Radiative corrections in MC Event Generators
From QCD to QED

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Introduction and disclaimer

This talk is inspired by work in the LHC event generator community
▶ Much of it will focus on QCD radiative corrections
▶ It will summarize a unified approach to QCD+QED evolution . . .
▶ . . . and a technique to match this resummation to fixed-order NLO
▶ There will be some results on QED radiative corrections in ep also
   These are courtesy of Stefan Prestel, who is the expert on this

The aim is to show how existing methods and codes can be useful
▶ Computational techniques developed for the LHC are generic
   ▶ QCD+QED resummation in the form of parton showers
   ▶ Matching to fixed-order calculations at NLO
   ▶ Incorporation of NLO electroweak corrections
   ▶ No difference between neutral and charged current
▶ Tools developed for the LHC are highly automated
   ▶ Parton showers are validated against $e^+e^-$, DIS and pp data
   ▶ Matching to fixed order NLO is now a push-button operation
   ▶ One-loop corrections are computed fully automatically
Conceptual design of general-purpose event generators

Modular structure of physics simulation

- **Hard interaction**
  LO, NLO QCD/EW\(^1\), NNLO QCD\(^2\)
  Generic matrix-element generators

- **Radiative corrections**
  Parton Showers, YFS resummation

- **Hadronization & Decays**
  Cluster / String model
  Phase space or EFTs + YFS

Comparison to fixed order (FO)

- **Hard interaction**
  Lower precision than FO

- **Radiative corrections**
  Resummed & matched to FO

- **Hadronization & Decays**
  Not accessible at FO

\(^1\) via interfaces to 1-loop generators
\(^2\) for selected processes

[Buckley at al.] arXiv:1101.2599
QCD radiative corrections in DIS
Precision QCD calculations

- **Inclusive DIS at NLO QCD**
  - [Altarelli,Ellis,Martinelli] NPB143(1978)521
  - [Humpert,van Neerven] NPB184(1981)225

- **... at N^2LO QCD**

- **Di-jet production at NLO QCD**

- **... at N^2LO QCD**
  - [Currie,Gehrmann,Niehues] arXiv:1606.03991

- **DIS at N^3LO QCD, fully exclusive**
Peculiarities of DIS

- **Leading order** $e^\pm p$ - scattering in collinear factorization
  - Only one scale! Kinematical variables:
    \[ Q^2 = q^2 = (k' - k)^2 \text{ and } x = \frac{Q^2}{2q \cdot p} \]
  - Hadronic cm energy
    \[ W = Q \sqrt{\frac{1 - x}{x}} \]

- **Dynamics at higher orders**
  - Multiple scales, e.g. $E_{T,B}^2$
  - $e^\pm q \rightarrow e^\pm q$ if $E_{T,B}^2 \lesssim Q^2$
  - $\gamma^* g \rightarrow jets$ if $Q^2 \lesssim E_{T,B}^2$

- What makes DIS different from $e^+ e^- \rightarrow jj$ and $pp \rightarrow e^+ e^-$ is that the virtuality of the exchanged photon tends to be close to zero.

- This necessitates a more sophisticated approach to radiative corrections which is not specific to QCD but applies to all massless gauge theories.
Combining fixed-order calculations and resummation

Exact

Approximate

inner structure of hard objects

correlations between objects
Combining fixed-order calculations and resummation

- Dynamics of the QCD multi-jet final state must be reflected accurately when matching fixed-order to the resummation

  [Carli,Gehrmann,SH] arXiv:0912.3715

  - $e^\pm q \rightarrow e^\pm q$ if $E_{T,B}^2 \lesssim Q^2$
  - $\gamma^* g \rightarrow$ jets if $Q^2 \lesssim E_{T,B}^2$
  - $qg \rightarrow$ jets if $Q^2 \ll E_{T,B}^2$

- Similar to taking direct and fragmentation component into account in hard photon production at hadron colliders

  [Schumann,Siegert,SH] arXiv:0912.3501
Comparison to HERA data

Variation of highest multiplicity in fixed-order calculation, \( N_{\text{max}} \)

[Carli,Gehrmann,SH] arXiv:0912.3715
Matching fixed-order NLO calculations to resummation

Exact

Approximate

correlations between hard objects

inner structure of hard objects

BVI

R − S

BVI

R − S

BVI

R − S

BVI

R − S

BVI

R − S
Matching fixed-order NLO calculations to parton-showers

Two possible ways to match NLO calculations and parton showers

**MC@NLO**
- Use parton-shower splitting kernel as infrared subtraction term
- Multiply LO event weight by Born-local K-factor including integrated subtraction term and virtual corrections
- Add hard remainder function consisting of subtracted real-emission correction

**POWHEG**
- Use matrix-element corrections to replace parton-shower splitting kernel by full real-emission matrix element in first shower branching
- Multiply LO event weight by Born-local NLO K-factor (integrated over real corrections that can be mapped to Born according to parton-shower kinematics)

Both cases: Beware of sub-leading color terms and spin correlations!
Matching vs Merging


[Carli, Gehrmann, SH] arXiv:0912.3715
Matching at NNLO accuracy

- Highest QCD accuracy in the simulation of inclusive DIS
- Projection-to-Born method for fully differential fixed order predictions
  [Bern,Dixon,Kosower] hep-ph/9708239
- UN^2LOPS matching to parton shower for particle-level simulations
- Scale choice appropriate for simultaneous description of inclusive DIS
  and inclusive jet / di-jet / tri-jet production \rightarrow \mu_{R/F}^2 = (Q^2 + (H_T/2)^2)/2

\[ \mu_{R/F}^2 = \frac{Q^2 + \left(\frac{H_T}{2}\right)^2}{2} \]

- Good agreement with H1 measurements in both high-\(Q^2\)
NNLO particle-level simulation vs. H1 high-$Q^2$ data

NNLO particle-level simulation vs. H1 low-$Q^2$ data

Inclusive jet selection

- H1 Data
- H1SysErr
- UNLOPS
- $\frac{1}{2}p_{T,F}$...2$p_{T,F}$
- ME+PS uncertainty
- NLO incl. jet
- $\frac{1}{2}p_{T,F}$...2$p_{T,F}$

Di-jet selection

- H1 Data
- H1SysErr
- UNLOPS
- $\frac{1}{2}p_{T,F}$...2$p_{T,F}$
- ME+PS uncertainty
- NLO incl. jet
- $\frac{1}{2}p_{T,F}$...2$p_{T,F}$

Tri-jet selection

- H1 Data
- H1SysErr
- UNLOPS
- $\frac{1}{2}p_{T,F}$...2$p_{T,F}$
- ME+PS uncertainty
- NLO incl. jet
- $\frac{1}{2}p_{T,F}$...2$p_{T,F}$

$5.5 < Q^2 < 8$ GeV$^2$ ($\times 1$)

$8 < Q^2 < 11$ GeV$^2$ ($\times 10^{-1}$)

$11 < Q^2 < 16$ GeV$^2$ ($\times 10^{-2}$)

$16 < Q^2 < 22$ GeV$^2$ ($\times 10^{-3}$)

$22 < Q^2 < 30$ GeV$^2$ ($\times 10^{-4}$)

$30 < Q^2 < 42$ GeV$^2$ ($\times 10^{-5}$)

$42 < Q^2 < 60$ GeV$^2$ ($\times 10^{-6}$)

$60 < Q^2 < 80$ GeV$^2$ ($\times 10^{-7}$)

$\langle p_{T,2j} \rangle [\text{GeV}]$

$\langle p_{T,3j} \rangle [\text{GeV}]$

$\langle p_{T,2j} \rangle [\text{GeV}]$

$\langle p_{T,3j} \rangle [\text{GeV}]$

Availability of simulations

▶ Herwig
  ▶ External 1-loop providers & built-in loop library
  ▶ Merging in modified unitarized approach
  ▶ QED & mixed higher-order corrections work in progress

▶ Pythia
  ▶ QED radiative corrections in preparation [Giele, Prestel]
  ▶ NLO QCD Matching via interface to POWHEG / MC@NLO

▶ Sherpa
  ▶ External 1-loop providers & built-in loop library
QCD$+$QED radiative corrections in pp collisions
The standard perturbative QCD approach

**“Direct” component**
Fixed-order calculation with isolated $\gamma$

- $\gamma + \text{jet}$ hep-ph/0602133
- $\gamma\gamma$ hep-ph/9911340
- $\gamma\gamma + \text{jet}$ hep-ph/0303012
- $gg \rightarrow \gamma\gamma g$ hep-ph/9905283

**“Fragmentation” component**
Factorization of $q\gamma$ collinear singularity

- Gives rise to $\gamma$ fragmentation function PLB79(1978)83
- Contribution depends on photon isolation criterion

Based on assumption that dominant higher-order corrections of QCD type
Direct and fragmentation component both needed for a complete prediction
Fragmentation function depends on nonperturbative input
The democratic approach used in MC event generators

Procedure

▶ Treat partons and photons fully democratically
▶ Combine fixed-order results of varying $\gamma$/parton multiplicity . . .
▶ . . . with interleaved QCD+QED parton shower evolution

Advantages

▶ Parton-to-hadron transition via fragmentation models
▶ No new parameters related to $\gamma$ fragmentation
▶ Unified treatment of QCD & QED radiative corrections
▶ Non-prompt $\gamma$-production comes for free

Corresponding parton shower modifications available in all major LHC generators (Herwig, Pythia, Sherpa)
Performance of QED parton showers in $e^+e^- \rightarrow$ hadrons

[Schumann,Siegert,SH] arXiv:0912.3501
Example result: Prompt photon production at the Tevatron

Transverse energy of isolated prompt photon

\[ \frac{d\sigma}{dE_{\gamma}} \] [pb/GeV]

\( E_{\gamma} \) [GeV]

CDF data

Do data

2 \( \rightarrow \) 2 total

\( jj \rightarrow jj \)

\( jj \rightarrow \gamma j \)

\( jj \rightarrow \gamma \gamma \)

MC/data

\[ \frac{d\sigma}{dp_{\gamma\perp}} \] [pb/GeV]

\( p_{\perp} \) [GeV]

\( p_{\gamma\perp} \) [GeV]

MC/data
Matching to fixed-order NLO calculations

[\text{d'Errico, Richardson}] \hspace{1cm} \text{arXiv:1106.3939}

(CDF data)

RESBOS
DIPHOX
Hw++ POWHEG
Hw++ LO

\(d\sigma/dM_{\gamma\gamma} \; [\text{pb/GeV}]\)

10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}

\(M_{\gamma\gamma} \; [\text{GeV}]\)

\(d\sigma/d\Delta\phi_{\gamma\gamma} \; [\text{pb/rad/GeV}]\)

10^{-1}
10^{-2}
10^{-3}

\(\Delta\phi_{\gamma\gamma} \; [\text{rad}]\)

(a) 50 GeV < \(M_{\gamma\gamma}\) < 80 GeV

(CDF - data)
QED radiative corrections DIS
Plots & figures courtesy of Stefan Prestel
Soft radiation pattern emerges from semi-classical soft current

\[ \mathcal{M}_{X+\gamma} \propto \mathcal{M}_{X,\mu} \sum_i \eta_i Q_i \frac{2p_i^\mu}{2p_{iq}} \]

\[ \rightarrow |\mathcal{M}_{X+\gamma}|^2 \propto |\mathcal{M}_X|^2 \sum_{i,j} \eta_i Q_i \eta_j Q_j \frac{4(p_i p_j)}{2(p_{iq})2(p_{jq})} \]

Individual contributions can be negative, but sum is positive

Resummed calculation implemented as a parton shower with matrix-element corrections \(\rightarrow\) recovers all interferences
Example: *ep* collisions at $E_e = E_p = 10$ GeV, $Q > 2$ GeV

Energy of highest-$E$ bare electron; $\langle E_{\nu} \rangle \approx 3$ GeV, $Q^2 > 2$ GeV^2

Photon energy

Photon multiplicity

Radiation pattern harder w/o interferences.
Interference effects

- Even in neutral current reactions at low $Q^2$ factorization does not work
- Naively: At small $Q^2$ the reconfiguration of the EM radiation field occurs at longer distances $\rightarrow$ interferences more important

Plots (ep collisions at $E_e = E_p = 10\ Ge\ V, Q > 2\ Ge\ V$)

Full ME
Factorized lepton/hadron currents

$\Delta R_{e\gamma}$

Separation of highest-energy bare electron and photons

$\frac{d\sigma}{d\Delta R_{e\gamma}} \ [pb]$

$\kappa^2$

$\frac{p_{\perp}}{\gamma}$ w.r.t. highest-energy $\ell$ and hardest jet $\kappa^2 = \frac{(p_e + p_j + p_\gamma)^2}{(p_e + p_j + p_\gamma)^2}$

Cross-talk between electron and jet

- Angular separation between lepton and photon
- Lepton and $\gamma$ closer to each other when including interferences
- Relative $\gamma$ transverse momentum w.r.t. axis defined by hardest electron and hardest jet
- At low $Q^2$ more radiation between $l$ and $j$, interferences of $\mathcal{O}(10\%)$
Summary

▶ DIS simulations available in all major event generation frameworks
▶ QCD NLO matching & merging of fixed-order and resummation standard
▶ QED matching & merging being developed in Pythia (S. Prestel)
  ▶ Offers unified treatment of radiative corrections in QCD+QED
  ▶ Includes interferences between radiation off leptons & quarks
  ▶ Works for both charged an neutral current interactions