Second-Order Radiative Corrections for ep Scattering

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SFB 1044
THE LOW-ENERGY FRONTIER
OF THE STANDARD MODEL

Overview

- Motivation
- Properties of radiative corrections
    - folding and unfolding
    - leptonic radiation
    - kinematical effects
- Second-order leptonic radiative corrections

➜ Implementation in POLARES
(with R. Bucoveanu, EPJA55, arXiv:1811.04970)

Motivation

Lepton-nucleon scattering – the goal:

High-precision measurements of the 'nucleon structure'

➜ measure form factors, structure functions, (generalized) parton distribution functions, ...

- at low Q^2 elastic and quasi-elastic scattering
  ➜ form factors, polarizabilities, ...
- at high Q^2 deep inelastic scattering
  ➜ parton distribution functions, GPDs, GDAs, ...

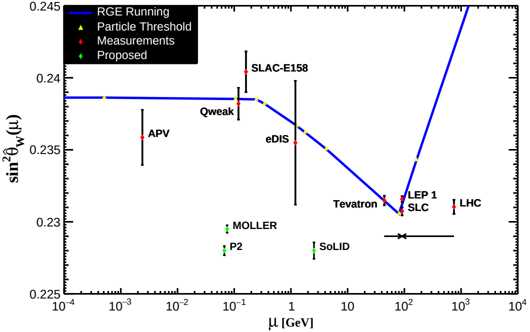
The interesting physics is encoded in FFs, PDFs, ...
test the dynamics of the strong interaction
QCD at high energies; effective theories, χpT at low energies

Lepton scattering: only via electromagnetic and weak interaction

➜ well-controlled and separable perturbative treatment

Also Electroweak physics: SoLid, EIC, HERA, LHeC, ...

P2@MESA: polarized ep scattering, measure asymmetry to determine the weak mixing angle



Measure FFs, PDFs, etc by comparing data with theoretical predictions:

$$\sigma_{\mathrm{exp}} = \sigma_{\mathrm{theory}}[F_n(x, Q^2, \ldots)]$$

High precision requires knowledge of higher-order corrections

$$\sigma_{\mathrm{theory}} = \sigma^{(0)}[F_n] + \alpha_{\mathrm{em}}\sigma^{(1)}[F_n, \ldots] + \ldots$$

- Emission of real photons
  experimentally often not distinguished from non-radiative processes:
  soft photons, collinear photons
  ➜ "radiative corrections"
- Virtual corrections: loop diagrams
  needed to cancel infrared divergences (Bloch-Nordsieck)
- Electroweak effects
  Z-, W-boson exchange ($O(G_F)$)
  and higher-order electroweak corrections ($O(\alpha G_F)$)

Classification of $O(\alpha)$ corrections

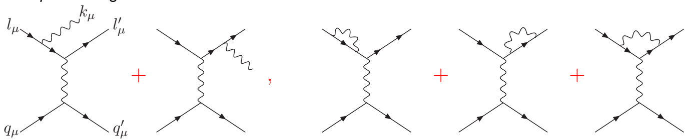
- Radiation from the lepton
  model independent (universal)
- Radiation from the hadronic initial/final state
  parton model: radiation from quarks
  to be considered as a part of the nucleon structure
- Interference of leptonic and hadronic radiation
  2γ exchange
  new structure
- vacuum polarization (γ and Z-boson self energy)
  universal
- purely weak corrections

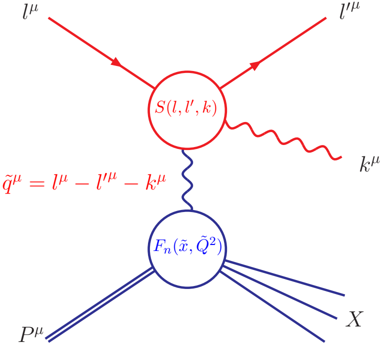
Note: for NC-scattering straightforward separation
Rule: respect gauge invariance
IR divergences: need to combine real and virtual radiation

Leptonic radiation

Feynman diagrams for leptonic radiation at $O(\alpha)$ (NC)

for *eq* scattering:





radiative leptonic tensor $S_{\mu\nu}(l, l', k)$ is

- gauge invariant
- infrared finite
- universal

(includes Born + loops: $\delta^{(4)}(k^\mu)$)

Observed cross section:

Convolution of true cross section $\otimes$ radiator function

$$\mathrm{d}\sigma^{\mathrm{obs}}(P,q) = \int \frac{d^3k}{2k^0} \sum_n R_n(l,l',k)\,\mathrm{d}\hat{\sigma}_n^{\mathrm{true}}(P,q-k)$$

Shifted kinematics

observed momentum transfer: $Q^2 = -q^2$, $q^\mu = l^\mu - l'^\mu$,
➜ true, shifted momentum transfer: $\tilde{Q}^2 = -(q-k)^2$
➜ correction factor: enhancement by $Q^2/\tilde{Q}^2$ ➜ radiative tail

➜ expect strong dependence on experimental prescriptions for measuring kinematic variables

➜ need full Monte-Carlo modelling

Can be extended to include higher-order effects: multi-photon emission, soft-photon exponentiation, e^+e^--pair creation

Determination of the true form factors F_n from the measured ones

involves unfolding, i.e. invert

$$F_n^{\rm obs}(Q^2) = \int \mathrm{d}\tilde{Q}^2 \, \hat{R}_n(Q^2; \tilde{Q}^2) F_n^{\rm true}(\tilde{Q}^2)$$

But, the problem is “ill-defined”,
i.e., no unique solution, large uncertainties, numerically unstable

A typical approach in practice: iterative solution
need a priori information, regularization

Difficult to treat radiative and detector effects separately (acceptance cuts, efficiencies, ...)

with partial fractioning, write: $R_n(l, l', k) = \frac{J}{k \cdot l} + \frac{F}{k \cdot l'} + \frac{C}{\tilde{Q}^2} + \ldots$

- initial state radiation, $k \cdot l$ small for $\sphericalangle(\mathrm{e_{in}}, \gamma) \to 0$
- final state radiation, $k \cdot l'$ small for $\sphericalangle(\mathrm{e_{out}}, \gamma) \to 0$
- Compton peak, $\tilde{Q}^2$ small for $p_T(\mathrm{e_{out}}) \simeq p_T(\gamma)$

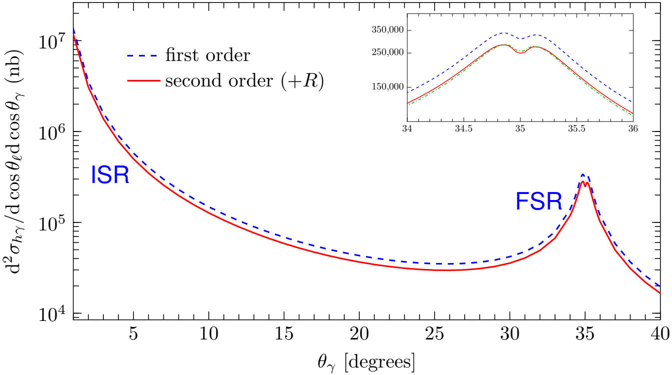
ISR, FSR: narrow peaks, width $\simeq \sqrt{m_l/E_l}$: collinear or mass singularities

upon angular integration: large logarithm $\propto \frac{\alpha}{\pi} \log \frac{Q^2}{m_e^2} \simeq 10\,\%$

Note: additional large logarithms from experimental cuts $\propto \log \frac{\Delta E}{E_{max}}$

A technical remark about numerical integration:
Do partial fractioning, choose denominator as integration variable

Leptonic radiation: collinear peaks



ep P2@MESA:

$E_e = 155$ MeV

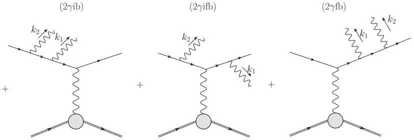
$\Delta = 10$ MeV

$\theta_l = 35°$

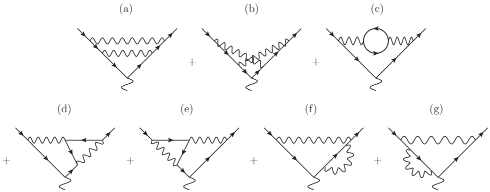
$E'_l > 45$ MeV

(important: keep full mass dependence!)

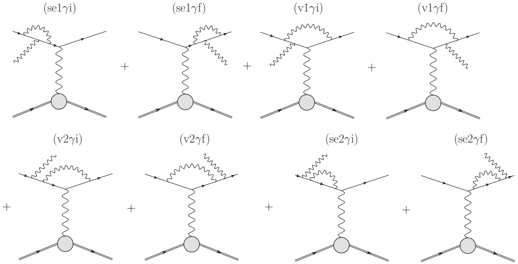
Second-order corrections



2-photon radiation



2-loop



1-loop corrected 1-photon radiation

Phase space slicing: soft radiation with $E_\gamma < \Delta$, hard radiation with $E_\gamma > \Delta$

First order:

$$\sigma^{(1)}_{\text{non-rad}}(\Delta) = \sigma^{(1)}_{1-\text{loop}} + \sigma^{(1)}_{1s\gamma}(\Delta)$$

$$\sigma^{(1)} = \sigma^{(1)}_{\text{non-rad}}(\Delta) + \sigma^{(1)}_{1h\gamma}(\Delta)$$

Second order:

$$\sigma^{(2)}_{\text{non-rad}}(\Delta) = \sigma^{(2)}_{2-\text{loop}} + \sigma^{(2)}_{1-\text{loop}+1s\gamma}(\Delta) + \sigma^{(2)}_{2s\gamma}(\Delta)$$

$$\sigma^{(2)}_{1h\gamma}(\Delta) = \sigma^{(2)}_{1-\text{loop}+1h\gamma}(\Delta) + \sigma^{(2)}_{1s\gamma+1h\gamma}(\Delta)$$

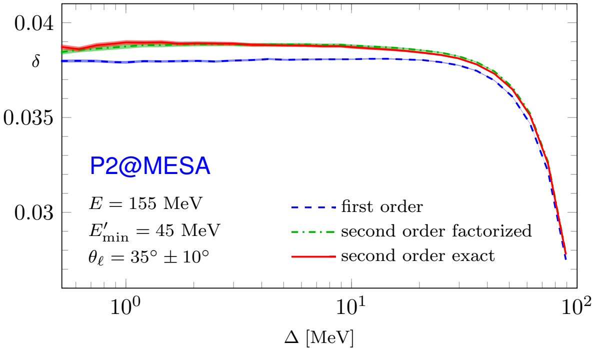
$$\sigma^{(2)} = \sigma^{(2)}_{\text{non-rad}}(\Delta) + \sigma^{(2)}_{1h\gamma}(\Delta) + \sigma^{(2)}_{2h\gamma}(\Delta)$$

Unphysical IR cutoff Δ cancels in the sum

Soft photon approximation, $E_\gamma \to 0$, used in soft-radiation parts, i.e. for $E_\gamma < \Delta$

➜ analytic cancellation of IR divergences

Check:
numerical cancellation of Δ-dependent terms in soft- and hard radiation



IR divergences

Use a finite photon mass λ as IR regulator $\quad (v = \sqrt{1 + 4m_\ell^2/Q_\ell^2}$ and $L = \ln(Q_\ell^2/m_\ell^2)$)

$$\delta^{(1)}_{\text{1-loop}} = \delta_{\text{1-loop,fin}} + \delta_{\text{IR}}, \qquad \delta_{\text{IR}} = \frac{\alpha}{\pi} \ln\left(\frac{\lambda^2}{m_\ell^2}\right) \left[\frac{v^2+1}{2v} \ln\left(\frac{v+1}{v-1}\right) - 1\right]$$

$$\delta^{(1)} = \delta_{\text{1-loop,fin}} + \delta_{1s\gamma,\text{fin}} + \delta_{1h\gamma}$$

$$\delta^{(2)}_{\text{2-loop}} = \frac{1}{2}\left(\delta^{(1)}_{\text{1-loop}}\right)^2 + \left(\frac{\alpha}{4\pi}\right)^2 \left[-\frac{8}{9}L^3 + \frac{76}{9}L^2 + \left(-\frac{979}{27} - \frac{44\pi^2}{9} + 48\zeta(3)\right)L + \frac{4252}{27} + \frac{47\pi^2}{3} - 16\pi^2\ln(2) - 72\zeta(3) - \frac{64\pi^4}{45} + \mathcal{O}\left(\frac{m_\ell^2}{Q_\ell^2}\right)\right]$$

Hill 2017

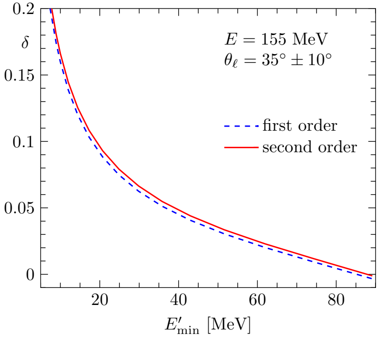
Soft-photon corrections exponentiate: Yennie, Frautschi, Suura

$$\delta^{(2)}_{2s\gamma} = \frac{1}{2!}\left(\delta^{(1)}_{1s\gamma}\right)^2 \quad \rightarrow \quad \delta^{(\infty)}_{s\gamma} = \exp\left(\delta^{(1)}_{1s\gamma}\right)$$

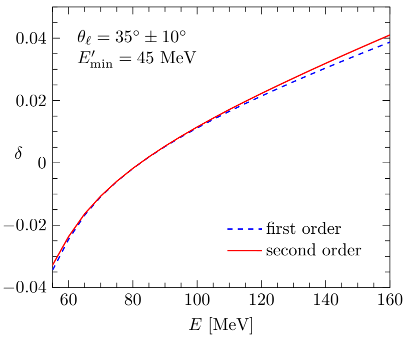
provided the phase space is factorized, i.e. $E_\gamma < \Delta$ for all photons independently
extra term if $E_\gamma + E'_\gamma < \Delta$: $-\frac{\alpha^2}{3}(L-1)^2$

Examples: P2@MESA

Dependence on cutoff for E'_e



Dependence on beam energy



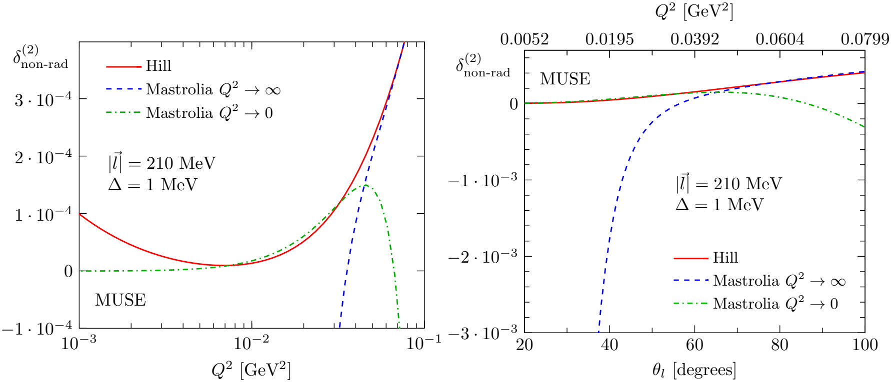
Strong dependence on kinematic conditions, i.e.
Large corrections due to shift of kinematics

➜ Small second-order corrections

YFS

for MUSE: μp scattering

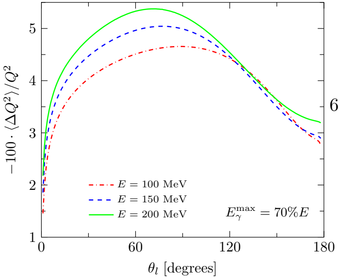
2-loop corrections in the low-Q^2 or large-Q^2 limits
(power expansion in $\mu = Q^2/m_\ell^2$) Mastrolia 2003/4, Hill 2017



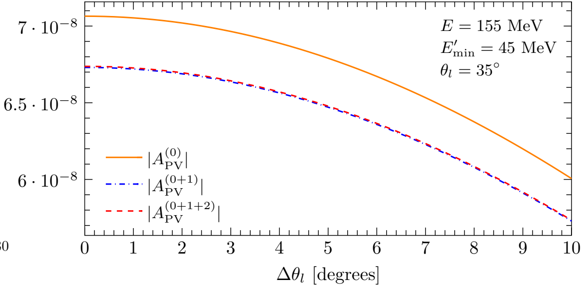
➜ Need calculation including the full mass dependence at 2^{nd} order
for MUonE: Carloni Calame et al, Banerjee et al

for P2@MESA: polarized electrons, need also Z-exchange diagrams

$$A_{\rm PV} = \frac{d\sigma^+_{\rm ep} - d\sigma^-_{\rm ep}}{d\sigma^+_{\rm ep} + d\sigma^-_{\rm ep}} = \frac{-G_F Q^2}{4\pi\alpha_{\rm em}\sqrt{2}} \left[Q_W(p) - F(E, Q^2)\right]$$



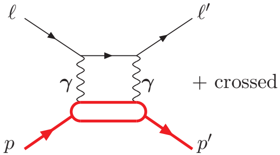
kinematic shift



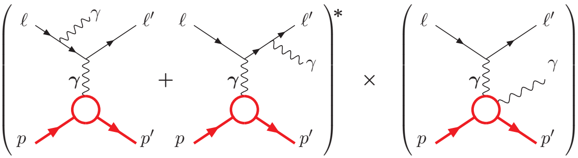
corrected asymmetry

2-photon exchange

carries both
Q^2- and E-dependence

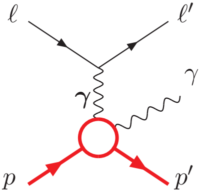


IR divergences cancel against real radiation:
Interference of leptonic and hadronic radiation



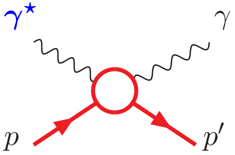
Mass singularites (large logs, $\ln(Q^2/m_e^2)$) cancel
see talks by Ahmed, Blunden, Afanasev, ...

Radiation from the proton



(+ loops)

➜



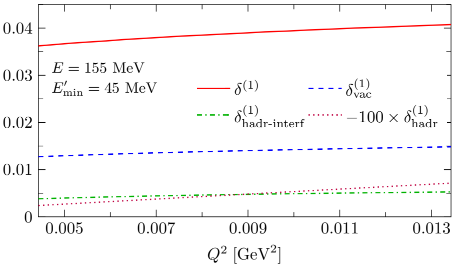
virtual Compton scattering

No large log expected

Model dependence

Partly absorbed into form factors

(cf. QCD: factorization and renormalization of PDFs)



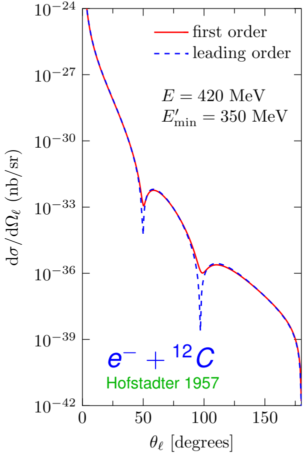
Monte Carlo event simulation and integrator POLARES for elastic electron scattering including polarization

Event generation for
ep, $ep\gamma$, $ep\gamma\gamma$

code written by R. Bucoveanu

User friendly
many input options
easy to extend

e.g. for electron-carbon scattering



Monte-Carlo approach in HERACLES and DJANGOH:
QCD-based event generation, valid at large Q^2: parton model

- Complete QED and electroweak corrections at $O(\alpha)$
- NC and CC scattering, polarized lepton, polarized nucleon
- Parton Distribution Functions from LHAPDF, models for low Q^2 structure functions
- Elastic tail
- Polarized nuclei
- Heavy nuclei: models for nuclear shadowing, nuclear parton distribution functions
- Interface to LEPTO, JETSET
- Jets, parton showers, hadronic final state
- SOPHIA for low-mass hadronic final states

Used for HERA, EIC

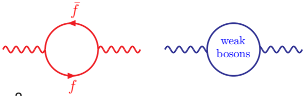
Concluding remarks

- NNLO corrections in POLARES, code to be published
- How to correct data?
  Radiative corrections $\otimes$ experimental conditions
  Unfolding
- To Do: Include full mass dependence in $\delta^{(2)}_{\text{2-loop}}$
- To Do: Improve model for 2-photon exchange
  and:
- To Do: Radiation from the proton
  Definition of form factors at NLO

Appendix

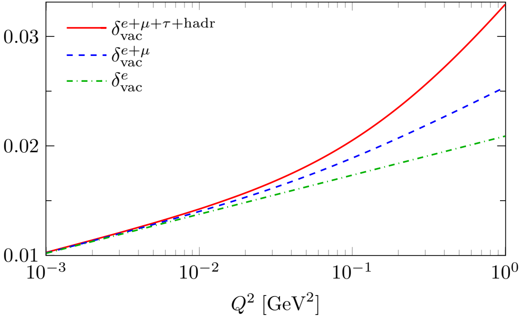
Vacuum polarization

Self energy diagrams of the exchanged boson (γ and Z)



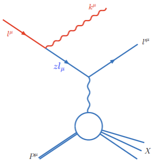
$\propto \log \frac{Q^2}{m_f^2} \to$ several % small

Photon self energy = vacuum polarization, absorbed in the running fine structure constant: $\alpha \to \alpha(Q^2) = \frac{\alpha}{1 - \Pi_\gamma(Q^2)}$



Collinear approximation (peaking approximation)

e.g., for initial-state radiation: assume $k^\mu = (1-z)l^\mu$
➜ Radiator function



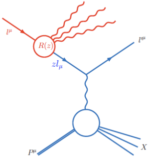
$$R_{\rm ISR} = \frac{\alpha}{2\pi}\,\frac{1+z^2}{1-z}\,\log\frac{Q^2}{m_e^2}$$

$(+\delta(1-z)$ from loops $\to$ $+$-distribution $1/(1-z)_+)$

$$\mathrm{d}\sigma_{\rm ISR} = \int \frac{\mathrm{d}z}{z} R_{\rm ISR}(z)\,\mathrm{d}\sigma_{\rm Born}(l^\mu \to zl^\mu)$$

(similar for final-state radiation)

Can be extended to include multi-photon emission:



$$R^{(2)}_{\rm ISR}(z) = \int_z^1 \frac{\mathrm{d}z'}{z'} R^{(1)}_{\rm ISR}(z')R^{(1)}_{\rm ISR}(z/z') + \ldots$$

Solution of evolution equations like DGLAP
Known at $O(\alpha^2)$ (complete) and partially at $O(\alpha^3)$

Corrections due to soft photons are universal

sum of real and virtual contributions: δ_{IR} (finite and gauge invariant)

$$1 + \delta_{\mathrm{tot}} = 1 + \bar{\delta}_{\mathrm{IR}} + \delta_{\mathrm{fin}} \rightarrow \exp(\bar{\delta}_{\mathrm{IR}})(1 + \delta_{\mathrm{fin}})$$

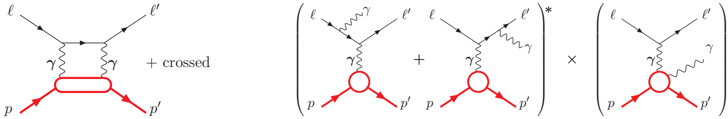
$\bar{\delta}_{\mathrm{IR}}$ contains $\log(\Delta)$ and $L = \log(Q^2/m_\ell^2)$:

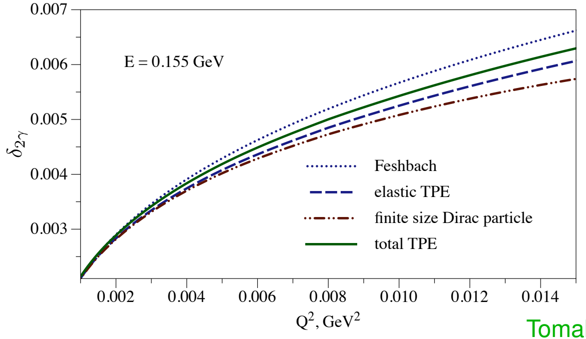
$$1 + \frac{\alpha}{2\pi}(L-1)\ln\frac{\Delta}{E_e} + \ldots \rightarrow \left(\frac{\Delta}{E_e}\right)^{\frac{\alpha}{2\pi}(L-1)}(1+\ldots)$$

(for DIS,in the $\gamma^* p$ cms: $\Delta = E_\gamma^{\mathrm{max}} = \frac{1}{2}\sqrt{y(1-x)S}$,
i.e. important at low y and large x)

Soft photon exponentiation — Yennie, Frautschi, Suura, 1961

2-photon exchange

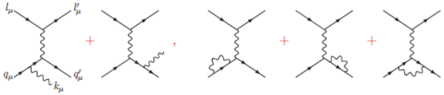




Tomalak

Hadronic radiation

at large Q^2: DIS, parton model
emission of photons
like emission of gluons



infrared divergences (soft photons / gluons) cancel with loops, collinear emission gives rise to corrections $\frac{\alpha}{2\pi} \log m_q^2$, but quark masses are ill-defined

➜ factorization: absorb collinear divergences into parton distribution functions

$$d\sigma = \sum_f d\hat{\sigma}_f (1 + \delta_f(Q^2; m_q^2)) q_f(x)$$

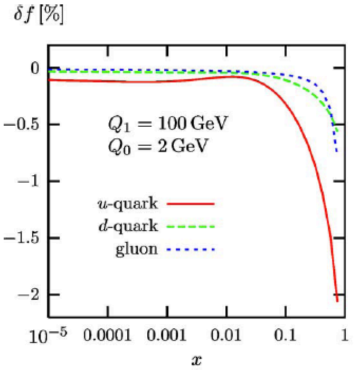
$$d\sigma = \sum_f d\hat{\sigma}_f (1 + \delta_f(Q^2; m_q^2)) q_f(x) = \sum_f d\hat{\sigma}_f \hat{q}_f(x, Q^2)$$

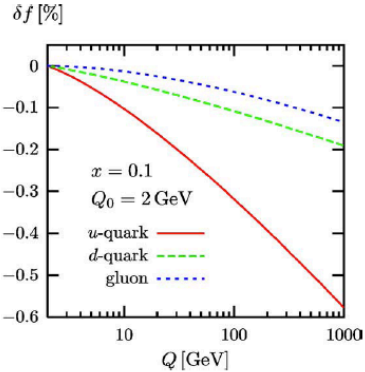
renormalized parton distribution functions

$$\hat{q}_f(x, Q^2) = (1 + \delta_f(Q^2; m_q^2)) q_f(x)$$

➜ modified scaling violations

well-known in QCD, $\overline{\text{MS}}$ factorization





implemented in MRST2004, NNPDF HS'95; Roth, Weinzierl, PLB590

relevant for precision predictions, e.g. *W* production at the LHC

different charges of *u*- and *d*-quarks ➜ isospin-violating effect

In practice: Do not include corrections due to radiation from quarks