JETSCAPE Framework for Medium Evolution

Initial collision geometry from the T$_{R}$ENTo model

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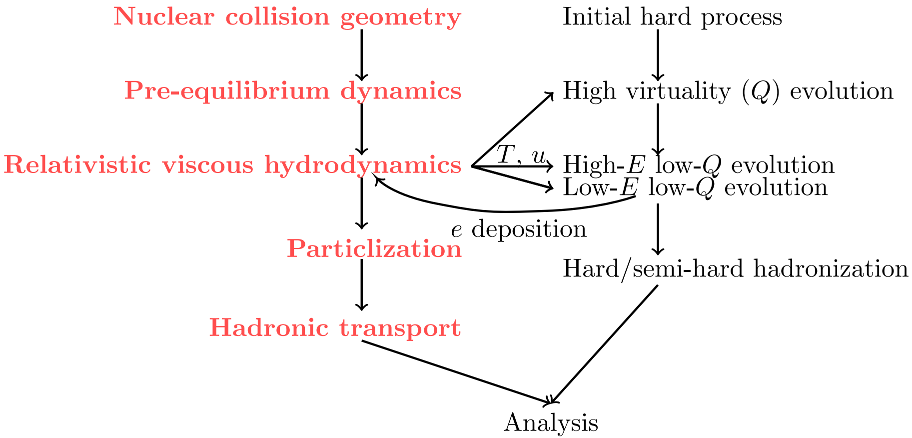
Today's plan:

- 30mins lecture on the TRENTo initial condition model.
- 2h30mins hands-on session on Bayesian analysis using TRENTo prediction as an example

This talk:

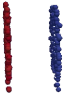
The JETSCAPE framework for medium evolution in heavy-ion collisions

JETSCAPE[1]: modular event generator for **jet** and **medium evolution** in relativistic heavy-ion collision. Develop statistical package to calibrate model parameters.

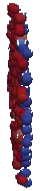


[1] http://jetscape.org/

Initializing the dynamical evolution of QGP

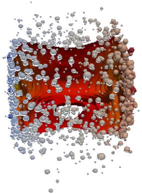


- Two approaching, Lorentz-contracted, nuclei A and B.
  - Nucleonic d.o.f. are slow and "freeze" on the collision time scale.
  - An event-by-event fluctuating nuclear geometry.



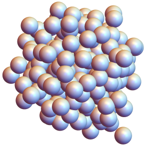
- Initial collisions: complicated many-body dynamics, needs modeling.
  - Simplication: only model energy deposition at $\tau \to 0^+$.
  - What is the **theoretical uncertainty** of a parametric $\frac{dE_T}{dx_\perp^2 \, d\eta_s}$?

- Early-time dynamics.
  - A 3+1D freestreaming model.
  - Evolve initial energy density.

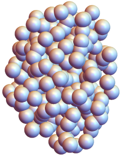


- Relativistic viscous hydrodynamics.
  - Matching energy-momentum tensor from early-time dynamics

The TRENTo initial condition model[2]: nuclear configuration



^{208}Pb



^{238}U

^{3}He

p

- Sample nucleon positions according to the (deformed) Woods-Saxon parametrization of heavy nuclei[1],

$$\frac{dN}{dr d\Omega} \propto \frac{1}{1+e^{\frac{r-r_0(\theta,\phi))}{a}}},$$
$$r_0(\theta,\phi) = R(1+\beta_2 Y_{20}+\beta_4 Y_{40})$$

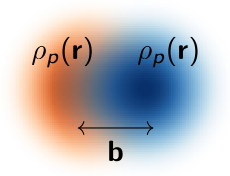
  - β_2, β_4 nuclear quadruple and hexadecapole moments.
  - R: radius parameter.
  - a: diffusiveness parameter.

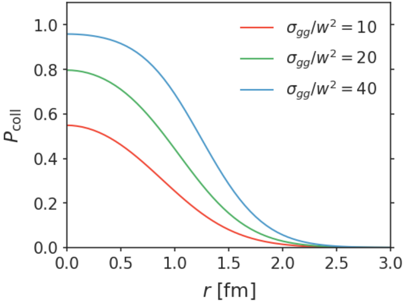
- Sample with minimum nucleon-nucleon distance d_{min} to mimic short-range repulsion.

[1] Atom. Data Nucl. Data Tabl. 36, 495–536 (1987) .For light nuclei like ^{3}He, one can load configurations from nuclear structure calculation PLB 680, 225–230 (2009).

[2] http://qcd.phy.duke.edu/trento/index.html

Nucleon profile and pairwise nucleon inelastic collisions





- 3D Gaussian model of proton density, $\mathbf{r} = (x, y)$

$$\rho_p(\mathbf{r}, z) = \frac{e^{-\frac{\mathbf{r}^2+z^2}{2w^2}}}{(2\pi w)^{3/2}} \xrightarrow{\int dz} \rho_p(\mathbf{r}) = \frac{e^{-\frac{\mathbf{r}^2}{2w^2}}}{2\pi w^2}$$

- Probability of two nucleons to collide depends on the density overlap $T_{pp}(b)$, b is the impact parameter.

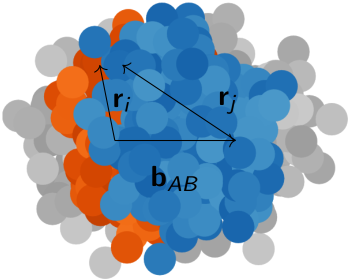
$$T_{pp}(b) = \int \rho_p(\mathbf{r} - \mathbf{b}/2)\rho_p(\mathbf{r} + \mathbf{b}/2) d\mathbf{r}^2$$

$$P_{\mathrm{coll}}(b) = 1 - \exp\{-\sigma_{gg} T_{pp}(b)\}$$

- σ_{gg}: an effective opacity parameter tuned to reproduce the pp inelastic cross-section at given beam energy ($\sqrt{s}$)

$$\sigma_{\mathrm{inel}}(\sqrt{s}) = \int P_{\mathrm{coll}}(b; \sigma_{gg}(\sqrt{s})) db^2 \to \frac{\sigma_{gg}}{w^2} = f\left(\frac{\sigma_{NN}}{w^2}\right)$$

Participants and binary collisions

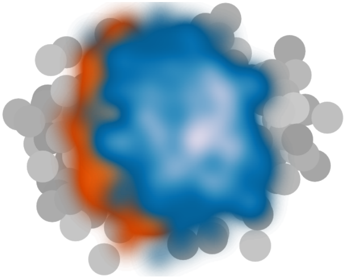


- Check each pair of nucleons (i, j) from the two collision nuclei A and B separated by impact-parameter b_{AB},

$$\text{Collision criterion: sample } P_{\text{coll}}(b = |\mathbf{r}_j - \mathbf{b}_{AB} - \mathbf{r}_i|)$$

- Participants: nucleons suffer at least one inelastic collision.

Participants and binary collisions



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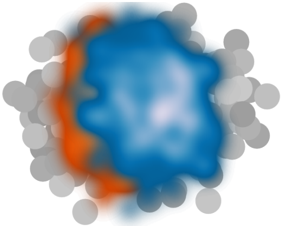
$$\text{Collision criterion: sample } P_{\text{coll}}(b = |\mathbf{r}_j - \mathbf{b}_{AB} - \mathbf{r}_i|)$$

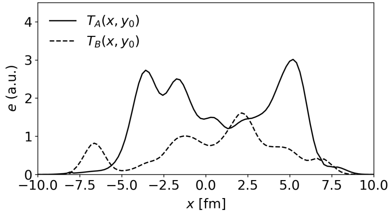
- Participants: nucleons suffer at least one inelastic collision.
- Smoothed participant density:

$$T_{A \text{ or } B}(\mathbf{r}) = \sum_{i \in \text{Part. } A \text{ or } B} \gamma_i \rho_p(\mathbf{r} - \mathbf{r}_i)$$

- γ_i draw from Γ distribution $\sim \gamma^{k-1} e^{-k\gamma}$. Important to simulate huge multiplicity fluctuation in pp collisions.

Energy density production at mid-rapidity





- Energy density deposition at mid-rapidity is assumed to be a local function of participant densities:

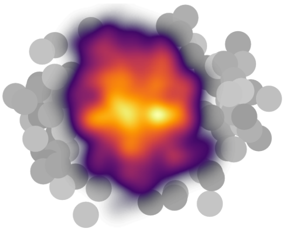
$$\frac{dE_T}{dx_\perp^2 d\eta_s}(x_\perp, \eta_s = 0) = \mathrm{Norm} \times f(T_A(x_\perp), T_B(x_\perp))$$

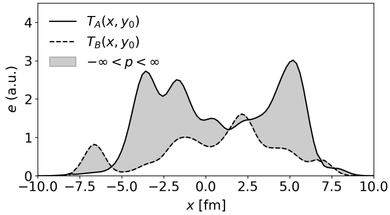
- A particular parametric form of f is used in TRENTo

$$f(T_A, T_B) = \left(\frac{T_A^p + T_B^p}{2}\right)^{1/p}$$

known as "generalized mean" ansatz.

Energy density production at mid-rapidity





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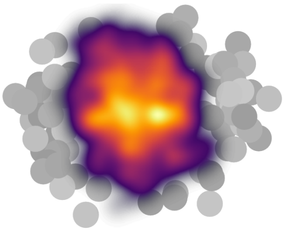
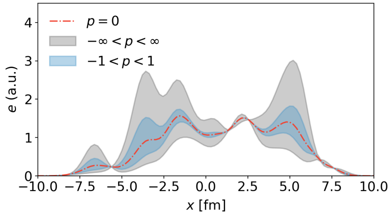
$$\left(\frac{T_A^p + T_B^p}{2}\right)^{1/p} = \begin{cases} \max\{T_A, T_B\}, & p \to \infty \\ \frac{T_A + T_B}{2}, & p = 1 \\ \sqrt{T_A T_B}, & p = 0 \\ \frac{2 T_A T_B}{T_A + T_B}, & p = -1 \\ \min\{T_A, T_B\}, & p \to -\infty \end{cases}$$

p: a model DoF to be constrained by data.

Energy density production at mid-rapidity (II)





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p: a model DoF to be constrained by data.

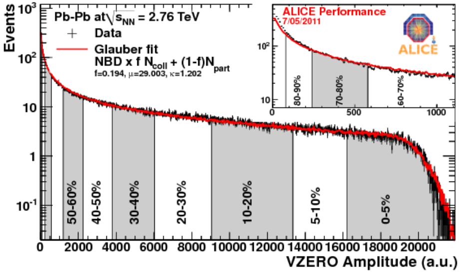
1. Review of the medium evolution model in JETSCAPE
2. Ingredients of the TRENTo initial condition model
3. Event properties at the initial condition level
4. Summary
5. Warm-up for the hands-on session.

Properties of initial conditions

- Impact-parameter: b.
- Number of participants: N_{part} nucleons that suffers at least one inelastic collisions.
- Number of binary collisions: N_{coll} the total number inelastic collisions.
- Total transverse energy: $dE_T/d\eta_s$ transverse summed energy per unit space-time rapidity.
- Eccentricity: $\epsilon_n e^{in\phi_n}$

None of the above are observables, even though some are strongly correlated with final-state properties.

Centrality definition



Centrality is an approximate indicator of geometry in AA collisions.

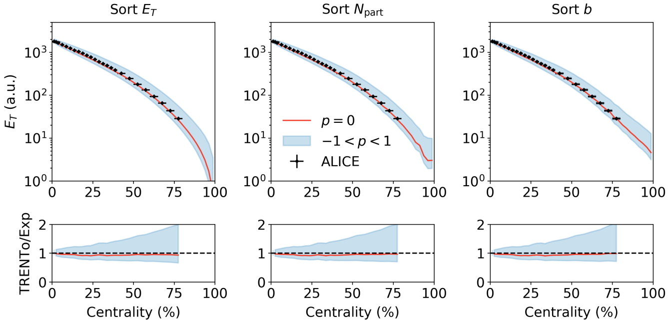
- Experimentally, one sorts the events according to how strong the detector responses to a collisions (# of particles, energy deposition in detector ...).
- Ideally, we should define centrality as close to experiments as possible, e.g, final-state charged particle multiplicity after the dynamical evolution[1].
- Initial condition can also used to estimate centrality approximately:
  Initial energy deposition strongly correlates with final-state energy / multiplicity.

[1] This definition is used in JETSCAPE medium calibration (in preparation).

Total transverse energy: centrality dependence in AA

Different way of defining centrality at the level of initial condition:

1. Sort events by $dE/d\eta_s$. Nucleon position, collision, and Γ-fluctuation (more realistic).
2. Sort events by $N_{\rm part}$. Nucleon position + collision fluctuations.
3. Sort events by b. Least fluctuation (less realistic).



E_T v.s. different centrality are similar in AA except for peripheral events.

Other observables can be sensitive to how you define centrality event in central collisions!

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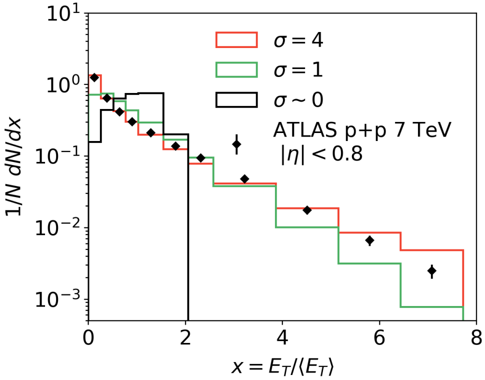
Energy deposition in small system: pp



ATLAS: JHEP11(2012)033

$$\gamma_i \sim \gamma^{k-1} e^{-k\gamma}$$

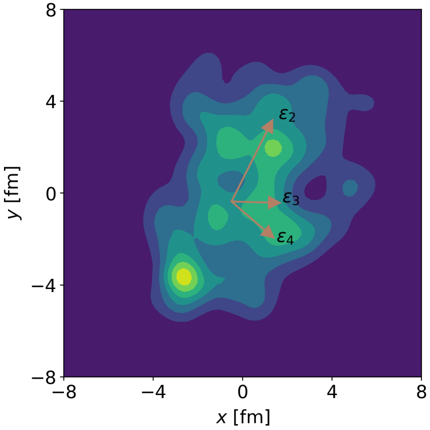
$$\sigma^2 \equiv \frac{1}{k} : \text{variance of } \Gamma \text{ fluctuation}$$

- Proton-proton collision produces huge total transverse energy E_T fluctuation.

$$E_T = \sum_{|\eta_i|<0.8} e_{T,i}$$

- TRENTo mimics part of the fluctuation by proton-proton geometry and the Γ fluctuation.
- Collision geometry does not explain the distribution. Dominated by the Γ fluctuation we put in.

Initial spatial eccentricity



- Finally state anisotropic flow are linear (+non-linear) responses of the hydrodynamic evolution to the initial spatial eccentricity[1].

$$v_n \propto c_n \times \epsilon_n + \cdots$$

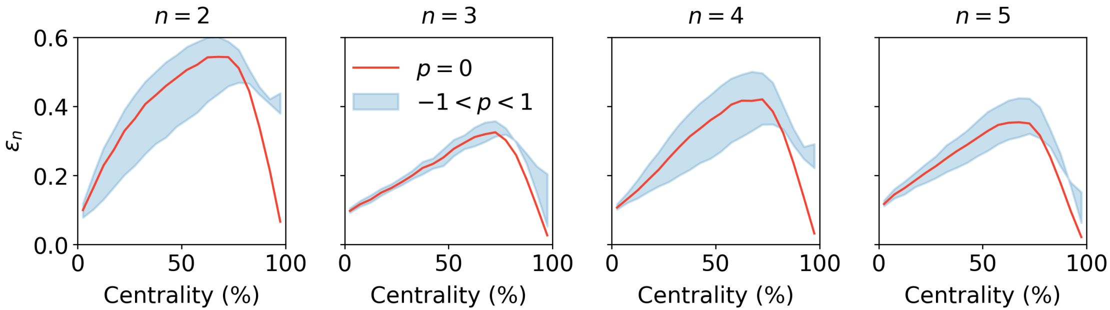
- In TRENTo, eccentricity is defined with energy density distribution $e(x, y)$.

$$\epsilon_n e^{in\phi_n} = \frac{\int e r^n e^{in\phi} dxdy}{\int e r^n dxdy}$$

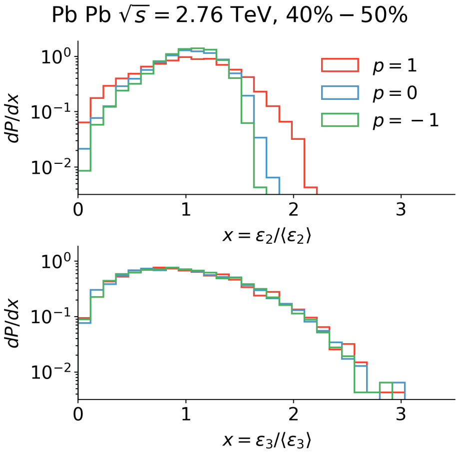
[1] The coefficients c_n also depends on transport coefficients, centrality, ...

Eccentricity

- The way energy production depends on participant density strongly affects the magnitude of initial spatial eccentricity.
- Notable initial condition uncertainty on the magnitude of anisotropic flow.



Eccentricity (II)



- It may not be too meaningful to look at the absolute magnitude of ϵ_n in initial condition.
- Linear reponses $v_n \sim c_n\epsilon_n + \cdots$ with unknown coefficients.
- The shape of the event-by-event distribution is more directly related to initial conditions, $x = \epsilon_n/\langle\epsilon_n\rangle$
- In $40 - 50\%$ centrality, ϵ_2 distribution is sensitive to the energy deposition ansatz (p-parameter).

Summary

- The soft sector initial condition in JETSCAPE uses a parametric model TRENTo.
- Initial condition is big source of uncertainty for understanding soft-sector evolution quantitatively from data. TRENTo parametrizes
  - a class of energy deposition relation $e = f(T_A, T_B)$ at mid-rapidity.
  - proton compactness through the width parameter.
  - variation in energy production fluctuation.
- We have seen these unconstrained DoF of initial condition affects
  - total energy v.s centrality.
  - centrality selection.
  - magnitude of spatial eccentricity.
- These uncertainties should be propagated to the extraction of interested physical parameters such as η/s and ζ/s.

Bayes' parameter extraction

The problem:

1. Given a model $\mathcal{M}$: compute quantities $\mathbf{y}$ with input parameters $\mathbf{x}$.
2. Given a prior belief of $\mathbf{x}$' true value's distribution $P_0(\mathbf{x}_{\rm true})$
3. Given observations $\mathbf{y}_{\rm exp}$.

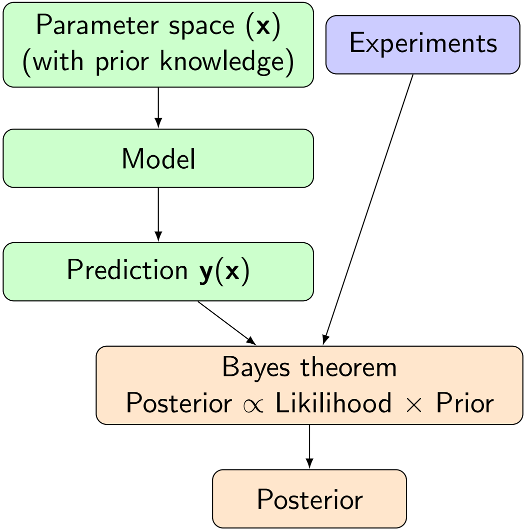
!! Ask for the updated probability distribution of $\mathbf{x}_{\rm true}$: $P(\mathbf{x}_{\rm true})$.

$$\text{Bayes' theorem:} \quad \underbrace{P(\mathbf{x}_{\rm true}|\mathcal{M}, \mathbf{y}_{\rm exp})}_{\rm Posterior} = \frac{\overbrace{P_L(\mathbf{y}_{\rm exp}|\mathcal{M}, \mathbf{x}_{\rm true})}^{\rm Likelihood}\overbrace{P_0(\mathbf{x}_{\rm true})}^{\rm Prior}}{\underbrace{\int P_L(\mathbf{x})P_0(\mathbf{x})d\mathbf{x}}_{\rm Normalization\ (evidence)}}$$

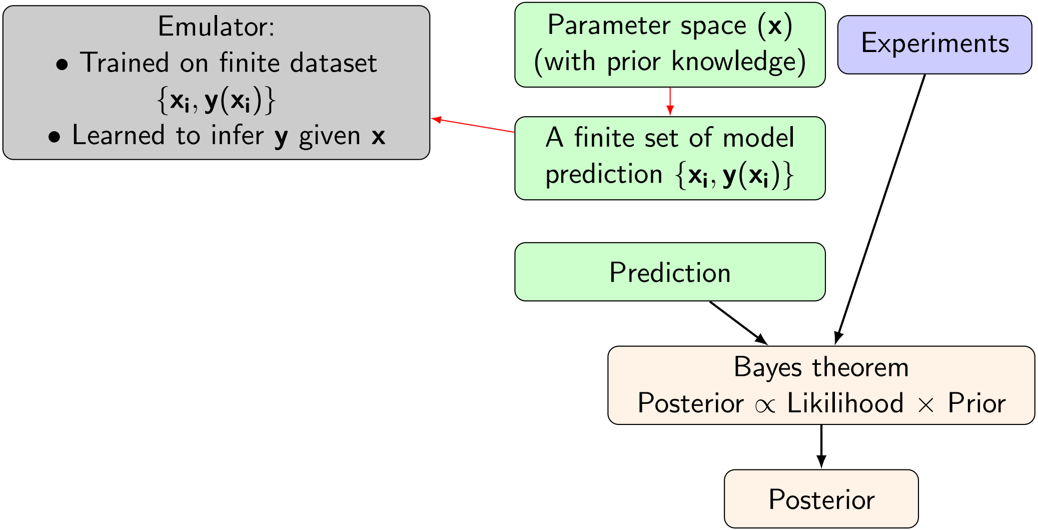
Often the form P_L is unknown. Commonly assumed to take the form:

$$\ln P_L = C - \frac{1}{2}\Delta\mathbf{y}\Sigma^{-1}\Delta\mathbf{y}^T, \quad \Delta\mathbf{y} = \mathbf{y}_{\rm exp} - \mathbf{y}(\mathbf{x};\mathcal{M}), \quad \Sigma : \text{Contains uncertainties}$$

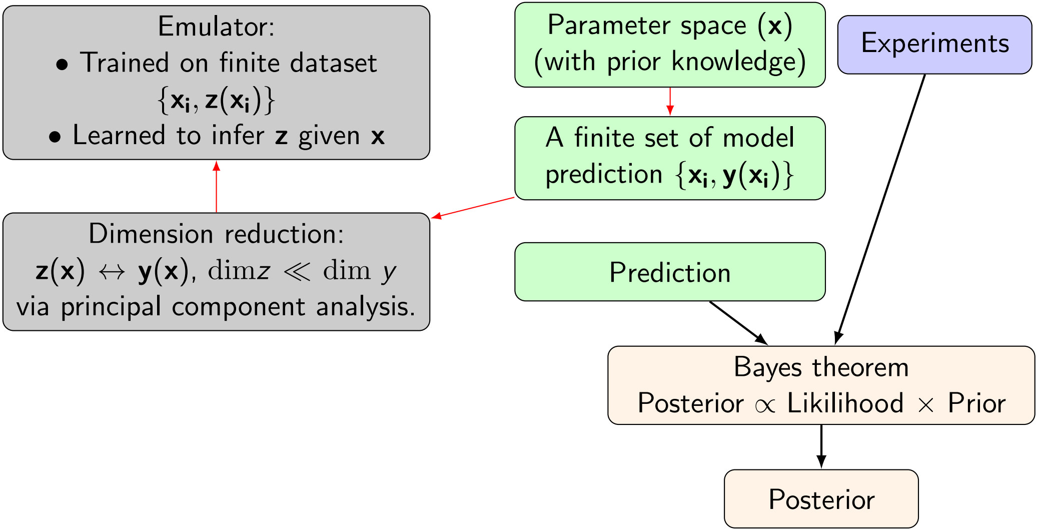
For simple models:



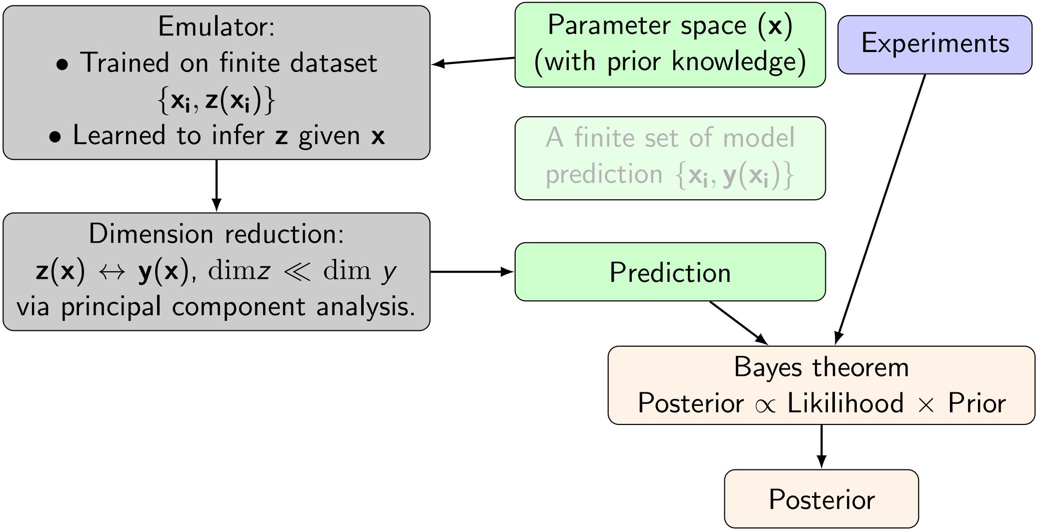
For computational intensive models:



For computational intensive models + high-dimensional output



Finally, the workflow of the emulator-assisted Bayesian analysis



Hands-on exercises for Bayesian model calibration:

We will have

- An exercise on Gaussian process emulator (1D input and 1D output).
- An exercises on principal component analysis to get familiar with dimensional reduction.
- Applying the above flow chart to a real problem: "Fitting" initial condition parameter by comparing initial condition prediction directly to a selection of experimental data.

Backup: TRENTo installation (standalone)

- Language: c++
- Dependence: boost, hdf5, gsl, c++11
- Download:

```
$ git clone https://github.com/Duke-QCD/trento.git
```

- Build and install (default path $HOME/.local)

```
$ mkdir build && cd build
$ cmake ..
$ make install
```

- Usage

```
$ trento [options] projectile target [number-events = 1]
```

Backup: TRENTo usage (standalone)

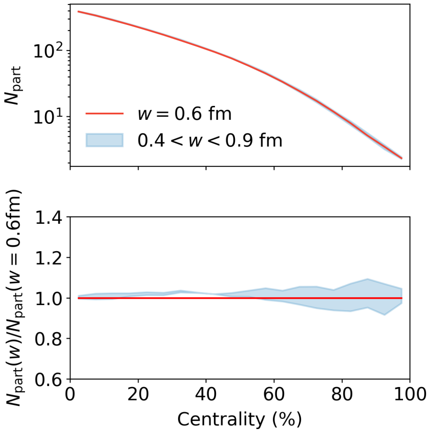
Example: generate 3 "minimum bias" Pb+Pb collisions.

```
trento Pb Pb 3
```

A subset of options:

- Nuclei: p, d, Cu, Xe, Au, Pb, U
- Nuclear configuration:
  - "-w": width [fm] of the Gaussian shape of the proton $\frac{1}{2\pi w^2} e^{-r^2/(2w^2)}$.
  - "-d": minimum nucleon distance [fm], short-range repulsion.
  - "-k": nucleonic fluctuation. Unit-mean Γ distribution, with variance $\sigma = 1/k^2$.
- Collision
  - "-x": set $\sigma_{pp}^{\mathrm{inel}}$ $[\mathrm{fm}^2]$, $\sim$ 4.2 fm^2 @200 GeV, 6.4 fm^2@2.76 TeV, 7.0 fm^2 @ 5 TeV.
  - "--ncoll": compute binary collision number (slow down the program a little).
- Energy deposition
  - "-p": set the p value in energy deposition ansatz $\sim (T_A^p + T_B^p)^{1/p}$.

Bonus: Number of participants



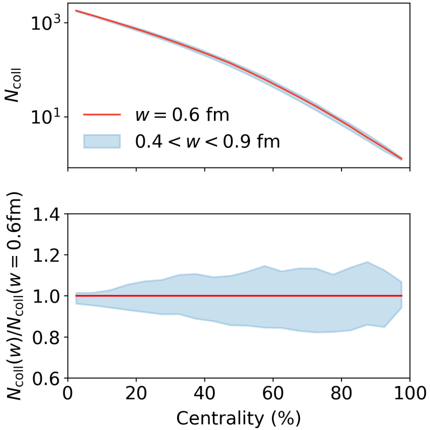
- Number of participants is rather insensitive to the size of proton.
- Probability for a nucleon "i" to be a participant depends on the overlap of the proton density and the thickness of the whole other nuclei T_{pA}.

$$1 - \prod_j (1 - P_{\rm coll}(b_{ij})) = 1 - e^{-\sigma_{gg}\sum_j T_{pp}(b_{ij})}$$

$$\approx \quad 1 - \exp\left\{-\#f\left(\frac{\sigma_{NN}}{w^2}\right)w^2 T_{pA}\right\}$$

- For large nuclei, a different nucleon width only affects this probability at the boundary, so $N_{\rm part}$ is not sensitive to w.

Bonus: Number of binary collisions



- Number of binary collisions: a measure of # of hard process (tiny fraction of the inelastic collision) in AA collisions.
- It is determined by T_{pp},

$$N_{\rm coll} = \left\langle \sum_{i\in A, j\in B} \left(1 - e^{-\frac{1}{4\pi} f\left(\frac{\sigma_{NN}}{w^2}\right) e^{-\frac{b_{ij}^2}{4w^2}}}\right)\right\rangle$$

- Sensitive to proton-proton overlap, and therefore sensitive to nucleon width parameter w.