Radiative correction studies for the PRad and planned PRad-II, DRad and SoLID experiments at Jefferson Laboratory

Vladimir Khachatryan

Medium Energy Physics Group,
Department of Physics, Duke University, Durham, NC 27708

"Radiative Corrections"
Workshop at Stony Brook, Jul 9-10, 2020

Content

1) **Overview of the results on radiative corrections (RC) and RC systematics for the PRad experiment ?**
   - ***The results from the recent studies***
   - ***The future plans for the planned PRad-II experiment***

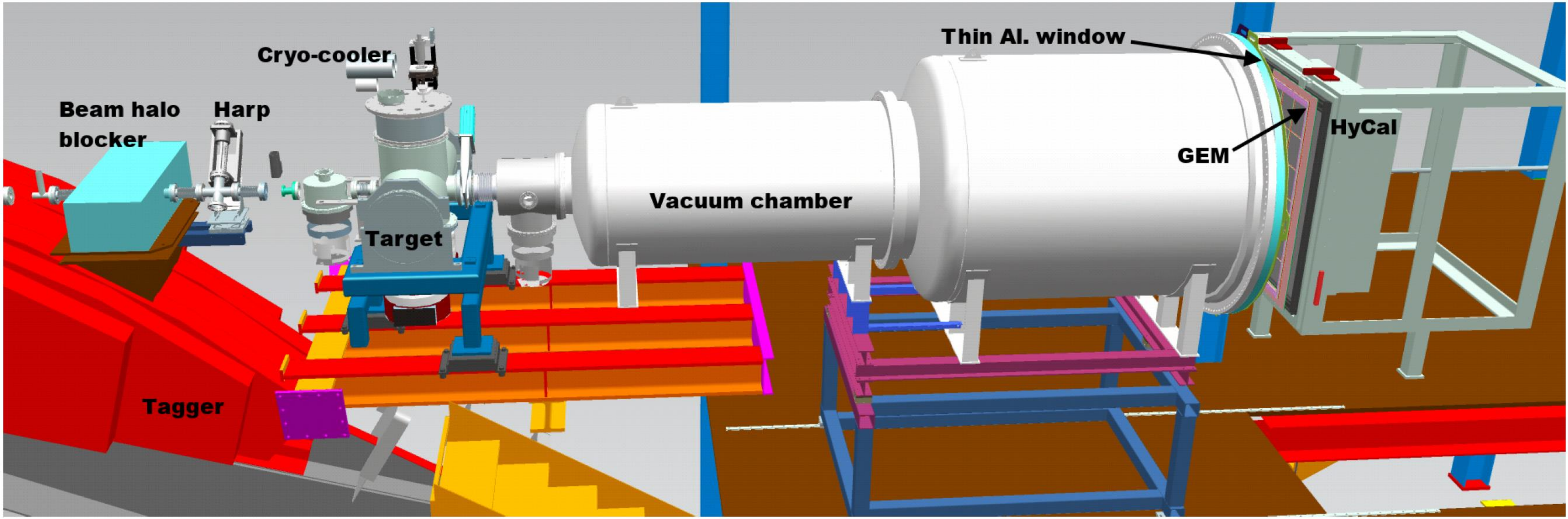
2) **Lowest order RC studies for SoLID SIDIS measurements**
   - ***Current theoretical developments***
   - ***A versatile Monte Carlo (MC) event generator including RC***

3) **Upcoming RC Monte Carlo studies in the e+d elastic scattering for the planned DRad experiment**

Part 1

1) **Overview of the results on radiative corrections (RC) and RC systematics for the PRad experiment ?**
   - ***The results from the recent studies***
   - ***The future plans for the planned PRad-II experiment***

- The PRad experiment was performed in 2016 at JLab, with both 1.1 and 2.2 GeV unpolarized electron beams on a windowless H_2 gas-flow target
- The experiment measured the elastic e+p scattering cross section and the proton electric form factor G_E^p in the Q^2 range $2\cdot10^{-4}$ (GeV/c)^2 – 0.06 (GeV/c)^2
- The luminosity was monitored by simultaneously measuring the Møller scattering process (e+e scattering)
- The absolute e+p elastic scattering cross section was normalized to that of the Møller scattering in order to cancel out the luminosity



Schematic layout of the PRad experimental apparatus in Hall B at Jefferson Lab

1)

- This figure is from the paper

I. Akushevich, H. Gao, A. Ilyichev, and M. Meziane, EPJ. A **51**, (2015) 1

- It shows complete analytical calculations of one-loop radiative corrections for e+p and Møller scatterings

- The vertical axes are

$$\delta^{ep} = \frac{\sigma^{ep}}{\sigma_0^{ep}} - 1 \qquad \delta^{ee} = \frac{\sigma^{ee}}{\sigma_0^{ee}} - 1$$

- For both e+p and e+e plots, we see three inelasticity cuts, v_{max}, imposed as three colored curves, which are functions of Q^2

- So in the data analysis the simulated cross section, including RC, has been used

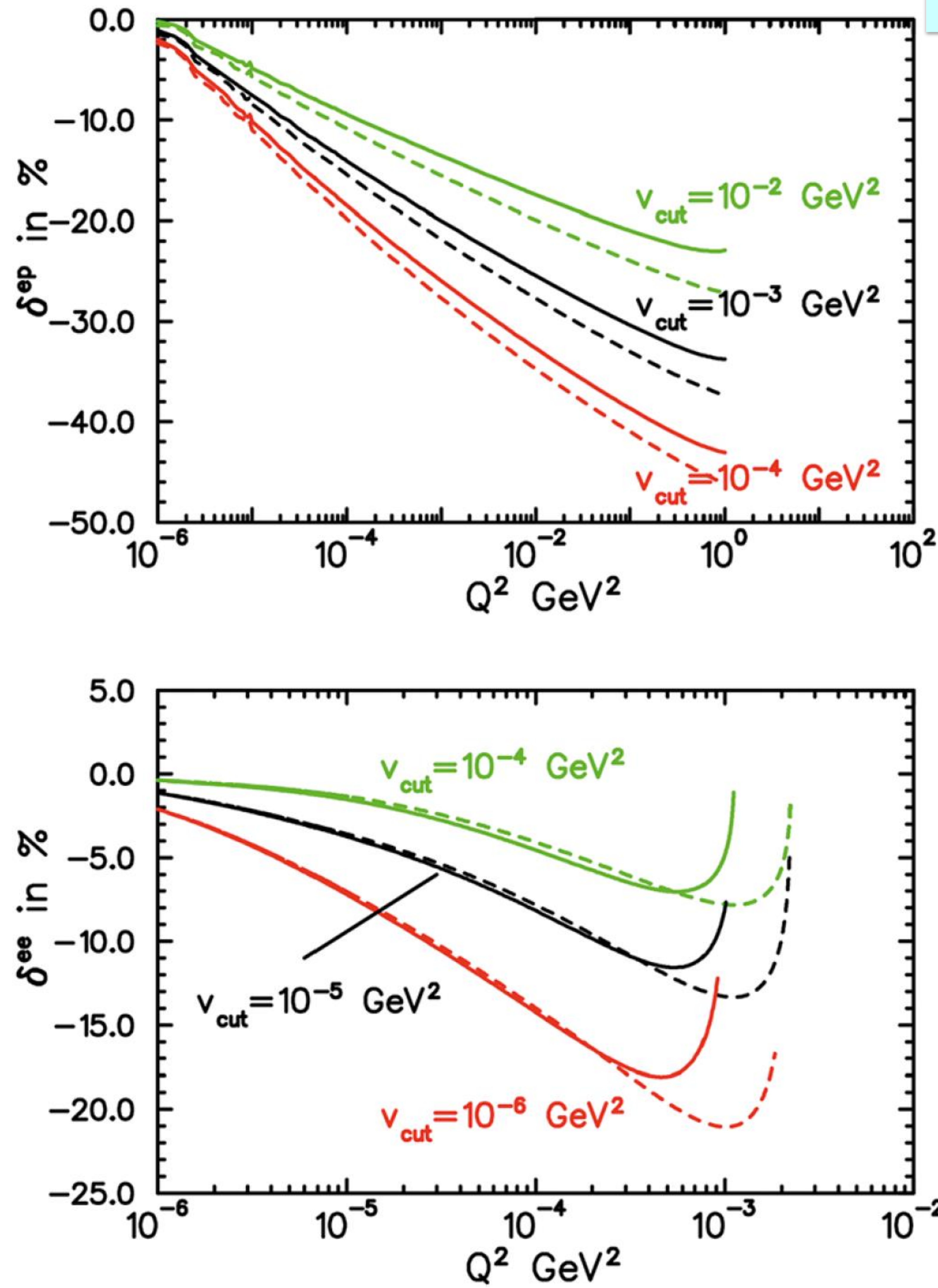
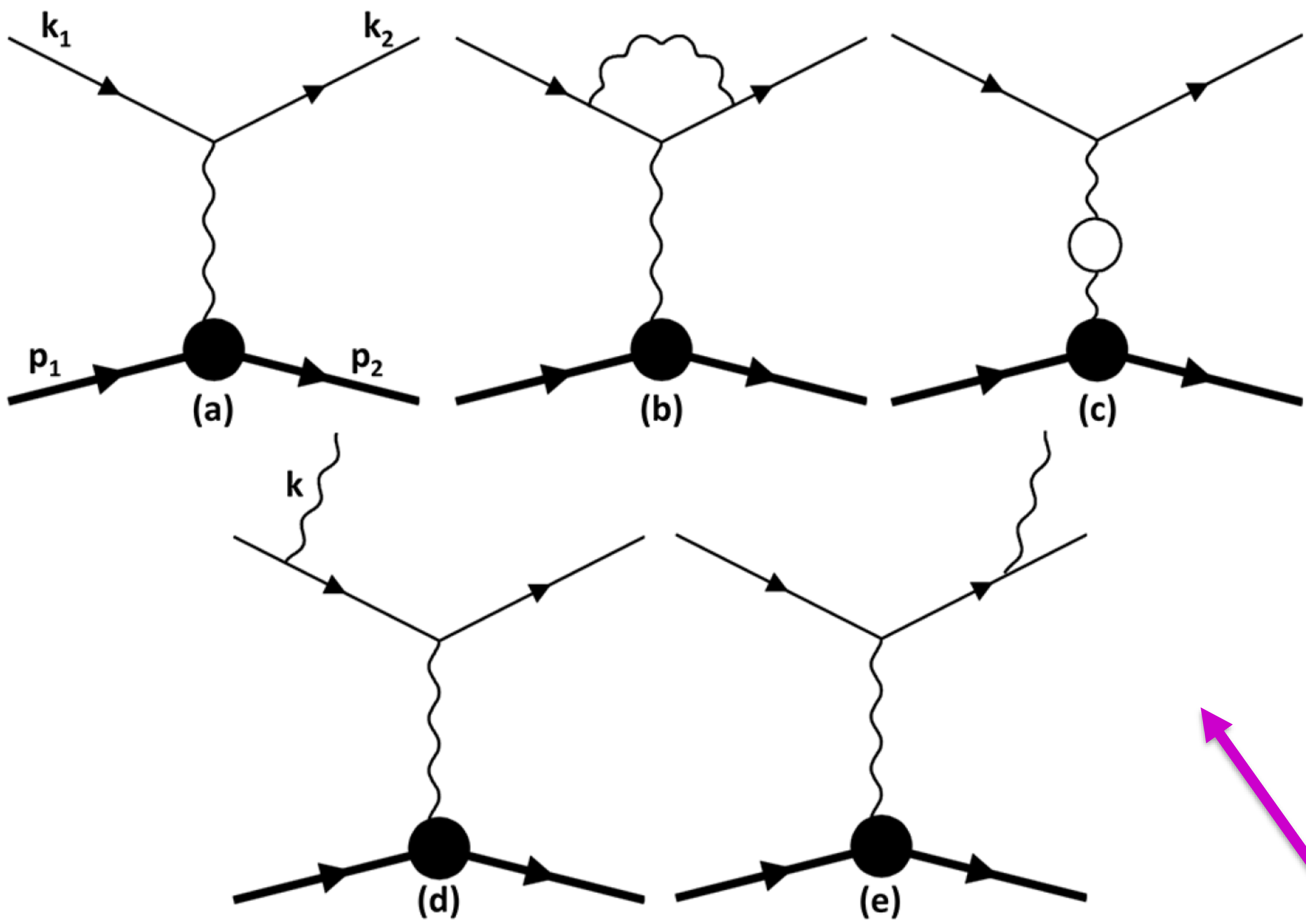
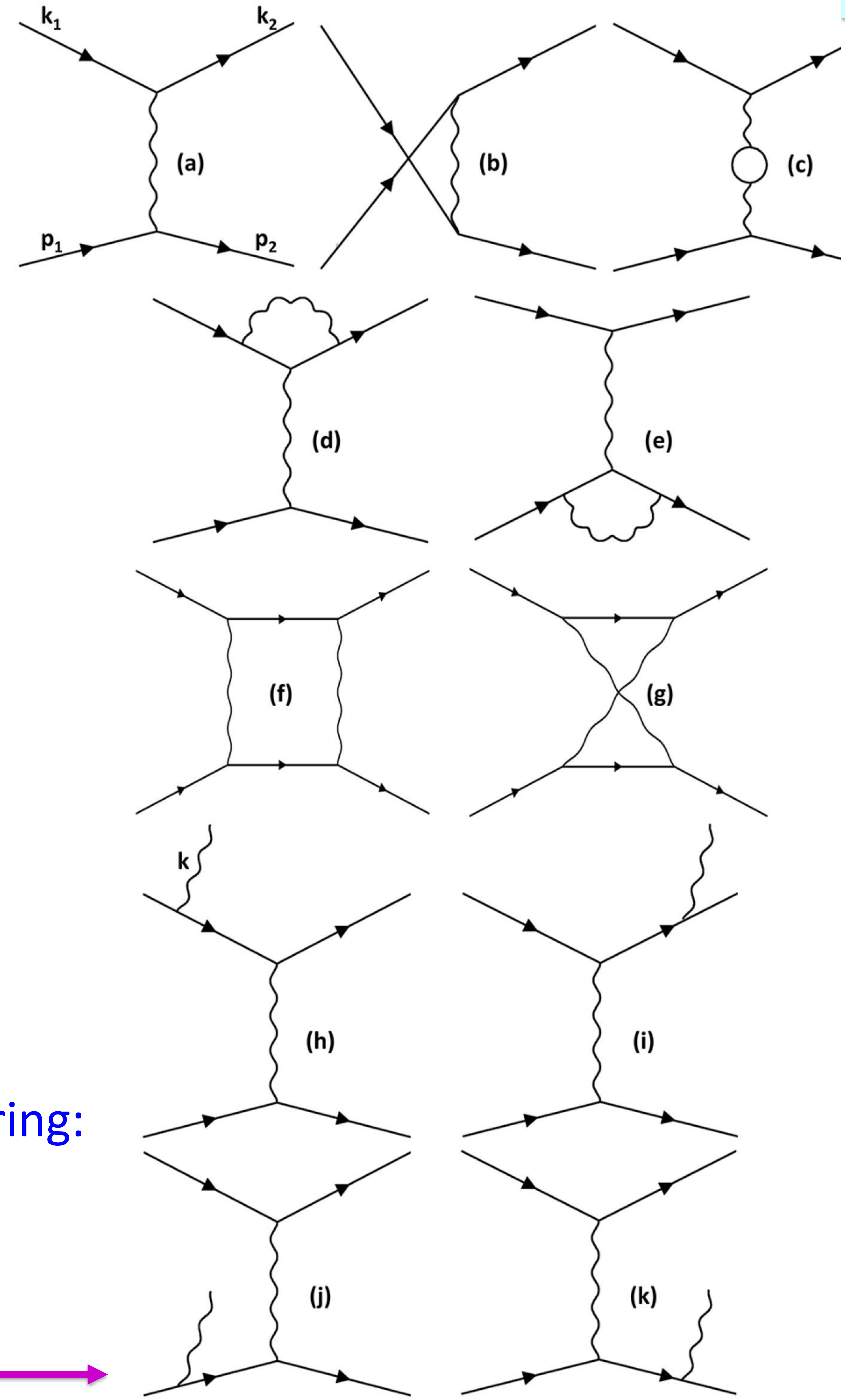


Fig. 3. (Color online) *ep* (top) and Møller (bottom) radiative correction as a function of Q^2 for different values of inelasticity cut and for $E_{\text{beam}} = 1.1\,\text{GeV}$ (solid lines) and $E_{\text{beam}} = 2.2\,\text{GeV}$ (dashed lines).



- Feynman diagrams contributing to the Born and RC cross sections in e+p elastic scattering: (a) The Born process, (b) Vertex correction, (c) vacuum polarization, (d)-(e) Bremsstrahlung

- Feynman diagrams contributing to the Born (a)-(b) and RC cross sections for Møller scattering: (c)-(e) Vacuum polarization and vertex correction, (f)-(g) box contributions (h)-(k) Bremsstrahlung



- For the e+p scattering the complete cross section is given by

$$\sigma = \sigma_0 \left(1 + \frac{\alpha}{\pi}(\delta_{VR} + \delta_{\rm vac} - \delta_{\rm inf})\right) e^{\frac{\alpha}{\pi}\delta_{\rm inf}} + \sigma_{\rm AMM} + \sigma_F,$$

where σ_0 is the Born cross section, σ_{AMM} - the anomalous magnetic moment and σ_F - the infrared divergence free contributions to the cross section.

δ_{VR} is the infrared divergent contribution, δ_{vac} is the vacuum polarization contribution, δ_{inf} term is to account for multi-photon emission at $Q^2 \to 0$

- For the Møller scattering the complete cross section is given by

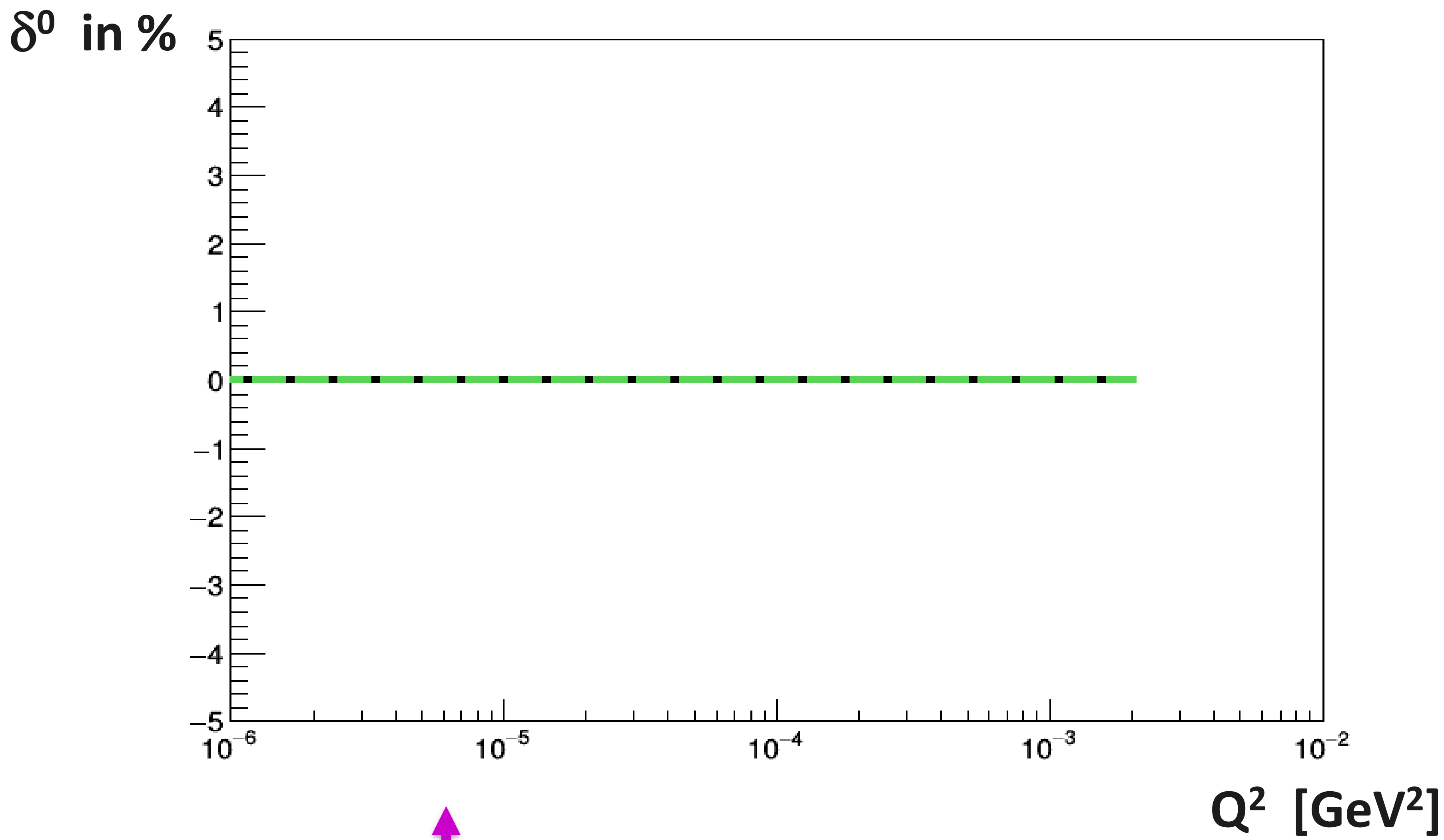
$$\sigma^{ee} = \left(1 + \frac{\alpha}{\pi}\left(J_0 \log\frac{v_{\rm max}}{m^2} + \delta_1^H + \delta_1^S\right)\right)\sigma_0 + \sigma_S + \sigma^F_{\rm vert} + \sigma^F_B + \sigma_F,$$

where the infrared divergent contribution of bremsstrahlung is represented as a sum of three factorized corrections

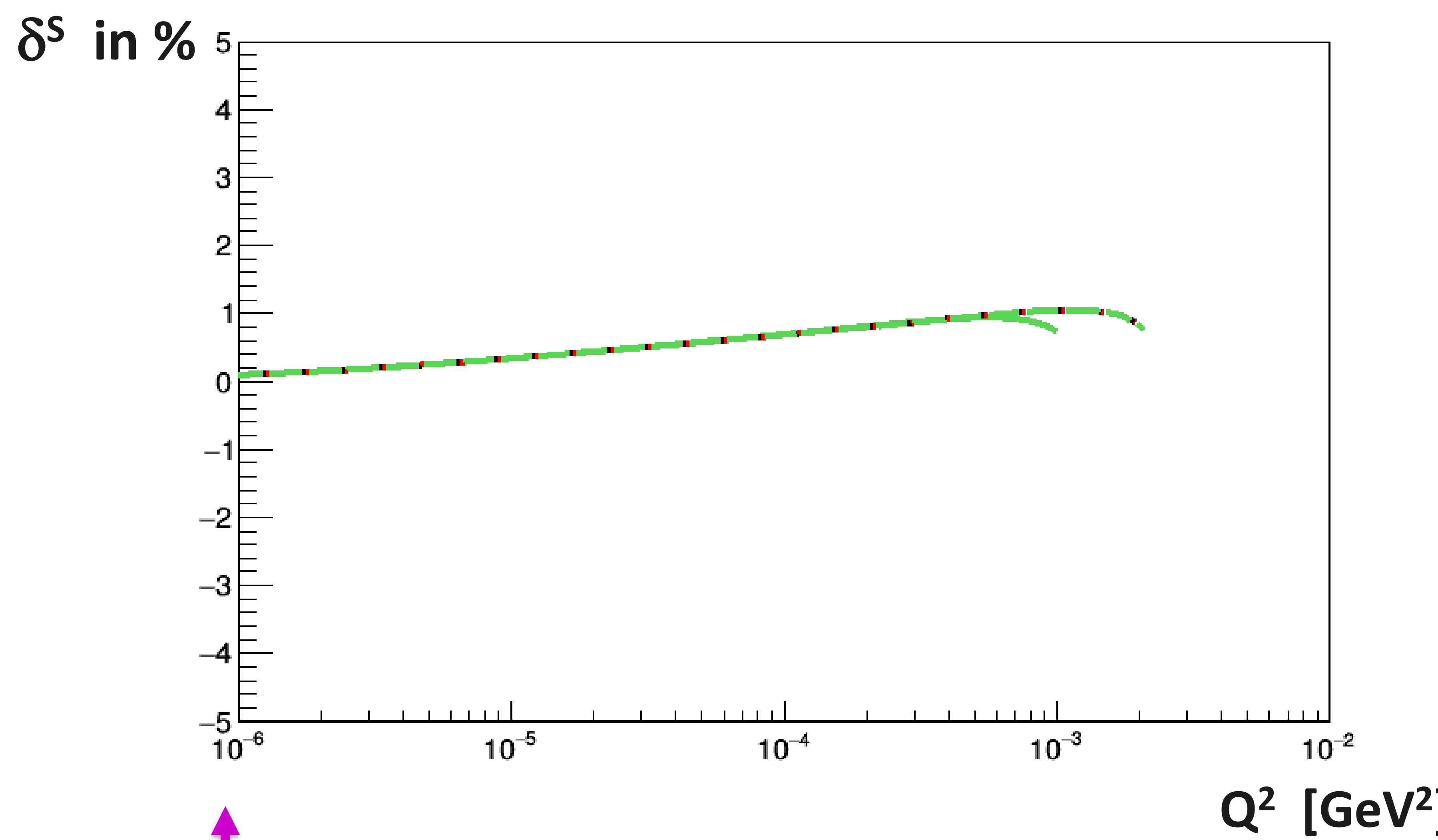
$$\sigma_{IR} = \frac{\alpha}{\pi}\left(J_0 \log\frac{v_{\rm max}}{m\lambda} + \delta_1^H + \delta_1^S\right)\sigma_0$$

➢ Here are different Møller cross section contributions

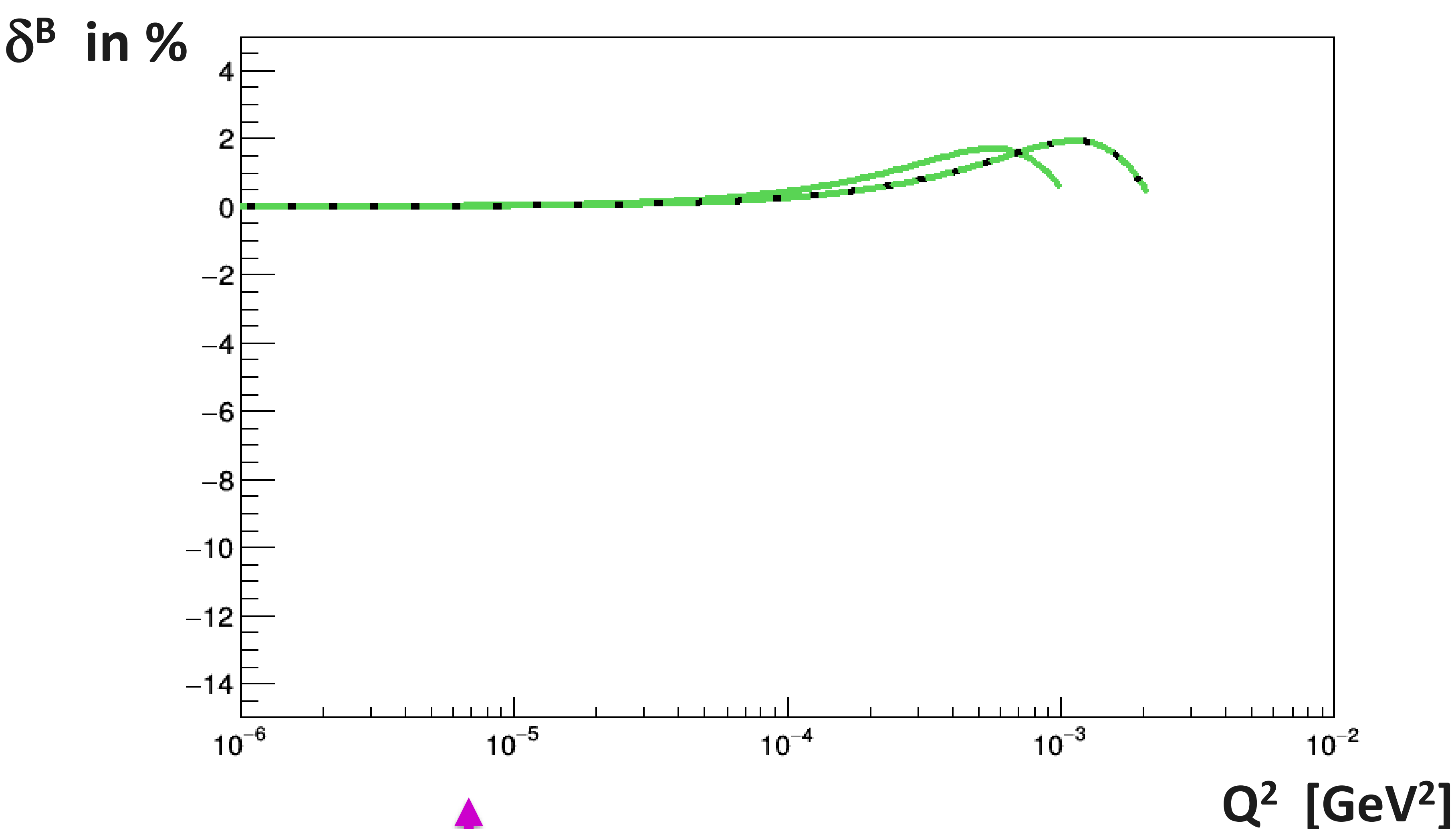
1)



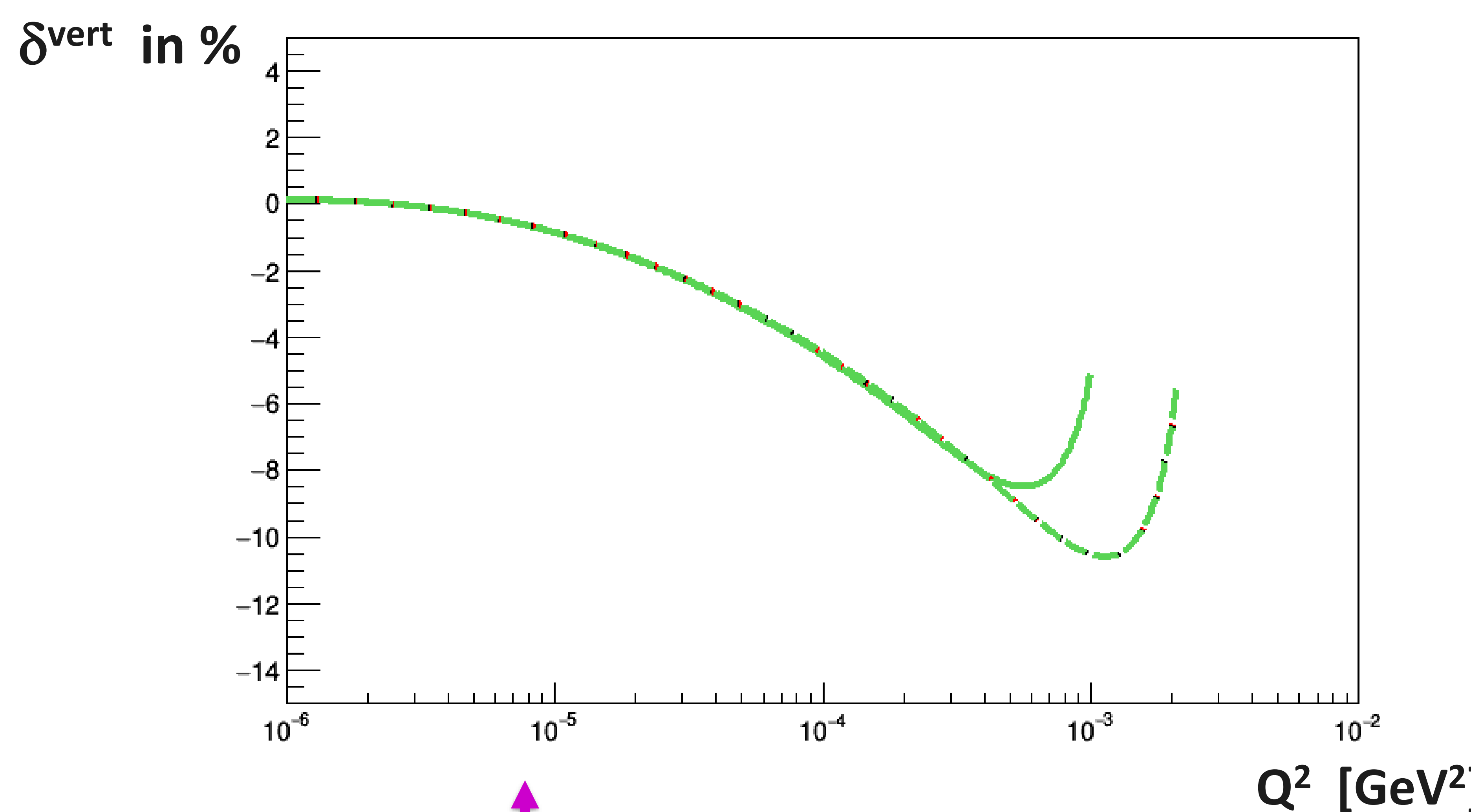
Born contribution (σ_0)



Vacuum polarization contribution (σ_S)



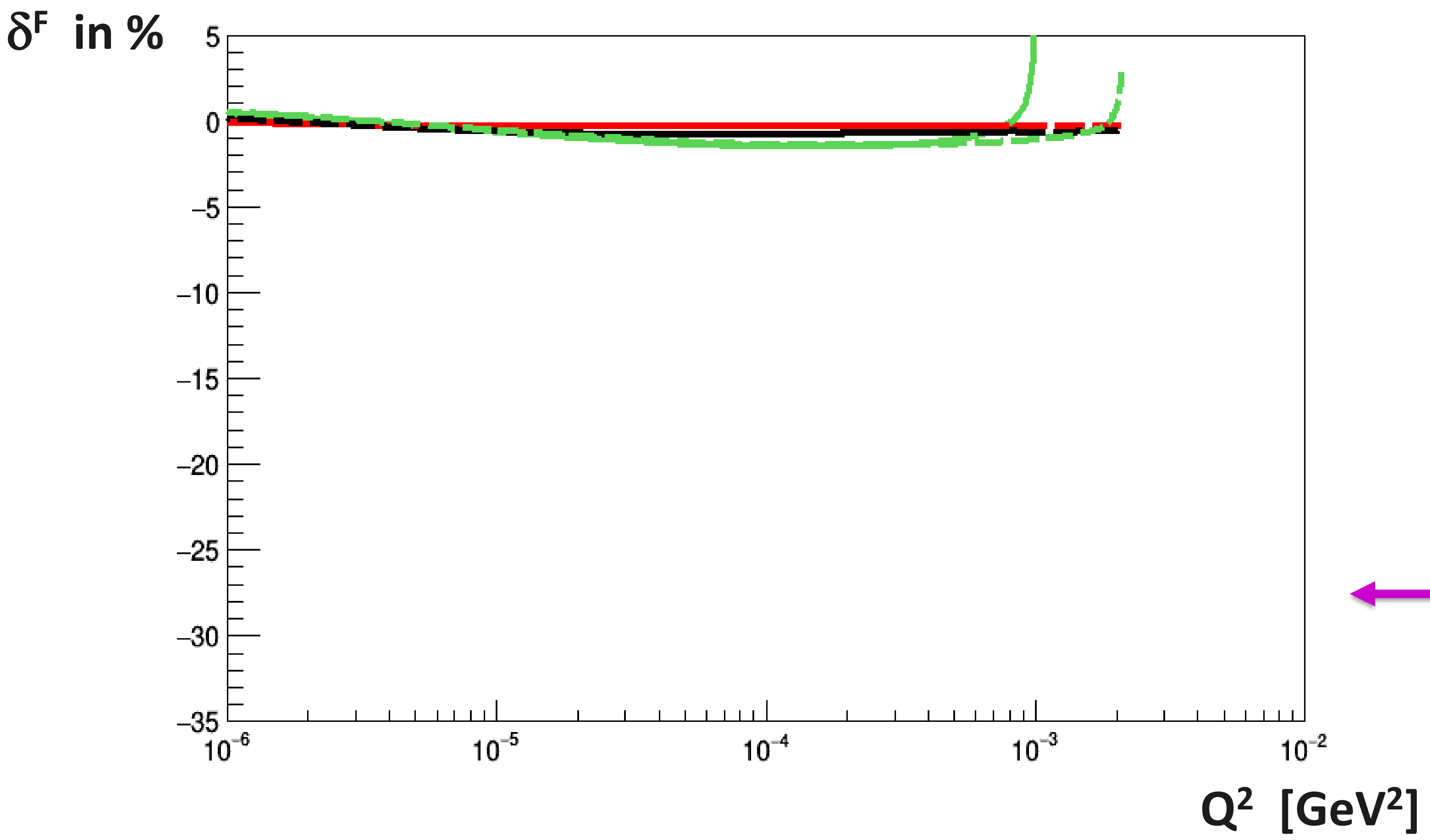
Box contribution (σ_B)



Vertex contribution (σ_{vert})

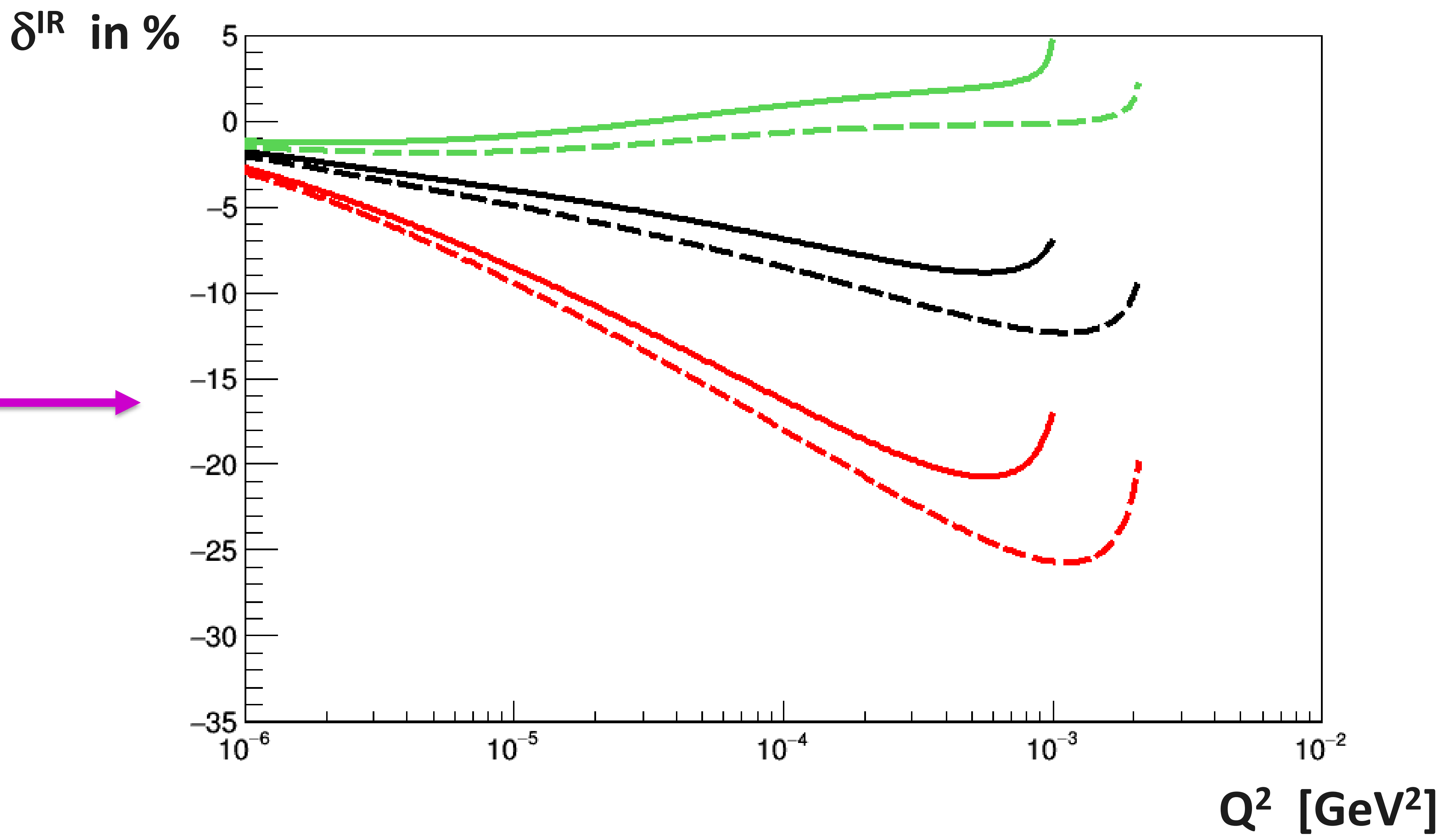
1)

- Focus on the Møller cross section contributions because they give the dominant part of the RC systematic uncertainty for the proton radius, r_p



Finite part of the cross section due to the infrared divergence (σ_F)

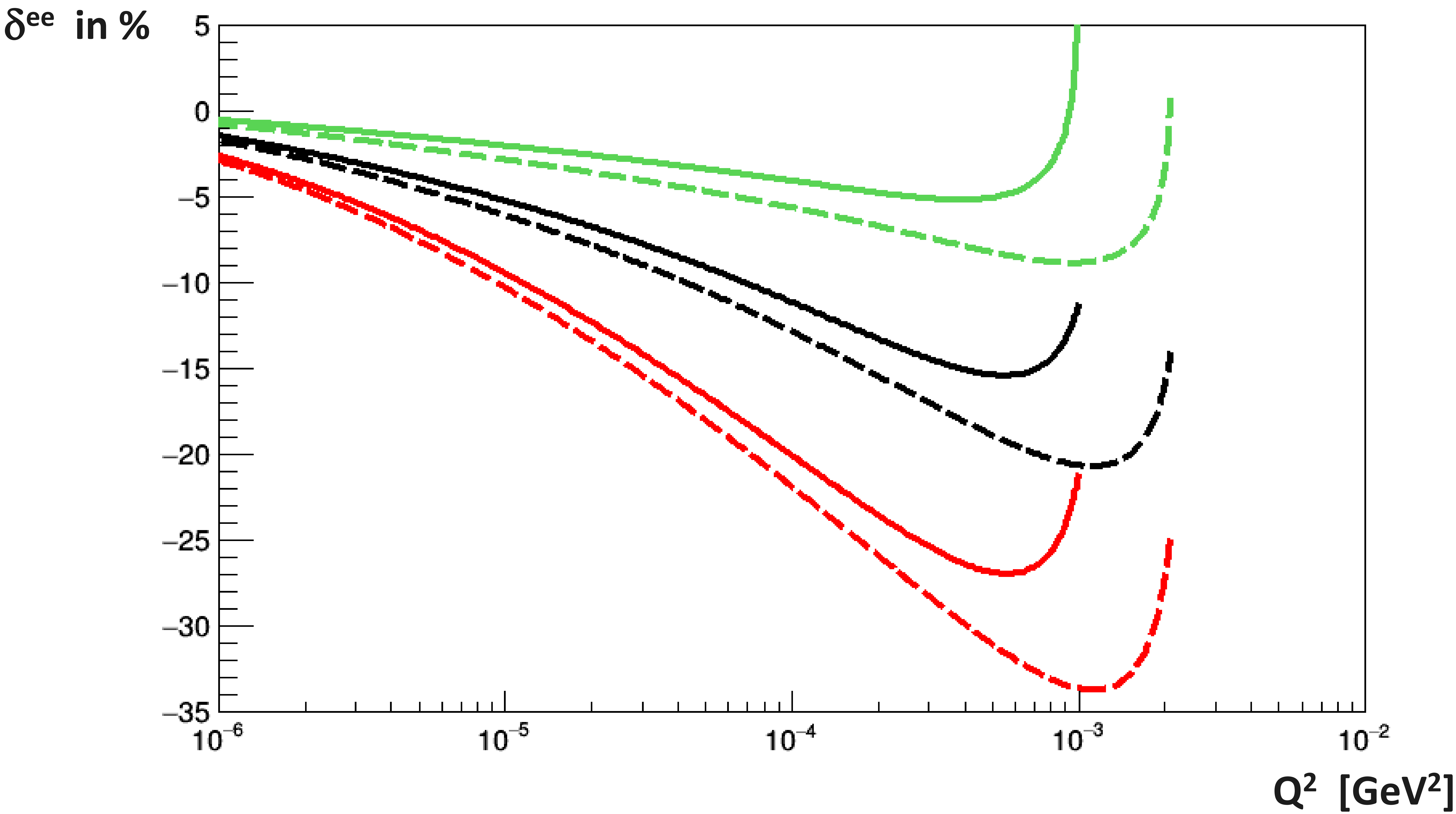
The infrared divergent contribution of bremsstrahlung (σ_{IR})



- Here we have another figure from our Monte Carlo generator

1)

- All the Møller cross section contributions are summed up



1)

Item	r_p uncertainty (fm)
Event selection	0.0070
Radiative correction	0.0069
Detector efficiency	0.0042
Beam background	0.0039
HyCal response	0.0029
Acceptance	0.0026
Beam energy	0.0022
Inelastic ep	0.0009
Total	0.0116

Systematic uncertainties for PRad

Different uncertainty contributions to the proton radius total uncertainty are given in terms of fm

The measured radius is

$$r_p = (0.831 \pm 0.007_{stat} \pm 0.012_{syst})\ \text{fm}$$

W. Xiong et al., Nature 575, 147 (2019)

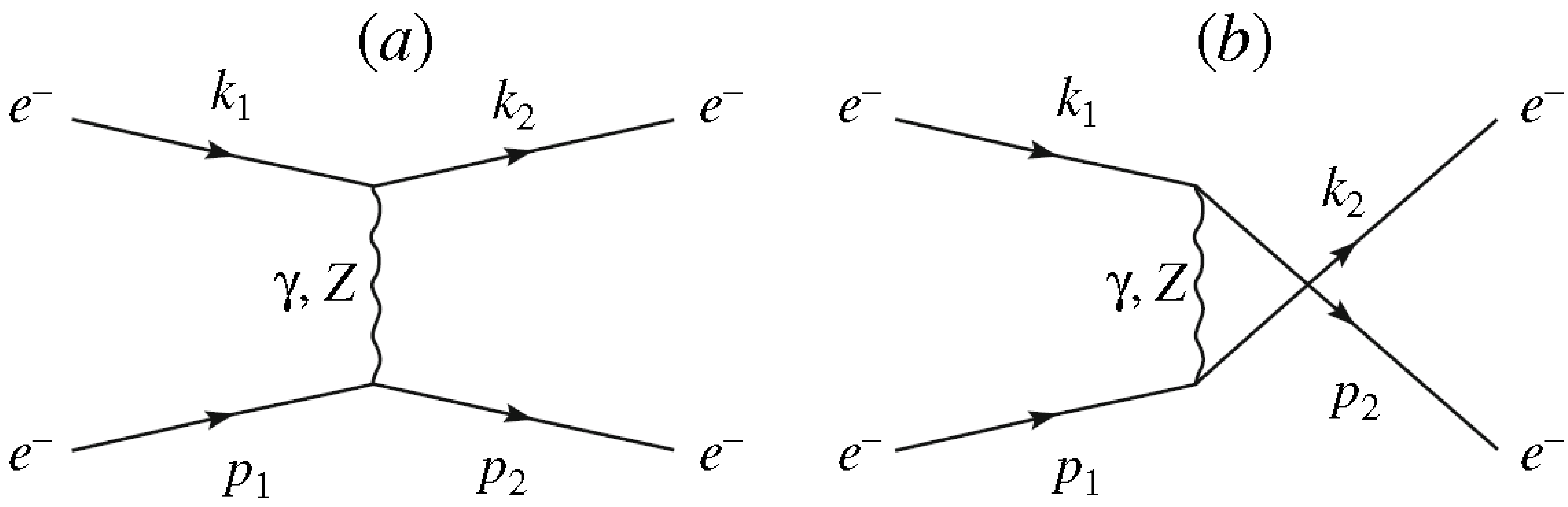
Our goal is to exactly calculate this contribution for the PRad-II experiment

For that purpose we should continue the studies of one-loop (NLO) and two-loop (NNLO) radiative corrections from the Møller scattering

- Let's now follow the method of the following paper shown bellow, by which we estimated the Møller RC from another perspective

"Estimating Two-Loop Radiative Effects in the MOLLER Experiment"
A. G. Aleksejevs, S. G. Barkanova, V. A. Zykunov and E. A. Kuraev,
Physics of Atomic Nuclei, **76**, **No7**, 888 (2013)

- We follow Aleksejevs'es nomenclature and terminology, but focusing on the electromagnetic component of the electroweak radiative corrections, which is dominant over the weak component in the region of moderate and small energies
- Like the one, which is shown in this Born (tree) diagram in the *t*- and *u*-channels



Or one can write the differential cross section of the Møller scattering approximately as the sum of the

(i) leading order (LO) Born term

(ii) next-to-leading order (NLO) one-loop term

(iii) next-to-next-leading order (NNLO) two-loop terms

$$\sigma = d\sigma/d\,cos\theta \cong (\,\alpha^2 M_0 M_0^+ [\mathrm{LO}] + \alpha^3\; 2\mathrm{Re}(M_1 M_0^+)[\mathrm{NLO}] +$$

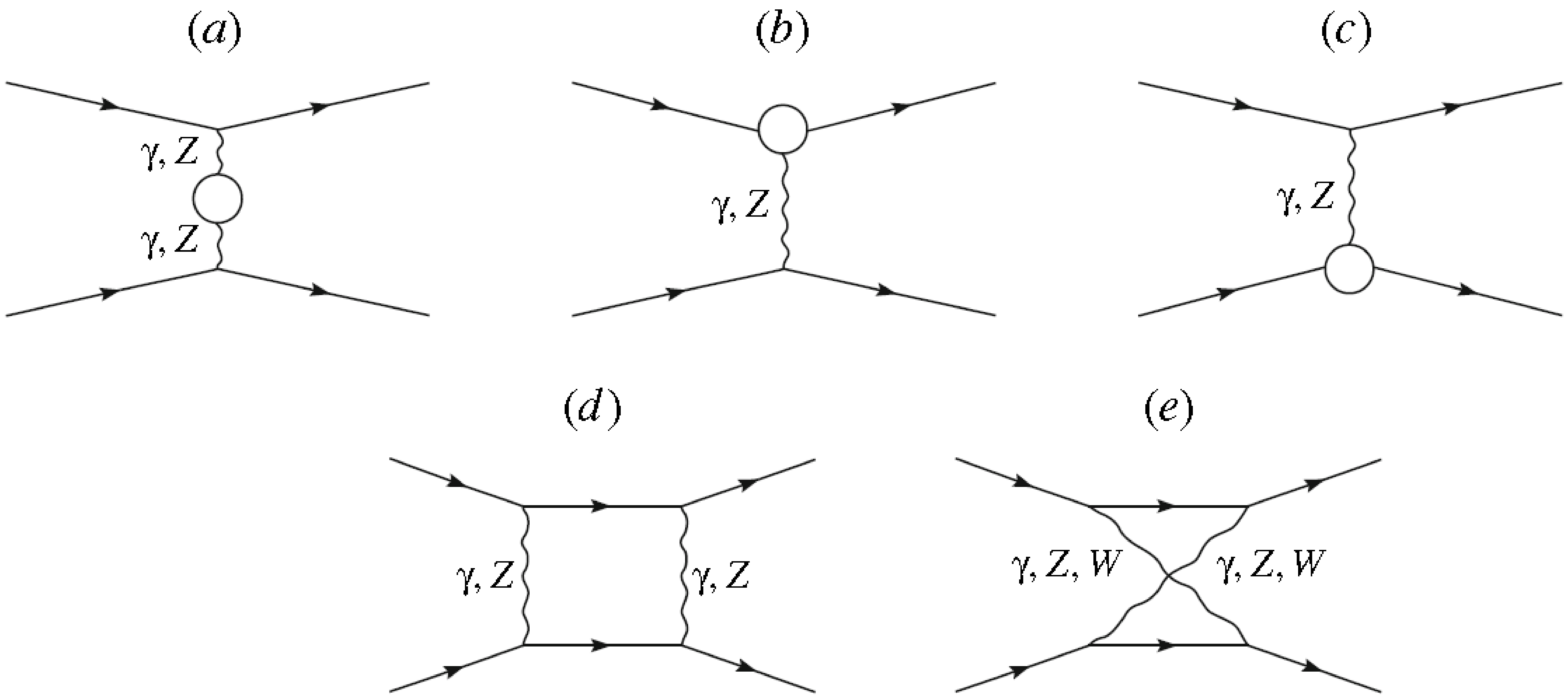
$$+\,\alpha^4 (M_1 M_1^+ + 2\mathrm{Re}(M_2 M_0^+)[\mathrm{NNLO}]\,)$$

The NNLO term is divided into two classes:

(a) Q-part induced by quadratic one-loop amplitudes ($\sim M_1 M_1^+$)

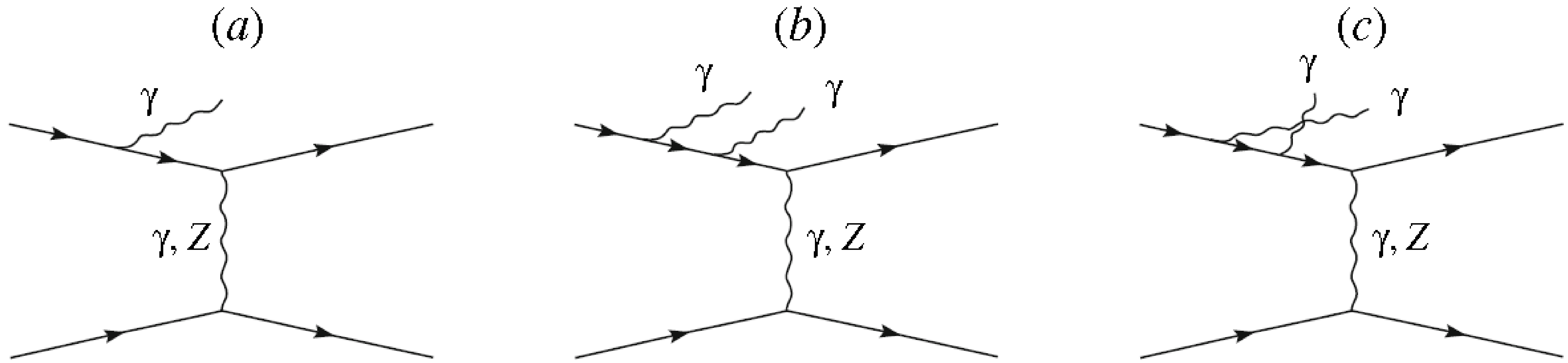
(b) T-part corresponding to the interference of the Born and two-loop amplitudes ($\sim 2\mathrm{Re}(M_2 M_0^+)$)

- Here are one-loop *t*-channel diagrams for the Møller scattering process
  - This one-loop amplitude M_1 consists of the
    - (i) boson self energy (BSE) diagram (a)
    - (ii) vertex (Ver) diagram (b, c)
    - (iii) box (B) diagram (d, e)

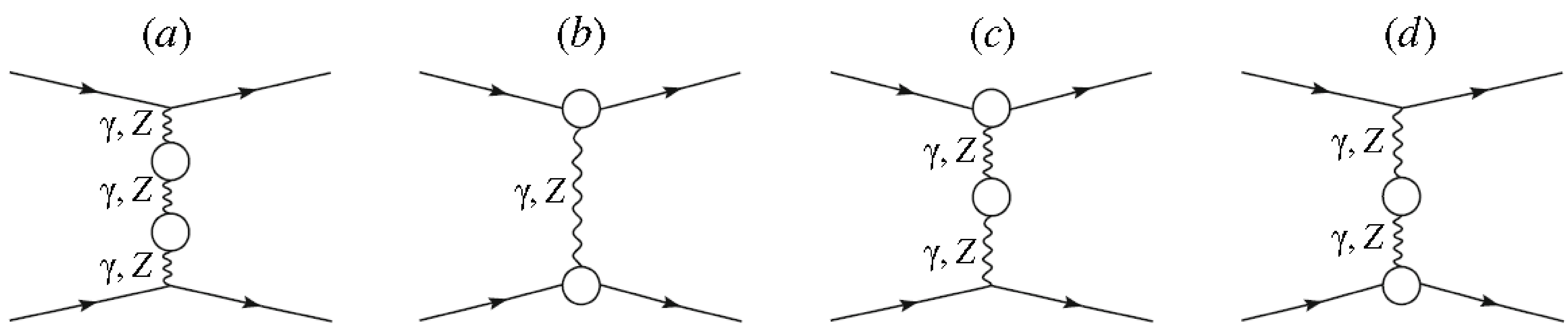


- The respective *u*-channel diagrams are obtained by the substitutions $k_2 \leftrightarrow p_2$

- Here are NLO (a) and NNLO (b, c) bremsstrahlung *t*-channel diagrams
- Those are needed to cancel the infrared divergences in the first order and second order amplitudes, respectively
- Radiation from only one electron line is shown in the figures but in the calculations all four electron lines are accounted for



- Two-loop *t*-channel diagrams from the gauge-invariant set of vertices and boson self-energies
- The circles represent the contributions of self-energies and vertex functions



1)

- First, the structure of the Born amplitude is given in what follows

$$M_0 = M_{0,t} - M_{0,u} \qquad M_{0,t} = i\frac{\alpha}{\pi}\left(I_\mu^\gamma\, D^{\gamma t}\, J^{\mu,\gamma}\right) \qquad M_{0,u} = i\frac{\alpha}{\pi}\left(I_\mu^\gamma\, D^{\gamma u}\, J^{\mu,\gamma}\right)$$

$$I_\mu^\gamma = \bar{u}(k_2)\,\gamma_\mu(v^\gamma - a^\gamma\gamma_5)u(k_1) \qquad J^{\mu,\gamma} = \bar{u}(p_2)\,\gamma^\mu(v^\gamma - a^\gamma\gamma_5)u(p_1)$$

- The self-energy, vertex and box diagrams have more complicated but kind of similar structures

- One can represent the one-loop amplitude as the sum of the infrared and infrared-finite contributions

$$M_1 = M_1^\lambda + M_1^f \qquad M_1^\lambda = \frac{\alpha}{2\pi}\delta_1^\lambda M_0, \quad M_1^f = M_{BSE} + M_{Ver}^f + M_{Box}^f + M_{Add}$$

$$\delta_1^\lambda = 4B \times \ln\left(\frac{\lambda}{\sqrt{s}}\right) \qquad B = \ln\left(\frac{tu}{m^2 s}\right) - 1 - i\pi$$

- The amplitudes M_{BSE}, M_{Ver}^f, M_{Box}^f, M_{Add} all are calculated analytically

- In analogy to the one-loop amplitude, one can construct the two-loop amplitude as the sum of the infrared and infrared-finite contributions

$$M_2 = M_2^\lambda + M_2^f \qquad \text{where for example} \qquad M_2^\lambda = \frac{\alpha}{2\pi}\delta_1^\lambda M_1^f + \frac{1}{8}\left(\frac{\alpha}{\pi}\right)^2 \left(\delta_1^\lambda\right)^2 M_0$$

- Thereby, we should have M_1^f and δ_1^f as well as two more quantities shown below (all for the PRad kinematics)

$$\sigma_Q^f = \frac{\pi^3}{2s} M_1^f \left(M_1^f\right)^+ = \left(\frac{\alpha}{\pi}\right)^2 \delta_Q^f\,\sigma^0 \qquad \sigma_T^f = \frac{\pi^3}{s}\mathrm{Re}\left(M_2^f M_0^+\right) = \left(\frac{\alpha}{\pi}\right)^2 \delta_T^f\,\sigma^0$$

- Thus, by having δ_1^f, δ_Q^f, and δ_T^f we will have the NLO and NNLO radiative correction cross sections free from the infrared divergences

$$\sigma_{NLO} = \frac{\alpha}{\pi}\left(R_1 + \delta_1^f\right)\sigma^0 \qquad \text{one-loop}$$

$$\sigma_{NNLO} = \left(\frac{\alpha}{\pi}\right)^2 \left(R_1\delta_1^f + \frac{1}{2}R_1^2 - \frac{1}{2}R_2 + \sigma_Q^f + \sigma_T^f\right)\sigma^0 \qquad \text{two-loops}$$

$$R_1 = -4B \times \ln\left(\frac{\sqrt{s}}{2\omega_{cut}}\right) - \left(\ln\left(\frac{s}{m^2}\right) - 1\right)^2 + 1 - \frac{\pi^2}{3} + \left(\ln\left(\frac{u}{t}\right)\right)^2 \qquad R_2 = \frac{8}{3}\pi^2 B^2$$

- The table below shows the effect on the r_p based on this estimation for Møller RC
- The different rows show the extracted r_p using 1.1 + 2.2 GeV beam energy
- The RC factor depends on the photon energy cut so we tried five cases with some reasonable values
- The first column shows the extracted r_p in each case, the second column is the difference between the default r_p and r_p from each case
- The r_p values are well within the systematic uncertainty we assigned due to Møller RC based on PRad's estimate, which is about 0.0065 fm.

	r_p (fm)	Δr_p (fm)
Default	0.8306	0.0000
$\omega_{\mathrm{cut}} = 20$ MeV	0.8259	0.0047
$\omega_{\mathrm{cut}} = 25$ MeV	0.8270	0.0036
$\omega_{\mathrm{cut}} = 30$ MeV	0.8278	0.0026
$\omega_{\mathrm{cut}} = 40$ MeV	0.8288	0.0018
$\omega_{\mathrm{cut}} = 50$ MeV	0.8295	0.0011
$\omega_{\mathrm{cut}} = 60$ MeV	0.8299	0.0007
$\omega_{\mathrm{cut}} = 70$ MeV	0.8302	0.0004

1)

- We wish to obtain a factor of two and half improvement of the total PRad uncertainties for a new planned proton radius measurement in the PRad-II experiment
- One of our priority goals is to calculate exactly the NNLO RC in <u>unpolarized elastic e+p</u> and <u>Møller</u> scatterings beyond ultrarelativistic limit
- An outstanding problem will be to calculate the corresponding one-loop and two-loop Feynman diagrams systematically
- It is highly desirable to develop methods for numerical semi-analytic evaluation of such diagram functions, like Feynman integrals
- A new method is under development where the calculated results, namely amplitudes or cross sections, can be represented as power series that are convergent on the chain of integration
- The current MC event generator that we used for PRad, can be modified accordingly, along with upcoming new results

See the talk of Stanislav Srednyak tomorrow !

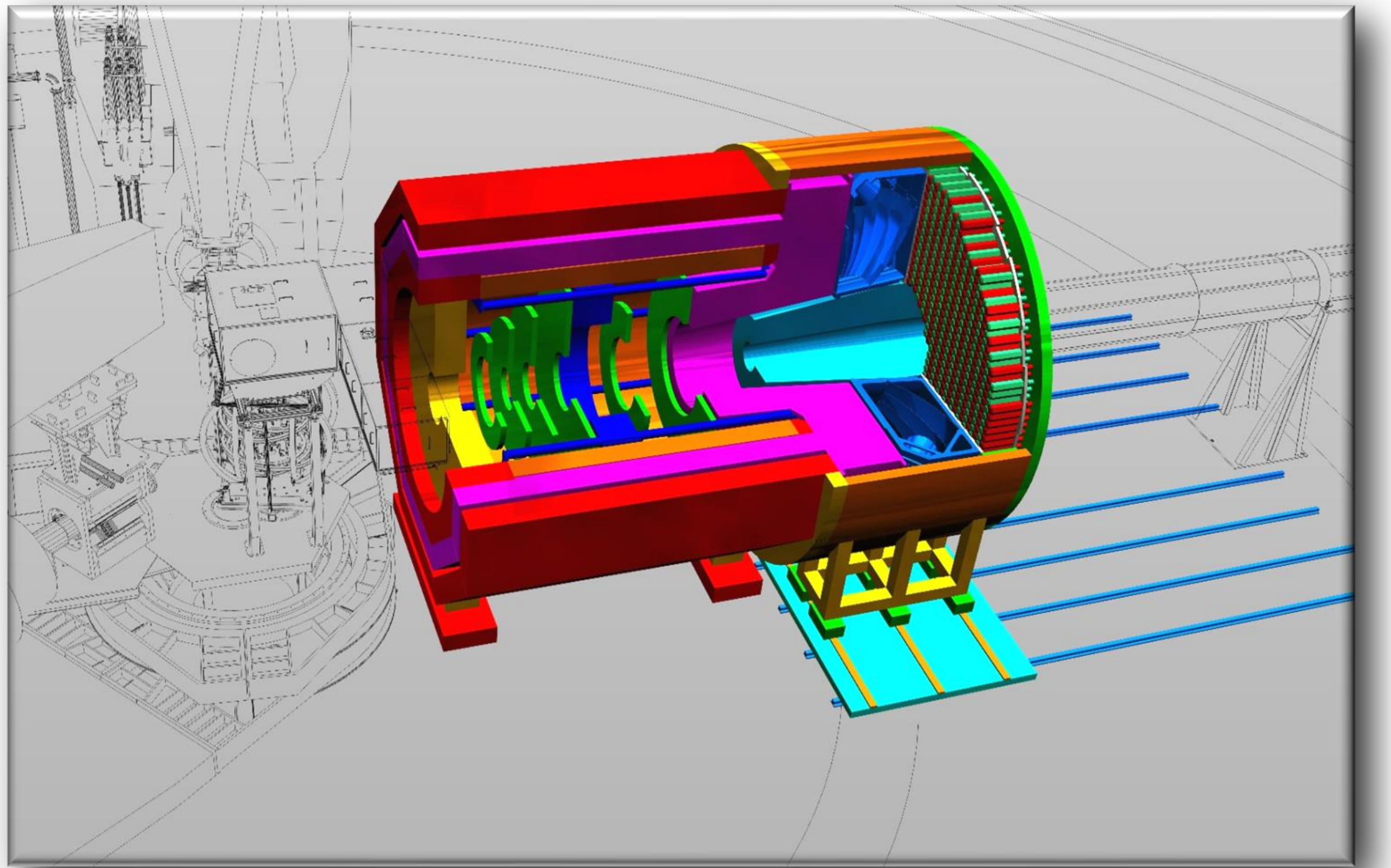
Part 2

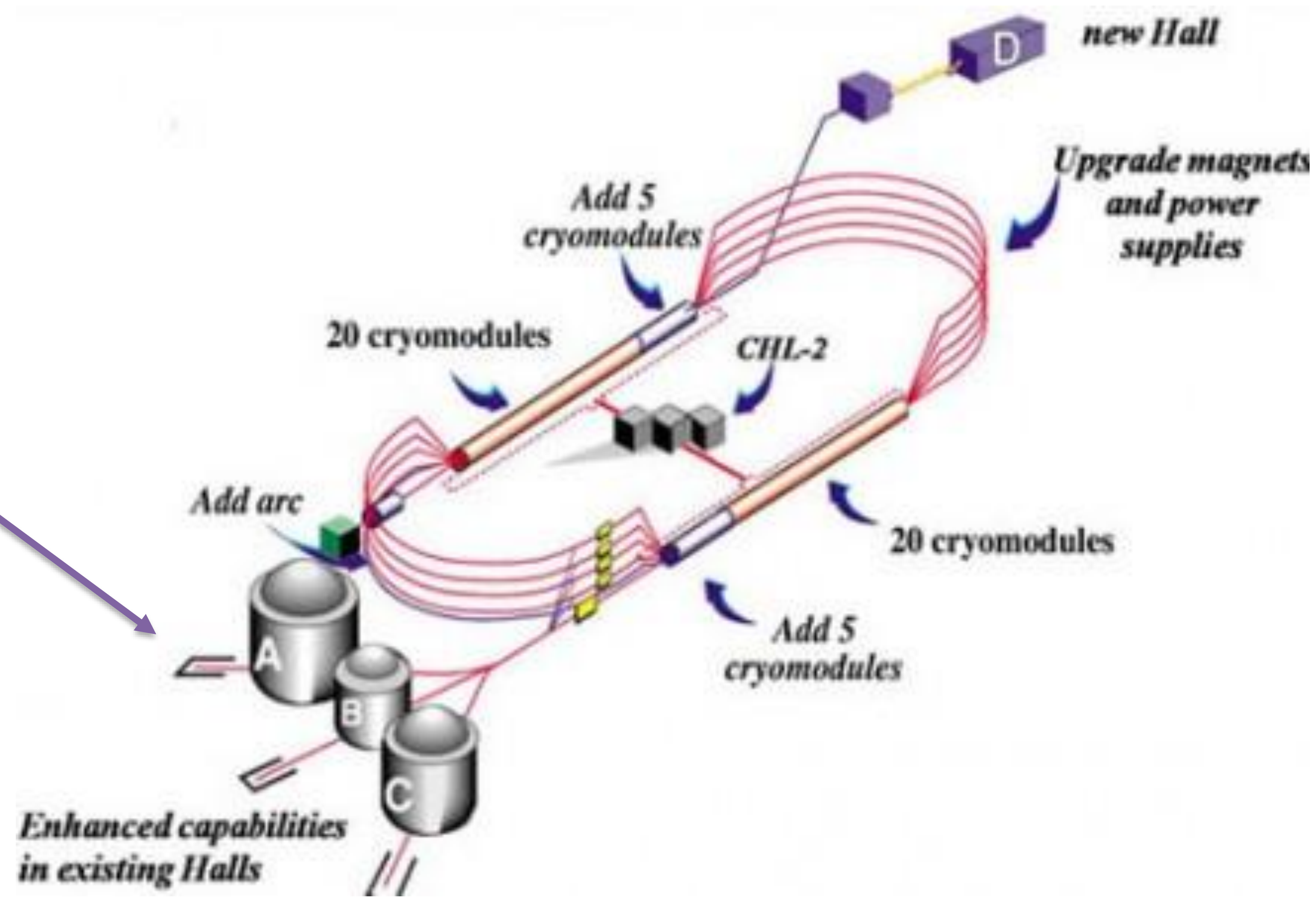
2) Lowest order RC studies for SoLID SIDIS measurements

- ***Current theoretical developments***
- ***A versatile Monte Carlo (MC) event generator including RC***

SoLID@12-GeV JLab: QCD at the intensity frontier

SoLID provides unique capability combining high luminosity ($10^{37\text{-}39}$ /cm^2/s) (more than 1000 times of EIC) and **large acceptance** with full ϕ coverage to maximize the science return of the 12-GeV CEBAF upgrade





SoLID with unique capability for rich physics programs

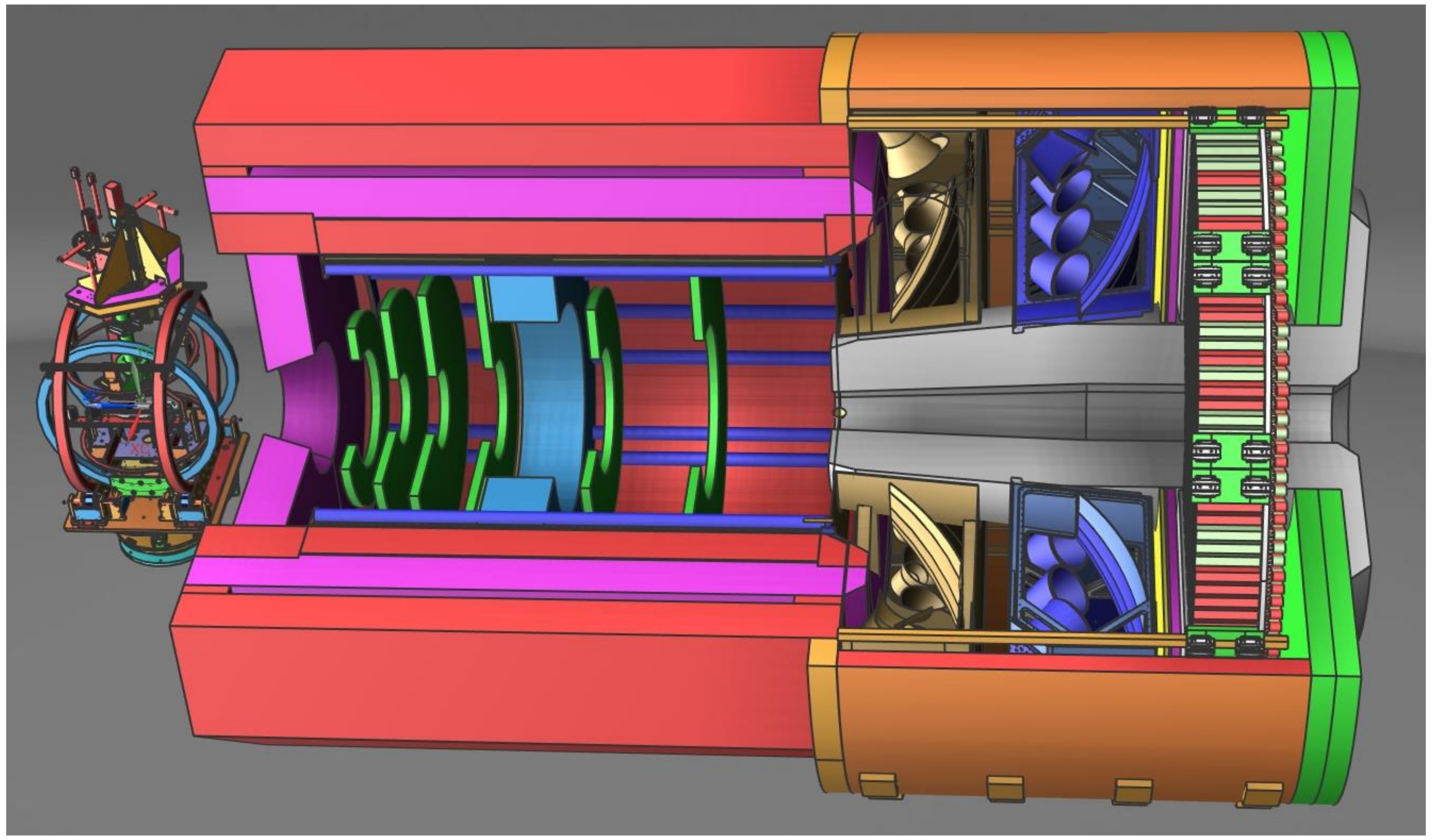
- Pushing the phase space in the search of new physics and of hadronic physics
- 3D momentum imaging of a relativistic strongly interacting confined system (nucleon spin)
- Superior sensitivity to the differential electro- and photo- production cross section of J/ψ near threshold (proton mass)

SoLID physics complementary and synergistic with the EIC science (proton spin and mass, two important EIC science questions) – high-luminosity SoLID unique for valence quark tomography (separation of structure from collision) and precision J/ψ production near the threshold

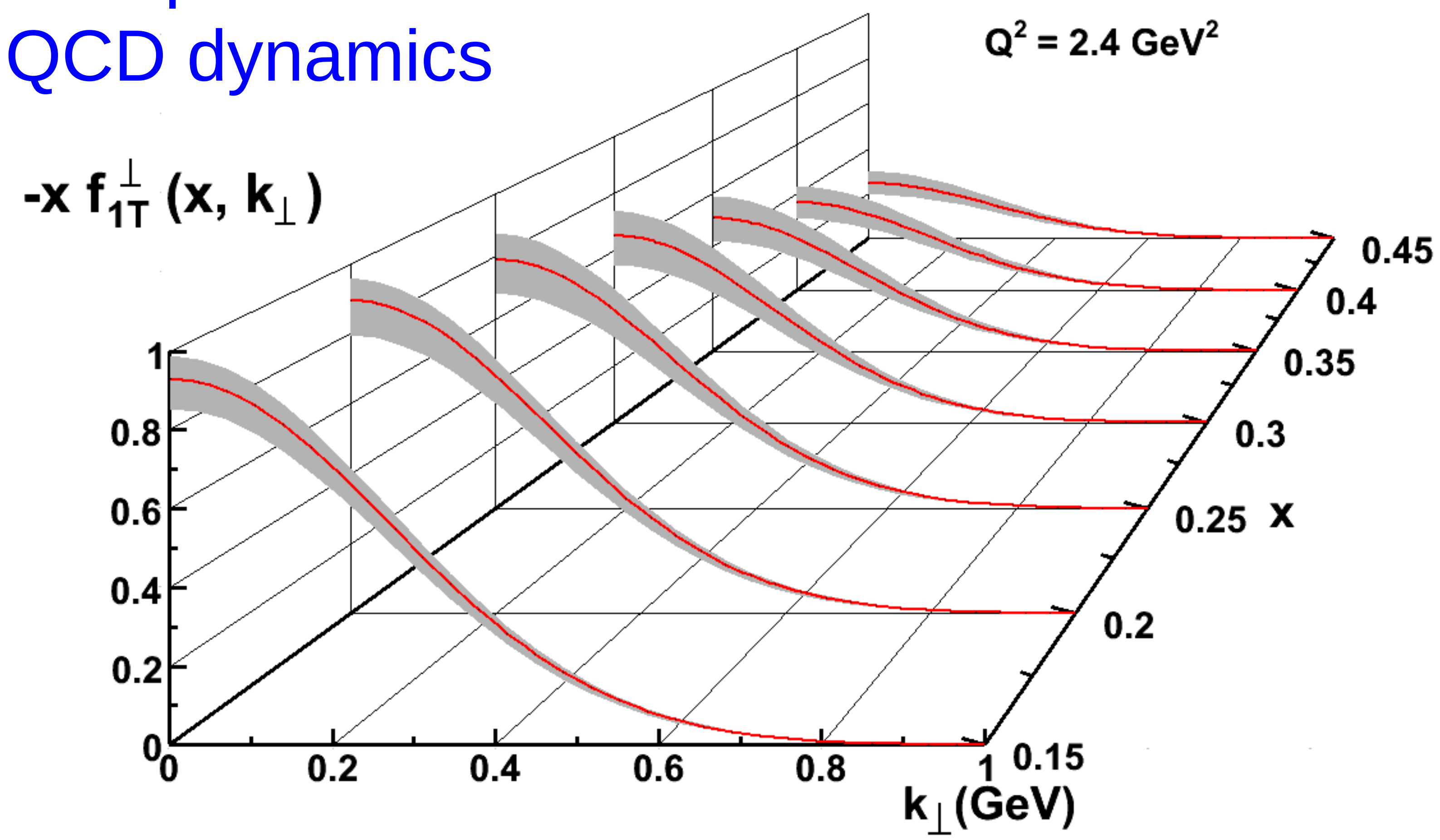
Nucleon momentum tomography and confined motion

2)

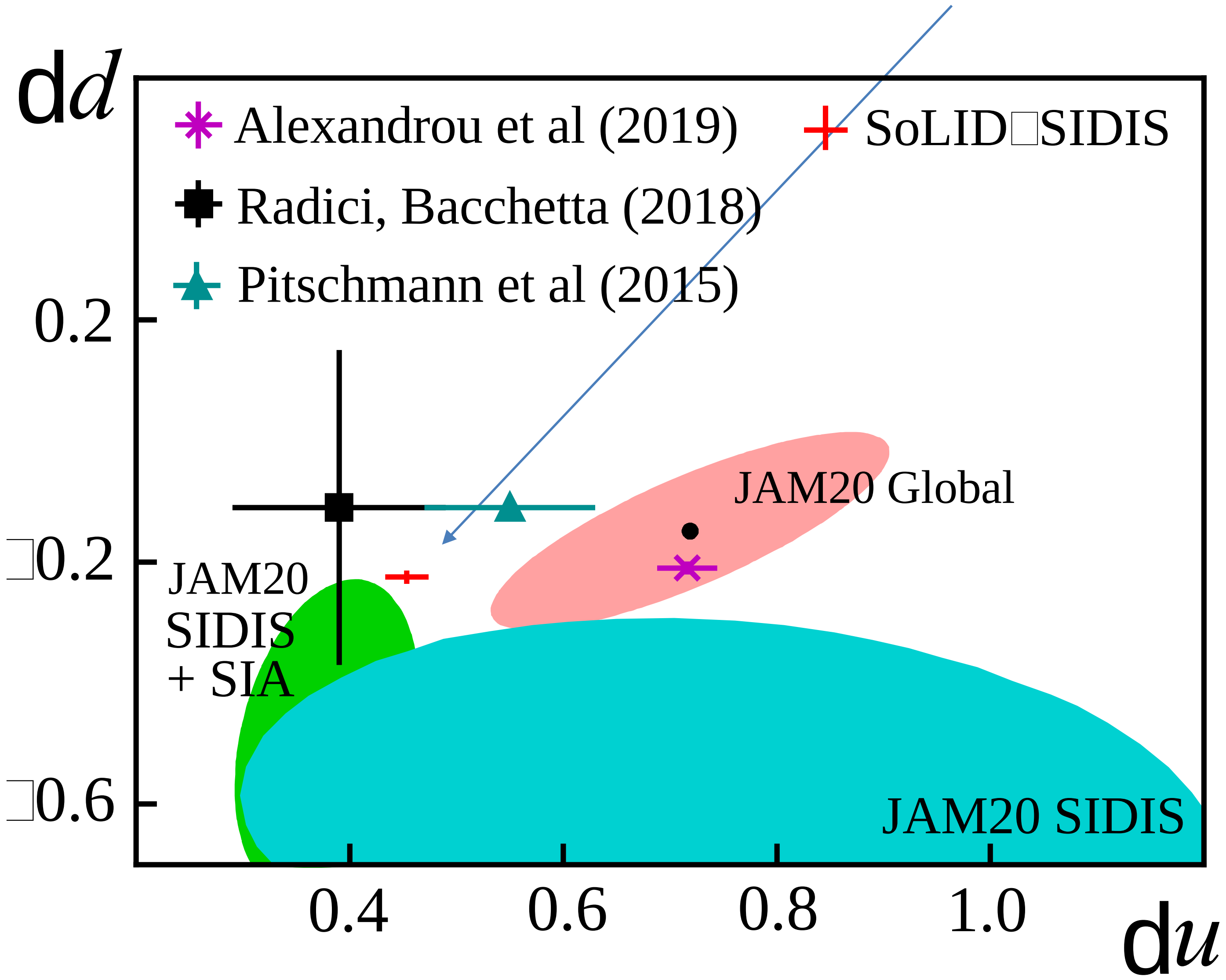
Polarized ³He (``neutron”) @ SoLID



- Sivers: an example of TMDs
- Confined quark motion inside nucleon
- Quantum correlations between nucleon spin and quark motion
- QCD dynamics



SoLID impact on tensor charge



Transversity distribution (another TMD) and tensor charge

$$\delta_T q = \int_0^1 \left[h_1^q(x) - h_1^{\bar{q}}(x)\right] \mathrm{d}x$$

- Tensor charge: a fundamental QCD quantity
- Moment of the transversity distribution, valence quark dominant
- Calculable in lattice QCD
- New physics combined with EDMs

JAM20: arxiv:2002.08384

2)

- The QED RC is one of the important sources of systematic uncertainties of SIDIS processes that will be investigated in SoLID experiments
- For example, the systematic uncertainties on asymmetry measurements for SIDIS are summarized in this table

SIDIS Systematic (rel.)	
Target polarization	3%
Nuclear effects	$(4-5)\%$
Random coincidence	0.2%
Radiative corrections	$(2-3)\%$
Diffractive meson contam.	3%
Total	$(6-7)\%$

- Important results came out last year from the following paper:

I. V. Akushevich and A. Ilyichev, Phys. Rev. D, **100**, 3, 033005 (2019)

- It shows exact analytical expressions obtained for the lowest order RC to semi-inclusive DIS scattering of polarized particles
- The contribution of the exclusive radiative tail is also derived
- The infrared divergence from real photon emission is extracted and cancelled by using the Bardin-Shumeiko approach

- We have a sixfold differential cross section of SIDIS with polarized particles

$$d\sigma/dxdydzdp_{hT}^2 d\phi_h d\phi$$

- The tree level (Born) process is given by

$$e(k_1,\xi) + n(p,\eta) \longrightarrow e(k_2) + h(p_h) + x(p_x)$$

where ξ/η is the initial lepton/nucleon polarized vector, and some variables are given by

$$k_1^2 = k_2^2 = m_l^2, \quad p^2 = M^2, \quad p_h^2 = m_h^2$$

$$x = -\frac{q^2}{2qp}, \ y = \frac{qp}{k_1 p}, \ z = \frac{p_h p}{pq},$$
$$t = (q - p_h)^2, \ \phi_h, \ \phi.$$

- The real photon emission in the semi-inclusive process is given by

$$e(k_1,\xi) + n(p,\eta) \longrightarrow e(k_2) + h(p_h) + x(\tilde{p}_x) + \gamma(k)$$

where *k* is the real photon four-momentum

- The exclusive radiative tail process is given by

$$e(k_1,\xi) + n(p,\eta) \longrightarrow e(k_2) + h(p_h) + u(p_u) + \gamma(k)$$

where *u* is the single unobservable hadron

2)

- The Feynman diagrams contributing to the lowest order SIDIS RC

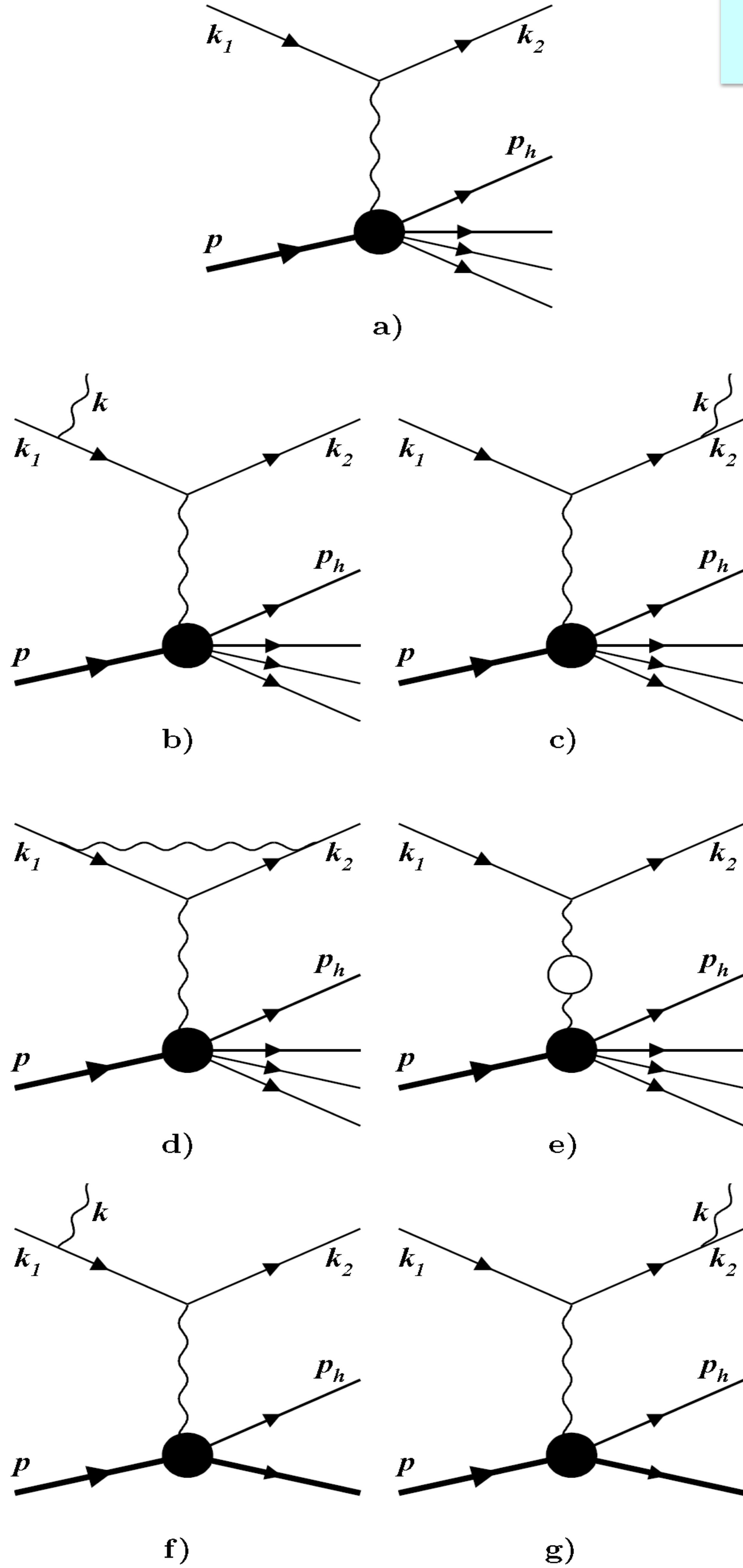
a) The Born contribution to SIDIS

b)-e) SIDIS RC with unobservable real photon emission from the lepton legs, with lepton vertex correction and vacuum polarization;

we have an observable hadron and continuum of unobservable particles

f)-g) The exclusive radiative tail contribution to the lowest order SIDIS RC with unobservable real photon from the lepton legs;

we have an observable hadron and a single unobservable hadron



- The total contribution of the inelastic tail to the SIDIS cross section plus the exclusive tail read as

$$\sigma^{in} = \frac{\alpha}{\pi}(\delta_{VR} + \delta^l_{\rm vac} + \delta^h_{\rm vac})\sigma^B + \sigma^F_R + \sigma^{\rm AMM}$$

$$\sigma^{ex}_R = -\frac{\alpha^3 S S_x^2}{2^9\pi^5 M p_l \lambda_S \sqrt{\lambda_Y}} \int\limits_{\tau_{\min}}^{\tau_{\max}} d\tau \int\limits_0^{2\pi} d\phi_k \times \sum_{i=1}^{9}\sum_{j=1}^{k_i} \frac{\tilde{\mathcal{H}}^{ex}_i \theta_{ij} R^{j-2}_{ex}}{(1+\tau-\mu)\tilde{Q}^4}$$

- The Born contribution has the following form:

$$\sigma^B \equiv \frac{d\sigma^B}{dx dy dz dp_t^2 d\phi_h d\phi} = \frac{\alpha^2 S S_x^2}{8 M Q^4 p_l \lambda_S} \sum_{i=1}^{9} \theta^B_i \mathcal{H}_i$$

where H_i are expressed via SIDIS structure functions

$$\begin{aligned}
H^{(0)}_{00} &= C_1 F_{UU,L},\\
H^{(0)}_{01} &= -C_1(F^{\cos\phi_h}_{UU} + iF^{\sin\phi_h}_{LU}),\\
H^{(0)}_{11} &= C_1(F^{\cos 2\phi_h}_{UU} + F_{UU,T}),\\
H^{(0)}_{22} &= C_1(F_{UU,T} - F^{\cos 2\phi_h}_{UU}),\\
H^{(S)}_{002} &= C_1 F^{\sin(\phi_h-\phi_s)}_{UT,L},\\
H^{(S)}_{012} &= C_1(F^{\sin\phi_s}_{UT} - F^{\sin(2\phi_h-\phi_s)}_{UT}\\
&\quad -i(F^{\cos\phi_s}_{LT} - F^{\cos(2\phi_h-\phi_s)}_{LT})),\\
H^{(S)}_{021} &= C_1(F^{\sin(2\phi_h-\phi_s)}_{UT} + F^{\sin\phi_s}_{UT}\\
&\quad -i(F^{\cos(2\phi_h-\phi_s)}_{LT} + F^{\cos\phi_s}_{LT})),
\end{aligned}$$

$$\begin{aligned}
H^{(S)}_{023} &= C_1(F^{\sin\phi_h}_{UL} - iF^{\cos\phi_h}_{LL}),\\
H^{(S)}_{121} &= C_1(-F^{\sin(3\phi_h-\phi_s)}_{UT} - F^{\sin(\phi_h+\phi_s)}_{UT}\\
&\quad +iF^{\cos(\phi_h-\phi_s)}_{LT}),\\
H^{(S)}_{123} &= C_1(-F^{\sin 2\phi_h}_{UL} + iF_{LL}),\\
H^{(S)}_{112} &= C_1(F^{\sin(3\phi_h-\phi_s)}_{UT} + F^{\sin(\phi_h-\phi_s)}_{UT,T}\\
&\quad -F^{\sin(\phi_h+\phi_s)}_{UT}),\\
H^{(S)}_{222} &= C_1(F^{\sin(\phi_h+\phi_s)}_{UT} + F^{\sin(\phi_h-\phi_s)}_{UT,T}\\
&\quad -F^{\sin(3\phi_h-\phi_s)}_{UT}),
\end{aligned}$$

$$C_1 = \frac{4 M p_l (Q^2 + 2xM^2)}{Q^4}.$$

- Some of the SIDIS structure functions can be expressed in terms of Fourier-transformed TMD PDFs and FFs

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = -x_B \sum_a e_a^2 \int \frac{d|b_T|}{(2\pi)} |b_T|^2 J_1(|b_T||P_{h\perp}|)\, Mz\, \tilde{f}_{1T}^{\perp(1)}(x, z^2 b_T^2)\, \tilde{D}_1(z, b_T^2)$$

$$F_{LL} = x_B \sum_a e_a^2 \int \frac{d|b_T|}{(2\pi)} |b_T|\, J_0(|b_T||P_{h\perp}|)\, \tilde{g}_{1L}(x, z^2 b_T^2)\, \tilde{D}_1(z, b_T^2)$$

$$F_{LT}^{\cos(\phi_h-\phi_s)} = x_B \sum_a e_a^2 \int \frac{d|b_T|}{(2\pi)} |b_T|^2 J_1(|b_T||P_{h\perp}|)\, Mz\, \tilde{g}_{1T}^{\perp(1)}(x, z^2 b_T^2)\, \tilde{D}_1(z, b_T^2)$$

$$F_{UT}^{\sin(\phi_h+\phi_S)} = x_B \sum_a e_a^2 \int \frac{d|b_T|}{(2\pi)} |b_T|^2 J_1(|b_T||P_{h\perp}|)\, M_h z\, \tilde{h}_1(x, z^2 b_T^2)\, \tilde{H}_1^{\perp(1)}(z, b_T^2)$$

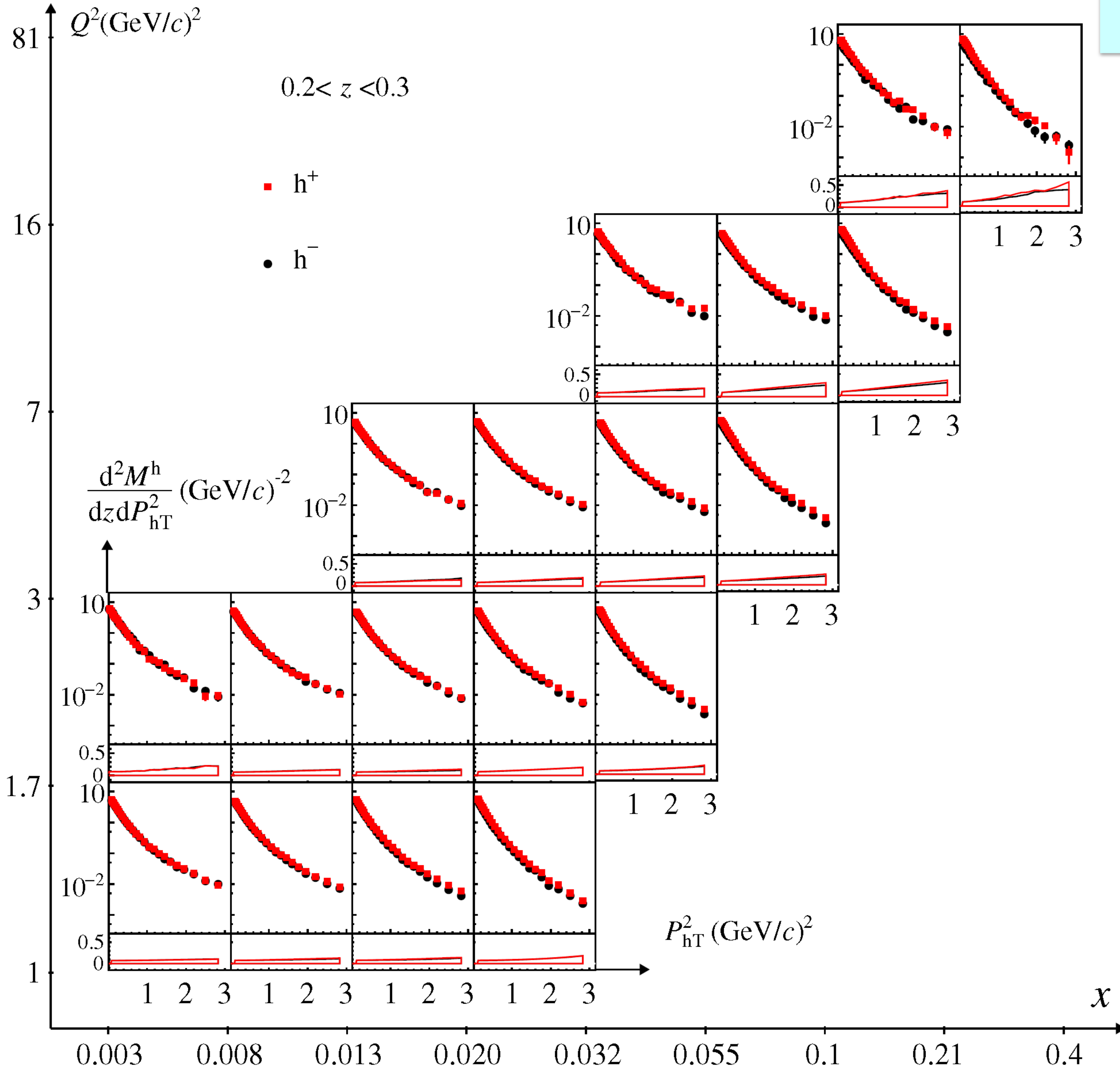
$$F_{UU}^{\cos(2\phi_h)} = x_B \sum_a e_a^2 \int \frac{d|b_T|}{(2\pi)} |b_T|^3 J_2(|b_T||P_{h\perp}|) M M_h z^2\, \tilde{h}_1^{\perp(1)}(x, z^2 b_T^2)\, \tilde{H}_1^{\perp(1)}(z, b_T^2)$$

$$F_{UL}^{\sin(2\phi_h)} = x_B \sum_a e_a^2 \int \frac{d|b_T|}{(2\pi)} |b_T|^3 J_2(|b_T||P_{h\perp}|) M M_h z^2\, \tilde{h}_{1L}^{\perp(1)}(x, z^2 b_T^2)\, \tilde{H}_1^{\perp(1)}(z, b_T^2)$$

$$F_{UT}^{\sin(3\phi_h-\phi_S)} = x_B \sum_a e_a^2 \int \frac{d|b_T|}{(2\pi)} |b_T|^4 J_3(|b_T||P_{h\perp}|) \frac{M^2 M_h z^3}{4} \tilde{h}_{1T}^{\perp(2)}(x, z^2 b_T^2)\, \tilde{H}_1^{\perp(1)}(z, b_T^2)$$

D. Boer, L. Gamberg, B. U. Musch and A. Prokudin, JHEP, **10**, 021 (2011)

- We work on making a MC event generator and can use it first for comparison of available data
- Like using semi-inclusive measurement data of charged hadron multiplicities in muon SIDIS by COMPASS
- For this quantity we can use the ratio between the differential semi-inclusive cross section and the differential inclusive cross section
- The hadron multiplicities are measured in the four-dimensional space



$$\frac{\mathrm{d}^2 M^{\mathrm{h}}(x, Q^2, z, P^2_{\mathrm{hT}})}{\mathrm{d}z\mathrm{d}P^2_{\mathrm{hT}}} = \left(\frac{\mathrm{d}^4\sigma^{\mathrm{h}}}{\mathrm{d}x\mathrm{d}Q^2\mathrm{d}z\mathrm{d}P^2_{\mathrm{hT}}}\right) \Big/ \left(\frac{\mathrm{d}^2\sigma^{\mathrm{DIS}}}{\mathrm{d}x\mathrm{d}Q^2}\right)$$

Multiplicities of positively and negatively charged hadrons as a function of p^2_{hT} in $(\mathrm{x}, \mathrm{Q}^2)$ bins for $0.2 < z < 0.3$

Part 3

3) **Upcoming RC Monte Carlo studies in the e+d elastic scattering for the planned DRad experiment**

- Our next goal is to make another MC event generator for elastic e+d scattering including the lowest order RC effects

- It will be based upon the following paper:

I. V. Akushevich and N. M. Shumeiko,
J. Phys. G: Nuclear Particle Physics, **20**, 513 (1994)

- The results are obtained for beyond and within ultrarelativistic approximaton

- We will consider the general polarized case, from which it is straightforward to obtained the unpolarized case

- The old form factors used in the paper should be substituted by the most recent ones, which are the charge, magnetic and quadrupole form factors of deuteron

- Here the same covariant formalism and similar variables have been used as in the case of SIDIS RC paper that we already discussed

- As such, let us look at the tree level (Born) process and the cross section:

$$e(k_1,\xi) + n(p,\eta) \longrightarrow e(k_2) + x(p_x)$$

$$\begin{aligned}\frac{d^2\sigma}{dx\,dy} = \frac{4\pi\,\alpha^2}{\sqrt{\lambda_s}}\frac{S_x}{Q^4}\Bigg\{&(Q^2-2m^2)\left[F_1+\frac{Q_N}{6}(1+k_n\epsilon)b_1\right]\\ &+\frac{SX-M^2Q^2}{S_x}\left[F_2+\frac{Q_N}{6}\left(b_2+k_n\epsilon\left(\frac{b_2}{3}+b_3+b_4\right)\right)\right]\\ &+\frac{mM}{S_x}P_N\left[(Q^2\xi\eta-q\eta k_2\xi)(g_1+g_2)+\left(k_2\xi-\frac{2\xi pQ^2}{S_x}\right)q\eta\,g_2\right]\\ &-\frac{Q_N}{12}\epsilon\Bigg[(Q^2+4m^2+12\eta k_1\eta k_2)\left(\frac{b_2}{3}-b_3\right)\\ &-\frac{3q\eta}{S_x}(X\eta k_1+S\eta k_2)\left(\frac{b_2}{3}-b_4\right)\Bigg]\Bigg\}\end{aligned}$$

where ξ/η is the initial lepton/nucleon polarized vector; P_N and Q_N define the polarization and quadrupolarization degrees, respectively

All the parameters, constants and functions are given in the paper !

- The photon emission is represented by the following process:

$$e(k_1,\xi) + n(p,\eta) \longrightarrow e(k_2) + \gamma(k) + x(p_x)$$

- The contribution to the inelastic radiative tail is given by

$$\sigma_{in} = -\frac{\alpha^3 S_x}{\lambda_s^{\frac{1}{2}}} \int_{\tau_{min}}^{\tau_{max}} d\tau \sum_{i=1}^{8} \sum_{j=1}^{k_i} \left\{ \theta_{ij}(\tau) \int_0^{R_{max}} dR \frac{R^{j-2}}{t^2} I_i(R,\tau) \right.$$

where $I_i(R,\tau)$ are defined as some combinations of DIS structure functions

- The infrared divergence occurs in the integral for R→0, in the term with j=1, and is cancelled by adding a virtual photon contribution

$$\sigma_v + \sigma_{in} = \frac{\alpha}{\pi}\left(\sigma_R^{IR} + \sigma_{vert} + \delta_{vac}^{l} + \delta_{vac}^{h}\right)\sigma^B + \sigma_R^F$$

- We are interested in the elastic radiative tail, which can be obtained from the formula of σ_{in}, by an appropriate substitution of structure functions by form factors, and by doing some more math

- it will result in

$$\sigma_{el} = -\frac{\alpha^3 S_x}{\lambda_s^{\frac{1}{2}}} \int_{\tau_{min}}^{\tau_{max}} d\tau \sum_{i=1}^{8} \sum_{j=1}^{k_i} \{\theta_{ij}(\tau) \frac{2M_d^2 R_{el}^{j-2}}{(1+\tau)t^2} \, F_i^{el}(R_{el}, \tau)$$

with the F_i functions, expressed in terms of the charge monopole, $G_C(Q^2)$, magnetic dipole, $G_M(Q^2)$, and charge quadrupole, $G_Q(Q^2)$, form factors:

$$F_1^{el} = \frac{1}{6}\, \eta_d \, G_M^2(4(1+\eta_d) + \eta_d Q_N);$$

$$F_2^{el} = \left(G_c^2 + \frac{2}{3}\eta_d G_M^2 + \frac{8}{9}\eta_d^2 G_Q^2\right) + \frac{Q_N}{6}\left(\eta_d G_M^2 + \frac{4\eta_d^2}{1+\eta_d}\left(\frac{\eta_d}{3} G_Q + G_C - G_M\right) G_Q\right);$$

$$F_3^{el} = -\frac{P_N}{2}(1+\eta_d) G_M \left(\frac{\eta_d}{3} G_Q + G_C\right); \qquad F_4^{el} = \frac{P_N}{4} G_M \left(\frac{1}{2} G_M - G_C - \frac{\eta_d}{3} G_Q\right);$$

$$F_5^{el} = \frac{Q_N}{24} G_M^2; \qquad F_6^{el} = \frac{Q_N}{24}\left(G_M^2 + \frac{4}{1+\eta_d}\left(\frac{\eta_d}{3} G_Q + G_C + \eta_d G_M\right) G_Q\right);$$

$$F_7^{el} = \frac{Q_N}{6}\eta_d(1+\eta_d) G_M^2; \qquad F_8^{el} = -\frac{Q_N}{6}\eta_d G_M (G_M + 2G_Q).$$

- The results of this paper are obtained in compact form convenient for the numerical analysis

- Based upon it, we will make a new generator

- And we can go forward and try to exactly calculate higher order RC effects in addition to the lowest order, and include the resulting cross section into the generator

Outlook

1) We will be working hard to calculate exactly the NNLO RC in unpolarized elastic e+p and Møller scatterings beyond ultrarelativistic limit, for the new proton radius measurements in the planned PRad-II experiment ?

2) We are already working to make an event generator (using C++) based on recent RC SIDIS studies

3) We also plan to make another event generator (using C++) for the elastic e+d scattering, for the deuteron radius measurements in the planned DRad experiment

Thank you !