

QCD and the proton spin

Yoshitaka Hatta

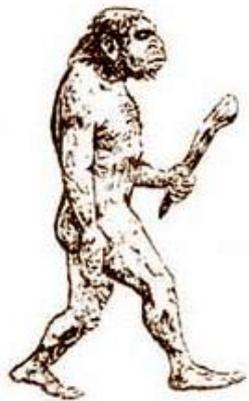
Nuclear theory group

Brookhaven National Laboratory

Outline

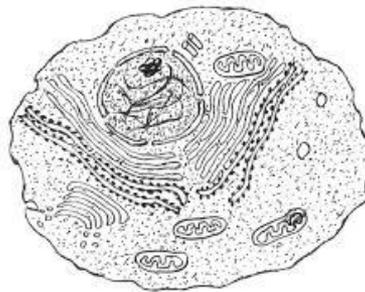
- Inside the microscopic world of hadrons
- The strong interaction and QCD
- Spin in quantum mechanics
- Physics of proton spin

The smallest unit in Nature



1m

human



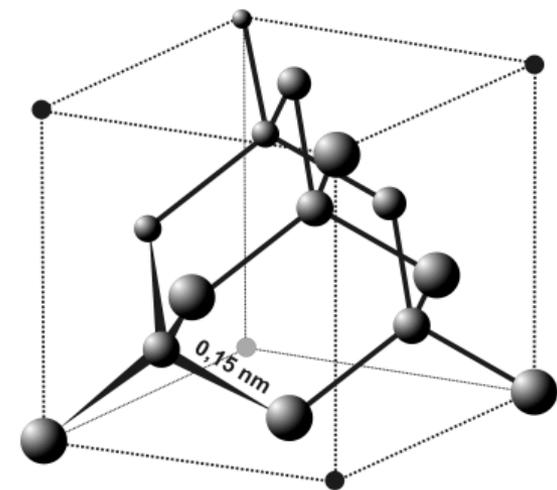
cell, bacteria,..

micrometer

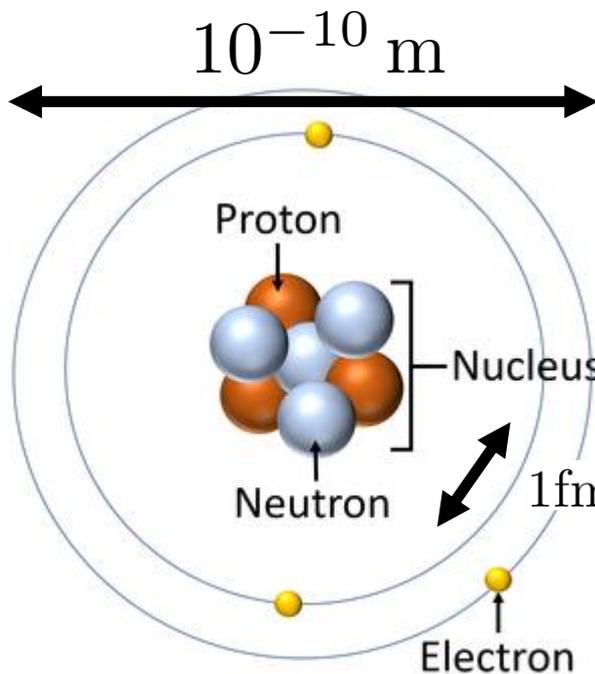
$$1\mu\text{m} = 10^{-6}\text{m}$$

nanometer

$$1\text{nm} = 10^{-9}\text{m}$$



molecule



atom

femtometer

cf. Planck length 10^{-35}m

Proton

Rutherford discovered the atomic nucleus in 1911.

Then in 1919, he discovered the proton.

In 1932, his student Chadwick discovered the neutron.

They are the building block of all known elements.

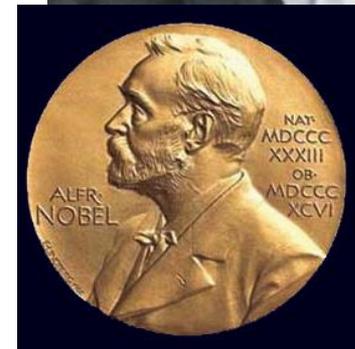
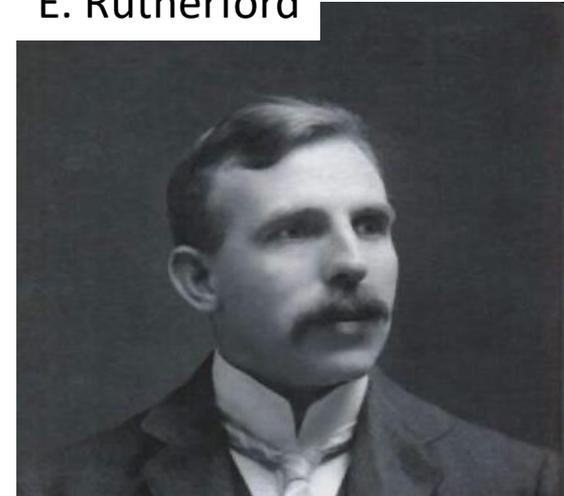
Mass $m_{proton} = 938\text{MeV}/c^2$

$$1\text{MeV}/c^2 = 1.78 \times 10^{-30}\text{kg}$$

Charge $+e$

Spin $\frac{1}{2}$

E. Rutherford

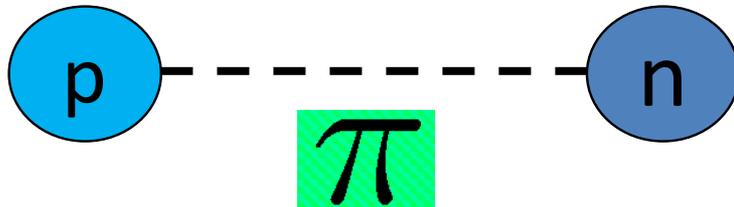


ザフォード

Meson

Force between particles arises from the exchange of another particle,
Like 'playing catch'.

Protons and neutrons are bound together to form a nucleus by exchanging **pions**.

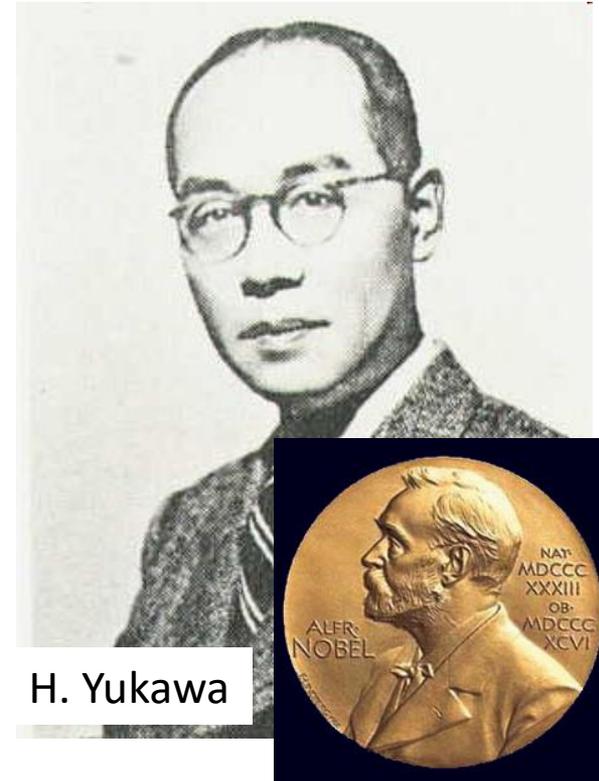


Much stronger than the electromagnetic interaction
Reach of the interaction
=inverse of the mass of the exchanged particle.

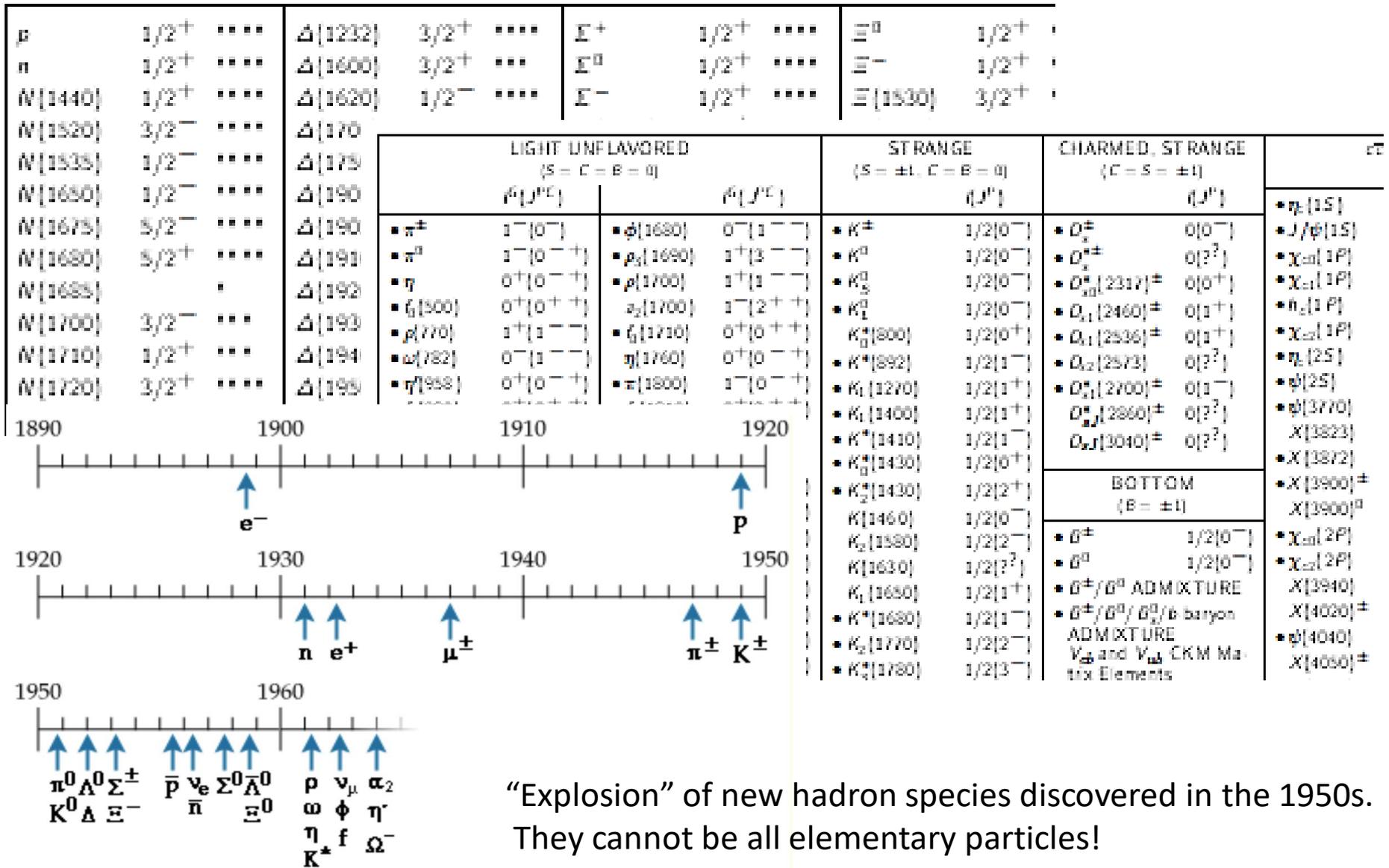
$$m_{\pi} \sim \frac{1}{1\text{fm}} \sim 200\text{MeV}/c^2$$

Predicted in 1934, discovered in 1947

$$m_{\pi} \approx 140\text{MeV}/c^2$$

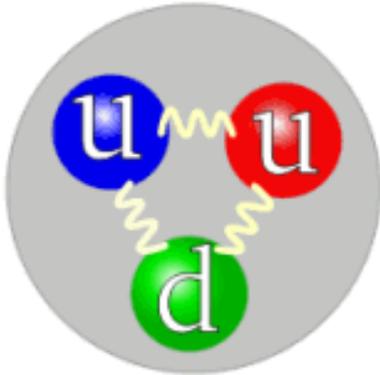


Hadrons (baryons and mesons)

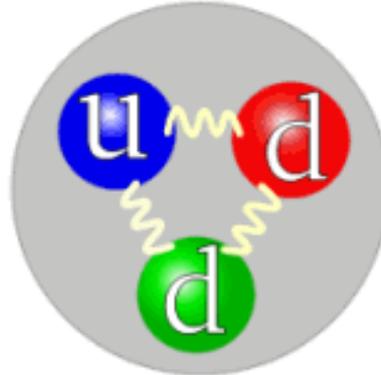


“Explosion” of new hadron species discovered in the 1950s. They cannot be all elementary particles!

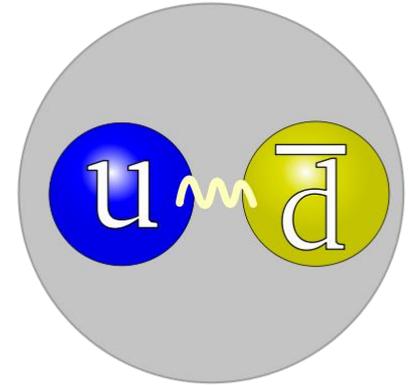
Quarks (1964)



proton



neutron

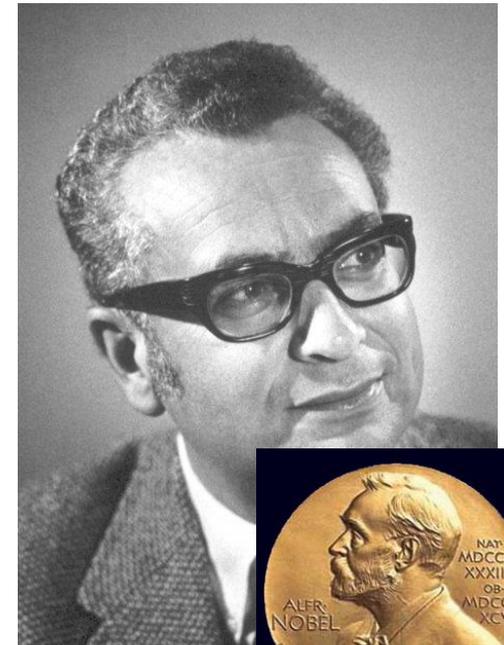


pion

Quarks have fractional electric charges

$$1 = \frac{2}{3} + \frac{2}{3} - \frac{1}{3}$$

$$0 = \frac{2}{3} - \frac{1}{3} - \frac{1}{3}$$

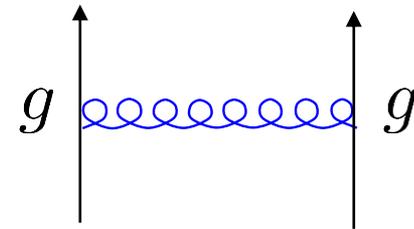
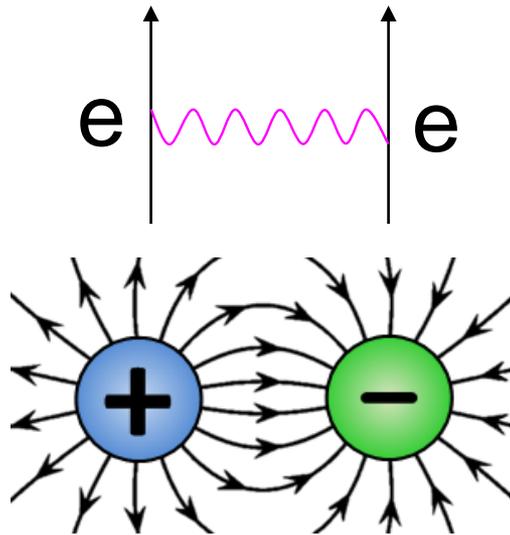


M. Gell-Mann

First generation	Second generation	Third generation
Up	Charm	Top
Down	Strange	Bottom

Gluons

Quarks interact with each other by exchanging massless particles called 'gluons', just like charged particles interact by exchanging photons.



Quarks have a **color** charge (in addition to electric charge)

All the four known forces of Nature are explained by the exchange of **gauge** particles.

Electromagnetic interaction (QED) → photon

Strong interaction (QCD) → gluon

Weak interaction → W,Z bosons

Gravity → graviton

Theory of the strong interaction

Quantum Chromodynamics

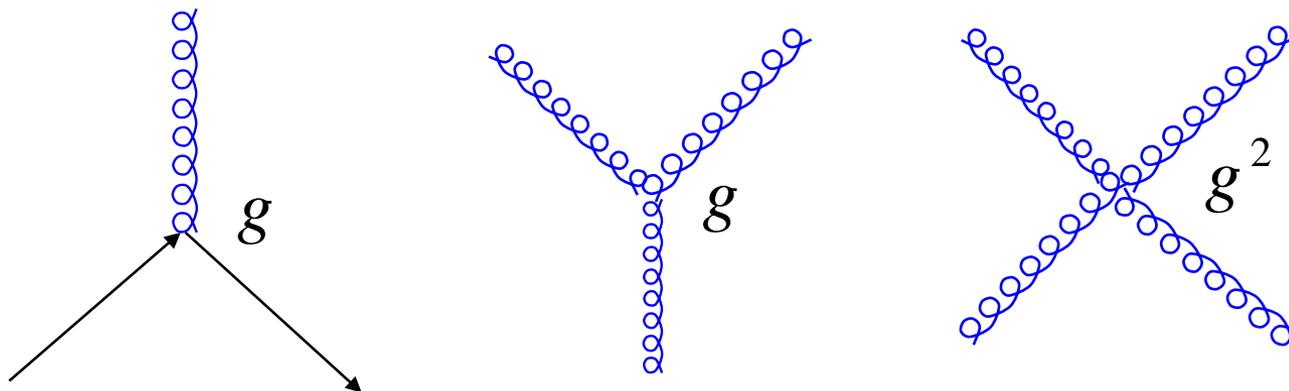
Lagrangian

Gauge theory based on the color SU(3) group

$$L = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a + \bar{q}(i\gamma^\mu \partial_\mu - m)q + g\bar{q}\gamma^\mu A_\mu q$$

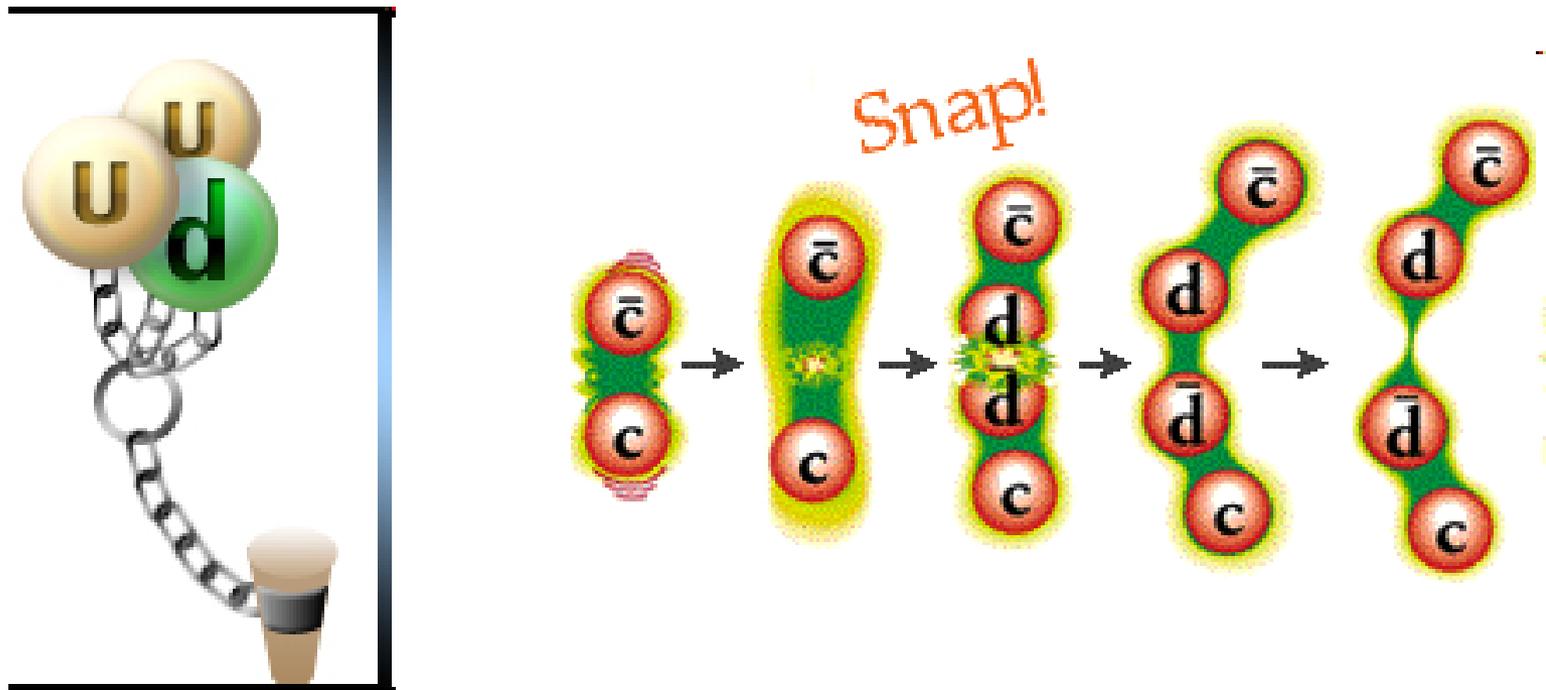
$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \leftarrow u = \begin{pmatrix} r \\ g \\ b \end{pmatrix}$$

red
green
blue



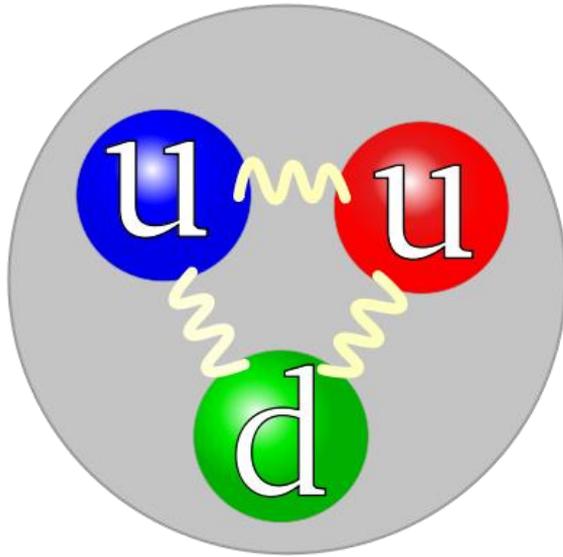
Confinement of color

Quarks are permanently confined into hadrons.
Cannot be detected in isolation



Only color neutral particles (hadrons) can be directly observed.

The mystery of proton mass



$$m_u = 3\text{MeV}/c^2$$

$$m_d = 5\text{MeV}/c^2$$

$$m_g = 0$$

$$m_{\text{proton}} = 938\text{MeV}/c^2$$

$$1\text{MeV}/c^2 = 1.78 \times 10^{-30}\text{kg}$$

$$3 + 3 + 5 + 0 = 938 \text{ !?}$$

Higgs mechanism can explain the quark masses, but not the proton mass!

Millennium Problems

Yang–Mills and Mass Gap

Experiment and computer simulations suggest the existence of a "mass gap" in the solution to the quantum versions of the Yang–Mills equations. But no proof of this property is known.

Riemann Hypothesis

The prime number theorem determines the average distribution of the primes. The Riemann hypothesis tells us about the deviation from the average. Formulated in Riemann's 1859 paper, it asserts that all the 'non-obvious' zeros of the zeta function are complex numbers with real part $1/2$.

P vs NP Problem

If it is easy to check that a solution to a problem is correct, is it also easy to solve the problem? This is the essence of the P vs NP question. Typical of the NP problems is that of the Hamiltonian Path Problem: given N cities to visit, how can one do this without visiting a city twice? If you give me a solution, I can easily check that it is correct. But I cannot so easily find a solution.

Navier–Stokes Equation

This is the equation which governs the flow of fluids such as water and air. However, there is no proof for the most basic questions one can ask: do solutions exist, and are they unique? Why ask for a proof? Because a proof gives not only certitude, but also understanding.

Hodge Conjecture

The answer to this conjecture determines how much of the topology of the solution set of a system of algebraic equations can be defined in terms of further algebraic equations. The Hodge conjecture is known in certain special cases, e.g., when the solution set has dimension less than four. But in dimension four it is unknown.

Poincaré Conjecture

In 1904 the French mathematician Henri Poincaré asked if the three dimensional sphere is characterized as the unique simply connected three manifold. This question, the Poincaré conjecture, was a special case of Thurston's geometrization conjecture. Perelman's proof tells us that every three manifold is built from a set of standard pieces, each with one of eight well-understood geometries.

Birch and Swinnerton-Dyer Conjecture

Supported by much experimental evidence, this conjecture relates the number of points on an elliptic curve mod p to the rank of the group of rational points. Elliptic curves, defined by cubic equations in two variables, are fundamental mathematical objects that arise in many areas: Wiles' proof of the Fermat Conjecture, factorization of numbers into primes, and cryptography, to name three.

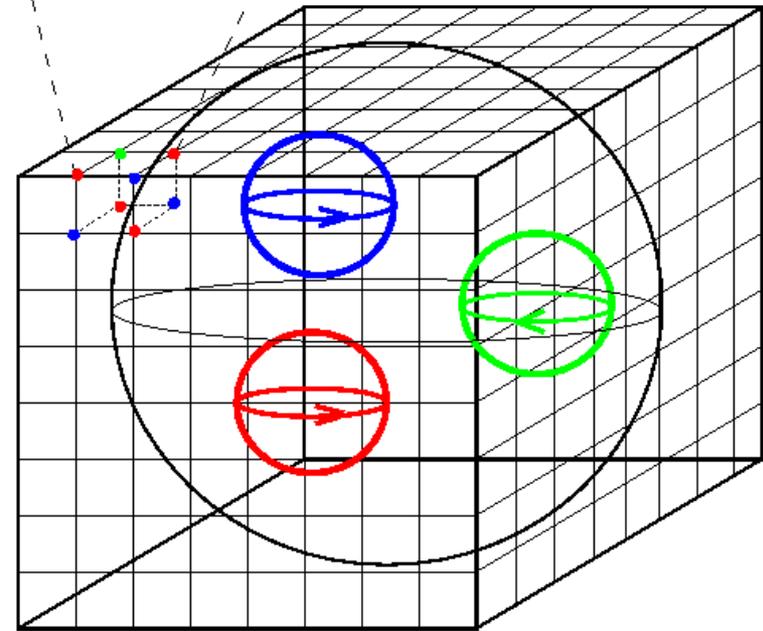
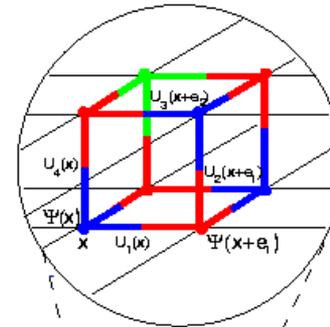
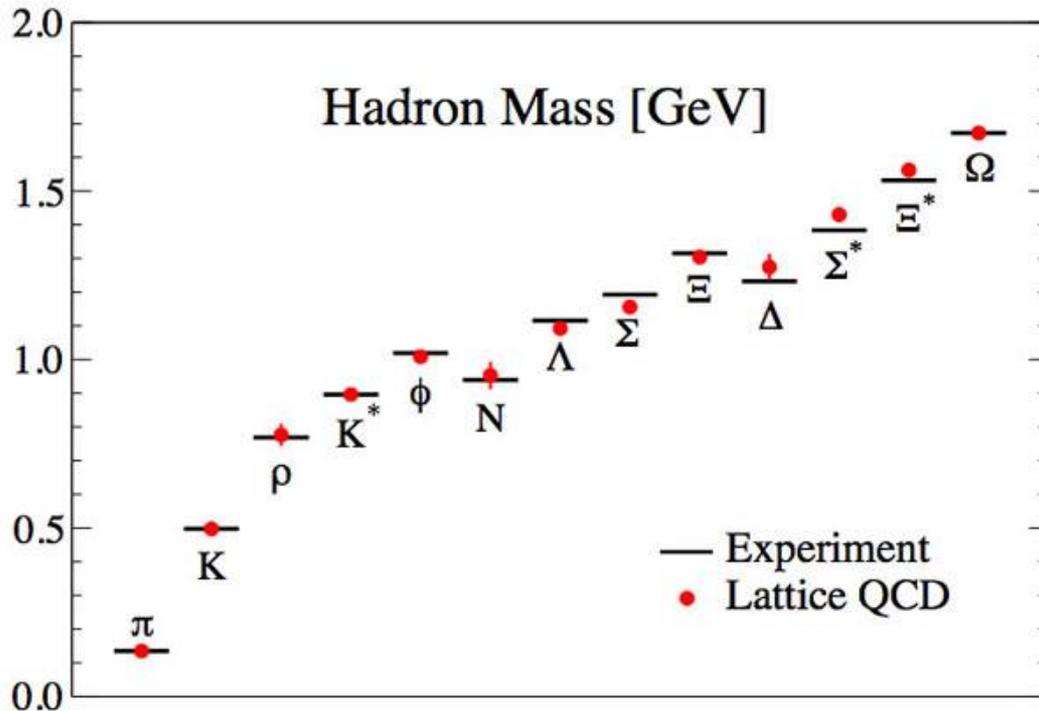
Millennium prize problems
Clay mathematics institute
(2000)



← Solved by G. Perelman

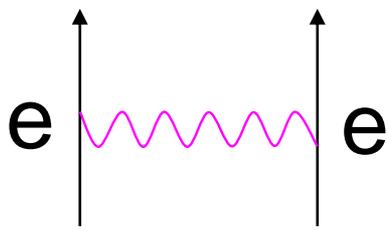
Hadron mass from lattice QCD simulations

Solve QCD on a discrete lattice using a supercomputer.



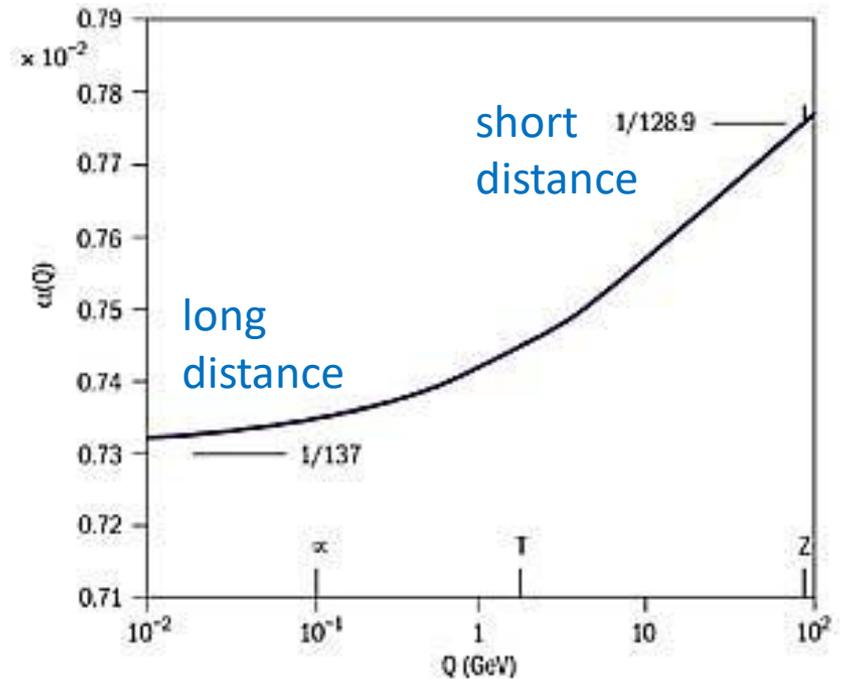
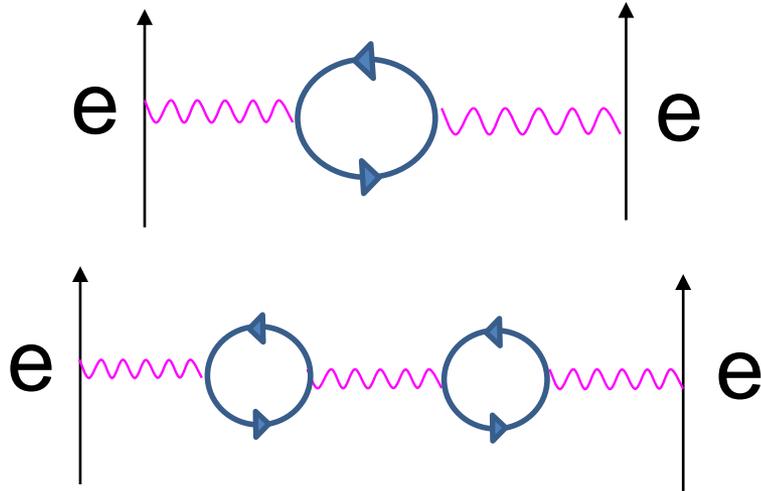
Coupling 'constant' is not a constant

Coulomb interaction



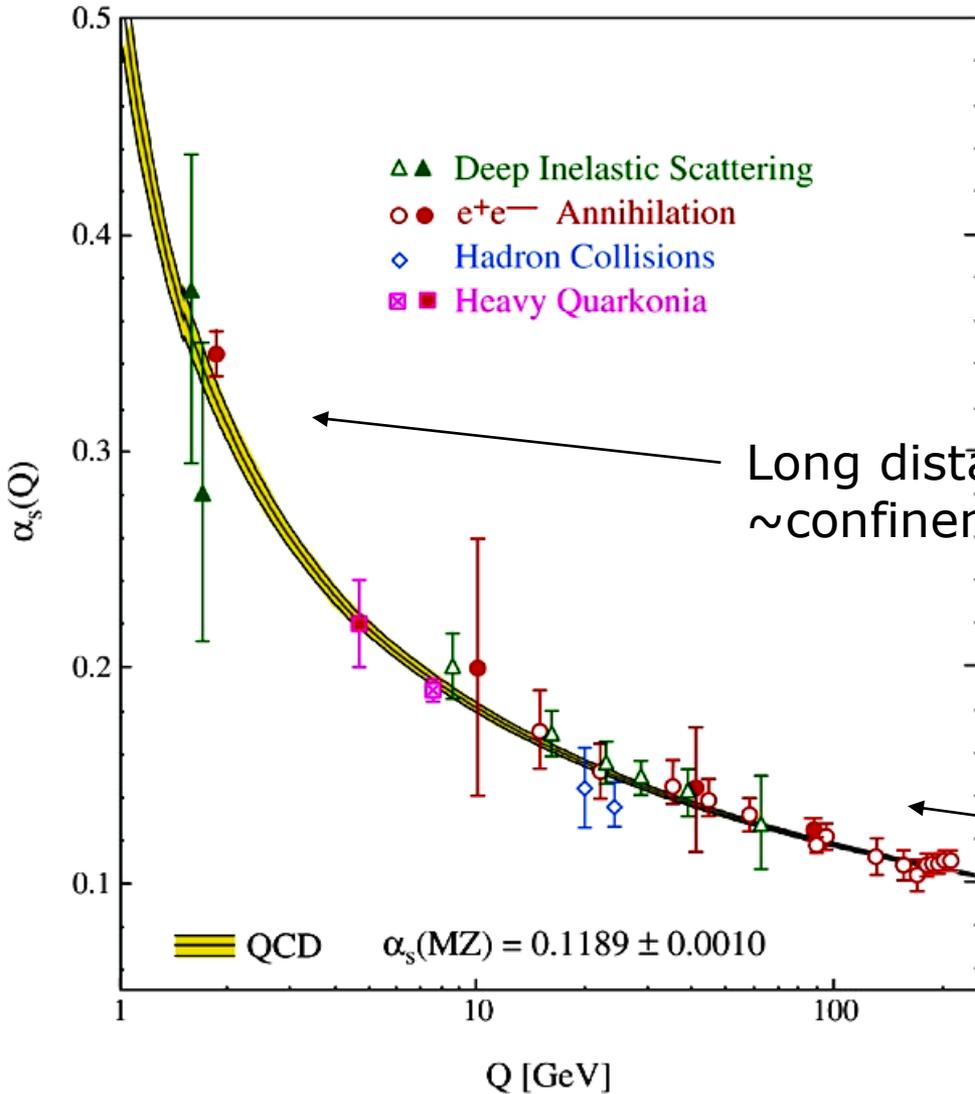
$$\vec{F} \sim \alpha_{em} \frac{\vec{r}}{r^3}$$

$$\alpha_{em} = \frac{e^2}{4\pi} \approx \frac{1}{137}$$

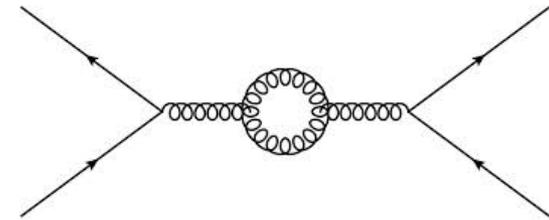


Coupling constant gets weaker at longer distances. → Charge screening

Asymptotic freedom



In QCD, the slope has an opposite sign due to a graph like this.



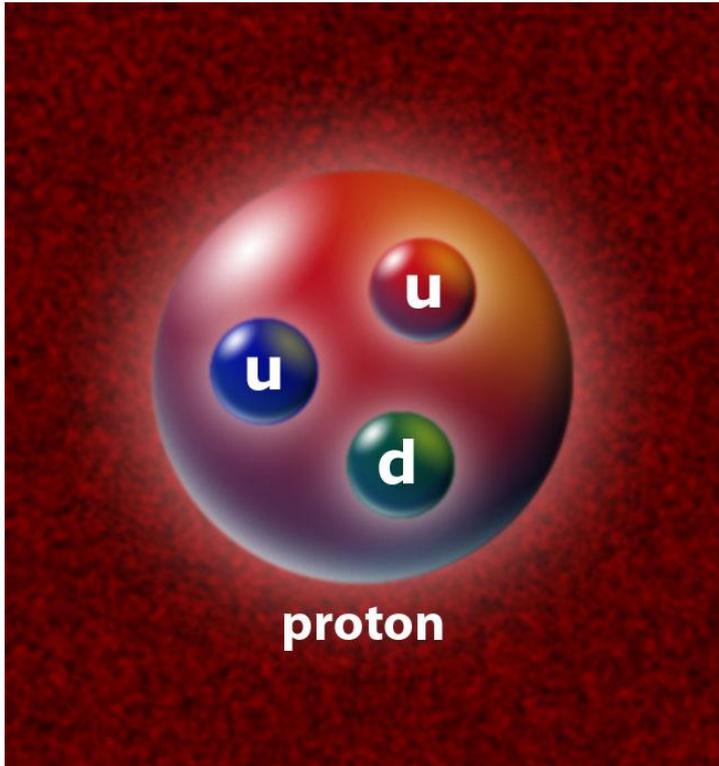
Gross-Wilczek-Politzer,



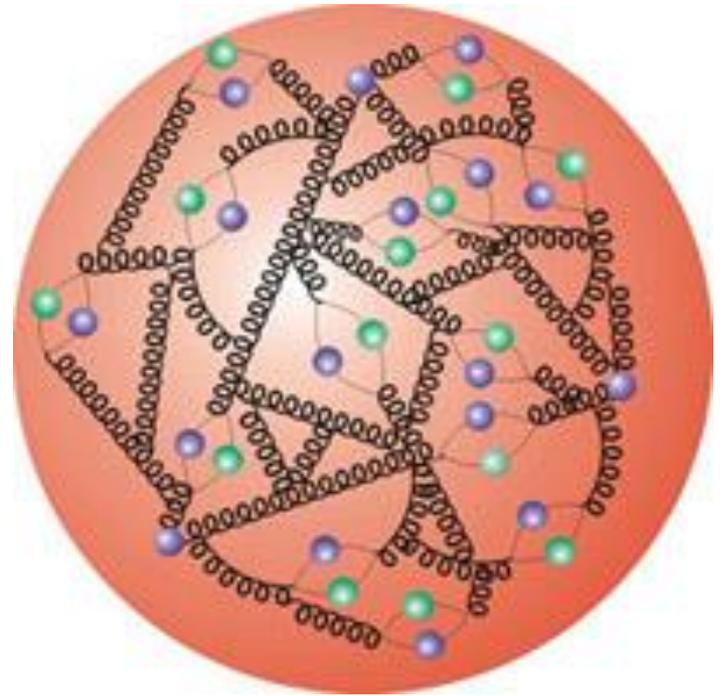
Long distance
 \sim confinement of quarks

Short distance
 \sim perturbation theory

Probing the internal structure of proton



OR



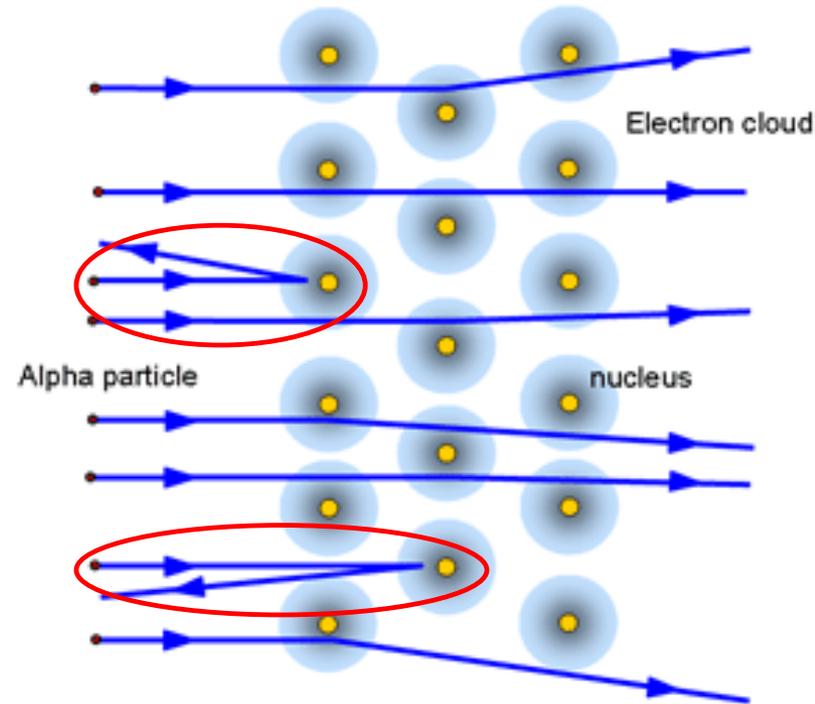
Quarks cannot be directly observed,
but can we probe them indirectly?

Discovery of nucleus (Rutherford, 1911)

Bombard a gold foil with a beam of alpha particles.

Most of the alpha particles pass through, but some bounced back 180 degrees.

There is a small and hard 'core' inside an atom → gold nucleus



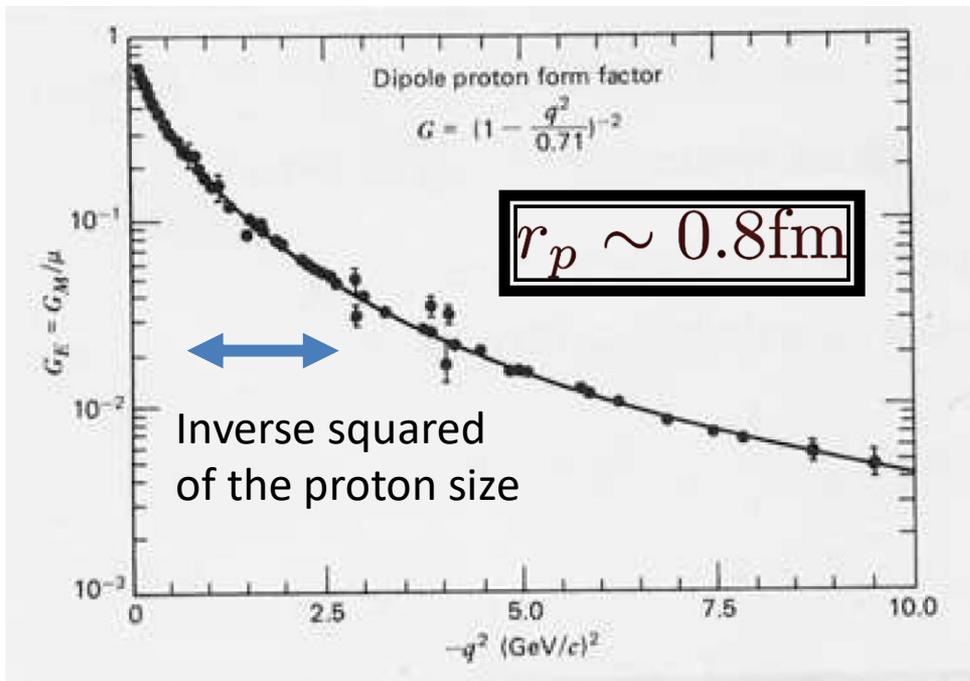
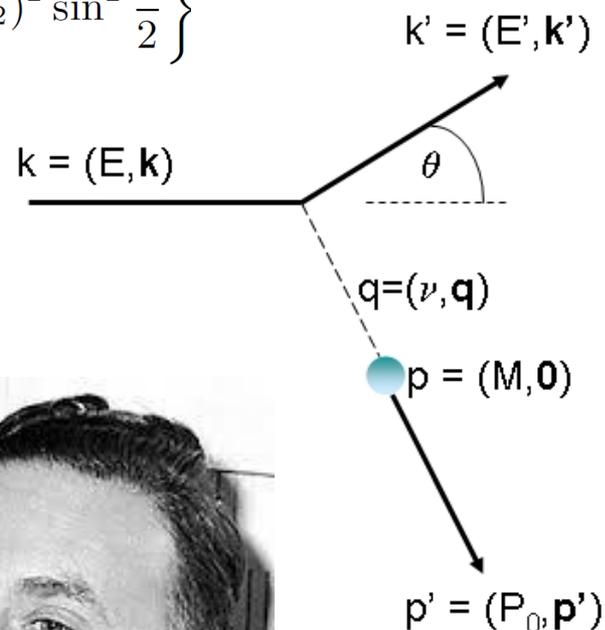
“It was quite the most incredible event that has ever happened to me in my life. It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you.”

Measuring the proton size in electron scattering

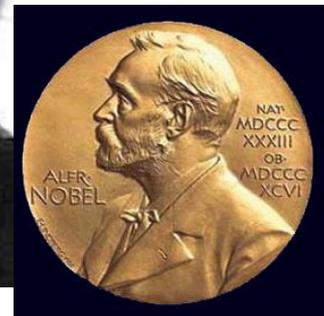
Cross section (probability of scattering)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4 \frac{\theta}{2}} \frac{E_3}{E_1} \left\{ \left(F_1^2 - \frac{\kappa_p^2 q^2}{4m_p^2} F_2^2 \right) \cos^2 \frac{\theta}{2} - \frac{q^2}{2m_p^2} (F_1 + \kappa_p F_2)^2 \sin^2 \frac{\theta}{2} \right\}$$

$F_{1,2}(q = k - k')$ **Form factors**



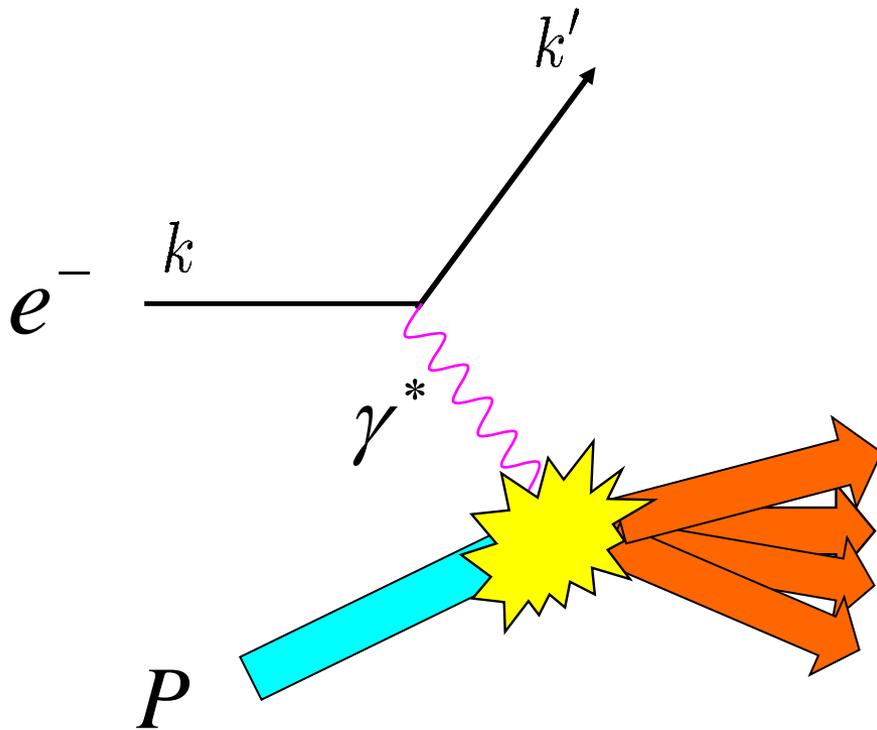
R. Hofstadter



Deep inelastic scattering (DIS)

At higher energy elastic scattering becomes inelastic.

In this regime, the incoming proton behaves like a bunch of pointlike particles, or 'partons'. [R. Feynman](#)



Two important variables to remember:

Resolution scale $Q \sim |k - k'|$
(magnifying power of a microscope)

Probe distances of order $1/Q$

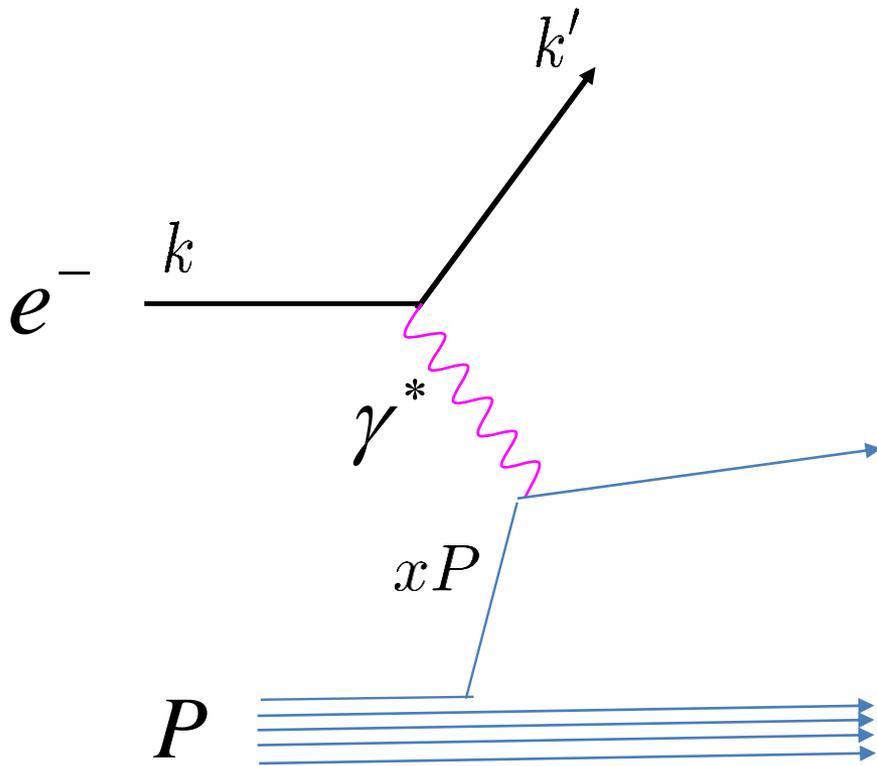
Momentum fraction carried by each parton

$$x = \frac{E_{parton}}{E_{proton}}$$

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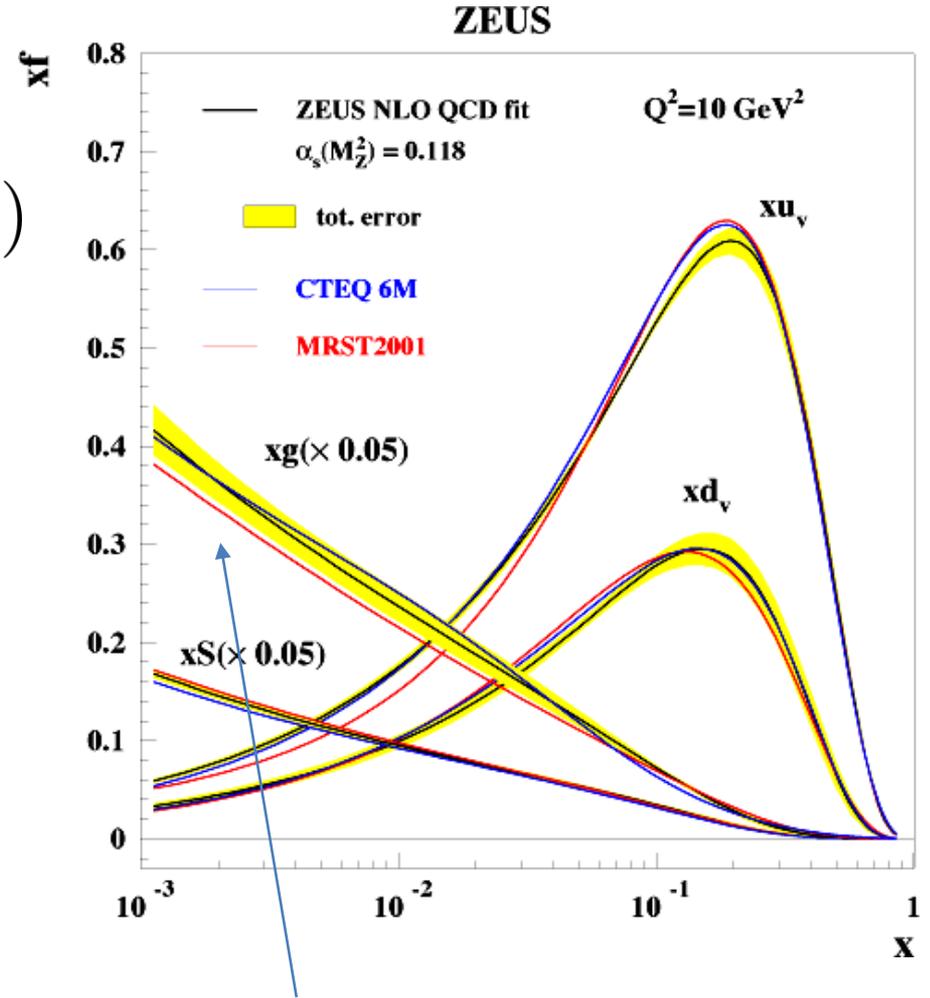
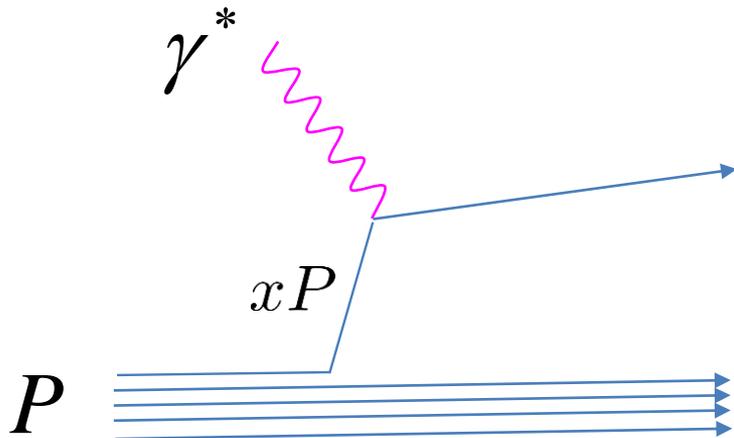
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Parton distribution function

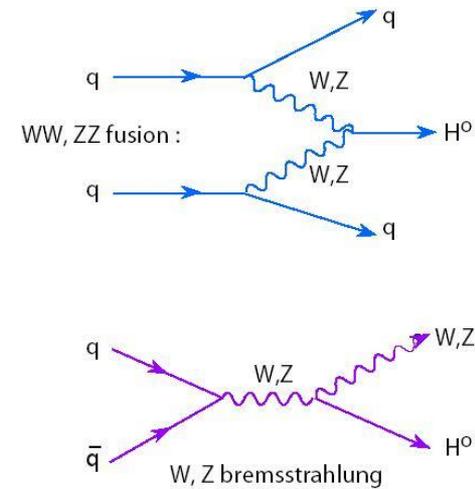
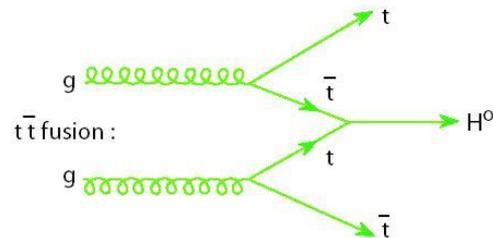
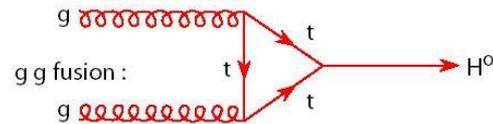
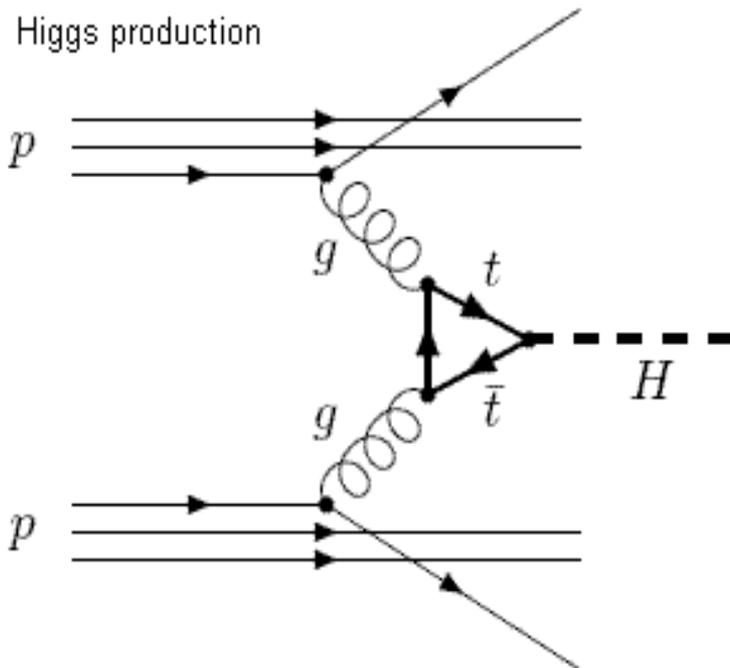
$$\sigma = \sum_i^{u,d,s,\dots} \int dx f_i(x) \sigma_i(xP)$$

Number density of partons with energy fraction x in a proton.



Notice the number of gluons with small values of x explodes.

Higgs production at the LHC



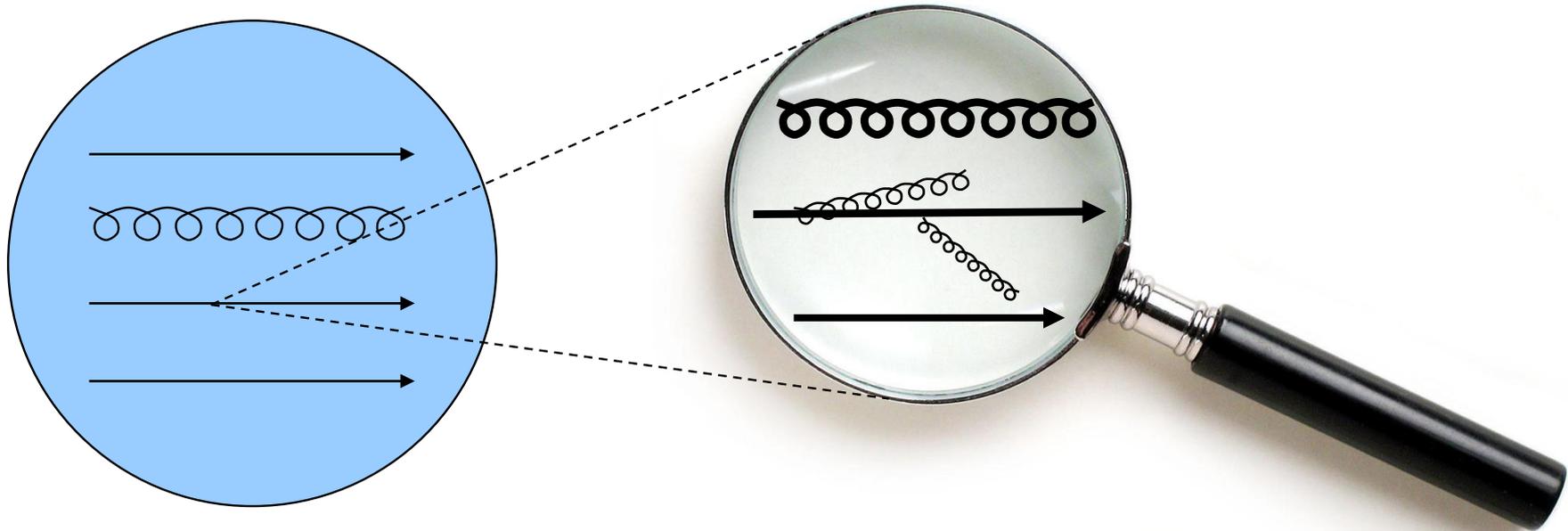
$$\sigma = \sum_{ij} \int dx_1 dx_2 f_i(x_1) f_j(x_2) \sigma_{ij}(x_1, x_2)$$

The same PDF as extracted from DIS can be used \rightarrow **universality** of PDF.

Parton evolution

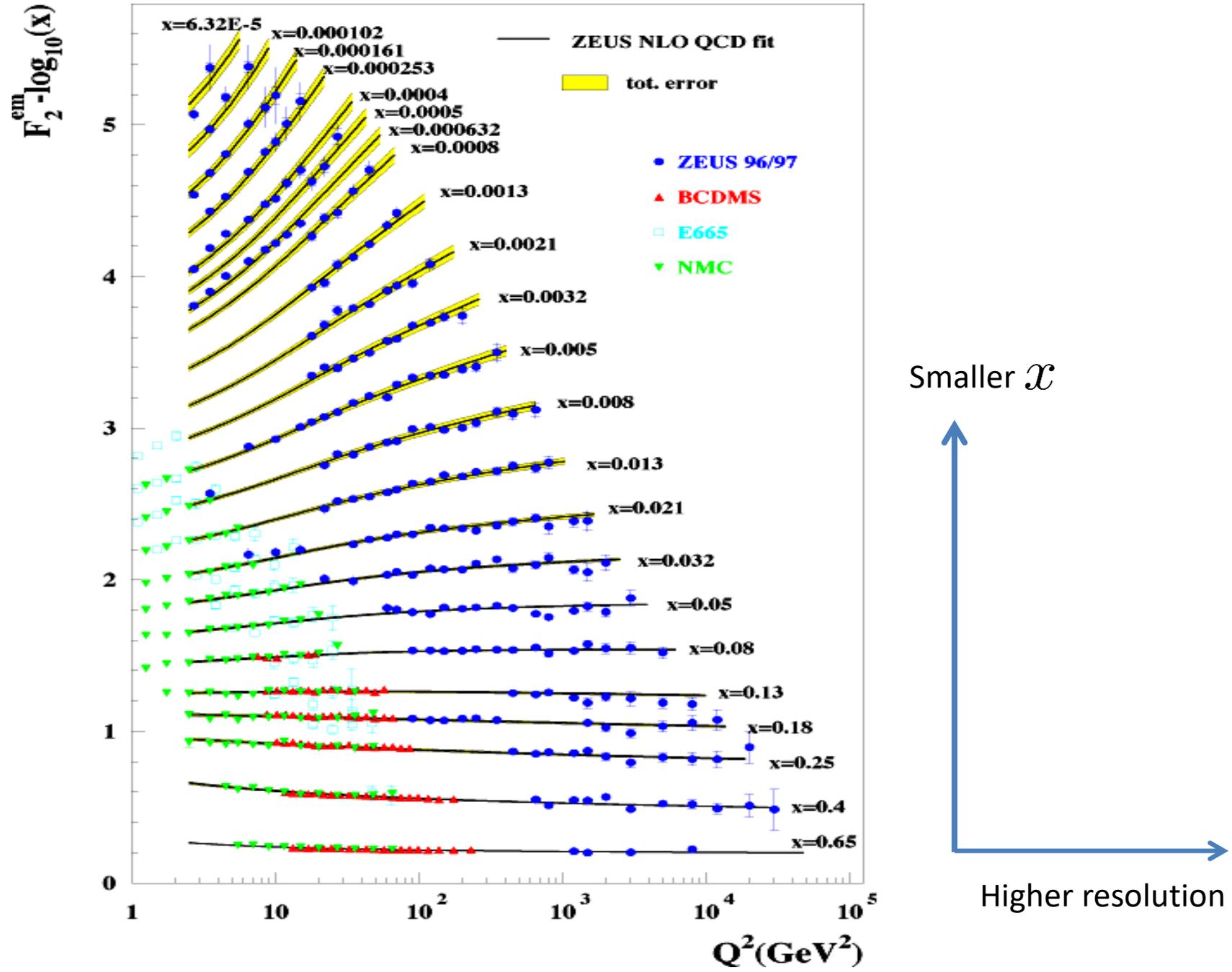
Partons can radiate. The distribution changes as the resolution scale Q is varied.

$$\frac{\partial}{\partial \ln Q^2} f(x, Q^2) = \int_x^1 dx' P(x') f(x/x', Q^2)$$



Dial up the magnification (increase Q) and you can see the finer structure of the proton.

ZEUS



Spin physics

Angular momentum in classical mechanics

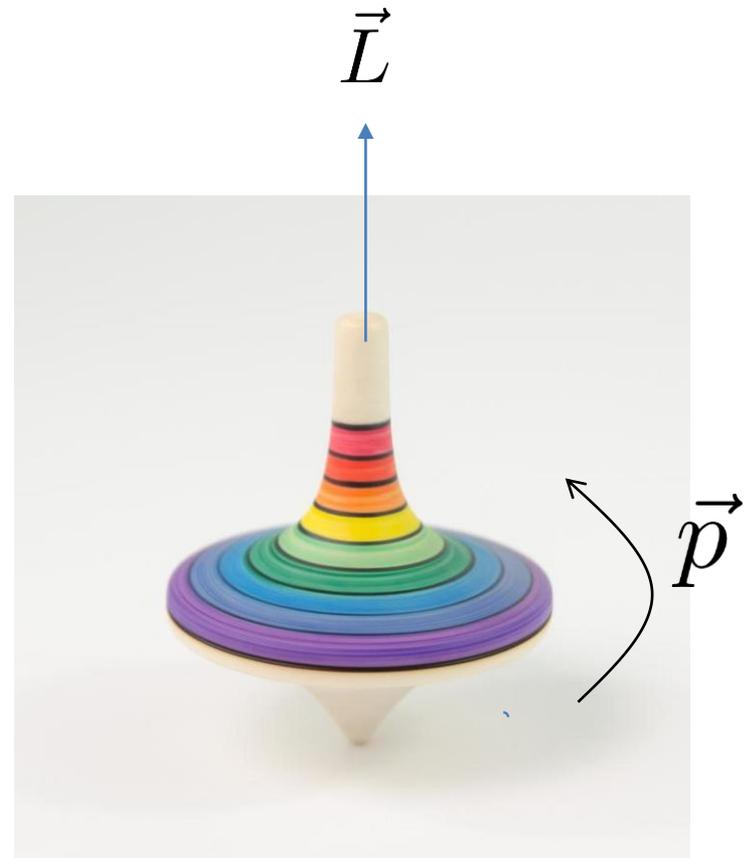
In classical mechanics, angular momentum is a vector perpendicular to the direction of rotation

$$\vec{L} = \vec{r} \times \vec{p}$$

It is **conserved** in the absence of external force (torque) ← rotational symmetry

$$\frac{d\vec{L}}{dt} = \dot{\vec{r}} \times \vec{p} + \vec{r} \times \dot{\vec{p}} = 0$$

and it can take any (continuous) value.



Spin in quantum mechanics

In quantum mechanics, the (orbital) angular momentum $\vec{L} = \vec{r} \times \vec{p}$ is **quantized** and takes **integer** values $|\vec{L}| = 0, 1, 2, \dots$ in units of $\hbar = \frac{h}{2\pi}$ (Planck constant)

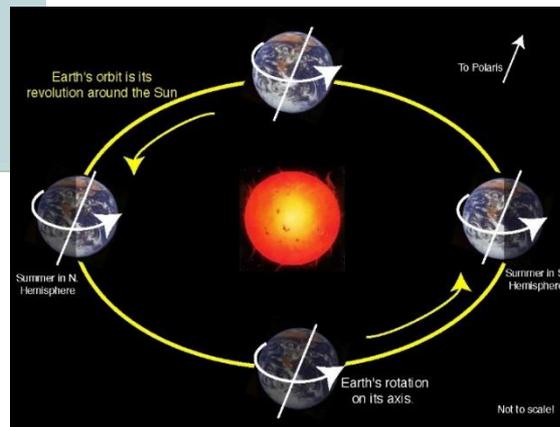
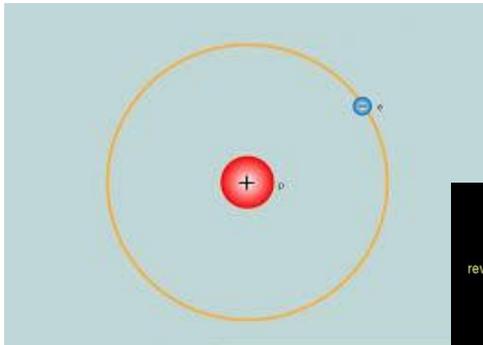
Separately to this, an electron has an intrinsic, **half-integer** $|\vec{s}| = \frac{1}{2}$ angular momentum called **spin**.

Electron wavefunction in a hydrogen atom

$$\psi(\vec{r}, s) = R(r)Y_{\ell m}(\theta, \phi)\chi(s)$$

spin ('rotation')

orbital ('revolution')



Beware, an electron is a pointlike particle, not really like a 'ball' spinning around some axis.

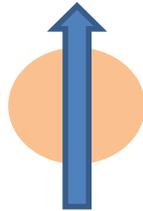
Still, the analogy to a spinning object is convenient for the sake of visualization.

Conception of spin

In 1925, G. Uhlenbeck and S. Goudsmit proposed the notion of spin to explain some puzzling phenomena. Completely ad hoc, but it worked.

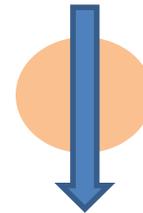
`spin-up'

$$s_z = \frac{1}{2}$$



`spin-down'

$$s_z = -\frac{1}{2}$$



“This is a good idea. Your idea may be wrong, but since both of you are so young without any reputation, you would not lose anything by making a stupid mistake.” — P. Ehrenfest

Earlier that same year, R. Kronig came up with the same idea, but he did not publish because

“It is indeed very clever but of course has nothing to do with reality.”

— W. Pauli

Today, spin is a fundamentally important concept in particle, nuclear and condensed matter physics.

Spin in particle physics

To really understand spin, you need to learn

1. special relativity
2. relativistic quantum mechanics
3. group theory (Lorentz group)

Every elementary particle has a unique value of **spin**

$$s = 0, \frac{1}{2}, 1, \dots$$

Integer spin \rightarrow **bosons** (photon, gluon, W,Z boson, Higgs,...)

Half-integer spin \rightarrow **fermions** (quark, electron, neutrino...)



P. Dirac



Dirac equation

$$\longrightarrow (i\gamma^\mu (\partial_\mu - igA_\mu) - m)q = 0$$

4x4 matrices

Understanding the origin of proton spin

The proton has spin-1/2

The proton is not an elementary particle.

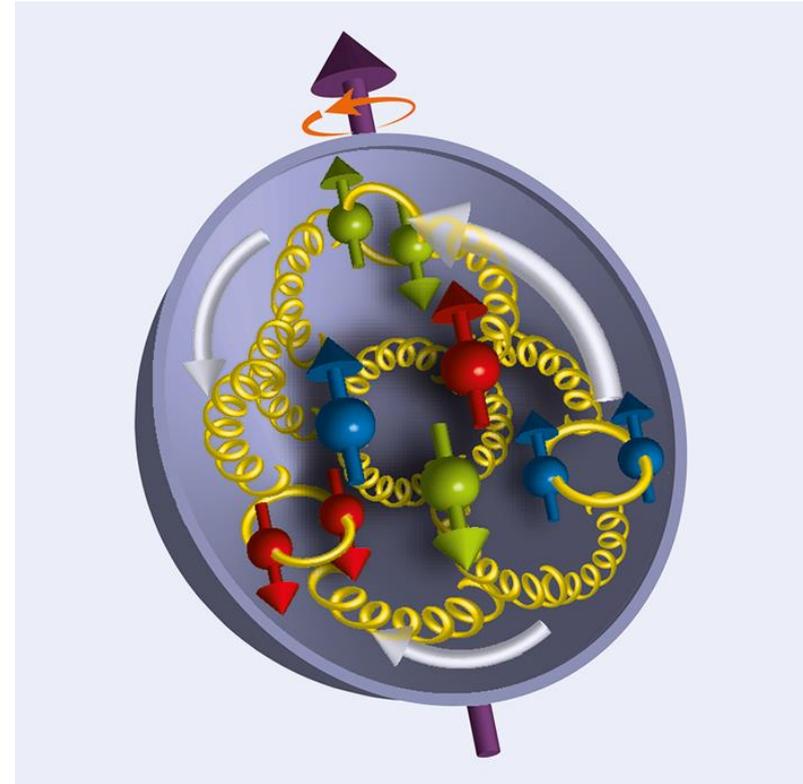


$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + L_z$$

quark spin

gluon spin

orbital angular
momentum



The spin of a composite particle can be ultimately understood by combining the angular momenta (spin+orbital) of elementary constituent particles.

Electron-Ion Collider (EIC)

Next-generation nuclear physics facility to be built at BNL

→ Lecture by E. Aschenauer next week



Brookhaven Picked as Site for Electron-Ion Collider

Publication date: 17 January 2020

Number: 5

The Department of Energy has selected Brookhaven National

News

\$2.6bn US electron collider promises fundamental insights into the atom

\$1.6

13 January 2020 | By

The National Academies of SCIENCES ENGINEERING MEDICINE

≡ wbur Here & Now

New Particle Accelerator In New York T Protons And Neutrons

09:34 </>

A Domestic Electron Ion Collider Would Unlock Scientific Mysteries of Atomic Nuclei, Maintain U.S. Leadership in Accelerator Science, New Report Says

News Release | July 24, 2018

SHARE f t in ✉

Finding 1: An EIC can uniquely address three profound questions about nucleons—protons—and how they are assembled to form the nuclei of atoms:

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?
- What are the emergent properties of dense systems of gluons?

A naive expectation

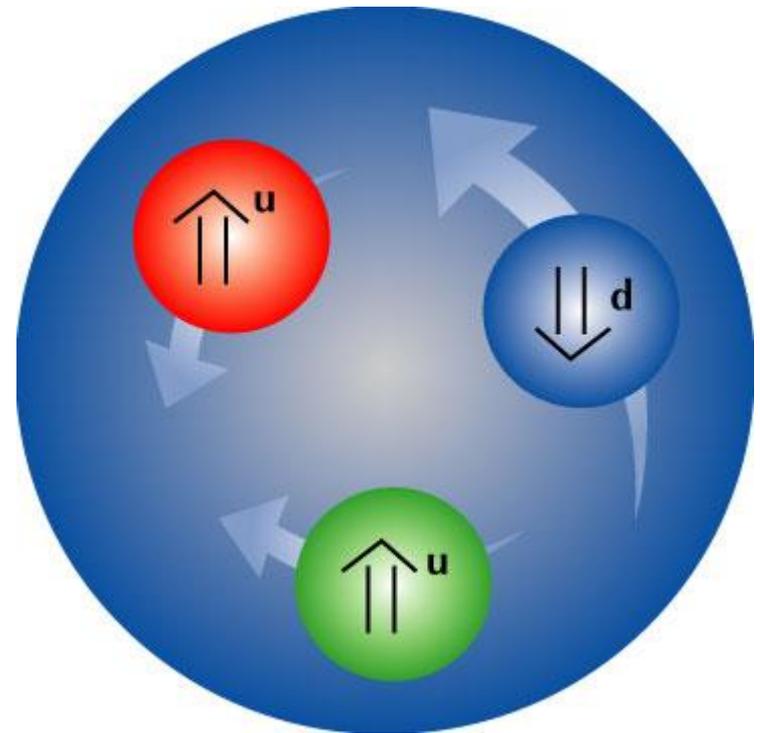
A proton consists of 2 up-quarks and 1 down-quark.

$$\frac{1}{2} = \frac{1}{2} + \frac{1}{2} - \frac{1}{2}$$

➔ $\Delta\Sigma = 1$

With relativistic effects,

$$\Delta\Sigma \approx 0.7$$

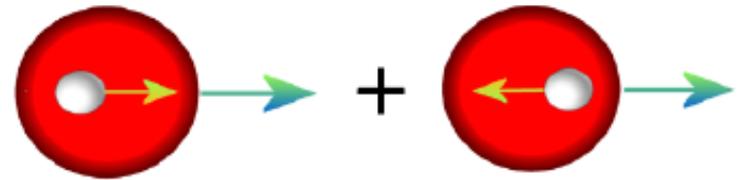


Polarized parton distribution function

Polarize : orient spins in one particular direction

Parton distribution function (PDF)

$$f(x) = f_{\uparrow}(x) + f_{\downarrow}(x)$$



Polarized PDF for a longitudinally polarized proton
(distribution of **net** spin in the direction of motion)

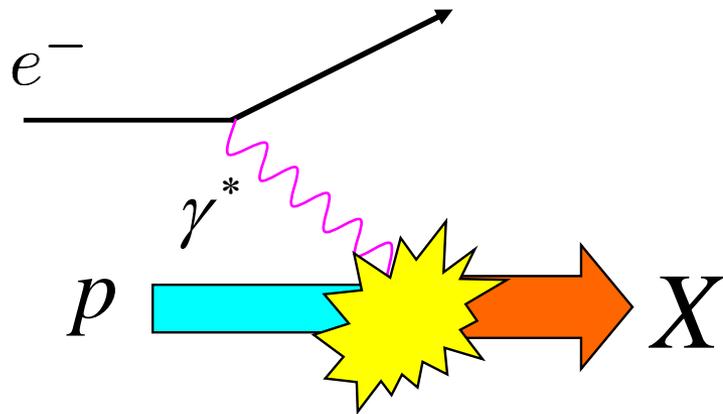
$$\Delta f(x) = f_{\uparrow}(x) - f_{\downarrow}(x)$$



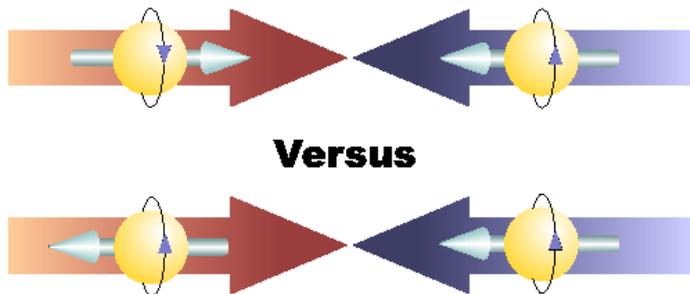
$$\Delta\Sigma = \sum_f^{u,d,s,\dots} \int_0^1 dx \Delta f(x)$$

Measurement of $\Delta\Sigma$ in polarized DIS

Both electron and proton are **longitudinally** polarized



$$A_{LL} = \frac{e^\uparrow p^\downarrow - e^\uparrow p^\uparrow}{e^\uparrow p^\downarrow + e^\uparrow p^\uparrow} \sim \left(1 + \frac{\sigma_L}{\sigma_T}\right) \frac{2xg_1}{F_2}$$



Spin opposite direction

Spin same direction

$$\int_0^1 dx g_1(x) = \frac{1}{9} \Delta\Sigma + \dots$$

'Spin crisis'

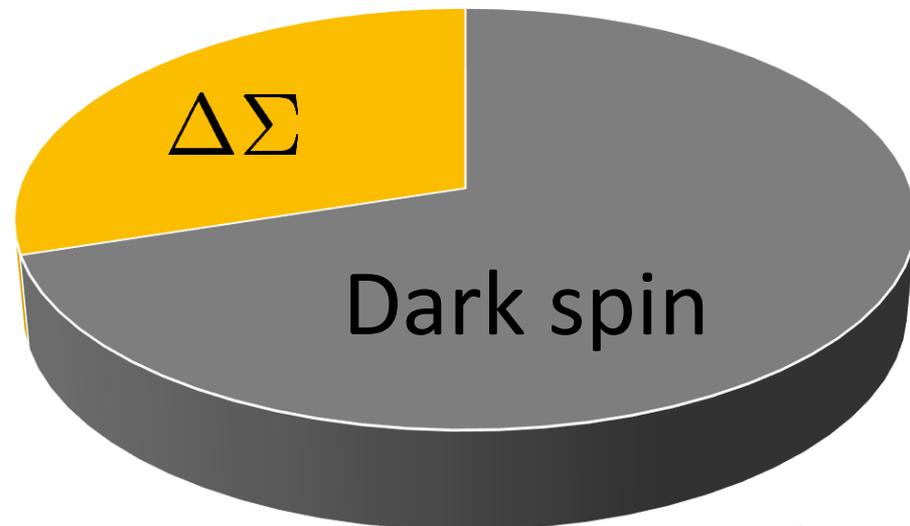
In 1987, the European Muon Collaboration at CERN announced a very small value of the quark spin contribution to proton spin

$$\Delta\Sigma = 0.12 \pm 0.09 \pm 0.14 \text{ !?}$$

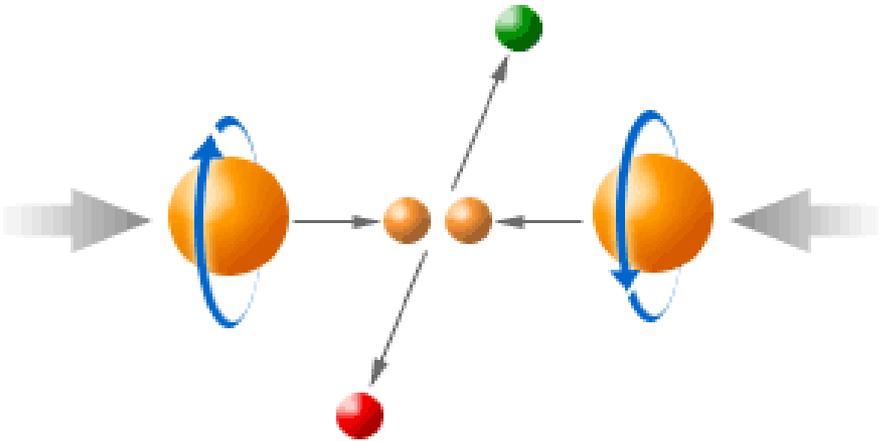
Recent value

$$\Delta\Sigma = 0.25 \sim 0.3$$

Still significantly less than 1.



RHIC spin project



For 20 years, RHIC (relativistic heavy-ion collider) has been colliding polarized protons to study the spin structure of the proton (in parallel with the heavy-ion program [← lecture by R. Pisarski](#)).

RHIC is the only machine in the world that can produce a high energy polarized proton beam.

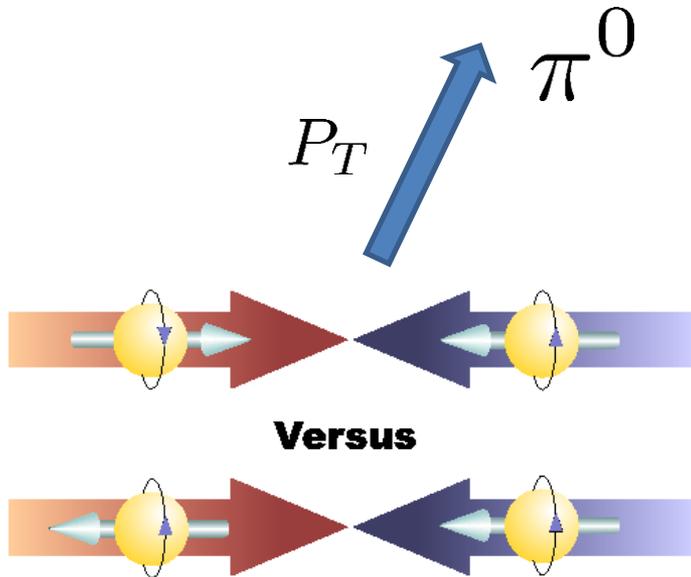
The main physics goal is to pin down the gluon spin contribution

$$\frac{1}{2} = \frac{1}{2} \Delta\Sigma + \Delta G + L_z$$



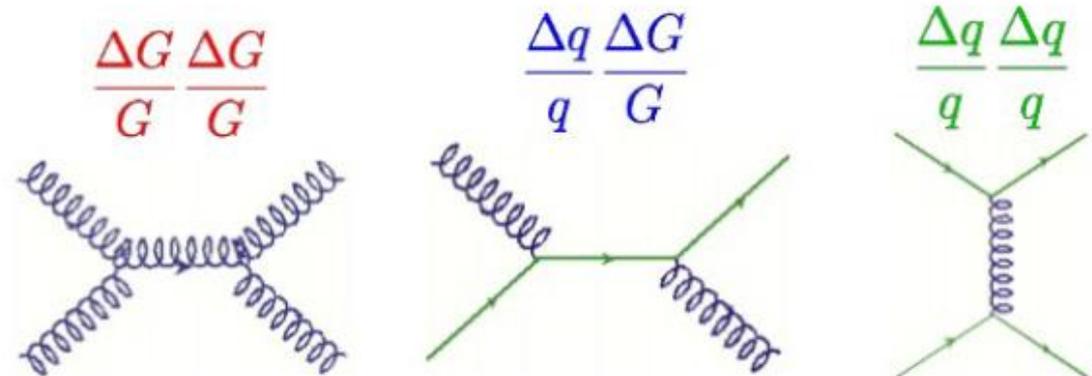
Determination of ΔG at RHIC

Pion production in polarized proton-proton collisions



$$A_{LL} = \frac{d\sigma^{++} - d\sigma^{+-}}{d\sigma^{++} + d\sigma^{+-}}$$

$$\propto \sum_{a,b} \Delta f_a \otimes \Delta f_b(x) \otimes \Delta\sigma_{ab}$$



The result

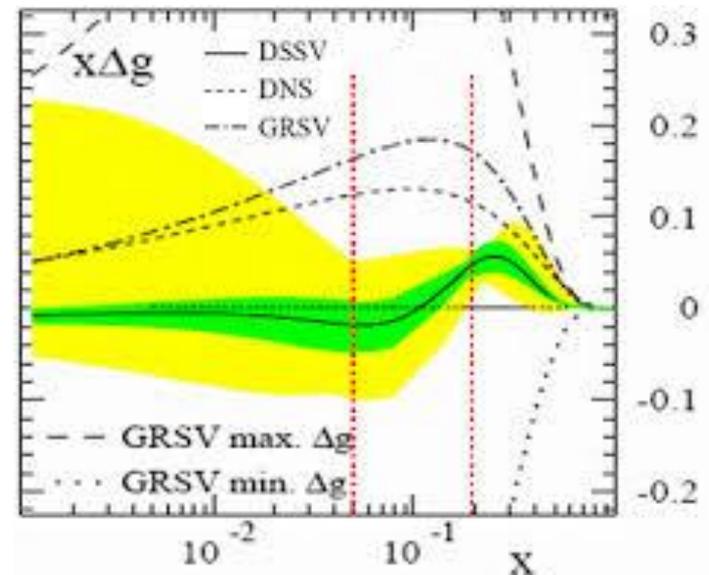
$$\int_{0.001}^1 dx \Delta g(x, Q^2 = 10 \text{ GeV}^2) = 0.20^{+0.06}_{-0.07} \quad \text{DSSV++}$$

$$\int_{0.05}^{0.2} dx \Delta g(x, Q^2 = 10 \text{ GeV}^2) = 0.17 \pm 0.06 \quad \text{NNPDFpol1.1}$$

$$\int_{0.001}^{0.05} dx \Delta g(x, Q^2 = 1 \text{ GeV}^2) = 0.5 \pm 0.4 \quad \text{JAM15}$$

Huge uncertainties from the small- x region. This will be settled at the EIC.

EIC should also address the orbital angular momentum contribution.



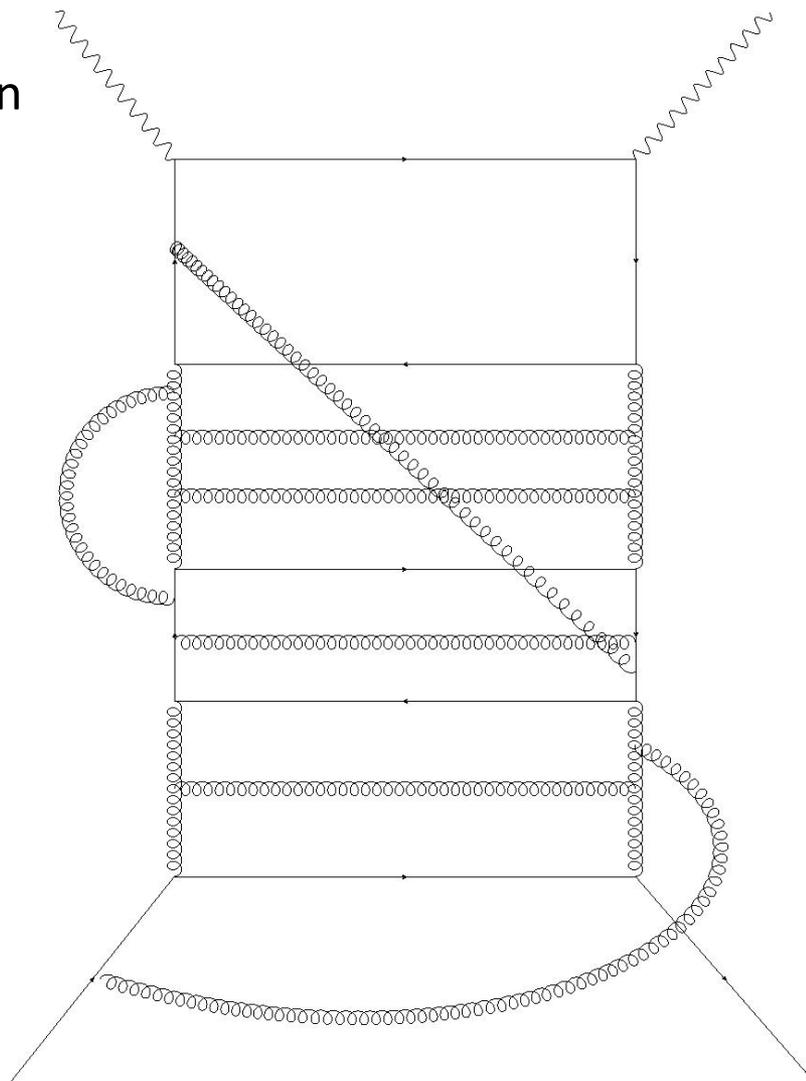
Computing (polarized) PDF at small-x

The behavior of $\Delta G(x)$ etc. in the small-x region can be calculated in perturbation theory (expansion in powers of $\alpha_s = \frac{g^2}{4\pi}$)

Add one gluon \rightarrow add one power of α_s

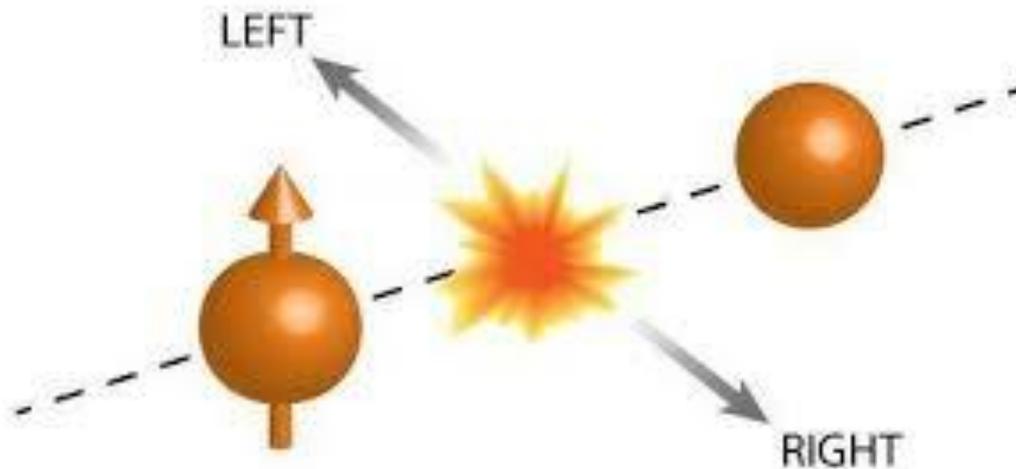
One has to sum infinitely many gluons.

$$\Delta G(x) \sim \left(\frac{1}{x} \right)^{\# \sqrt{\alpha_s}}$$



Physics of transverse spin

Polarize spin perpendicularly to the direction of motion and hit an unpolarized target.



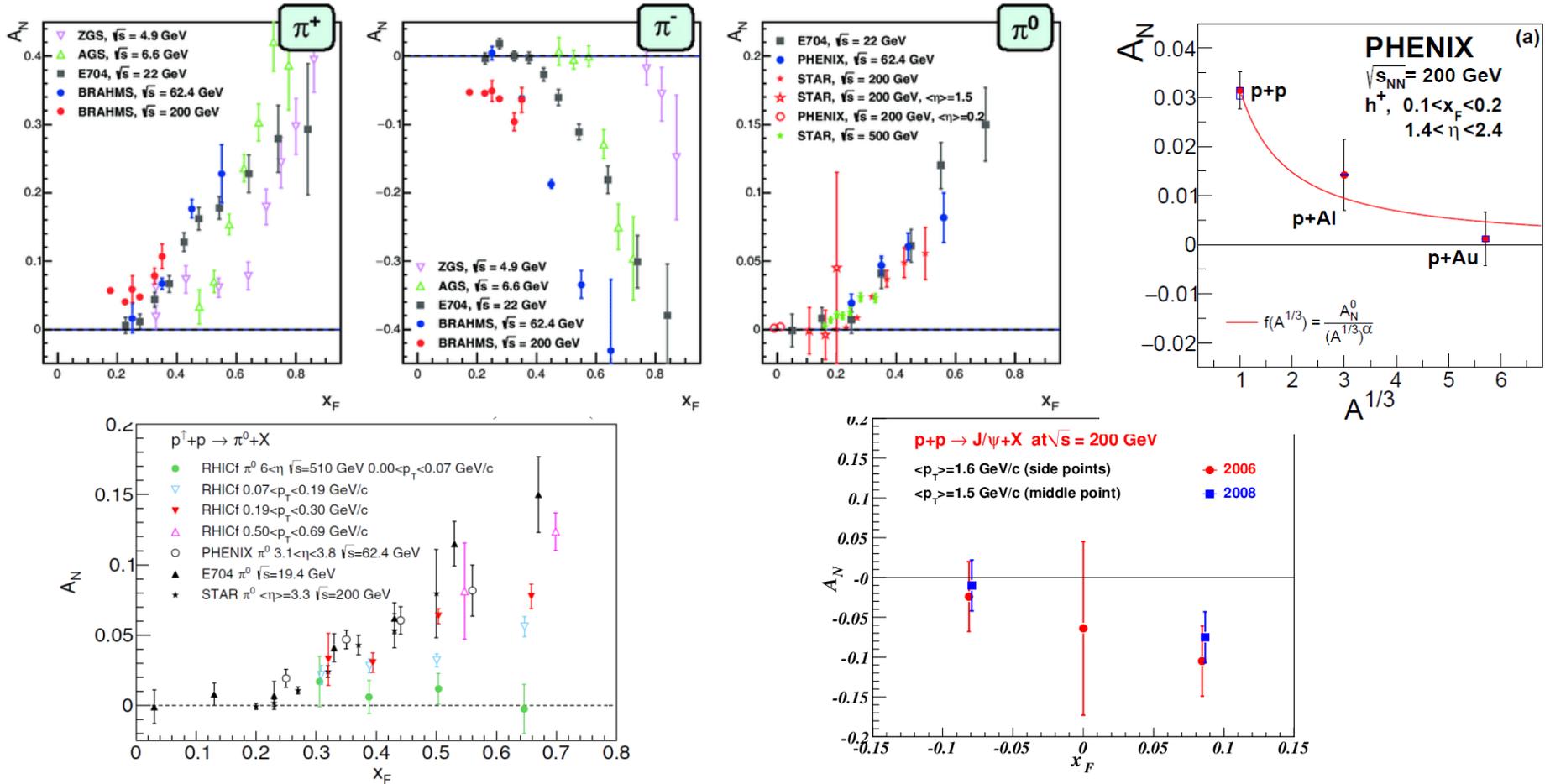
The number of particles produced on the left hand side and right hand side are **different**.

Single spin asymmetry

Single spin asymmetry at RHIC

Up to 40% left-right asymmetry

$$A_N = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$$

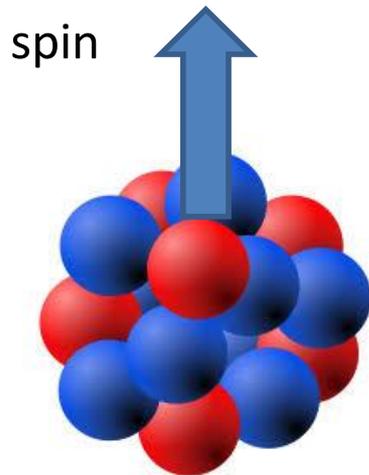
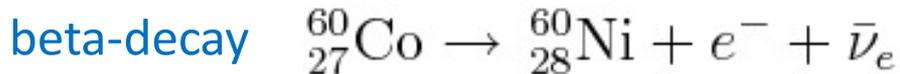


...but why is there asymmetry in the first place?

Parity

The laws of physics are unchanged under the inversion of coordinates $\vec{r} \rightarrow -\vec{r}$

The first experiment which demonstrated parity violation in weak interaction (Wu, 1956)

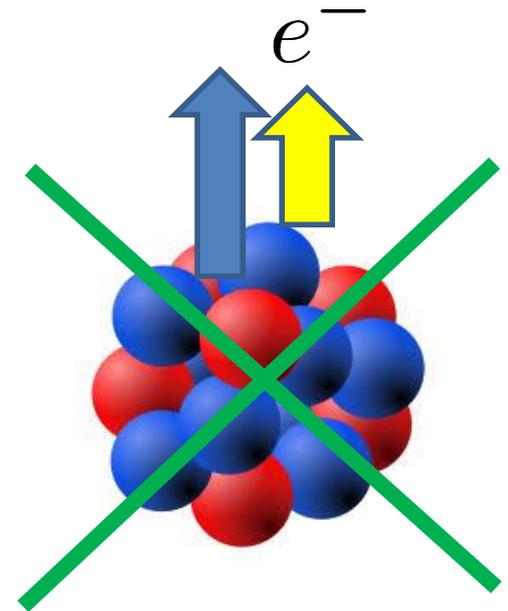


Parity transformation



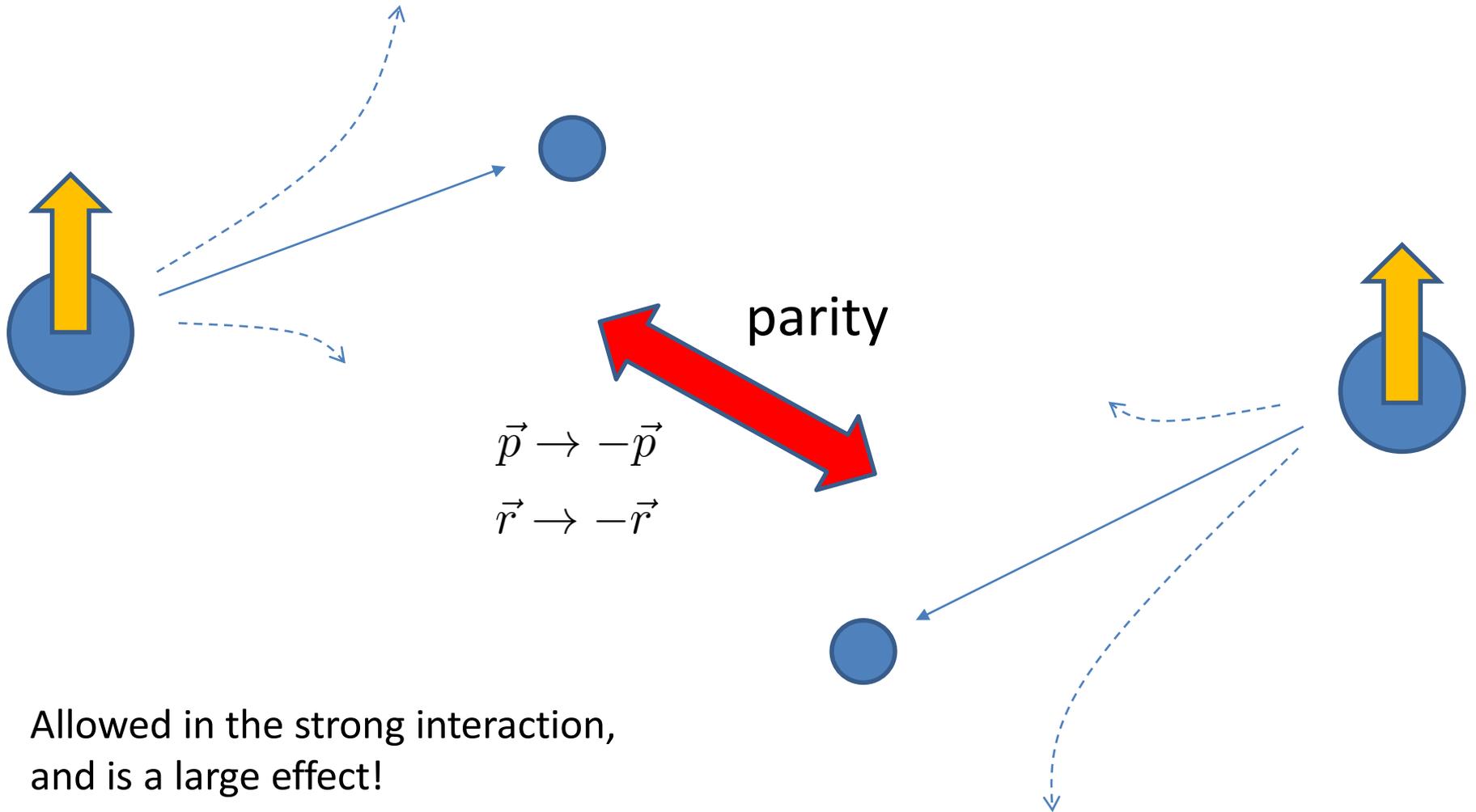
$$\vec{r} \rightarrow -\vec{r}, \vec{p} \rightarrow -\vec{p}$$

$$\vec{L} = \vec{r} \times \vec{p} \rightarrow \vec{L}$$



Spin vector does not flip signs \rightarrow 'pseudovector'
Electron preferentially emitted opposite to the nucleus spin.
 \rightarrow Violation of parity

Unlike up-down asymmetry,
left-right asymmetry does **not** violate parity.

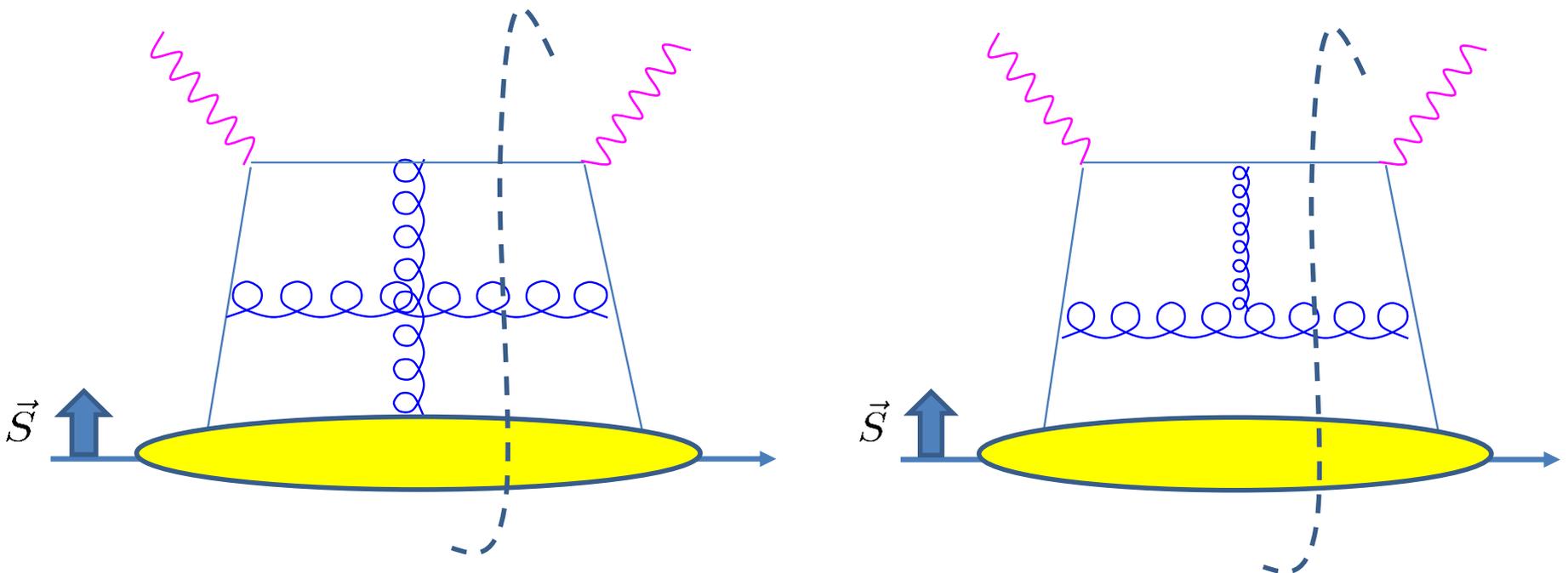


Allowed in the strong interaction,
and is a large effect!

Calculating single spin asymmetry

Several mechanisms for generating SSA are known.

Spin-dependent effects are usually subleading, challenging to compute



Benic, Hatta, Li, Yang (2019)

Conclusion

- QCD as the theory of the strong interaction, full of mysteries even 40 years after the discovery.
- EIC is the next-generation (YOUR generation) machine that will uncover the partonic structure of the proton and nuclei. The spin of the proton is a vital part of the program.

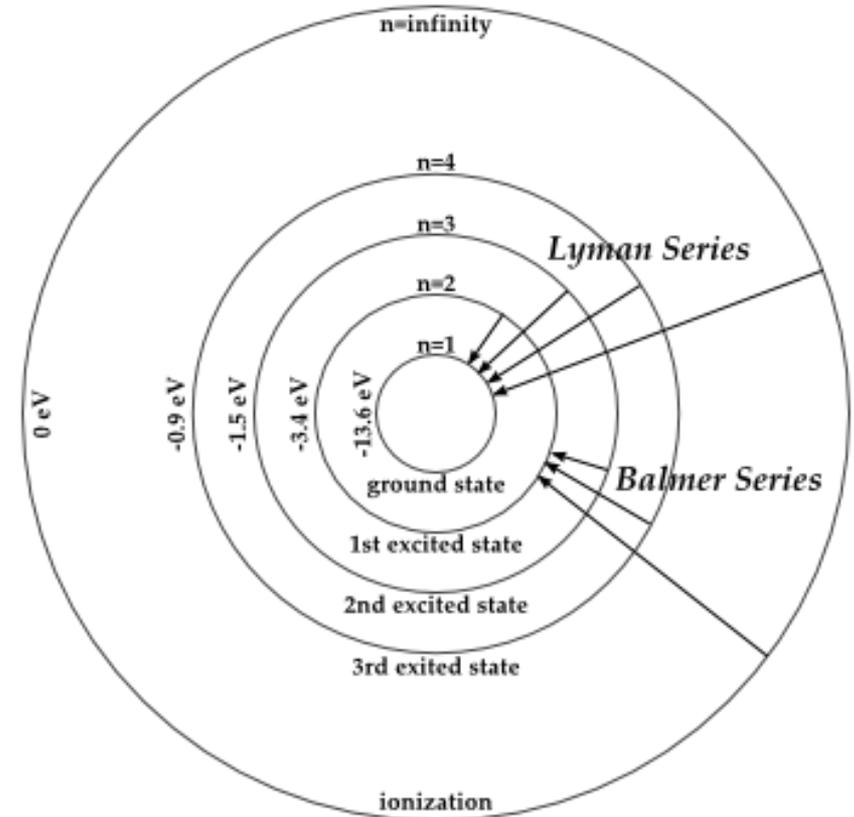
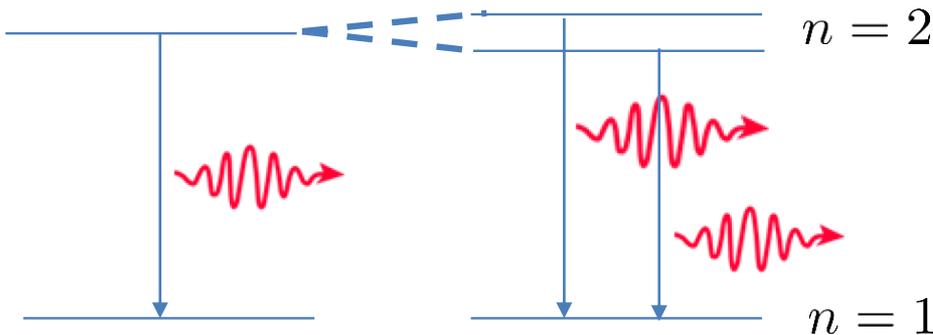
Fine structure

Energy levels of an electron in a hydrogen atom

$$E_n = -\frac{13.6\text{eV}}{n^2} \quad n = 1, 2, 3, \dots$$

A photon is emitted when the electron in an outer orbit drops into an inner orbit.

The energy of the emitted photon takes discrete values.



The photon energy splits into two values which differ only by less than 0.01%