Radiative Corrections for low energy lepton-proton scattering in *Heavy Baryon Chiral Perturbation Theory*. [1]

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The aim of the MUSE precision experiment at PSI is to extract the proton's r.m.s. radius.

At MUSE energies we know the muons are not radiating the bremsstrahlung photons according to the "peaking" approximation.

The muon mass is at the order of the beam momenta used by MUSE.

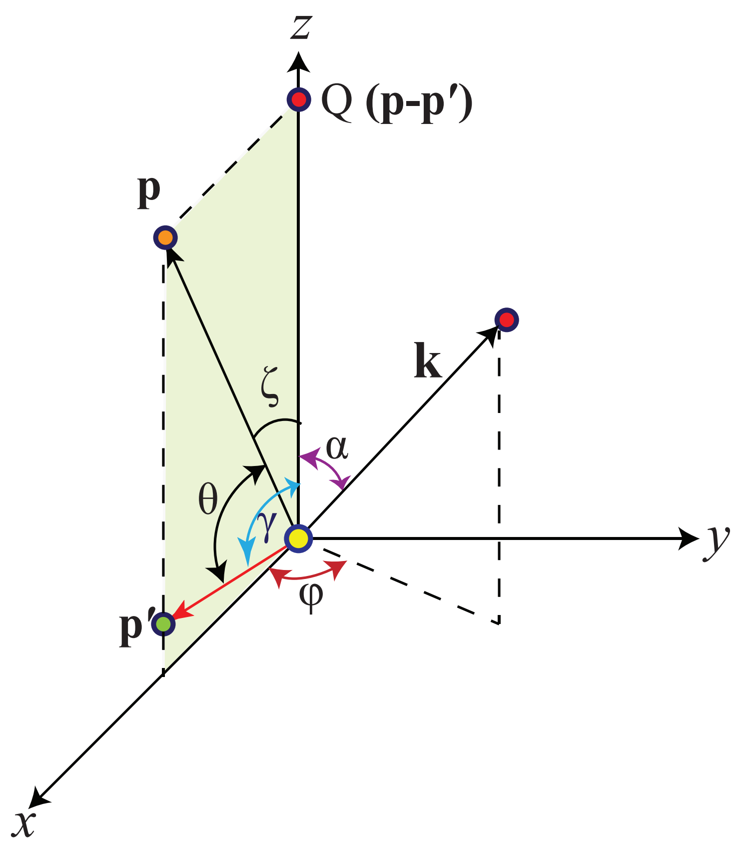
Needed radiation corrections $\delta_{2\gamma}$ have been completed and will be presented:

$$\left[\frac{\mathrm{d}\sigma_{el}^{(\mathrm{NLO})}(Q^2)}{\mathrm{d}\Omega_l'}\right]_\gamma = \left[\frac{\mathrm{d}\sigma_{el}(Q^2)}{\mathrm{d}\Omega_l'}\right](1+\delta_{2\gamma})$$

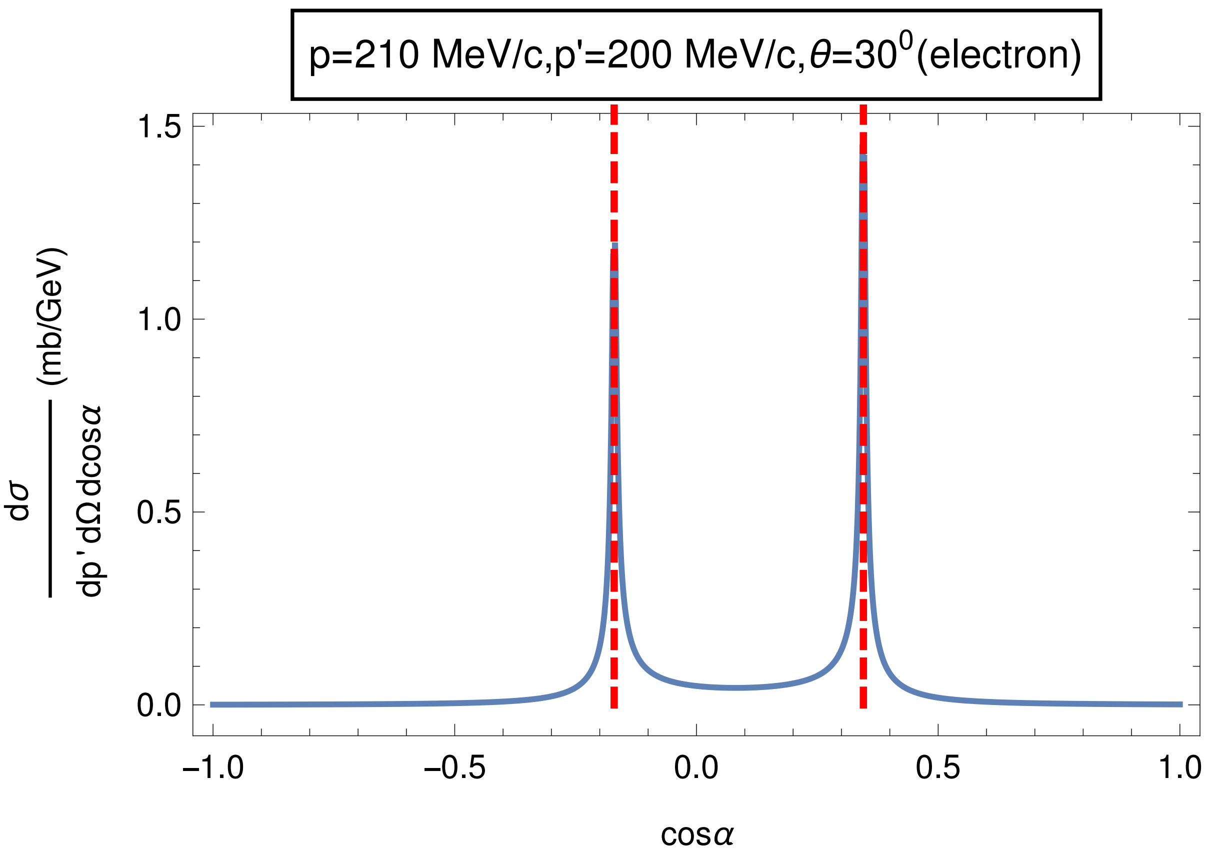
The low-energy lepton proton elastic scattering studied by MUSE require the masses of the relativistic leptons to be non-zero.

No polarization observables are measured by MUSE

All evaluations are done in the lab. frame, i.e. $\vec{p}_p = 0$.

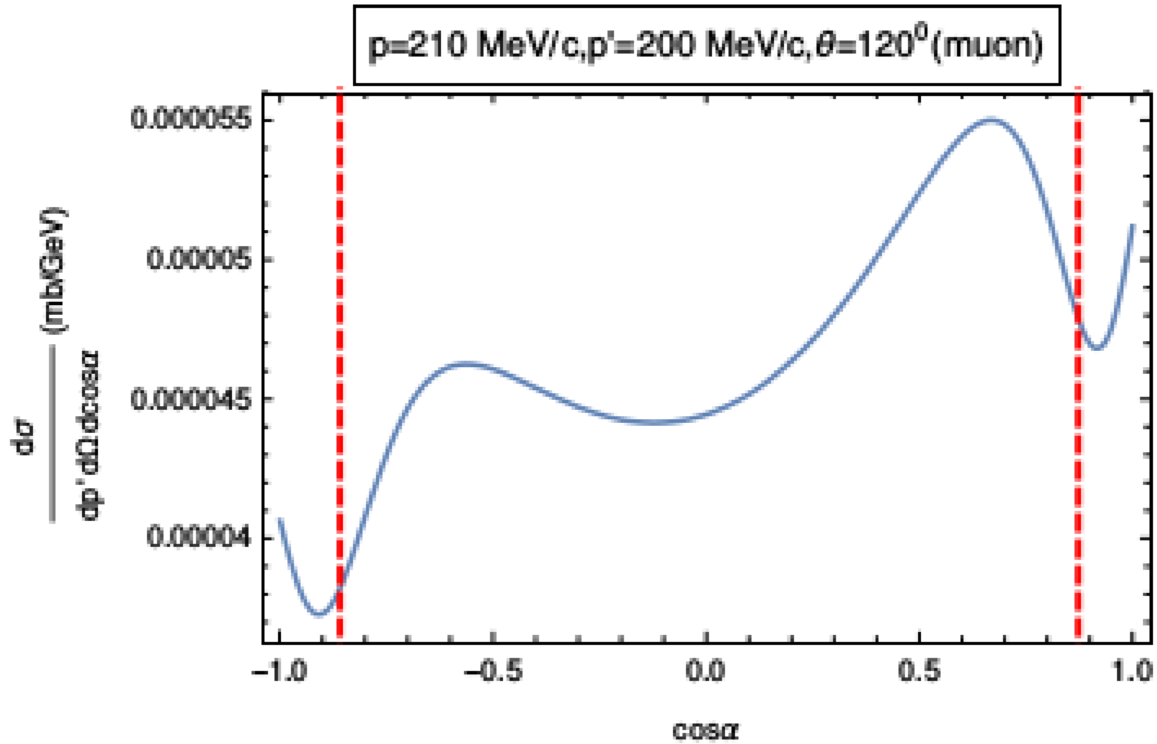


Coordinate system used in some evaluations:
$\vec{Q} = \vec{p} - \vec{p}\,'$ **is along the** z**-axis, while** $\vec{p}$ **and** $\vec{p}\,'$ **lie in** xz**-plane.**



The two peaks occur for the photon angle α coincides with the incoming or outgoing electron direction in our coordinate system.

The momentum transfer $Q = p - p'$ is well defined.



The bremsstrahlung photon's angular distribution for muon scattering at a backward scattering angle θ.

The two red vertical lines indicate the incoming and outgoing muon angles. The momentum transfer $Q = p - p'$ is <u>not</u> well defined.

Fortunately, the muon is relatively "heavy".

(1): At the relatively small MUSE momenta the recoil of the target proton is moving non-relativistically.

(2): At these low energies we keep the lepton mass in all our expressions.

(3): The proton mass, $m_p \sim 1$ GeV is large compared to lepton momenta, and $m_p \sim \Lambda_\chi \sim 4\pi f_\pi$, which is the chiral scale.

As is standard in heavy baryon chiral perturbation theory (χPT), the expansion of the Lagrangian in powers of m_p^{-1} is an integral part of the chiral expansion in p/Λ_χ.

χPT is a low-energy model-independent theory where gauge invariance is naturally included.

We evaluate all radiative corrections for elastic lepton-proton scattering using χPT, which is an effective low-energy field theory of QCD.

The effective hadronic Lagrangian $\mathcal{L}_\chi$ is expanded in increasing chiral order.

$$\mathcal{L}_\chi = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi\pi}^{(2)} + \cdots \ ,$$

The leading order (LO) Lagrangian is:

$$\mathcal{L}_{\pi N}^{(1)} \; = \; \bar{\psi}_N (iv \cdot D + g_A S \cdot u) \psi_N$$

We chose the nucleon velocity $v^\mu = (1, \vec{0})$,
which gives the covariant nucleon spin $S^\mu = (0, \frac{1}{2}\vec{\sigma})$.

This satisfy the requirement of the heavy nucleon expansion: $v \cdot S = 0$.

We evaluate the LO and next-to-leading order (NLO) contributions.

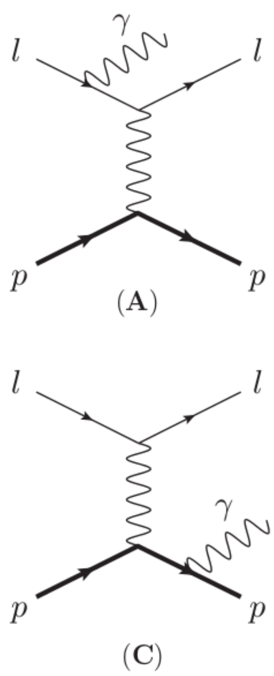
The pion degrees of freedom do not enter at this order,
i.e., the pion Lagrangian $\mathcal{L}_{\pi\pi}^{(2)}$ enter only at NNLO.

In χPT the pion-loops contribute to the proton form factor
(and the charge r.m.s. radius)!
(More on this topic at the end).

Some parts of the NLO Lagrangian $\mathcal{L}_{\pi N}^{(2)}$ will contribute.

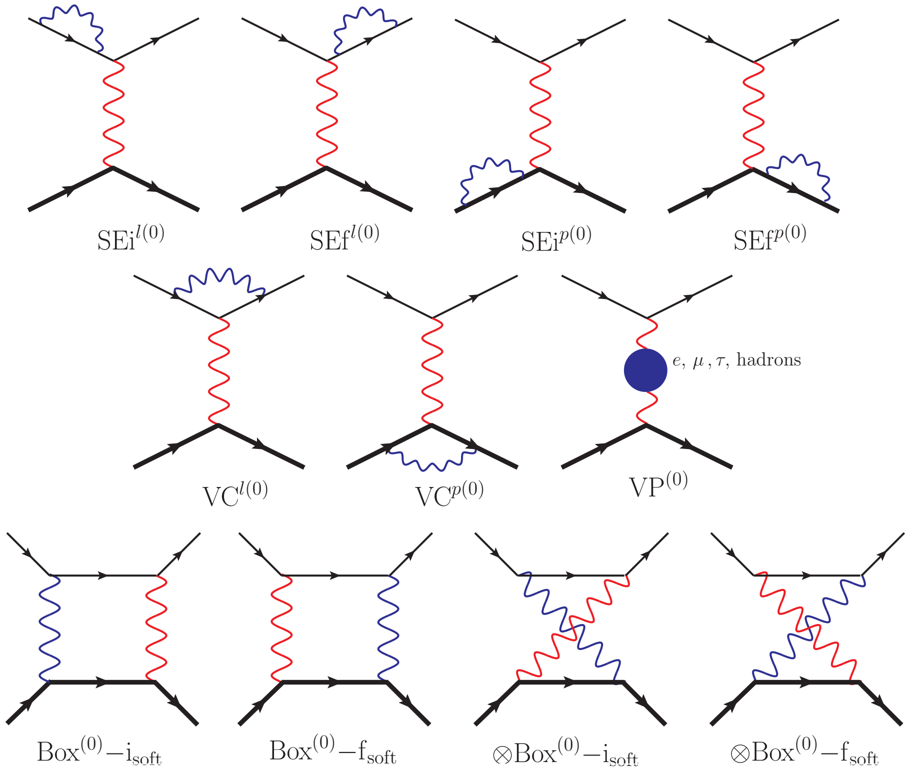
$$\mathcal{L}_{\pi N}^{(2)} \; = \; \frac{1}{m_N} \bar{\psi}_N [(v \cdot \mathcal{D})^2 - \mathcal{D} \cdot \mathcal{D} + \cdots] \psi_N \, ,$$

where for the non-relativistic proton $\mathcal{D}_\mu = \partial_\mu + e A_\mu$.



Feynman diagrams considered in this χPT evaluation.
Only diagrams (A) and (B) contribute to the LO bremsstrahlung cross section in the Coulomb gauge.

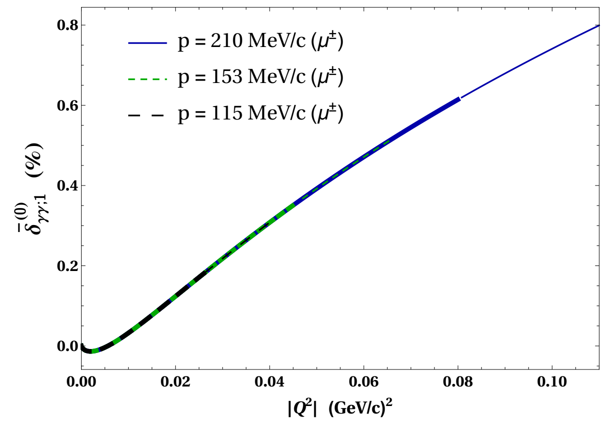
At leading order (LO) the proton does not radiate in Coulomb gauge.

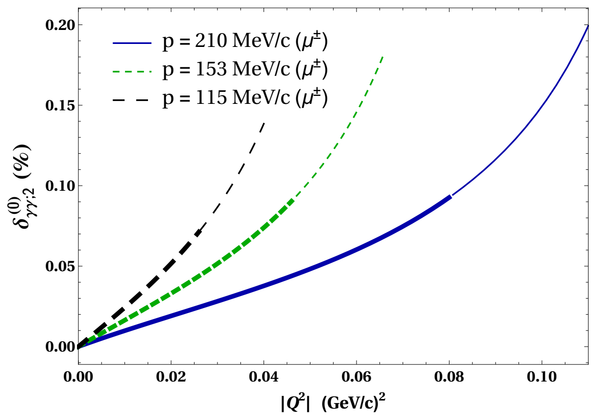


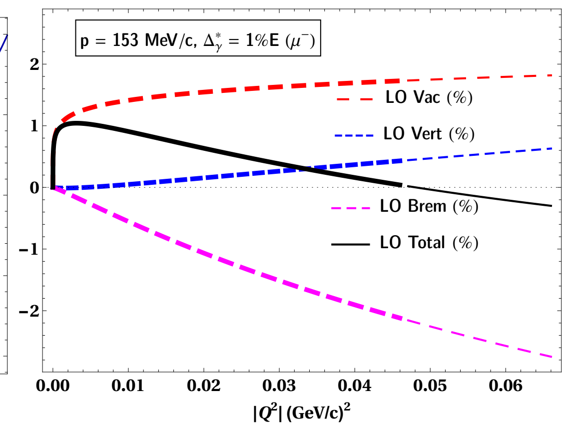
The radiative corrections to elastic lepton proton scattering at LO.
The vacuum polarization blob diagram has leptonic and hadronic contributions.

(1): At LO there are <u>no</u> radiative <u>proton</u> corrections at LO.
(2): The Two-Photon-Exchange (TPE) contribution is <u>zero</u> at LO.

In essence, at LO in χPT
the lepton scatters off a static Coulomb potential.





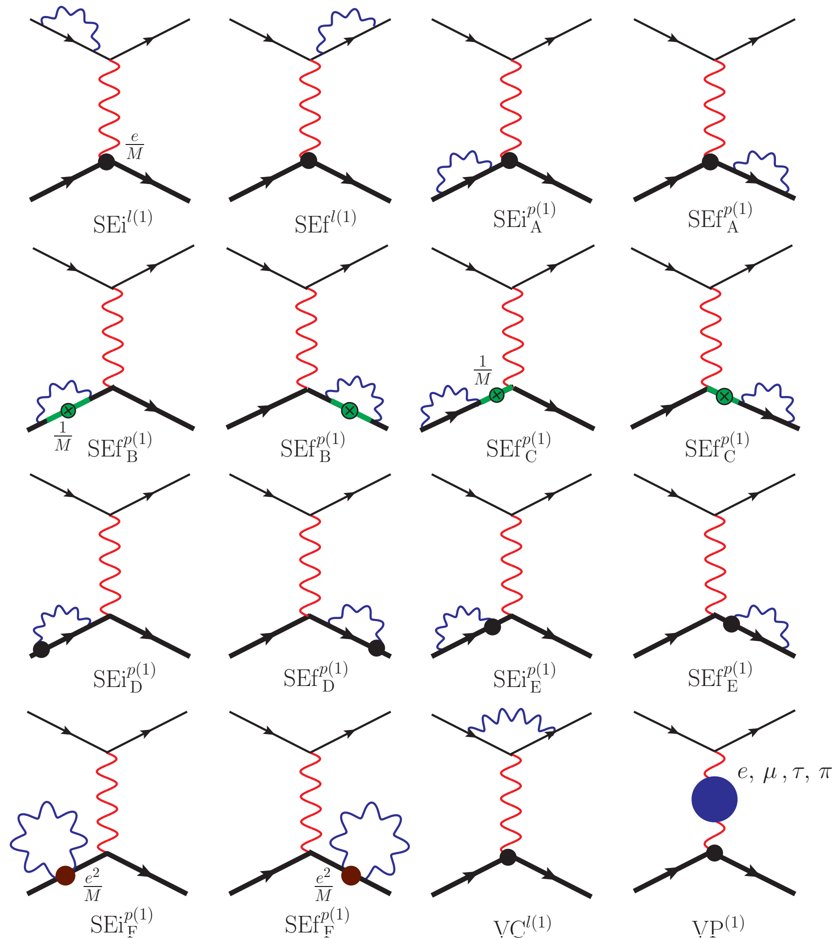


One-loop LO vertex contributions $\overline{\delta}^{(0)}_{\gamma\gamma;1}$ and $\delta^{(0)}_{\gamma\gamma;2}$ for muon scattering.

A few comments of the LO evaluations of $\delta^{(0)}_{2\gamma}$:

- **All radiative corrections vanish in the limit $Q^2 \to 0$.**
- **The lepton Dirac form factor, F_1, is almost independent of the beam momentum.**
- **The lepton Pauli form factor, F_2, depends strongly on the beam momentum.**
- **Whereas the bremsstrahlung correction is negative, the vacuum polarization and the vertex corrections are positive.**

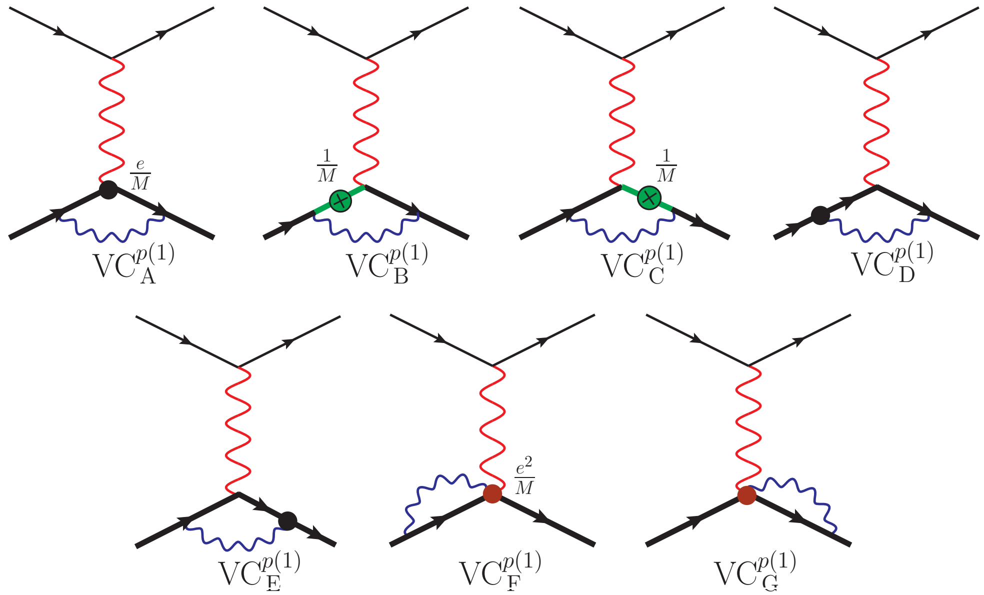
Next we will present the NLO radiative corrections.



Some NLO contributions to lepton proton elastic scattering.

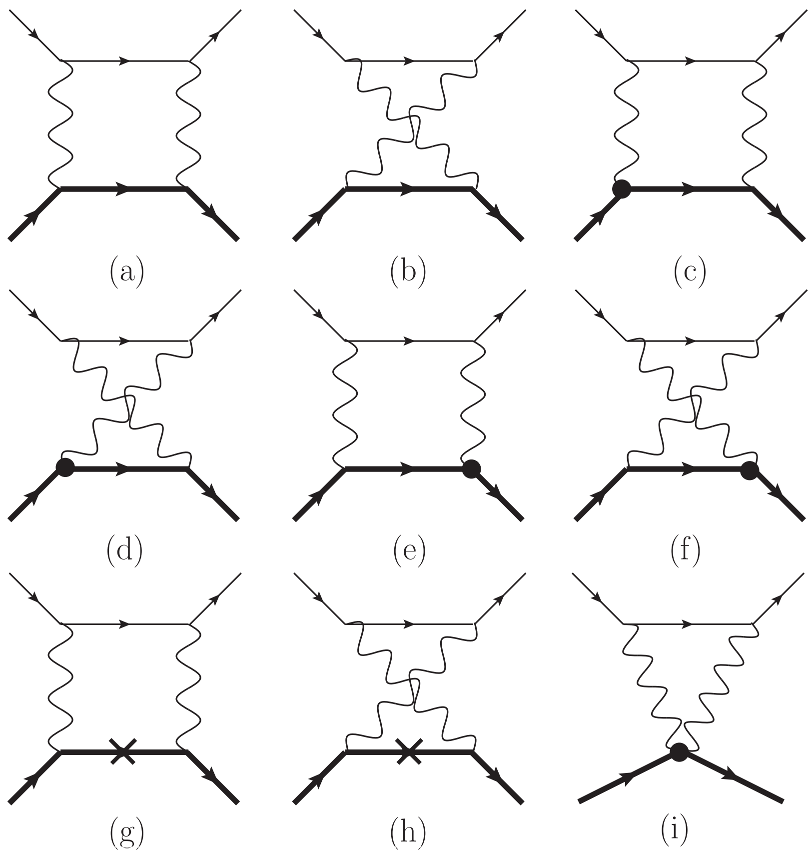
The NLO photon-nucleon vertex from $\mathcal{L}^{(2)}_{\pi N}$ is the filled circle.

Only the four lepton diagrams in *dimensional regularizations* are <u>non-zero</u>. However, since they <u>interfere</u> with the LO elastic lepton-proton scattering amplitude, they are $\mathcal{O}(M^{-2})$.



The Proton vertex corrections at NLO are displayed.

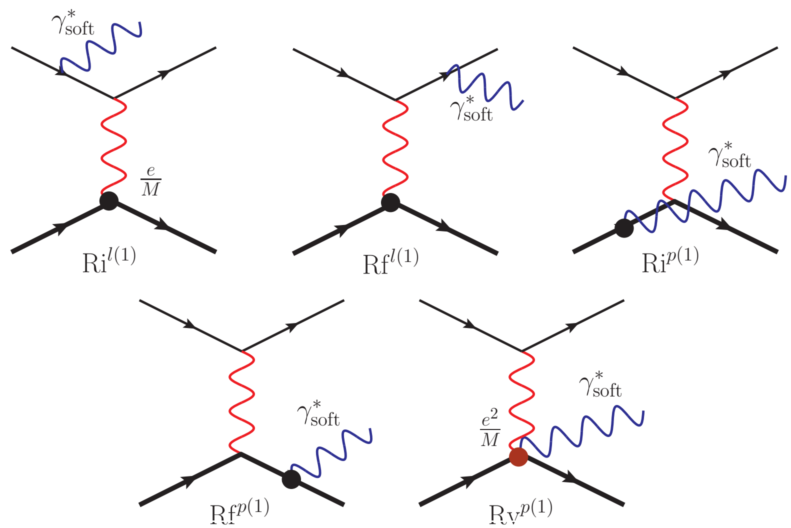
In *dimensional regularizations* **we show that these diagrams give** **<u>zero</u>** **contributions**



The two-photon exchange contributions. **Diagrams (a) and (b) are LO contributions. The other seven diagrams are NLO contributions in χPT.**

The TPE diagrams at NLO [except diagram (i)] were evaluated using the *soft photon approximation*.

Diagram (i) was evaluated exactly (no approximations). *It has no divergences*.



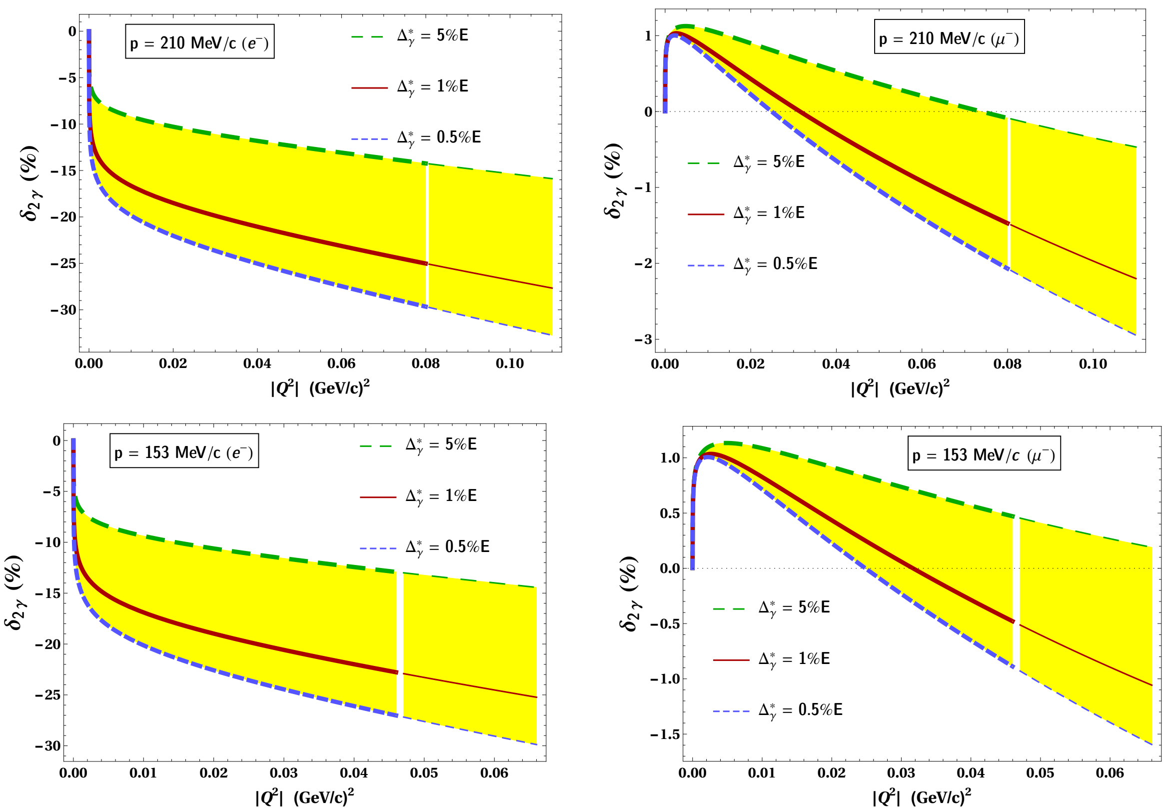
The NLO bremsstrahlung contributions.

The NLO diagrams [except the last one] were evaluated using the *soft photon approximation* **and** *dimensional regularizations***.**

The last diagram labelled $Rv^{p(1)}$ was evaluated exactly (no approximations). It has no IR singularity.

A few comments regarding the NLO corrections $\delta_{2\gamma}^{(1)}$ in χPT

- **The NLO TPE <u>does depend</u> on the lepton charge.**
  The total correction $\delta_{2\gamma} = \delta_{2\gamma}^{(0)} + \delta_{2\gamma}^{(1)}$ is smaller for ℓ^+-p than the ℓ^--p scattering.
- **Both the TPE and the bremsstrahlung contributions are roughly linear with increasing Q^2.**
- **Both NLO contributions are of comparable magnitude but of opposite sign.**
- **Radiative corrections are evaluated using the *soft photon approximation*.**
- **We have only one free parameter in our evaluations:**
  the upper limit of the *soft (undetected) photons* in the lab frame is $\sim \Delta_\gamma^*$.
- **The parameter Δ_γ^* depends on the detector sensitivity.**



The lower limit of the photon energy detection $\sim \Delta_\gamma^*$.

The (yellow) band are the variation of the *lab*-frame detector resolution when $1\,\% < \Delta_\gamma^* < 5\,\%$ of the incident lepton energy E.

We have presented all QED radiative corrections, i.e.,
(a) particle self-energies,
(b) photon-loop vertex corrections,
(c) vacuum-polarization of the exchanged photon, and
(d) two-photon exchange corrections using *soft proton approximation*.
(e) Bremsstrahlung photons using *soft proton approximation*.

All contributions are evaluated in χPT using *dimensional regularizations*.
The LO and NLO contributions in χPT are included.

The evaluated lepton vertex corrections give the lowest order QED Dirac form factors, $F_1(q^2)$ and $F_2(q^2)$, at small q^2 including the lepton mass dependence.

At $Q^2 = 0$, due to the Ward identity for F_1, as $Q^2 \to 0$, we find the expected lepton gyromagnetic moment

$$g \;=\; 2 + 2F_2(Q^2 = 0) = 2 + \frac{\alpha}{\pi} + \mathcal{O}(\alpha^2)$$

For $q^2 < 0$ and $-q^2 \gg m^2$ we find the usual expression for $F_2^{loop}(q^2)$.

We find that the IR divergences of all the radiative photon-loop corrections do <u>cancel</u> the IR divergences of the bremsstrahlung diagrams.

To LO in χPT we find that the <u>two-photon</u>, box and cross-box diagrams, cancel.

The NLO $\mathcal{L}_{\pi N}^{(2)}$ terms $\propto m_N^{-1}$ give a two-photon exchange contribution within χPT.

At $\mathcal{L}_{\pi N}^{(2)}$ two photons can couple to the heavy proton in one vertex, i.e., we have a TPE triangle diagram and a similar bremsstrahlung diagram. Both can be evaluated <u>exactly</u> and give <u>finite</u> contributions to the radiative cross section.

All the UV divergences require the usual counter-terms in the Lagrangian.

Summary

A successful MUSE experiment requires precision and reasonable estimates of all possible corrections including the bremsstrahlung corrections.

The electron behaves relativistically.
The "peaking approximation" and radiative corrections by, for example,
Maximon and Tjon, PRC **62**, *054320 (2000)*,
applies to electron scattering, except.

The exception occur when p' is at the order of 20 − 40 MeV/c.
Then cross section increases rapidly with smaller p' values below 30 MeV/c.
This increase is due to the small electron mass.

Theoretically, as $m_e \to 0$ you could have a singularity in the diff. cross section at a finite p' value.

This work does not included the "hard" photon components in the TPE evaluations and the "hard" bremsstrahlung contributions.

In χPT at NNLO the pion-loops, meaning the hadronic proton form factors including the proton r.m.s. radius, will contribute, as will $\mathcal{O}(M^{-2})$ terms.

In the MUSE experiment the bremsstrahlung photons and p' are not detected.

How precise can you measure the momentum transfer q^2
when only the outgoing lepton's scattering angle θ is determined?

The square of the momentum transfer:

$$-q^2(\theta,\alpha) \;=\; 2\left[EE' - |\vec{p}||\vec{p}\,'|\cos\theta\right] + 2E_\gamma(E-E') - 2m^2 - 2E_\gamma\cos\alpha\sqrt{|\vec{p}|^2+|\vec{p}\,'|^2-2|\vec{p}||\vec{p}\,'|\cos\theta}\,,$$

i.e., q^2 depends on cosα, where α is the outgoing bremsstrahlung photon angle.

Note: <u>if</u> $p-p'=\Delta p \ll p$, <u>and</u> the lepton mass can be ignored, i.e., $E\simeq p$, <u>then</u>

$$-q^2(\theta,\alpha) \;\simeq\; 2\,|\vec{p}||\vec{p}\,'|\left[1-\cos\theta\right] + \mathcal{O}(\Delta p)\,,$$

i.e., linear in p' and independent of cosα.

The cross section in the lab. frame, $\vec{p}_p = 0$:

$$\mathrm{d}\sigma \; = \; \int \frac{\mathrm{d}^3\vec{p}\,' \; \mathrm{d}^3\vec{k}}{(2\pi)^5 8 E_l' E_\gamma} \frac{\delta\left(E_l - E_l' - E_\gamma - \frac{(\vec{p}-\vec{p}\,'-\vec{k})^2}{2m_p}\right)}{4 m_p E_l \left(m_p + \frac{(\vec{p}-\vec{p}\,'-\vec{k})^2}{2m_p}\right)} \frac{1}{4} \sum_{spin} |M|^2 \,,$$

We expand the phase-space expression of the final proton's non-relativistic recoil energy.

$$E_p' = \sqrt{m_p^2 + (\vec{p}_p\,')^2} = m_p + \frac{(\vec{p}_p + \vec{p} - \vec{p}\,' - \vec{k})^2}{2m_p} + \cdots \,.$$