Small-x Improved TMD Factorization for Jets

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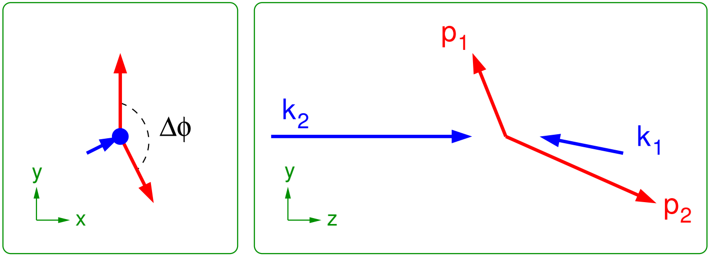
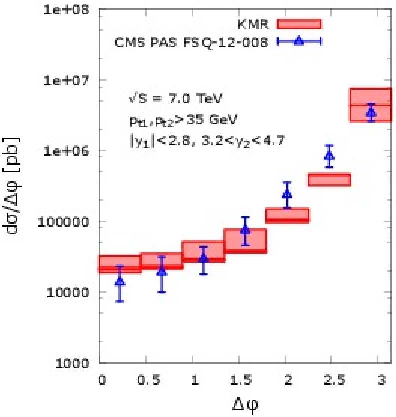
Forward-central dijet decorrelations pp $\longrightarrow$ 2j

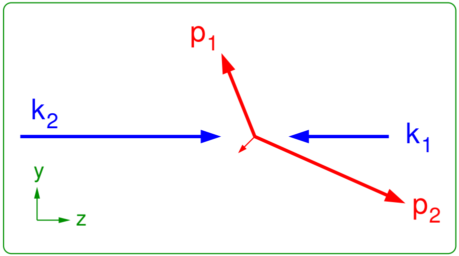
AvH, Kutak, Kotko, Sapeta 2014

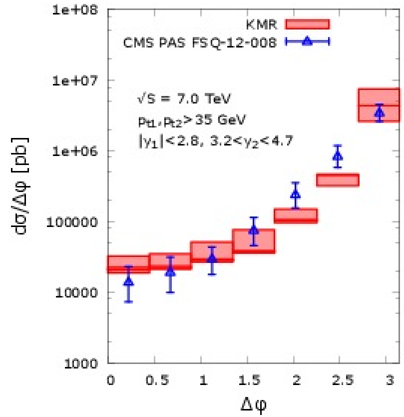




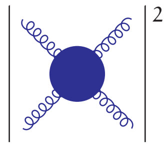
Forward-central dijet decorrelations pp $\to$ 2j

Collinear factorization

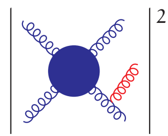




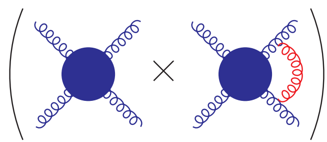
LO: $\hat{\sigma}^{\mathrm{LO}}_{a,b\to n} = \int d\Phi_n \left|\mathcal{M}^{(0)}_{a,b\to n}\right|^2 \mathcal{O}^{\mathrm{LO}}_n$



NLO: $\hat{\sigma}^{\mathrm{NLO}}_{a,b\to n} = \int d\Phi_{n+1} \left|\mathcal{M}^{(0)}_{a,b\to n+1}\right|^2 \mathcal{O}^{\mathrm{NLO}}_{n+1}$

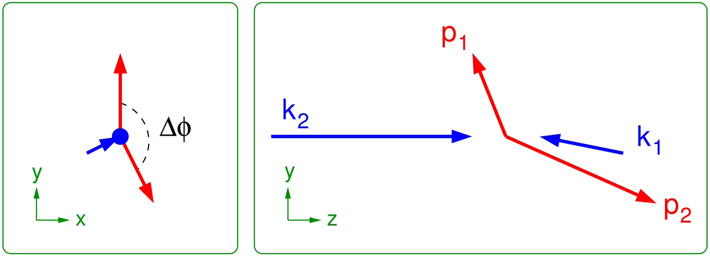


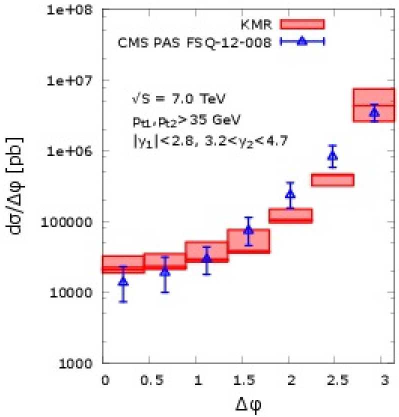
$+ \int d\Phi_n \, 2\mathrm{Re}\left(\mathcal{M}^{(0)}_{a,b\to n} \, \mathcal{M}^{(1)}_{a,b\to n}\right) \mathcal{O}^{\mathrm{LO}}_n$

2Re 

Forward-central dijet decorrelations $pp \to 2j$

AvH, Kutak, Kotko, Sapeta 2014





Hybrid factorization:

$$d\sigma_{pp\to X} = \int dk_T^2 \int dx_A \int dx_B \sum_b \mathcal{F}_{g^*}(x_A, k_T, \mu)\, f_b(x_B, \mu)\, d\hat{\sigma}_{g^*b\to X}(x_A, x_B, k_T, \mu)$$

$$k_1^\mu = x_A P_A^\mu + k_T^\mu \qquad P_A^2 = 0 \qquad k_1^2 = k_T^2$$
$$k_2^\mu = x_B P_B^\mu \qquad P_B^2 = 0 \qquad k_2^2 = 0$$

$$x_B \gg x_A \qquad \left|\vec{p}_1 + \vec{p}_2\right| = \left|\vec{k}_T\right|$$

Trijets in k_T-factorization

Van Haevermaet, AvH, Kotko, Kutak, Van Mechelen 2020

$$pp \to 3j$$

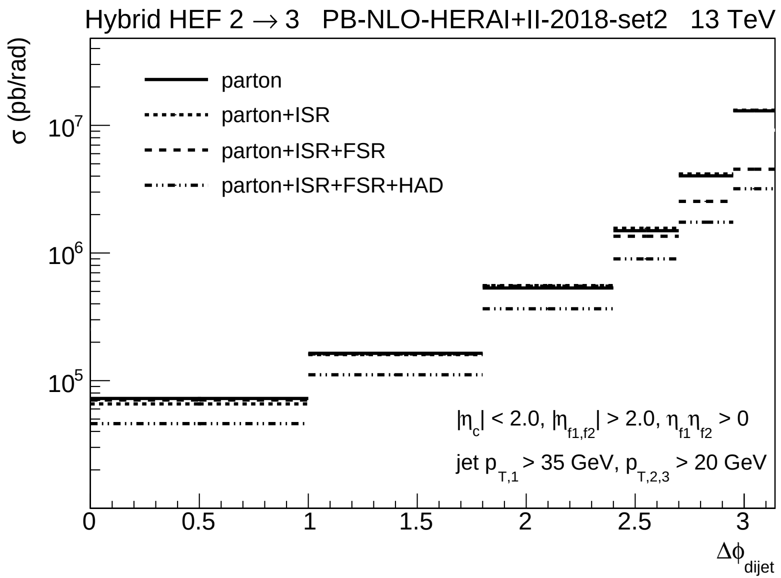
Require two jets with pseudorapidity

$$2 < |\eta| < 4.7$$

and a third one with

$$|\eta| < 2$$

$\Delta\phi_{\text{dijet}}$ is the angle between the sum of the 2 hardest jets, and the 3th jet.



Calculations performed with parton-level event generator KaTie (AvH 2016) and parton shower Monte Carlo Cascade (Jung et al. 2010), using TMDs from the parton-branching method Hautmann et al, 2018, Bermudez Martinez et al. 2019.

Hardly any difference between parton and parton+ISR

High Energy Factorization

a.k.a. k_T-factorization

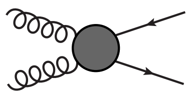
Catani, Ciafaloni, Hautmann 1991 | Collins, Ellis 1991

$$\sigma_{h_1,h_2\to QQ} = \int d^2k_{1\perp}\,\frac{dx_1}{x_1}\,\mathcal{F}(x_1,k_{1\perp})\,d^2k_{2\perp}\,\frac{dx_2}{x_2}\,\mathcal{F}(x_2,k_{1\perp})\,\hat{\sigma}_{gg}\left(\frac{m^2}{x_1x_2s},\frac{k_{1\perp}}{m},\frac{k_{2\perp}}{m}\right)$$

- reduces to collinear factorization for $s \gg m^2 \gg k_\perp^2$, but holds al so for $s \gg m^2 \sim k_\perp^2$
- typically associated with small-x physics
- $k_\perp$-dependent $\mathcal{F}$ imagined to satisfy BFKL-eqn, CCFM-eqn, ...
- allows for higher-order kinematical effects at leading order
- requires matrix elements with *off-shell* initial-state partons with $k_i^2 = k_{i\perp}^2 < 0$

$k_1 = x_1p_1 + k_{1\perp}$

$k_2 = x_2p_2 + k_{2\perp}$



- Can this be generalized to "arbitrary" processes, with higher multiplicities in the final state?
- With well-defined gauge-invariant matrix elements?

- parton level event generator, like ALPGEN, HELAC, MADGRAPH, etc.
- arbitrary processes within the standard model (including effective Higgs-gluon coupling) with several final-state particles.
- 0, 1, or 2 off-shell intial states.
- produces (partially un)weighted event files, for example in the LHEF format.
- requires LHAPDF. TMD PDFs can be provided as files containing rectangular grids, or with TMDlib.
- a calculation is steered by a single input file.
- employs an optimization stage in which the pre-samplers for all channels are optimized.
- during the generation stage several event files can be created in parallel.
- event files can be processed further by parton-shower program like CASCADE.
- (evaluation of) matrix elements now separately available, including C++ interface.

k_T-dependent factorization with KaTie

Hadron-scattering process Y with partonic processes y contributing to multi-jet final state

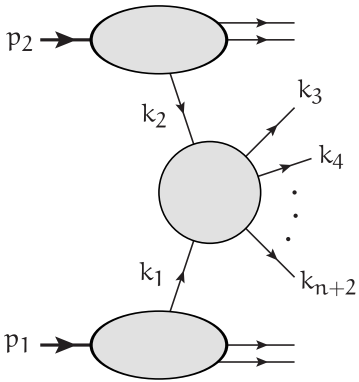
$$d\sigma_Y(p_1,p_2;k_3,\dots,k_{2+n}) = \sum_{y\in Y}\int d^4k_1\,\mathcal{P}_{y_1}(k_1)\int d^4k_2\,\mathcal{P}_{y_2}(k_2)\,d\hat{\sigma}_y(k_1,k_2;k_3,\dots,k_{2+n})$$

Collinear factorization:

$$\mathcal{P}_{y_i}(k_i) = \int\frac{dx_i}{x_i}\,f_{y_i}(x_i,\mu)\,\delta^4(k_i - x_ip_i)$$

k_T-dependent factorization factorization:

$$\mathcal{P}_{y_i}(k_i) = \int\frac{d^2\mathbf{k}_{iT}}{\pi}\int\frac{dx_i}{x_i}\,\mathcal{F}_{y_i}(x_i,|\mathbf{k}_{iT}|,\mu)\,\delta^4(k_i - x_ip_i - k_{iT})$$



Differential partonic cross section:

$$d\hat{\sigma}_y(k_1,k_2;k_3,\dots,k_{2+n}) = d\Phi_Y(k_1,k_2;k_3,\dots,k_{2+n})\,\Theta_Y(k_3,\dots,k_{2+n}) \times \mathsf{flux}(k_1,k_2)\times\mathcal{S}_y\,|\mathcal{M}_y(k_1,\dots,k_{2+n})|^2$$

KaTie creates tree-level event files corresponding to $d\sigma_Y$,
requires LHAPDF, TMDlib or grid files to evaluate f_y and/or $\mathcal{F}_y$.

k_T-dependent factorization with KaTie

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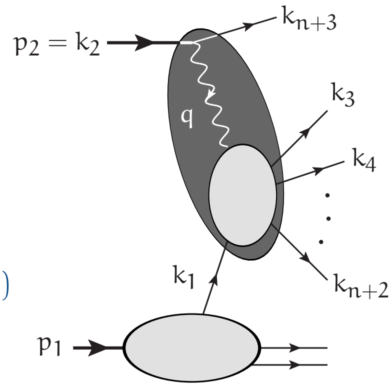
Collinear factorization:

$$\mathcal{P}_{y_i}(k_i) = \int \frac{dx_i}{x_i}\,f_{y_i}(x_i,\mu)\,\delta^4(k_i - x_i p_i)$$

k_T-dependent factorization factorization:

$$\mathcal{P}_{y_i}(k_i) = \int \frac{d^2\mathbf{k}_{iT}}{\pi}\int\frac{dx_i}{x_i}\,\mathcal{F}_{y_i}(x_i,|\mathbf{k}_{iT}|,\mu)\,\delta^4(k_i - x_i p_i - k_{iT})$$



Differential partonic cross section:

$$d\hat{\sigma}_y(k_1,k_2;k_3,\ldots,k_{3+n}) = d\Phi_Y(k_1,k_2;k_3,\ldots,k_{3+n})\,\Theta_Y(k_3,\ldots,k_{3+n}) \times \mathrm{flux}(k_1,k_2)\times\mathcal{S}_y\,|\mathcal{M}_y(k_1,\ldots,k_{3+n})|^2$$

KaTie creates tree-level event files corresponding to $d\sigma_Y$,
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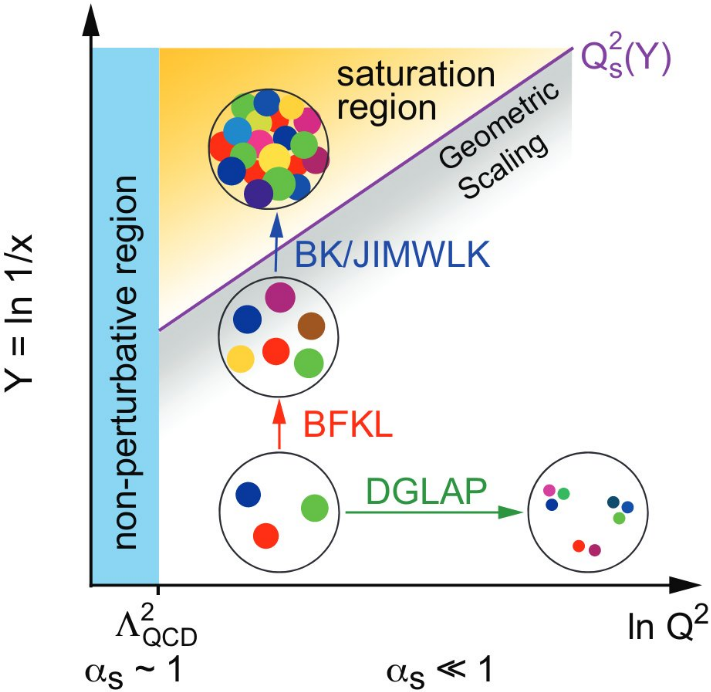
QCD evolution, dilute vs. dense



A dilute system carries a few high-x partons contributing to the hard scattering.

A dense system carries many low-x partons.

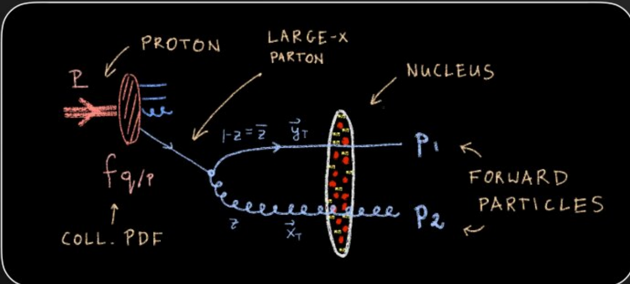
At high density, gluons are imagined to undergo recombination, and to saturate.

This is modeled with non-linear evolution equations, involving explicit non-vanishing k_T.

Saturation implies the turnover of the gluon density, stopping it from growing indefinitely for small x

P. Kotko, EPSHEP2019

pA (dilute-dense) collisions within CGC



$$\frac{d\sigma_{qA\to 2j}}{d^3p_1 d^3p_2} \sim \int \frac{d^2x}{(2\pi)^2}\frac{d^2x'}{(2\pi)^2}\frac{d^2y}{(2\pi)^2}\frac{d^2y'}{(2\pi)^2}\, e^{-i\vec{p}_{T1}\cdot(\vec{x}_T-\vec{x}'_T)}e^{-i\vec{p}_{T2}\cdot(\vec{y}_T-\vec{y}'_T)}$$

$$\times \psi_z^*\left(\vec{x}'_T-\vec{y}'_T\right)\psi_z\left(\vec{x}_T-\vec{y}_T\right)$$

QUARK WAVE FUNCTION

$$\times\Big\{ S_x^{(6)}\left(\vec{y}_T,\vec{x}_T,\vec{y}'_T,\vec{x}'_T\right) - S_x^{(4)}\left(\vec{y}_T,\vec{x}_T,\bar{z}\vec{y}'_T+z\vec{x}'_T\right)$$
$$-S_x^{(4)}\left(\bar{z}\vec{y}_T+z\vec{x}_T,\vec{y}'_T,\vec{x}'_T\right) - S_x^{(2)}\left(\bar{z}\vec{y}_T+z\vec{x}_T,\bar{z}\vec{y}'_T+z\vec{x}'_T\right)\Big\}$$

CORRELATORS OF WILSON LINES

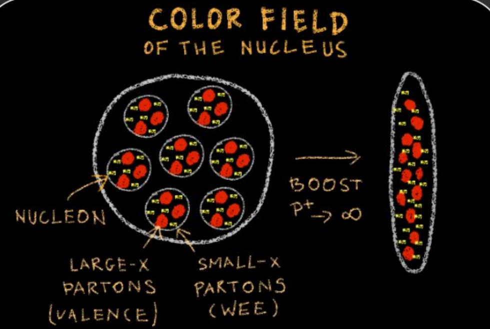
$$S_x^{(2)}\left(\vec{y}_T,\vec{x}_T\right) = \frac{1}{N_c}\left\langle \mathrm{Tr}\, U(\vec{y}_T)U^\dagger(\vec{x}_T)\right\rangle_x$$

$$S_x^{(4)}\left(\vec{z}_T,\vec{y}_T,\vec{x}_T\right) = \frac{1}{2C_F N_c}\left\langle \mathrm{Tr}\left[U(\vec{z}_T)U^\dagger(\vec{y}_T)\right]\mathrm{Tr}\left[U(\vec{y}_T)U^\dagger(\vec{x}_T)\right]\right\rangle_x - S_x^{(2)}\left(\vec{z}_T,\vec{x}_T\right)$$

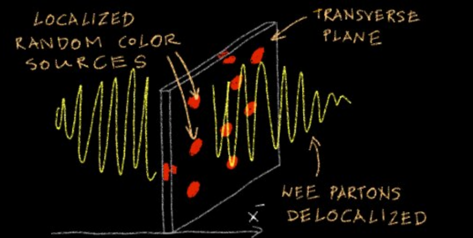
etc...

$$U(\vec{x}_T) = \mathscr{P}\exp\left\{ ig\int_{-\infty}^{+\infty} dx^+\, A_a^-\left(x^+,\vec{x}_T\right)t^a\right\}$$

[C. Marquet, 2007]



[L. McLerran, R. Venugopalan, 1993]



Large-x partons — the color source for wee partons:

$$(D_\mu F^{\mu\nu})_a(x^-,\vec{x}_T) = \delta^{\nu+}\rho_a(\vec{x}_T)\delta(x^-)$$

RANDOM DISTRIBUTION OF COLOR SOURCES

AVERAGE OVER COLOR SOURCES

GAUSSIAN FUNCTIONAL $\longrightarrow$ $\mathscr{W}_x[\rho]$

B-JIMWLK EVOLUTION IN X

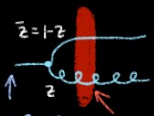
[Balitsky-Jalilian-Marian-Iancu-McLerran -Weigert-Leonidov-Kovner, 1996-2002]

CGC & TMD Leading twist study

P. Kotko, EPSHEP2019

FORWARD DIJET PRODUCTION IN CGC

[F. Dominguez, C. Marquet, B. Xiao, F. Yuan, 2011]



$\vec{P}_{T1}$, $\vec{P}_{T2}$

$\vec{k}_T = \vec{P}_{T1} + \vec{P}_{T2}$

$\vec{P}_T = \bar{z}\vec{P}_{T1} - z\vec{P}_{T2}$

LARGE-X PARTON (x_1)

EXCHANGE OF SMALL-X GLUONS (x_2)

$x_2 \ll x_1$

LEADING POWER LIMIT

$$Q_s \sim k_T \ll P_T \sim \mu$$

SATURATION SCALE

HARD SCALE

$$\frac{d\sigma_{pA\to 2j+X}}{dy_1 dy_2 d^2p_{T1} d^2p_{T2}} \sim \sum_{a,c,d} f_{a/p}(x_1,\mu) \sum_i H^{(i)}_{ag\to cd}(k_T=0)\, \mathcal{F}^{(i)}_{ag}(x_2,k_T)$$

ON-SHELL HARD FACTORS

TMD GLUON DISTRIBUTIONS (SMALL-X LIMIT)

Equivalence of leading power CGC and TMD ‘factorization’ was recently shown for dijet+photon process.

[T. Altinoluk, R. Boussarie, C. Marquet, P. Taels, 2018]

SMALL-X LIMIT OF TMD GLUON DISTRIBUTIONS

$$\mathcal{F}^{(i)}_{ag}(x,k_T) \sim \int \frac{d\xi^+ d^2\xi_T}{(2\pi)^3}\, e^{ixP^-\xi^+ - i\vec{k}_T\cdot\vec{\xi}_T} \langle P|\,\mathrm{Tr}\left[\hat{F}^{i-}\left(\xi^+,\vec{\xi}_T,\xi^-=0\right)\mathcal{U}_{C_1}\hat{F}^{i-}(0)\,\mathcal{U}_{C_2}\right]|P\rangle$$

$x \to 0$

DEPENDENCE ON X IS ONLY VIA THE SMALL-X EVOLUTION

For example:

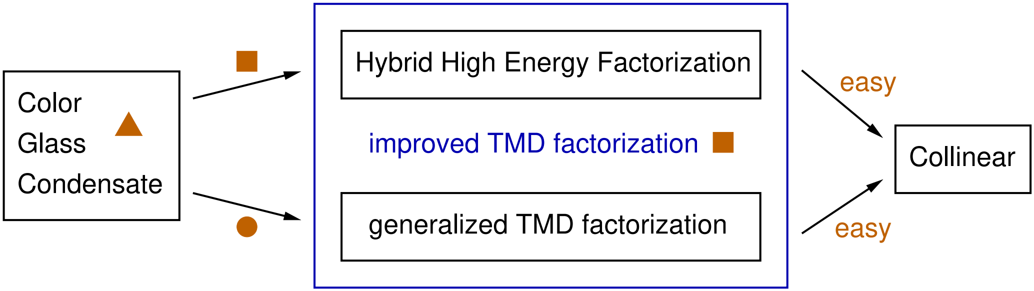
$$\mathcal{F}^{(1)}_{qg} \sim \int \frac{d^2x_T d^2y_T}{(2\pi)^4}\, k_T^2 e^{-i\vec{k}_T\cdot(\vec{x}_T-\vec{y}_T)} \left\langle \mathrm{Tr}\left[U(\vec{x}_T)U^\dagger(\vec{y}_T)\right]\right\rangle_x$$

Intensively studied:

[D. Kharzeev, Y. Kovchegov, K. Tuchin, 2003]
[B. Xiao, F. Yuan, 2010]
[F. Dominguez, C. Marquet, B. Xiao, F. Yuan, 2011]
[A. Metz, J. Zhou, 2011]
[E. Akcakaya, A. Schafer, J. Zhou, 2012]
[C. Marquet, E. Petreska, C. Roiesnel, 2016]
[I. Balitsky, A. Tarasov, 2015, 2016]
[D. Boer, P. Mulders, J. Zhou, Y. Zhou, 2017]
[C. Marquet, C. Roiesnel, P. Taels, 2018]
[Y. Kovchegov, D. Pitonyak, M. Sievert, 2017,2018]
[T. Altinoluk, R. Boussarie, 2019]

ITMD Factorization

For forward dijet production
in dilute-dense hadronic collisions



▲ McLerran, Venugopalan 1994, Iancu, Venugopalan 2003

■ Kotko, Kutak, Marquet, Petreska, Sapeta, AvH 2015, Altinoluk, Boussarie, Kotko 2019

● Dominguez, Marquet, Xiao, Yuan 2011

Model interpolating between hybrid High Energy Factorization and Generalized TMD factorization and valid for kinematical regions with hard scale $\gtrsim k_T \gtrsim$ saturation scale.
Partonic cross section $d\hat{\sigma}^{(i)}_{gb}$ **depends on color-structure** i,
and is calculated with **space-like initial-state gluons**.

$$d\sigma_{AB\to X} = \int dk_T^2 \int dx_A \sum_i \int dx_B \sum_y \phi^{(i)}_{gy}(x_A, k_T, \mu)\, f_y(x_B, \mu)\, d\hat{\sigma}^{(i)}_{gy\to X}(x_A, x_B, k_T, \mu)$$

ITMD Factorization

ITMD formalism is obtained from the CGC formalism, by neglecting certain twist corrections (so-called genuine twist as opposed to kinematic twist). Antinoluk, Boussarie, Kotko 2019

Comparison of ITMD and CGC for forward quark dijets shows that the first is a good approximation of the second for reasonable large p_T (much larger than Q_s) of the final-state jets Fujii, Marquet, Watanabe 2020.

ITMD Factorization for 3 or more jets

Bury, Kotko, Kutak, 2019

Color decomposition of, for example, the amplitude $\mathcal{M}^{a_1 a_2 i_3 i_4}_{j_3 j_4}$ for two gluons and two quark-anti-quark pairs in terms of color factors and partial amplitudes $\mathcal{A}_\sigma$:

$$\tilde{\mathcal{M}}^{i_1 i_2 i_3 i_4}_{j_1 j_2 j_3 j_4} \equiv \left(\sqrt{2}T^{a_1}\right)^{i_1}_{j_1} \left(\sqrt{2}T^{a_2}\right)^{i_2}_{j_2} \mathcal{M}^{a_1 a_2 i_3 i_4}_{j_3 j_4} = \sum_{\sigma \in S_4} \delta^{i_1}_{j_{\sigma(1)}} \delta^{i_2}_{j_{\sigma(2)}} \delta^{i_3}_{j_{\sigma(3)}} \delta^{i_4}_{j_{\sigma(4)}} \mathcal{A}_\sigma$$

The sum over colors for the squared amplitude is facilitated by a **color matrix** $C_{\tau\sigma}$

$$\mathcal{M}^{a_1 a_2 i_3 i_4}_{j_3 j_4} \mathcal{M}^{*a_1 a_2 j_3 j_4}_{i_3 i_4} = \tilde{\mathcal{M}}^{i_1 i_2 i_3 i_4}_{j_1 j_2 j_3 j_4} \tilde{\mathcal{M}}^{*j_1 j_2 j_3 j_4}_{i_1 i_2 i_3 i_4} = \sum_{\tau,\sigma} \mathcal{A}_\tau C_{\tau\sigma} \mathcal{A}^*_\sigma$$

Each element of the matrix $C_{\tau\sigma}$ is a single power of N_c (Kanaki, Papadopoulos 2002). The cross section formula for ITMD is obtained by inserting color correlators like

$$\mathrm{TMD}_1 \times \tilde{\mathcal{M}}^{i_1 i_2 i_3 i_4}_{j_1 j_2 j_3 j_4} \tilde{\mathcal{M}}^{*j_1 j_2 j_3 j_4}_{i_1 i_2 i_3 i_4} \Rightarrow \left\langle\!\!\left\langle F^{j_1}_{i_1} \mathcal{U}^{k_2}_{i_2} \mathcal{U}^{k_3}_{i_3} \mathcal{U}^{k_4}_{i_4} F^{k_1}_{l_1} \mathcal{U}^{j_2}_{l_2} \mathcal{U}^{j_3}_{l_3} \mathcal{U}^{j_4}_{l_4} \right\rangle\!\!\right\rangle \tilde{\mathcal{M}}^{i_1 i_2 i_3 i_4}_{j_1 j_2 j_3 j_4} \tilde{\mathcal{M}}^{*l_1 l_2 l_3 l_4}_{k_1 k_2 k_3 k_4}$$

where the $\mathcal{U}^k_l$ are certain Wilson line operators, depending on the external partons, and F^k_l the field strenght. This leads to a

TMD-valued color matrix $\mathcal{C}_{\tau\sigma}(x, k_T)$

This has been implemented in KaTie.

Bury, Kotko, Kutak, 2019

ALL POSSIBLE OPERATORS FOR TMD GLUON DISTRIBUTIONS

$$\mathcal{F}_{qg}^{(1)} \sim \langle P|\mathrm{Tr}\left[\hat{F}^{i-}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i-}(0)\,\mathcal{U}^{[+]}\right]|P\rangle$$

← DIPOLE TMD

$$\mathcal{F}_{qg}^{(2)} \sim \langle P|\frac{\mathrm{Tr}\,\mathcal{U}^{[\square]}}{N_c}\mathrm{Tr}\left[\hat{F}^{i-}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i-}(0)\,\mathcal{U}^{[+]}\right]|P\rangle$$

$$\mathcal{F}_{qg}^{(3)} \sim \langle P|\mathrm{Tr}\left[\hat{F}^{i-}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i-}(0)\,\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\right]|P\rangle$$

$$\mathcal{F}_{gg}^{(1)} \sim \langle P|\frac{\mathrm{Tr}\,\mathcal{U}^{[\square]\dagger}}{N_c}\mathrm{Tr}\left[\hat{F}^{i-}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i-}(0)\,\mathcal{U}^{[+]}\right]|P\rangle$$

$$\mathcal{F}_{gg}^{(2)} \sim \langle P|\mathrm{Tr}\left[\hat{F}^{i-}(\xi)\,\mathcal{U}^{[\square]\dagger}\right]\mathrm{Tr}\left[\hat{F}^{i-}(0)\,\mathcal{U}^{[\square]}\right]|P\rangle$$

$$\mathcal{F}_{gg}^{(3)} \sim \langle P|\mathrm{Tr}\left[\hat{F}^{i-}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i-}(0)\,\mathcal{U}^{[+]}\right]|P\rangle$$

← WEISACKER-WILLIAMS TMD

$$\mathcal{F}_{gg}^{(4)} \sim \langle P|\mathrm{Tr}\left[\hat{F}^{i-}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i-}(0)\,\mathcal{U}^{[-]}\right]|P\rangle$$

$$\mathcal{F}_{gg}^{(5)} \sim \langle P|\mathrm{Tr}\left[\hat{F}^{i-}(\xi)\,\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}\hat{F}^{i-}(0)\,\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\right]|P\rangle$$

$$\mathcal{F}_{gg}^{(6)} \sim \langle P|\frac{\mathrm{Tr}\,\mathcal{U}^{[\square]}}{N_c}\frac{\mathrm{Tr}\,\mathcal{U}^{[\square]\dagger}}{N_c}\mathrm{Tr}\left[\hat{F}^{i-}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i-}(0)\,\mathcal{U}^{[+]}\right]|P\rangle$$

$$\mathcal{F}_{gg}^{(7)} \sim \langle P|\frac{\mathrm{Tr}\,\mathcal{U}^{[\square]}}{N_c}\mathrm{Tr}\left[\hat{F}^{i-}(\xi)\,\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}\hat{F}^{i-}(0)\,\mathcal{U}^{[+]}\right]|P\rangle$$

WILSON LOOP → $\mathcal{U}^{[\square]} = \mathcal{U}^{[+]}\mathcal{U}^{[-]\dagger}$

[M. Bury, PK , K. Kutak, 2018]

ore jets

$\mathcal{M}^{a_1a_2i_3i_4}_{j_3j_4}$ for two gluons and two

artial amplitudes $\mathcal{A}_\sigma$:

$$\sum_{\sigma\in S_4}\delta^{i_1}_{j_{\sigma(1)}}\delta^{i_2}_{j_{\sigma(2)}}\delta^{i_3}_{j_{\sigma(3)}}\delta^{i_4}_{j_{\sigma(4)}}\mathcal{A}_\sigma$$

litated by a **color matrix** $C_{\tau\sigma}$

$$\ldots^{j_3j_4}_{i_3i_4} = \sum_{\tau,\sigma}\mathcal{A}_\tau\, C_{\tau\sigma}\,\mathcal{A}^*_\sigma$$

N_c (Kanaki, Papadopoulos 2002).

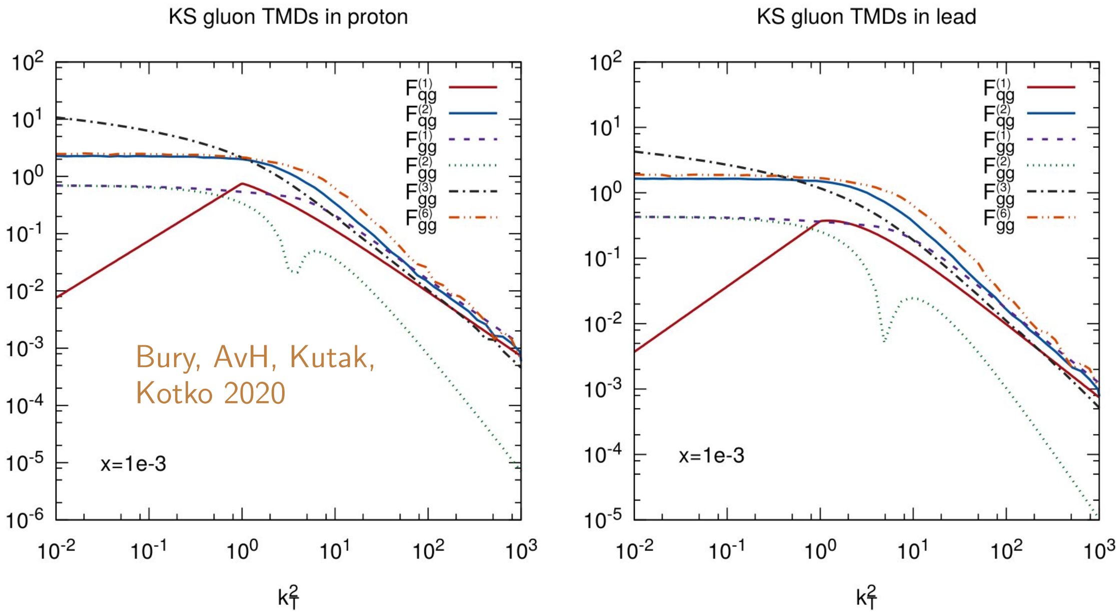
serting color correlators like

$$\ldots\,\mathcal{U}^{j_2}_{i_2}\,\mathcal{U}^{j_3}_{i_3}\,\mathcal{U}^{j_4}_{i_4}\Big\rangle\Big\rangle\,\tilde{\mathcal{M}}^{i_1i_2i_3i_4}_{j_1j_2j_3j_4}\,\tilde{\mathcal{M}}^{*l_1l_2l_3l_4}_{k_1k_2k_3k_4}$$

ding on the external partons, and F^k_l

$$\ldots\,\mathcal{C}_{\tau\sigma}(x,k_T)$$

ITMD gluons



- KS gluon (Kutak, Sapeta 2012) is the dipole-distribution as a solution to the BK equation (Balitsky 1996, Kovchegov 1999) formulated in the momentum space with corrections of higher order, and fitted to F_2 data.

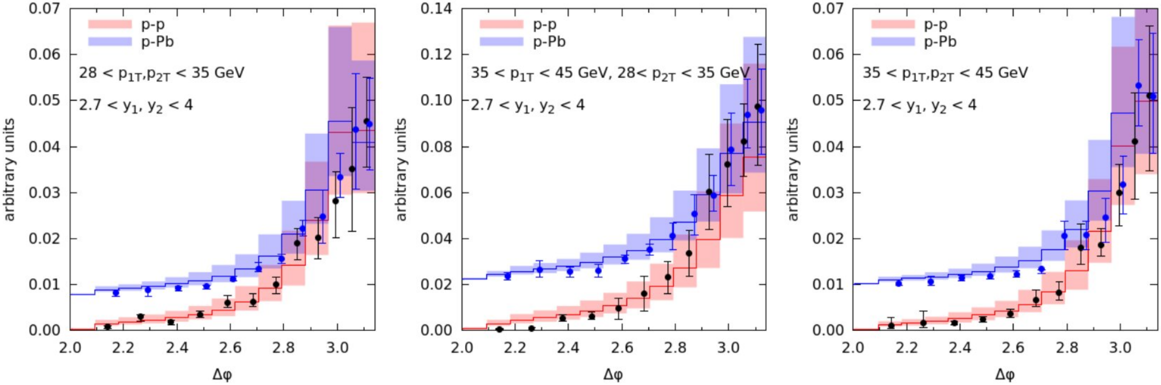
- ITMD gluons follow from the KS gluon

Saturation effects from forward jets

AvH, Kotko, Kutak, Sapeta 2019

Study of saturation using dijet production in p-p and p-pB collisions.
Angle $\Delta\varphi$ between the jets is particularly sensitive to saturation effects.

Data points from ATLAS 2019. Arbitrary normalization and relative shift to to accentuate the difference in shape between p-p and p-Pb.



Calculations where performed within ITMD factorization.
Besides saturation, the inclusion of resummed Sudakov logarithms are essential to reach this accuracy, included here via event-reweighting.
Both KaTie and LxJet (http://nz42.ifj.edu.pl/~pkotko/LxJet.html) were used for independent cross checks.

Saturation at EIC

Probing Gluon Saturation through Dihadron Correlations at an Electron-Ion Collider
Zheng, Aschenauer, Lee, Xiao
Phys. Rev. D 89, 074037 (2014)

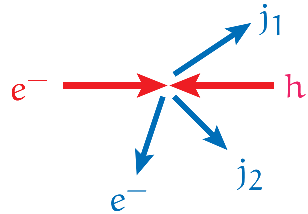
Multi-gluon correlations and evidence of saturation from dijet measurements at an Electron Ion Collider
Mäntysaari, Mueller, Salazar, Schenke
Phys. Rev. Lett. 124, 112301 (2020)

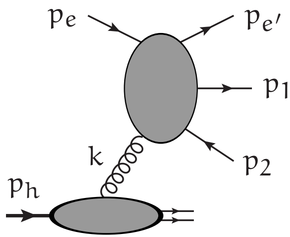
Forward jets in DIS

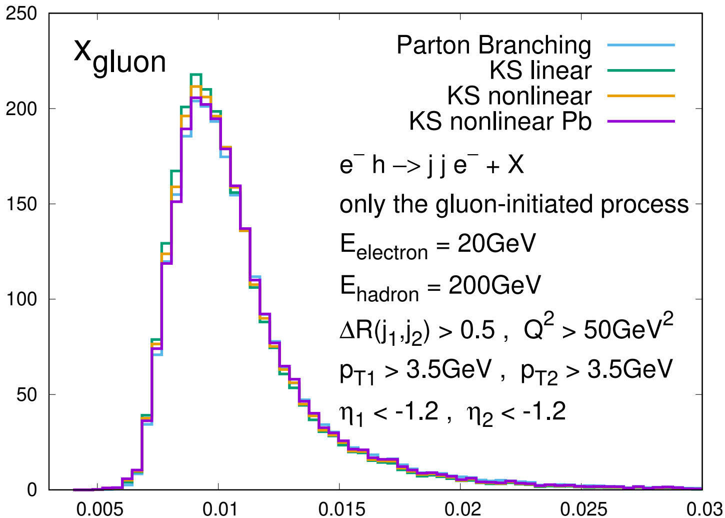
Try to study saturation like with forward jets in h-h scattering.

Need gluon as initial-state parton $\Longrightarrow$ consider dijets.

Need small x_{gluon} $\Longrightarrow$ consider forward(backward?) jets.





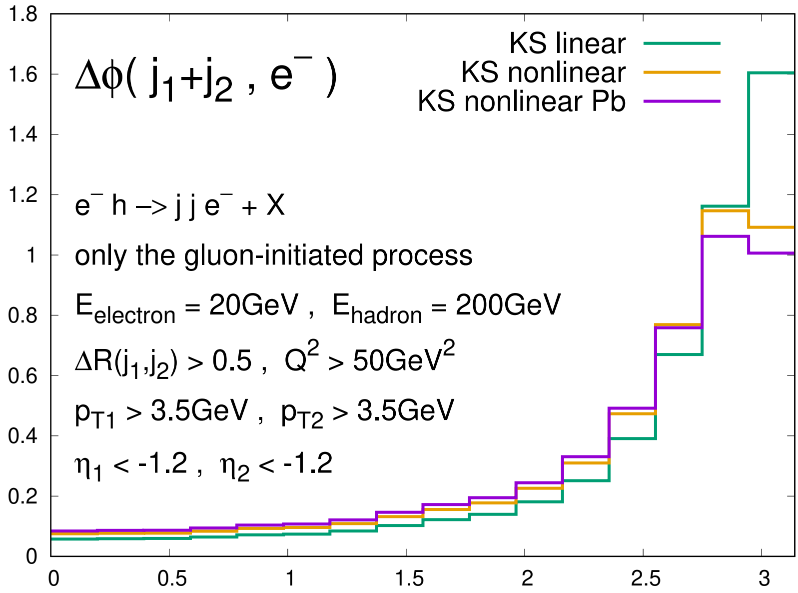


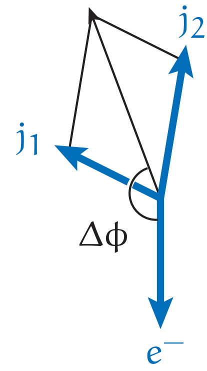
$$d\sigma = \int_0^1 dx_{\text{gluon}} \int \frac{d^2k_T}{\pi} \times \mathcal{F}(x_{\text{gluon}}, k_T, \mu) \times d\hat{\sigma}(x_{\text{gluon}}, k_T, \mu)$$

$$x_{\text{Bjorken}} = \frac{-(p_e - p_{e'})^2}{2p_h\cdot(p_e - p_{e'})}$$

Azimuthal angle at EIC energies

Need observable sensitive to final state momentum inbalance, eg. the angle between the electron and the jet pair.





Angle between final-state lepton and jets studied previously by H1 (2012), and in the context of azimuthal assymetries Jacobsson PhD-thesis 1994.

The shapes of the distribution are clearly different, for TMDs with different rates of saturation included.

Thank you for your attention.