70 YEARS OF CREATING TOMORROW

Los Alamos NATIONAL LABORATORY

R-Matrix Analysis of Reactions in the ^{17}O System using EDA

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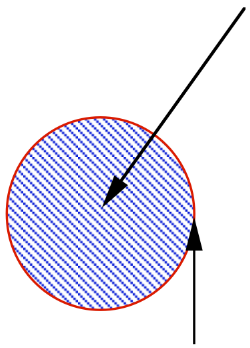
Outline

- Basic properties of scattering theory satisfied by the R-matrix formalism
  - Unitarity, reciprocity (TRI), causality
- Energy Dependent Analysis (EDA) code
  - Relativistic relations
- Recent results from the ^{17}O system analysis
  - Fits, data renomalizations, etc.
- Comparisons with Leal cross sections
- Summary and outlook

R-matrix Formalism

INTERIOR (Many-Body) REGION
(Microscopic Calculations)

$$\underbrace{H + \mathcal{L}_B}_{\substack{\text{compact, hermitian}\\ \text{operator with real,}\\ \text{discrete spectrum;}\\ \text{eigenfunctions in}\\ \text{Hilbert space}}}$$



SURFACE

$$|\psi^+) = (H + \mathcal{L}_B - E)^{-1} \mathcal{L}_B |\psi^+)$$

ASYMPTOTIC REGION
(S-matrix, phase shifts, etc.)

$$(r_{c'}|\psi_c^+\rangle = -I_{c'}(r_{c'})\delta_{c'c} + O_{c'}(r_{c'})S_{c'c}$$

or equivalently,

$$(r_{c'}|\psi_c^+\rangle = F_{c'}(r_{c'})\delta_{c'c} + O_{c'}(r_{c'})T_{c'c}$$

Measurements

$$\mathcal{L}_B = \sum_c |c)(c| \left(\frac{\partial}{\partial r_c} r_c - B_c \right),$$

$$(\mathbf{r}_c | c) = \frac{\hbar}{\sqrt{2\mu_c a_c}} \frac{\delta(r_c - a_c)}{r_c} [(\phi_{s_1}^{\mu_1} \otimes \phi_{s_2}^{\mu_2})_s^{\mu} \otimes Y_l^m(\hat{\mathbf{r}}_c)]_J^M$$

$$R_{c'c} = (c' | (H + \mathcal{L}_B - E)^{-1} | c) = \sum_\lambda \frac{(c' | \lambda)(\lambda | c)}{E_\lambda - E}$$

Basic Properties of Scattering Theory

1) Unitarity ($\boldsymbol{SS}^{\dagger} = \boldsymbol{S}^{\dagger}\boldsymbol{S} = 1$): enforced by $\boldsymbol{R_B}$ being real and symmetric ($H+\mathcal{L}_{\mathrm{B}}$ hermitian).

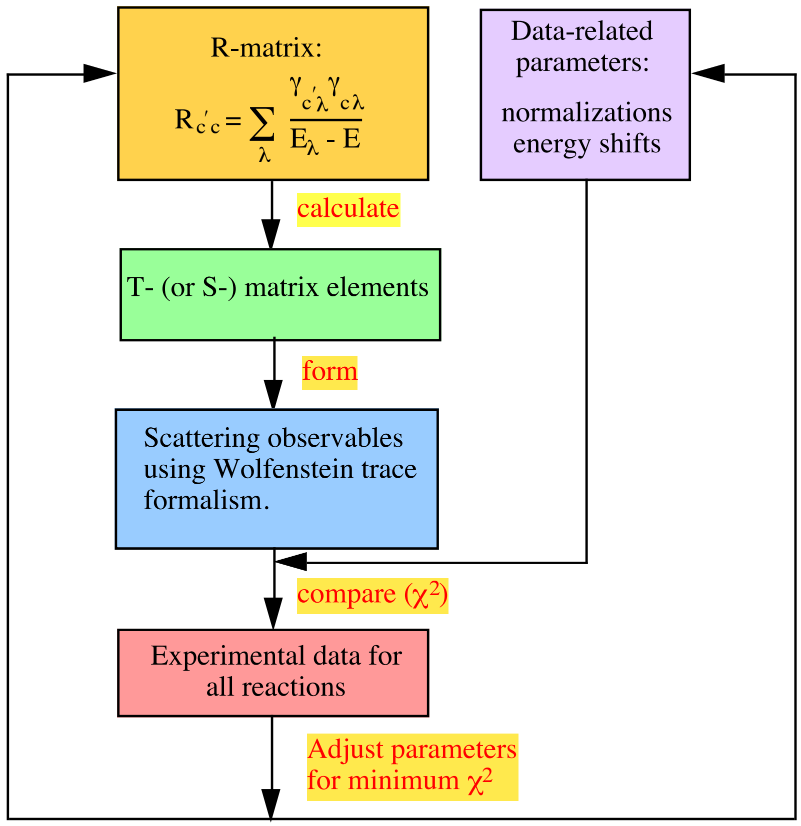
2) Reciprocity (TRI): enforced by the symmetry of $\boldsymbol{R_B}$ and all asymptotic matrices (such as $\boldsymbol{S}$) derived from it.

3) Causality: no poles of $\boldsymbol{S}$ in upper-half k-plane. Poles of $\boldsymbol{R}_{\mathrm{L}}$ are all in the lower half-plane, at $k = k_0$ and $-k_0^*$.

Note that the MLBW approximation violates *all* of these basic principles.

Scheme and Properties of the EDA Code

Energy Dependent Analysis Code



- Accommodates general (spins, masses, charges) two-body channels
- Uses relativistic kinematics and R-matrix formulation
- Calculates general scattering observables for 2 ➔2 processes
- Has rather general data-handling capabilities (but not as general as, e.g., SAMMY)
- Uses modified variable-metric algorithm that gives parameter covariances at a solution

Relativistic form(s) for *R* in EDA

$$R = \sum_{\lambda} \frac{\gamma_\lambda \gamma_\lambda^T}{E_\lambda(s) - E(s)},$$

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2 = (\varepsilon_{\text{rel}} + M)^2.$$

Forms for $E_{(\lambda)}(s)$:

a) $\sqrt{s} - M = \varepsilon_{\text{rel}}$

b) $\dfrac{s - M^2}{2M} = \left(1 + \dfrac{\varepsilon_{\text{rel}}}{2M}\right)\varepsilon_{\text{rel}}$

c) $\dfrac{(s - M^2)(s - \Delta^2)}{8s\mu}$ (Layson)

d) ε_{nr} (norel=1)

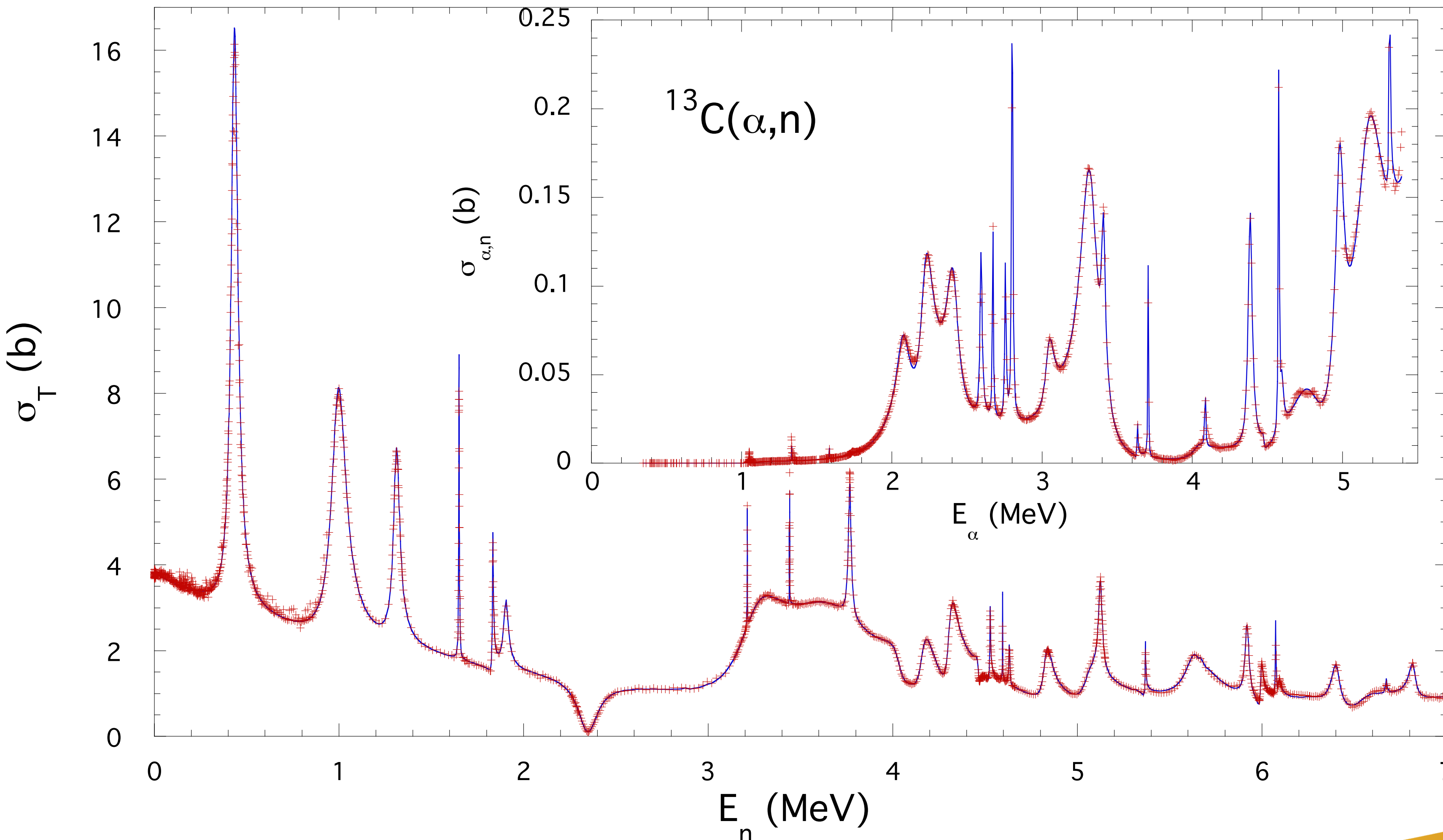
$$\begin{cases} M = m_1 + m_2 \\ \Delta = m_1 - m_2 \\ \mu = \dfrac{m_1 m_2}{m_1 + m_2} \end{cases}$$

R-Matrix Analysis of Reactions in the ^{17}O System

channel	a_c (fm)	l_{max}
n+^{16}O	4.4	4
α+^{13}C	5.4	5

Reaction	Energies (MeV)	# data points	Data types
^{16}O(n,n)^{16}O	$E_n = 0 – 7$	2718	σ_T, $\sigma(\theta)$, $P_n(\theta)$
^{16}O(n,α)^{13}C	$E_n = 2.35 – 5$	850	σ_{int}, $\sigma(\theta)$, $A_n(\theta)$
^{13}C(α,n)^{16}O	$E_\alpha = 0 – 5.4$	874	σ_{int}
^{13}C(α,α)^{13}C	$E_\alpha = 2 – 5.7$	1296	$\sigma(\theta)$
total		5738	8

Integrated (total) Cross Sections

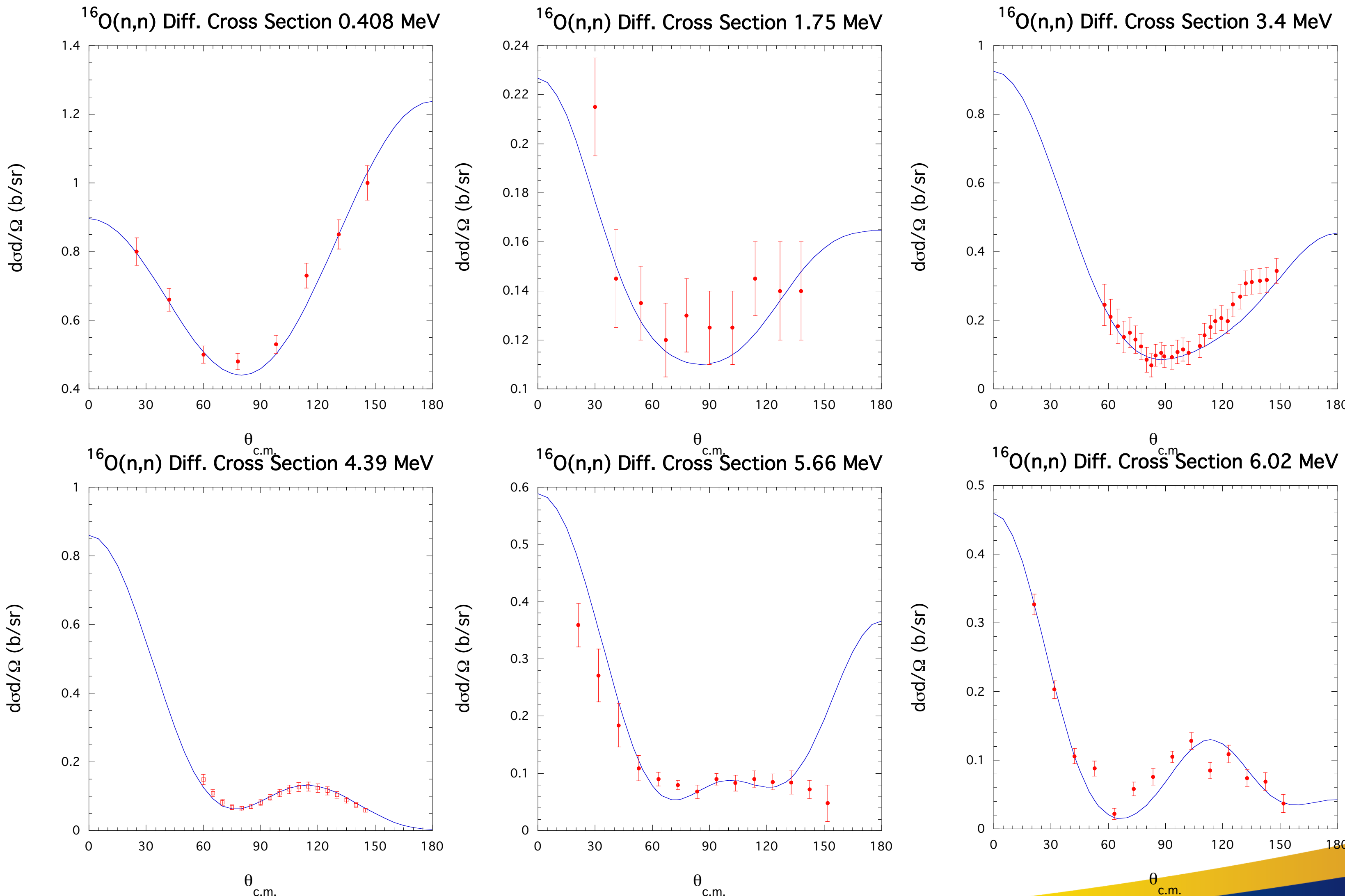


Total Cross Section Data

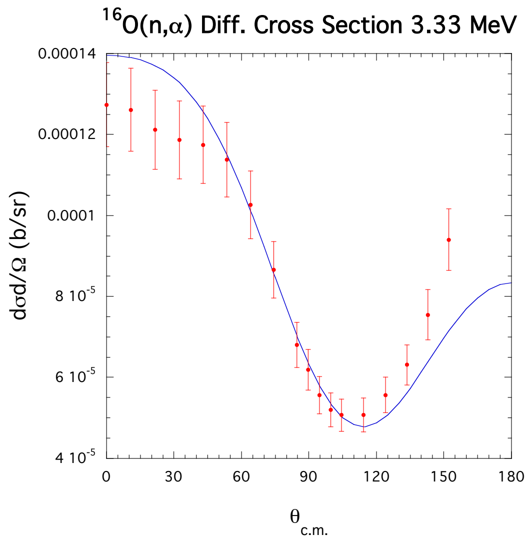
Authors	Energy Range	Energy Shift	Normalization
Dilg,Koester,Block	0.13 – 23.5 keV	0	1.0 (fixed)
Ohkubo (corr. for H)	0.8 – 935 keV	0	1.0009
Johnson & Fowler (including LOX)	49 – 3139 keV	0	0.9823
Cierjacks et al.	3.143 – 7.0 MeV	0	1.0414

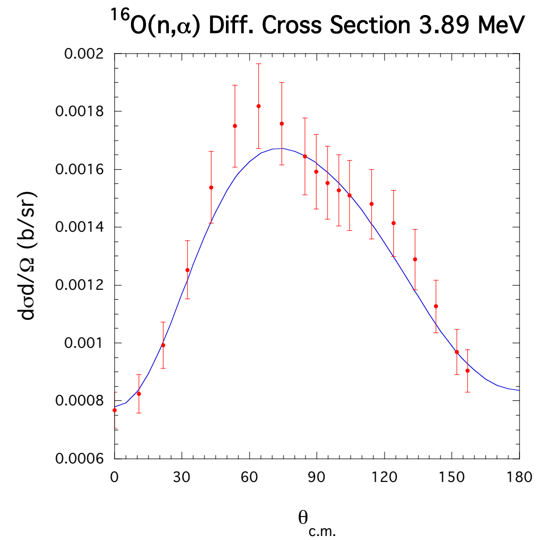
Authors	Energy Range	Energy Shift	Normalization
Drotleff et al.	346 – 1389 keV	0	1.0 (fixed)
Heil et al.	416–899 keV	0	1.0 (fixed)
Kellogg	445–1045 keV	0	1.512
Bair and Haas	0.997–5.402 MeV	-4 keV	0.9882

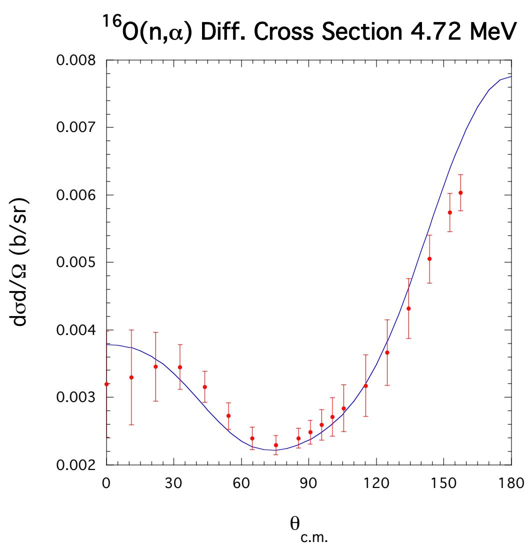
$^{16}O(n,n)$ Differential Cross Sections

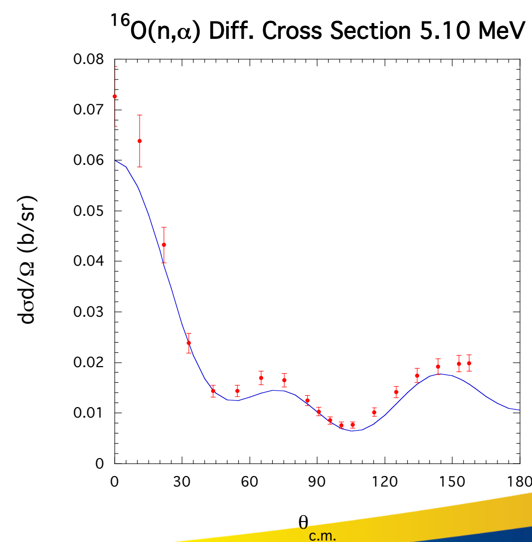


$^{16}O(n,\alpha)$ Differential Cross Sections

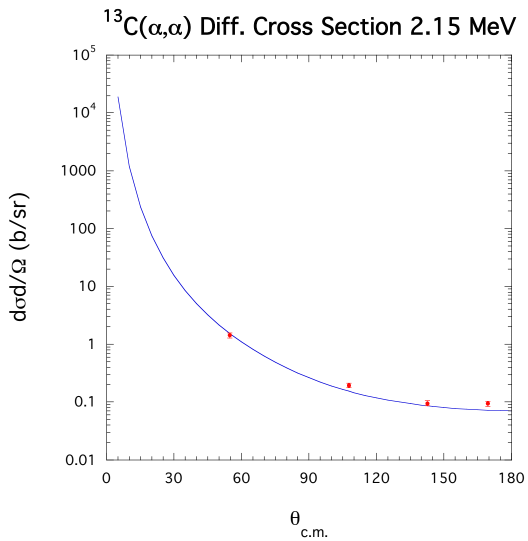


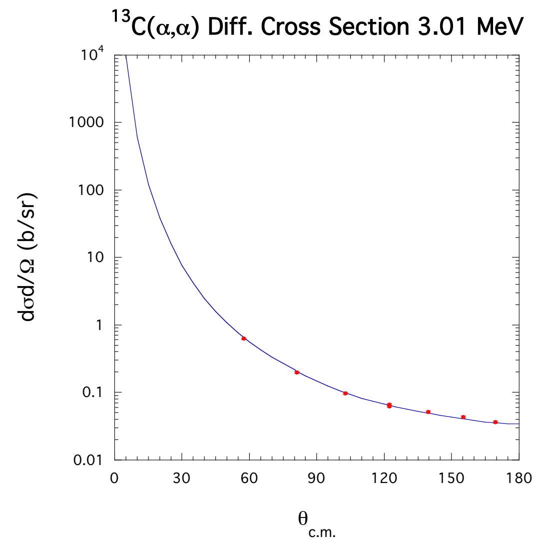


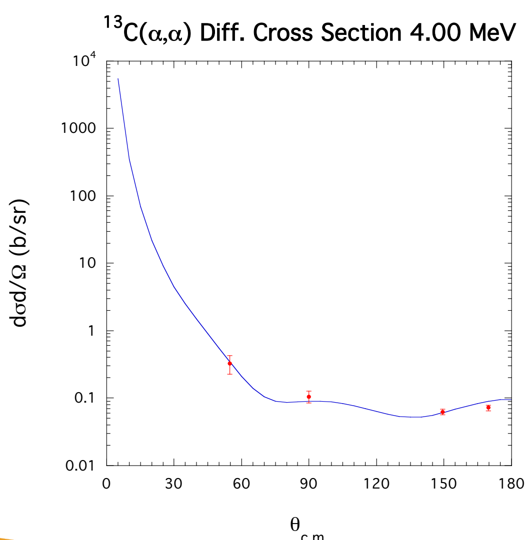


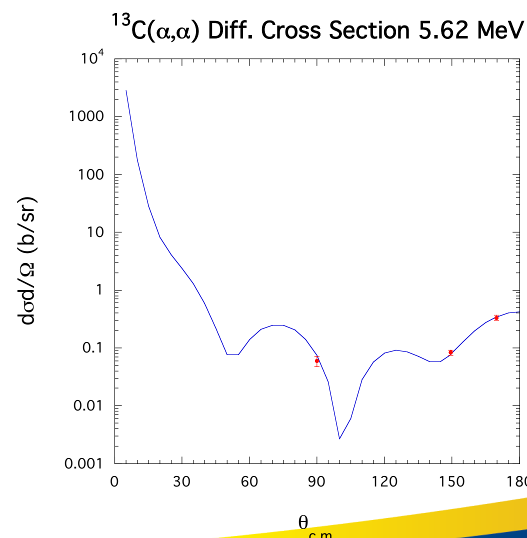


$^{13}C(\alpha,\alpha)$ Differential Cross Sections

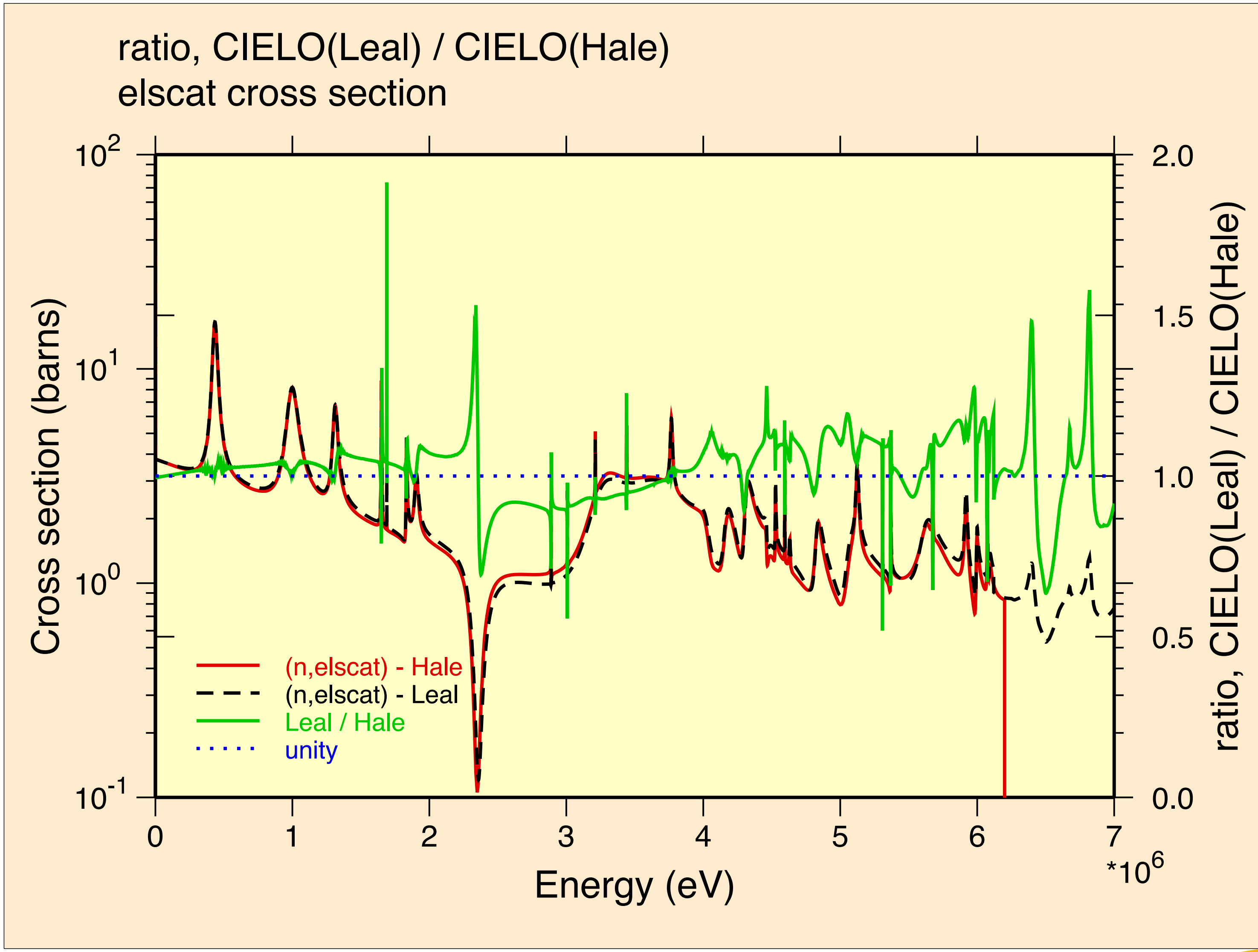




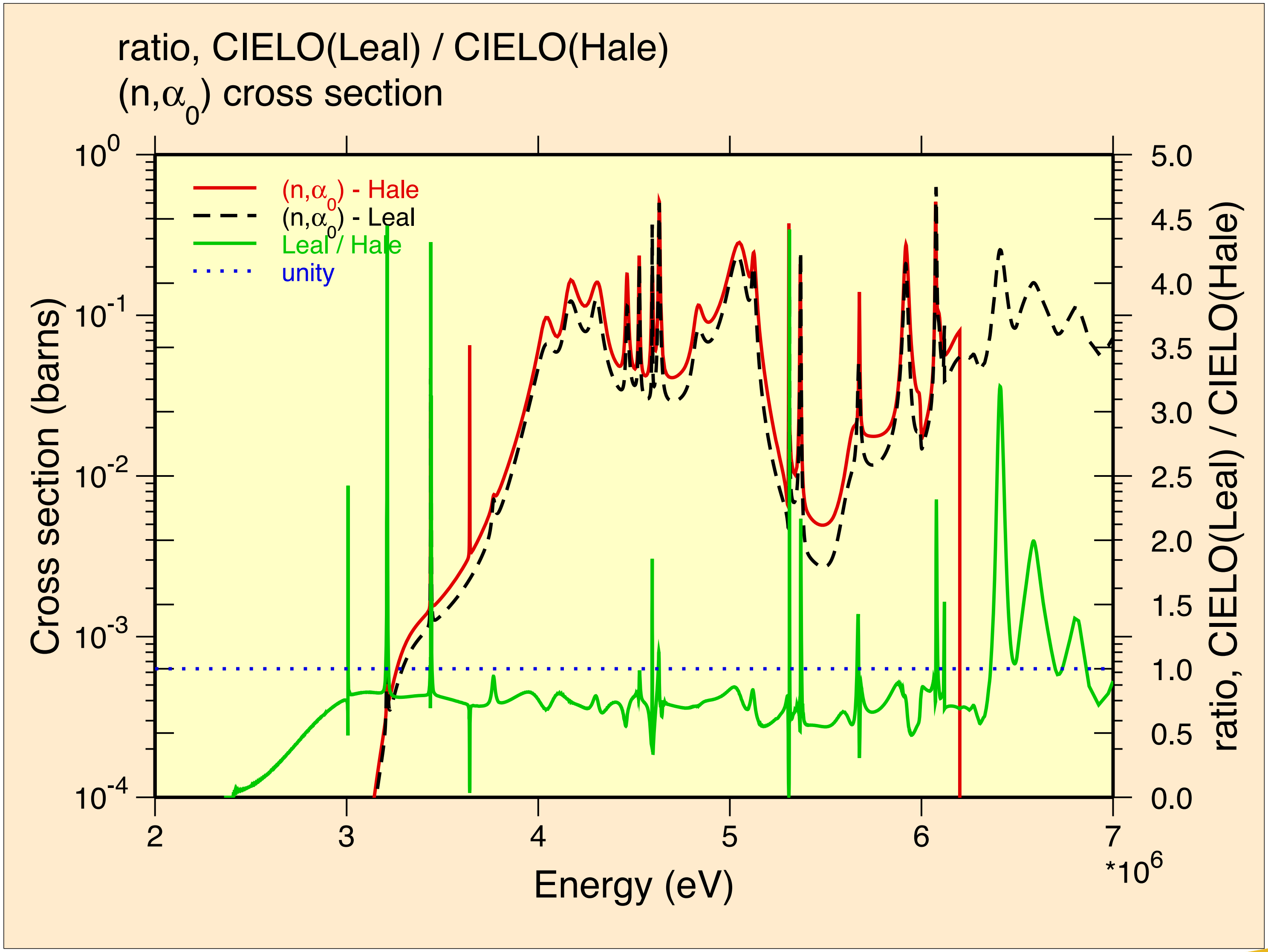




Comparison to Leal – (n,n)



Comparison to Leal - (n,α)



Summary and Outlook

- R-matrix descriptions for light systems can give very accurate and detailed fits to the experimental data, constrained by fundamental properties (unitarity, causality, TRI) of nuclear reaction theory.
- EDA R-matrix analyses of the ^{17}O system include data from all possible reactions, giving results that are highly constrained by the properties above (especially unitarity).
- The low-energy n+^{16}O scattering cross sections are now in better agreement with high-precision measurements, but the higher-energy cross sections did not change much.
- Differences with Leal's SAMMY analysis are significant in the MeV region. Do they come from using different data sets, or from differences in the R-matrix parametrization? Kunieda's results could provide insight into this question.
- We will continue the ^{17}O analysis with additional experimental data.