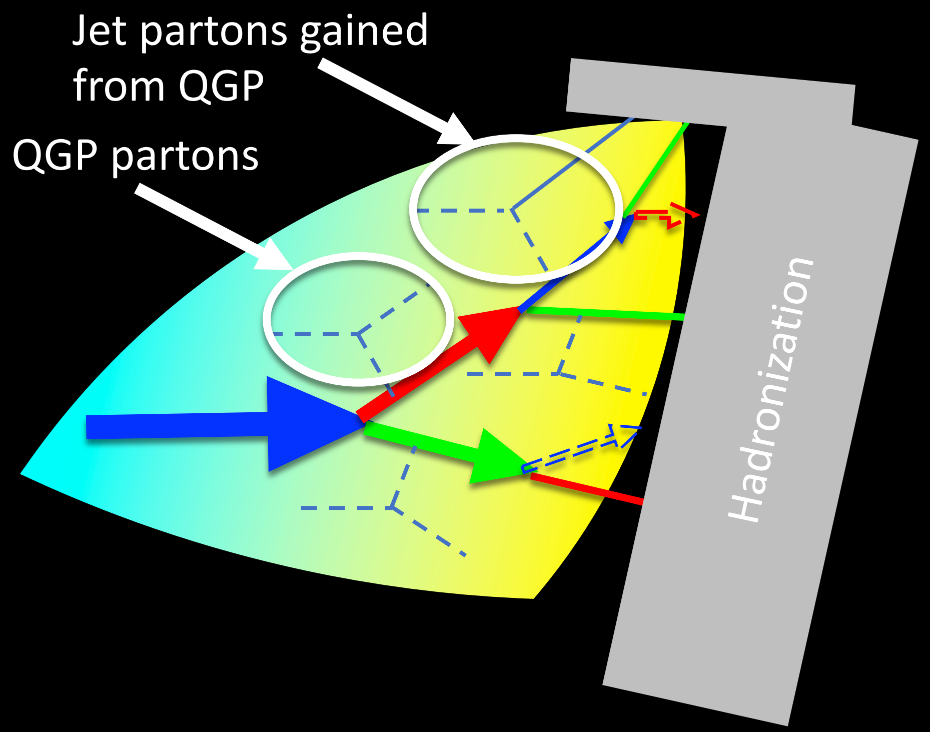


- The particles in a jet after hadronization occupy a wider cone.
- This widening is given by
  - “average” (i.e. $\hat{q}$) modifications to virtuality/energy evolution of partons in the jet due to presence of QGP.

Modified splitting inside the QGP



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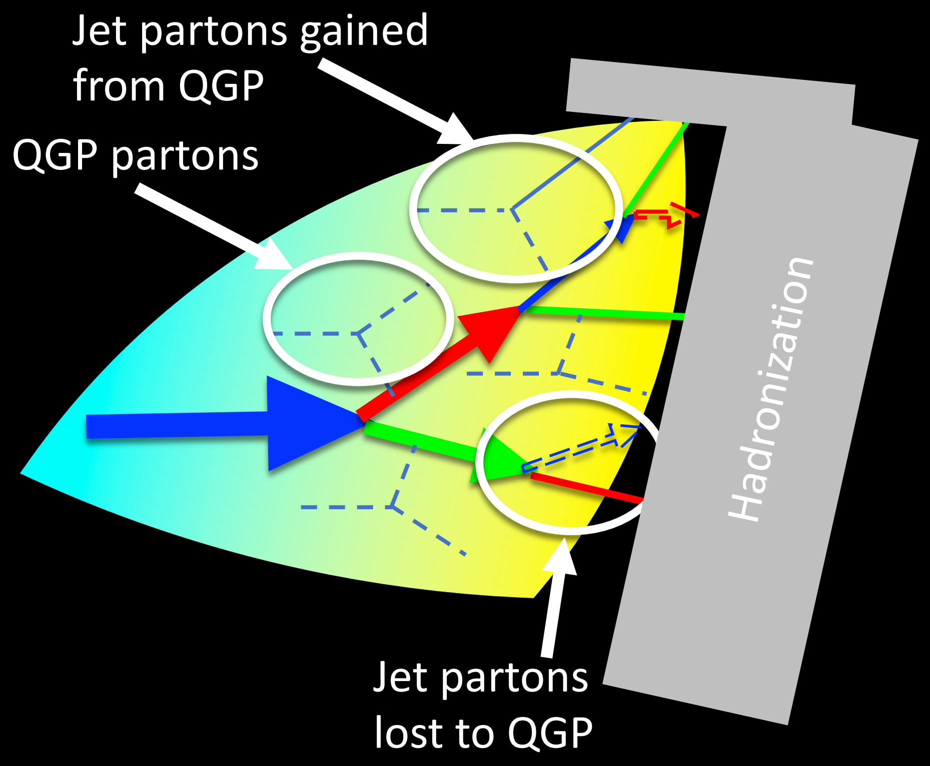
  - also includes partons picked up from the QGP that become part of the jet (see dashed to solid lines) vice versa.

Modified splitting inside the QGP



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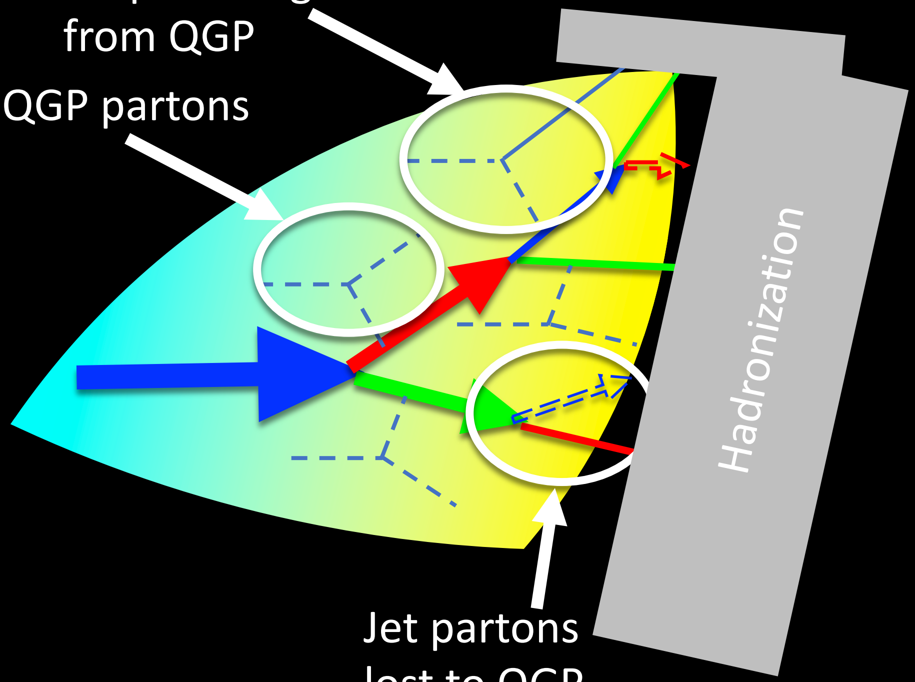

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The goal next is to explore the low Q and high E portion of the showering

Parton Energy “Loss”

The story of high energy and low virtuality partons

(Linear) Boltzmann Transport

- Valid for high E, assuming particles are on-shell
- Solves the time-ordered evolution for the phase space distribution function

$$p \cdot \partial f(x,p) = \Gamma_{el} + \mathcal{G}_{inel}$$

The case of $p \cdot \partial f(x,p) = \Gamma_{el}$ was already discussed on by Dmytro Oliinychenko

(Linear) Boltzmann Transport

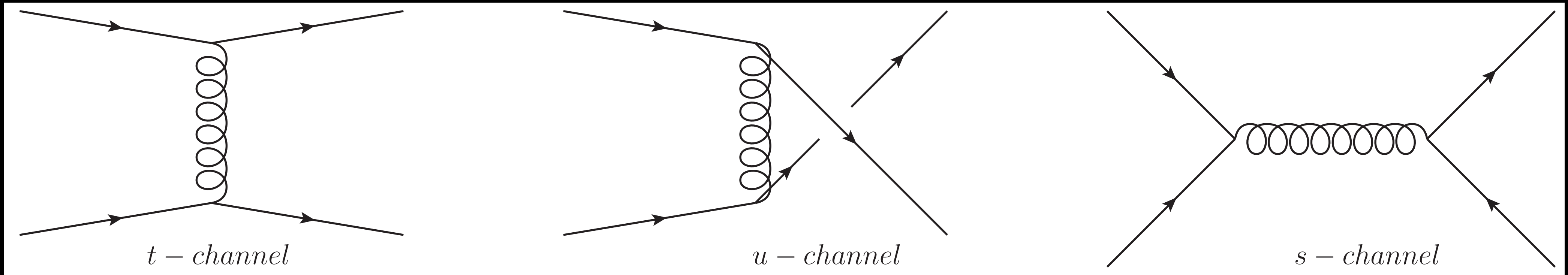
- Valid for high E, assuming particles are on-shell
- Solves the time-ordered evolution for the phase space distribution function

$$p \cdot \partial f(x,p) = \Gamma_{el} + \mathcal{G}_{inel}$$

- The LO pQCD $2 \to 2$ scattering matrix elements $\mathcal{M}$ is included in Γ_{el}

$$\Gamma_{el} = \int \frac{d^3k}{2k^0(2\pi)^3} \int \frac{d^3l}{2l^0(2\pi)^3} \int \frac{d^3q}{2q^0(2\pi)^3} f(p)f(k)|\mathcal{M}|^2 f'(l)f'(q)(2\pi)^4\delta^{(4)}(p+k-l-q)$$

$$\Gamma_{el} = \frac{dN}{d\tau} \text{ Rate of } 2 \to 2 \text{ collisions}$$



Note that channels where $q \to g$ are also included (though not shown)

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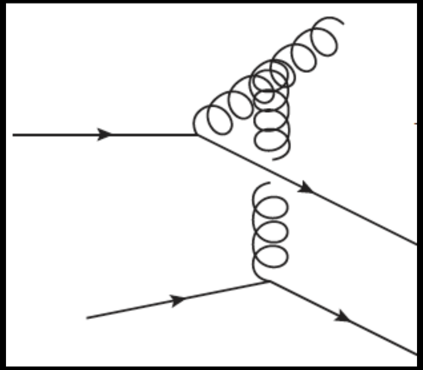
- The $\mathcal{G}_{inel}$ calculates medium-induced stimulated $1 \to 2$ emission at LO in $\left(\alpha_s, \frac{\mathrm{M}^2}{\mathrm{Q}^2}\right)$ [see **PRC 94, 054902 (2016)**]

$$\mathcal{G}_{inel} = \frac{dN}{d\tau} = \int \frac{d(Q^2)}{Q^2} \frac{\alpha_s(Q^2)}{2\pi} \int dy\, \tilde{\mathcal{P}}(y)$$

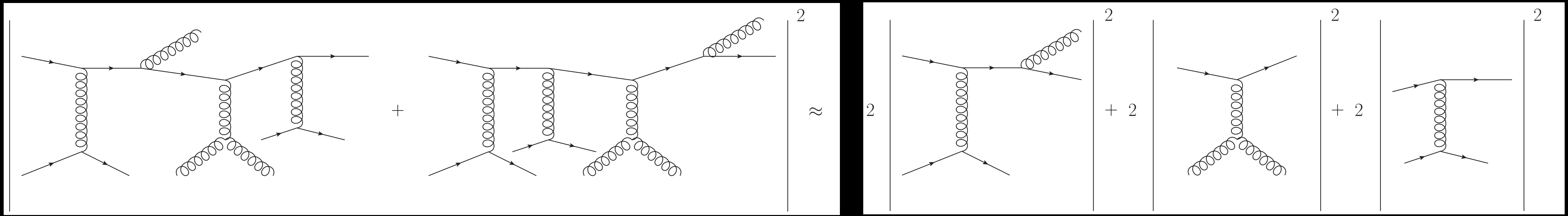
$$\tilde{\mathcal{P}}(y) = \cancel{P(y)} + \frac{P(y) \int_{\tau_i}^{\tau_f} dt\, \hat{q}(\tau) \left[4\sin^2\left[\frac{\tau - \tau_i}{2\tau_f}\right]\right]}{y(1-y)Q^2}$$

Recall



Incoherent scatterings

- The LO pQCD $1 \to 2$ rates are obtained from the Arnold Moore Yaffe (AMY) formalism [**JHEP 0111, 057 (2001); JHEP 0206, 030 (2002); JHEP 0301, 030 (2003**]. There are two contributions to AMY rates:
  - Incoherent emission (i.e. with large phase change between scatterings ⇒ negligible interreference between scatterings inducing radiation)
  - Incoherent emission: typically associated with large angle (and large E) of radiated partons.



  - Incoherent radiation and can be calculated via the usual $2 \to 2 + 1 \to 2$ matrix elements $\mathcal{M}$

Coherent Scattering & resummations

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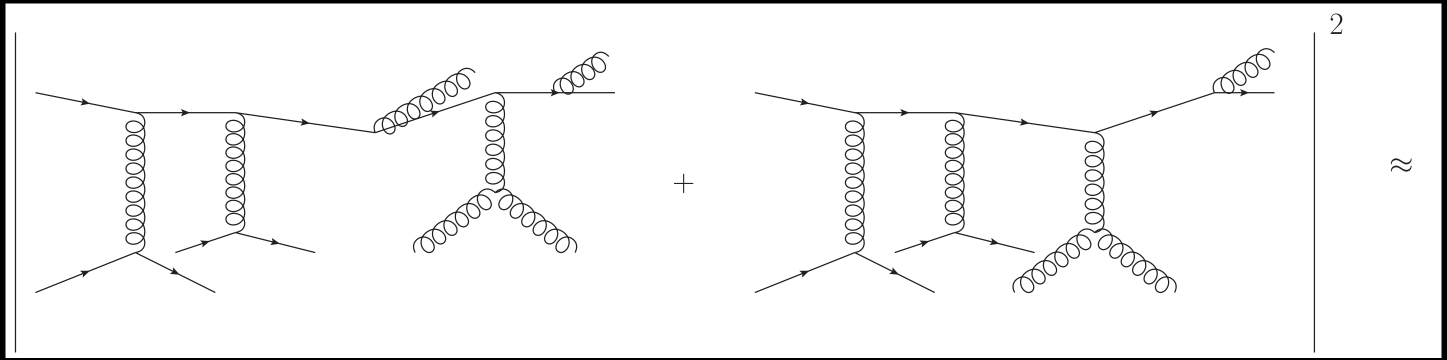
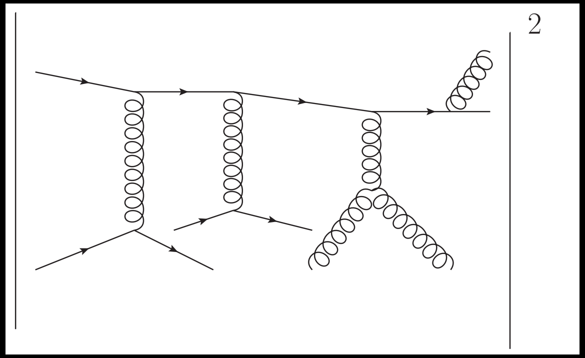
  - Coherent emission (i.e. with small phase change between scatterings): destructive interreference between scatterings and induced radiation ⇒ less radiation from the medium
  - Coherent emission: associated with small angle (and small E) of radiated partons





  - Need to sum all scattering diagrams taking phase into account

    ⇒ Landau-Pomeranchuk-Migal resummation

Arnold, Moore, and Yaffe resummations

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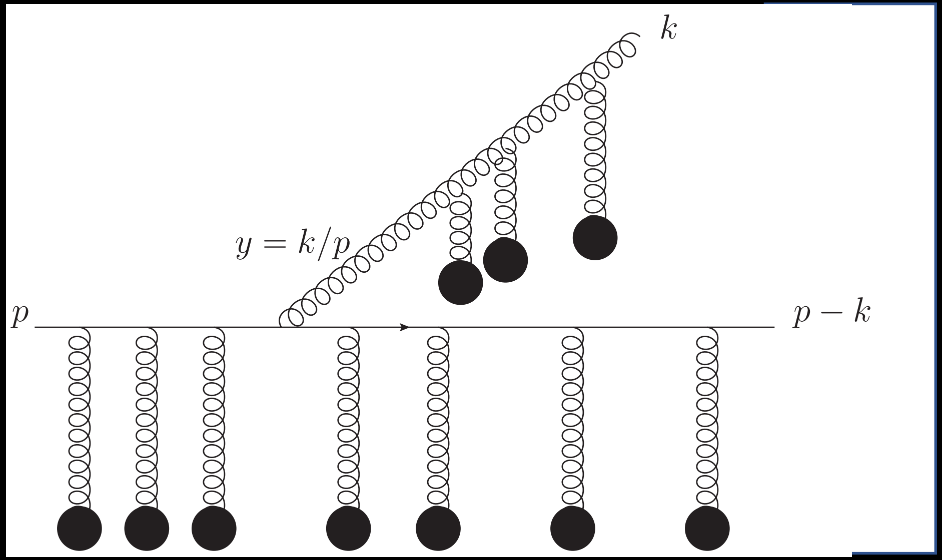
  - Need to sum all scattering diagrams ⇒ Landau-Pomeranchuk-Migal (LPM) resummation



Arnold, Moore, and Yaffe resummations

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  - Coherent emission (i.e. with small phase change between scatterings): destructive interreference between scatterings and induced radiation ⇒ less radiation from the medium
  - Coherent emission: associated with small angle (and small E) of radiated partons
  - After Landau-Pomeranchuk-Migal (LPM) resummation

$$\frac{d^2\Gamma_g}{dpdk} = \frac{\alpha_s}{4p^7}\frac{1}{1 \pm \exp\left[-\frac{k}{T}\right]}\frac{1}{1 \pm \exp\left[-\frac{p-k}{T}\right]}\left\{\begin{array}{ll} C_F\,\frac{1+(1-y)^2}{y^3(1-y)^2} & q \rightarrow qg \\ 2N_f T_f\,\frac{y^2+(1-y)^2}{y^2(1-y)^2} & g \rightarrow q\bar{q} \\ C_A\,\frac{1+y^4+(1-y)^4}{y^3(1-y)^3} & g \rightarrow gg \end{array}\right\}\int \frac{d^2\boldsymbol{h}}{(2\pi)^2}\; 2\boldsymbol{h}\cdot Re[\boldsymbol{F}(\boldsymbol{h},p,k)]$$

Phys. Rev. C 71, 034901 (2005)

LBT & MARTINI: Monte Carlo simulations at low Q, high E

- Linear Boltzmann Transport (LBT) solves the linearized Boltzmann equation containing:
  - LO pQCD by $2 \to 2$ scattering rates Γ_{el}
  - $1 \to 2$ radiation rate included in $\mathcal{G}_{inel}$
  - Using Γ_{el} and $\mathcal{G}_{inel}$ one can construct probabilities of scattering/radiation and use those in a Monte Carlo algorithm to solve the linearized Boltzmann equation
    - Note: linearized Boltzmann ⇒ no back-reaction onto the medium is included in *linearized* Boltzmann equations

- Modular Algorithm for Relativistic Treatment of heavy IoN Interaction (MARTINI) uses:
  - LO pQCD by $2 \to 2$ scattering rates Γ_{el}
  - LPM suppressed $1 \to 2$ radiation rate following Arnold, Moore, and Yaffe (AMY) formalism is included in $\frac{d^2\Gamma_g}{dpdk}$
  - Using Γ_{el} and $\frac{d^2\Gamma_g}{dpdk}$ one can construct probabilities that are be used in a Monte Carlo algorithm

Using JETSCAPE

Hands on session

The exercise

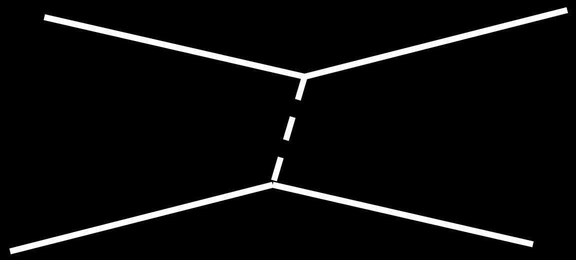
- Calculate the cross-section of pion production in a nuclear medium, i.e. $d\sigma_{AA}^{(\pi)}/dp_T d\eta$.
- In theory, one can calculate the nuclear modification function R_{AA} as:

$$R_{AA}^{(\pi)}(p_T,\eta) = \frac{d\sigma_{AA}^{(\pi)}/dp_T d\eta}{d\sigma_{pp}^{(\pi)}/dp_T d\eta}$$

- To obtain R_{AA} the most complicated piece is the calculation of the numerator, as the denominator can be calculated using Pythia.
- To calculate $\frac{d\sigma_{AA}^{(\pi)}}{dp_T d\eta}$ we need to bin particles in finite intervals

$$\frac{1}{\sigma_{inel}} \frac{d\sigma_{AA}^{(\pi)}}{2\pi p_T \Delta p_T \Delta\eta} = \sum_{k=1}^{N} \frac{dN\left(\hat{p}_{T;k}\right)}{2\pi p_T \Delta p_T \Delta\eta} \frac{\hat{\sigma}(\hat{p}_{T;k})}{\sigma_{inel}}$$

- What is $\hat{p}_T$? It's the p_T of the propagator – – – – – in a $2 \to 2$ scattering $\Rightarrow$



Preparing for the hands-on

- % = host machine
- $ = docker container
- Download and run the docker container
    - % docker run -it -v ~/MATTER_LBT_result:/home/jetscape-user/JETSCAPE/MATTER_LBT_results -p 8888:8888 gvujan/jetscape-school:latest
    - NOTE: no files outside of the folder MATTER_LBT_results will be saved once you exit the docker container.
- $ cd ~/JETSCAPE/config
    - jetscape_user.xml contains the .xml variables that we will be editing
- $ cd ~/JETSCAPE/examples/sample_hydro_files/event-0/
- That folder should contain 5 files 2 of which we will need:
    - JetData.h5 contains the entire hydro profile history
    - initial.hdf5 contains the locations of the scattering centers needed to insert the partons in the medium

jetscape_user.xml

Number of events/$\hat{p}_T$-bin

Number of jet events per hydro event

If seed=1, the same rand number list is generated. Use seed=0 for different lists of rand numbers

Range of $\hat{p}_T$ spanned by the $\hat{p}_T$-bin in GeV

$\sqrt{s_{NN}}$ in GeV

Path to where the hydro profile is located

Selector for MATTER shower in vacuum (in_vac=1) or in QGP (in_vac=0)

LBT in_vac=1 => LBT is off
LBT in_vac=0 => LBT is on

- Notes
  - nReuseHydro: there are more than one scattering center per hydro, so you can reuse the same hydro for different jet events.
  - We use seed=1 so that everyone can get the same result.
  - MATTER shower with in_vac=1 $\Rightarrow$ vacuum $P(y)$ in_vac=0 $\Rightarrow$ medium $P(y)$

```xml
<?xml version="1.0"?>

<jetscape>

  <nEvents> 200 </nEvents>
  <nReuseHydro> 200 </nReuseHydro>

  <Random>
    <seed>1</seed>
  </Random>

  <JetScapeWriterAscii> on </JetScapeWriterAscii>

  <!-- Inital State Module  -->
  <IS>
    <Trento> </Trento>
    <initial_profile_path>../examples/sample_hydro_files</initial_profile_path>
  </IS>

  <!-- Hard Process -->
  <Hard>
    <PythiaGun>
      <pTHatMin>50</pTHatMin>
      <pTHatMax>70</pTHatMax>
      <eCM>5020</eCM>
    </PythiaGun>
  </Hard>

  <!--Preequilibrium Dynamics Module -->
  <Preequilibrium>
    <NullPreDynamics> </NullPreDynamics>
  </Preequilibrium>

  <!-- Hydro  Module  -->
  <Hydro>
    <Brick bjorken_expansion_on="false" start_time="0.6"> </Brick>
    <hydro_from_file>
      <hydro_files_folder>../examples/sample_hydro_files</hydro_files_folder>
    </hydro_from_file>
  </Hydro>

  <!--Eloss Modules -->
  <Eloss>
    <Matter>
      <in_vac> 0 </in_vac>
    </Matter>
    <Lbt>
      <in_vac> 0 </in_vac>
    </Lbt>
  </Eloss>

  <!-- Jet Hadronization Module -->
  <JetHadronization>
    <name>colorless</name>
  </JetHadronization>

</jetscape>
```

- $ cd JETSCAPE/build
- $ ./runJetscape ../config/jetscape_user.xml
  - This will generate test_out.dat and cross_section.dat. We'll need the cross-section file later to construct $d\sigma_{AA}^{(\pi)}$
- $ cp test_out.dat cross_section.dat ../MATTER_LBT_results/
- $ ./FinalStateHadrons test_out.dat final_hadrons.dat
- $ cp final_hadrons.dat ../MATTER_LBT_results/
- Do a run with 4 $\hat{p}_T$ bins spanning 100-200 GeV with as many events as you can, e.g. :
  - <nEvents> 2000 </nEvents>
  - <nReuseHydro> 2000 </nReuseHydro>

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  </JetHadronization>

</jetscape>
```

Thank you!

Question?