Quantum Computation for Nuclear Physics

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What is quantum computation?

Richard Feynman (1981)
“Simulating Physics with Computers”

“Nature isn’t classical
... and if you want to make a simulation of Nature,
you’d better make it quantum mechanical,
and by golly it’s a wonderful problem,
because it doesn’t look so easy.”

“...What I hoped to do was to design a computer
in which I knew how every part worked with everything
specified down to the atomic level. In other words I wanted
to write down a Hamiltonian for a system that could
make a calculation.”
What is quantum computation?

- **Classical Computers** can efficiently simulate statistical processes, but not quantum mechanical ones — exponential resources!

\[ \sim \exp(V) \]

- **Unlike Quantum Computers** = Universal Quantum Simulators

\[ \sim V \]

Today: A (nuclear) theorists perspective
What is quantum computation?

Digital (universal) Quantum Computers

source: IBM
source: NIST
source: Google
source: Ion-Q, C. Monroe
source: Microsoft
source: NIST
What is quantum computation?

- Analog (non-universal) Quantum Simulators
- Quantum Communication and Cryptography
- Quantum Annealing

+ much more!
Current Status

Quantum Computers just learned to walk

Impressive Progress in recent years!

Noisy Intermediate-Scale Quantum (NISQ) technology
Resources

- Books, e.g. “Quantum Computation and Quantum Information”, Nielsen & Chuang
- Webpages of the big players, e.g. Department of Energy, Google, IBM, Ion-Q, Microsoft, NIST, Rigetti etc.

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visit: https://www.bnl.gov/physics/NTG/people/mueller.php
Outline of this lecture

1. Basics - Quantum 101
2. Quantum Computing (Nuclear) Physics
3. How to do it?
4. New Ideas
Basics - Quantum 101

• Qubit = spin 1/2

\[ |\uparrow\rangle = |0\rangle = (\uparrow) \]

\[ |\downarrow\rangle = |1\rangle = (\downarrow) \]
Basics - Quantum 101

• Qubit = spin 1/2
  \[ |\uparrow\rangle = |0\rangle = (\begin{pmatrix} 1 \\ 0 \end{pmatrix} \), \quad |\downarrow\rangle = |1\rangle = (\begin{pmatrix} 0 \\ 1 \end{pmatrix} \) \]

• Hilbert space \( \mathcal{H} = \text{span}(|0\rangle, |1\rangle) \)

\[ a |0\rangle + b |1\rangle \]
Basics - Quantum 101

- Qubit = spin 1/2

\[ |\uparrow\rangle = |0\rangle = \left(\begin{array}{c} 1 \\ 0 \end{array}\right), \quad |\downarrow\rangle = |1\rangle = \left(\begin{array}{c} 0 \\ 1 \end{array}\right) \]

- Hilbert space \( \mathcal{H} = \text{span}(|0\rangle, |1\rangle) \)

- Many qubits, big Hilbert space \( \mathcal{H}' = \mathcal{H} \otimes \mathcal{H} \ldots \otimes \mathcal{H} \)

\[ (a_0 |0\rangle + b_0 |1\rangle) \otimes (a_1 |0\rangle + b_1 |1\rangle) = a_0a_1 |00\rangle + a_0b_1 |01\rangle + b_0a_1 |10\rangle + b_0b_1 |11\rangle \]
Basics - Quantum 101

• Qubit = spin 1/2

\[ | \uparrow \rangle = | 0 \rangle = (0) \]
\[ | \downarrow \rangle = | 1 \rangle = (1) \]

• Hilbert space \( \mathcal{H} = \text{span}(|0\rangle, |1\rangle) \)

• Many qubits, big Hilbert space \( \mathcal{H}' = \mathcal{H} \otimes \mathcal{H} \ldots \otimes \mathcal{H} \)

\[
(a_0 | 0 \rangle + b_0 | 1 \rangle) \otimes (a_1 | 0 \rangle + b_1 | 1 \rangle) = a_0 a_1 | 00 \rangle + a_0 b_1 | 01 \rangle + b_0 a_1 | 10 \rangle + b_0 b_1 | 11 \rangle
\]

\[
\begin{pmatrix}
  a_0 \\
  b_0
\end{pmatrix} \otimes \begin{pmatrix}
  a_1 \\
  b_1
\end{pmatrix} =
\begin{pmatrix}
  a_0 a_1 \\
  a_0 b_1 \\
  b_0 a_1 \\
  b_1 a_1
\end{pmatrix}
\]

Size of this vector grows exponentially \( 2^N \) with quantum mechanical degrees of freedom.
Basics - Quantum 101
Basics - Quantum 101

- Information can be encoded \[|001001110\rangle\]
Basics - Quantum 101

• Information can be encoded \( |001001110\rangle \)

• Power of quantum: superposition of information

\[
\frac{1}{\sqrt{2}} \left( |001001110\rangle + |110001111\rangle \right)
\]

\[
\frac{1}{\sqrt{2}} \left( |\text{cat}\rangle + |\text{dog}\rangle \right)
\]
Basics - Quantum 101

• Information can be encoded \( |001001110\rangle \)

• Power of quantum: superposition of information

\[
\frac{1}{\sqrt{2}} \left( |001001110\rangle + |110001111\rangle \right)
\]

• Information processing via quantum circuit

\[ |001001110\rangle \rightarrow |111101110\rangle \]
Basics - Quantum 101

• Information can be encoded
  \[ |001001110\rangle \]

• Power of quantum: **superposition of information**
  \[
  \frac{1}{\sqrt{2}} \left( |001001110\rangle + |110001111\rangle \right)
  \]

• Information processing via quantum circuit
  \[ |001001110\rangle \rightarrow |111101110\rangle \]

  classically, think “matrix multiplication”
  (matrix size grows exponential)
  \[
  \begin{pmatrix}
  0 \\
  1
  \end{pmatrix}
  =
  M
  \begin{pmatrix}
  1 \\
  0
  \end{pmatrix}
  \]
  (M unitary)
Basics - Quantum 101

- Information can be encoded

\[ |001001110\rangle \]

- Power of quantum: superposition of information

\[ \frac{1}{\sqrt{2}} \left( |001001110\rangle + |110001111\rangle \right) \quad \frac{1}{\sqrt{2}} \left( |\text{cat}\rangle + |\text{mouse}\rangle \right) \]

- Information processing via quantum circuit

\[ |001001110\rangle \rightarrow |111101110\rangle \]

Classically, think "matrix multiplication"

\[ \begin{pmatrix} 0 \\ 1 \end{pmatrix} = M \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (M \text{ unitary}) \]

- Quantum parallelism

\[ x = 001001110, \quad f(x)? \]

\[ \frac{1}{\sqrt{2}} \left( |001001110\rangle + |111001111\rangle \right) \rightarrow \frac{1}{\sqrt{2}} \left( |101100010\rangle + |000110001\rangle \right) \]
Basics - Quantum 101
• Information can be entangled
  \[ |\psi\rangle = \frac{1}{\sqrt{2}} (|01\rangle + |10\rangle) \]

• **Entanglement** is a resource in Quantum Information Science!
Basics - Quantum 101

• Information can be entangled

\[ |\psi\rangle = \frac{1}{\sqrt{2}} \left( |01\rangle + |10\rangle \right) \]

• Entanglement is a resource in Quantum Information Science!

• Quantum Mechanics: Extract Information via Measurement
Basics - Quantum 101

• Information can be entangled

\[ |\psi\rangle = \frac{1}{\sqrt{2}} (|01\rangle + |10\rangle) \]

• Entanglement is a resource in Quantum Information Science!

• Quantum Mechanics: Extract Information via Measurement

• Extracting answer from Quantum Computer: subtile issue
Quantum Computation for Physics
Challenges
Quantum Computation for Physics

Challenges

- Quantum-Many Body systems

\[ |2\rangle = |1\rangle \otimes |n\rangle \otimes |1\rangle \otimes |1\rangle \ldots |1\rangle \]

Size of Hilbert space $S^N$ (Gold $N = 197$)
Quantum Computation for Physics

Challenges

• Quantum-Many Body systems

\[ i \hbar \partial_t \left| \psi \right\rangle = H \left| \psi \right\rangle \]

| Hilbert space \( S^N \) (Gold \( N = 197 \))

• Many degrees of freedom, exponentially large Hilbert space

\[ H = \sum_i \frac{\hat{p}_i^2}{2m} + \sum_{ij} V_{ij} + \sum_{ijk} V_{ijk} + \ldots \]

(Hamiltonian operator)
Quantum Computation for Physics

Challenges

• Quantum-Many Body systems

\[ \text{size of Hilbert space } S^N (\text{Gold } N = 197) \]

• Many degrees of freedom, exponentially large Hilbert space

\[ i\hbar \partial_t |\psi\rangle = H |\psi\rangle \]

\[ H = \sum_i \frac{\hat{p}^2}{2m} + \sum_{ij} V_{ij} + \sum_{ijk} V_{ijk} + \ldots \]

(Hamiltonian operator)

• Assuming \(|n\rangle\) and \(|p\rangle\) each had 2 states, Schroedinger equation for Gold = matrix equation of size \(2^{197} \sim 10^{59}\)

(earth consists of \(\sim 10^{50}\) atoms, yet only 197 qubits enough to represent Hilbert space of Gold)
Quantum Computation for Physics
Challenges
Quantum Computation for Physics

Challenges

• Quantum Many-Body Theory $\rightarrow$ Quantum Field Theory
Quantum Computation for Physics

Challenges

• Quantum Many-Body Theory $\rightarrow$ Quantum Field Theory

• Example (Quantum) Electrodynamics

$$H = \int d^3x \left\{ \frac{E^2(x)}{2} + \frac{B(x)^2}{2} + \psi^\dagger(x)\gamma^0(i\gamma \cdot \nabla + m)\psi(x) \right\}$$
Quantum Computation for Physics

Challenges

• Quantum Many-Body Theory → Quantum Field Theory

• Example (Quantum) Electrodynamics

\[ H = \int d^3x \left\{ \frac{E^2(x)}{2} + \frac{B(x)^2}{2} + \psi^\dagger(x)\gamma^0(i\gamma \cdot \nabla + m)\psi(x) \right\} \]

• Every \( x \) labels one quantum mechanical degree of freedom.

Infinitely many dof's in any volume \( V \)!
Quantum Computation for Physics

Challenges

• Quantum Many-Body Theory $\rightarrow$ Quantum Field Theory

• Example (Quantum) Electrodynamics

$$H = \int d^3x \left\{ \frac{E^2(x)}{2} + \frac{B(x)^2}{2} + \psi^\dagger(x)\gamma^0(i\gamma \cdot \nabla + m)\psi(x) \right\}$$

• Every $x$ labels one quantum mechanical degree of freedom.

Infinitely many dof's in any volume $V$!
Quantum Computation for Physics
Challenges
Quantum Computation for Physics
Challenges

- Quantum Field Theory $\rightarrow$ Lattice Quantum Field Theory

\[ \psi(n) \]

\[ x = na_s \quad n = (n_x, n_y, n_z) \]
Quantum Computation for Physics
Challenges

- Quantum Field Theory → **Lattice Quantum Field Theory**

\[ \psi(x) \rightarrow \psi(n) \]

\[ x = na_s \quad n = (n_x, n_y, n_y) \]

- Lattice Quantum Field Theory ~ Quantum Many Body Theory
Quantum Computation for Physics
Challenges
Quantum Computation for Physics

Challenges

• Gauge Theories (e.g. QED)

\[ H = \int d^3x \left\{ \frac{E^2(x)}{2} + \frac{B(x)^2}{2} + \psi^\dagger(x)\gamma^0(i\gamma \cdot \nabla + m)\psi(x) \right\} \]

• Electroweak Force
• Strong Force
• Gravity
Quantum Computation for Physics

Challenges

- **Gauge Theories (e.g. QED)**

  \[ H = \int d^3x \left\{ \frac{E^2(x)}{2} + \frac{B(x)^2}{2} + \psi^\dagger(x)\gamma^0(i\gamma \cdot \nabla + m)\psi(x) \right\} \]

- **Redundancy**, not all dofs are physical!
Quantum Computation for Physics

Challenges

- **Gauge Theories** (e.g. QED)

  \[
  H = \int d^3 x \left\{ \frac{E^2(x)}{2} + \frac{B(x)^2}{2} + \psi^\dagger(x) \gamma^0 (i \gamma \cdot \nabla + m) \psi(x) \right\}
  \]

- **Redundancy**, not all dofs are physical!

- **Gauss law** (operator) defines physical sector

  \[
  e^{iG(x)} |\psi^{\text{phys}}\rangle = |\psi^{\text{phys}}\rangle
  \]

  \[
  G(x) = \nabla_x E(x) - J(x)
  \]
How to compute something?
Quantum Computation for Nuclear Physics
Lattice QCD simulations
Quantum Computation for **Nuclear Physics**

Lattice QCD simulations

- From Hamiltonian to Lagrangian

\[ \mathcal{L}_{\text{QCD}} = \bar{\psi}_i (i (\gamma^\mu D_\mu)_{ij} - m \delta_{ij}) \psi_j - \frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} \]

\[ (G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - ig f^{abc} A_\mu^b A_\nu^c) \]
Quantum Computation for **Nuclear Physics**

**Lattice QCD simulations**

- From Hamiltonian to Lagrangian

\[ \mathcal{L}_{\text{QCD}} = \bar{\psi}_i \left( i (\gamma^\mu D_\mu)_{ij} - m \delta_{ij} \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a \]

\[ (G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - ig f^{abc} A^b_\mu A^c_\nu) \]

- … to path integral

\[ Z = \int dA \, e^{iS_{\text{QCD}}[A]} \]

(\[ S_{\text{QCD}} = \int d^4x \mathcal{L}_{\text{QCD}} \rightarrow a_s^3 \sum_n \mathcal{L}_{\text{lattice}} \])

\[ dA \equiv \prod_x dA(x) \rightarrow \prod_n dA(n) \]
Quantum Computation for **Nuclear Physics**

**Lattice QCD simulations**

- From Hamiltonian to Lagrangian

  \[ \mathcal{L}_{\text{QCD}} = \bar{\psi}_i (i(\gamma^\mu D_\mu)_{ij} - m \delta_{ij}) \psi_j - \frac{1}{4} G_{\mu\nu}^a G^{\mu\nu}_a \]

  \[(G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - ig f^{abc} A_\mu^b A_\nu^c)\]

- ... to path integral

  \[ Z = \int dA \, e^{i\mathcal{S}_{\text{QCD}}[A]} \]

- In Euclidean Spacetime: statistical mechanics problem

  \[ Z_E = \int dA \, e^{-S_E[A]} \]

  *Lattice Monte-Carlo simulations work in many dimensions!*
Quantum Computation for **Nuclear** Physics

Lattice QCD simulations

(*) over-simplification. Please don't be mad, lattice practitioners
Quantum Computation for \textbf{Nuclear Physics}

Lattice QCD simulations

- Very expensive

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Quantum Computation for **Nuclear** Physics

Lattice QCD simulations

- Very expensive
- Do not work for various interesting problems (*)!

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Quantum Computation for **Nuclear** Physics

Lattice QCD simulations

- Very expensive
- Do not work for various interesting problems

\[\text{Real-time physics, systems out of Equilibrium}\]

Nope, euclidean space time remember?

(*) over-simplification. Please don't be mad, lattice practitioners
Quantum Computation for **Nuclear Physics**

Lattice QCD simulations

- Very expensive
- Do not work for various interesting problems

- Real-time physics, systems out of equilibrium
- Systems at high density

\[ Z_E = \int dA e^{-S_E[A;\mu]} \]

(*) over-simplification. Please don’t be mad, lattice practitioners...
Quantum Computation for **Nuclear Physics**

Lattice QCD simulations

- Very expensive
- Do not work for various interesting problems

- Real-time physics, systems out of equilibrium
  
  *Nope, euclidean space time remember?*

- Systems at high density
  
  *Nope, “sign problem”*

\[ Z_E = \int dA \, e^{-S_E[A;\mu]} \]

(*) over-simplification. Please don’t be mad, lattice practitioners
How to do it?

Step 1: Digitization
How to do it?

Step 1: Digitization

\[ \text{OCD} \]
\[ \text{Hilbert space } \mathcal{H}_{\text{OCD}} \]

"digitization"

\[ \text{Controllable Hilbert space of qubits} \]
\[ |010010\rangle \]

OR

\[ \text{Controllable Hilbert space of some atoms etc. (quantum simulation)} \]
How to do it?

Step 1: Digitization

- Example 1D: fermions
How to do it?
Step 1: Digitization

- Example 1D: fermions

\[ H = -t \sum_{n} c_{n}^{\dagger} c_{n+1} + h.c. \]

\[ + m \sum_{n} (-1)^{n} c_{n}^{\dagger} c_{n} + \text{interactions} \]
How to do it?

Step 1: Digitization

- Example 1D: fermions

\[
H = -t \sum_n c_n^{\dagger} c_{n+1} + h.c. + m \sum_n (-1)^n c_n^{\dagger} c_n + \text{interactions}
\]

- Local Hilbert space \( \mathcal{H} = \bigotimes_n \mathcal{H}_n \)

- Fermion = 2 states (occupied/unoccupied)
- qubit = 2 states (|1\rangle, |0\rangle)
How to do it?

Step 1: Digitization

- Example 1D: fermions

\[ H = -t \sum_n c_n^\dagger c_{n+1} + h.c. + m \sum_n (-1)^n c_n^\dagger c_n \text{ + interactions} \]

- Local Hilbert space \( \mathcal{H} = \bigotimes_n \mathcal{H}_n \)

- Fermion = 2 states
  (occupied/unoccupied)

- Qubit = 2 states
  (\( |1\rangle, |0\rangle \))

\( \mathcal{H} \text{fermions} \leftrightarrow \mathcal{H} \text{qubits} \)
How to do it?

Step 2: Come up with an algorithm
How to do it?
Step 2: Come up with an algorithm

• Example: Real-time dynamics

How does state $|\psi\rangle$ evolve over time?
How to do it?

Step 2: Come up with an algorithm

- Example: Real-time dynamics

How does state $|\psi\rangle$ evolve over time?

$$|\psi(t)\rangle = U(t) |\psi\rangle$$
How to do it?

Step 2: Come up with an algorithm

- Example: Real-time dynamics

How does state $|\psi\rangle$ evolve over time?

$|\psi(t)\rangle = U(t) |\psi\rangle$

- Initial state
- Final state
- Time evolution operator

$U(t) = e^{-iHt} |\psi\rangle$
How to do it?
Step 2: Come up with an algorithm
How to do it?

Step 2: Come up with an algorithm

• Decompose $U(t)$ into circuit

$|y\rangle U(t) |y(t)\rangle$
How to do it?

Step 2: Come up with an algorithm

- Decompose $U(t)$ into circuit

$$ |4\rangle \quad U(t) \quad |4(t)\rangle $$

$$ |0\rangle \quad X \quad |1\rangle $$

$$(\sigma) = (|0\rangle \ 1) (1 \ 0)$$
How to do it?

Step 2: Come up with an algorithm

- Decompose $U(t)$ into circuit

- Figure out how to set up $|\psi\rangle$

- and how to extract information about $|\psi(t)\rangle$ through measurement
Operator formulation of Lattice Gauge Theory

Let’s dive a bit deeper. I will go a little faster now.
Let's dive a bit deeper. I will go a little faster now.

• **QED**$^{1+1}$: Schwinger-model

\[ H = \int dx \left[ \frac{E_x^2}{2} + \psi^\dagger \gamma^0 (i\gamma^1 D_x + m) \psi \right] \]
Let’s dive a bit deeper. I will go a little faster now.

- **QED\(_{1+1}\): Schwinger-model**
  \[ H = \int dx \left[ \frac{E_x^2}{2} + \psi^\dagger \gamma^0 (i \gamma^1 D_x + m) \psi_x \right] \]

- **Lattice theory**
  \[ H = a_s \sum_n \left[ \frac{E_n^2}{2} - \frac{i}{2a_s} (\psi_n^\dagger U_n \psi_{n+1} - h.c.) + m(-1)^n \psi_n^\dagger \psi_n \right] \]
Operator formulation of Lattice Gauge Theory

Let’s dive a bit deeper. I will go a little faster now.

- **QED$^{1+1}$: Schwinger-model**
  \[ H = \int d^4x \left[ \frac{E_x^2}{2} + \psi^\dagger \gamma^0 (i \gamma^1 D_x + m) \psi_x \right] \]

- **Lattice theory**
  \[ H = a_s \sum_n \left[ \frac{E_n^2}{2} - \frac{i}{2a_s} \left( \psi_n^\dagger U_n \psi_{n+1} - h.c. \right) + m(-1)^n \psi_n^\dagger \psi_n \right] \]
Operator formulation of Lattice Gauge Theory
Operator formulation of Lattice Gauge Theory

• Hilbert space, gauge sector

\[ \hat{E}_n |e_n\rangle = E_n |e_n\rangle \]
\[ \hat{U}_n |e_n\rangle = |e_n + 1\rangle \]
Operator formulation of Lattice Gauge Theory

- Hilbert space, gauge sector

\[ \hat{E}_n |e_n\rangle = E_n |e_n\rangle \]
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Operator formulation of Lattice Gauge Theory

• Hilbert space, gauge sector

\[ \hat{E}_n | e_n \rangle = E_n | e_n \rangle \]
\[ \hat{U}_n | e_n \rangle = | e_n + 1 \rangle \]

• Truncation / Digitization

Fermionic Hilbert space
2 states: occupied \( | \) unoccupied \( \rangle \)
\( \Rightarrow \) map onto qubits \( | 0 \rangle \) \( | 1 \rangle \)

Gauge field Hilbert space:
\[ \hat{E}_n | e_n \rangle = E_n | e_n \rangle \]
\( e_n = [-\infty, \infty] \)

Truncate and digitize:
Keep finitely many states
Cut off the tail of the spectrum.
Operator formulation of Lattice Gauge Theory

- Hilbert space, gauge sector

\[ \hat{E}_n |e_n\rangle = E_n |e_n\rangle \]
\[ \hat{U}_n |e_n\rangle = |e_n + 1\rangle \]

- Truncation / Digitization

- Map onto qubits
Operator formulation of Lattice Gauge Theory
Operator formulation of Lattice Gauge Theory

• A state of the full theory

\[ |y\rangle = |110100001100001010111111\rangle \]

(24 qubits)
Operator formulation of Lattice Gauge Theory

• A state of the full theory

\[ |\psi\rangle_n = |\psi^\dagger_n\psi_n + (-1)^n - 1\rangle_2 \]

(24 qubits)

• Most of Hilbert space is unphysical, Gauss law (*)

\[ G_n = E_n - E_{n-1} - e\left[\psi_n^\dagger\psi_n + \frac{(-1)^n - 1}{2}\right] \]

(can see this because in 1+1d can integrate out Gauss law to remove gauge fields, physical Hilbert space can be represented with 6 qubits, instead of 24)
Operator formulation of Lattice Gauge Theory

• A state of the full theory

\[ G_n = E_n - E_{n-1} - e \left[ \psi_n^+ \psi_n + \frac{(-1)^n - 1}{2} \right] \]

(can see this because in 1+1d can integrate out Gauss law to remove gauge fields, physical Hilbert space can be represented with 6 qubits, instead of 24)

• Most of Hilbert space is unphysical, Gauss law (*)

\[ |\psi(\text{t})\rangle = U(\text{t}) |\psi\rangle = e^{-iHt} |\psi\rangle \]

• Hamiltonian commutes with Gauss law

\[ [H, G_n] = 0 \] If initial state is physical, it will stay physical
Operator formulation of Lattice Gauge Theory
Operator formulation of Lattice Gauge Theory

\[ |\psi(t)\rangle = U(t) |\psi\rangle = e^{-iHt} |\psi\rangle \]
Operator formulation of Lattice Gauge Theory

\[ |\psi(t)\rangle = U(t) |\psi\rangle = e^{-iHt} |\psi\rangle \]

- \[ U(t) = \prod_{t} U(\delta t) \]

“Trotterization”
Operator formulation of Lattice Gauge Theory

- $|\psi(t)\rangle = U(t) |\psi\rangle = e^{-iHt} |\psi\rangle$

- $U(t) = \prod_t U(\delta t)$

“Trotterization”

Take a look at e.g. https://arxiv.org/pdf/2002.11146.pdf to see examples of actual circuits!
Operator formulation of Lattice Gauge Theory

New Ideas

Quantum Computation for the Electron Ion Collider

• Measured are “real-time correlation functions” \( \langle P | J_\mu(x)J_\nu(0) | P \rangle \)

• What is the structure of the proton, what is \( |P\rangle \)?
New Ideas

Quantum Computation for the Electron Ion Collider

The Department of Energy has selected Brookhaven National Laboratory as the site for its proposed Electron-Ion Collider, a flagship nuclear science facility that is estimated to cost between $1.6 billion and $2.6 billion.

- Measured are “real-time correlation functions” $\langle P | J_\mu(x)J_\nu(0) | P \rangle$
- What is the structure of the proton, what is $| P \rangle$?

Quantum computers can go where classical computing fails!
Very exciting times ahead! - (Second) Quantum Revolution!

Enjoy thinking differently about problem? Go quantum!
Your turn!

Any questions?

I will hang around on Bluejeans after this lecture and we can discuss on virtual whiteboard

(or email me: nmueller@bnl.gov)
PS: Bye, bye BNL and thanks for all the fun!
(this was my last talk as a BNL’er here)
## Backup: Elementary Circuits / Gates

<table>
<thead>
<tr>
<th>Operator</th>
<th>Gate(s)</th>
<th>Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pauli-X (X)</td>
<td>X</td>
<td>$\begin{bmatrix} 0 &amp; 1 \ 1 &amp; 0 \end{bmatrix}$</td>
</tr>
<tr>
<td>Pauli-Y (Y)</td>
<td>Y</td>
<td>$\begin{bmatrix} 0 &amp; -i \ i &amp; 0 \end{bmatrix}$</td>
</tr>
<tr>
<td>Pauli-Z (Z)</td>
<td>Z</td>
<td>$\begin{bmatrix} 1 &amp; 0 \ 0 &amp; -1 \end{bmatrix}$</td>
</tr>
<tr>
<td>Hadamard (H)</td>
<td>H</td>
<td>$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 &amp; 1 \ 1 &amp; -1 \end{bmatrix}$</td>
</tr>
<tr>
<td>Phase (S, P)</td>
<td>S, T</td>
<td>$\begin{bmatrix} 1 &amp; 0 \ 0 &amp; e^{i\pi/4} \end{bmatrix}$</td>
</tr>
<tr>
<td>$\pi/8$ (T)</td>
<td>T</td>
<td>$\begin{bmatrix} 1 &amp; 0 \ 0 &amp; e^{i\pi/8} \end{bmatrix}$</td>
</tr>
<tr>
<td>Controlled Not</td>
<td>$\text{CNOT}, \text{CX}$</td>
<td>$\begin{bmatrix} 1 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 1 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 1 \end{bmatrix}$</td>
</tr>
<tr>
<td>Controlled Z (CZ)</td>
<td>Z</td>
<td>$\begin{bmatrix} 1 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 1 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; -1 \end{bmatrix}$</td>
</tr>
<tr>
<td>SWAP</td>
<td></td>
<td>$\begin{bmatrix} 1 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 1 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; -1 \end{bmatrix}$</td>
</tr>
<tr>
<td>Toffoli</td>
<td></td>
<td>$\begin{bmatrix} 1 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 1 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 1 &amp; 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 &amp; 1 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 1 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 1 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 0 &amp; 1 \end{bmatrix}$</td>
</tr>
</tbody>
</table>

source: wikipedia

![IBM Quantum Experience](source: IBM Q Experience)
Backup: What is a path integral?
Backup: What is a path integral?

• Path integral for single particle

classically: \( \dot{x}(t), \dot{p}(t) \)

quantum mechanically: \( |x(t)\rangle \otimes |p(t)\rangle \)

one classical path

superposition of classical paths
Backup: What is a path integral?

- Path integral for single particle
  
  \[
  \text{classically: } \dot{x}(t), \dot{p}(t) \\
  \text{quantum mechanically: } |x(t)\rangle = |\hat{p}(t)\rangle
  \]

- Path integral for many body theory: (Quantum) Field Theory

  \[
  \phi(x) \rightarrow \phi(x) \\
  \Phi(x) \rightarrow \Phi(x)
  \]

  \[
  \text{Heisenberg Field Operator}
  \]

  \[
  \text{with each } \phi(x) \text{ comes a local Hilbert space} \\
  \text{eigenvalue at } x
  \]

  \[
  H = \bigotimes_x H_x \\
  H_x = \text{span} \{ |x\rangle^2 \} = \text{span} \{ |\hat{p}_x\rangle^2 \}
  \]

  \[
  \text{path integral: } \langle \Phi_1 | e^{-i\hat{H}t} | \Phi_0 \rangle
  \]

  \[
  \text{classically: } \Phi_0 \rightarrow \Phi_1 \\
  \text{quantum mechanically: superposition of a "path"}
  \]
Backup: What is a path integral?

not so crucial to understand now!
Backup: What is a path integral?

• Path Integral = Representation of quantum mechanical amplitude

\[
\langle \Phi_b | e^{-iHt} \Phi_a \rangle = \langle \Phi_b | e^{-i\Delta \int d\Pi \frac{d\Phi}{dt}} \langle \Pi_0 | \Phi_{\Phi} \rangle \langle \Phi_1 e^{-i\Delta \int d\Pi \frac{d\Phi}{dt}} \Phi_0 \rangle \langle \Phi_1 \cdots \Phi_n \rangle \\
= \int_{\Pi_t} d\Pi_{\Phi} d\Phi_{\Phi} \langle \Phi_0 e^{-i\Delta \int d\Pi \frac{d\Phi}{dt}} \Phi_0 \rangle \langle \Pi_{\Phi} | \Phi_{\Phi} \rangle \langle \Phi_1 \cdots \Phi_n \rangle \\
= \int D\Phi D\Pi e^{i\int dt (\Phi \Pi - H)} e^{i\Pi_0} \\
= \int D\Phi D\Pi e^{iS} \quad \text{"weighted average over field operator eigenvalue"}
\]

not so crucial to understand now!
Backup: Whiteboard space
Backup: Whiteboard space
Backup: Whiteboard space