Angular Distributions in EMPIRE

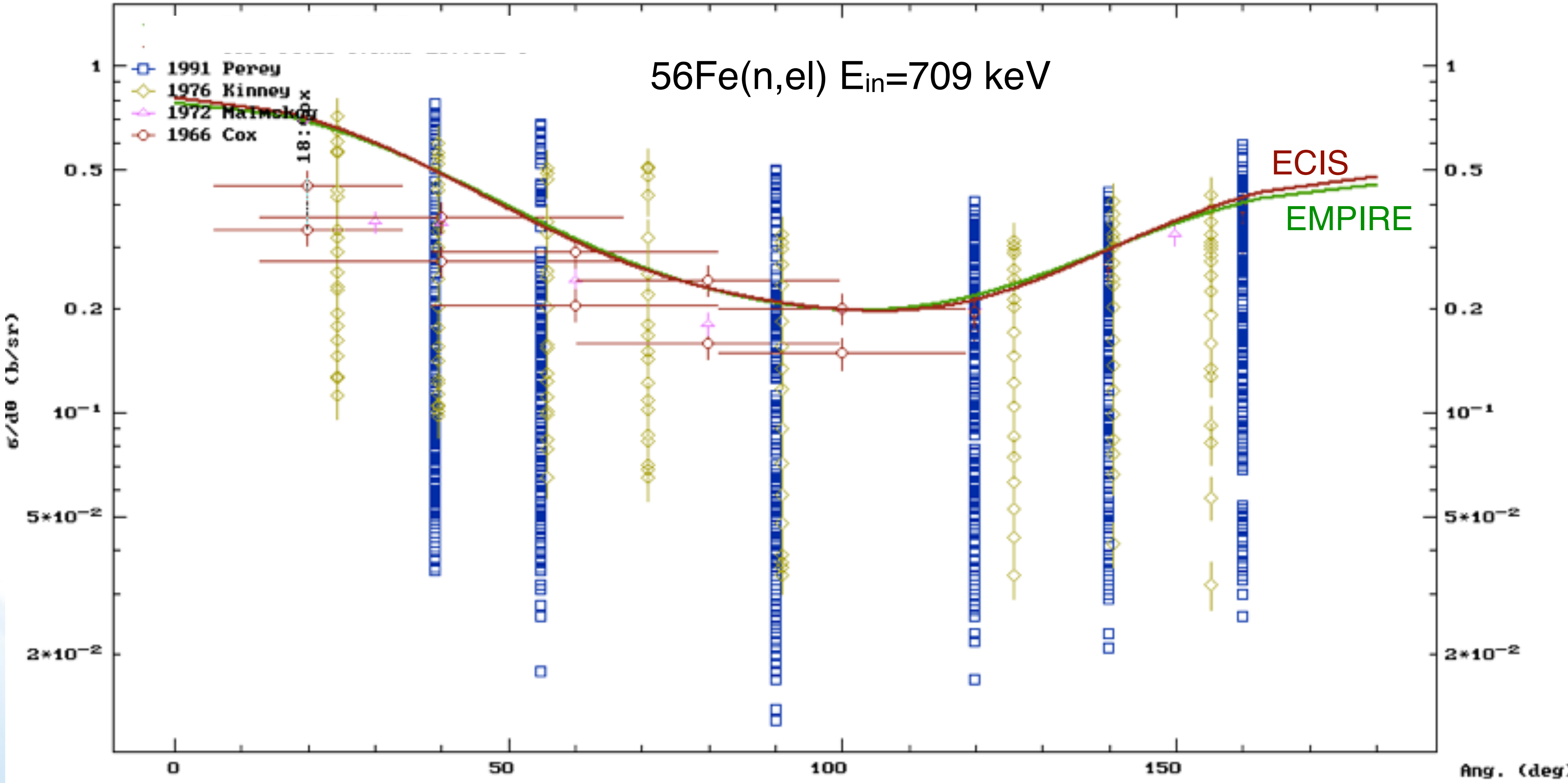
D. Brown, BNL

BROOKHAVEN NATIONAL LABORATORY
a passion for discovery

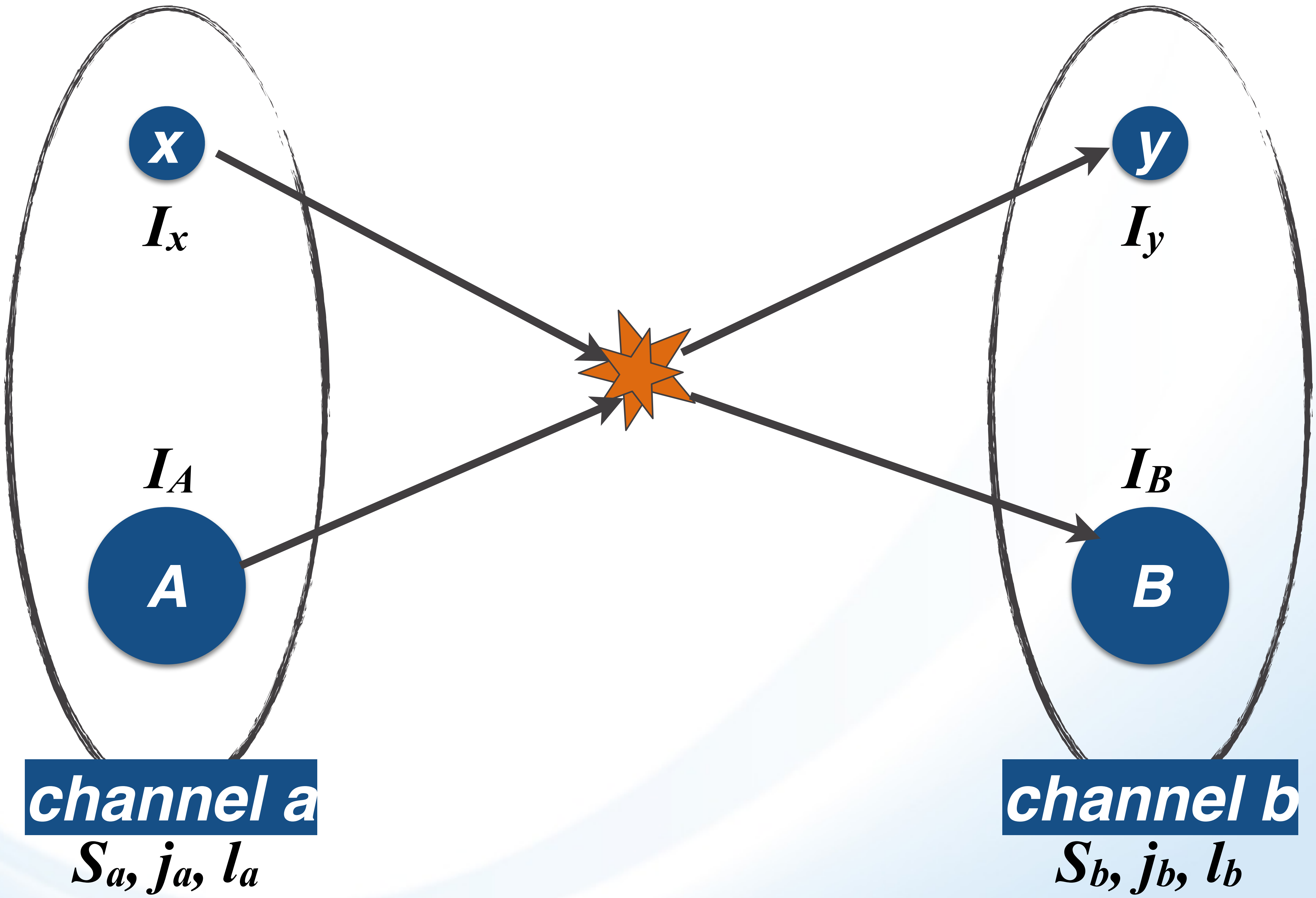
U.S. DEPARTMENT OF ENERGY | Office of Science

CN angular distributions in EMPIRE

- Previously CN angular distr. were calculated by rescaling ECIS results - not fully consistent and cumbersome
- Native EMPIRE calculations required replacing T_l with T_{lj}
- New HRTW subroutine was totally recoded in F90



Standard view of two-body reaction



Computing angular distributions is easy, right?

- The angle-differential cross section for a two body channel in EMPIRE is the usual

$$\overline{\frac{d\sigma_{a\to b}}{d\Omega_b}} = \frac{d\sigma^{dir}_{a\to b}}{d\Omega_b} + \frac{d\sigma^{CN}_{a\to b}}{d\Omega_b}$$

- where either term can be written

$$\frac{d\sigma_{a\to b}}{d\Omega_b} = \left(\frac{2\pi}{\hbar}\right)^4 \mu_a\mu_b \frac{k_b}{k_a}\frac{1}{2I_x+1}\frac{1}{2I_A+1}\sum_{L=0}^{\infty} B_L(\underline{b},\underline{a};E_a)P_L(\mu)$$

- textbooks give B_L in channel spin S_a-l_a ($S_a=I_A+I_x$) coupling:

$$B_L(\underline{b},\underline{a};E_a) = \sum_{S_a,S_b}\frac{(-)^{S_b-S_a}}{4}\sum_{J\ell_a\ell_b}\sum_{J'\ell'_a\ell'_b}\bar{Z}(\ell_a J\ell'_a J';S_a L)\bar{Z}(\ell_b J\ell'_b J';S_b L)\Re\left[T^{J*}_{\{\underline{a};\ell_a S_a\}\to\{\underline{b};\ell_b S_b\}}T^{J'}_{\{\underline{a};\ell'_a S_a\}\to\{\underline{b};\ell'_b S_b\}}\right]$$

(Note: here T is T-matrix, not transmission coefficient)

Textbook equations don't work for Hauser-Feshbach

- Natural coupling for transmission coefficients is T_{lj} so need j_a-I_A (j_a=I_x+l_a) coupling
- Result is

$$B_L(\underline{b},\underline{a};E_a) = \frac{1}{4}\sum_{J\ell_a\ell_b j_a j_b}\ \sum_{J'\ell'_a\ell'_b j'_a j'_b} \bar{Z}(\ell_a j_a \ell'_a j'_a; I_x L)\bar{Z}(\ell_b j_b \ell'_b j'_b; I_y L)\Re\left[T^{J*}_{\{\underline{a};\ell_a j_a\}\to\{\underline{b};\ell_b j_b\}} T^{J'}_{\{\underline{a};\ell'_a j'_a\}\to\{\underline{b};\ell'_b j'_b\}}\right]$$
$$\times(-1)^{-I_A-I_x+I_B+I_y}(2J+1)(2J'+1)\begin{Bmatrix} j_a & J & I_A \\ J' & j'_a & L \end{Bmatrix}\begin{Bmatrix} j_b & J & I_B \\ J' & j'_b & L \end{Bmatrix}$$

(Note: here T is T-matrix, not transmission coefficient)

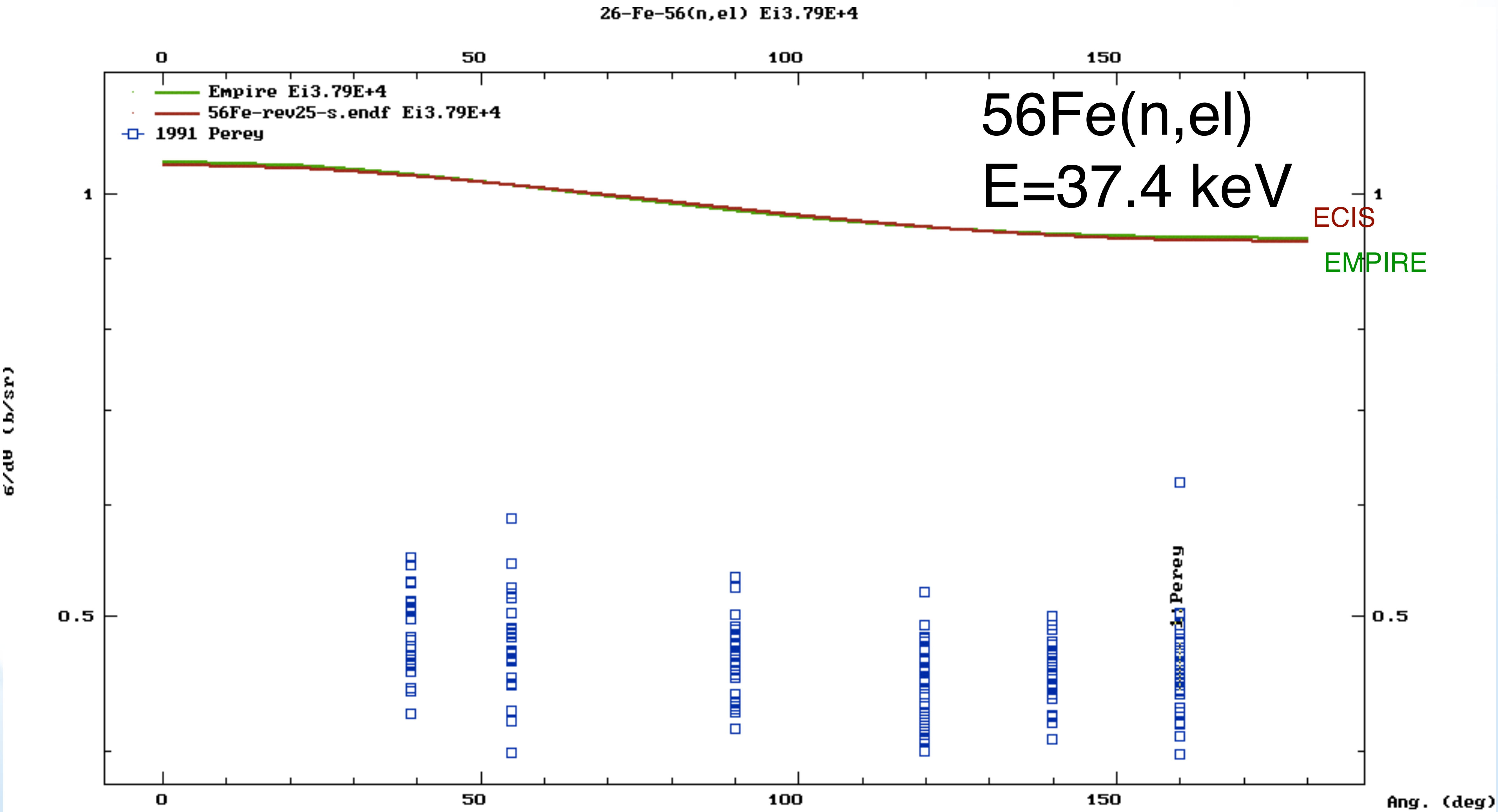
- I lost two weeks of my life (re)deriving this
- Simplifying to the HF case:

$$B_L(\underline{b},\underline{a};E_a) = \frac{1}{4}\sum_{J\ell_a\ell_b j_a j_b} \bar{Z}(\ell_a j_a \ell_a j_a; I_x L)\bar{Z}(\ell_b j_b \ell_b j_b; I_y L)\overline{\left|T^{J(fl)}_{\{\underline{a};\ell_a j_a\}\to\{\underline{b};\ell_b j_b\}}\right|^2}$$
$$\times(-1)^{-I_A-I_x+I_B+I_y}(2J+1)^2\begin{Bmatrix} j_a & J & I_A \\ J & j_a & L \end{Bmatrix}\begin{Bmatrix} j_b & J & I_B \\ J & j_b & L \end{Bmatrix}$$
$$= \frac{1}{4}\sum_{J\ell_a\ell_b j_a j_b} \bar{Z}(\ell_a j_a \ell_a j_a; I_x L)\bar{Z}(\ell_b j_b \ell_b j_b; I_y L)\overline{\left|T^{J(fl)}_{\{\underline{a};\ell_a j_a\}\to\{\underline{b};\ell_b j_b\}}\right|^2}$$
$$\times(-1)^{-I_A-I_x+I_B+I_y+2(j_a+j_b)}(2J+1)^2\mathcal{W}(j_a J j_a J; I_A L)\,\mathcal{W}(j_b J j_b J; I_B L)$$

As I said, we coded this in EMPIRE

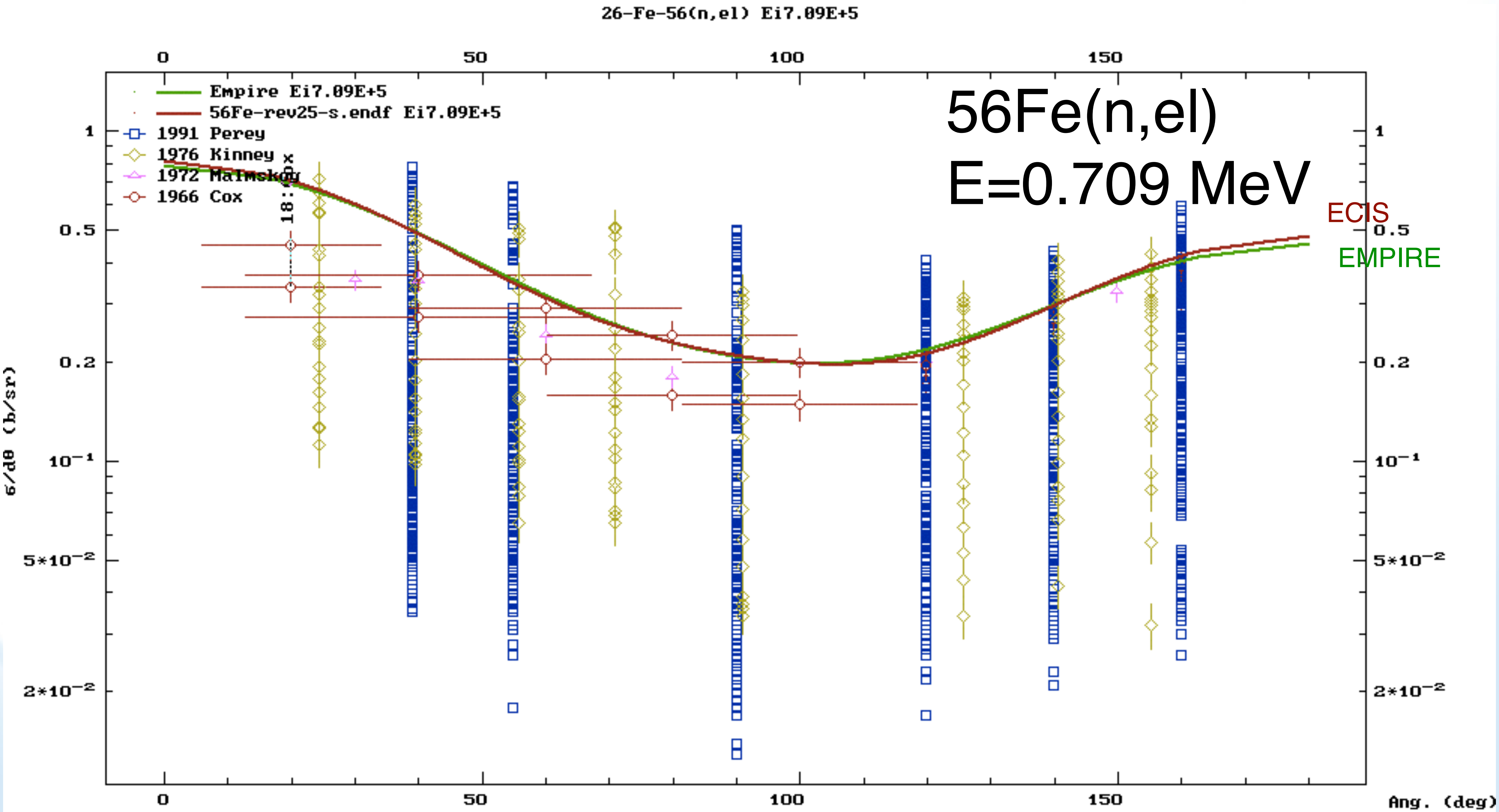
- To make sure it was right, we wrote F95 unit tests for
  - 3-j symbols
  - 6-j symbols
  - Racah coefficients
  - 9-j symbols
  - Blatt-Biedenharn Z and Z-bar coefficients
- Along the way, we found bugs in the equivalent routines in Fudge (Python) and CoH (C++) and made fixes
  - (T.K., I still have to get you yours)
- We couldn't figure out TALYS implementation and suspect it is wrong

Proof the darn thing works



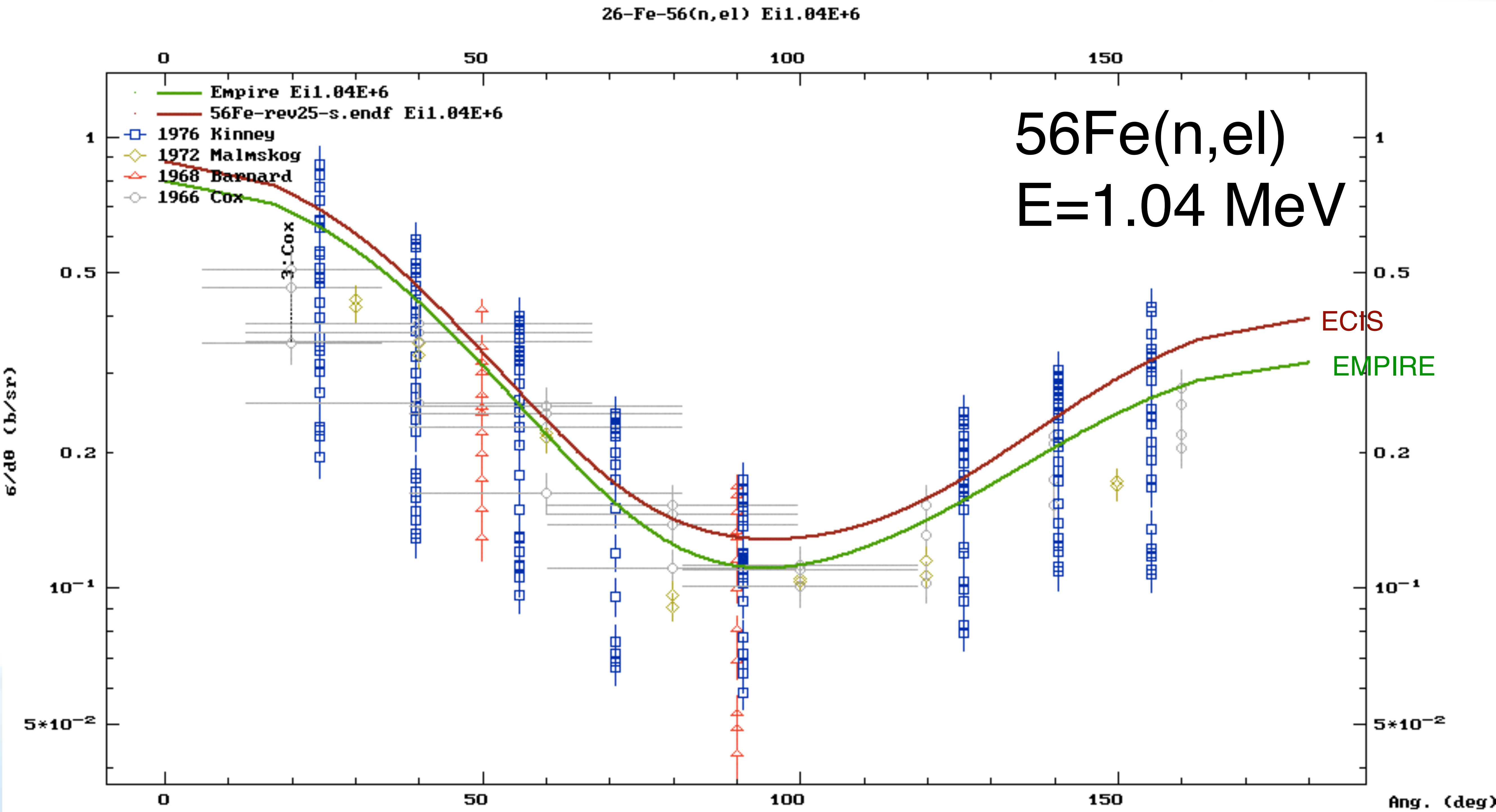
BROOKHAVEN NATIONAL LABORATORY

Proof the darn thing works

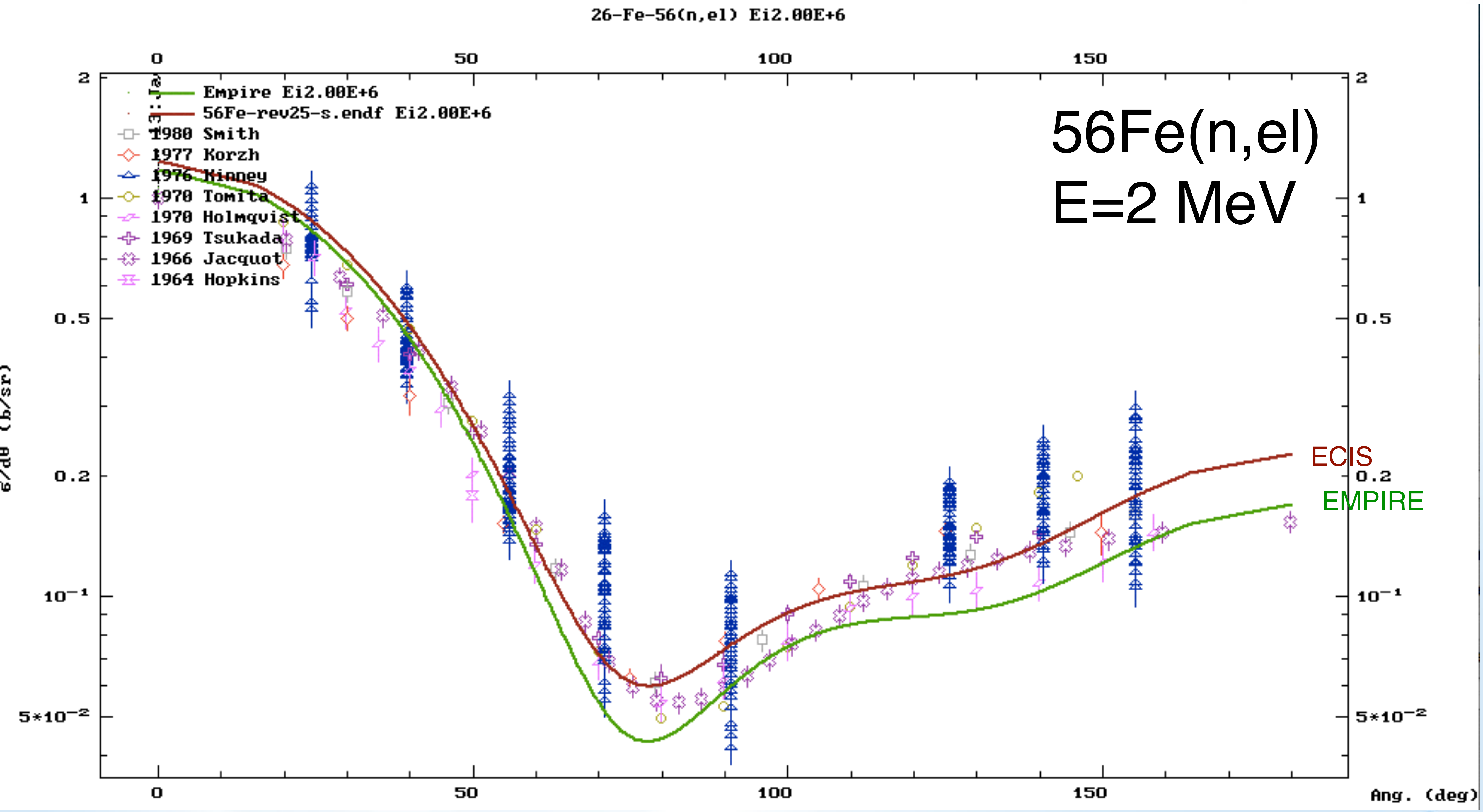


BROOKHAVEN
NATIONAL LABORATORY

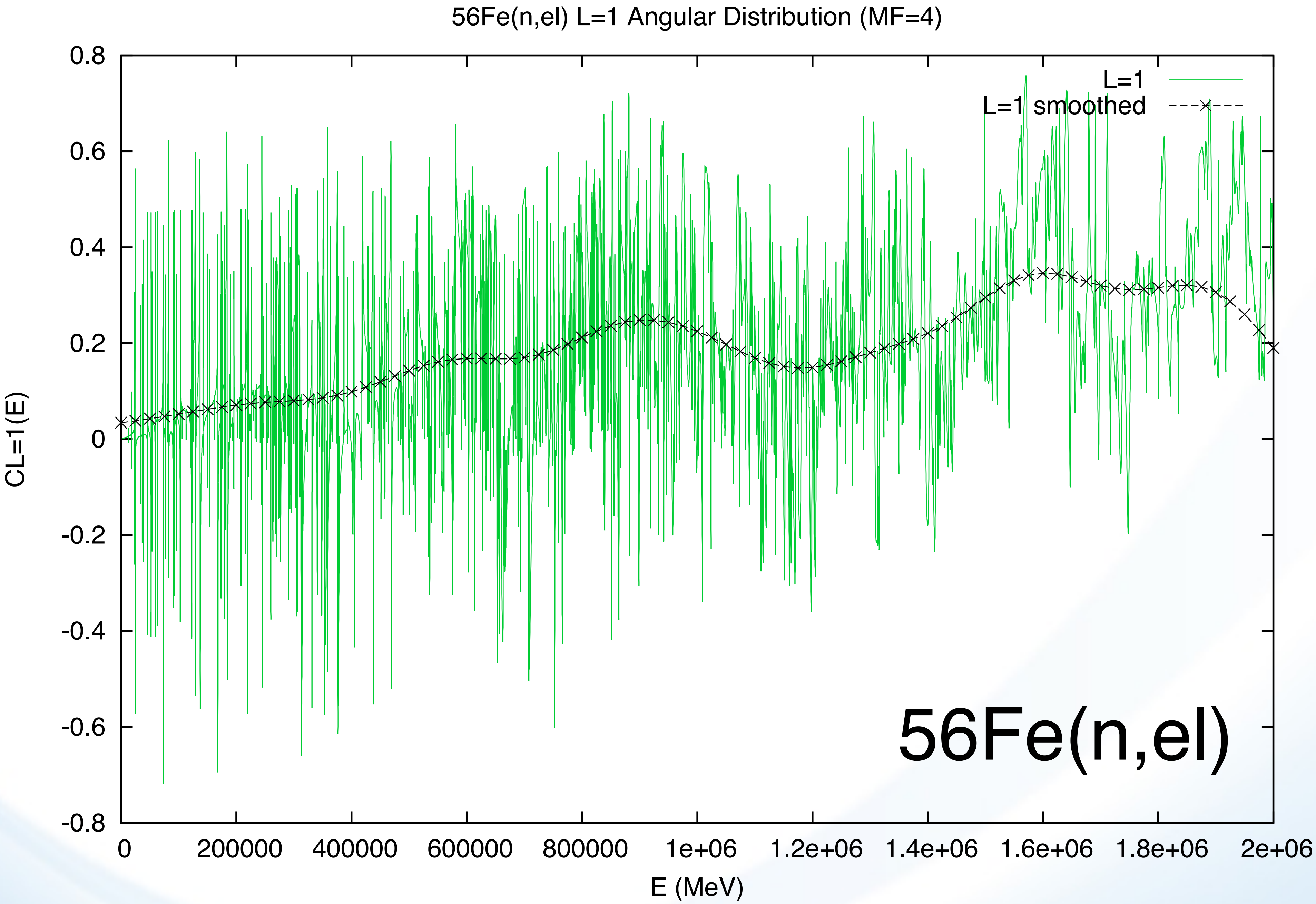
Proof the darn thing works



Proof the darn thing works



This wasn't enough testing, so we compared EMPIRE to RRR data



- HF only works on smooth cross section, so we smoothed the angular distributions from Leal's 56Fe resonances

How did we smooth?

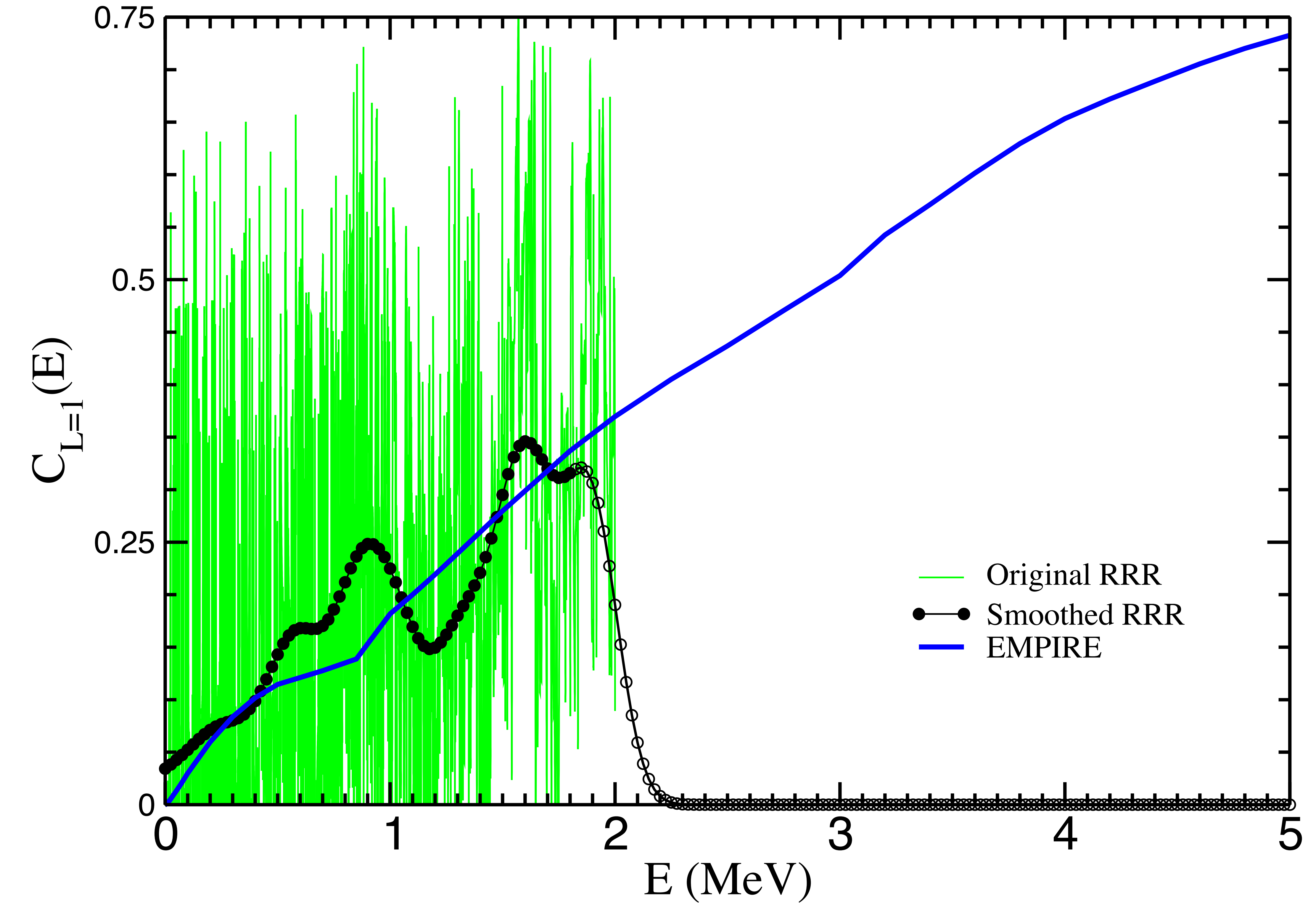
- Assume we can smooth cross section & Legendre moments of angular PDF:

$$\left\langle \frac{d\sigma(E)}{d\Omega} \right\rangle \approx \frac{\langle \sigma(E) \rangle}{4\pi} \sum_{L=0}^{\infty} P_L(\mu) \langle C_L(E) \rangle$$

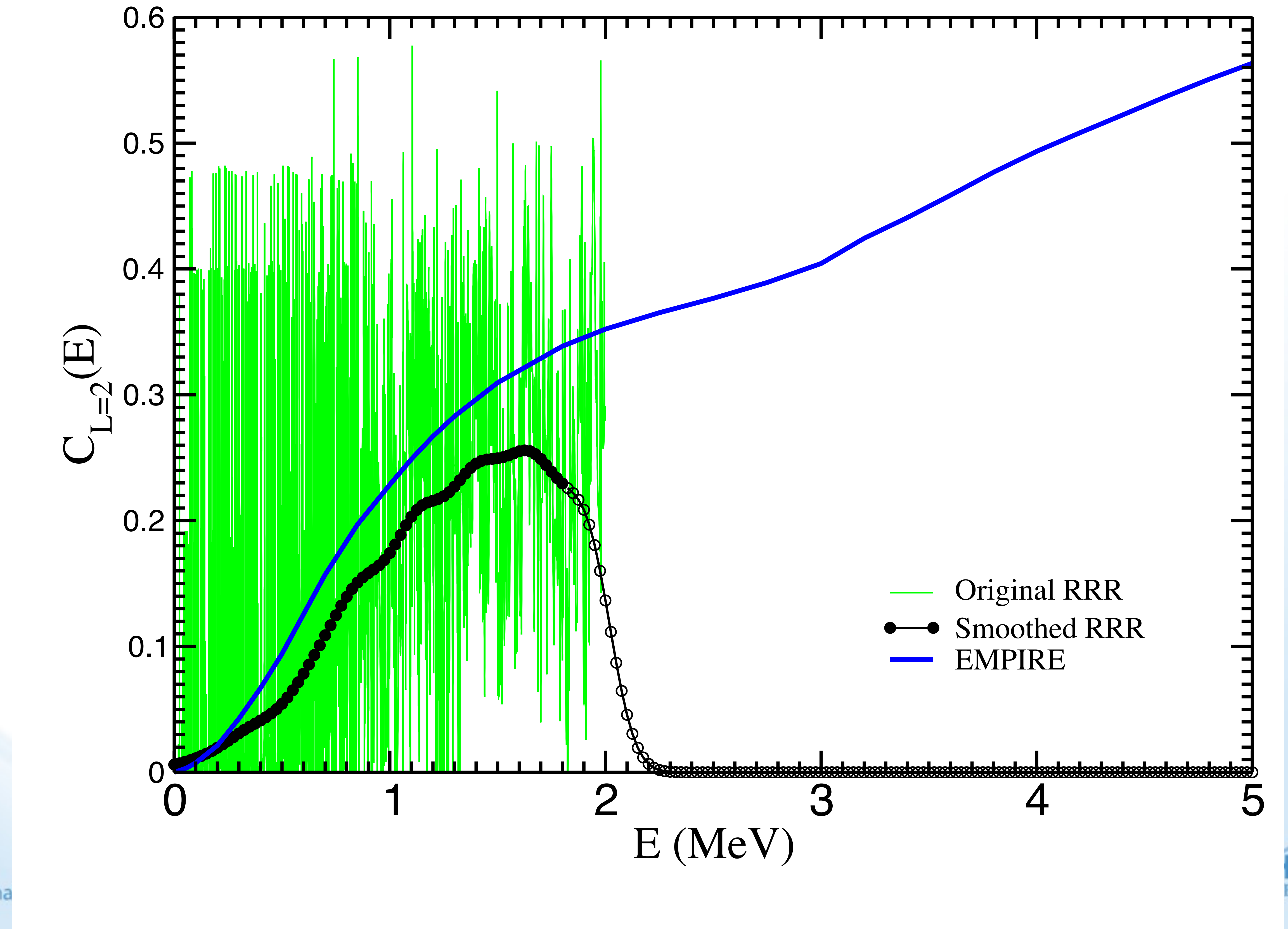
- where can use Lorenzian (or Gaussian it turns out)

$$\begin{aligned} \langle f(E) \rangle &= \int_{-\infty}^{\infty} dE' L(E,E') f(E') \\ &= f(E+iI) \end{aligned}$$

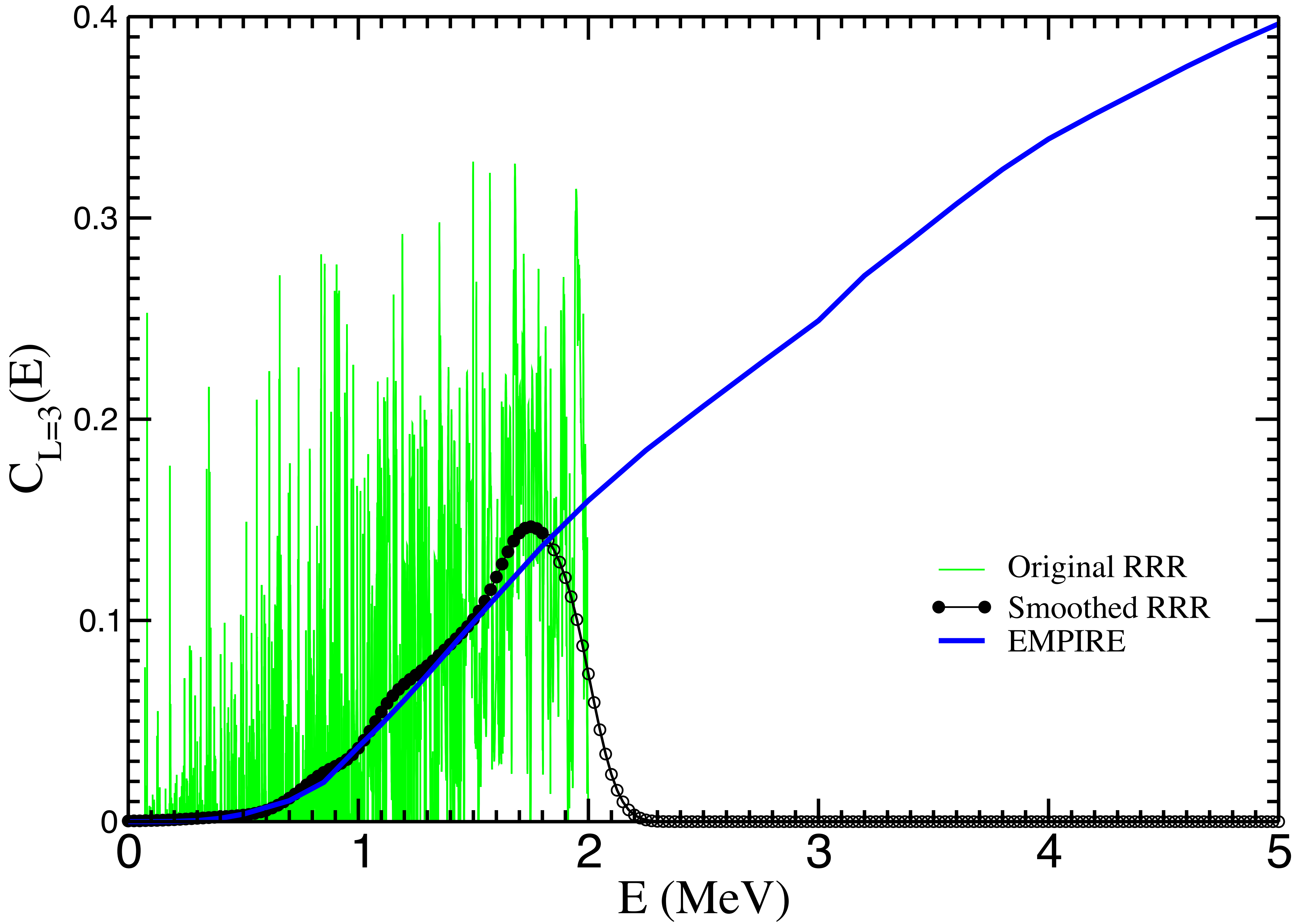
More proof the darn thing works (L=1)



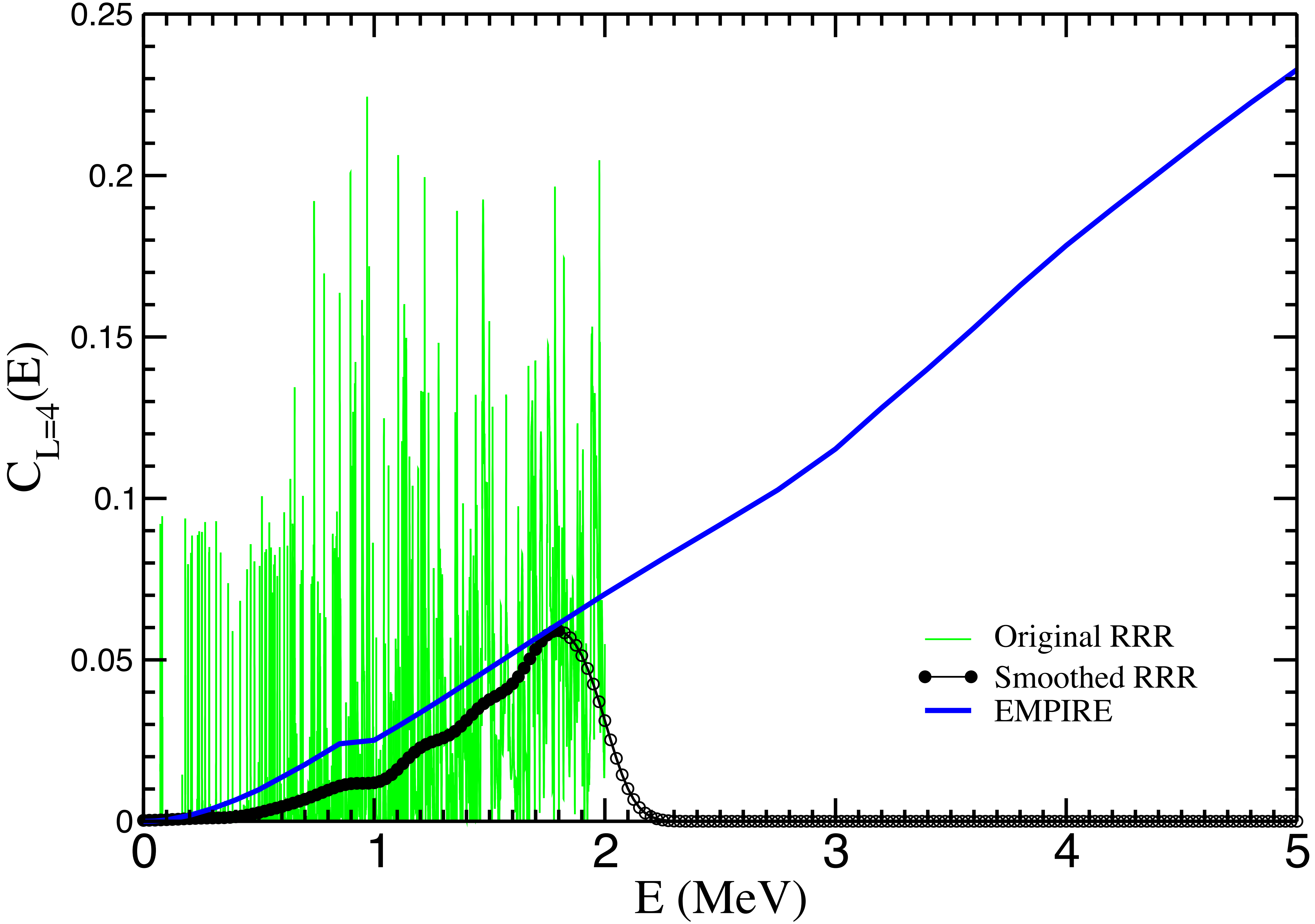
More proof the darn thing works (L=2)



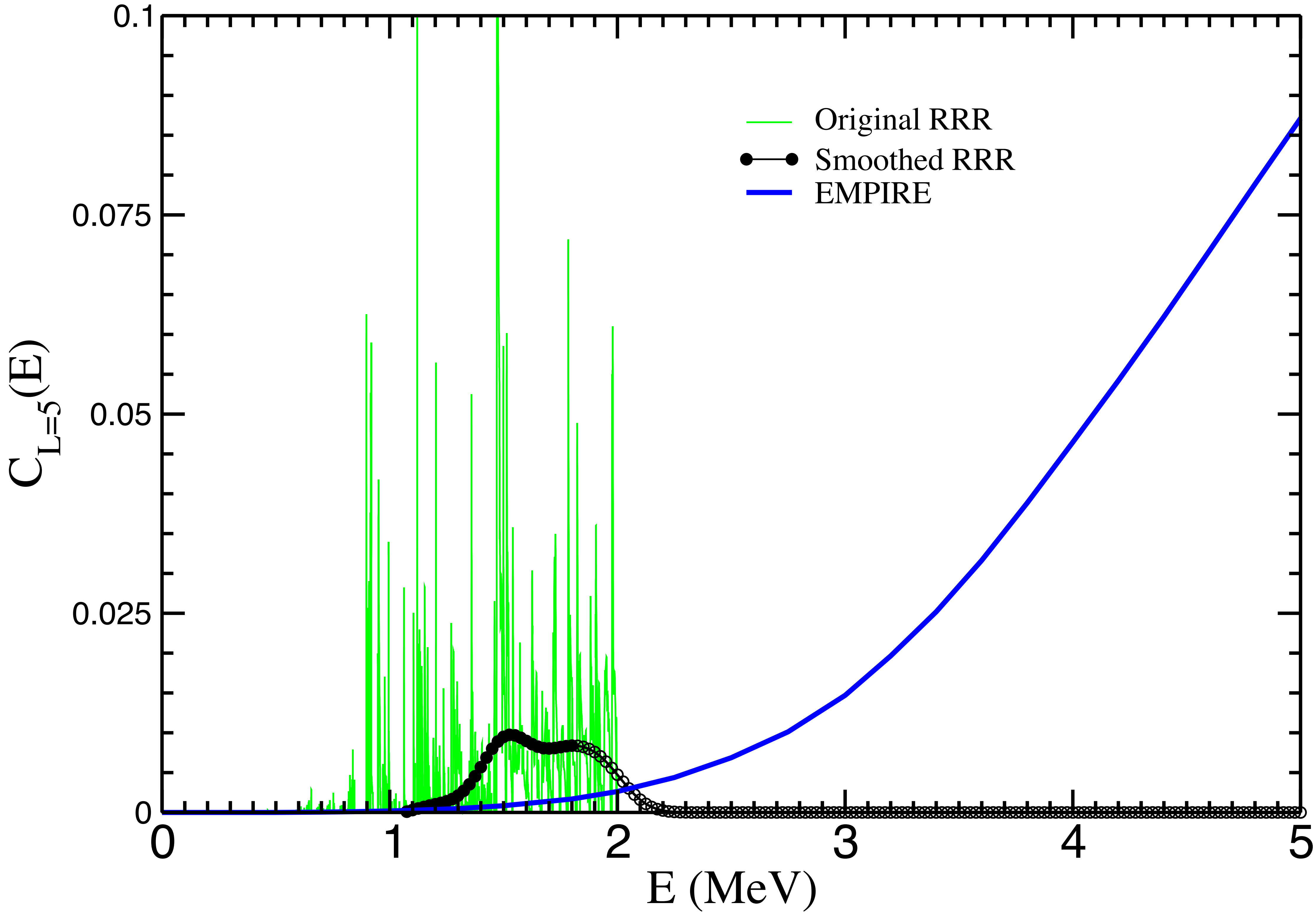
More proof the darn thing works (L=3)



More proof the darn thing works (L=4)



More proof the darn thing works (L=5)



More proof the darn thing works (L=6)