Radiative Corrections for Lepton-Nucleon Scattering: Current State of Affairs

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9 July 2020
Radiative Corrections Workshop

- Intro: accelerated charge radiates – at times as a nuisance
- Formalism, conventions, approximations
- Higher orders and exponentiation, Heitler & Co.
- Numerics, software, Monte Carlo

Accelerated charge radiates: motion of electrons in an antenna

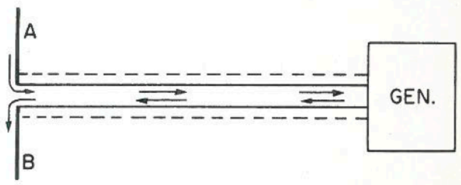


Fig. 28–1. A high-frequency signal generator drives charges up and down on two wires.

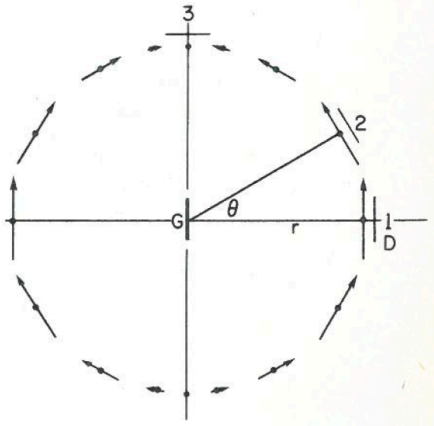


Fig. 28–2. The instantaneous electric field on a sphere centered at a localized, linearly oscillating charge.

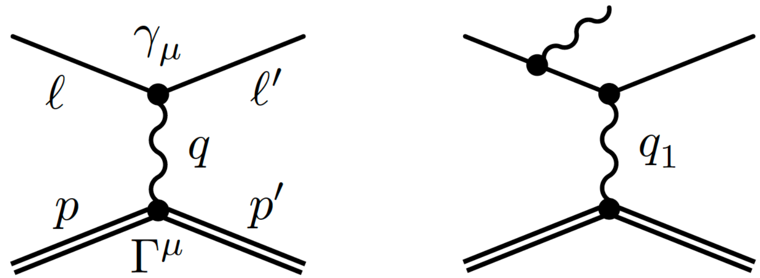
R. Feynman, Lectures on Physics I, in ch. 32 on (classical) "Radiation damping" (1963):

in phase with the motion. That is, if we have the electron standing still, we know that "action equals reaction." So the various internal forces are equal, and there is no net force. But if the electron is accelerating, then because of the time delay across it, the force which is acting on the front from the back is not exactly the same as the force on the back from the front, because of the delay in the effect. This delay in the timing makes for a lack of balance, so, as a net effect, the thing holds itself back by its bootstraps! This model of the origin of the resistance to acceleration, the radiation resistance of a moving charge, has run into many difficulties, because our present view of the electron is that it is *not* a "little ball"; this problem has never been solved. Nevertheless we can calculate exactly, of course, what the net radiation resistance force must be, i.e., how much loss there must be when we accelerate a charge, in spite of not knowing directly the mechanism of how that force works.

Nobel Prize in Physics 1965 (Tomonaga, Schwinger, Feynman)

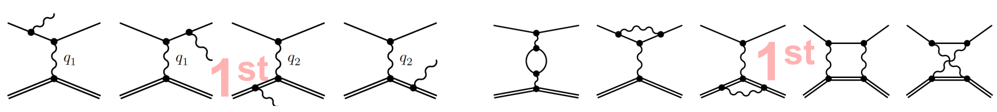
Prize motivation: "for their fundamental work in quantum electrodynamics, with deep-ploughing consequences for the physics of elementary particles."

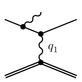
Accelerated charge radiates: correction to elastic lepton-nucleon scattering

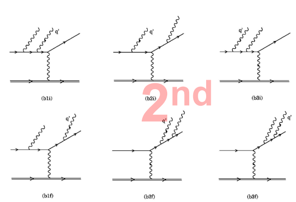


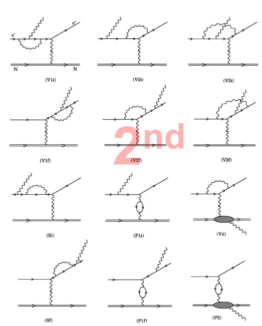
figs. from: Gramolin et al., arXiv:1401.2959

$$d\sigma_{Exp} = d\sigma_{Born}(1+\delta)$$



 includes:





real

virtual

internal corrections

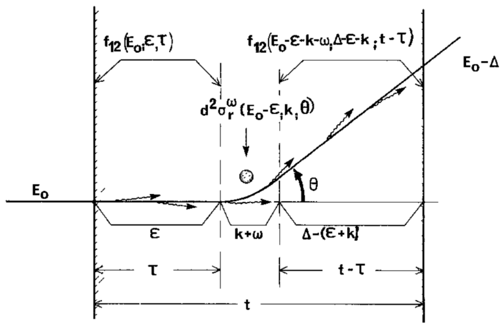
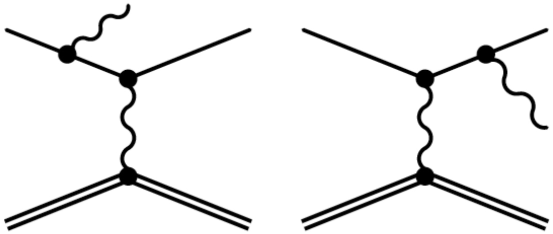


Fig. 1. The path of an electron with incident energy E_0 through a target of thickness t. Energy loss before, during and after the large-angle scattering (which occurs at target depth τ and which is shown enlarged) is ϵ, $k+\omega$ and $\Delta-(\epsilon+k+\omega)$, respectively. For further details of the nomenclature see text.

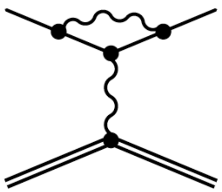
from: Pieroth et al., NIM B36 (1989)

external bremsstrahlung

1st order internal corrections

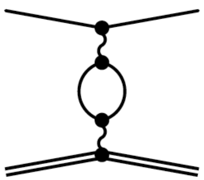


$\delta_{vertex} + \delta_R$



$Q^2 \gg m^2$

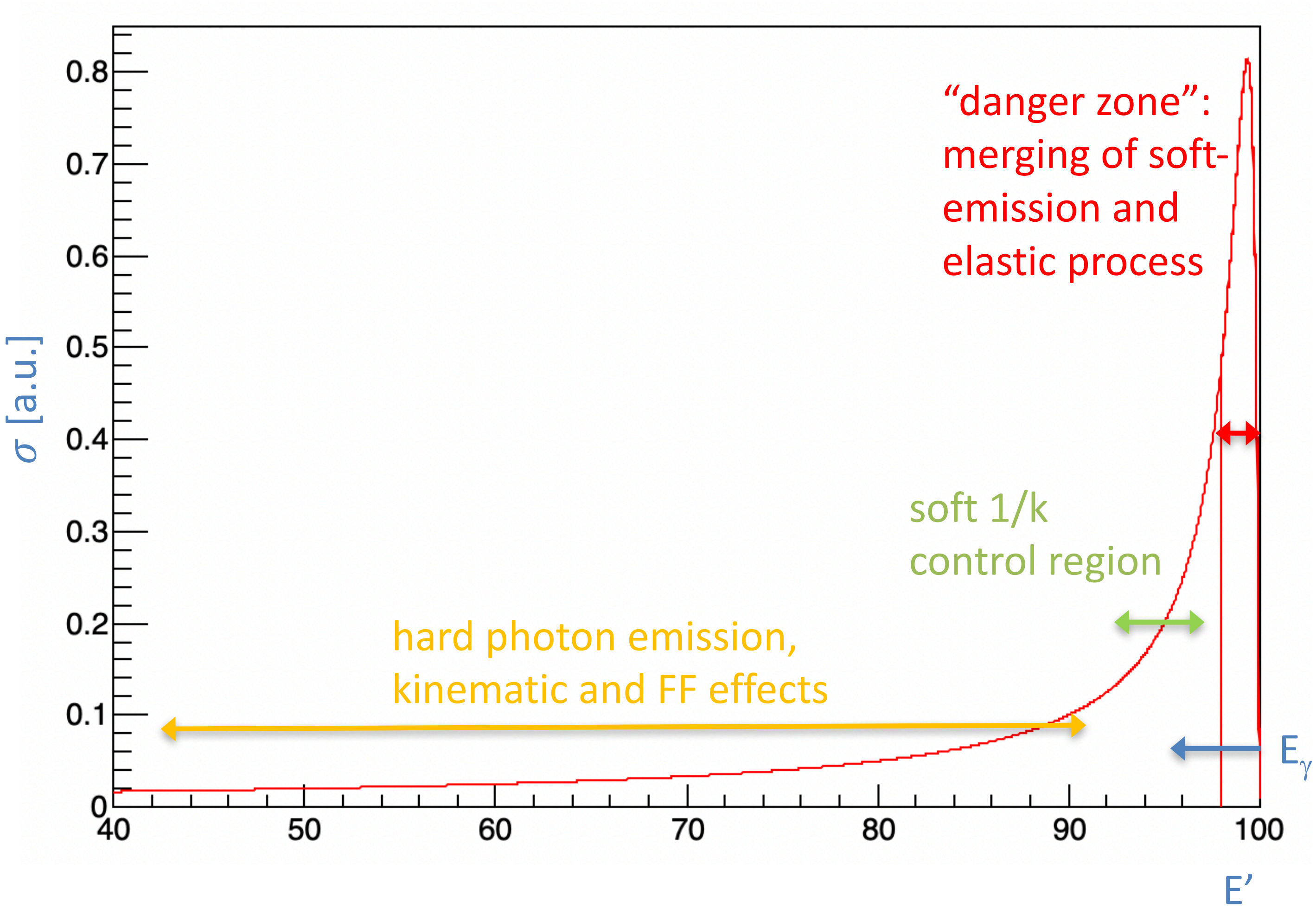
$$\delta_{vac} + \delta_{vertex} + \delta_R = \frac{\alpha_{em}}{\pi}\left\{ \ln\left(\frac{(\Delta E_s)^2}{E_e E_e'}\right)\left[\ln\left(\frac{Q^2}{m^2}\right) - 1\right] + \frac{13}{6}\ln\left(\frac{Q^2}{m^2}\right) - \frac{28}{9} - \frac{1}{2}\ln^2\left(\frac{E_e}{E_e'}\right) - \frac{\pi^2}{6} + Sp\left(\cos^2\frac{\theta_e}{2}\right)\right\},$$



δ_{vac}

- the first-order real and virtual corrections need to be merged on cross-section level
- formally: $\delta_R = +\infty$ and $\delta_R = -\infty$
- the underlying "infrared" divergence is not related to the regularization scheme
- under certain kinematic conditions and depending on the choice of the cut-off energy ΔE, parts of the corrections may cancel (or become even zero) – this does not imply that the correction is "really small"
- uncertainty has to be estimated in any case, and can be larger than the correction

Shape of the elastic peak



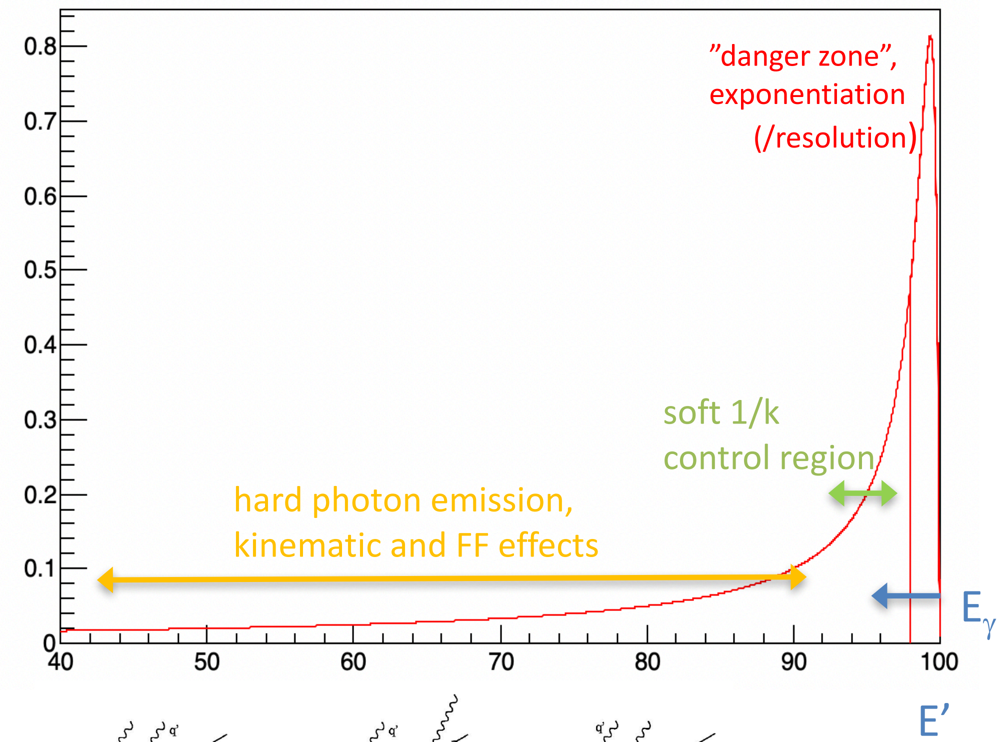
Peak shape at vanishing photon energy

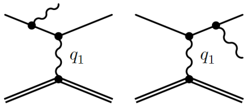
- an often-copied but still wrong "excuse" for motivating this merging procedure is e.g. found in Bjorken & Drell, Relativistic Quantum Mechanics (1964):

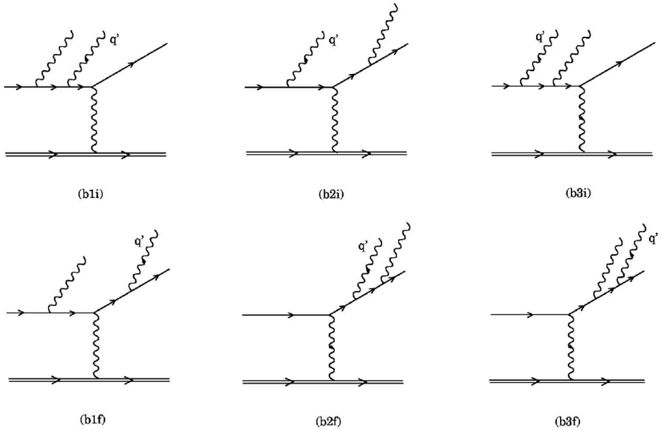
> by considering all vertex graphs involving the exchange of two virtual photons.
>
> The other terms of (8.62) and (8.63) lead to infrared divergent contributions to electron scattering. These, however, disappear when the contribution from bremsstrahlung of soft photons is included in the cross section. Any experimental apparatus has finite resolution; if electrons are detected with a given energy resolution ΔE, the number of observed events corresponds to the elastic cross section plus the bremsstrahlung cross section leading to electrons whose energy is within ΔE of the elastic value.
>
> We verify to order e^2 that this sum of elastic plus inelastic cross

Peak shape with no experimental (resp. external) smearing

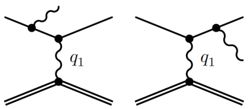
- the correction $\delta_R \xrightarrow{\Delta E \to 0} +\infty$ was originally introduced as "small correction"
- it expresses the probability to emit one real photon along the Born process
- if the emission of a photon with a certain energy is large, it is plausible that two or more photons are emitted:

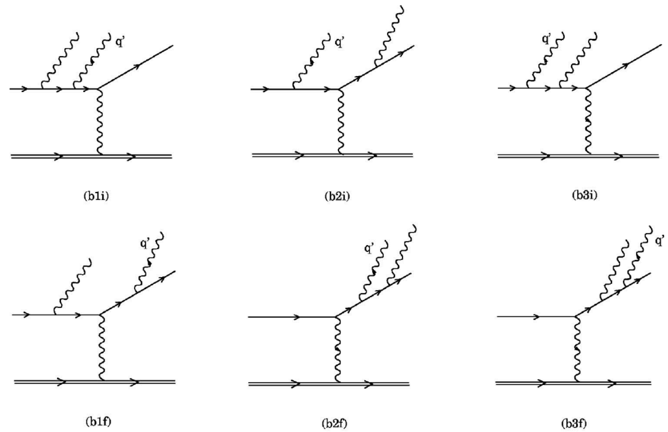






Exponentiation procedure





- if the emission of a photon with a certain energy is large, it is plausible that two (and more) photons are emitted

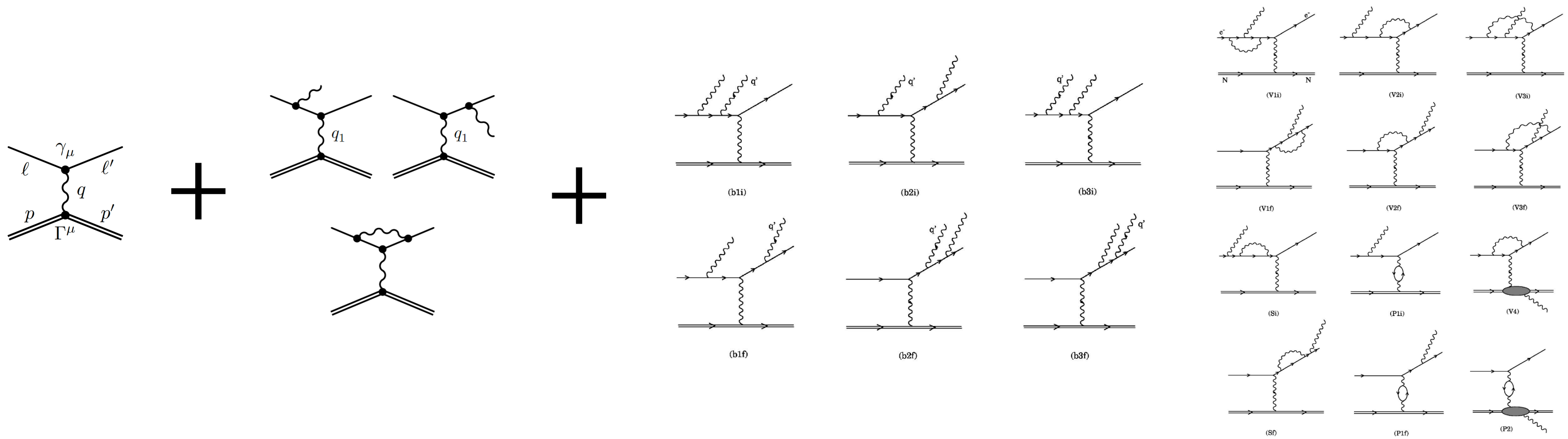
$$\left(\frac{d\sigma}{d\Omega'_e}\right)_{VIRTUAL\,\gamma} + \left(\frac{d\sigma}{d\Omega'_e}\right)_{REAL\,SOFT\,\gamma} = \left(\frac{d\sigma}{d\Omega'_e}\right)_{BORN} (1 + \delta_{vac} + \delta_{vertex} + \delta_R)$$

$$\longrightarrow$$

$$\left(\frac{d\sigma}{d\Omega'_e}\right)_{VIRTUAL\,\gamma} + \left(\frac{d\sigma}{d\Omega'_e}\right)_{REAL\,SOFT\,\gamma} = \left(\frac{d\sigma}{d\Omega'_e}\right)_{BORN} \frac{e^{\delta_{vertex} + \delta_R}}{(1 - \delta_{vac}/2)^2}.$$

- inspired by the higher-order divergence cancellation proof (Jennie, Frautschi, Suura 1961): infinitely soft photon emission / absorption becomes independent
- unclear for finite ΔE (no cheap way around calculating the higher orders)

Exponentiation procedure

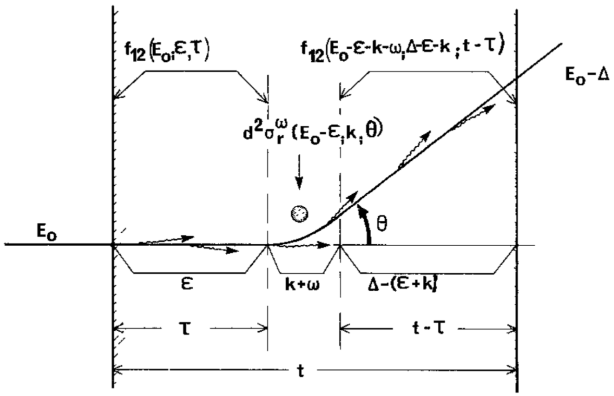


$$1+\delta(\Delta E) \;\rightarrow\; e^{\delta(\Delta E)} = 1+\delta(\Delta E) + \frac{\delta^2(\Delta E)}{2} + \cdots$$

- unclear for finite ΔE (no cheap way around the calculation of the higher orders)
- theory homework for 2nd order Feynman diagrams is done, check integral (over 4-particle f.s.)

External bremsstrahlung

- calculus of the total bremsstrahlung probability down to zero scattering angle
  - screening by atomic electrons
  - long-wavelength limit: contribution from different scattering centers
  - coherent bremsstrahlung in crystals
- "sublimation" of all effects into Tsai's radiation lengths X_0 may not be the full answer to describe correctly the external bremsstrahlung in a given setup
- best way: measure it



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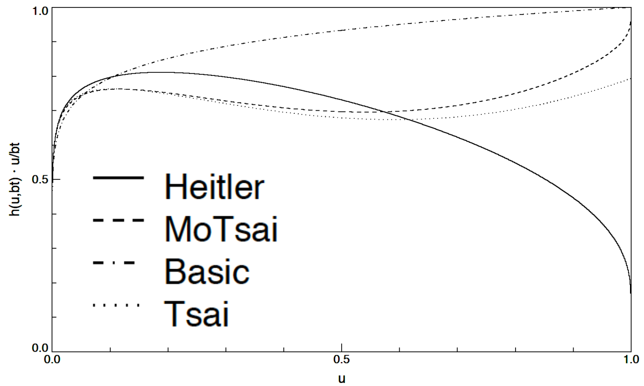


$$h_H(u,bt)\;du = \frac{1}{\Gamma(bt)}\left(-\ln[1-u]\right)^{bt-1}\;du$$

$$h_{MT}(u,bt) = n_{MT}\cdot\frac{bt}{u}\left(1-u+0.75u^2\right)\left(-\ln[1-u]\right)^{bt}$$

$$h_B(u,bt)\;du = bt\cdot u^{bt-1}\;du$$

$$h_T(u,bt) = n_T\left(1-u+0.75u^2\right)u^{bt-1}$$

from: J.F. PhD thesis 2000

Planned, ongoing, recent scattering experiments of the proton form factor

The discrepancy between the results – the proton radius puzzle - triggered many new proposals and experiments:

- e^- scattering radiative: ISR electron scattering at MAGIX-MESA
- e^- scattering at medium *E* with active-target TPC at MAMI
- e^- scattering at higher *E*: PRad at Jefferson Lab

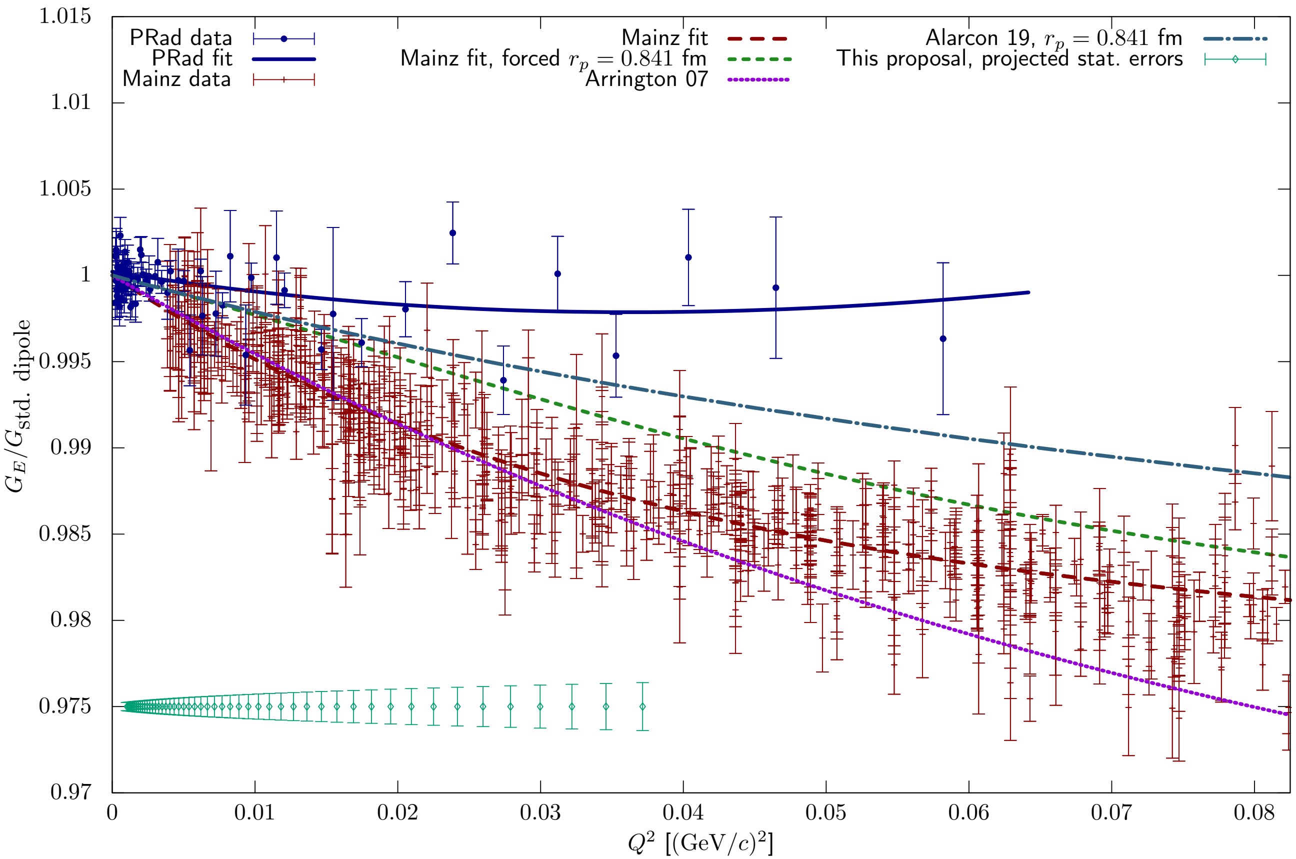
- $\mu^{+/-}, e^{+/-}$ scattering at low energy: MUSE / PSI

our Proposal:

- $\mu^{+/-}$ at high *E* at CERN (COMPASS++/AMBER)

different, in several ways favourable systematics

Mainz vs JLab data

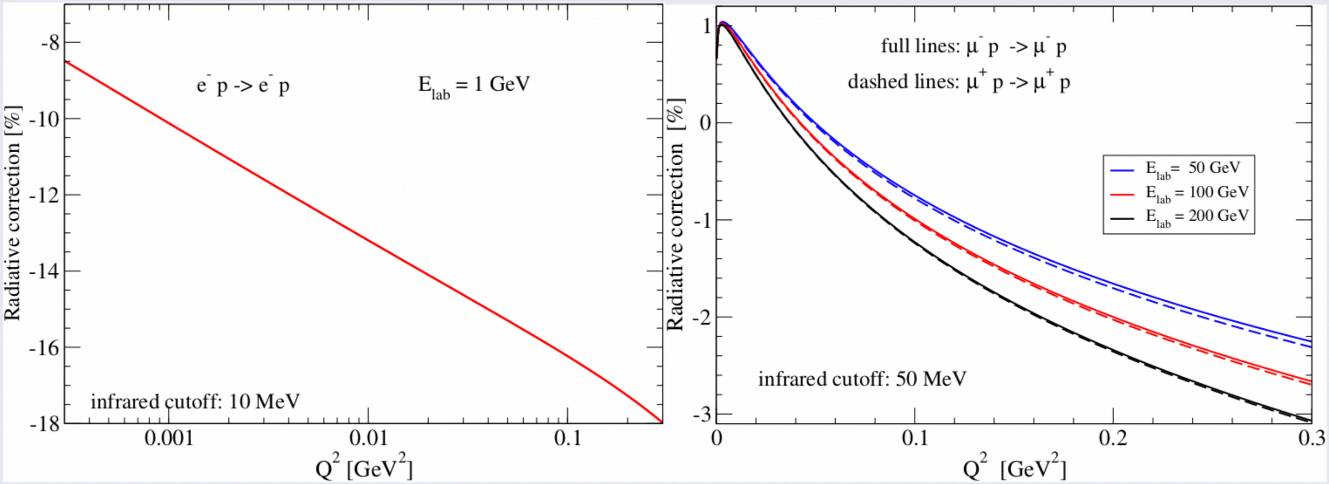


uncertainties for the COMPASS++/AMBER proposal

- program for 200 days of beam
- precision on the proton radius < 0.01 fm

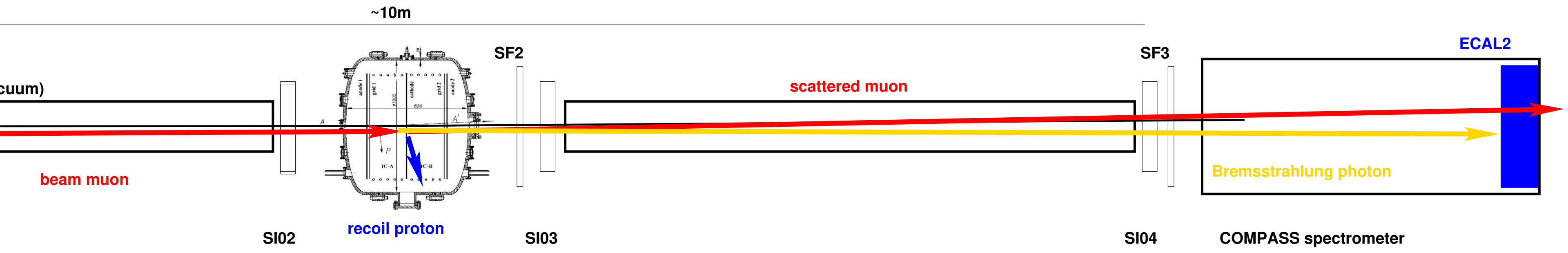
Radiative corrections for electron and muon scattering

QED radiative corrections



- for soft bremsstrahlung photon energies ($E_\gamma / E_{beam} \sim 0.01$), QED radiative corrections amount to $\sim$ 15-20% for electrons, and to $\sim$ 1.5% for muons
- important contribution to the uncertainty of elastic scattering intensities: *change* of this correction over the kinematic range of interest
- check: impact of exponantiation procedure (stricty valid only for vanishing photon energies): e^-: $2-4\%$, μ^-: 0.1%
- integrating the radiative tail out to large fraction of beam energy: shifts the correction to smaller values, but only *increases* the uncertainty

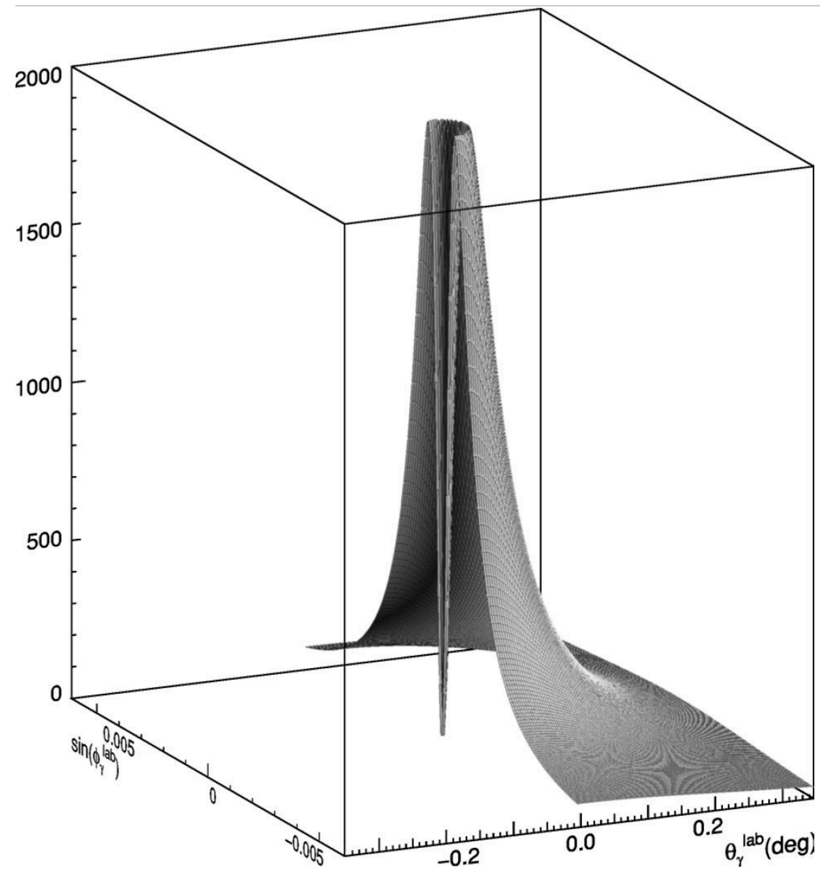
Bremsstrahlung: real-photon emission and observation along muon-proton scattering



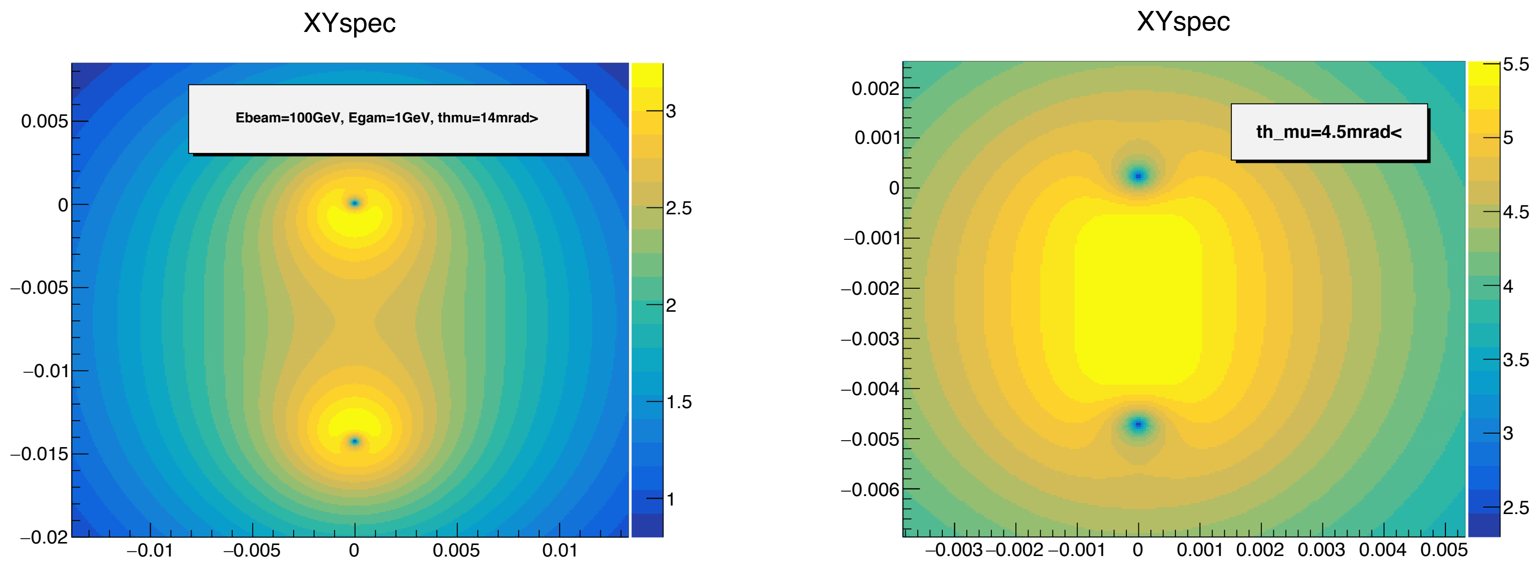
- Bremsstrahlung accompanies the elastic process
- for low-energy photons roughly $1/E_\gamma$ ('infrared divergence')
- angular spectrum: peaking in the relativistic case, opening angle $1/\gamma$ [Lorentz factor]
- 100 GeV beam: E_γ between 50 MeV and 5 GeV emission probability at θ_μ =0.3mrad (Q^2=0.001): 5 $\times 10^{-4}$
- Bremsstrahlung events for 7e7 elastic events in Q^2=0.001…0.04 GeV^2/c^2 are about 38000

Peaking and forward effect for small Q^2

- Bremsstrahlung of ultra-relativistic moving charges is peaked with opening angle $1/\gamma$
- emission probability in exact forward direction practically vanishes
- if the lepton scattering angle is in the order of the radiation opening angle ($Q^2 \approx m^2$), interference becomes important



Bremsstrahlung emission angle, E=100GeV



- forward cancellation in case of 100 GeV muon scattering at $Q^2 < m^2 \approx 0.01\,\text{GeV}^2$ ($\vartheta_\mu \approx 1\text{mrad}$)
- similar effect discussed in Fadin & Gerasimov, PLB 795 (2019) (however "neglect m^2 compared with M^2 and Q^2")

Generators

- for a concise description of the experimental conditions, the simulation must include all effects from
  - atomic collision energy loss (Landau straggling)
  - external bremsstrahlung
  - internal radiative corrections

- for the internal 1st order corrections, the ESEPP generator became available (arXiv-1401.2959)
  - implementation of full corrections including the real-photon distributions
  - usage of the TFoam (CERN/root) library for importance sampling

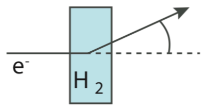
from arXiv-1401.2959 about higher orders:

$$\frac{\mathrm{d}\sigma_{\mathrm{meas}}}{\mathrm{d}\Omega_\ell} = \exp(\delta)\,\frac{\mathrm{d}\sigma_{\mathrm{Born}}}{\mathrm{d}\Omega_\ell}. \tag{2.3}$$

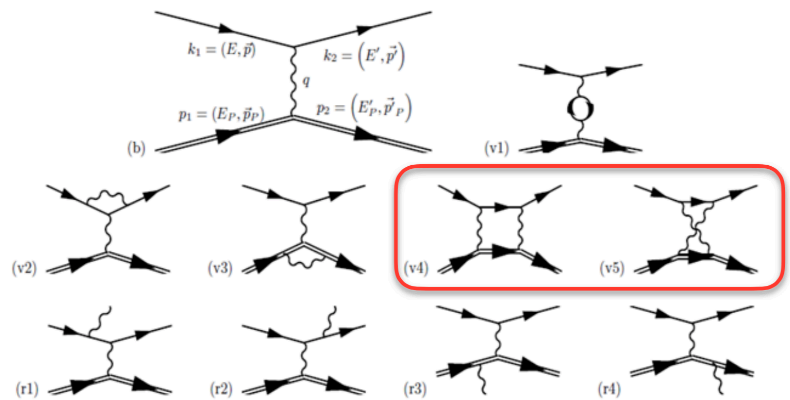
This exponentiation procedure is incompatible with our approach, but we can use the formula (2.3) to make a rough estimation of the contribution of higher-order bremsstrahlung. To do this, we choose the following numerical parameters approximately corresponding to the Novosibirsk TPE experiment: $E_\ell = 1$ GeV, $-q^2 = 1$ GeV^2, and $\Delta E = 0.1$ GeV.

do the calculation with $\Delta E = 0.01\ \mathrm{GeV}$ *and get the uncertainty orders of magnitude higher.*

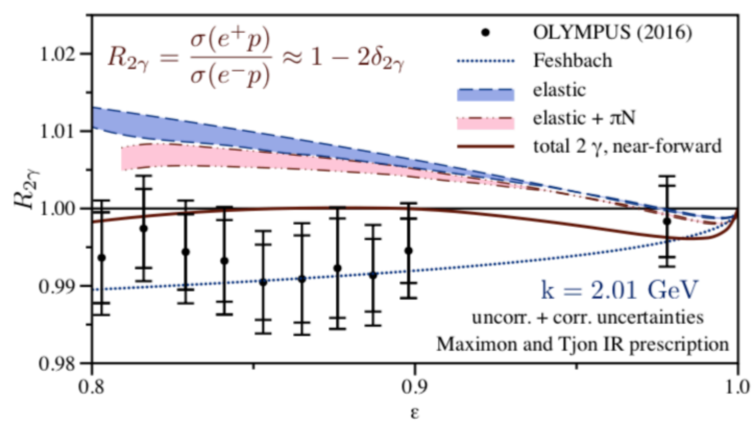
two-photon exchange
Radiative corrections



e⁻-p scattering



$$\sigma^{exp} \equiv \sigma_{1\gamma}(1 + \delta_{soft} + \delta_{2\gamma})$$



near-forward 2γ agree with data
multi-particle 2γ, e.g. ππN, is important

Tomalak, Pasquini, Vdh (2017)
Pasquini, Vdh, Ann.Rev.Nucl.Part.Sci(2018)

Conclusion from a different corner…

The important subject of emission of radiation by accelerated point charges is discussed in detail in Chapters 14 and 15. Relativistic effects are stressed, and expressions for the frequency and angular dependence of the emitted radiation are developed in sufficient generality for all applications. The examples treated range from synchrotron radiation to bremsstrahlung and radiative beta processes. Cherenkov radiation and the Weizsäcker–Williams method of virtual quanta are also discussed. In the atomic and nuclear collision processes semiclassical arguments are again employed to obtain approximate quantum-mechanical results. I lay considerable stress on this point because I feel that it is important for the student to see that radiative effects such as bremsstrahlung are almost entirely classical in nature, even though involving small-scale collisions. A student who meets bremsstrahlung for the first time as an example of a calculation in quantum field theory will not understand its physical basis.

J. D. Jackson
Urbana, Illinois, January, 1962

Summary

- my caveats:
  - conclude on the exponentiation (*i.e.* higher-order) issue
  - verify the validity of external bremsstrahlung description
  - estimation of the radiative correction uncertainty is even more involved than the correction itself
  - mixing *e.g.* of standard simulation tools with single-photon generators may turn to an incorrect description resp. interpretation of experimental data
  - internal structure effects (*e.g.* of the proton as scatterer) add their uncertainty to the correction

- my wish:
  - a versatile, precise, fast generator including all internal and external radiation, energy loss and straggling effects

… and learn a lot more along this workshop!

Charge radius: definition and model dependence

Determination of the rms radius from a form factor measurement

- the rms radius of a charge distribution seen in lepton scattering is *defined* as the slope of the electric form factor at vanishing momentum transfer Q^2

$$\langle r_E^2 \rangle = -6\hbar^2 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2 \to 0}$$

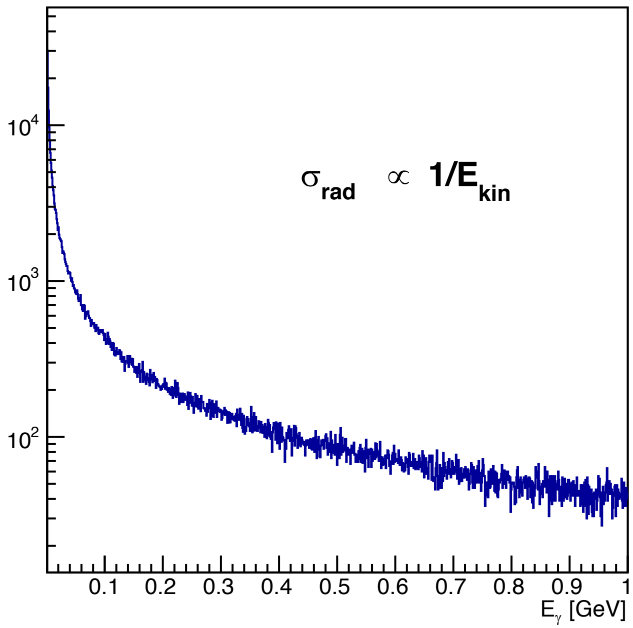
- elastic scattering experiments provide data for G_E at non-vanishing Q^2 and thus require an extrapolation procedure towards zero
  $\rightarrow$ mathematical ansatz may take more or less bounds into account (physics/theory/whatever motivated)
- Any approach (Padé, CF, DI, CM,...) *must* boil down to a series expansion

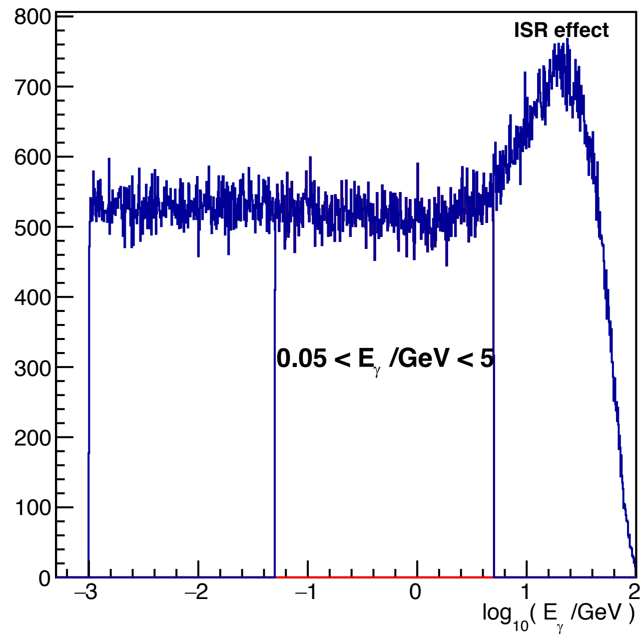
$$G_E(Q^2) = 1 + c_2 Q^2 + c_4 Q^4 + \ldots$$

  introducing possibly very different assumptions on the coefficients c_i
- recipe for experimenters: measure a sufficiently large range of Q^2 down to values as small as possible and as precise as possible

Real-photon energy spectrum

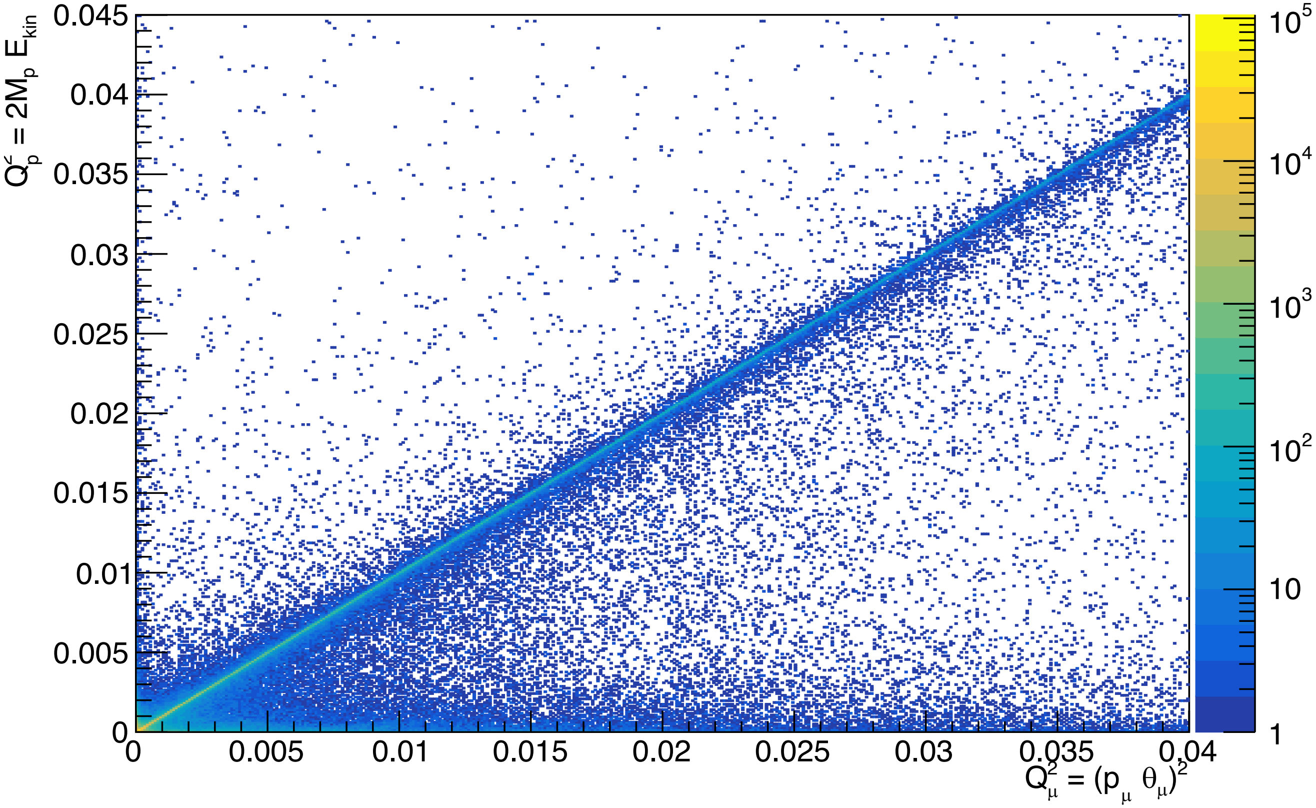
MC simulation of 500k events in θ_μ =0.3 … 2 mrad, E_γ > 1 MeV





ISR effect: if incoming muon loses much of its energy, the scattering off the proton under a specific scattering angle happens at lower average Q^2 and accordingly a larger cross section

Impact on Q2 reconstruction



real-photon emission distorts the kinematics, correlation of reconstruction from muon and recoil proton becomes blurred

Nuclear Instruments and Methods
Volume 129, Issue 2, 15 November 1975, Pages 505-514

Radiation tail and radiative corrections for elastic electron scattering ☆

J. Friedrich

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https://doi.org/10.1016/0029-554X(75)90745-4

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Abstract

A set of formulae, which is currently used to calculate radiative effects in electron scattering experiments, is discussed. Since these formulae contain approximations which fail to describe radiation losses properly a modified set is proposed. The radiation unfolding procedure of elastically scattered electrons is investigated in detail, account is taken of the different sets of formulae and of the finite energy resolution. The cross sections extracted from measured spectra depend on the formulae

The quest for the extension of the proton – a look at the origins

PHYSICAL REVIEW VOLUME 79, NUMBER 4 AUGUST 15, 1950

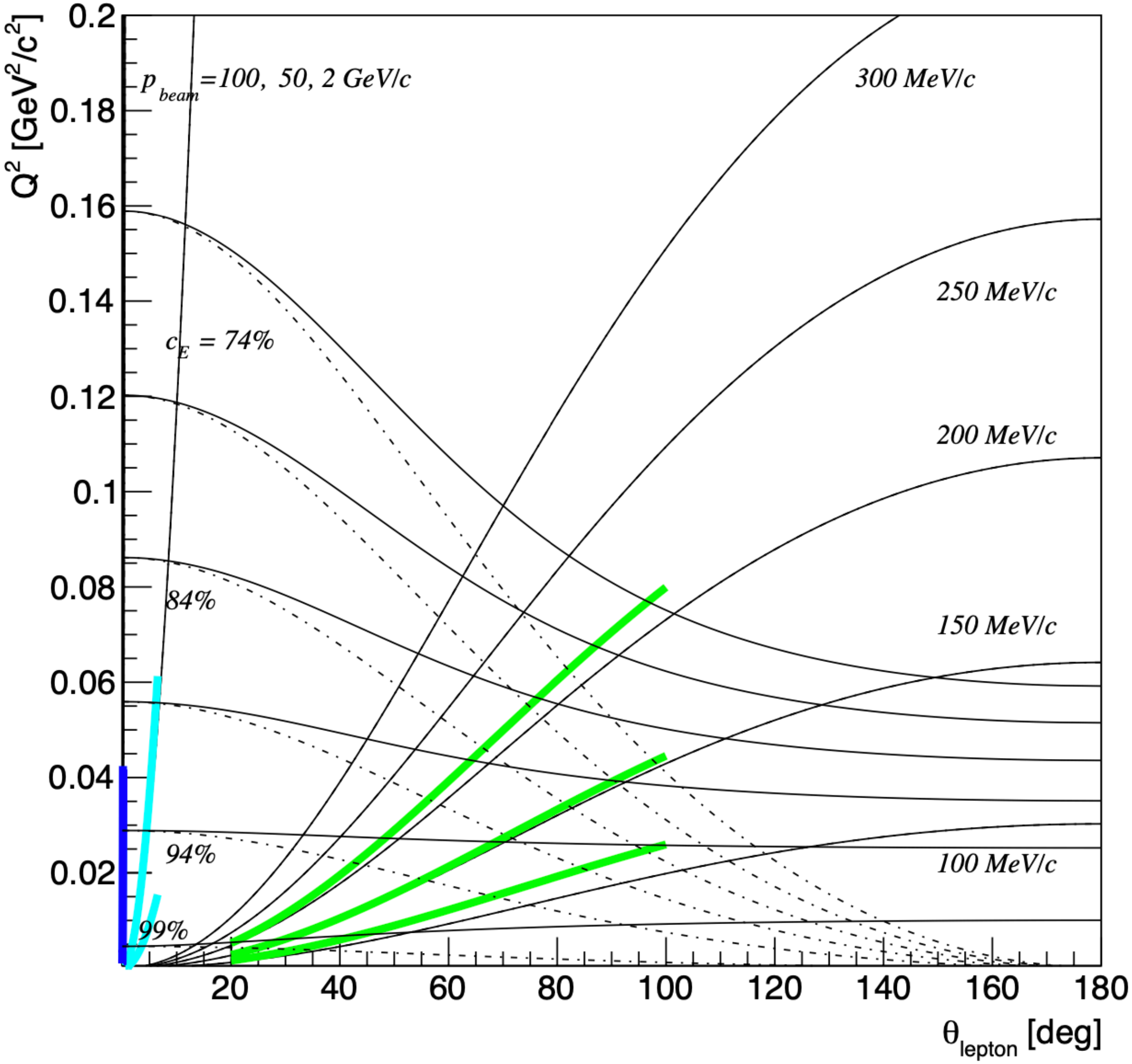
High Energy Elastic Scattering of Electrons on Protons

M. N. ROSENBLUTH
Stanford University, Stanford, California

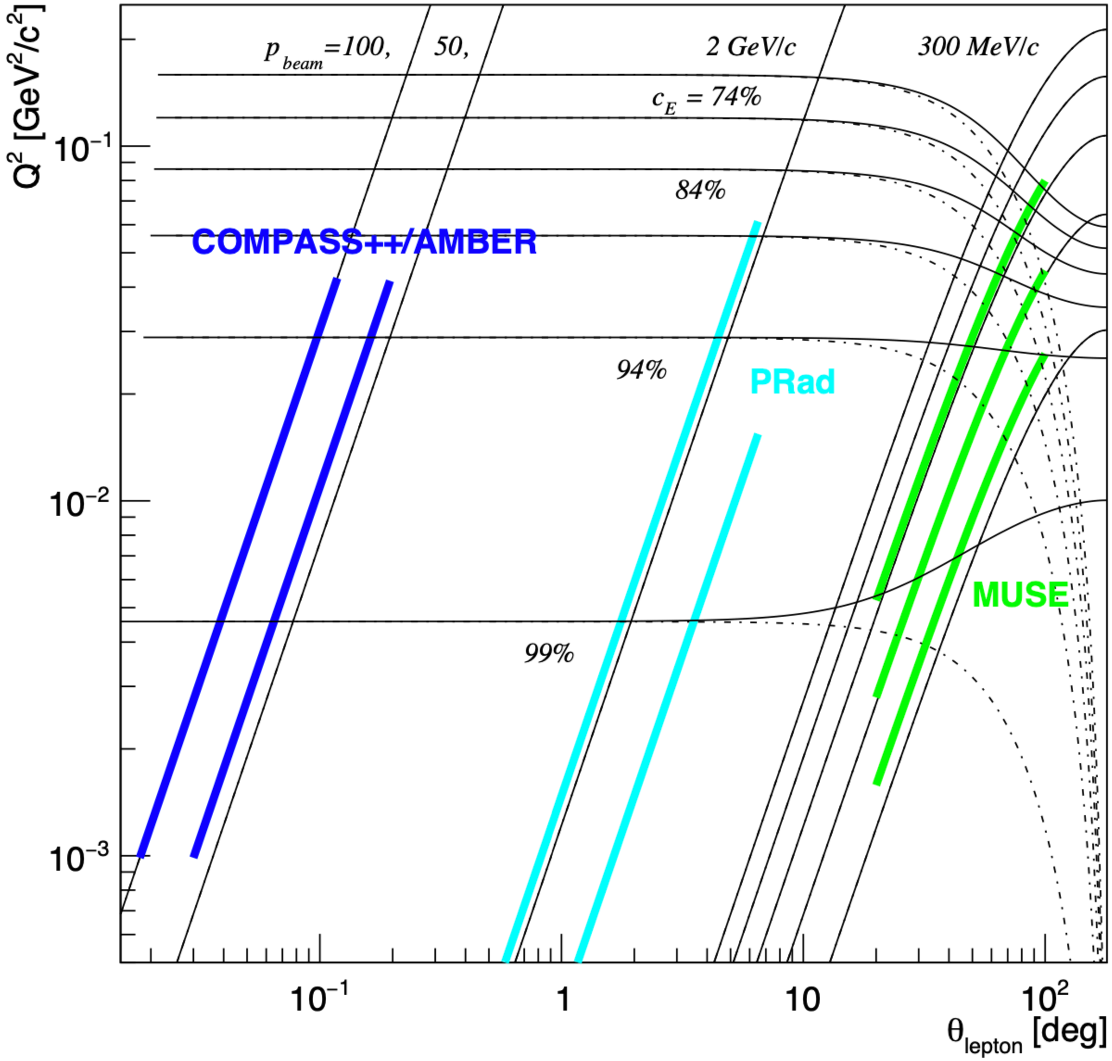
(Received March 28, 1950)

The theory of the elastic scattering of electrons on protons at very high energies is discussed in detail. A formula is given for the cross section. This formula contains certain parameters which depend on the action of the virtual photon and meson fields. In particular, curves have been calculated on the assumption of scalar and pseudoscalar meson theory. While these perturbation theory calculations are not very trustworthy, and the results depend on the choice of coupling constants, it is felt that qualitative features can be checked with experiment. It is concluded that at low relativistic energies ($E < 50$ Mev) the experiment provides a valuable check on quantum electrodynamics. At higher energies it should yield data on the nature of the meson cloud of the proton.

Kinematic ranges



Kinematic ranges



Models for the Nucleon Form Factors employing Dispersion Relations

Nuclear Physics A 596 (1996) 367–396

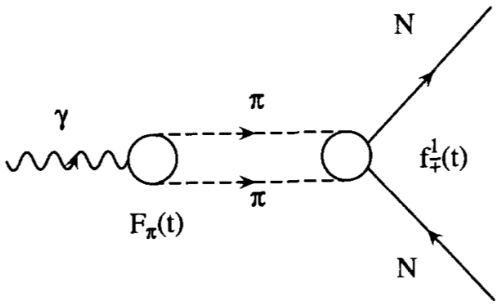
Dispersion-theoretical analysis of the nucleon electromagnetic form factors ★

P. Mergell [a,1], Ulf-G. Meißner [b,2], D. Drechsel [a,3]

[a] *Universität Mainz, Institut für Kernphysik, J.-J.-Becher Weg 45, D-55099 Mainz, Germany*

[b] *Universität Bonn, Institut für Theoretische Kernphysik, Nussallee 14-16, D-53115 Bonn, Germany*

Received 21 June 1995



ig. 1. Two-pion cut contribution to the isovector nucleon form factors.

Table 2
Proton and neutron radii

	r_E^p [fm]	r_M^p [fm]	r_M^n [fm]	r_1^p [fm]	r_2^p [fm]	r_2^n [fm]
Best fit	0.847	0.836	0.889	0.774	0.894	0.893
Ref. [21]	0.836	0.843	0.840	0.761	0.883	0.876

accurate values from a few-parameter fit to all-Q² data

For the data in the low-energy region, the contribution of the Q^4 term to the proton electric form factor is marginal ($< 0.3\%$). This leads to an rather accurate value for $\langle r_E^2 \rangle_p$,

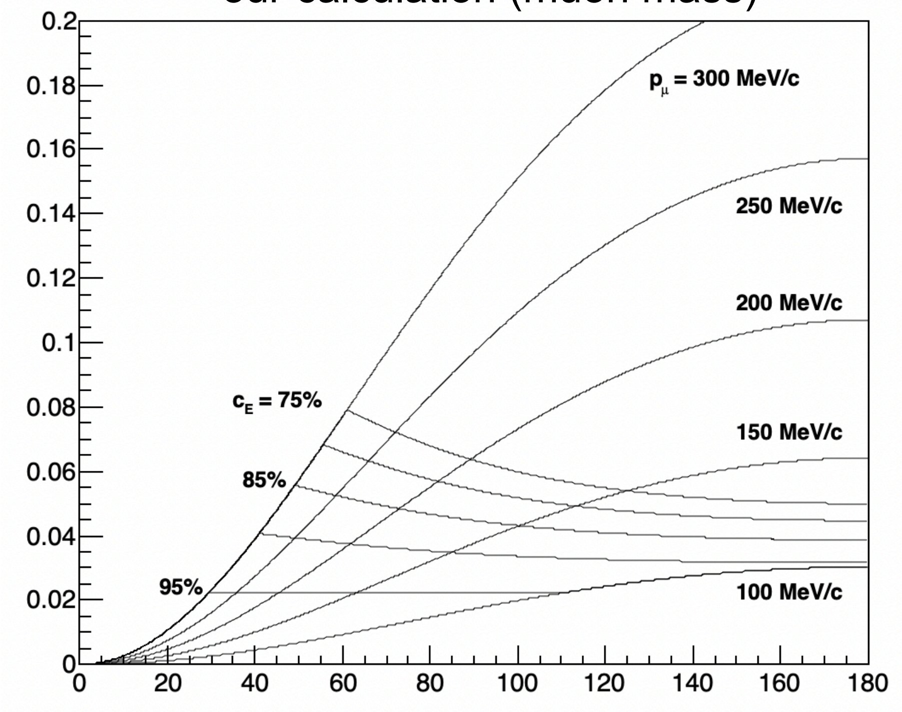
$$\langle r_E^2 \rangle_p = (0.862 \pm 0.012)^2 \mathrm{fm}^2 . \qquad (29)$$

low-Q² experimental of-the-time value discussed

With that constraint, the authors of Ref. [15] performed a four-pole fit (with two masses fixed at $M_\rho = 0.765$ GeV and $M_{\rho'} = 1.31$ GeV) to the available data for the proton electric and magnetic form factors up to $Q^2 \simeq 5$ GeV^2. This allowed to reconstruct the

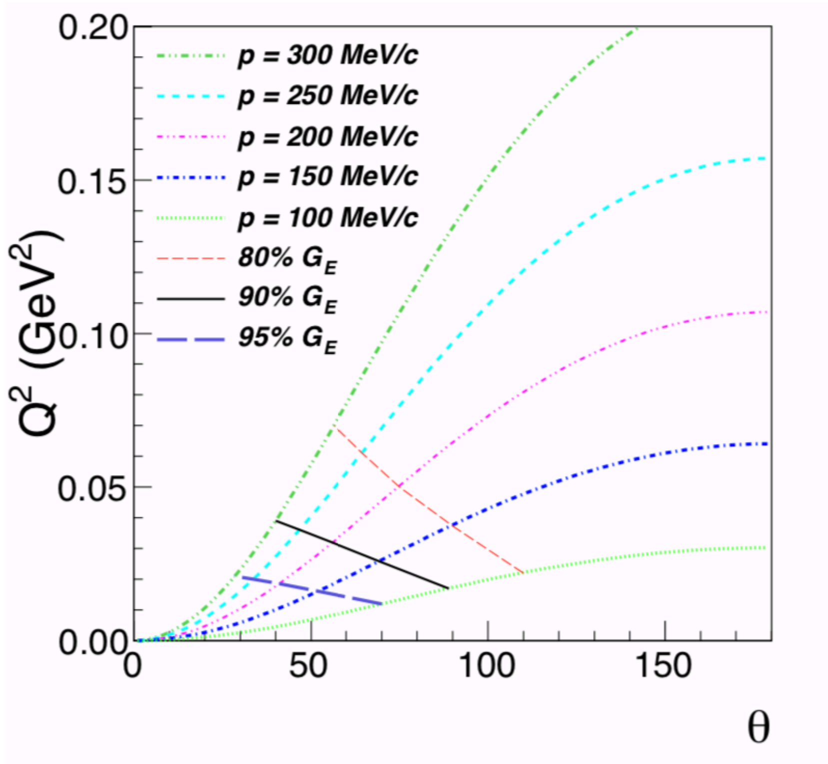
MUSE – kinematics of low-energy elastic muon scattering

our calculation (muon mass)

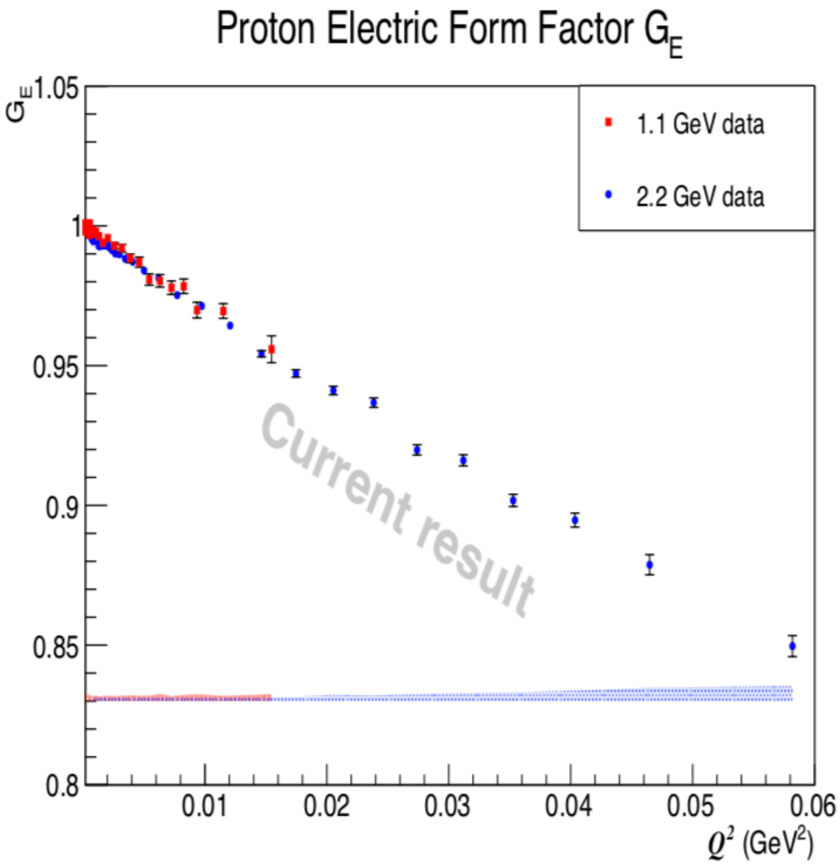


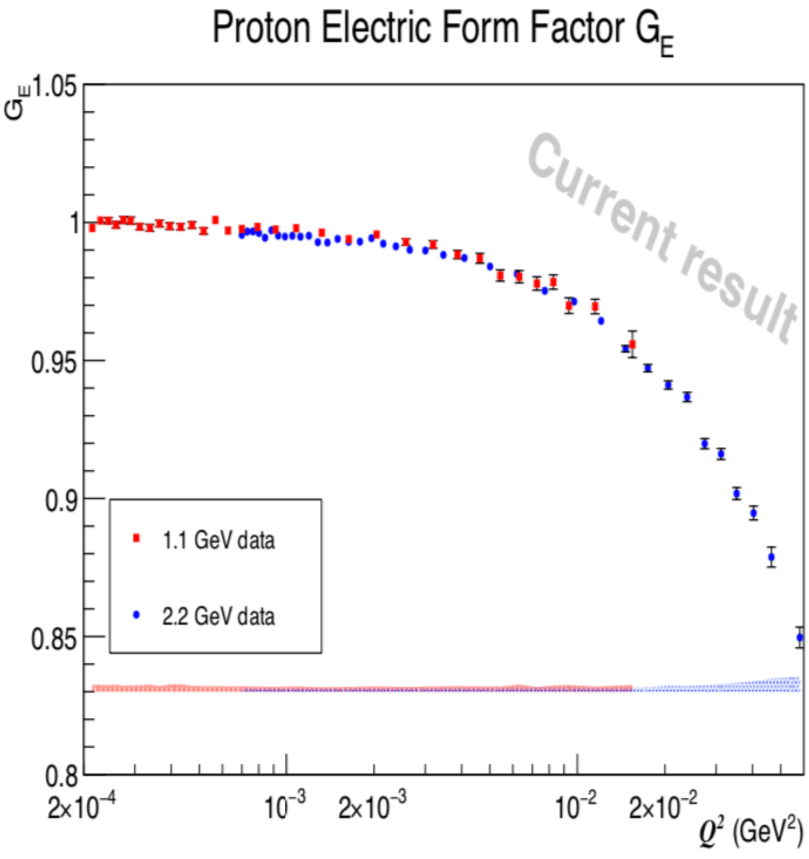
A Proposal for the Paul Scherrer Institute πM1 beam line

Studying the Proton "Radius" Puzzle with μp Elastic Scattering



PRad





Lowest Q^2 ever achieved from ep elastic scattering

from: H. Gao, ICSAC2019, Losinj, Croatia

General cross-section behavior

- steep increase towards smaller Q^2 with $1/Q^4$
- forever rising?
- not for scattering off atoms / molecules: