Semi-inclusive heavy quark production at EIC

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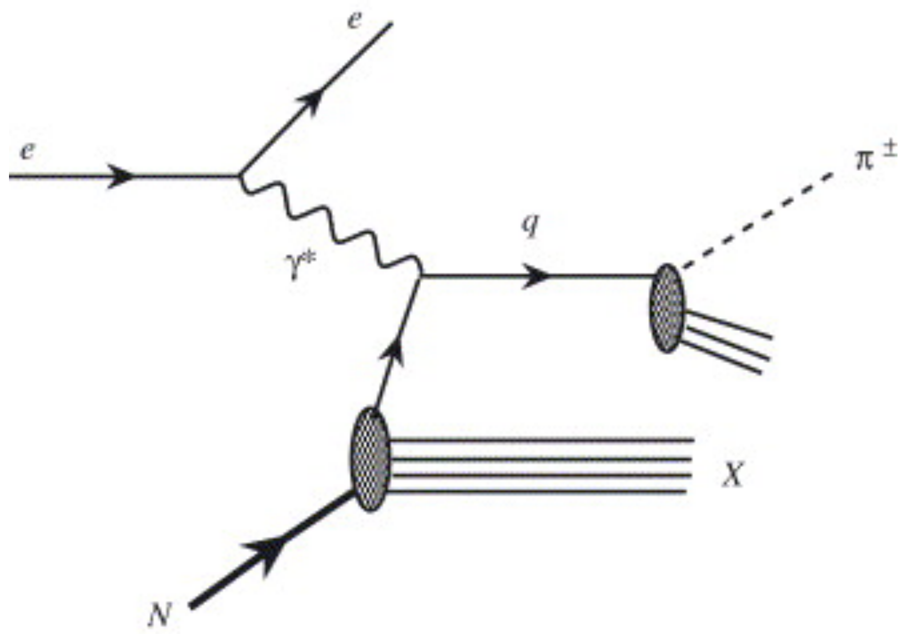
university of groningen

Typical TMD processes

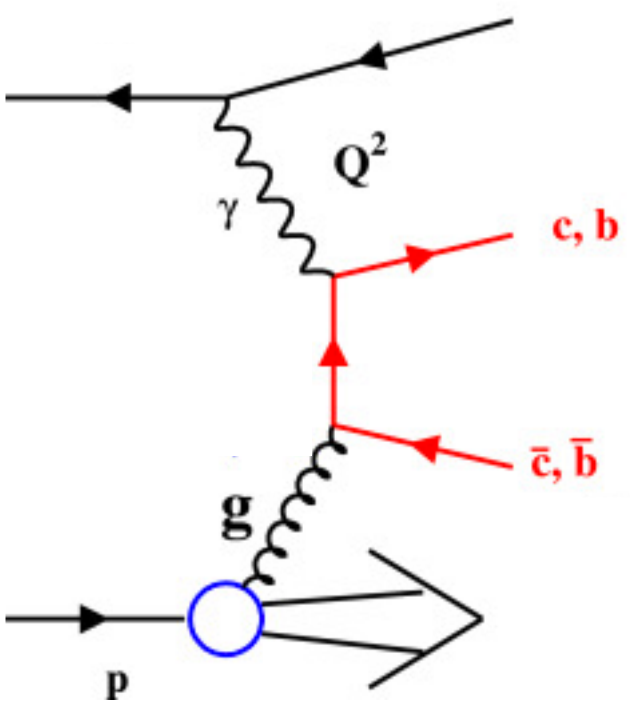
Semi-inclusive DIS is a process sensitive to the transverse momentum of quarks

$$e\,p \to e'\,h\,X$$

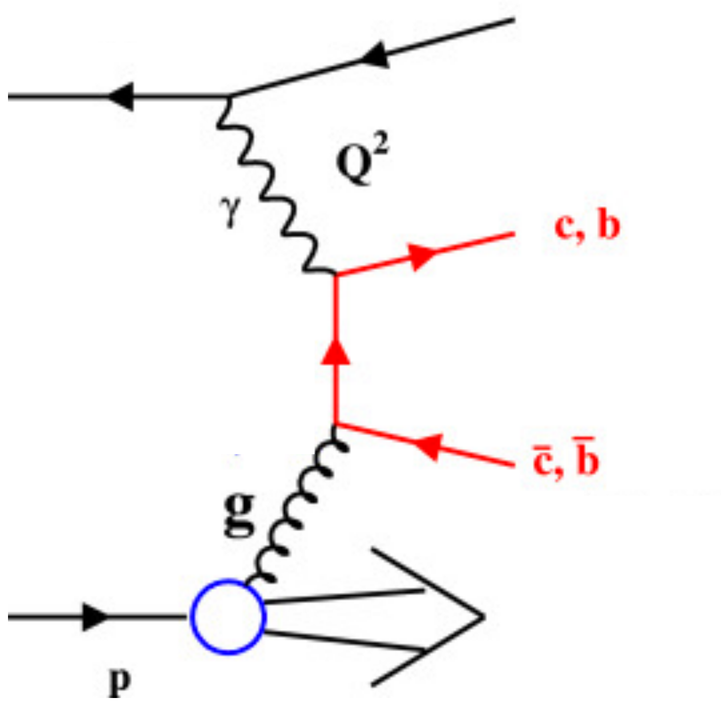


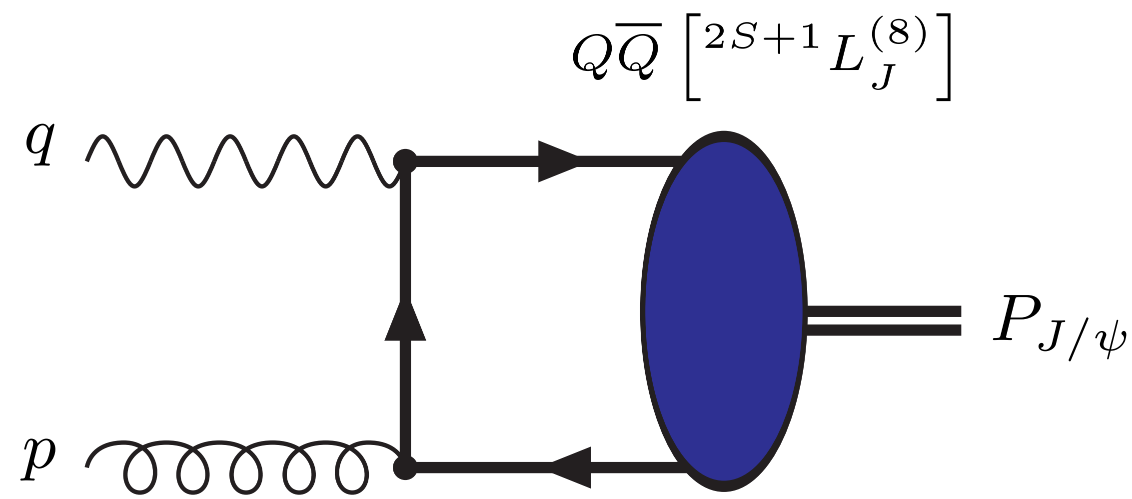
Heavy quark pair production is sensitive to transverse momentum of gluons

$$ep \to e'Q\bar{Q}X$$



Probing gluon transverse momentum at EIC





$$ep \to e'Q\bar{Q}X \qquad\qquad e\,p \to e'\,\mathcal{Q}\,X$$

Open heavy quark pair production and quarkonium production are processes at EIC that are sensitive to the transverse momentum of gluons

This aspect was not studied at HERA, but is possible at EIC

Gluons TMDs

The gluon correlator:

$$\Gamma_g^{\mu\nu[\mathcal{U},\mathcal{U}']}(x,k_T) \equiv \mathrm{F.T.}\langle P|\mathrm{Tr}_c\left[F^{+\nu}(0)\,\mathcal{U}_{[0,\xi]}\,F^{+\mu}(\xi)\,\mathcal{U}'_{[\xi,0]}\right]|P\rangle$$

For unpolarized protons:

$$\Gamma_U^{\mu\nu}(x,\boldsymbol{p}_T) = \frac{x}{2}\left\{-g_T^{\mu\nu}f_1^g(x,\boldsymbol{p}_T^2) + \left(\frac{p_T^\mu p_T^\nu}{M_p^2} + g_T^{\mu\nu}\frac{\boldsymbol{p}_T^2}{2M_p^2}\right)h_1^{\perp\,g}(x,\boldsymbol{p}_T^2)\right\}$$

unpolarized gluon TMD

linearly polarized gluon TMD

[Mulders, Rodrigues '01]

Gluons inside *unpolarized* protons can be polarized!

For transversely polarized protons:

gluon Sivers TMD

$$\Gamma_T^{\mu\nu}(x,\boldsymbol{p}_T) = \frac{x}{2}\left\{g_T^{\mu\nu}\frac{\epsilon_T^{\rho\sigma}p_{T\rho}S_{T\sigma}}{M_p}f_{1T}^{\perp\,g}(x,\boldsymbol{p}_T^2) + \ldots\right\}$$

Gluon TMDs in unpolarized scattering

Perhaps surprisingly, no gluon TMD has been extracted from experiments yet

$p\,p \to \mathcal{Q}\,\gamma\,X$ could be a good process to extract $f_1^g(x, p_T^2)$ at LHC

Den Dunnen, Lansberg, Pisano, Schlegel, 2014

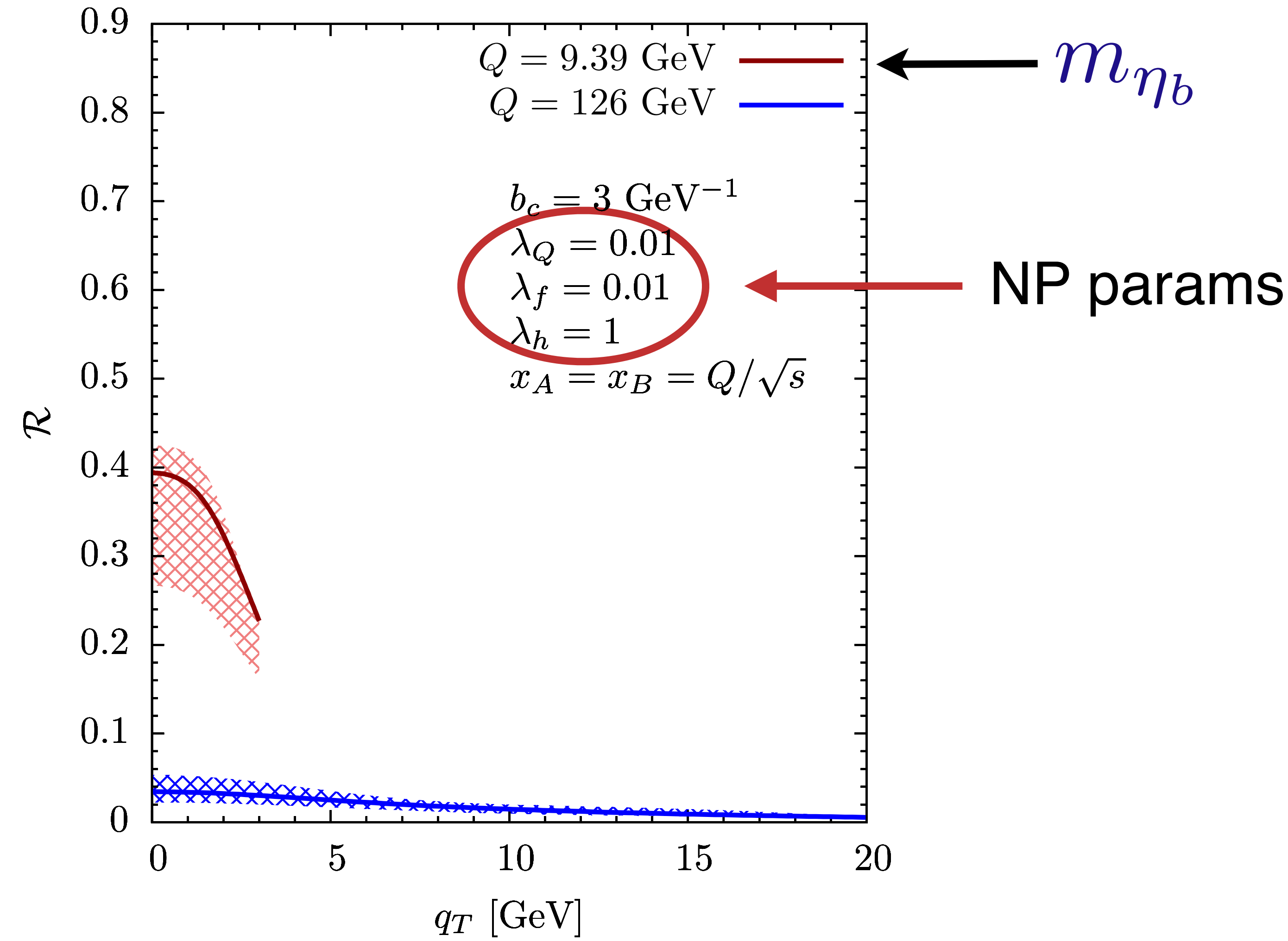
$h_1^{\perp g}$ can be probed for instance in $p\,p \to \eta_{c,b}\,X$

Boer, Pisano, 2012;
Echevarria, Kasemets, Mulders, Pisano, 2015

η_c even larger, but uncertainties big

At LHC also possible (5% level)
in $p\,p \to J/\psi\,J/\psi\,X$

Scarpa, Boer, Echevarria, Lansberg, Pisano, Schlegel, 2019

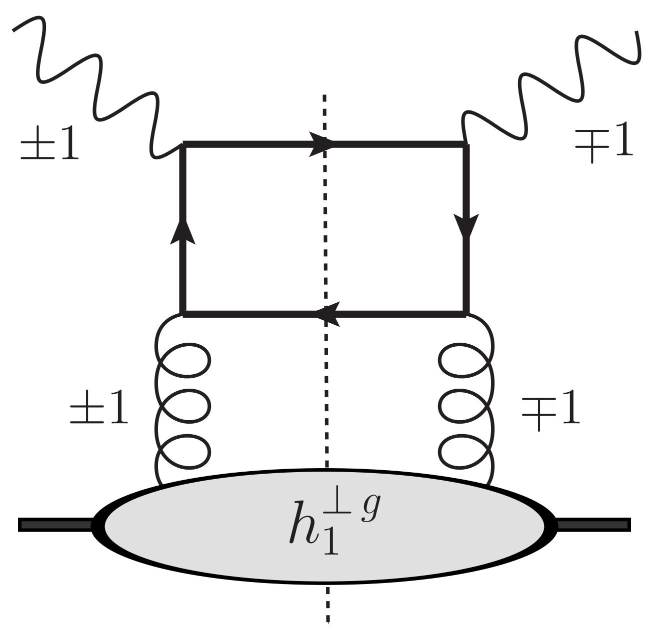


At RHIC and LHC always convolution of 2 TMDs → advantage of EIC

Open heavy quark electro-production

Unpolarized open heavy quark production at EIC allows to probe $h_1^{\perp\, g}(x, p_T^2)$

$$ep \to e' Q\bar{Q} X$$



no convolution!

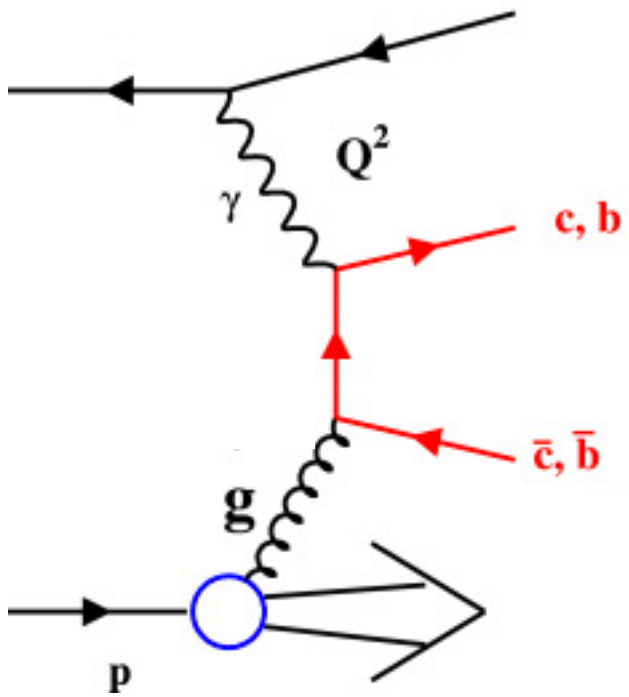
[Boer, Brodsky, Mulders & Pisano, 2010]

The individual transverse momenta have to be large but their sum has to be small

The sum q_T is then related to the transverse momentum of the initial gluon

The linear polarization of gluons will show up as an angular modulation

Open heavy quark electro-production



The heavy quarks will not be exactly back-to-back in the transverse plane:

$$K_\perp = (K_{Q\perp} - K_{\bar{Q}\perp})/2$$

$$q_T = K_{Q\perp} + K_{\bar{Q}\perp}$$

$$|q_T| \ll |K_\perp|$$

$\phi_T, \phi_\perp$ are the angles of q_T, $K_\perp$

Linear gluon polarization shows up as a $\cos 2\phi_T$ or $\cos 2(\phi_T - \phi_\perp)$ distribution

The gluon TMD asymmetries are analogous to the quark asymmetries in SIDIS

Parallels between SIDIS and HQ pair production

LO asymmetries in HQ pair production: [Boer, Pisano, Mulders, Zhou, 2016]

$$|\langle \cos 2\phi_T \rangle| = \left| \frac{\int \mathrm{d}\phi_\perp \mathrm{d}\phi_T \, \cos 2\phi_T \, \mathrm{d}\sigma}{\int \mathrm{d}\phi_\perp \mathrm{d}\phi_T \, \mathrm{d}\sigma} \right| = \frac{\boldsymbol{q}_T^2 \, |B_0^U|}{2\, A_0^U} = \frac{\boldsymbol{q}_T^2}{2M^2} \frac{\left| h_1^{\perp\, g}(x, \boldsymbol{p}_T^2) \right|}{f_1^g(x, \boldsymbol{p}_T^2)} \frac{\left| \mathcal{B}_0^{eg \to eQ\overline{Q}} \right|}{\mathcal{A}_0^{eg \to eQ\overline{Q}}}$$

$$A_N^{\sin(\phi_S - \phi_T)} = \frac{|\boldsymbol{q}_T|}{M_p} \frac{A_0^T}{A_0^U} = \frac{|\boldsymbol{q}_T|}{M_p} \frac{f_{1T}^{\perp\, g}(x, \boldsymbol{q}_T^2)}{f_1^g(x, \boldsymbol{q}_T^2)}$$

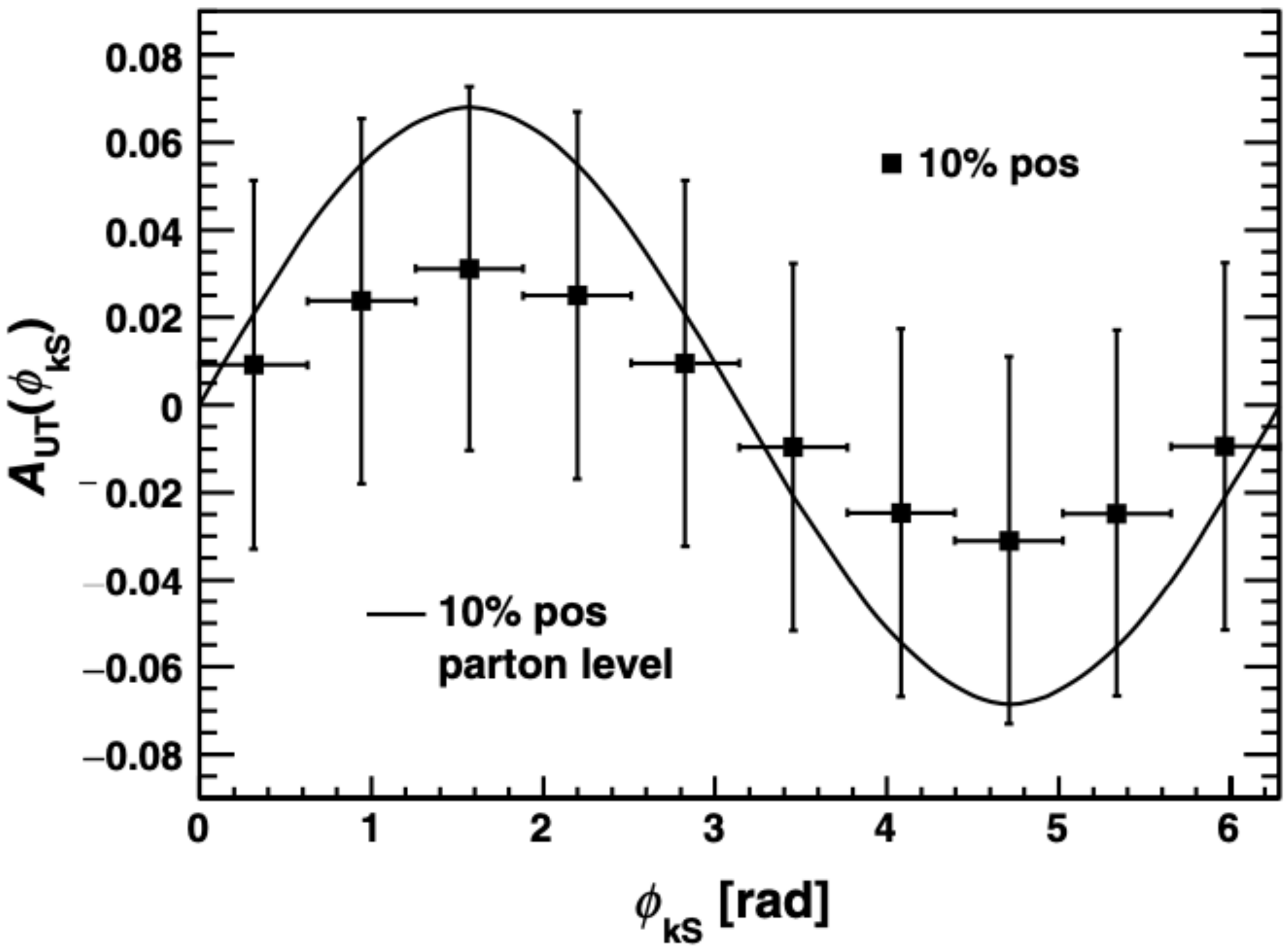
$$A_N^{\sin(\phi_S + \phi_T)} = \frac{|\boldsymbol{q}_T|}{M_p} \frac{B_0'^T}{A_0^U} = \frac{2(1-y)\, \mathcal{B}_{0T}^{\gamma^* g \to Q\overline{Q}}}{[1 + (1-y)^2]\, \mathcal{A}_{U+L}^{\gamma^* g \to Q\overline{Q}} - y^2 \mathcal{A}_L^{\gamma^* g \to Q\overline{Q}}} \frac{|\boldsymbol{q}_T|}{M_p} \frac{h_1^g(x, \boldsymbol{q}_T^2)}{f_1^g(x, \boldsymbol{q}_T^2)}$$

$$A_N^{\sin(\phi_S - 3\phi_T)} = -\frac{|\boldsymbol{q}_T|^3}{M_p^3} \frac{B_0^T}{2A_0^U} = -\frac{2(1-y)\, \mathcal{B}_{0T}^{\gamma^* g \to Q\overline{Q}}}{[1 + (1-y)^2]\, \mathcal{A}_{U+L}^{\gamma^* g \to Q\overline{Q}} - y^2 \mathcal{A}_L^{\gamma^* g \to Q\overline{Q}}} \frac{|\boldsymbol{q}_T|^3}{2M_p^3} \frac{h_{1T}^{\perp\, g}(x, \boldsymbol{q}_T^2)}{f_1^g(x, \boldsymbol{q}_T^2)}$$

SIDIS - Fragmentation functions HQ pairs - calculable amplitudes

Asymmetries in heavy quark pair production

Sivers asymmetry in D-meson pair production at EIC:



Assumes gluon Sivers TMD that is 10% of the positivity bound

Luminosity of 10 fb^{-1}, which for transversely polarized running is considered realistic

Zheng, Aschenauer, Lee, Xiao, Jin, 2018

$\langle x_B \rangle = 0.0012$

This (f-type) Sivers TMD lacks the 1/x growth of the unpolarized gluon TMD, at least in the perturbative k_T regime, hence 10% at x=0.001 may be too optimistic

Boer, Echevarria, Mulders, J. Zhou, PRL 2016

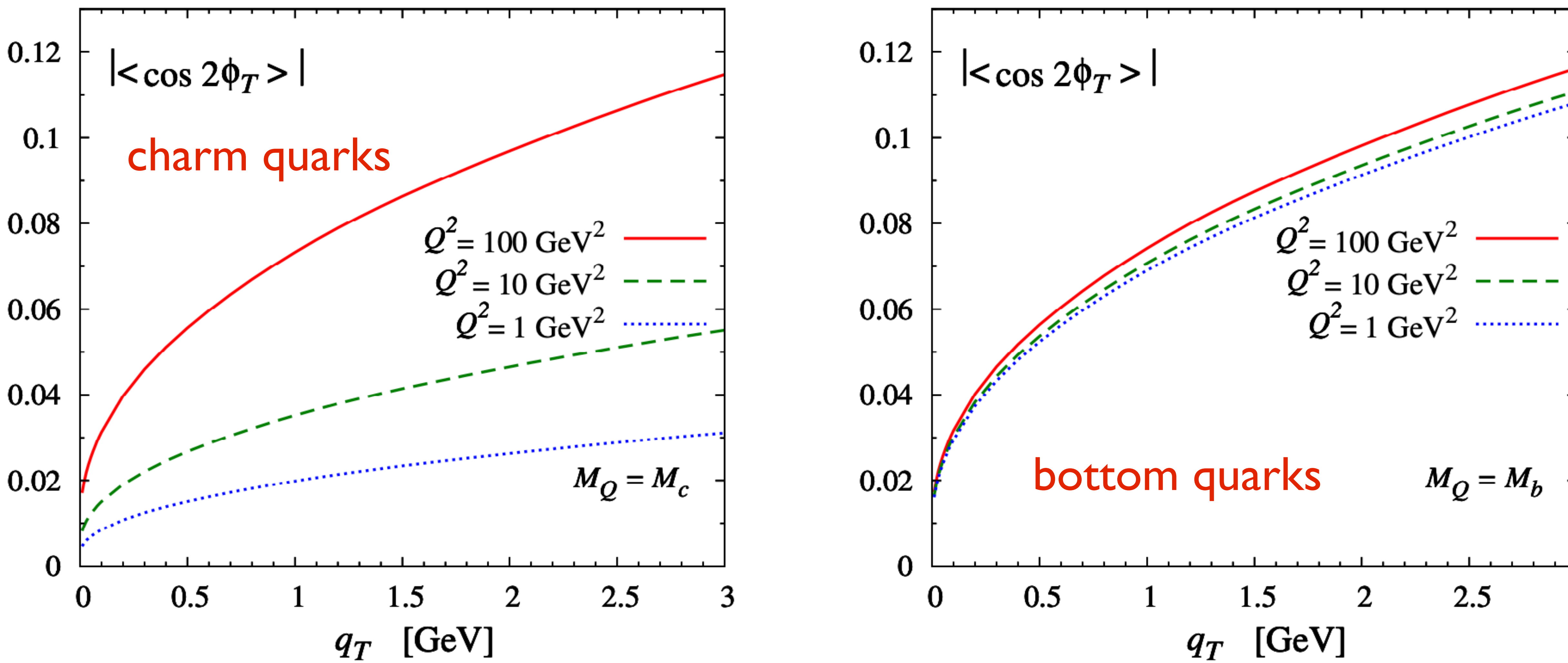
Jet pair production more promising, but some contamination from quark TMDs

Asymmetries in heavy quark pair production

More promising is the $\cos(2\phi_T)$ asymmetry in unpolarized ep and eA collisions

$$|\langle \cos 2\phi_T \rangle| = \left| \frac{\int \mathrm{d}\phi_\perp \mathrm{d}\phi_T \, \cos 2\phi_T \, \mathrm{d}\sigma}{\int \mathrm{d}\phi_\perp \mathrm{d}\phi_T \, \mathrm{d}\sigma} \right| = \frac{\boldsymbol{q}_T^2 \, |B_0^U|}{2\, A_0^U} = \frac{\boldsymbol{q}_T^2}{2M^2} \frac{\left| h_1^{\perp\, g}(x, \boldsymbol{p}_T^2) \right|}{f_1^g(x, \boldsymbol{p}_T^2)} \frac{\left| \mathcal{B}_0^{eg \to eQ\overline{Q}} \right|}{\mathcal{A}_0^{eg \to eQ\overline{Q}}}$$

$h_1^{\perp\, g}$ expected to keep up with growth of the unpolarized gluons TMD as x → 0



MV model prediction for $|K_\perp|$=6 GeV, z=0.5, y=0.1

[Boer, Pisano, Mulders, Zhou, 2016]

Conclusion on heavy quark pair production

Sizable $\cos(2\phi_T)$ and $\cos 2(\phi_T - \phi_\perp)$ asymmetries are expected

No projections for EIC have been given yet

Linear gluon polarization was not discussed in the YR meetings at all so far

The WW distribution probed at EIC is also the one in Higgs or $\eta_{c,b}$ production in proton-proton collisions → opportunity for synergy with LHC

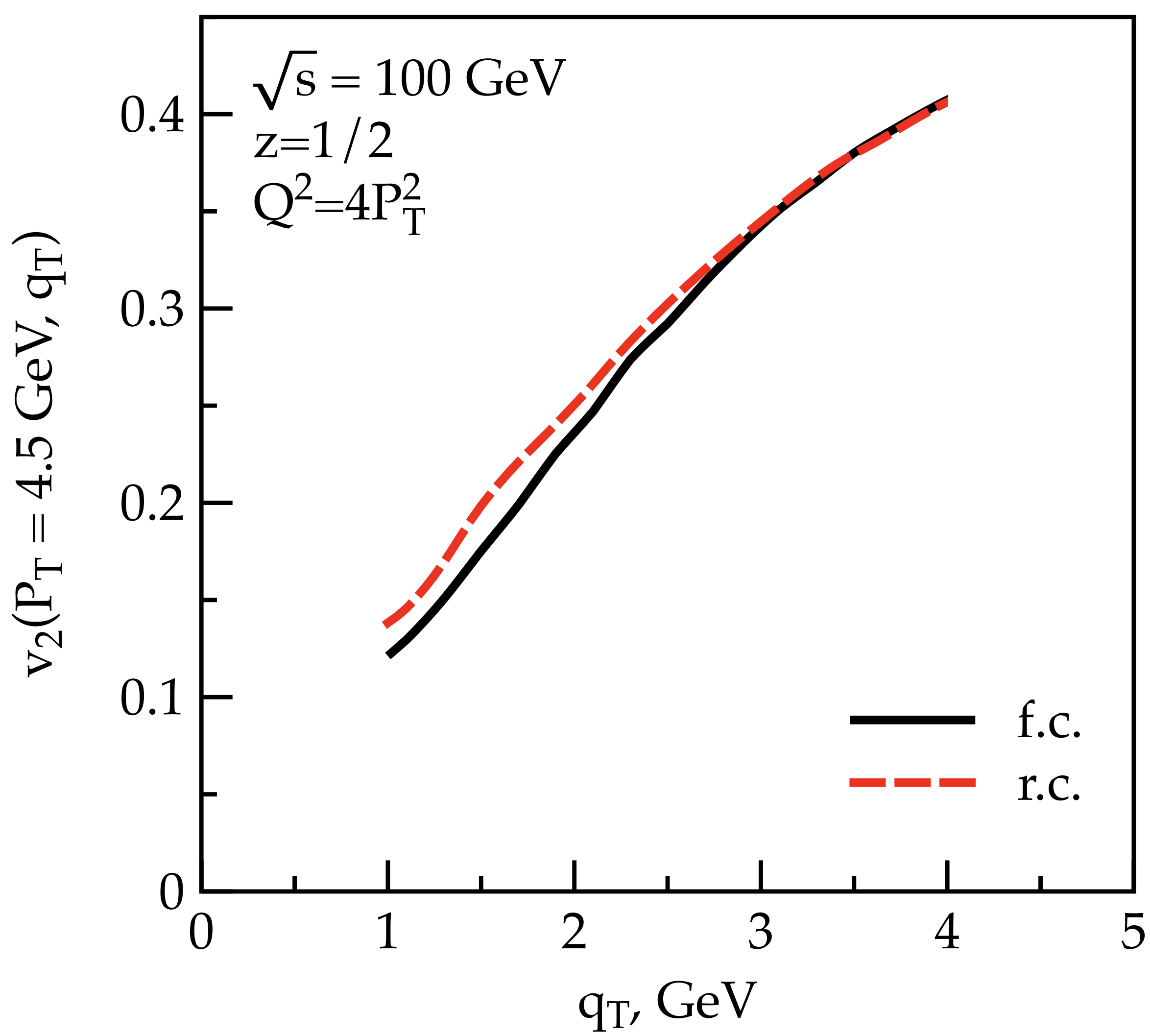
Accessing TMDs in HQ pair production requires (besides standard requirements for the detection of heavy quarks, like vertex detectors) a minimal transverse momentum resolution of a few hunderd MeV on q_T in the small transverse momentum region up to a few GeV

Dijet production at EIC

$h_1^{\perp g}$ (WW) is also accessible in dijet production at EIC

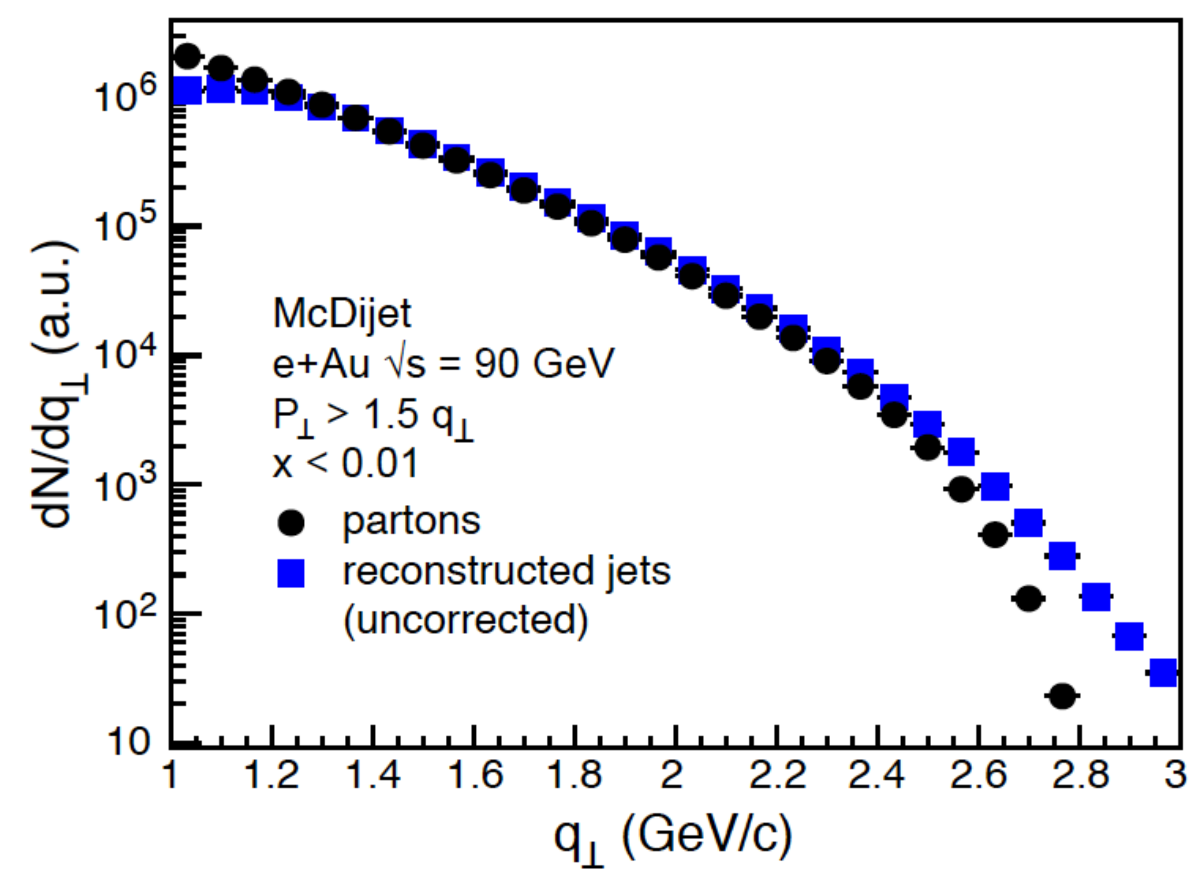
[Metz, Zhou 2011; Pisano, Boer, Brodsky, Buffing, Mulders, 2013; Boer, Pisano, Mulders, Zhou, 2016]

Linear gluon polarization shows itself through a cos2ϕ distribution ("v_2")



Large effects are found

Dumitru, Lappi, Skokov, 2015



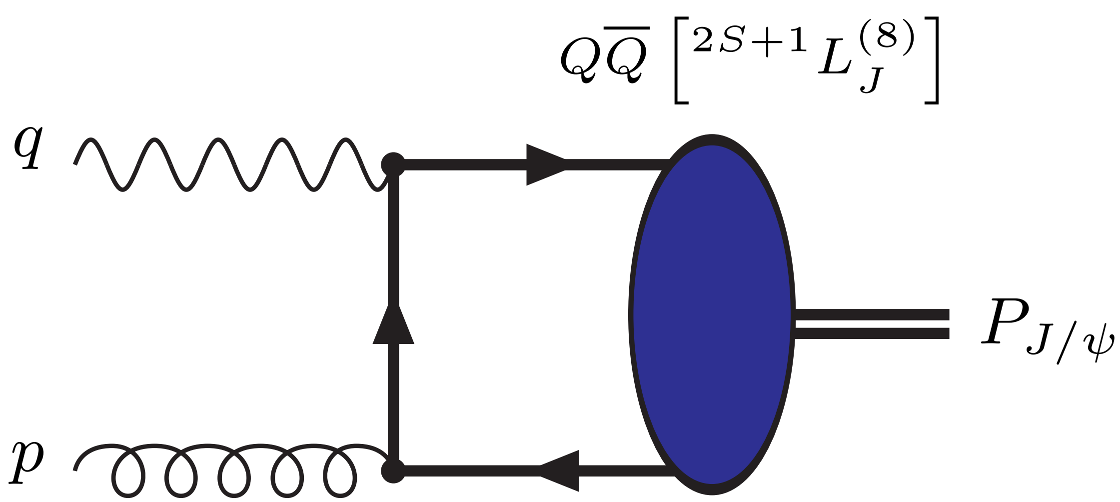
Jets are reasonable proxies for outgoing quarks concerning the q_T distribution

Dumitru, Skokov, Ullrich, 2018

Quarkonium production

$$e\,p \to e'\,\mathcal{Q}\,X \text{ with } \mathcal{Q} \text{ either a } J/\psi \text{ or a } \Upsilon \text{ meson}$$

[Godbole, Misra, Mukherjee, Rawoot, 2012/3; Godbole, Kaushik, Misra, Rawoot, 2015; Mukherjee, Rajesh, 2017; Rajesh, Kishore, Mukherjee, 2018]



A cos(2ϕ_T) asymmetry probes $h_1^{\perp\,g}$

$$\langle \cos 2\phi_T \rangle = \frac{(1-y)\,\mathcal{B}_T^{\gamma^* g \to \mathcal{Q}}}{[1+(1-y)^2]\mathcal{A}_{U+L}^{\gamma^* g \to \mathcal{Q}} - y^2 \mathcal{A}_L^{\gamma^* g \to \mathcal{Q}}} \times \frac{\boldsymbol{q}_T^2}{2M_p^2} \frac{h_1^{\perp\,g}(x, \boldsymbol{q}_T^2)}{f_1^g(x, \boldsymbol{q}_T^2)}.$$

In LO NRQCD the prefactor of the asymmetry depends on y, Q, M_Q and on two quite uncertain Color Octet (CO) Long Distance Matrix Elements (LDMEs)

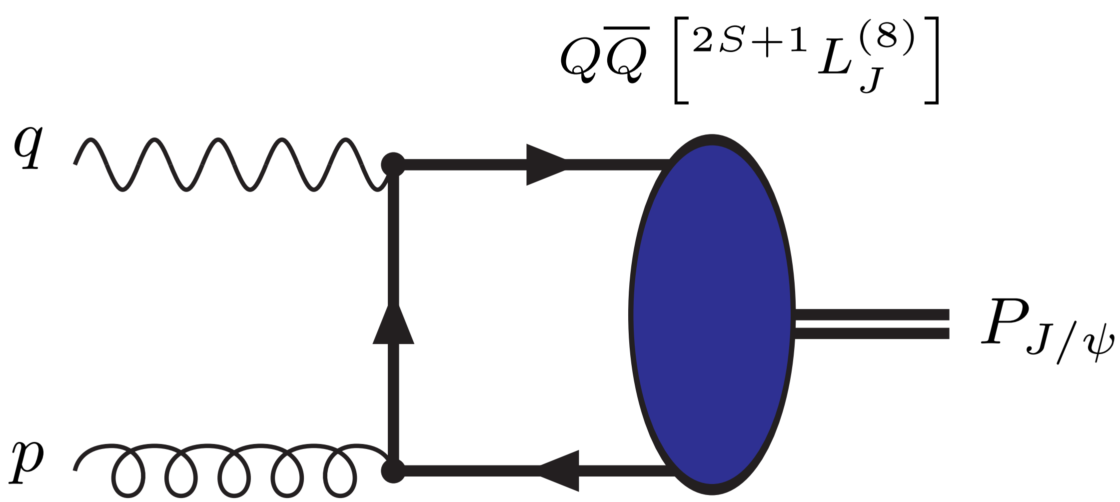
One can cancel out the CO LDMEs by considering ratios with spin asymmetries

[Bacchetta, Boer, Pisano, Taels, 2018]

Quarkonia

$$e\,p^{\uparrow} \to e'\,\mathcal{Q}\,X \text{ with } \mathcal{Q} \text{ either a } J/\psi \text{ or a } \Upsilon \text{ meson}$$

[Godbole, Misra, Mukherjee, Rawoot, 2012/3; Godbole, Kaushik, Misra, Rawoot, 2015; Mukherjee, Rajesh, 2017; Rajesh, Kishore, Mukherjee, 2018]



Using LO NRQCD the cos($2\phi_T$) asymmetry depends on quite uncertain Color Octet (CO) Long Distance Matrix Elements (LDMEs)

One can cancel out the CO LDMEs by considering ratios with spin asymmetries

[Bacchetta, Boer, Pisano, Taels, 2018]

$$\frac{A^{\cos 2\phi_T}}{A^{\sin(\phi_S+\phi_T)}} = \frac{\boldsymbol{q}_T^2}{M_p^2}\,\frac{h_1^{\perp\,g}(x,\boldsymbol{q}_T^2)}{h_1^g(x,\boldsymbol{q}_T^2)}$$

$$\frac{A^{\cos 2\phi_T}}{A^{\sin(\phi_S-3\phi_T)}} = -\frac{1}{2}\,\frac{h_1^{\perp\,g}(x,\boldsymbol{q}_T^2)}{h_{1T}^{\perp\,g}(x,\boldsymbol{q}_T^2)}$$

$$\frac{A^{\sin(\phi_S-3\phi_T)}}{A^{\sin(\phi_S+\phi_T)}} = -\frac{\boldsymbol{q}_T^2}{2M_p^2}\,\frac{h_{1T}^{\perp\,g}(x,\boldsymbol{q}_T^2)}{h_1^g(x,\boldsymbol{q}_T^2)}$$

This requires considering a system of 3 asymmetries (with 3 unknowns)

Color Octet LDMEs from EIC

One can also consider ratios where the TMDs cancel out

Allows to obtain new experimental information on the poorly known CO LDMEs

This requires a comparison of $e\,p \to e'\,\mathcal{Q}\,X$ and $ep \to e'Q\bar{Q}X$

$$\mathcal{R}^{\cos 2\phi} = \frac{\int d\phi_T \cos 2\phi_T \, d\sigma^{\mathcal{Q}}(\phi_S, \phi_T)}{\int d\phi_T \, d\phi_\perp \cos 2\phi_T \, d\sigma^{Q\overline{Q}}(\phi_S, \phi_T, \phi_\perp)}$$

$$\mathcal{R} = \frac{\int d\phi_T \, d\sigma^{\mathcal{Q}}(\phi_S, \phi_T)}{\int d\phi_T \, d\phi_\perp \, d\sigma^{Q\overline{Q}}(\phi_S, \phi_T, \phi_\perp)}$$

Two observables depending on two unknowns:

$$\mathcal{O}_8^S \equiv \langle 0|\mathcal{O}_8^{\mathcal{Q}}(^1S_0)|0\rangle$$

$$\mathcal{O}_8^P \equiv \langle 0|\mathcal{O}_8^{\mathcal{Q}}(^3P_0)|0\rangle$$

$$\mathcal{R}^{\cos 2\phi_T} = \frac{27\pi^2}{4}\frac{1}{M_Q}\left[\mathcal{O}_8^S - \frac{1}{M_Q^2}\,\mathcal{O}_8^P\right]$$

$$\mathcal{R} = \frac{27\,\pi^2}{4}\frac{1}{M_Q}\frac{[1+(1-y)^2]\,\mathcal{O}_8^S + (10-10y+3y^2)\,\mathcal{O}_8^P/M_Q^2}{26-26y+9y^2}$$

$z = 1/2$

To avoid evolution we chose $K\perp = Q = 2M_Q$

[Bacchetta, Boer, Pisano, Taels, 2018]

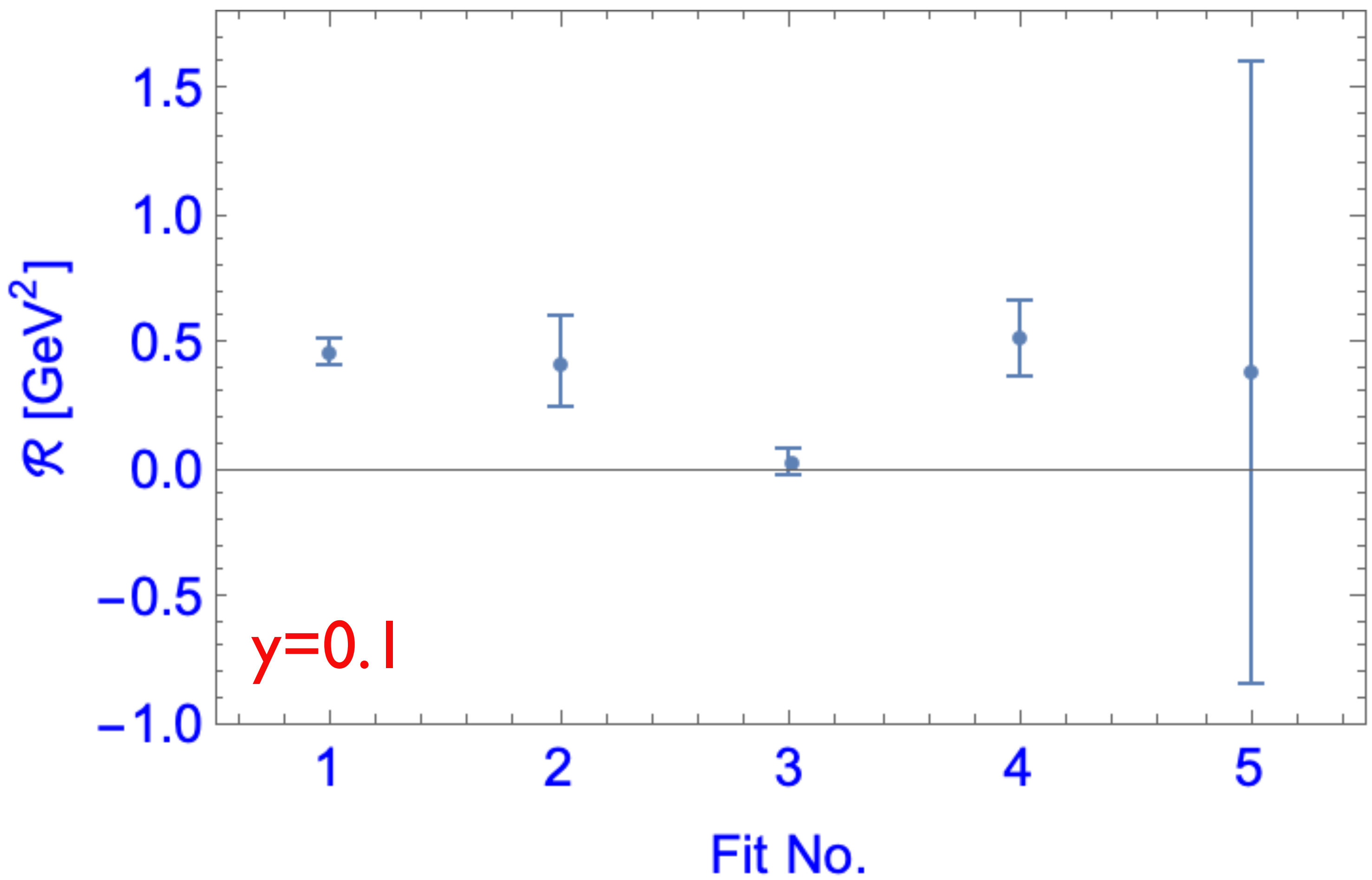
CO LDMEs

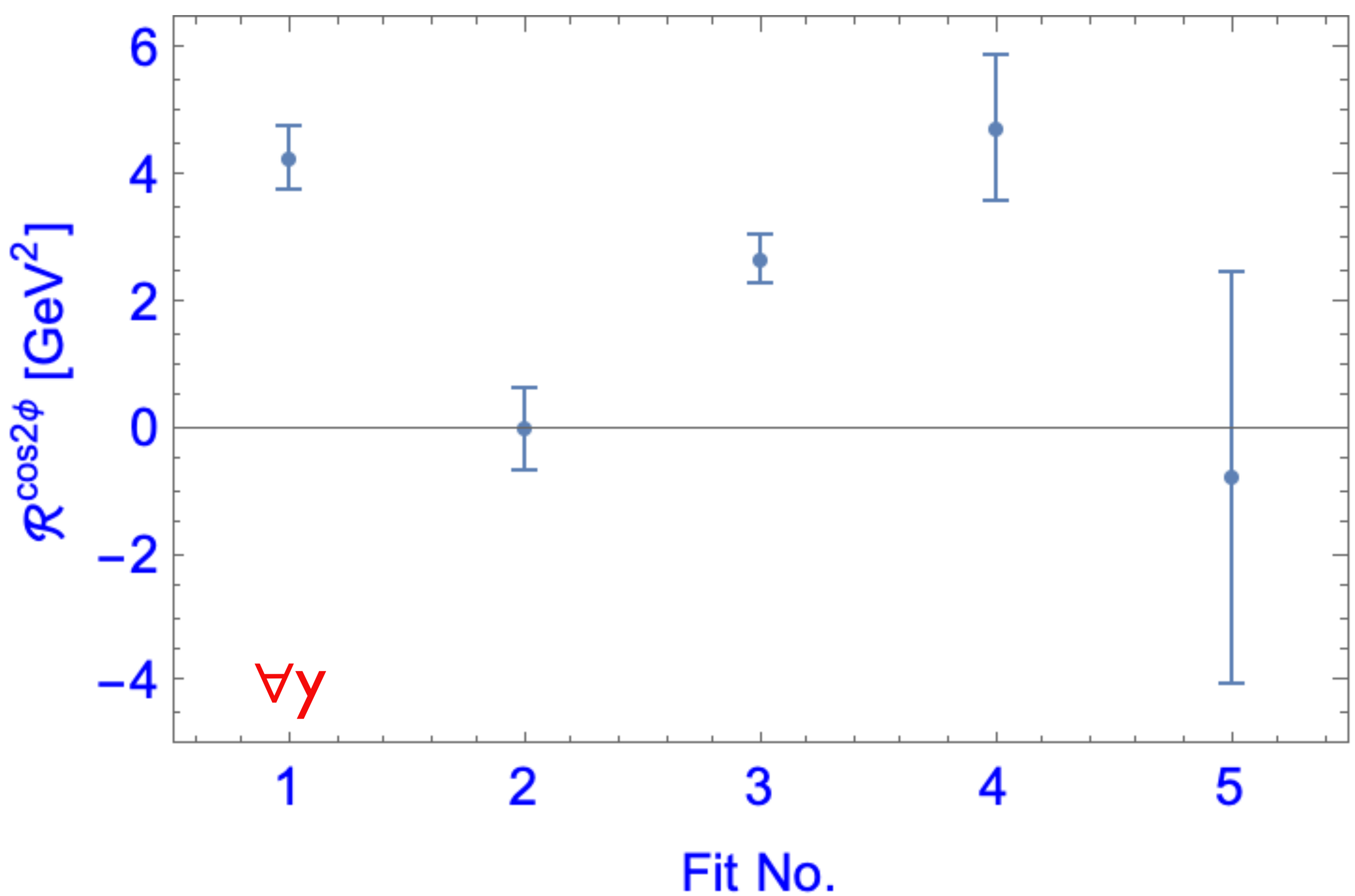
Extractions of the Color Octet Long Distance Matrix Elements

	J/ψ	$\langle 0\|\mathcal{O}_8^{J/\psi}(^1S_0)\|0\rangle$	$\langle 0\|\mathcal{O}_8^{J/\psi}(^3P_0)\|0\rangle/M_c^2$	
Fit 1	Chao et al. [40]	8.9 ± 0.98	0.56 ± 0.21	$\times 10^{-2}$ GeV ^{3}
Fit 2	Sharma et al. [41]	1.8 ± 0.87	1.8 ± 0.87	$\times 10^{-2}$ GeV ^{3}
Fit 3	Butenschoen et al. [39]	4.50 ± 0.72	-0.72 ± 0.21	$\times 10^{-2}$ GeV ^{3}
Fit 4	Bodwin et al. [42]	9.9 ± 2.2	-0.07 ± 0.06	$\times 10^{-2}$ GeV ^{3}

	$\Upsilon(1S)$	$\langle 0\|\mathcal{O}_8^{\Upsilon(1S)}(^1S_0)\|0\rangle$	$\langle 0\|\mathcal{O}_8^{\Upsilon(1S)}(^3P_0)\|0\rangle/(5M_b^2)$	
Fit 5	Sharma et al. [41]	1.21 ± 4.0	1.21 ± 4.0	$\times 10^{-2}$ GeV ^{3}

Fit 1 - Chao et al., PRL 108, 242004 (2012)
Fit 2 - Sharma & Vitev, PRC 87, 044905 (2013)
Fit 3 - Butenschoen & Kniehl, PRL106, 022003 (2011)
Fit 4 - Bodwin et al., PRL 113, 022001 (2014)
Fit 5 - Sharma & Vitev, PRC 87, 044905 (2013)



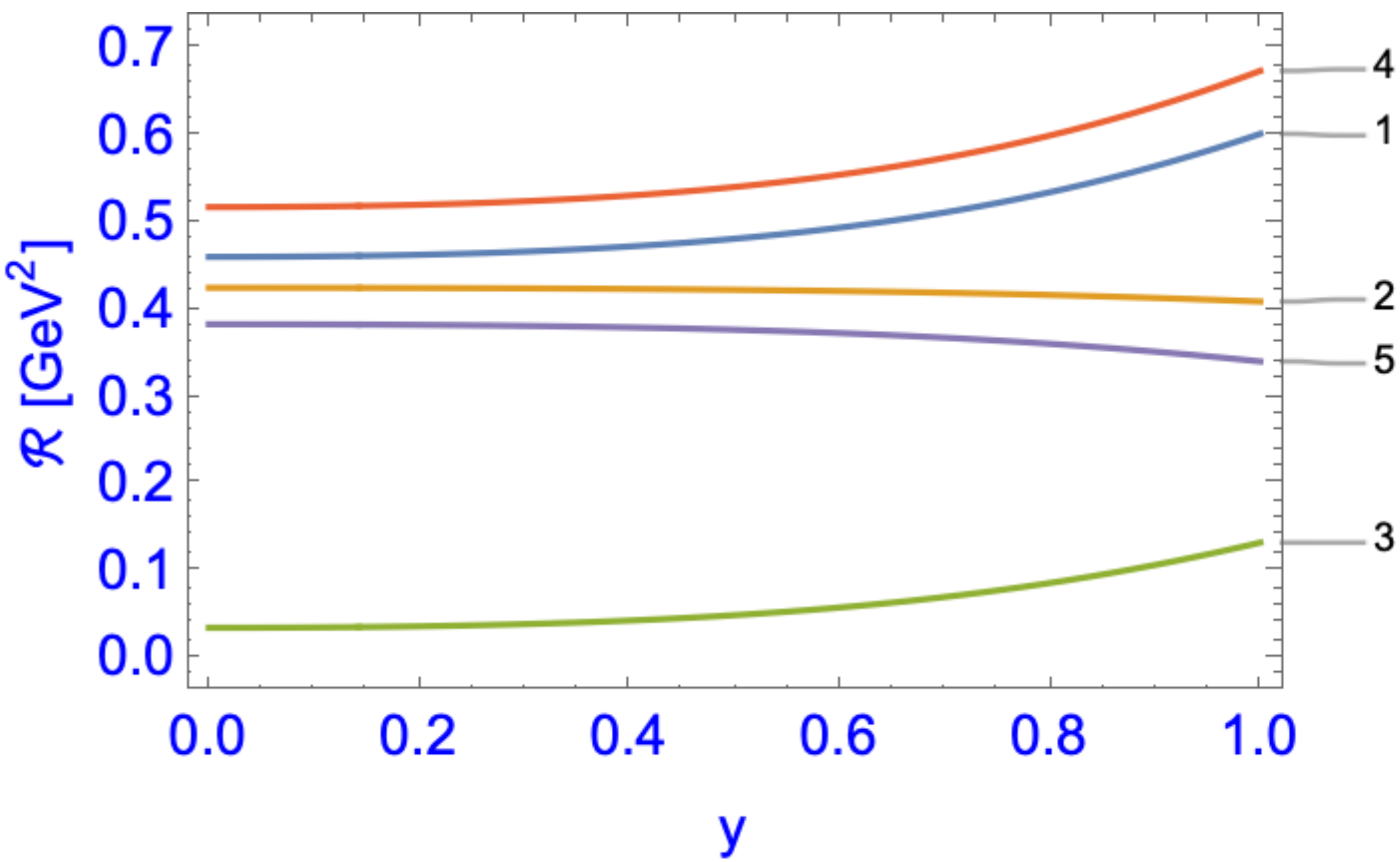


Ratios not normalized to [0,1] for $\mathcal{R}$ or [-1,1] for $\mathcal{R}^{\cos(2\phi)}$

$$\frac{\mathcal{R}^{\cos 2\phi_T}}{\mathcal{R}} = \frac{\langle \cos 2\phi_T \rangle_{\mathcal{Q}}}{\langle \cos 2\phi_T \rangle_{Q\overline{Q}}}$$

Based on fits the ratio of asymmetries could be anywhere between 0 and ∞

But rough average of the fits would indicate that the $\cos(2\phi_T)$ asymmetry could be of $\mathcal{O}(10)$ times larger in J/Ψ production



central values shown only

moderate y dependence

But numerator and denominator of $\mathcal{R}^{\cos(2\phi)}$ have prefactor (1-y) so vanish at y=1

No projections for EIC available, i.e. there is no information on how well a $\cos(2\phi_T)$ asymmetry in J/Ψ could be determined

Exploiting polarization

There are different equations for polarized quarkonium production that involve the same two unknowns:

$$\mathcal{O}_8^S \equiv \langle 0|\mathcal{O}_8^{\mathcal{Q}}(^1S_0)|0\rangle \qquad \mathcal{O}_8^P \equiv \langle 0|\mathcal{O}_8^{\mathcal{Q}}(^3P_0)|0\rangle$$

$$\mathcal{R}_L = \frac{9\pi^2}{4}\frac{1}{M_Q}\frac{[1+(1-y)^2]\mathcal{O}_8^S+3(6-6y+y^2)\mathcal{O}_8^P/M_Q^2}{26-26y+9y^2}$$

$$\mathcal{R}_T = \frac{9\pi^2}{2}\frac{1}{M_Q}\frac{[1+(1-y)^2]\mathcal{O}_8^S+3(2-2y+y^2)\mathcal{O}_8^P/M_Q^2}{26-26y+9y^2}$$

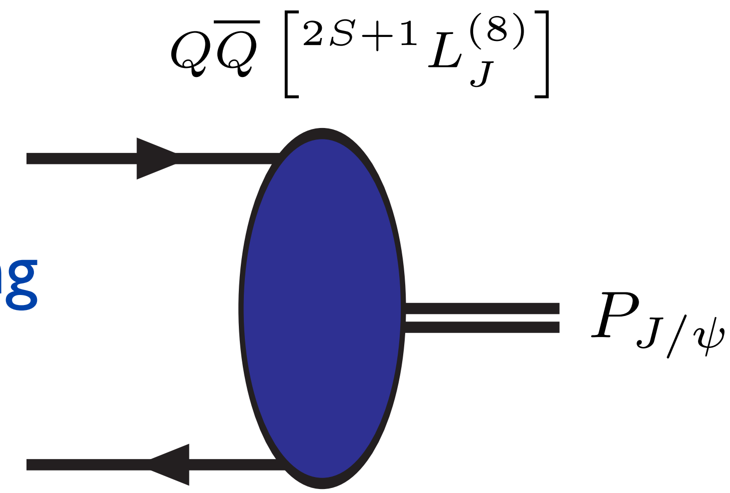
$$\mathcal{R}_L^{\cos 2\phi_T} = \frac{27\pi^2}{4}\frac{1}{M_Q}\left[\frac{1}{3}\mathcal{O}_8^S - \frac{1}{M_Q^2}\mathcal{O}_8^P\right] \qquad \mathcal{R}_T^{\cos 2\phi_T} = \frac{9\pi^2}{2}\frac{1}{M_Q}\mathcal{O}_8^S$$

$$\mathcal{R}_L^{\cos 2\phi_T} + \mathcal{R}_T^{\cos 2\phi_T} = \mathcal{R}^{\cos 2\phi_T}$$
$$\mathcal{R}_L + \mathcal{R}_T = \mathcal{R}.$$

Overconstrained system allows to cross check the extraction and to estimate the uncertainty

[Bacchetta, Boer, Pisano, Taels, 2018]

Effect of smearing



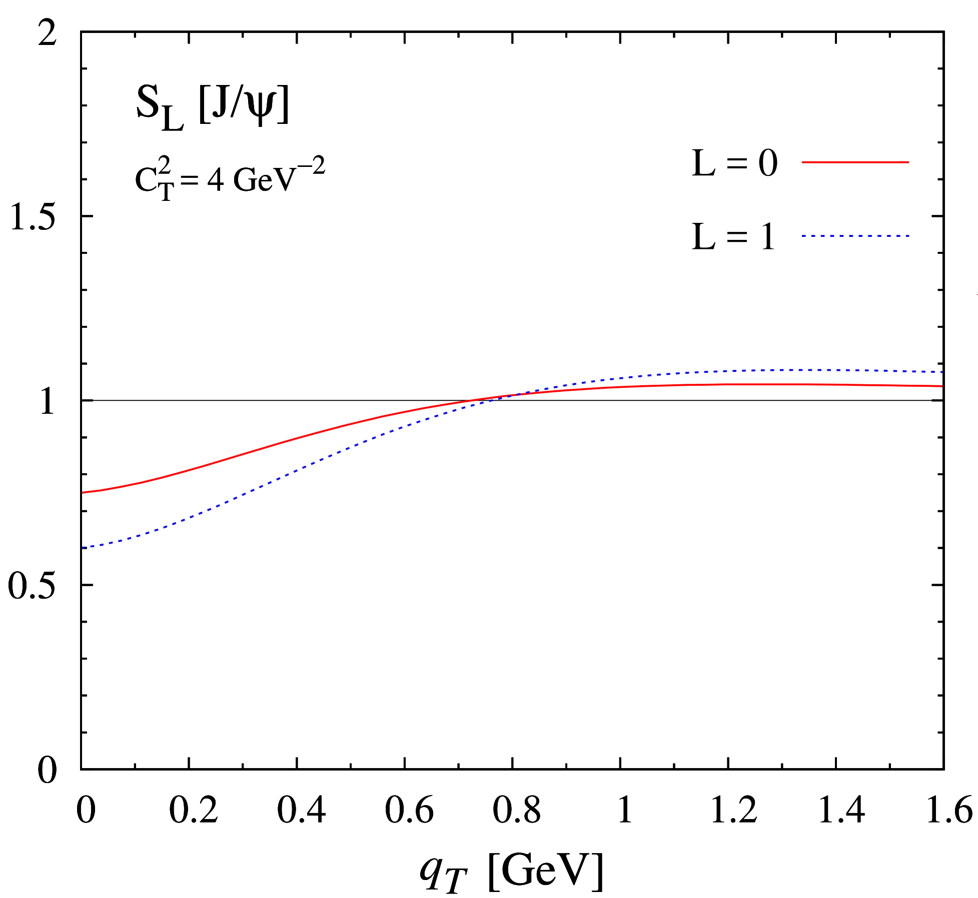
In reality the process of $Q\bar{Q} \to J/\Psi$ involves some k_T-smearing

The factorization involves still unknown "shape functions"

[Echevarria, 2019; Fleming, Makris & Mehen, 2019]

If L dependent this smearing would affect the extraction of CO LDMEs:

$$\mathcal{R} = \frac{27\,\pi^2}{4}\,\frac{1}{M_Q}\,\frac{[1+(1-y)^2]\,\mathcal{O}_8^S\,S_0(x,\boldsymbol{q}_T^2) + (10-10y+3y^2)\,\mathcal{O}_8^P/M_Q^2\,S_1(x,\boldsymbol{q}_T^2)}{26-26y+9y^2}$$



$$S_L(x,\boldsymbol{q}_T^2) = \frac{\mathcal{C}[f_1^g\,\Delta_L](x,\boldsymbol{q}_T^2)}{f_1^g(x,\boldsymbol{q}_T^2)}$$

← Example study using positronium-like functions

[Bacchetta, Boer, Pisano, Taels, 2018]

Note that these are not the wave functions of the quarkonia however ($\Delta \neq \Psi$)

Perturbative tail is L independent

[Boer, D'Alesio, Murgia, Pisano, Taels, 2020]

Conclusions

Conclusions

- Gluon TMDs can be studied at EIC using heavy quark production processes, offering synergy with complementary studies at LHC

- The linear polarization of gluons inside unpolarized hadrons is expected to lead to sizable $\cos 2\phi$ asymmetries, especially at smaller x and higher Q^2

- J/ψ or Υ production in ep/eA collisions allow to probe gluon TMDs, but involve (in LO NRQCD) two Color Octet LDMEs that are still poorly known

- These CO LDMEs can be extracted from the comparison to open heavy quark pair production. The polarization of the quarkonia allow for cross checks

- Heavy quark studies form a promising part of the rich physics program of EIC but projections are still lacking to a large extent

A brief accompanying document has been prepared by Cristian Pisano and me, summarizing these opportunities and the projections for EIC (if available)

Back-up slides

Parallels between quarks and gluons

$$\Phi_U(x,\boldsymbol{k}) = \frac{1}{2}\left[\not{n}\, f_1(x,\boldsymbol{k}^2) + \frac{\sigma_{\mu\nu}k_T^\mu \bar{n}^\nu}{M}\, h_1^\perp(x,\boldsymbol{k}^2)\right],$$

$$\Phi_L(x,\boldsymbol{k}) = \frac{1}{2}\left[\gamma^5 \not{n}\, S_L\, g_1(x,\boldsymbol{k}^2) + \frac{i\sigma_{\mu\nu}\gamma^5 \bar{n}^\mu k_T^\nu S_L}{M}\, h_{1L}^\perp(x,\boldsymbol{k}^2)\right],$$

$$\Phi_T(x,\boldsymbol{k}) = \frac{1}{2}\left[\frac{\not{n}\,\epsilon_T^{S_T k_T}}{M}\, f_{1T}^\perp(x,\boldsymbol{k}^2) + \frac{\gamma^5 \not{n}\, \boldsymbol{k}\cdot\boldsymbol{S}_T}{M}\, g_{1T}(x,\boldsymbol{k}^2) + i\sigma_{\mu\nu}\gamma^5 \bar{n}^\mu S_T^\nu\, h_1(x,\boldsymbol{k}^2) - \frac{i\sigma_{\mu\nu}\gamma^5 \bar{n}^\mu k_T^{\nu\rho} S_{T\rho}}{M^2}\, h_{1T}^\perp(x,\boldsymbol{k}^2)\right]$$

For quarks the BM & Sivers TMDs are T-odd and the h-type functions are chiral-odd

$$\Gamma_U^{ij}(x,\boldsymbol{k}) = x\left[\delta_T^{ij}\, f_1(x,\boldsymbol{k}^2) + \frac{k_T^{ij}}{M^2}\, h_1^\perp(x,\boldsymbol{k}^2)\right],$$

$$\Gamma_L^{ij}(x,\boldsymbol{k}) = x\left[i\epsilon_T^{ij} S_L\, g_1(x,\boldsymbol{k}^2) + \frac{\epsilon_T^{\{i}{}_{\alpha} k_T^{j\}\alpha} S_L}{2M^2}\, h_{1L}^\perp(x,\boldsymbol{k}^2)\right],$$

$$\Gamma_T^{ij}(x,\boldsymbol{k}) = x\left[\frac{\delta_T^{ij}\,\epsilon_T^{S_T k_T}}{M}\, f_{1T}^\perp(x,\boldsymbol{k}^2) + \frac{i\epsilon_T^{ij}\,\boldsymbol{k}\cdot\boldsymbol{S}_T}{M}\, g_{1T}(x,\boldsymbol{k}^2) - \frac{\epsilon_T^{k_T\{i} S_T^{j\}} + \epsilon_T^{S_T\{i} k_T^{j\}}}{4M}\, h_1(x,\boldsymbol{k}^2) - \frac{\epsilon_T^{\{i}{}_{\alpha} k_T^{j\}\alpha S_T}}{2M^3}\, h_{1T}^\perp(x,\boldsymbol{k}^2)\right]$$

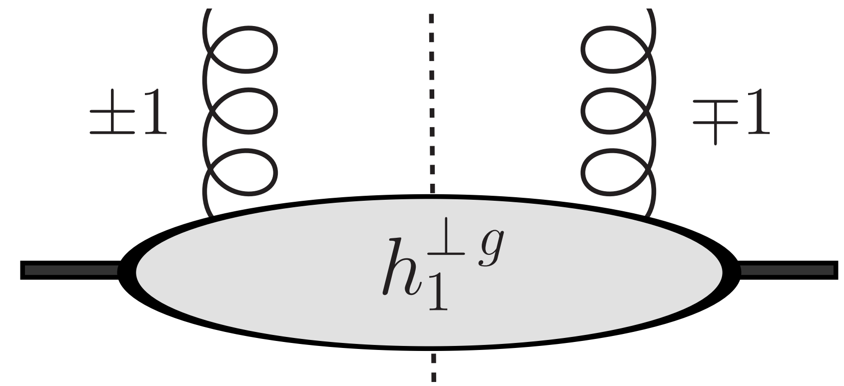
For gluons $h_1^\perp$ is T-even and h_1 is k_T-odd, T-odd and unrelated to transversity

Gluon polarization inside unpolarized protons

$$\Gamma_U^{\mu\nu}(x,\boldsymbol{p}_T) = \frac{x}{2}\left\{ -g_T^{\mu\nu}\, f_1^g(x,\boldsymbol{p}_T^2) + \left(\frac{p_T^\mu p_T^\nu}{M_p^2} + g_T^{\mu\nu}\frac{\boldsymbol{p}_T^2}{2M_p^2}\right) h_1^{\perp\, g}(x,\boldsymbol{p}_T^2)\right\}$$

Linearly polarized gluons can exist in **unpolarized** hadrons

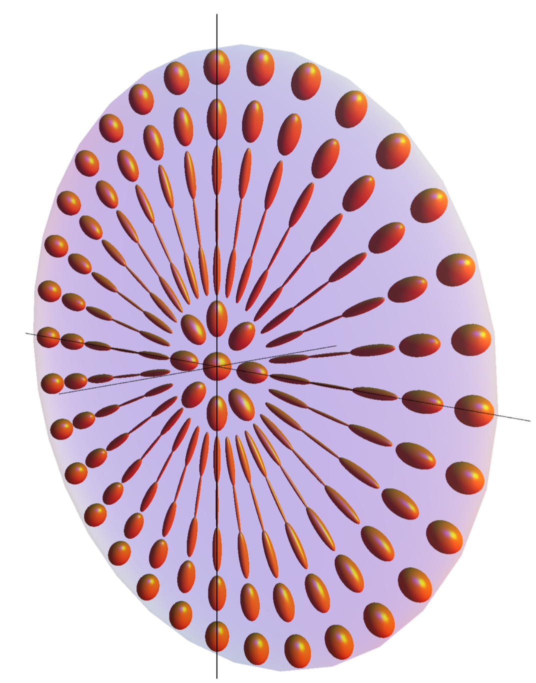
[Mulders, Rodrigues, 2001]



an interference between ±1 helicity gluon states

It requires nonzero transverse momentum: TMD

It is k_T-even and T-even



For $h_1^{\perp\, g} > 0$ gluons prefer to be polarized along k_T, with a $\cos 2\phi$ distribution of linear polarization around it, where $\phi = \angle(k_T, \varepsilon_T)$

Probes of linear gluon polarization

$h_1^{\perp g}$ is difficult to extract, as it cannot be probed in DIS, DY, SIDIS, nor in inclusive hadron or γ+jet production in pp or pA collisions

Processes that probe the linearly polarized gluon TMD:

	$pp \to \gamma\,\gamma\,X$	$pA \to \gamma^*\,\mathrm{jet}\,X$	$e\,p \to e'\,Q\,\overline{Q}\,X$ $e\,p \to e'\,j_1\,j_2\,X$	$pp \to \eta_{c,b}\,X$ $pp \to H\,X$	$pp \to J/\psi\,\gamma\,X$ $pp \to \Upsilon\,\gamma\,X$
$h_1^{\perp\,g\,[+,+]}$ (WW)	$\surd$	$\times$	$\surd$	$\surd$	$\surd$
$h_1^{\perp\,g\,[+,-]}$ (DP)	$\times$	$\surd$	$\times$	$\times$	$\times$

1% level at RHIC
Qiu, Schlegel, Vogelsang, 2011

5% level at RHIC
Boer, Mulders, J. Zhou, Y. Zhou, 2017

10% level at EIC
Boer, Brodsky, Pisano, Mulders, 2011;
Dumitru, Lappi, Skokov, 2015;
Boer, Pisano, Mulders, J. Zhou, 2016

10% level for η_Q and
1% level for Higgs at LHC
Boer & den Dunnen, 2014;
Echevarria, Kasemets, Mulders, Pisano, 2015

At LHC also possible (5% level) in $p\,p \to J/\psi\,J/\psi\,X$

[Scarpa, Boer, Echevarria, Lansberg, Pisano, Schlegel, 2019]

At RHIC and LHC always convolution of 2 TMDs

Small gluon Sivers effect?

Experiments suggest gluon Sivers is small, but not necessarily tiny:

- Burkardt sum rule already (approximately) satisfied by up and down quarks

$$\sum_{a=q,g} \int f_{1T}^{\perp(1)a}(x)\, dx = 0$$

- small Sivers asymmetry in SIDIS on deuteron target by COMPASS
  [Brodsky & Gardner, 2006]

- small A_N at midrapidity at RHIC (small gluon Sivers function in the GPM)
  [Anselmino, D'Alesio, Melis & Murgia, 2006; D'Alesio, Murgia, Pisano, 2015]

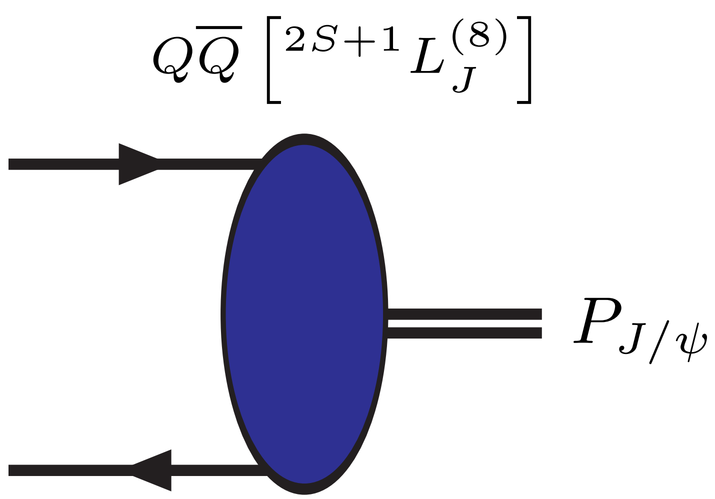
- COMPASS using high-p_T hadron pairs measured the gluon Sivers asymmetry:
  $A^{Siv} = -0.23 \pm 0.08$ (stat) ± 0.05 (syst) at $\langle x_g \rangle = 0.15$
  [C. Adolph et al., PLB 2017]

Gluon Sivers function is constrained to be $\lesssim$ 30% of nonsinglet quark Sivers function

D.B., Lorcé, Pisano & J. Zhou, 2015

This is its natural size, being $1/N_c$ suppressed at $x \sim 1/N_c$, like the flavor singlet u+d
[Efremov, Goeke, Menzel, Metz, Schweitzer, 2005]

Effect of smearing



$$\mathcal{R} = \frac{27\,\pi^2}{4}\,\frac{1}{M_Q}\,\frac{[1+(1-y)^2]\,\mathcal{O}_8^S\,S_0(x,\boldsymbol{q}_T^2)+(10-10y+3y^2)\,\mathcal{O}_8^P/M_Q^2\,S_1(x,\boldsymbol{q}_T^2)}{26-26y+9y^2}$$

$$S_L(x,\boldsymbol{q}_T^2) = \frac{\mathcal{C}[f_1^g\,\Delta_L](x,\boldsymbol{q}_T^2)}{f_1^g(x,\boldsymbol{q}_T^2)}$$

To the best of our knowledge, no parametrization is so far available for the smearing functions Δ_L. Therefore we propose a model based on the properties of the radial wave function of the hydrogen atom in momentum space, namely:

- For large $\boldsymbol{p}_T$, Δ_L vary as $(\boldsymbol{p}_T^2)^{-(L+4)}$, with $L=0,1$, independently of the heavy quark mass.
- For small $\boldsymbol{p}_T$, Δ_L vary as $(\boldsymbol{p}_T^2)^L$, hence Δ_1 vanishes at $\boldsymbol{p}_T=0$, while Δ_0 does not.

Furthermore, the normalization is fixed by imposing

$$\int \mathrm{d}^2\boldsymbol{k}_T\,\Delta_L(\boldsymbol{k}_T^2) = 1\,. \tag{75}$$

Explicitly we have

$$\Delta_0(\boldsymbol{k}_T^2) = \frac{3C_T^2}{\pi}\,\frac{1}{(1+\boldsymbol{k}_T^2\,C_T^2)^4}\,, \qquad \Delta_1(\boldsymbol{k}_T^2) = \frac{12C_T^4}{\pi}\,\frac{\boldsymbol{k}_T^2}{(1+\boldsymbol{k}_T^2\,C_T^2)^5}\,, \tag{76}$$

where C_T is taken to be independent of L and equal to the width of the TMD distribution in Eq. (74). This guarantees that the transverse momentum distribution for a heavier quarkonium state falls off less fast, reflecting its smaller spatial extent.

[Bacchetta, Boer, Pisano, Taels, 2018]