Radiative corrections for precision electron-proton scattering

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In collaboration with Wally Melnitchouk
arXiv:2006.12543 (2020)

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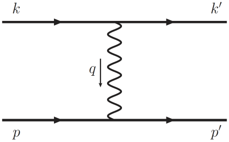
Born Approximation



$$\mathcal{M}_\gamma = \frac{e^2}{Q^2}\,\bar{u}_e(k')\gamma_\mu u_e(k)\;\bar{u}_N(p')\left[F_1(Q^2)\gamma^\mu + F_2(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M}\right]u_N(p)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \frac{\varepsilon G_E^2\left(Q^2\right)+\tau G_M^2\left(Q^2\right)}{\varepsilon\left(1+\tau\right)}$$

$$Q^2 \equiv -q^2 = 4EE'\sin^2(\theta/2)$$
$$\tau = Q^2/4M^2$$
$$\varepsilon = \left[1+2\left(1+\tau\right)\tan^2(\theta/2)\right]^{-1}$$
$$G_E(0) = 1; \quad G_M(0) = \mu_p$$

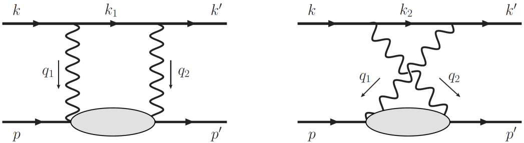
- $G_E = F_1 - \tau F_2; \quad G_M = F_1 + F_2$

Experimental cross section is not Born cross section !

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{exp}} = \left(\frac{d\sigma}{d\Omega}\right)_0 (1+\delta); \qquad \delta \to \text{Radiative Corrections.}$$

Two Photon Exchange

TPE Amplitude $\mathcal{M}_{\gamma\gamma} = \mathcal{M}^{\text{box}}_{\gamma\gamma} + \mathcal{M}^{\text{xbox}}_{\gamma\gamma}$ $\quad\quad$ $\mathcal{M}^{\text{xbox}}_{\gamma\gamma} \to$ purely real!



TPE cross-section, $\delta_{\gamma\gamma} = \dfrac{2\,\mathrm{Re}(\mathcal{M}^{\dagger}_{\gamma}\mathcal{M}_{\gamma\gamma})}{|\mathcal{M}_{\gamma}|^2}$

$$\begin{aligned}\mathcal{M}_{\gamma\gamma} \quad = \quad & \frac{e^2}{Q^2}\,\bar{u}_e(k')\gamma_\mu u_e(k)\ \bar{u}_N(p')\left[F_1'(Q^2,\nu)\gamma^\mu + F_2'(Q^2,\nu)\frac{i\sigma^{\mu\nu}q_\nu}{2M}\right]u_N(p)\\ & + \frac{e^2}{Q^2}\,\bar{u}_e(k')\gamma_\mu\gamma_5 u_e(k)\ \bar{u}_N(p')G_a'(Q^2,\nu)\gamma^\mu\gamma_5 u_N(p)\end{aligned}$$

$$\delta_{\gamma\gamma} = 2\mathrm{Re}\frac{\varepsilon G_E(F_1' - \tau F_2') + \tau G_M(F_1' + F_2') + \nu(1-\varepsilon)G_M G_a'}{\varepsilon G_E^2 + \tau G_M^2}.$$

Approaches

- GPD approach: (PhysRevLett.93.122301)

  Valid only in large momentum transfer; $|s, t, u| >> M^2$
- Hadronic Degrees of Freedom:
  - Low to moderate ($Q^2 \lesssim 5\ \mathrm{GeV}^2$)
  - Direct Loop integration (real part)
  - Sums/products of monopole form factors
  - Half off-shell form factor ambiguity
  - Divergence in the forward angles (high energy) limit for Δ resonance !
- Dispersive approach:

  M. Gorchtein (Phys.Lett.B644,322(2007)),
  Borisyuk and Kobushkin (Phys. Rev. C78, 025208 (2008)),
  Tomalak and Vanderhaeghen (Eur. Phys. J. A51, 24 (2015))
  - Sum of monopole form factors.

    Blunden et al. (PRC 95, 065209 (2017)) $\Rightarrow$ More generalized class of form factors

Applied for all 3 and 4-star resonance intermediate states with

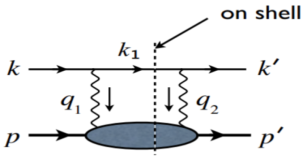
$$W < 1.8\ \mathrm{GeV}$$

Dispersive Method

$$\mathcal{S} = 1 + i\mathcal{M}$$

$$\mathcal{S}^\dagger\mathcal{S} = (1 - i\mathcal{M}^\dagger)(1 + i\mathcal{M}) = 1$$

Unitarity $\rightarrow -i(\mathcal{M} - \mathcal{M}^\dagger) = 2\,\mathrm{Im}\,\mathcal{M} = \mathcal{M}^\dagger\mathcal{M}$



$$\mathrm{Im}\,\langle f|\mathcal{M}|i\rangle = \frac{1}{2}\int \frac{d^3k_1}{(2\pi^3)\,2E_{k_1}} \sum_n \langle f|\mathcal{M}^*|n\rangle\langle n|\mathcal{M}|i\rangle$$

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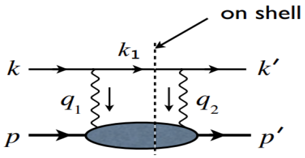

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- On-shell intermediate lepton & hadron: $\mathrm{Im}\,\mathcal{M}_{\gamma\gamma} \Rightarrow \mathrm{Im}(F_1', \; F_2', \; Ga')$
- On-shell parametrization of hadronic transition current
- **Resonance intermediate states**: $\Delta(1232)\,3/2^+$, $N(1440)\,1/2^+$, $N(1520)\,3/2^-$, $N(1535)\,1/2^-$, $\Delta(1620)\,1/2^-$, $N(1650)\,1/2^-$, $\Delta(1700)\,3/2^-$, $N(1710)\,1/2^+$ and $N(1720)\,3/2^+$
- CLAS exclusive meson electroproduction data for $A_{1/2}, A_{3/2}$ and $S_{1/2}$

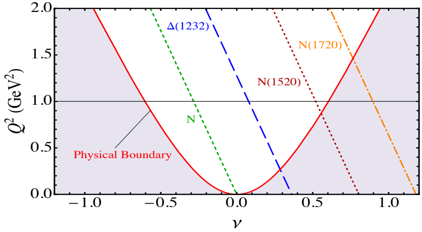
Dispersion Relations

$$\mathrm{Re}\, F_1'(Q^2,\nu) = \frac{2}{\pi}\mathcal{P}\int_{\nu_{\min}}^{\infty} d\nu' \; \frac{\nu}{\nu'^2-\nu^2} \; \mathrm{Im}\, F_1'(Q^2,\nu')$$

$$\mathrm{Re}\, F_2'(Q^2,\nu) = \frac{2}{\pi}\mathcal{P}\int_{\nu_{\min}}^{\infty} d\nu' \; \frac{\nu}{\nu'^2-\nu^2} \; \mathrm{Im}\, F_2'(Q^2,\nu')$$

$$ReG_a'(Q^2,\nu) = \frac{2}{\pi}\mathcal{P}\int_{\nu_{\min}}^{\infty} d\nu' \; \frac{\nu'}{\nu'^2-\nu^2} \; \mathrm{Im}\, G_a'(Q^2,\nu')$$

- **Integrals extend into unphysical region** $(\cos\theta < -1)$



e.g. $\Delta(1232)3/2^+$
$Q^2 = 1$ GeV^2
$\nu_{\mathrm{th}} = 0.604,$
$\nu_{\min} = 0.078$

- Analytically continued into the unphysical region (**PRC 95, 065209**)

Technical Overview

$$I_\delta = \frac{\alpha}{4\,\pi} Q^2 \frac{1}{i\pi^2} \int d^4 q_1 \frac{\mathrm{Im}\, L_{\alpha\mu\nu} H^{\alpha\mu\nu}}{(q_1^2 - \lambda^2)(q_2^2 - \lambda^2)}$$

L & H

$\Downarrow$

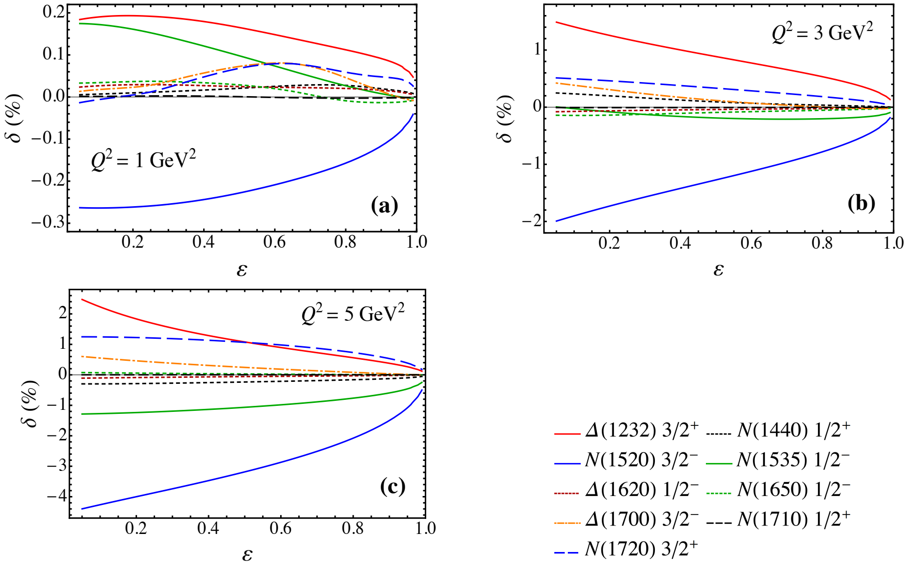
leptonic & hadronic tensors

Unitarity : $\mathrm{I}_\delta = \frac{\mathrm{s} - \mathrm{W}^2}{4\mathrm{s}} \int \mathrm{d}\Omega_{\mathrm{k}_1} \frac{\mathrm{G_i}(\mathrm{Q}_1^2)\, \mathrm{G_j}(\mathrm{Q}_2^2)\, \mathrm{f_{ij}}(\mathrm{Q}_1^2, \mathrm{Q}_2^2)}{(\mathrm{Q}_1^2 + \lambda^2)(\mathrm{Q}_2^2 + \lambda^2)}$

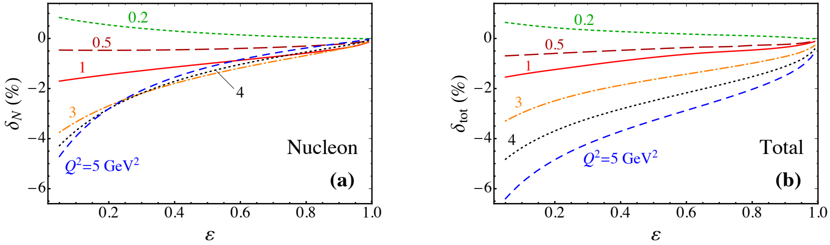
- f_{ij} are polynomial of combined degree 4 in $Q^2_{1,2}$
- $G_{i,j}(Q^2_{1,2})$: transition form factors at hadronic vertices; $(i, j = 1, 2, 3)$
- $G_{i,j}(Q^2_{1,2}) \Leftrightarrow A_h$; direct fit to CLAS A_h data
- Numerical contour integration (**PRC 95, 065209 (2017)**)

  $\Rightarrow$ Arbitrary functional forms for $G_{i,j}(Q^2_{1,2})$

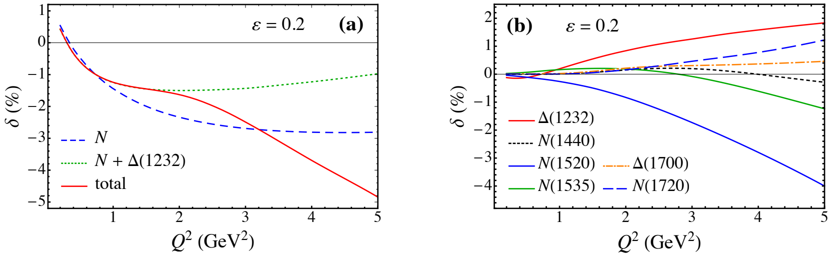
- TPE cross section correction from individual resonances:



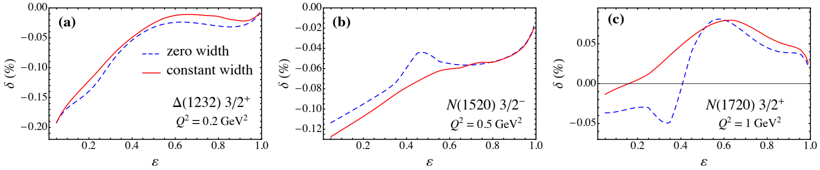
- Total TPE cross-section correction Vs. virtual photon polarization ε



- TPE cross section correction as a function of Q^2:



- Assumed continuum of W^2 as an infinite set of $\delta(W^2 - W_i^2)$, for each resonance. A Breit-Wigner distribution is used.



- Constant total decay width washes away the cusps!

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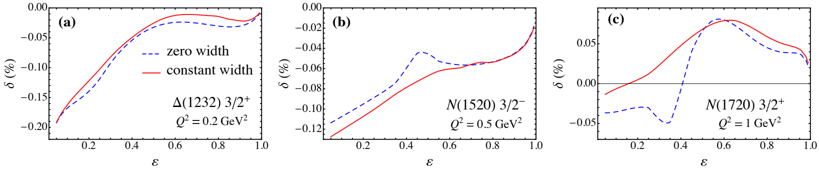

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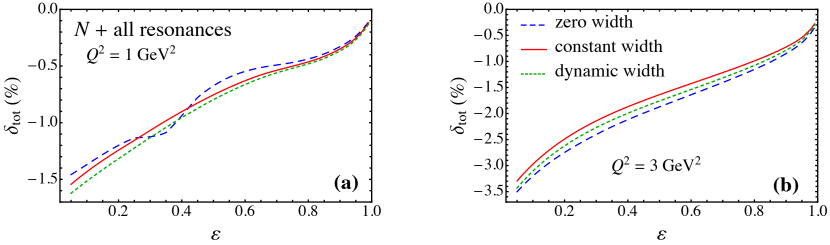
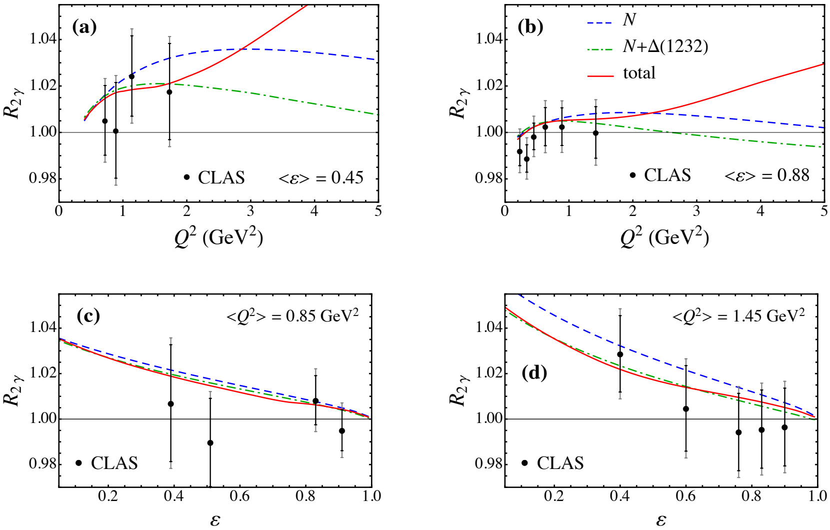
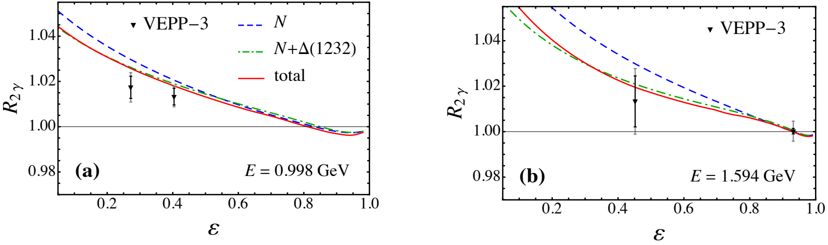


- Negligible width effect on total TPE cross section magnitude and slope. arXiv:2006.12543 (2020)

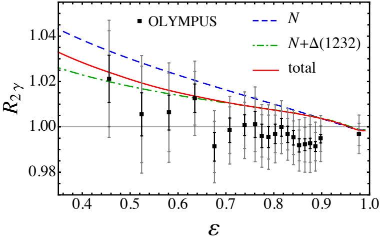
$$R_{2\gamma} = \sigma(e^+p)/\sigma(e^-p) \approx 1 - 2\delta_{\gamma\gamma}$$



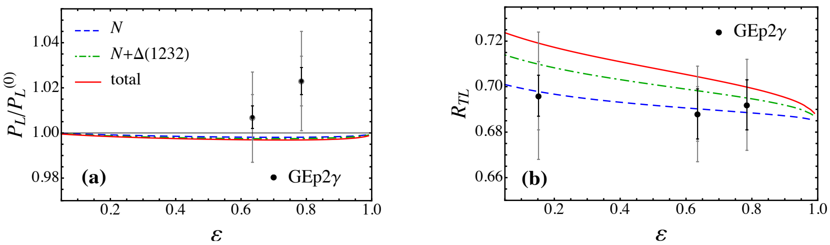
- VEPP3 result:



- OLYMPUS result:



Polarization observable: $R_{TL} = -\mu_p \sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}} \frac{P_T}{P_L}$



$\mu_p G_E/G_M$

$\Rightarrow$ Dr. Peter Blunden

Beam/target normal Single Spin Asymmetry (SSA)

- Beam/target normal single spin asymmetry :

$$\mathrm{SSA} = \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow}$$

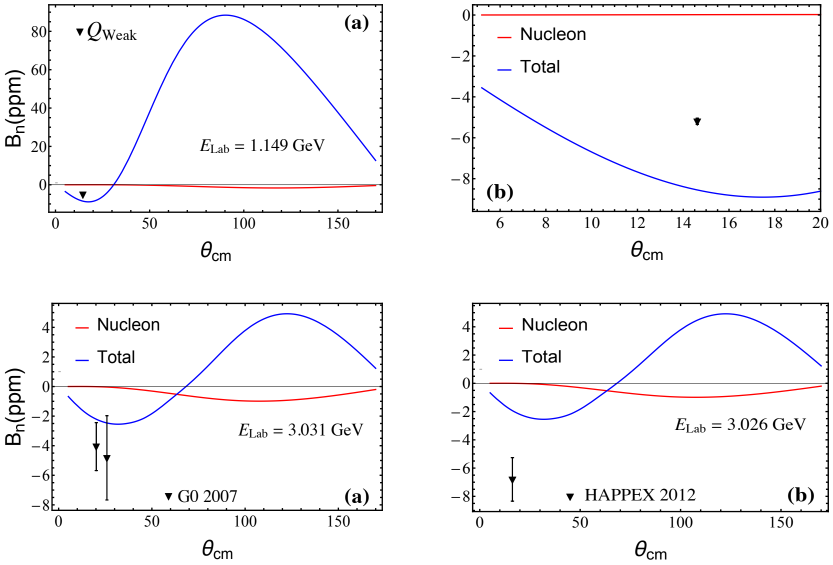
- Polarization vector parallel or anti-parallel to $\mathbf{S}_\mathrm{n} = \dfrac{\mathbf{k} \times \mathbf{k}'}{|\mathbf{k} \times \mathbf{k}'|}$

- Time reversal invariance and unitarity:

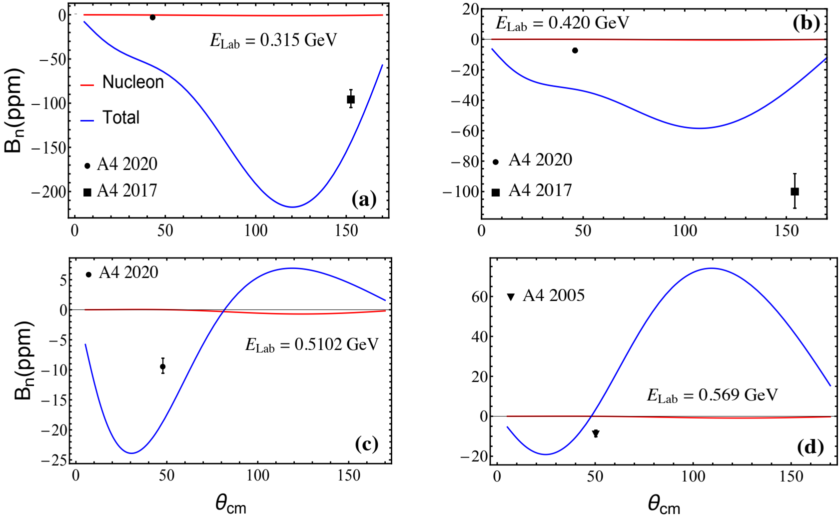
$$\mathrm{SSA} = \frac{2\,\mathrm{Im}\big(\sum\limits_{\mathrm{spins}} \mathcal{M}_\gamma^* . \mathrm{Abs}\mathcal{M}_{\gamma\gamma}\big)}{\sum\limits_{\mathrm{spins}} |\mathcal{M}_\gamma|^2}$$

- SSA $\Rightarrow B_n$ or $\mathrm{A_n}$

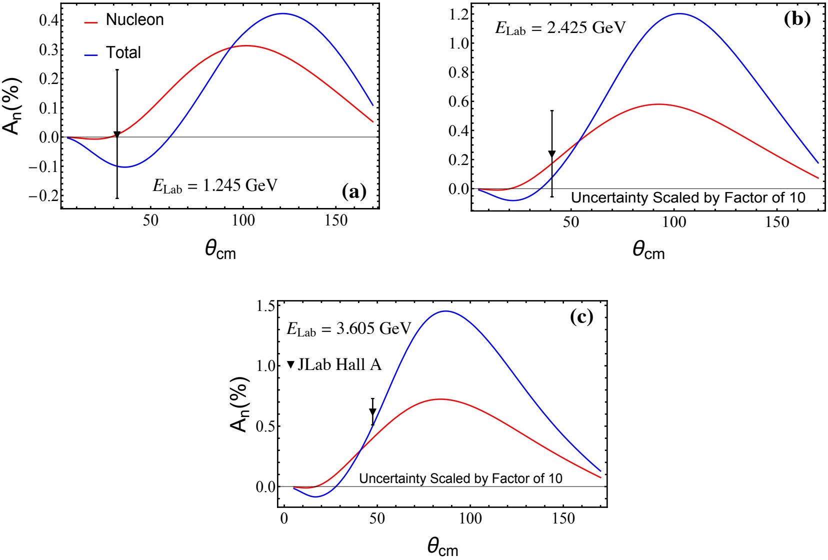
Beam Normal SSA:



Beam Normal SSA:



Target Normal SSA



Summary

- $N(1520)3/2^-$ is the major contributor for higher Q^2
- Elastic nucleon alone is a good approximation for $Q^2 < 1\ \mathrm{GeV}^2$
- Overall enhancement in the TPE cross section correction at $Q^2 > 3\ \mathrm{GeV}^2$
- Width effect is negligible
- Proper inclusion of TPE resolves $\mu_p G_E/G_M$ discrepancy
- Need more data in the higher Q^2 region
- Follow up work: inclusion of non-resonant background and spin $5/2$ resonances.

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Thanks !