

Light meson form factors from exclusive measurements

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Exclusive Reactions Working Group

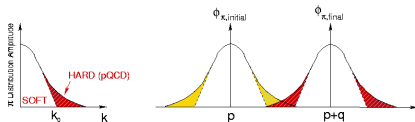
# Outline

- Charged Meson Form Factors
- JLab 12 GeV data
- EIC Measurement and Simulation

# Charged Meson Form Factors

- Simple  $q\bar{q}$  valence structure of mesons makes them an excellent testing ground
- Pion form factor,  $F_\pi$ , is the overlap integral -

$$F_\pi(Q^2) = \int \phi_\pi^*(p) \phi_\pi(p+q) dp$$

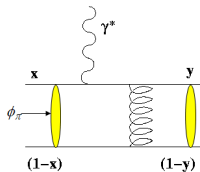


- Meson wave function can be split into  $\phi_\pi^{\text{soft}}$  ( $k < k_0$ ) and  $\phi_\pi^{\text{hard}}$ , the hard tail
  - Can treat  $\phi_\pi^{\text{hard}}$  in pQCD, cannot with  $\phi_\pi^{\text{soft}}$
- Study of  $Q^2$  dependence of form factor focuses on finding description of hard and soft contributions

# The Pion in pQCD 1/2

- At very large  $Q^2$ ,  $F_\pi$  can be calculated using pQCD via -

$$F_\pi(Q^2) = \frac{4_F \alpha_s(Q^2)}{Q^2} \left| \sum_{n=0}^{\infty} a_n \left( \log \left( \frac{Q^2}{\Lambda^2} \right) \right)^{-\gamma_n} \right|^2 \left[ 1 + O \left( \alpha_s(Q^2), \frac{m}{Q} \right) \right]$$



# The Pion in pQCD 2/2

- At asymptotically high  $Q^2$  ( $Q^2 \rightarrow \infty$ ), the pion distribution amplitude becomes -

$$\phi_\pi(x) \rightarrow \frac{3f_\pi}{\sqrt{n_c}} x(1-x)$$

- With  $f_\pi = 93 \text{ MeV}$ , the  $\pi^+ \rightarrow \mu^+ \nu$  decay constant
- $F_\pi$  takes the form -

$$Q^2 F_\pi \rightarrow 16\pi\alpha_s(Q^2) f_\pi^2$$

- This only relies on asymptotic freedom in QCD, i.e.  $(\partial\alpha_s/\partial\mu) < 0$  as  $\mu \rightarrow \infty$
- $Q^2 F_\pi$  should behave as  $\alpha_s(Q^2)$ , even for moderately large  $Q^2$
- Pion form factor seems to be the best tool for experimental study of the nature of the quark-gluon coupling constant renormalisation

Eqns - G.P. Lepage, S.J. Brodsky, PLB 87, p359, 1979 | Closing Statement - A.V. Efremov, A.V. Radyushkin PLB 94, p245, 1980

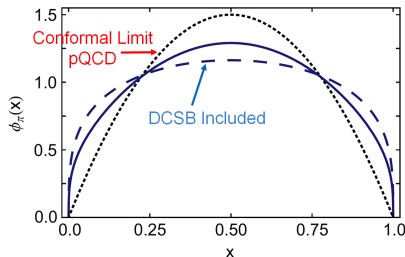
# Implications for Pion Structure 1/2

- Previous pQCD derivation used normalisation of  $F_\pi$  based on the conformal limit of the pion's twist 2-PDA -

$$\phi_\pi^{cl}(x) = 6x(1-x)$$

- Gives  $F_\pi$  that are “too small”
- Incorporating the DCSB effects yields Pion PDA -

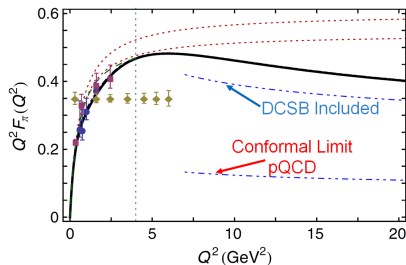
$$\phi_\pi(x) = \frac{8}{\pi} \sqrt{x(1-x)}$$



L. Chang, et al., PRL110(2013) 132001

# Implications for Pion Structure 2/2

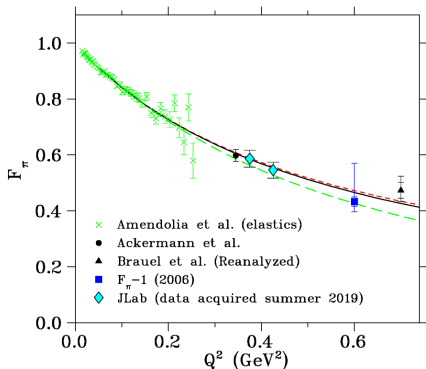
- Using this  $\phi_\pi(x)$  in the pQCD expression brings the  $F_\pi$  calculation much closer to the data
- Underestimates the full computation by  $\sim 15\%$  for  $Q^2 \geq 8 \text{ GeV}^2$



L. Chang, et al., PRL111(2013) 141802

# Measurement of $F_\pi$ - Low $Q^2$

- At low  $Q^2$ ,  $F_\pi$  can be measured model independently
  - High energy elastic  $\pi^-$  scattering from atomic electrons in  $H$
- CERN SPS used 300 GeV pions to measure  $F_\pi$  up to  $Q^2 = 0.25 \text{ GeV}^2$
- Used data to extract pion charge radius -  
 $r_\pi = 0.657 \pm 0.012 \text{ fm}$
- Maximum accessible  $Q^2$  approximately proportional to pion beam energy
  - $Q^2 = 1 \text{ GeV}^2$  requires 1 TeV pion beam (!)

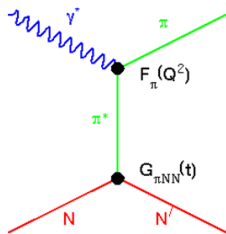


Amendolia, et al., NPB 277(1986) p168, P. Brauel, et al., ZPhysC (1979), p101, H. Ackerman, et al., NPB137 (1978), p294



# Measurement of $F_\pi$ - Larger $Q^2$

- To access higher  $Q^2$ , must measure  $F_\pi$  indirectly
  - Use the “pion cloud” of the proton via pion electroproduction  
 $p(e, e'\pi^+)n$
  - At small  $-t$ , the pion pole process dominates the longitudinal cross section,  $\sigma_L$
- In the Born term model,  $F_\pi^2$  appears as -
$$\frac{d\sigma_L}{dt} \propto \frac{-tQ^2}{(t - m_\pi^2)} g^2(t) F_\pi^2(Q^2, t)$$
- Drawbacks of this technique -
  - Isolating  $\sigma_L$  experimentally challenging
  - Theoretical uncertainty in  $F_\pi$  extraction  
→ Model dependent



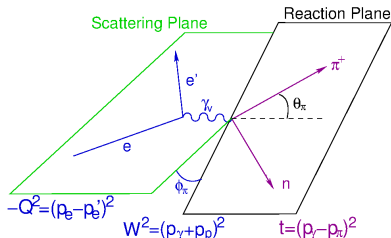
# Measurement of $F_\pi$ at JLab

- The physical cross section for the electroproduction process is given by -

$$2\pi \frac{d^2\sigma}{dtd\phi} = \epsilon \frac{d\sigma_L}{dt} + \frac{d\sigma_T}{dt} + \sqrt{2\epsilon(\epsilon+1)} \frac{d\sigma_{LT}}{dt} \cos\phi + \epsilon \frac{d\sigma_{TT}}{dt} \cos 2\phi,$$

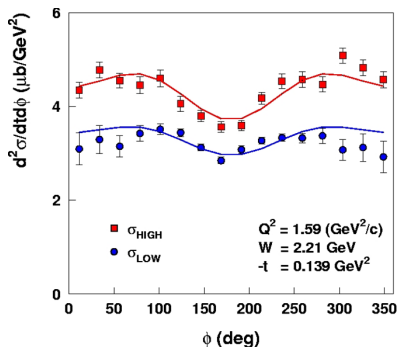
$$\epsilon = \left( 1 + 2 \frac{(E_e - E_{e'})^2 + Q^2}{Q^2} \tan^2 \frac{\theta_{e'}}{2} \right)^{-1}$$

- $\epsilon \rightarrow$  Virtual photon polarisation
- L-T separation required to isolate  $\sigma_L$  from  $\sigma_T$
- Need data at lowest  $-t$  possible,  $\sigma_L$  has maximum pole contribution here



# Measuring $\frac{d\sigma_L}{dt}$ at JLab

- Rosenbluth separation required to isolate  $\sigma_L$ 
  - Fix  $W$ ,  $Q^2$  and  $-t$ , measure cross section at two beam energies
  - Carry out simultaneous fit at two different  $\epsilon$  values to determine interference terms
- Careful control of point-to-point systematics crucial,  $1/\Delta\epsilon$  error amplification in  $\sigma_L$
- Spectrometer acceptance, kinematics and efficiencies must all be carefully studied and understood



T. Horn, et al., PRL 97(2006) 192001

# Extracting $F_\pi$ at JLab

- Only reliable approach for extracting  $F_\pi$  from  $\sigma_L$  is to use a model that incorporates the  $\pi^+$  production mechanism and the spectator nucleon
- JLab  $F_\pi$  experiments use the VGL Regge model
  - Reliably describes  $\sigma_L$  across a wide kinematic domain
- Ideally, want a better understanding of the model dependence of the result
- There has been considerable recent interest
  - T.K. Choi, K.J. Kong, B.G. Yu, arXiv 1508.00969
  - T. Vrancx, J. Ryckebusch, PRC 89(2014)025203
  - M.M. Kaskulov, U. Mosel, PRC 81(2010)045202
  - S.V. Goloskokov, P.Kroll, EPJC 65(2010)137
- We aim to publish our experimentally measured cross section data so that updated values of  $F_\pi$  can be extracted as the models improve

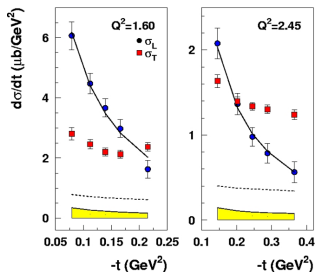
VGL - Vanderhaeghen-Guidal-Laget Model - Vanderhaeghen, Guidal, Laget, PRC 57(1998) 1454

# $F_\pi(Q^2)$ from JLab Data

VGL model incorporates  $\pi^+$  production mechanism and spectator neutron effects

- Feynman propagator -  $\frac{1}{t-m_\pi^2}$  replaced by  $\pi$  and  $\rho$  Regge propagators
- Represents the exchange of a **series** of particles, compared to a **single** particle
- Free parameters -  $\Lambda_\pi, \Lambda_\rho$  - Trajectory cutoff parameters
- **At small  $-t$ ,  $\sigma_L$  only sensitive to  $F_\pi$**

$$F_\pi = \frac{1}{1 + Q^2/\Lambda_\pi^2}$$



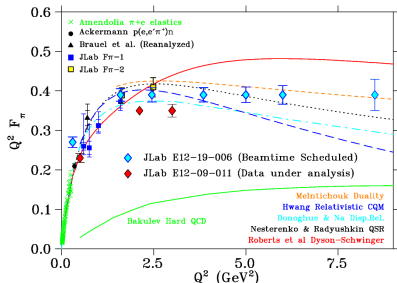
Error bars indicate statistical and random (pt-pt) systematic uncertainties in quadrature. Yellow band indicates the correlated (scale) and partly correlated (t-corr) systematic uncertainties.

$\Lambda_\pi^2 = 0.513, 0.491 \text{ GeV}^2, \Lambda_\rho^2 = 1.7 \text{ GeV}^2$

T. Horn, et al., PRL 97(2006) 192001

# Current and Projected JLab $F_\pi$ Data

- JLab 12 GeV program includes measurements of  $F_\pi$  to higher  $Q^2$
- No other facility worldwide can perform this measurement
- New overlap points at  $Q^2 = 1.6, 2.45$  will be closer to pole to constrain  $-t_{min}$  dependence
- Check  $\pi^+/\pi^-$  ratios at modest  $Q^2$  to test  $t$ -channel dominance



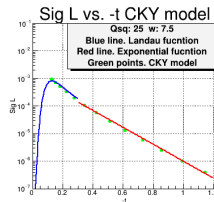
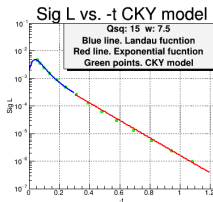
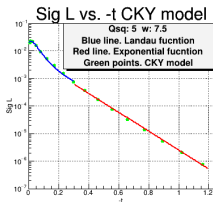
- New low  $Q^2$  point will provide best comparison of the electroproduction extraction of  $F_\pi$  vs elastic  $\pi + e$  data

# DEMP Studies at the EIC

- Measurements of the  $p(e, e'\pi^+n)$  reaction at the EIC have the potential to extend the  $Q^2$  reach of  $F_\pi$  measurements even further
- A challenging measurement however
  - Need good identification of  $p(e, e'\pi^+n)$  triple coincidences
  - Conventional L-T separation not possible  $\rightarrow$  would need lower than feasible proton energies to access low  $\epsilon$
- Utilise new EIC software framework to assess the feasibility of the study with updated design parameters
  - Feed in events generated from a DEMP event generator

# DEMP Event Generator

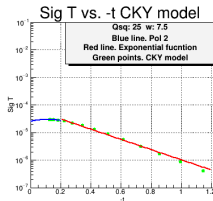
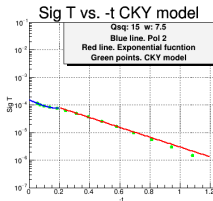
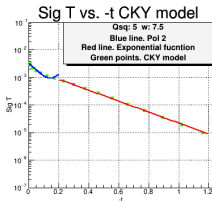
- Want to examine **exclusive** reactions
  - $p(e, e'\pi^+n)$  **exclusive reaction** is reaction of interest
    - $\rightarrow p(e, e'\pi^+)X$  SIDIS events are background
- Generator uses Regge-based  $p(e, e'\pi^+)n$  model from T.K. Choi, K.J. Kong and B.G. Yu (CKY) - arXiv 1508.00969
  - MC event generator created by parametrising CKY  $\sigma_L, \sigma_T$  for  $5 < Q^2 < 35, 2 < W < 10, 0 < -t < 1.2$





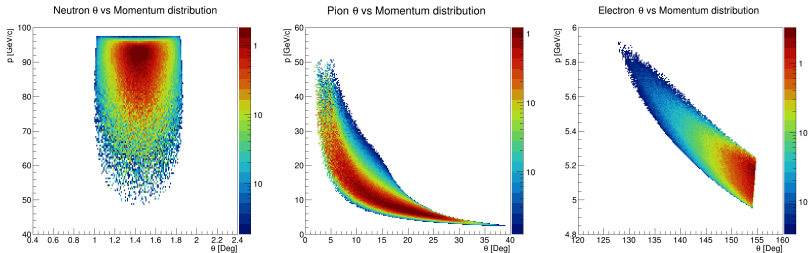
# DEMP Event Generator

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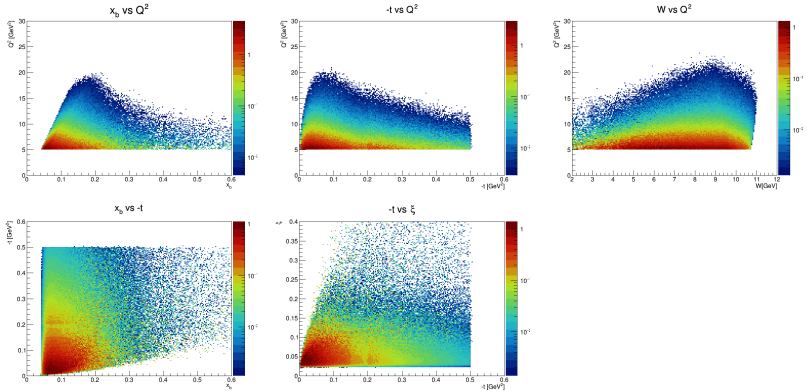
# DEMP Acceptance for $-t < 0.5 \text{ GeV}^2$

- $5(e^-)$  on  $100(p)$   $\text{GeV}$  collisions,  $25 \text{ mrad}$  crossing angle
- Events weighted by cross section
- No smearing



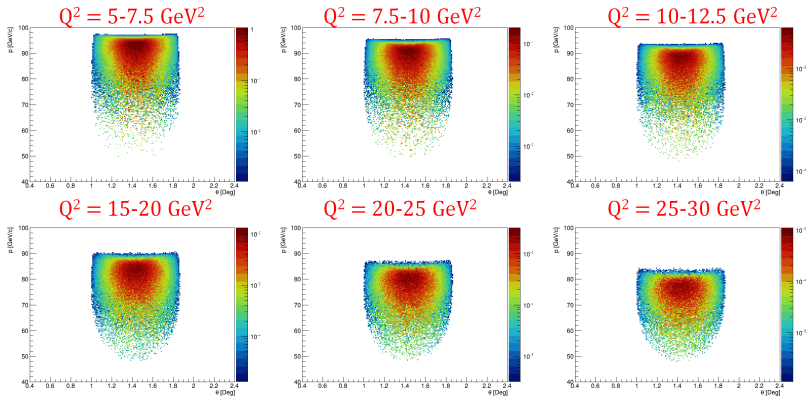
- Neutrons within  $0.2^\circ$  of outgoing proton beam, offset is due to the crossing angle ( $25 \text{ mrad} \approx 1.4^\circ$ )

# DEMP Kinematic Coverage - 5 on 100

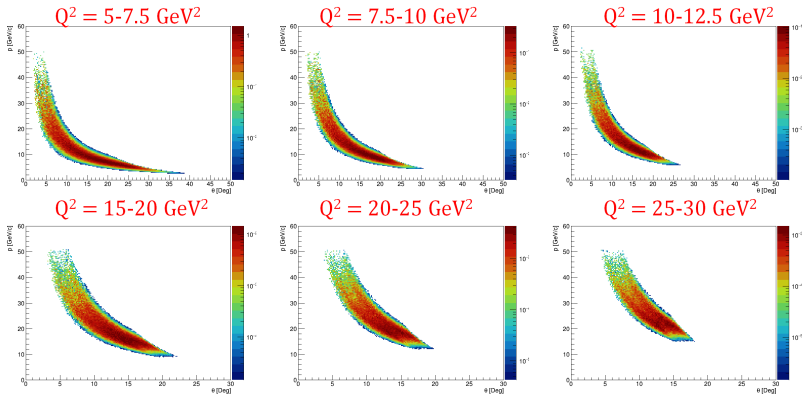


$\xi$  = skewness

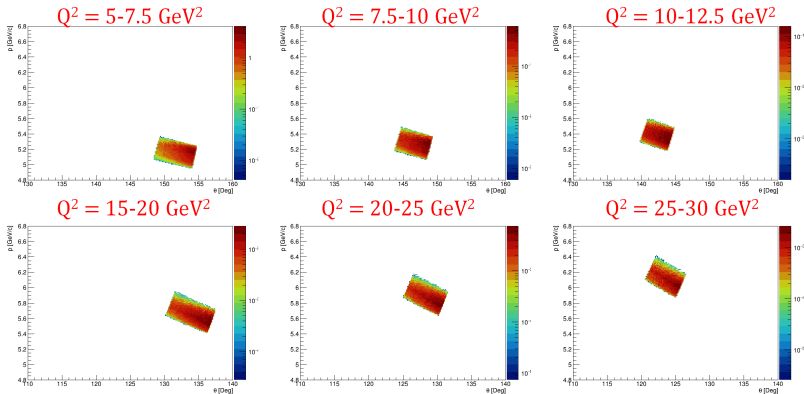
# Neutron Acceptance Across $Q^2$ - 5 on 100



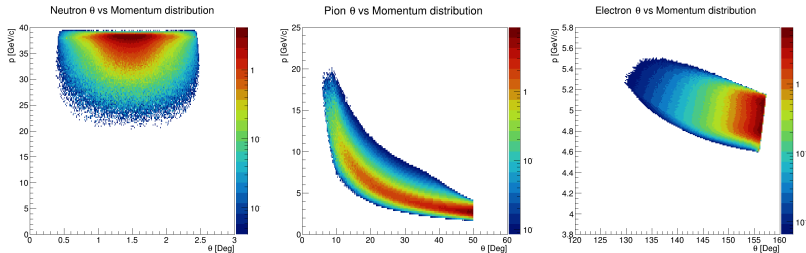
# Pion Acceptance Across $Q^2$ - 5 on 100



# Electron Acceptance Across $Q^2$ - 5 on 100

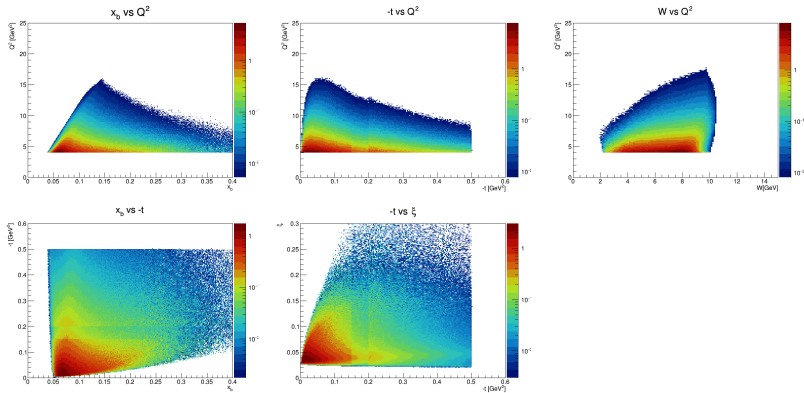


# DEMP Acceptance - 5 on 41



- $Q^2 > 4 \text{ GeV}^2$  cut applied, low  $Q^2$  events dominate otherwise
  - High weight on low  $Q^2$  events
- Neutron distribution broader in  $\theta$ 
  - May miss ZDC? Need to run full simulation and see

# DEMP Kinematic Coverage - 5 on 41

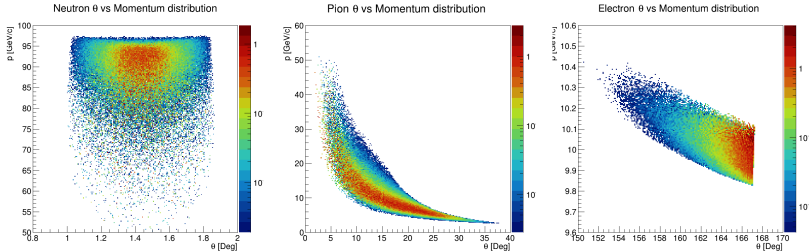


$\xi$  = skewness

- $Q^2 > 4 \text{ GeV}^2$  cut applied
- Similar kinematic coverage to 5 on 100

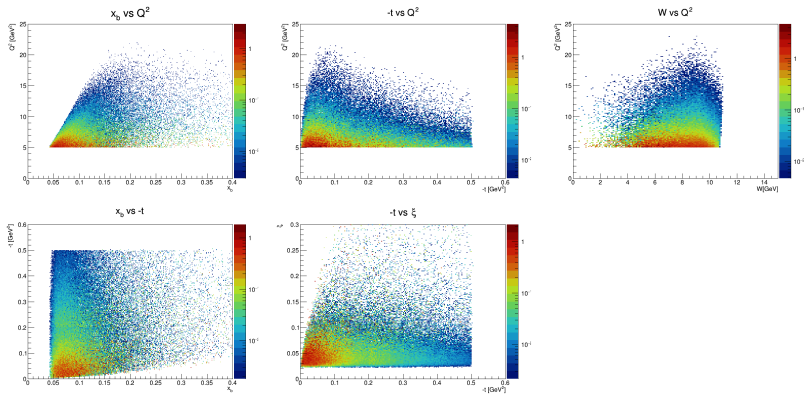


# DEMP Acceptance - 10 on 100



- Distributions broadly similar to 5 on 100
- Fewer events
- Electrons at higher momentum and wider angle

# DEMP Kinematic Coverage - 10 on 100



$\xi = \text{skewness}$

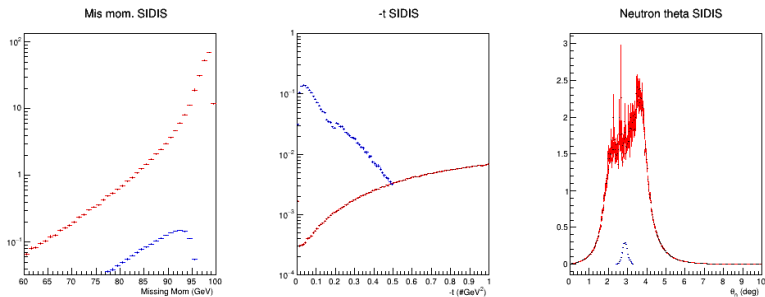
- Similar to other energies, events shifted to higher  $W$

# Background Events

- Want to isolate a clean sample of  $p(e, e'\pi^+n)$  events by detecting the neutron
- **SIDIS  $p(e, e'\pi^+)X$  events a large source of background**
  - Utilised the EIC SIDIS event generator by Duke University to generate SIDIS background events
  - [/work/eic/evgen\\_DUKE/e5p100 on the JLab farm](#)
- Both the DEMP and SIDIS generators produce LUND format files that can be interpreted within the EIC software container

# DEMP vs SIDIS Kinematics

- DEMP events are  $e'\pi^+n$  triple coincidence
- SIDIS events are  $e'\pi^+$  double coincidence,  $p_{miss}$  reconstructed
  - $p_{miss} = |\underline{p}_e + \underline{p}_p - \underline{p}_{e'} - \underline{p}_{\pi^+}|$



- SIDIS events overwhelm foreground exclusive events, *but* distributed over wider momentum range and at larger  $-t$
- Note - Plots from earlier study with smearing included

# Isolating $\sigma_L$ from $\sigma_T$ in an e-p Collider

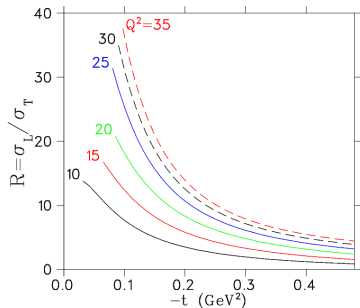
- For a collider -

$$\epsilon = \frac{2(1-y)}{1+(1-y)^2} \quad \text{with} \quad y = \frac{Q^2}{x(s_{tot} - M_N^2)}$$

- $y$  is the fractional energy loss
- **Systematic uncertainties in  $\sigma_L$  magnified by  $1/\Delta\epsilon$** 
  - Ideally,  $\Delta\epsilon > 0.2$
- To access  $\epsilon < 0.8$  with a collider, need  $y > 0.5$ 
  - Only accessible at small  $s_{tot}$
  - **Requires low proton energies ( $\sim 10$  GeV), luminosity too low**
- Conventional L-T separation not practical, need another way to determine  $\sigma_L$

## $\sigma_L$ Isolation with a Model

- QCD scaling predicts  $\sigma_L \propto Q^{-6}$   
and  $\sigma_T \propto Q^{-8}$
- At the high  $Q^2$  and  $W$  accessible at the EIC, phenomenological models predict  $\sigma_L \gg \sigma_T$  at small  $-t$
- Can attempt to extract  $\sigma_L$  by using a model to isolate dominant  $d\sigma_L/dt$  from measured  $d\sigma_{UNS}/dt$
- **Critical to confirm the validity of the model used!**



Predictions are assuming  $\epsilon > 0.9995$  with the kinematic ranges seen earlier

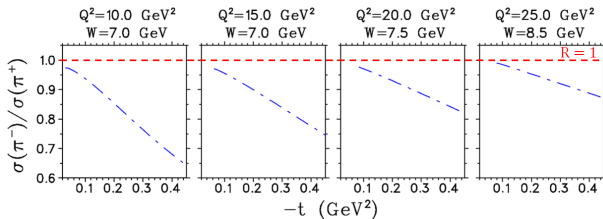
T.Vrancx, J. Ryckebusch, PRC 89(2014)025203

# Model Validation via $\pi^-/\pi^+$ ratios

- Measure exclusive  ${}^2H(e, e'\pi^+n)n$  and  ${}^2H(e, e'\pi^-p)p$  in same kinematics as  $p(e, e'\pi^+n)$
- $\pi$   $t$ -channel diagram is purely isovector  $\rightarrow$  G-Parity conserved

$$R = \frac{\sigma[n(e, e'\pi^-p)]}{\sigma[p(e, e'\pi^+n)]} = \frac{|A_V - A_S|^2}{|A_V + A_S|^2}$$

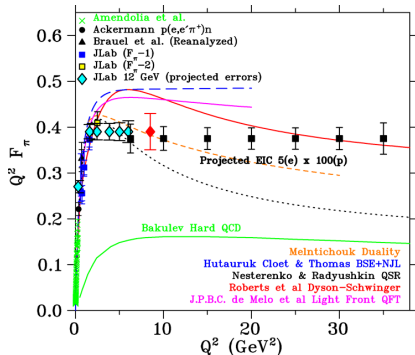
- $R$  will be diluted if  $\sigma_T$  not small or if there are significant non-pole contributions to  $\sigma_L$
- Compare  $R$  to model expectations



# EIC Kinematic Reach

## Assumptions

- $5(e^-)$  on  $100(p)$
- $\int \mathcal{L} = 20 \text{ fb}^{-1} \text{ yr}^{-1}$
- Clean identification of  $\rho(e, e' \pi^+ n)$
- Syst.Unc:  
2.5% pt-pt, 12% scale
- $R = \sigma_L / \sigma_T = 0.013 - 0.14$   
at lowest  $-t$  from VR model
- $\delta R = R$  Syst.Unc in model  
subtraction to isolate  $\sigma_L$
- $\pi$  pole dominance at small  
 $-t$  confirmed in  $^2H \pi^+ / \pi^-$   
ratios



- Results look promising, but  
need further studies and  
further energy combinations



# Outlook and Future Plans

- Higher  $Q^2$  data on  $F_\pi$  vital for our understanding of hadronic physics
  - Pion properties connected to DCSB
  - $F_\pi$  is our best hope of observing QCD's transition from confinement-dominated physics to perturbative QCD
- Measurement of  $F_\pi$  at the EIC will be challenging
  - Conventional L-T separation not possible
  - Should be possible to use a model to separate  $\sigma_L$  from the unseparated cross section
  - Can use  $\pi^-/\pi^+$  ratio in  $e + d$  collisions to validate model
  - Process files through full geant simulation, process other beam energy combinations, contribute to yellow report
- Building on our current event generator, may examine possibility of a Kaon event generator based on VR model
  - Could attempt to measure  $F_K$  in a similar manner
  - Further challenges to address for such a study!

Thanks for listening, any questions?



University  
of Regina



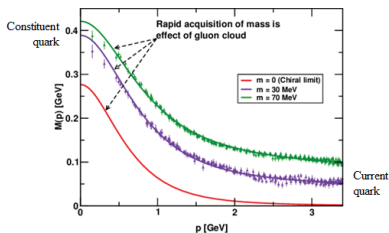
**NSERC**  
**CRSNG**

S.J.D. Kay, G.M. Huber, Z. Ahmed, Daniele Binosi, Huey-Wen Lin, Timothy Hobbs,  
Arun Tadepalli, Rachel Montgomery, Paul Reimer, David Richards, Rik Yoshida, Craig Roberts, Thia Keppel,  
John Arrington, Lei Chang, Ian L. Pegg, Jorge Segovia, Carlos Ayerbe Gayoso, Wenliang Li, Yulia Furletova,  
Dmitry Romanov, Markus Diefenthaler, Richard Trotta, Tanja Horn, Rolf Ent, Tobias Frederico

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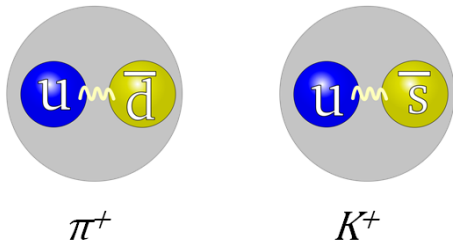
# Recent Theoretical Advances

- Have a much better understanding of how **D**ynamical **C**hiral **S**ymmetry **B**reaking (**DCSB**) generates hadron mass
- Evolution of the current-quark of pQCD into constituent quark was observed as its momentum becomes smaller
- The constituent quark mass arises from a cloud of low momentum gluons attaching themselves to the current quark
- Non-perturbative effect that generates a quark mass from nothing, occurs in even in the chiral ( $m = 0$ ) limit



M.S. Bhagwat, et al., PRC 68(2003) 015203, L. Chang, et al., Chin.J.Phys. 49(2011)955

## A 2<sup>nd</sup> Test Case - The Charged Kaon



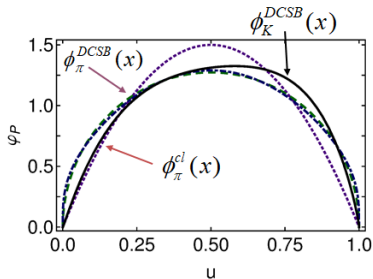
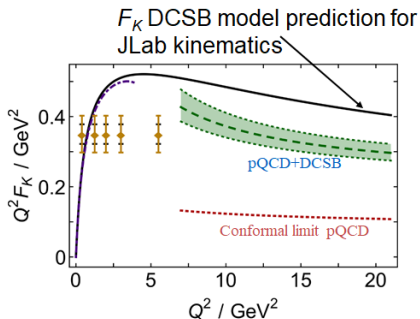
- In the hard scattering limit, pCQD predicts  $F_\pi$  and  $F_K$  will behave similarly -

$$\frac{F_K(Q^2)}{F_\pi(Q^2)} \rightarrow \frac{f_K^2}{f_\pi^2}$$

- Should compare the magnitude and  $Q^2$  dependences of both form factors

# Effects of DCSB on $K^+$ Properties

- $K^+$  PDA is also broad, concave and asymmetric
- Heavier  $s$  quark carries more bound state momentum than the  $u$  quark, shift is less than one might expect based on the difference in current quark masses.



C. Shi, et al., PRD 92 (2015) 014035, F. Guo, et al., PRD 96(2017) 034024 (Full calculation)

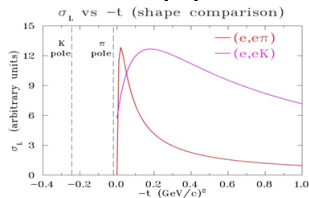
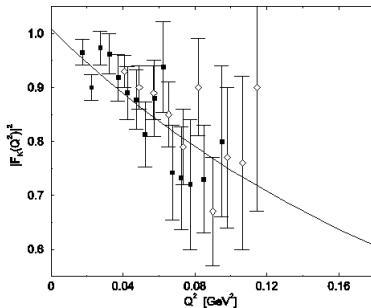
# $F_K$ Measurement at JLab

- Similar to  $F_\pi$ , elastic  $K^+$  scattering from electrons used to determine  $F_K$  at low  $Q^2$
- Can “kaon cloud” of the proton be used in the same way as the pion to extract  $F_K$  from electroproduction?
- Kaon pole further from kinematically allowed region

$$\frac{d\sigma_L}{dt} \propto \frac{-tQ^2}{(t - m_K^2)} g_K^2(T) F_K^2(Q^2, t)$$

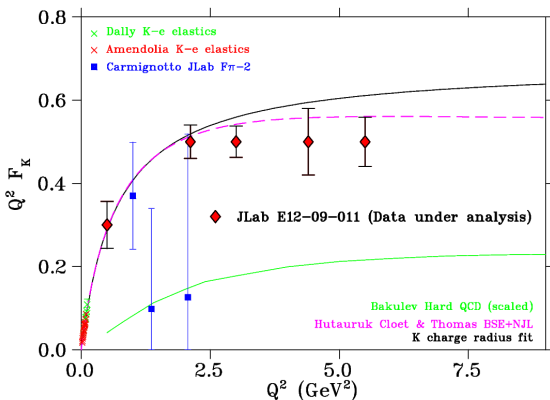
- Issues are being explored and tested in JLab E12-09-011

Amendolia, et al., PLB178(1986)435



# $F_K$ Measurement at JLab - Projections

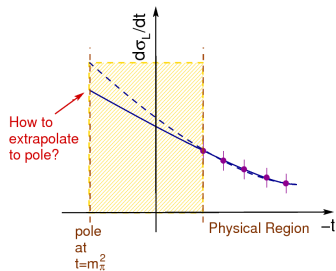
- Points with projected errors shown below
- Data has all been acquired and analysis is in progress
- $y$  positioning of points arbitrary



# Chew-Low Method to determine $F_\pi$

- $p(e, e'\pi^+)n$  data obtained away from  $t = m_\pi^2$  pole
- “Chew Low” extrapolation method - must know analytical dependence of  $d\sigma_L/dt$  in unphysical region
- Extrapolation method last used in 1972 by Devenish and Lyth
- Very large systematic uncertainties
- Failed to produce a reliable result
- Different polynomial fits equally likely in physical region
  - Form factor values divergent when extrapolated

We do not use the Chew-Low method



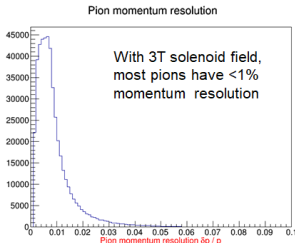
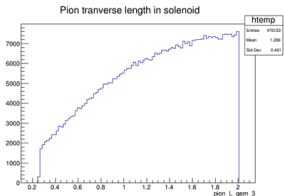
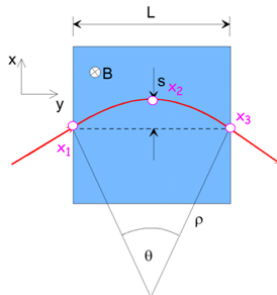


# Old Momentum Resolution Estimate

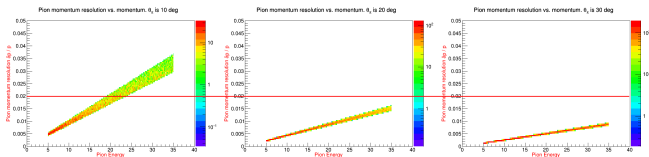
- Intrinsic momentum resolution from  $n$  equidistant measurements

$$\frac{\delta p}{p} = \frac{p}{0.3 B L^2} \frac{\sigma_{r\phi}}{\sqrt{n+4}}$$

- R. L. Gluckstern, NIM24(1963), p381
- $B$  = central field (T),  $\sigma_{r\phi}$  = position resolution (m),  $L$  = length of transverse path through field (m),  $N$  = number of measurements
- Assumed  $n = 5$ ,  $B = 3$  T

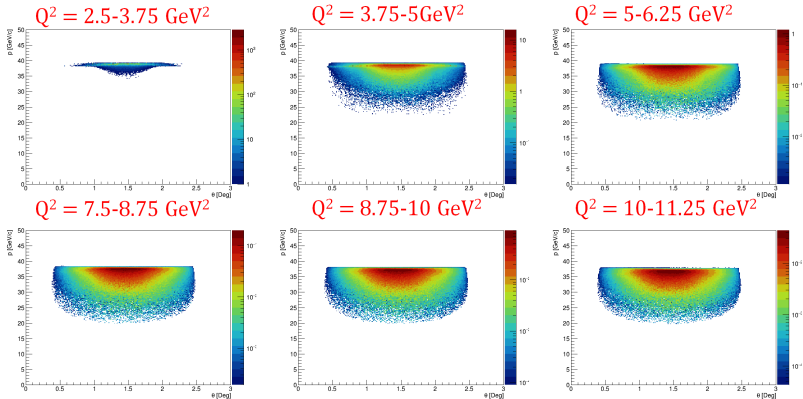


# Old $\pi$ Momentum Resolution with 3 T Solenoid

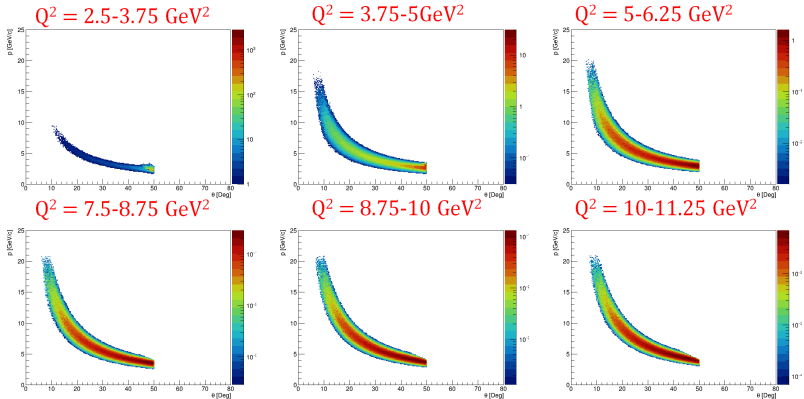


- Pion momentum resolution suffers when the pion is emitted at a shallow angle to the solenoidal field
- To simplify the MC study, assumed  $\delta p/p = 2\%$  for all angles, for both pion and electron
  - Typical  $\pi^+$  angles:  $7 - 30^\circ$
  - Typical  $e^-$  angles:  $25 - 45^\circ$

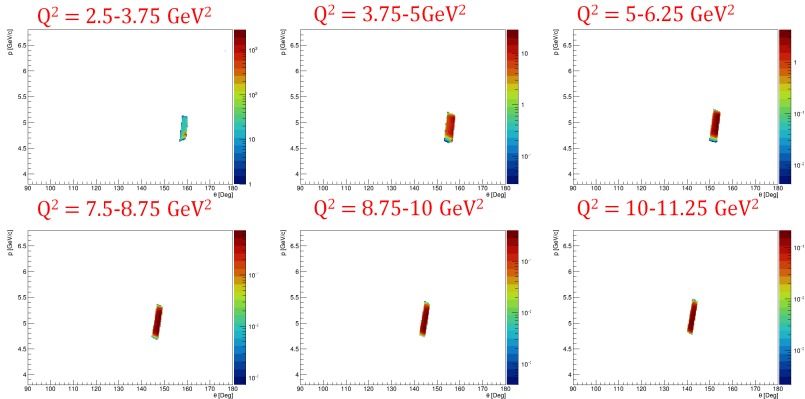
# Neutron Acceptance Across $Q^2$ - 5 on 41



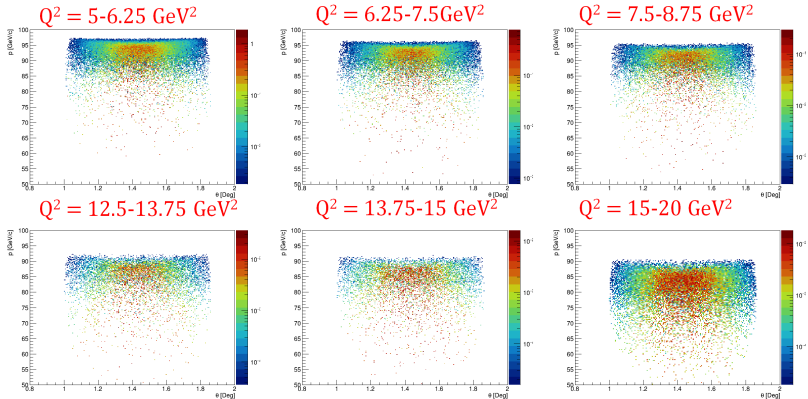
# Pion Acceptance Across $Q^2$ - 5 on 41



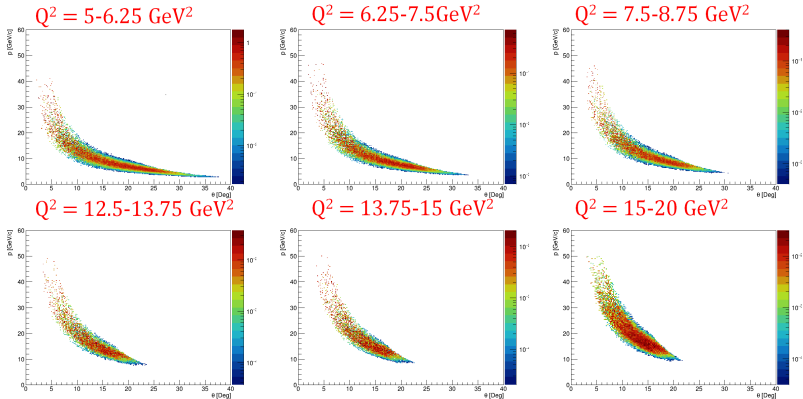
# Electron Acceptance Across $Q^2$ - 5 on 41



# Neutron Acceptance Across $Q^2$ - 10 on 100



# Pion Acceptance Across $Q^2$ - 10 on 100



# Electron Acceptance Across $Q^2$ - 10 on 100

