Semi-inclusive heavy quark production at EIC
Input for the EIC Yellow Report

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We summarize semi-inclusive open and bound heavy quark pair production processes that can be studied at EIC with the goal of measuring gluon TMDs and aspects of NRQCD. This topic belongs to the "Physics/Imaging" part of the EIC Yellow Report and is at the overlap of the "Semi-Inclusive Reactions" and "Jets and Heavy Quarks" areas. We do not address exclusive or diffractive heavy quark production and consider only the leading TMD and NRQCD frameworks, not the Generalized Parton Model studies, pQCD corrections or higher twist effects. We indicate clearly for which observables projections have been given or not.

I. PROCESS 1: HEAVY QUARK PAIR PRODUCTION

The process $e\,p \to e'\,Q\,\overline{Q}\,X$, where Q and $\overline{Q}$ denote a heavy quark and antiquark, is considered to be a promising probe of transverse momentum dependent parton distributions (TMDs). For this to be the case, a particular kinematic regime should be considered, the so-called **back-to-back correlation limit**. In this kinematical region the transverse momenta of the heavy quarks (transverse to the virtual photon - hadron direction) have to be almost back-to-back. The sum of the two transverse momenta is related to the transverse momentum entering the TMDs, therefore it should be small, much smaller than the difference of the two transverse momenta which should be large. In this process there are therefore two hard scales: the photon virtuality (given by Q^2) and the relative transverse momentum of the two heavy quarks. The heavy quark mass itself may also be considered a large scale, but plays a minor role (see the subsection on TMD evolution). The sum of the two transverse momenta is typically considered in the range up to a few GeV for EIC energies. Therefore, the transverse momentum resolution in the small transverse momentum region should be on the order of a few hundred MeV, such that sufficient bins can be selected to map out this region. One should also keep in mind the uncertainty arising from the difference in momentum between the heavy quark and the heavy meson it will form and that will be detected. TMD shape functions (analogues of fragmentation functions, but now for pairs of quarks) should be considered. It is expected that these are less important than the smearing effects in dijet production. Moreover, radiative corrections will also lead to a modification of the transverse momentum distribution, but all TMD studies thus far consider consider tree level production.

Notation: Consider the momentum assignments as follows [6, 7]:

$$e(\ell) + p(P,S) \to e(\ell') + Q(K_1) + \overline{Q}(K_2) + X\,, \tag{1}$$

then define $q_T \equiv K_{1\perp} + K_{2\perp}$ and $K_\perp \equiv (K_{1\perp} - K_{2\perp})/2$, where $K_{i\perp}$ $(i = 1,2)$ are the transverse momenta of the heavy quark and antiquark, such that $K_{i\perp}^2 = -\boldsymbol{K}_{i\perp}^2$. The back-to-back correlation limit is when $|q_T| \ll |K_\perp|$. Furthermore, we consider a reference frame in which azimuthal angles are measured w.r.t. the lepton plane ($\phi_\ell = \phi_{\ell'} \equiv 0$). ϕ_S, ϕ_T and $\phi_\perp$ are the azimuthal angles of the three-vectors $\boldsymbol{S}_T$, $\boldsymbol{q}_T$ and $\boldsymbol{K}_\perp$, respectively,

In summary, accessing TMDs in the process $e\,p \to e'\,Q\,\overline{Q}\,X$ requires (besides standard requirements for the detection of heavy quarks, like vertex detectors) a minimal transverse momentum resolution of a few hunderd MeV on q_T in the small transverse momentum region up to a few GeV.

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A. Physics objective 1: Gluon Sivers function measurements at EIC

The single spin asymmetry (SSA) in $e\,p^{\uparrow} \to e'\,D^0\,\overline{D}^0\,X$ in the back-to-back correlation limit has been put forward as an ideal probe of the gluon Sivers function at EIC, see for instance [4], section 2.3.1. This particular process has not been studied experimentally before, and, in general, the gluon Sivers function has not been extracted from any TMD factorizing process yet. For a review of what is currently known about the gluon Sivers function see e.g. [1].

The process $e\,p^{\uparrow} \to e'\,D^0\,\overline{D}^0\,X$ is viewed as the open charm production process $e\,p^{\uparrow} \to e'\,c\,\bar{c}\,X$ or on the leading order (LO) partonic scattering level as $\gamma^* g \to c\bar{c}$. The subprocesses $\gamma^* q \to qg$ and $\gamma^* \bar{q} \to \bar{q}g$ that generally contribute to hadron pair production do not contribute here.

The gluon Sivers function is a TMD and therefore process dependent. The process $e\,p^{\uparrow} \to e'\,c\,\bar{c}\,X$ probes the so-called f-type gluon Sivers TMD $f_{1T}^{\perp\,g\,(f)}$ (with two future-pointing (+) links [6]), which is the one entering the Burkardt sum rule (BSR) [5] and related to the one probed in $p^{\uparrow}p \to \gamma\gamma X$ but with an opposite sign [7].

The gluon Sivers function gives rise to an SSA $A_{UT}^{\sin(\phi_S-\phi_T)}$ that can be maximally unity for a gluon Sivers function that saturates its positivity bound. In reality it is not likely that large. **Projections for EIC are given in [9]** assuming a gluon Sivers function that is 10% of the bound and an integrated luminosity of 10 fb^{-1} which requires already a substantial amount of running. Their Fig. 9a shows that the maximally $\sim$ 6% asymmetries are not discernible within statistics in this case. The aim should thus be to resolve percent level asymmetries, which is challenging with this particular process. The situation is different for hadron and jet pair production, but those receive contributions from more partonic subprocesses and are less straightforward to analyze.

High-p_T hadron pair production: The analogous Sivers asymmetry have been studied by COMPASS for high-p_T charged hadron pair production in muon-deuteron and muon-proton scattering [8], where photon-gluon fusion is expected to dominate. The same kinematics was used for deuteron and proton: $Q^2 > 1$ (GeV$/c)^2$, $W > 5$ GeV$/c^2$, $0.003 < x_{\rm Bj} < 0.7$, $0.1 < y < 0.9$. From the measured asymmetry $A_{UT}^{\sin(\phi_{2h}-\phi_S)}$ the photon-gluon fusion contribution is extracted and found to be $A_{\rm PGF}^{{\rm Siv},d} = -0.14 \pm 0.15({\rm stat.}) \pm 0.10({\rm syst.})$ at $\langle x_G \rangle = 0.13$ for the deuteron and $A_{\rm PGF}^{{\rm Siv},p} = -0.26 \pm 0.09({\rm stat.}) \pm 0.06({\rm syst.})$ at $\langle x_G \rangle = 0.15$ for the proton and for the combination of proton and deuteron: $A_{\rm PGF}^{\rm Siv} = -0.23 \pm 0.08({\rm stat.}) \pm 0.05({\rm syst.})$. For the interpretation of the data in terms of the gluon Sivers effect, the p_T of each hadron needs to be sufficiently large. COMPASS took $p_{1T} > 0.7$ GeV$/c$ and $p_{2T} > 0.4$ GeV$/c$, where the bulk of the data was for $p_{1T} < 1.7$ GeV$/c$ and $p_{2T} < 1.1$ GeV$/c$, thus only moderately large. The gluon contribution has been estimated using MC. The Sivers TMD itself has not been extracted from these COMPASS data.

At EIC this process can also be studied, but at higher p_T and smaller $\langle x_G \rangle$. Since $f_{1T}^{\perp\,g\,(f)}$ is not expected to keep up with the rise of the unpolarized gluon distribution towards small x, x should be small to enhance the gluon component w.r.t. the quark one, but not too small either. Lower x also means lower Q^2 so an optimum should be sought here as well. The x range of EIC, which is roughly $[10^{-4}, 10^{-1}]$ seems well suited for this study. A comparison between the heavy quark pair production and the lighter meson pair production will be interesting and helpful. For COMPASS the QCD Compton contribution ($\gamma^* q \to qg$) is significant leading to an asymmetry for deuterons of $-0.12 \pm 0.11({\rm stat.}) \pm 0.08({\rm syst.})$ and protons of $-0.13 \pm 0.05({\rm stat.}) \pm 0.03({\rm syst.})$. **Projections for EIC are given in [9]** assuming a gluon Sivers function that is 5% of the bound and an integrated luminosity of 10 fb^{-1}. Their Fig. 11 shows that the few permille level asymmetries are on the border of being discernible within statistics in this case, which is promising.

Dijet production: Similar considerations apply to dijet production [7]. Typically this will require higher Q^2 and higher transverse momenta than for light hadron or heavy meson pairs. The resolution of the sum transverse momentum will generally be less good. For dijets the $\gamma^* q \to qg$ and $\gamma^* \bar{q} \to \bar{q}g$ subprocesses matter. Dijets are more promising w.r.t. heavy quarks because of the smaller statistical error that can be achieved. **Projections for EIC are given in [9]** assuming a gluon Sivers function that is 5% of the bound and an integrated luminosity of 10 fb^{-1}. Their Fig. 13 shows that the $\sim$ 5 permille level asymmetries are discernible within statistics which probably makes this the most promising channel to probe the f-type gluon Sivers TMD.

The SSA in the process of **single D-meson detection**, $e\,p^{\uparrow} \to e'\,D\,X$ [2, 3], for large p_T D-mesons will not allow to probe the gluon Sivers TMD. It will be sensitive to twist-3 functions that may in part be related to the gluon Sivers TMD. A Generalized Parton Model (GPM) study has been done and will be interesting to compare to, but it should be stressed that it does not give a theoretically solid extraction of the gluon Sivers TMD, just as for A_N in $p^{\uparrow}p \to \pi X$. Fig. 9b of [9] shows percent level asymmetries are expected for a gluon Sivers function that is 10% of the bound and for an integrated luminosity of 10 fb^{-1}, which is discernible within the statistics.

B. Physics objective 2: Accessing linearly polarized gluons at EIC

The process $e\,p \to e'\,D^0\,\overline{D}^0\,X$ for unpolarized protons is also considered interesting because it allows to probe the distribution of linearly polarized gluons, described by the TMD $h_1^{\perp\,g}$ [10]. Again the function with two future pointing Wilson lines is probed, which *in the small-x limit* is also referred to as the Weiszäcker-Williams (WW) distribution. This is the same one that contributes to Higgs or $\eta_{c,b}$ production in proton-proton collisions.

In [7] it was pointed out that there are different asymmetries that can be considered: $\cos 2\phi_T$ and $\cos 2(\phi_T - \phi_\perp)$. The former depends on the orientation w.r.t. the lepton scattering plane, whereas the latter is independent from it. Estimates for the maximum allowed asymmetry and from a small-x model are not very different: they indicate that the asymmetries may be on the 10% level for both charm and bottom production. Especially at larger transverse momentum the WW-type $h_1^{\perp\,g}$ is expected to keep up with the growth of the unpolarized gluon distribution towards small x. For the measurement of these asymmetries considerations regarding the kinematic region are very similar to those for the gluon Sivers function. However, since one is dealing with a higher harmonic (the Sivers asymmetry is just a left-right asymmetry), the binning in the angular dependence is more important now and ideally there should be full 2π coverage. **No projections for EIC have been given yet**, only model predictions or maximal asymmetries.

Dijet production: The differential cross section expressions were given in [7] and the small-x study of [11] predicted large rising asymmetries from 10-40% for q_T in the range $[1, 4]$ GeV. In [12] it was shown that for the q_T distribution the jets form a good proxy for the partons that lead to the jets, except at the edges (for $q_T < 1$ GeV and $q_T > 2.5$ GeV). Moreover, it is pointed out that the asymmetry is expected to change sign depending on the polarization of the virtual photon, which cannot be studied on an event-to-event basis, but should be possible when averaged over many events. **Projections for EIC are given in [12].** For example, the S/B shows an optimum around $Q^2 \sim 9$ GeV^2 for $\sqrt{s} = 90$ GeV, an integrated luminosity of 10 fb$^{-1}/A$, and cuts $1.25 < q_T < 1.75$ GeV/c, $3 < K_\perp < 3.5$ GeV/c, $4 < Q^2 < 12$ GeV^2, and $1 < \eta < 2.5$. Again sufficient resolution and range in q_T and $K_\perp$ are required.

C. Physics objective 3: TMD evolution studies at EIC

When one considers heavy quark pair production the invariant mass of the combined system depends not only on the heavy quark mass but also on the transverse momentum of the heavy quarks. In this way one can study the scale evolution of the observables. One can vary in this invariant mass and in Q^2. They typically should be considered of the same magnitude in order not to generate large logarithms of two large scales that are very different in size. The possibility to study the TMD evolution is possible in heavy quark pair production, in hadron pair and dijet production, but not in quarkonium production. **No TMD evolution predictions for pair production processes have been given yet.**

II. PROCESS 2: QUARKONIUM PRODUCTION

Heavy quarkonium production in SIDIS can be considered for all ranges in transverse momentum, but small and large transverse momentum require different analyses. Depending on the quantum numbers of the state and the kinematic region the heavy quarkonium can be in a color singlet and/or in a color octet state. Irrespective of the interpretation of the process, it is clear that at EIC one would like to study various quarkonia $(J/\psi, \psi(2S), \Upsilon, \eta_{c,b}, \ldots)$ over a wide range of transverse momenta. It is not yet clear what can be done precisely. Considering low p_T of the quarkonium is theoretically not a problem in electron-proton scattering as opposed to proton-proton scattering. At high p_T of the quarkonium one is actually considering quarkonium plus jet production in SIDIS [14] or in photoproduction [15], allowing a study of the shape functions [16–18], of TMDs through the determination of q_T and of TMD evolution. **No projections for EIC have been given yet.**

A. Physics objective 1: Gluon Sivers function measurements at EIC

At low p_T of the quarkonium the Sivers asymmetry has a similar simple form as in open heavy quark production as a function of q_T [13]. Good p_T resolution will thus be important here as well. At high p_T the expressions are given in [14]. **No projections for EIC have been given**, only model predictions or maximal asymmetries.

B. Physics objective 2: Accessing linearly polarized gluons at EIC

Unlike the Sivers asymmetry the $\cos 2\phi_T$ and $\cos 2(\phi_T - \phi_\perp)$ asymmetries are not only ratios involving kinematic factors and TMDs, but also of the NRQCD Color Octet (CO) Long Distance Matrix Elements (LDMEs). However, one can construct ratios of other SSA (different from the Sivers one) to cancel out these quite uncertain factors, cf. [13]. **No projections for EIC have been given**, only model predictions or maximal asymmetries. Dijet cross sections as a function of the transverse momentum of the two jets has been studied at ZEUS [19]. No conclusions about gluon TMDs can be drawn from those data. The results are in agreement with collinear NLO predictions within sizable errors.

C. Physics objective 3: Extracting NRQCD color octet LDMEs at EIC

The comparison of the $\cos 2\phi_T$ asymmetries in open heavy quark pair production and quarkonium production allows to extract the still poorly known NRQCD CO LDMEs as pointed out in [13]:

$$\mathcal{R}^{\cos 2\phi_T} = \frac{\int \mathrm{d}\phi_T \cos 2\phi_T \, \mathrm{d}\sigma^{\mathcal{Q}}(\phi_S, \phi_T)}{\int \mathrm{d}\phi_T \, \mathrm{d}\phi_\perp \cos 2\phi_T \, \mathrm{d}\sigma^{Q\overline{Q}}(\phi_S, \phi_T, \phi_\perp)} = \frac{27\pi^2}{4} \frac{1}{M_Q} \left[\mathcal{O}_8^S - \frac{1}{M_Q^2} \mathcal{O}_8^P \right], \tag{2}$$

$$\mathcal{R} = \frac{\int \mathrm{d}\phi_T \, \mathrm{d}\sigma^{\mathcal{Q}}(\phi_S, \phi_T)}{\int \mathrm{d}\phi_T \, \mathrm{d}\phi_\perp \, \mathrm{d}\sigma^{Q\overline{Q}}(\phi_S, \phi_T, \phi_\perp)} = \frac{27\,\pi^2}{4} \frac{1}{M_Q} \frac{[1 + (1-y)^2]\, \mathcal{O}_8^S + (10 - 10y + 3y^2)\, \mathcal{O}_8^P / M_Q^2}{26 - 26y + 9y^2}, \tag{3}$$

where $\mathcal{O}_8^S = \langle 0 | \mathcal{O}_8^{\mathcal{Q}}(^1S_0) | 0 \rangle$ and $\mathcal{O}_8^P = \langle 0 | \mathcal{O}_8^{\mathcal{Q}}(^3P_0) | 0 \rangle$ denote the CO LDMEs that can be probed. Their determinations thus far are listed in tables I and II.

J/ψ	$\langle 0 \vert \mathcal{O}_8^{J/\psi}(^1S_0) \vert 0 \rangle$	$\langle 0 \vert \mathcal{O}_8^{J/\psi}(^3P_0) \vert 0 \rangle / M_c^2$	
Fit No. 1 by Chao *et al.* [20]	8.9 ± 0.98	0.56 ± 0.21	$\times 10^{-2}$ GeV^3
Fit No. 2 by Sharma & Vitev [21]	1.8 ± 0.87	1.8 ± 0.87	$\times 10^{-2}$ GeV^3
Fit No. 3 by Butenschön & Kniehl [22]	4.50 ± 0.72	-0.72 ± 0.21	$\times 10^{-2}$ GeV^3
Fit No. 4 by Bodwin *et al.* [23]	9.9 ± 2.2	0.07 ± 0.06	$\times 10^{-2}$ GeV^3

TABLE I: Fit values of the CO LDMEs from J/ψ production.

$\Upsilon(1S)$	$\langle 0 \vert \mathcal{O}_8^{\Upsilon(1S)}(^1S_0) \vert 0 \rangle$	$\langle 0 \vert \mathcal{O}_8^{\Upsilon(1S)}(^3P_0) \vert 0 \rangle / (5M_b^2)$	
Fit No. 5 by Sharma & Vitev [21]	1.21 ± 4.0	1.21 ± 4.0	$\times 10^{-2}$ GeV^3

TABLE II: Fit values of the CO LDMEs from $\Upsilon(1S)$ production.

These results translate into the values for $\mathcal{R}$ and $\mathcal{R}^{\cos 2\phi_T}$ as in the figures below.

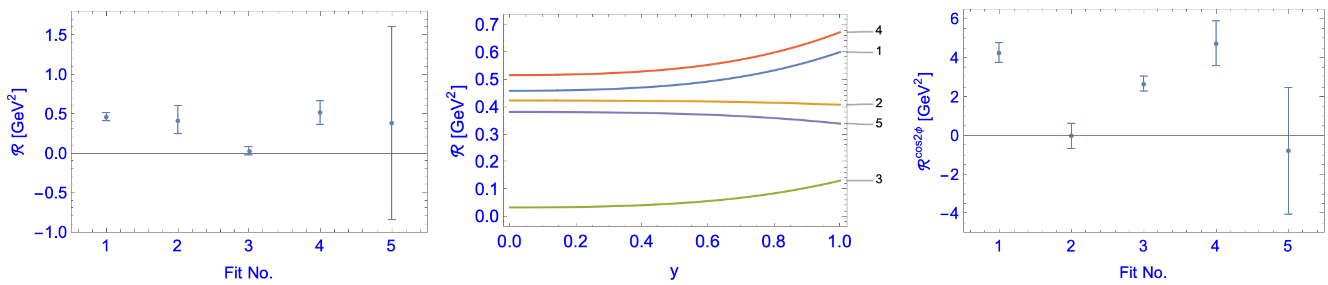

FIG. 1: The values for $\mathcal{R}$ and $\mathcal{R}^{\cos 2\phi_T}$ obtained using the fits for the CO LDMEs given in the tables. No possible correlations between the errors of the two LDMEs have been taken into account. The left figure shows $\mathcal{R}$ for $y = 0.1$ and the middle figure as function of y for the central values of the 5 fits. The right figure shows $\mathcal{R}^{\cos 2\phi_T}$ and holds for all y.

Here $K_\perp = Q = 2M_Q$ is chosen in order to avoid having to consider TMD evolution in the comparison of the differential cross sections and $z = 1/2$. For the more general expressions we refer to [13]. Note that the ratios are not normalized. Also, although $\mathcal{R}^{\cos 2\phi_T}$ has no y dependence, both numerator and denominator have a prefactor

$(1-y)$, hence vanish at $y = 1$. Furthermore, there is some additional uncertainty from final state smearing effects, but those are expected to enter as an overall q_T-dependent prefactor [13, 16]. Moreover, there is a way to cross check the extracted CO LDMEs by measuring the polarization state of the quarkonium, cf. [13]. **No projections for EIC have been given and no studies of how well the polarization of the quarkonium will be able to be determined have been performed.**

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