Radiative corrections for precision electron-proton scattering

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**Born Approximation**

\[ M_\gamma = \frac{e^2}{Q^2} \bar{u}_e(k') \gamma_\mu u_e(k) \bar{u}_N(p') \left[ F_1(Q^2) \gamma^\mu + F_2(Q^2) \frac{i\sigma^{\mu\nu} q_\nu}{2M} \right] u_N(p) \]

\[ \left( \frac{d\sigma}{d\Omega} \right)_0 = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \frac{\varepsilon G_E^2(Q^2) + \tau G_M^2(Q^2)}{\varepsilon (1 + \tau)} \]

\[ Q^2 \equiv -q^2 = 4EE' \sin^2(\theta/2) \]
\[ \tau = Q^2/4M^2 \]
\[ \varepsilon = [1 + 2(1 + \tau) \tan^2(\theta/2)]^{-1} \]
\[ G_E(0) = 1; \quad G_M(0) = \mu_p \]

\[ G_E = F_1 - \tau F_2; \quad G_M = F_1 + F_2 \]

Experimental cross section is not Born cross section!

\[ \left( \frac{d\sigma}{d\Omega} \right)_{\text{exp}} = \left( \frac{d\sigma}{d\Omega} \right)_0 (1 + \delta); \quad \delta \rightarrow \text{Radiative Corrections.} \]
Two Photon Exchange

TPE Amplitude $\mathcal{M}_{\gamma\gamma} = \mathcal{M}_{\gamma\gamma}^{\text{box}} + \mathcal{M}_{\gamma\gamma}^{\text{xbox}}$

$\mathcal{M}_{\gamma\gamma}^{\text{xbox}} \rightarrow$ purely real!

TPE cross-section, $\delta_{\gamma\gamma} = \frac{2 \text{Re}(\mathcal{M}_{\gamma}^\dagger \mathcal{M}_{\gamma\gamma})}{|\mathcal{M}_\gamma|^2}$

\[
\mathcal{M}_{\gamma\gamma} = \frac{e^2}{Q^2} \bar{u}_e(k') \gamma_\mu u_e(k) \bar{u}_N(p') \left[ F'_1(Q^2, \nu) \gamma^\mu + F'_2(Q^2, \nu) \frac{i\sigma^{\mu\nu} q_\nu}{2M} \right] u_N(p) + \frac{e^2}{Q^2} \bar{u}_e(k') \gamma_\mu \gamma_5 u_e(k) \bar{u}_N(p') G'_a(Q^2, \nu) \gamma^\mu \gamma_5 u_N(p)
\]

$\delta_{\gamma\gamma} = 2\text{Re} \frac{\varepsilon G_E (F'_1 - \tau F'_2) + \tau G_M (F'_1 + F'_2) + \nu (1 - \varepsilon) G_M G'_a}{\varepsilon G_E^2 + \tau G_M^2}$
Approaches

- **GPD approach**: (PhysRevLett.93.122301)
  
  Valid only in large momentum transfer; $|s, t, u| >> M^2$

- **Hadronic Degrees of Freedom**:
  - Low to moderate ($Q^2 \lesssim 5$ GeV$^2$)
  - Direct Loop integration (real part)
  - Sums/products of monopole form factors
  - Half off-shell form factor ambiguity
  - Divergence in the forward angles (high energy) limit for $\Delta$ resonance!

- **Dispersive approach**:
  - Sum of monopole form factors.
    - Blunden et al. (PRC 95, 065209 (2017)) $\Rightarrow$ More generalized class of form factors

  Applied for all 3 and 4-star resonance intermediate states with $W < 1.8$ GeV
Dispersive Method

\[ S = 1 + iM \]
\[ S^\dagger S = (1 - iM^\dagger)(1 + iM) = 1 \]

Unitarity → \(-i(M - M^\dagger) = 2 \text{Im} \ M = M^\dagger M\)

\[ \text{Im} \langle f|M|i \rangle = \frac{1}{2} \int \frac{d^3 k_1}{(2\pi^3) 2E_{k_1}} \sum_n \langle f|M^*|n \rangle \langle n|M|i \rangle \]
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\]

- On-shell intermediate lepton & hadron: \( \text{Im} M_{\gamma\gamma} \Rightarrow \text{Im}(F'_1, F'_2, G\alpha') \)
- On-shell parametrization of hadronic transition current

**Resonance intermediate states:** \( \Delta(1232) \frac{3}{2}^+, N(1440) \frac{1}{2}^+, N(1520) \frac{3}{2}^−, N(1535) \frac{1}{2}^−, \Delta(1620) \frac{1}{2}^−, N(1650) \frac{1}{2}^−, \Delta(1700) \frac{3}{2}^−, N(1710) \frac{1}{2}^+, \text{ and } N(1720) \frac{3}{2}^+ \)

- CLAS exclusive meson electroproduction data for \( A_{1/2}, A_{3/2} \) and \( S_{1/2} \)
**Dispersion Relations**

\[
\text{Re } F_1'(Q^2, \nu) = \frac{2}{\pi} \mathcal{P} \int_{\nu_{\text{min}}}^{\infty} d\nu' \frac{\nu}{\nu'^2 - \nu^2} \text{Im } F_1'(Q^2, \nu')
\]

\[
\text{Re } F_2'(Q^2, \nu) = \frac{2}{\pi} \mathcal{P} \int_{\nu_{\text{min}}}^{\infty} d\nu' \frac{\nu}{\nu'^2 - \nu^2} \text{Im } F_2'(Q^2, \nu')
\]

\[
\text{Re } G_a'(Q^2, \nu) = \frac{2}{\pi} \mathcal{P} \int_{\nu_{\text{min}}}^{\infty} d\nu' \frac{\nu'}{\nu'^2 - \nu^2} \text{Im } G_a'(Q^2, \nu')
\]

- **Integrals extend into unphysical region** \((\cos \theta < -1)\)

\begin{itemize}
  \item e.g. \(\Delta(1232)3/2^+\)
  \[Q^2 = 1 \text{ GeV}^2\]
  \[\nu_{\text{th}} = 0.604, \nu_{\text{min}} = 0.078\]
\end{itemize}

- **Analytically continued into the unphysical region** \((\text{PRC 95, 065209})\)
Technical Overview

\[ I_\delta = \frac{\alpha}{4\pi} Q^2 \frac{1}{i\pi^2} \int d^4 q_1 \frac{\text{Im} \left[ L_{\alpha\mu\nu} H^{\alpha\mu\nu} \right]}{(q_1^2 - \lambda^2)(q_2^2 - \lambda^2)} \]

Unitarity:
\[ I_\delta = \frac{s - W^2}{4s} \int d\Omega_{k_1} \frac{G_i(Q_{1}^2) G_j(Q_{2}^2) f_{ij}(Q_{1}^2, Q_{2}^2)}{(Q_1^2 + \lambda^2)(Q_2^2 + \lambda^2)} \]

- \( f_{ij} \) are polynomial of combined degree 4 in \( Q_{1,2}^2 \)
- \( G_{i,j}(Q_{1,2}^2) \): transition form factors at hadronic vertices; \((i, j = 1, 2, 3)\)
- \( G_{i,j}(Q_{1,2}^2) \Leftrightarrow A_h \); direct fit to CLAS \( A_h \) data

Numerical contour integration (PRC 95, 065209 (2017))
\[ \Rightarrow \text{Arbitrary functional forms for } G_{i,j}(Q_{1,2}^2) \]
● TPE cross section correction from individual resonances:

- $Q^2 = 1 \text{ GeV}^2$
  - (a)

- $Q^2 = 3 \text{ GeV}^2$
  - (b)

- $Q^2 = 5 \text{ GeV}^2$
  - (c)

- $\Delta(1232) \ 3/2^+$
- $N(1440) \ 1/2^+$
- $N(1520) \ 3/2^-$
- $N(1535) \ 1/2^-$
- $\Delta(1620) \ 1/2^-$
- $N(1650) \ 1/2^-$
- $\Delta(1700) \ 3/2^-$
- $N(1710) \ 1/2^+$
- $N(1720) \ 3/2^+$
- Total TPE cross-section correction Vs. virtual photon polarization $\varepsilon$

- TPE cross section correction as a function of $Q^2$: 
• Assumed continuum of $W^2$ as an infinite set of $\delta(W^2 - W_i^2)$, for each resonance. A Breit-Wigner distribution is used.

• Constant total decay width washes away the cusps!
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• Constant total decay width washes away the cusps!

• Negligible width effect on total TPE cross section magnitude and slope. 

$$R_{2\gamma} = \frac{\sigma(e^+ p)}{\sigma(e^- p)} \approx 1 - 2\delta_{\gamma\gamma}$$
- **VEPP3 result:**

![Graph showing VEPP3 results with plots for different energies.](image)

- **OLYMPUS result:**

![Graph showing OLYMPUS results with plots for different energies.](image)
Polarization observable: $R_{TL} = -\mu_p \sqrt{\frac{\tau(1 + \varepsilon)}{2\varepsilon}} \frac{P_T}{P_L}$
$\mu_p G_E / G_M$

⇒ Dr. Peter Blunden
Beam/target normal Single Spin Asymmetry (SSA)

- Beam/target normal single spin asymmetry:
  \[ SSA = \frac{\sigma_{\uparrow} - \sigma_{\downarrow}}{\sigma_{\uparrow} + \sigma_{\downarrow}} \]

- Polarization vector parallel or anti-parallel to \( S_n = \frac{k \times k'}{|k \times k'|} \)

- Time reversal invariance and unitarity:
  \[ SSA = \frac{2 \text{Im}(\sum_{\text{spins}} M^*_\gamma \text{Abs} M_{\gamma\gamma})}{\sum_{\text{spins}} |M_\gamma|^2} \]

- SSA ⇒ \( B_n \) or \( A_n \)
Beam Normal SSA:

(a) $Q_{\text{we}}$

$B_n$(ppm) vs. $\theta_{\text{cm}}$

$E_{\text{Lab}} = 1.149$ GeV

(b) Nucleon and Total

$B_n$(ppm) vs. $\theta_{\text{cm}}$

$E_{\text{Lab}} = 3.031$ GeV

G0 2007

(c) Nucleon and Total

$B_n$(ppm) vs. $\theta_{\text{cm}}$

$E_{\text{Lab}} = 3.026$ GeV

HAPPEX 2012
Beam Normal SSA:

(a) $E_{\text{Lab}} = 0.315$ GeV

(b) $E_{\text{Lab}} = 0.420$ GeV

(c) $E_{\text{Lab}} = 0.5102$ GeV

(d) $E_{\text{Lab}} = 0.569$ GeV

$B_n$ (ppm) vs. $\theta_{\text{cm}}$ for different energies and years.
Target Normal SSA

(a) $E_{\text{Lab}} = 1.245$ GeV

(b) $E_{\text{Lab}} = 2.425$ GeV

(c) $E_{\text{Lab}} = 3.605$ GeV

Uncertainty Scaled by Factor of 10
Summary

- $N(1520)3/2^-$ is the major contributor for higher $Q^2$
- Elastic nucleon alone is a good approximation for $Q^2 < 1 \text{ GeV}^2$
- Overall enhancement in the TPE cross section correction at $Q^2 > 3 \text{ GeV}^2$
- Width effect is negligible
- Proper inclusion of TPE resolves $\mu_p G_E/G_M$ discrepancy
- Need more data in the higher $Q^2$ region
- Follow up work: inclusion of non-resonant background and spin $5/2$ resonances.
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Thanks!