

Correlating anisotropic flow with isotropic flow in heavy-ion collisions

Nuclear phenomenology at high energy
beyond the quark-gluon plasma

by

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with J. Jia and Y. Zhou

- **HEAVY-ION COLLISIONS: ESTABLISHED QGP PICTURE**

- Particle multiplicities ($dN/d\eta$).
- Anisotropic flow (V_n).
- Isotropic flow ($\langle p_t \rangle$)
- Status of soft sector.

- **NUCLEAR PHENOMENOLOGY AT HIGH ENERGY BEYOND THE QGP**

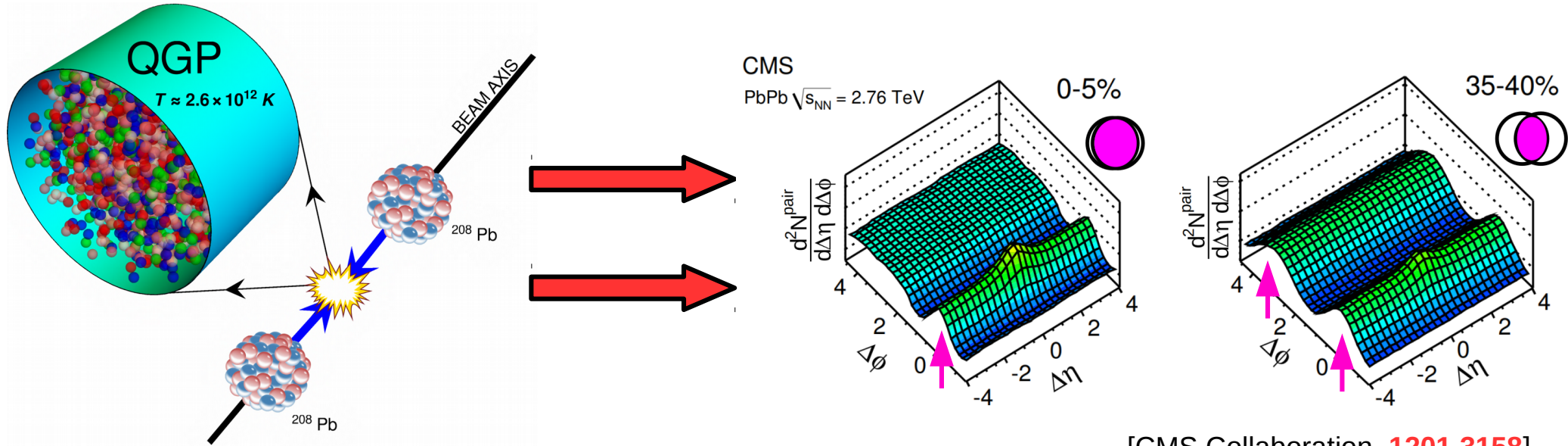
- Correlating V_n with $\langle p_t \rangle$ (at fixed $dN/d\eta$).
- Signatures of strong magnetic fields.
- Nuclear deformation in heavy-ion collisions.
- Primordial momentum anisotropies.

- **OUTLOOK**

- The tip of the iceberg.

HEAVY-ION COLLISIONS: **EMERGENT PHENOMENA AT HIGH ENERGY.** “More is different”
[Anderson, 1972]

- Particle density in the overlap region is huge: 1 to 10 fm⁻³. Nuclear matter: 0.16 fm⁻³.
- Regular structures in data: emergence of an **effective (collective) description.**



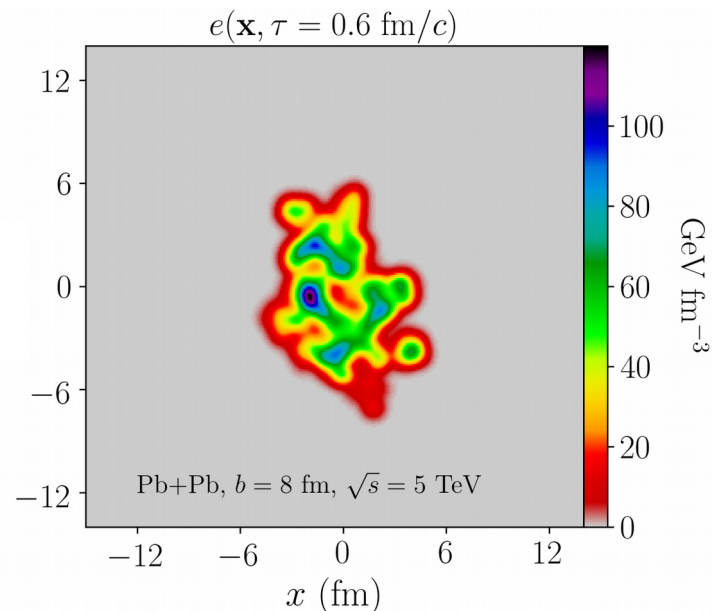
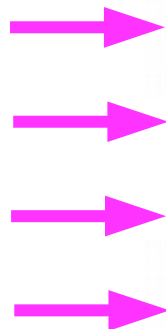
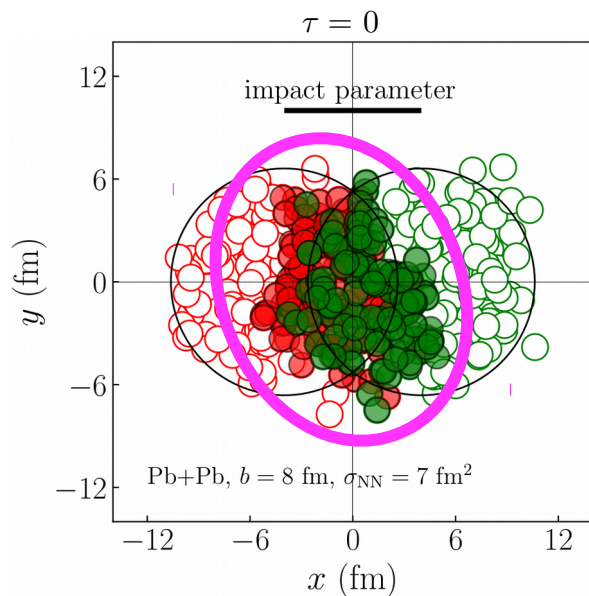
[Gardim, Giacalone,
Luzum, Ollitrault, [1908.09728](#)]

[CMS Collaboration, [1201.3158](#)]

System size much larger than “mean free path”. Equilibration on time scale of QCD, ~ 1 fm/c.

[Schlichting, Teaney, [1908.02113](#)] [Berges, Mazeliauskas, Spaliński, Venugopalan, [2005.12299](#)]

\implies **Effective description is that of a (relativistic) fluid.**



[Romatschke & Romatschke, [1712.05815](#)]

Quasi-ideal dynamics:

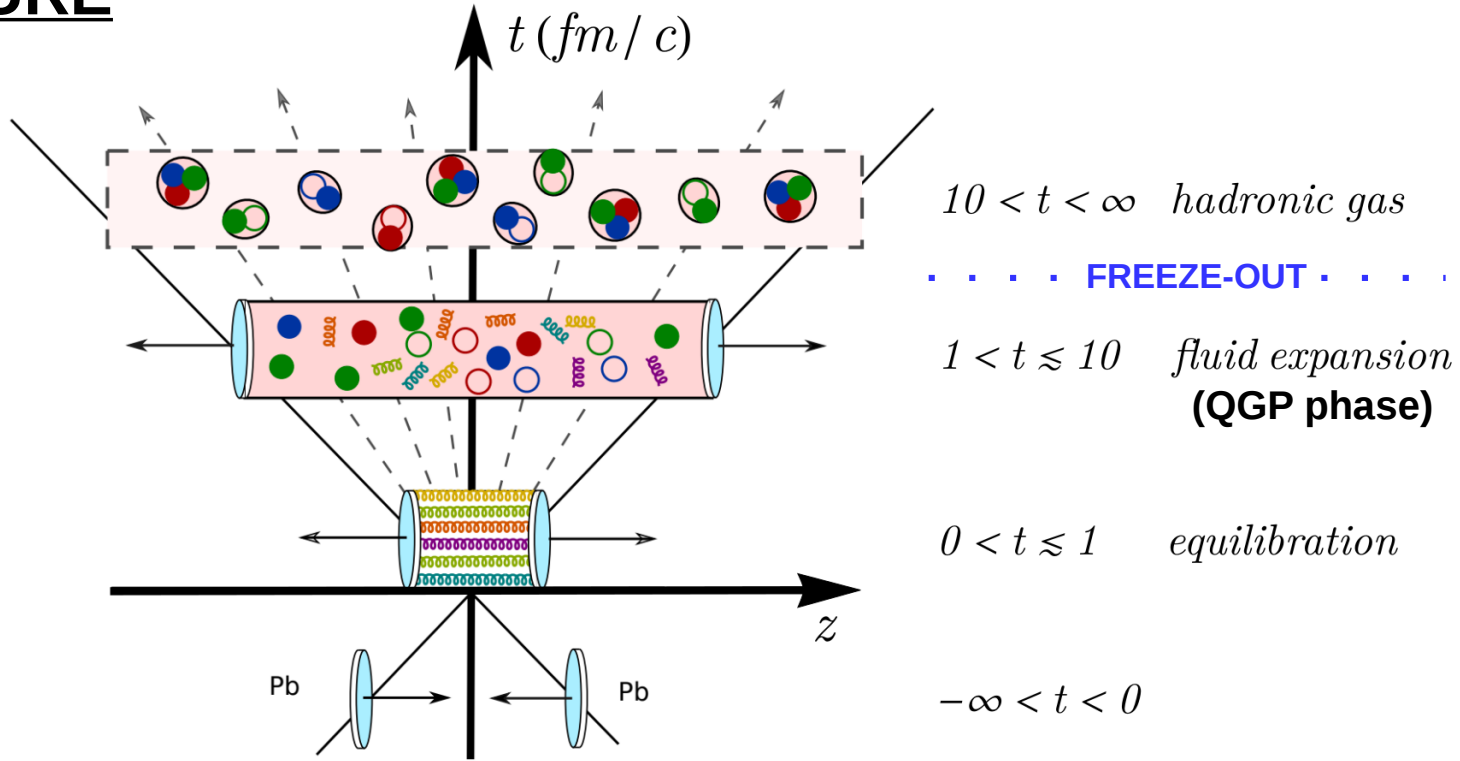
$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - P g^{\mu\nu} + \text{small viscous corrections } (\eta/s, \zeta/s, \dots) + \partial_\mu T^{\mu\nu} = 0$$

Equation of state of high-temperature QCD ($T > T_c \sim 156$ MeV). [HotQCD collaboration, [1407.6387](#)]

Large number of **DOF** (~ 40) due to parton liberation: **Quark-Gluon Plasma (QGP)**.

BIG PICTURE

(courtesy A. Mazeliauskas)



$t = \infty$



$$\frac{dN}{d^3 \mathbf{p}} \quad \text{or} \quad \frac{dN}{dy d^2 \mathbf{p}_t}$$

Hadron spectrum in momentum space.

Bulk of produced $O(10^3-10^4)$ particles is soft, $p_t < 2 \text{ GeV}$.

At ultrarelativistic energy, **particle yields are nearly independent of rapidity**.
Focus on the midrapidity slice:

$$\frac{dN}{dyd^2\mathbf{p}_t} \longrightarrow \frac{dN}{d^2\mathbf{p}_t} = \frac{dN}{p_t dp_t d\phi_p}$$

Fundamental quantities in the soft sector:

event multiplicity

$$N = \int_{\mathbf{p}_t} \frac{dN}{d^2\mathbf{p}_t}$$

anisotropic flow

$$V_n = \frac{1}{N} \int_{\mathbf{p}_t} \frac{dN}{d^2\mathbf{p}_t} e^{-in\phi_p}$$

average momentum

$$\langle p_t \rangle = \frac{1}{N} \int_{\mathbf{p}_t} p_t \frac{dN}{d^2\mathbf{p}_t}$$

Basics of QGP phenomenology: understanding these quantities.

[Giacalone, **2101.00168**]

1 - event multiplicity.

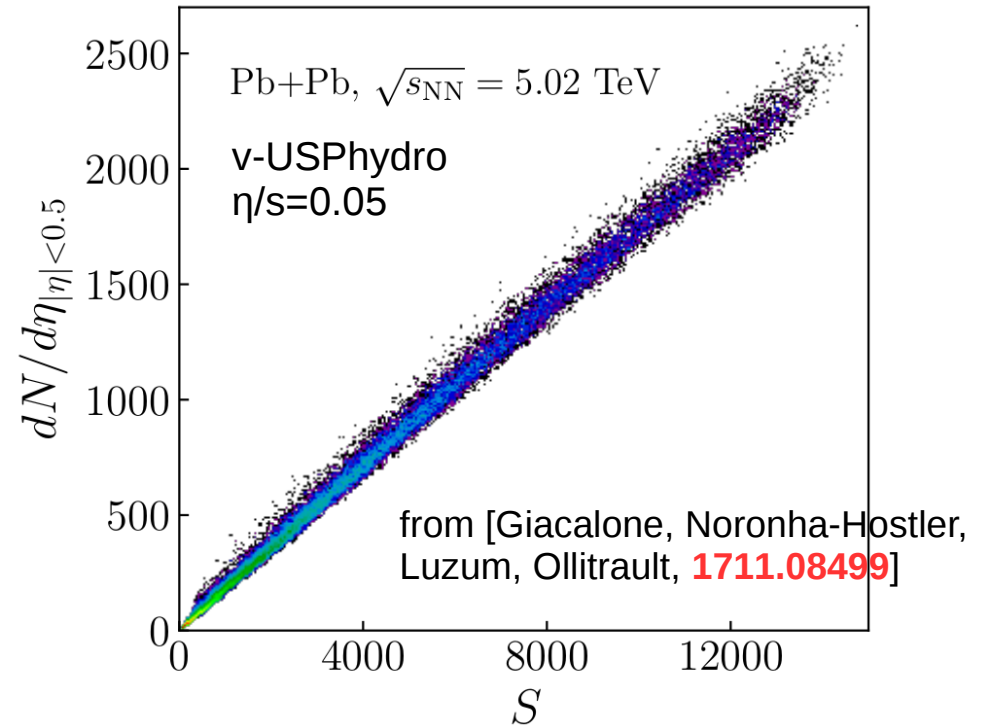
Underlying physics is that of an ideal gas of massless particles.

The number of particles determines the entropy. The QGP expansion is nearly isentropic.

Entropy, S , should thus be proportional to number of detected hadrons:

$$S \propto N \text{ (or } dN/d\eta\text{)}$$

Confirmed by full hydro calculations.
(including viscosity, freeze-out, etc.)



The event multiplicity is a measure of the entropy of the system.

2 - anisotropic flow.

Azimuthal anisotropy of particle emission.

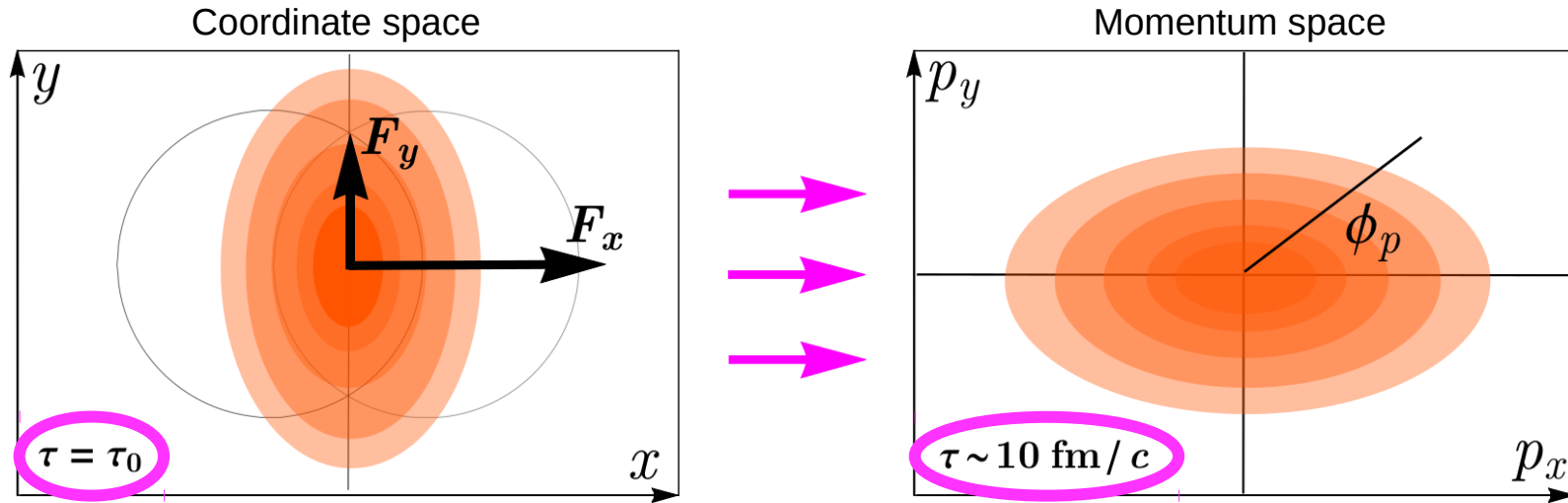
The most famous is **elliptic flow**, the 2nd harmonic.

$$\longrightarrow V_2 = \frac{1}{N} \int_{\mathbf{p}_t} \frac{dN}{d^2\mathbf{p}_t} e^{-i2\phi_p}$$

Geometric origin: imbalance of forces in anisotropic medium.

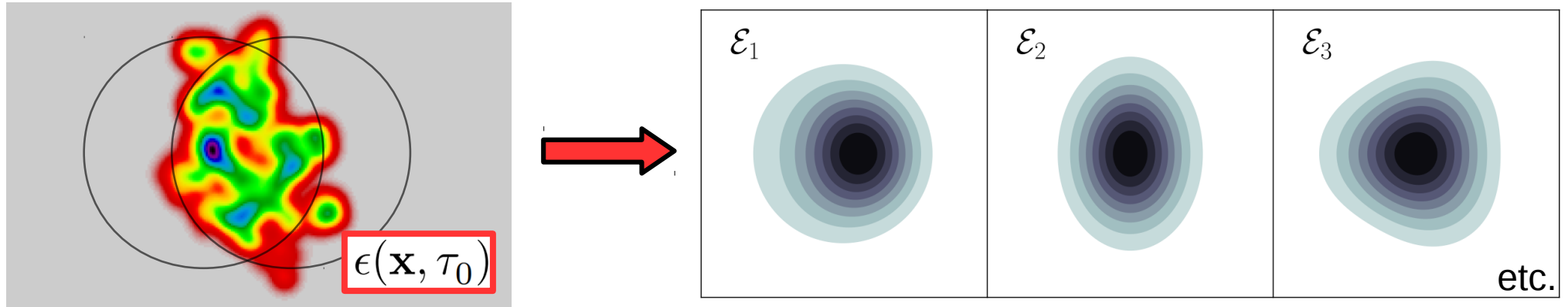
Shape-flow transmutation at finite impact parameter. $\vec{F} = -\vec{\nabla}P$

[Ollitrault, 1992]



Momentum anisotropy from spatial anisotropy.

More than ellipse: due to fluctuations (nucleon positions), all multipole moments are nonzero.



Multipole moments in two dimensions identified by [Teaney, Yan, [1010.1876](#)].

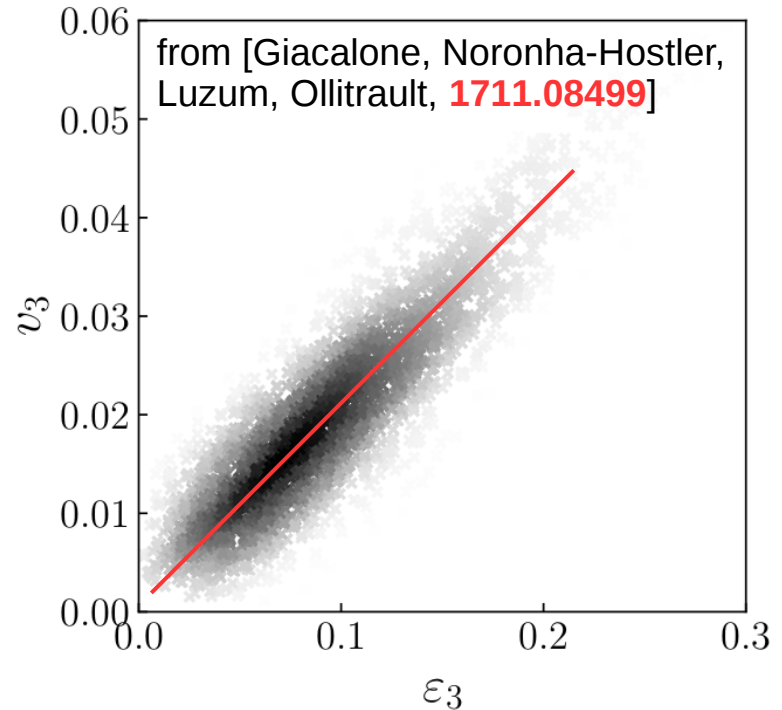
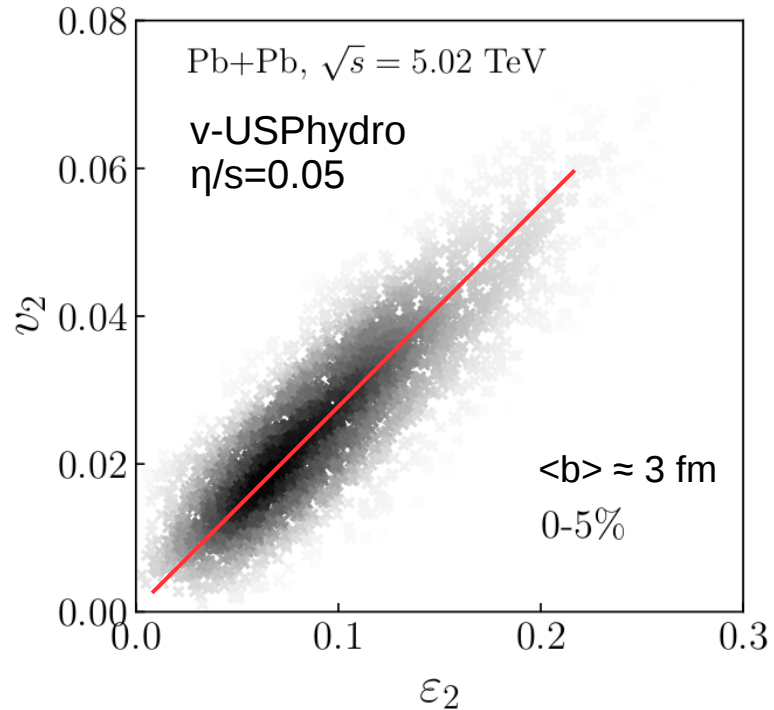
For $n=2$ or $n=3$, they read:

$$\mathcal{E}_n = - \frac{\int r dr d\phi r^n e^{in\phi} \epsilon(r, \phi)}{\int r dr d\phi r^n \epsilon(r, \phi)}$$

But recall: $\vec{F} = -\vec{\nabla}P$. Each E_n in the initial state leads to V_n in the final state.

Simple relation: $V_n \propto \mathcal{E}_n$

Verified in full hydrodynamic simulations ($\varepsilon_n = |\mathcal{E}_n|$, $v_n = |V_n|$)



Explains experimental data in both large and small systems.

The importance of initial conditions.

[Giacalone, Noronha-Hostler, Ollitrault, [1702.01730](#)]

3 – average momentum (isotropic flow)

Mean transverse momentum is the “energy per particle”. $\longrightarrow \langle p_t \rangle = \frac{1}{N} \int_{\mathbf{p}_t} p_t \frac{dN}{d^2\mathbf{p}_t}$

Energy per particle in the QGP is nontrivial:

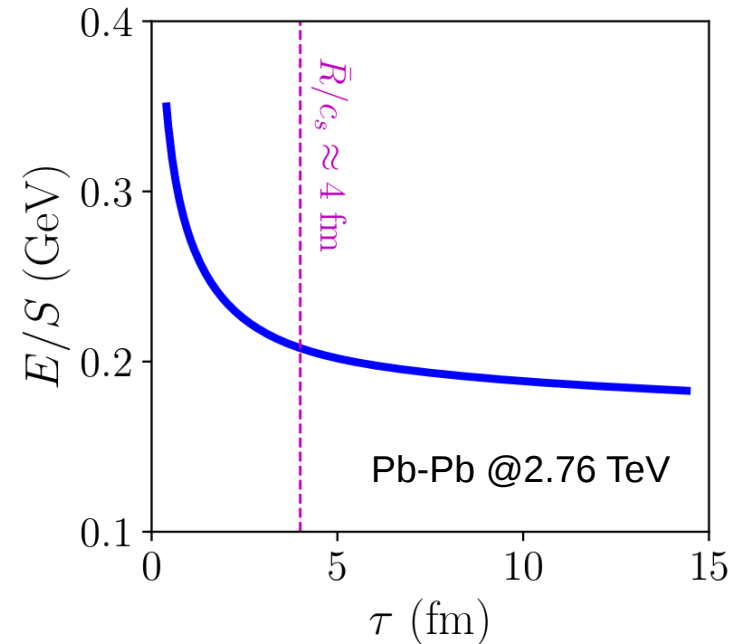
$$E/S$$

Evolution is quasi-isentropic, but the energy is not constant!

Longitudinal cooling in QGP: [Björken, 1982]

$$dE = -PdV. \quad \longrightarrow$$

Energy per particle is fixed only at the end of cooling.



Back to the ideal gas. Energy per particle:

$$p \simeq E = 3T$$

Therefore in a heavy-ion collision we expect:

$$\langle p_t \rangle \simeq 3T$$

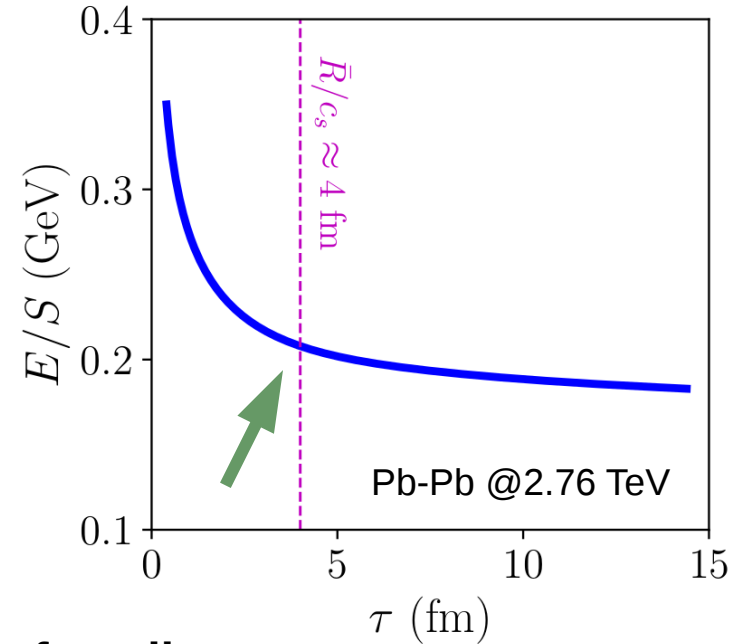
where T is the temperature of the system at the end of cooling.

This has been verified in hydrodynamic simulations. [Gardim, Giacalone, Luzum, Ollitrault, [1908.09728](#)]

Application:

Understanding $\langle p_t \rangle$ measured by the ALICE collaboration in 0-5% Pb+Pb.

They measure $\langle p_t \rangle \approx 680 \text{ MeV}$. The temperature is then $\langle p_t \rangle / 3 \approx 226 \text{ MeV} \sim 2.6 \times 10^{12} \text{ K}$.



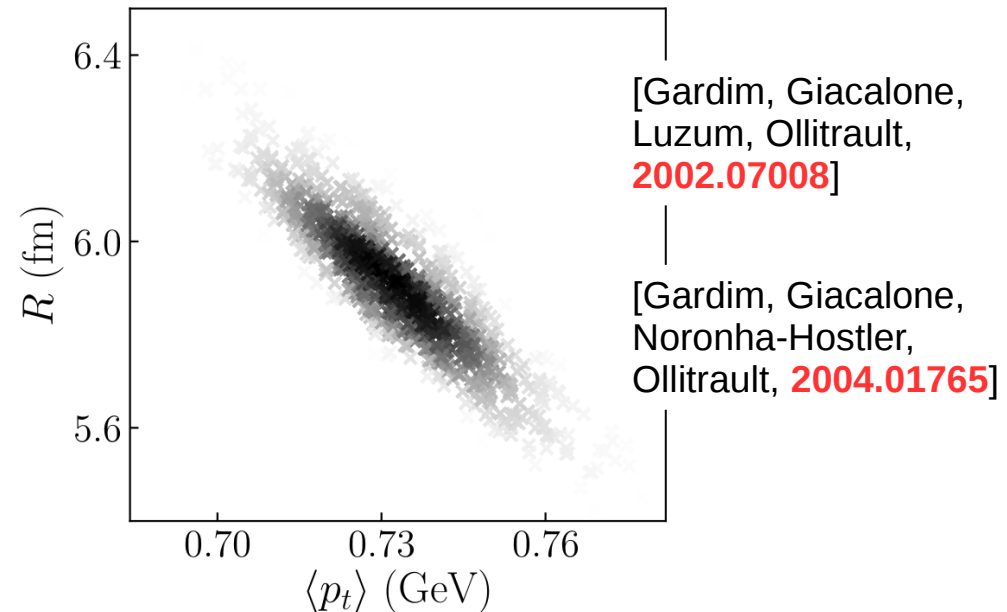
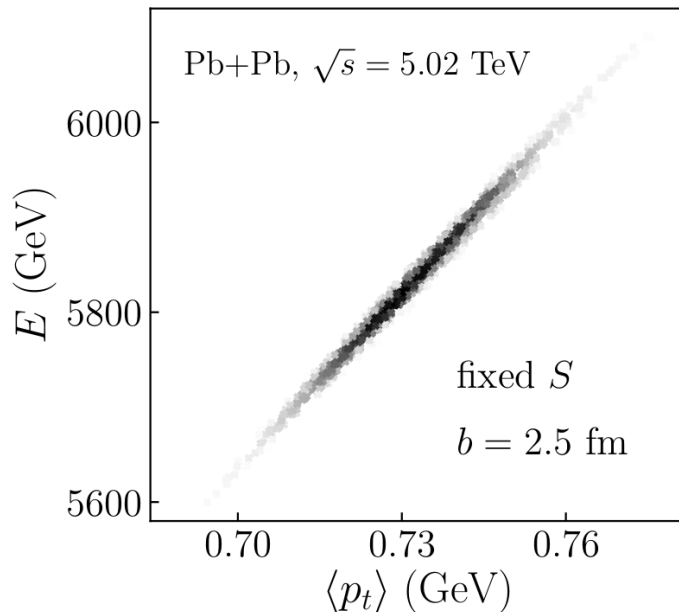
$\langle p_t \rangle$ fluctuations. Consider that $\langle p_t \rangle$ is proportional to the initial temperature, T .
 A few relations follow (E =initial energy, R =initial rms radius):

$$c_s^2 = \frac{dP}{d\epsilon} = \frac{d \ln T}{d \ln s} \quad \xrightarrow{\langle p_t \rangle \sim T} \quad \frac{d\langle p_t \rangle}{\langle p_t \rangle} \propto \frac{dE}{E} \quad \frac{d\langle p_t \rangle}{\langle p_t \rangle} \propto -\frac{dR}{R}$$

The proportionality factors depend on the equation of state.

[Gardim, Giacalone, Noronha-Hostler, Ollitrault, [2004.09799](#)]

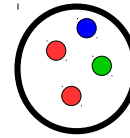
Verified in full hydrodynamic simulations at fixed entropy (multiplicity).



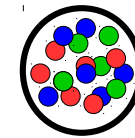
SUMMARY: basics of the soft sector of heavy-ion collisions.

- The **number of detected hadrons** (N or $dN/d\eta$) is a measure of the **entropy** of the QGP.

$$S \propto N$$



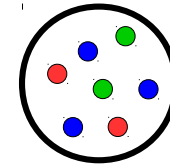
small S



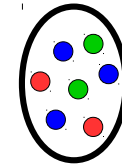
large S

- Anisotropic flow coefficients** (V_n) are a hydrodynamic response of the system to its **initial spatial anisotropies** (\mathcal{E}_n).

$$V_n \propto \mathcal{E}_n$$



small v_2



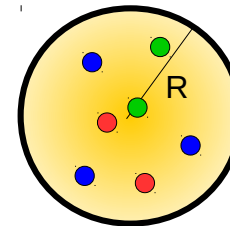
large v_2

- Momentum** of the outgoing hadrons, $\langle p_t \rangle$, depends on the temperature reached in the QGP. Its **fluctuations probe the thermodynamics** of the system (energy, volume).

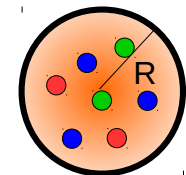
$$\frac{d\langle p_t \rangle}{\langle p_t \rangle} \propto \frac{dE}{E}$$

$$\frac{d\langle p_t \rangle}{\langle p_t \rangle} \propto -\frac{dR}{R}$$

small $\langle p_t \rangle$



large $\langle p_t \rangle$



Current status of the field. I think there are three main directions:

- **Clarifying the origin and limits of applicability of this picture.** How does thermalization occur? How small can a QGP be? When can you talk about 'hydro'?

[Berges, Mazeliauskas, Spaliński, Venugopalan, [2005.12299](#)]

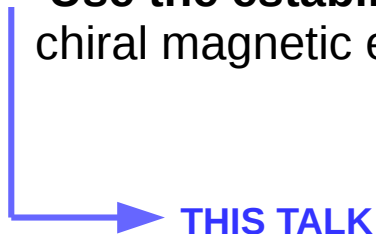
- **Refining the picture and challenging current descriptions.** Values of transport coefficients, medium properties, initial conditions, freeze-out, etc.

[Trajectum, [2010.15130](#), [2010.15134](#)] [JETSCAPE Collaboration, [2011.01430](#), [2010.03928](#)]

[Devetak, Dubla, Floerchinger, Grossi, Masciocchi, Mazeliauskas, Selyuzhenkov, [1909.10485](#)]

[Gardim, Giacalone, Ollitrault, [1909.11609](#)]

- **Use the established picture to study new phenomena at high energy.** Popular topics: chiral magnetic effect, color glass condensate, hydrodynamics with spin, nuclear structure.



Revealing nuclear phenomena at high energy with multi-particle correlations.

So far we have built observables from fluctuations of anisotropic flow:

$$\langle v_n^2 \rangle = \left\langle \frac{\int_{\mathbf{p}_1, \mathbf{p}_2} e^{in(\phi_1 - \phi_2)} \frac{dN}{d^2 \mathbf{p}_1 d^2 \mathbf{p}_2}}{\int_{\mathbf{p}_1, \mathbf{p}_2} \frac{dN}{d^2 \mathbf{p}_1 d^2 \mathbf{p}_2}} \right\rangle$$

Or fluctuations of the average transverse momentum, **which I also dub [p_t]**:

$$\langle [p_t]^2 \rangle - \langle [p_t] \rangle^2 = \left\langle \frac{\int_{\mathbf{p}_1, \mathbf{p}_2} (p_1 - \langle\langle p \rangle\rangle) (p_2 - \langle\langle p \rangle\rangle) \frac{dN}{d^2 \mathbf{p}_1 d^2 \mathbf{p}_2}}{\int_{\mathbf{p}_1, \mathbf{p}_2} \frac{dN}{d^2 \mathbf{p}_1 d^2 \mathbf{p}_2}} \right\rangle$$

Revealing nuclear phenomena at high energy with multi-particle correlations.

Breakthrough idea: Mix the previous two!

$$\langle v_n^2 \delta[p_t] \rangle \equiv \left\langle \frac{\int_{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3} (p_1 - \langle\langle p \rangle\rangle) e^{in(\phi_2 - \phi_3)} \frac{dN}{d^2 \mathbf{p}_1 d^2 \mathbf{p}_2 d^2 \mathbf{p}_3}}{\int_{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3} \frac{dN}{d^2 \mathbf{p}_1 d^2 \mathbf{p}_2 d^2 \mathbf{p}_3}} \right\rangle$$

First apparition as a byproduct of a principal component analysis.

[Mazeliauskas, Teaney, [1509.07492](#)]

Božek's formulation as a **Pearson correlation coefficient**:

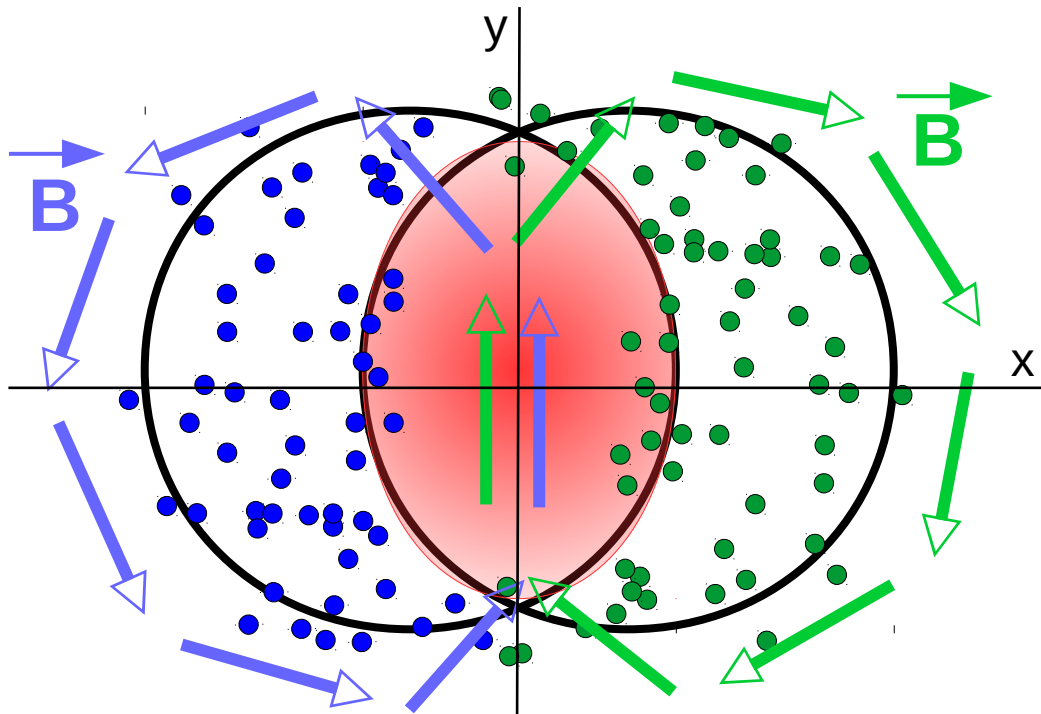
[Božek, [1601.04513](#)]

$$\rho(v_n^2, [p_t]) = \frac{\langle \delta v_n^2 \delta[p_t] \rangle}{\sqrt{\langle (\delta v_n^2)^2 \rangle \langle (\delta[p_t])^2 \rangle}}$$

With $\delta o = o - \langle o \rangle$ at fixed multiplicity (entropy).

[I show that this observable appears naturally in the study of several phenomena.](#)

#1 – Electromagnetic fields.



– Magnetic field of **spectator protons** sum up coherently in the region of overlap.

– The strongest field ever produced in the laboratory. $|\mathbf{B}| \sim 10^{14}$ T

– Typically probed by **charge-dependent dipolar flows** (very small effects).

[Oliva, 2007.00560] $\langle \cos(\phi_1^\pm - \phi_2^\pm) \rangle$

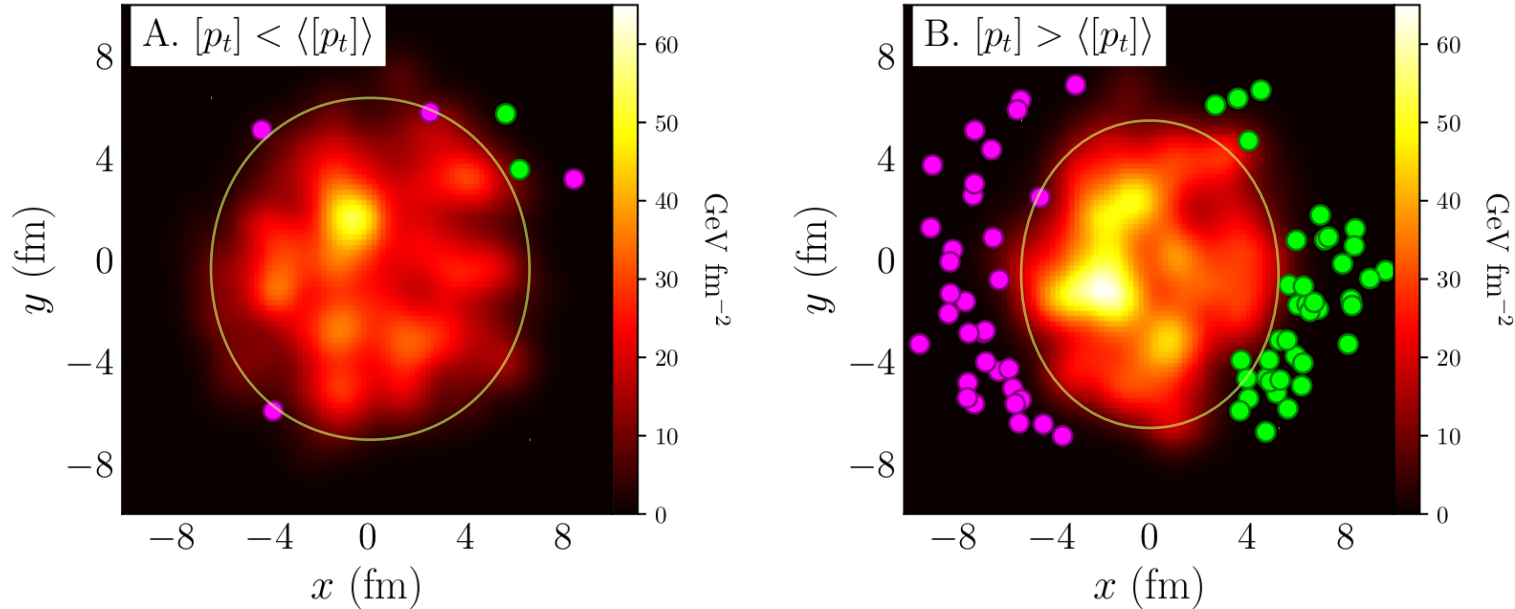
Motivation

- Strong fields.
- Parity violation (CME).
- Electric conductivity of QGP.

– **Elusive. Clean experimental evidence still missing. But we can use $\langle pt \rangle$!**

Select two **central events** (2-3%) at the **same multiplicity** but very **different $[p_t]$** .

It is an isentropic transformation of the QGP which increases T and reduces R.

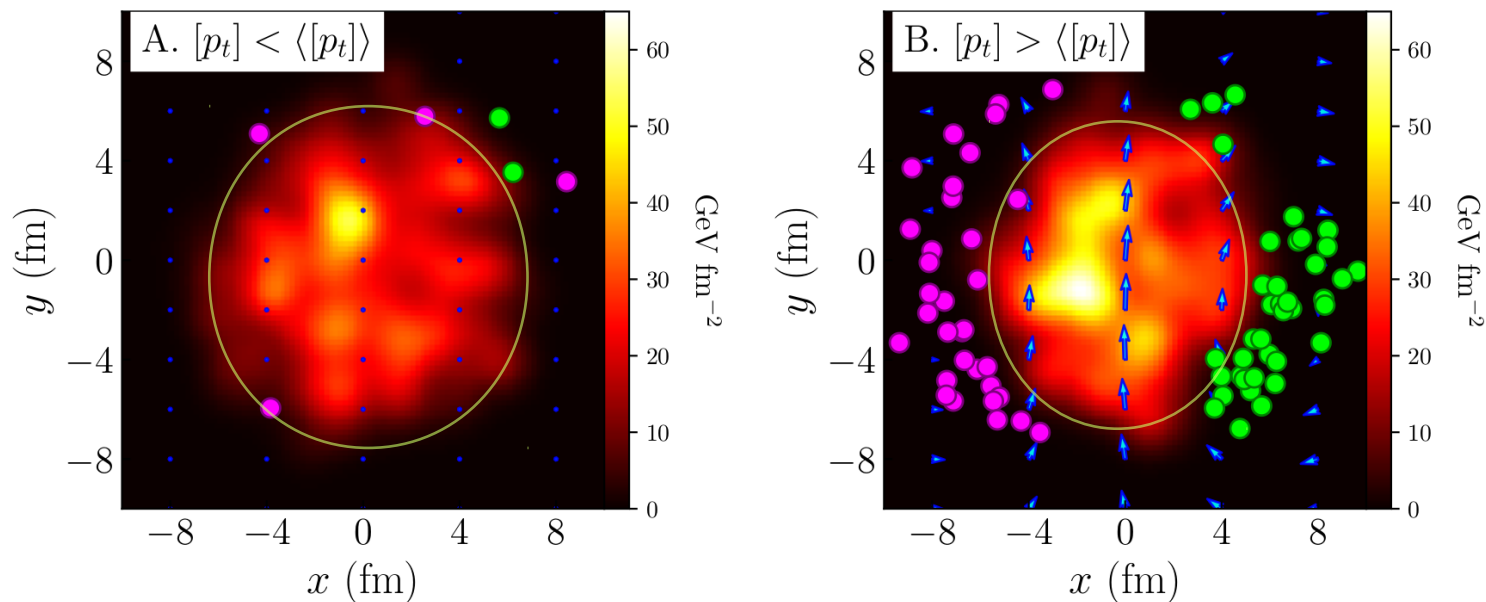


event	A	B
N_{ch}	2813	2791
b (fm)	0.49	4.41
N_s	6	72
$[p_t]/\langle [p_t] \rangle$	0.976	1.028
R (fm)	4.53	4.03
ε_2	0.073	0.153

Finding of [Giacalone, [2006.06269](#)]: **strong correlation between $[p_t]$ and $\langle N_s \rangle$.**
What about the magnetic field?

Event-by-event calculation of $\langle B_y \rangle$ with the framework developed in:

Consider only B field over overlap area: $\langle B_y \rangle = \frac{1}{E} \int d^2\mathbf{x} B_y(\mathbf{x}) e(\mathbf{x})$



event	A	B
N_{ch}	2813	2791
b (fm)	0.49	4.41
N_s	6	72
$[p_t]/\langle [p_t] \rangle$	0.976	1.028
$\langle B_y \rangle / m_\pi^2$	0.013	0.151
R (fm)	4.53	4.03
ε_2	0.073	0.153

The idea works! Increase $[p_t]$ and the B field appears!

AN OPTIMAL EVENT-SHAPE ENGINEERING (FOR CENTRAL COLLISIONS!)

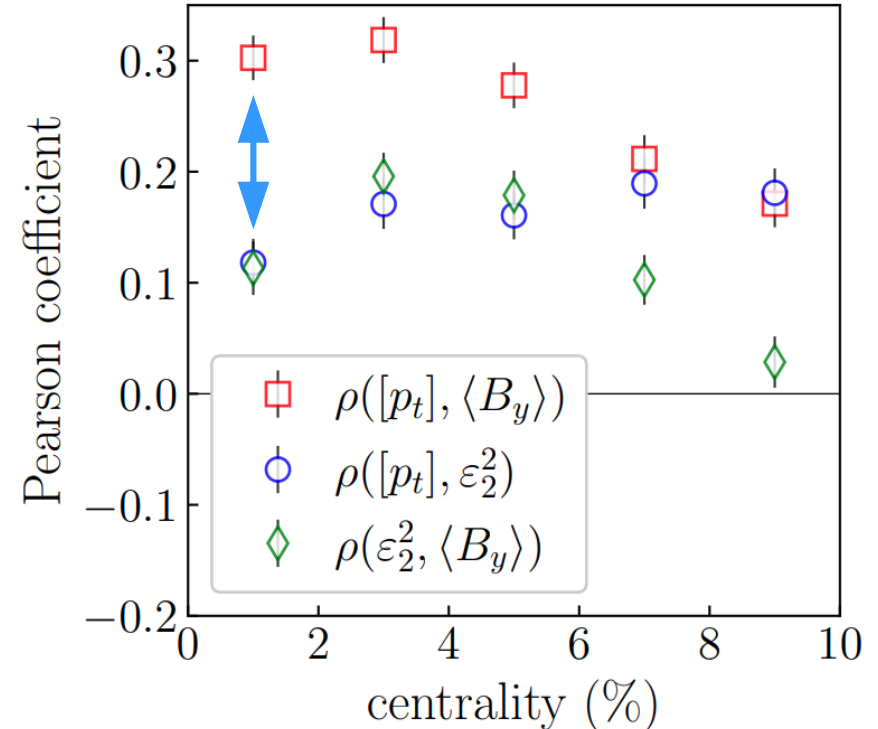
[Giacalone, Shen, [2104.xxxxx](#)]

$$- \rho([p_t], \langle B_y \rangle) > \rho(\varepsilon_2^2, \langle B_y \rangle)$$

Event selection based on $[p_t]$ provides a better handle on $\langle B_y \rangle$ than usual event selection based on q_2 vectors. ;-)

$$- \rho([p_t], \langle B_y \rangle) > \rho([p_t], \varepsilon_2^2)$$

Event selection based on $[p_t]$ increases $\langle B_y \rangle$ more than v_2 . Important for CME background.



NATURAL OBSERVABLE TO SEARCH FOR THE B FIELD (AND THE CME!)

[Giacalone, 2006.06269]

[Giacalone, Shen, 2104.xxxxx]

$$\left\langle \delta \langle p_t \rangle \cos (\phi_1^\pm - \phi_2^\pm) \right\rangle$$

- Correlation between charge-dependent v_1 and $\langle p_t \rangle$ at fixed multiplicity.
- Can be turned into a Pearson (Božek) coefficient:

$$\rho^\pm (\langle p_t \rangle, v_1^\pm) = \frac{\left\langle \delta \langle p_t \rangle \cos (\phi_1^\pm - \phi_2^\pm) \right\rangle}{\sqrt{\left\langle (\delta \langle p_t \rangle)^2 \right\rangle (g^\pm / v_2)}}$$

Where one should use:

$$g^\pm = \left\langle \cos (\phi_1^\pm + \phi_2^\pm - 2\phi_3) \right\rangle = (v_1^\pm)^2 v_2$$

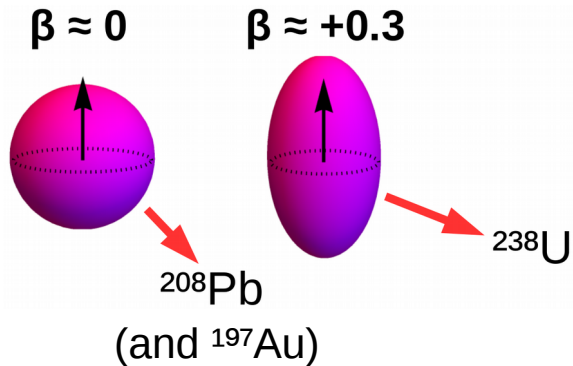
#2 – Nuclear structure: deformation.

Deformation: emergent property of nuclei (even for $J=0$). Nucleus as a deformed object. Quadrupole is the most important:

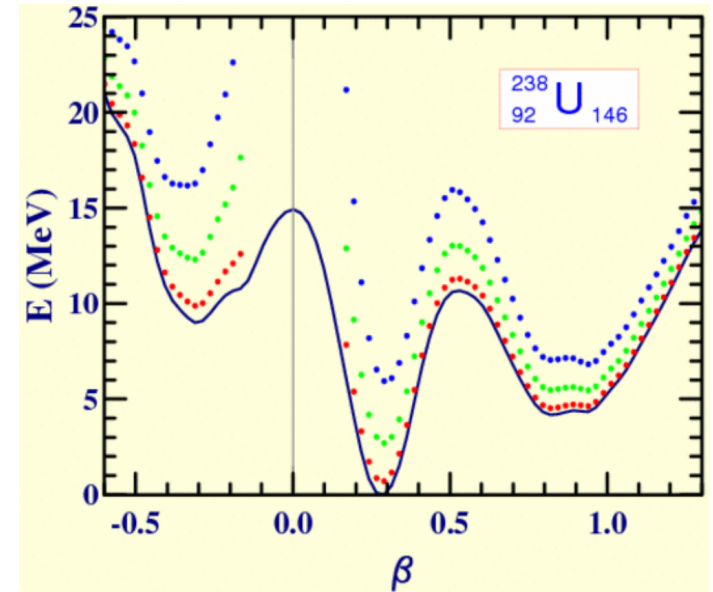
$$Q_2 \propto \langle Y_2^0(\Theta, \Phi) r^2 \rangle \neq 0$$

Quadrupole shape quantified by a coefficient:

$$\beta \propto \frac{Q_2}{\langle r^2 \rangle}$$



Hartree-Fock approach: in most nuclear systems, deformed configurations are energetically favored.

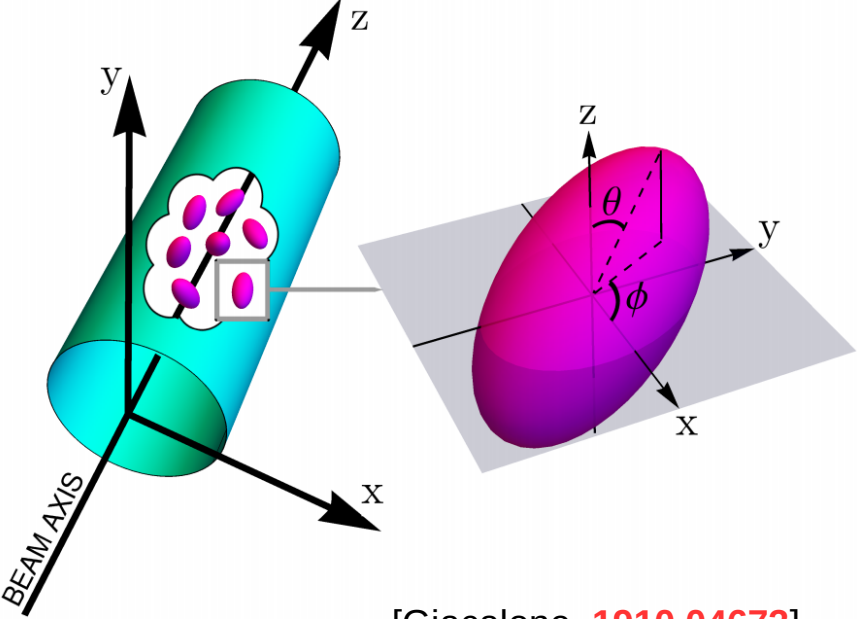


Rotational model: nucleus as an intrinsically deformed object with a random orientation.

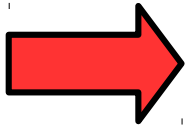
[Bohr, Mottelson 1957]

Nuclear structure and heavy-ion collisions: an inevitable marriage.

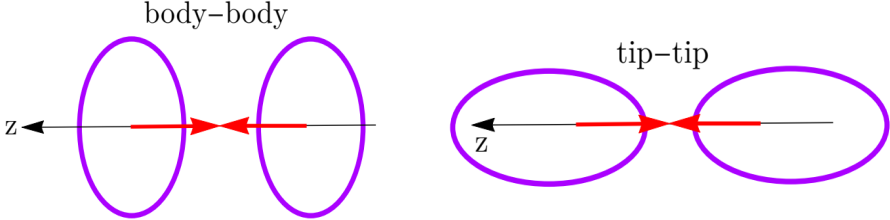
Deformed nuclei in the beampipe.



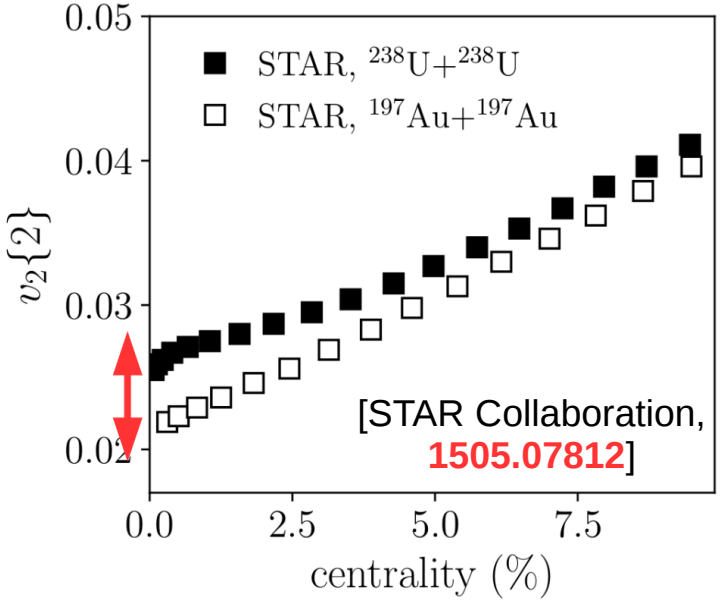
[Giacalone, 1910.04673]



New overlap geometries:

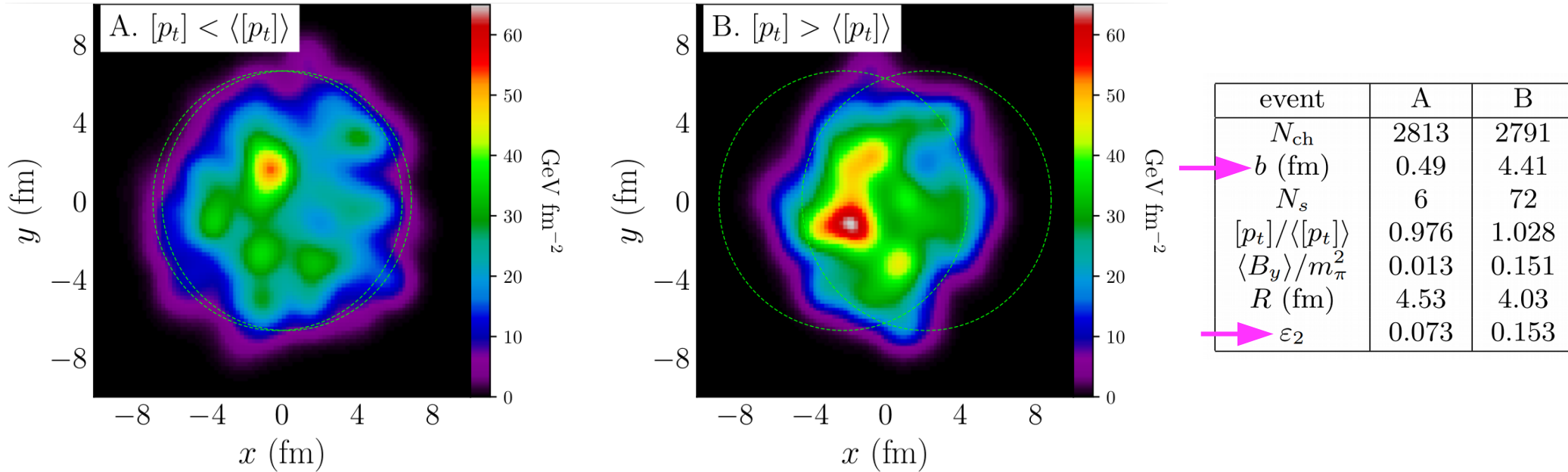


Prediction: contamination from body-body yields an excess v_2 in central U+U collisions. Verified in RHIC data!



Back to spherical nuclei.

Select two **central events** (2-3%) at the **same multiplicity** but very **different $[p_t]$** .

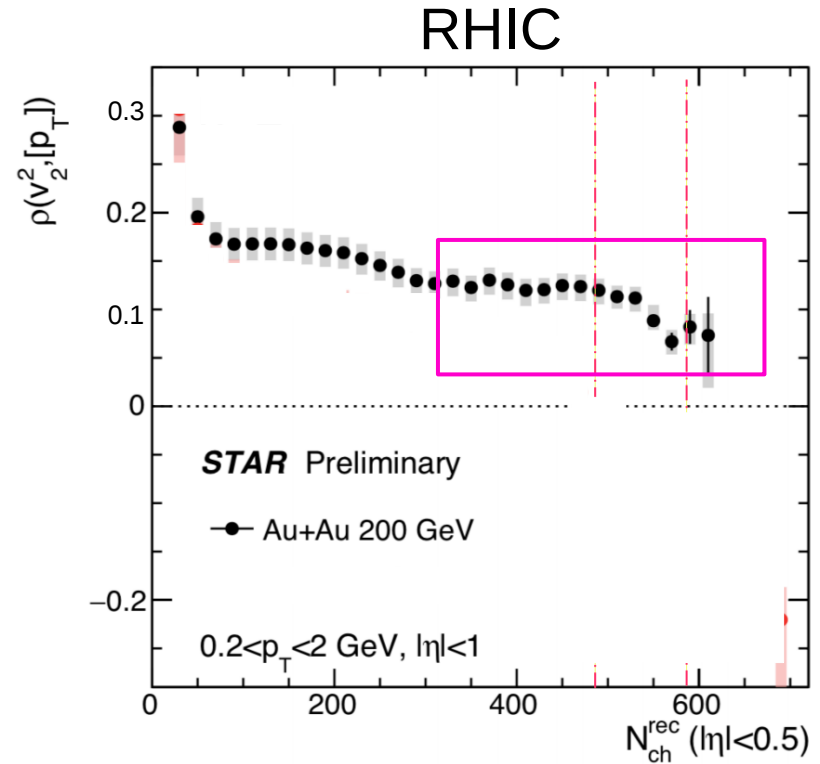
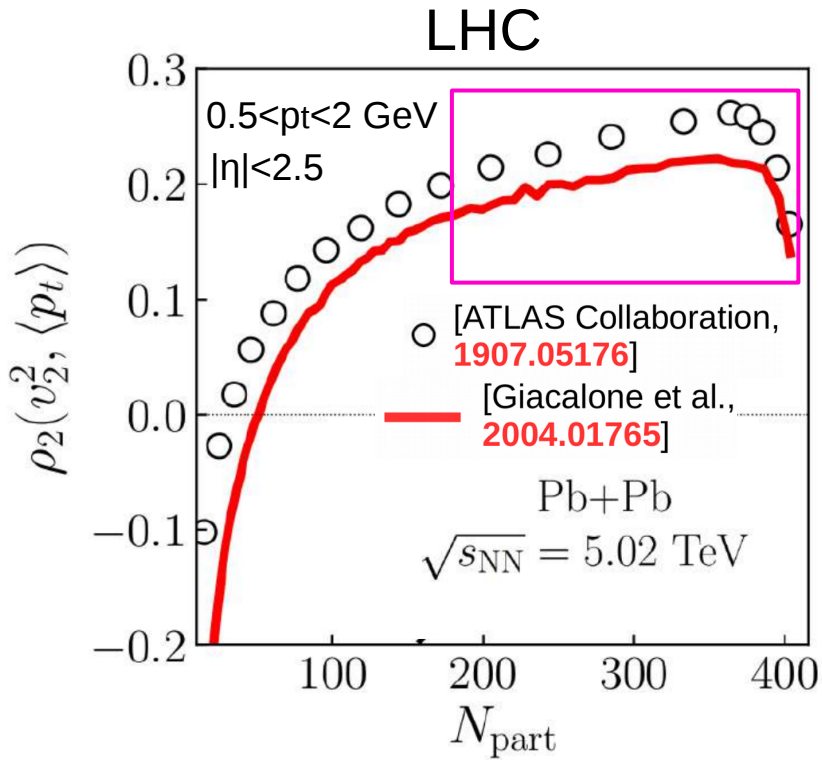


But also increases the impact parameter and eccentricity!

Prediction. In central heavy-ion collisions:

$$\rho(v_2^2, [p_t]) > 0$$

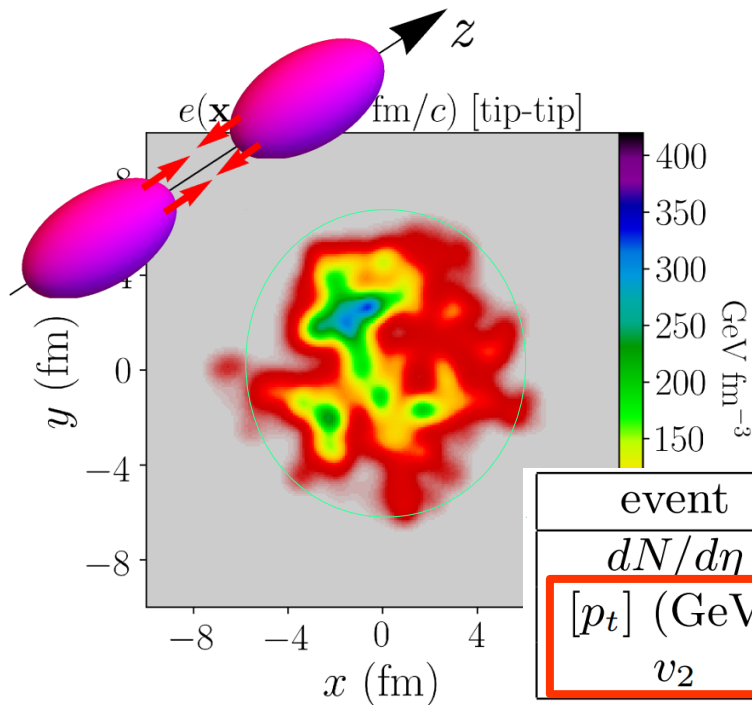
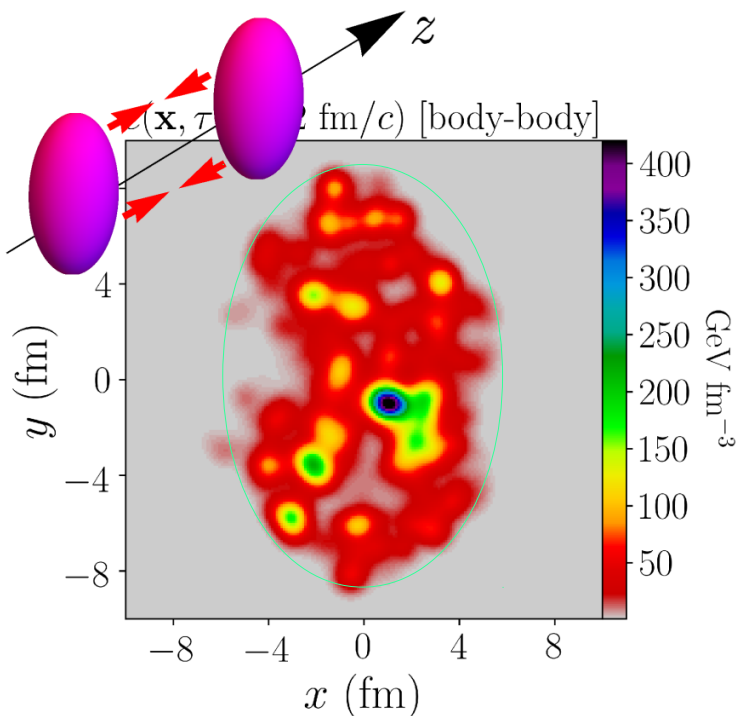
Prediction verified at LHC (Pb+Pb) and RHIC (Au+Au). Correlation is positive.



Insane differences between the two results... I'll get back to this.

WHAT IF THE COLLIDING NUCLEI ARE DEFORMED? $^{238}\text{U} + ^{238}\text{U}$ ($\beta=0.3$)

Select two **central events** (0.8-0.9%) at the **same multiplicity** but very **different** [p_t].



[Giacalone, [1910.04673](#)]

[Giacalone, [2004.14463](#)]

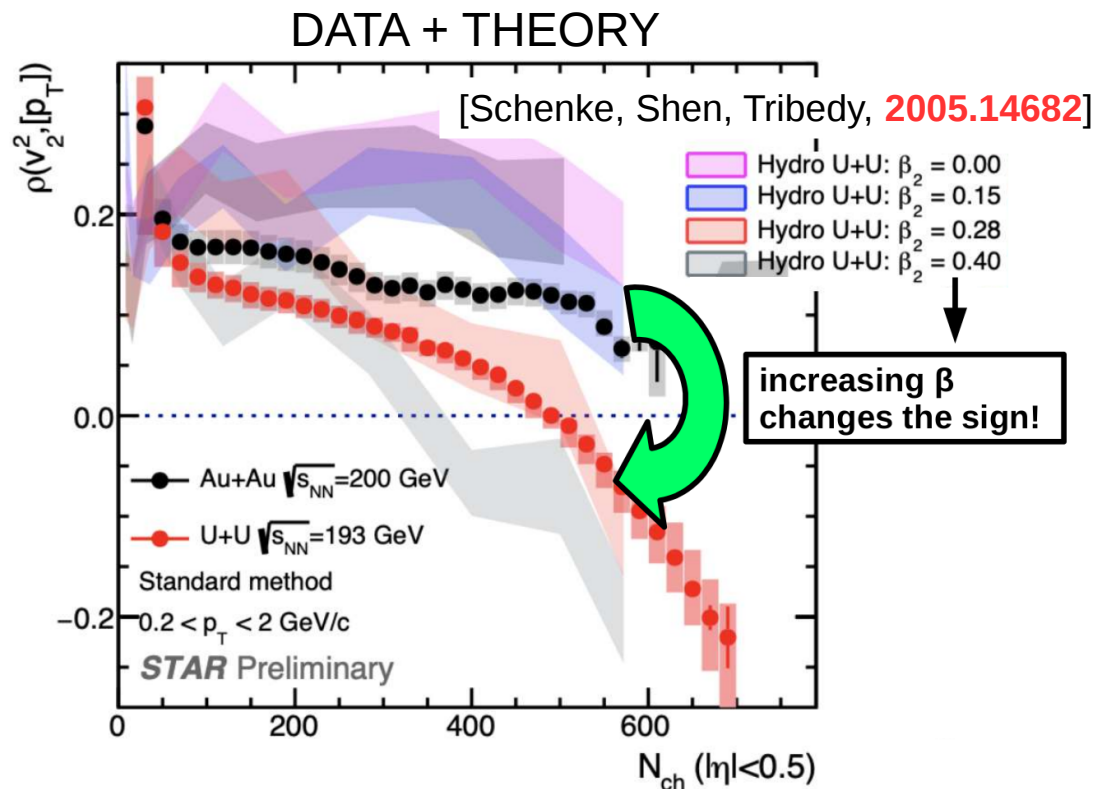
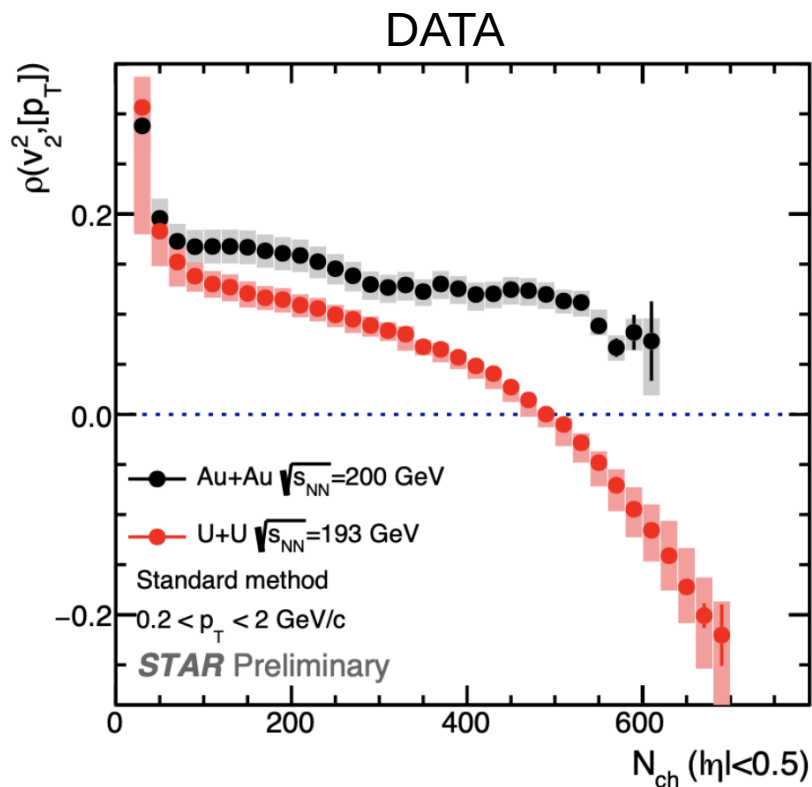
event	body-body	tip-tip
$dN/d\eta$	1296	1280
$[p_t]$ (GeV)	0.587	0.651
v_2	0.083	0.027

Body-Body: small $\langle p_t \rangle$, large v_2 . Tip-tip: large $\langle p_t \rangle$, small v_2 .

Prediction! In central collisions of deformed heavy ions:

$$\rho(v_2^2, [p_t]) < 0$$

Spectacular confirmation at RHIC. Correlation is positive in Au+Au, and negative in U+U.



Nuclear deformation interpretation confirmed by hydro calculations: $\beta \approx 0.3$
Brings consistency with low-energy nuclear structure theory.

We have isolated body-body collisions. A nontrivial result in nuclear physics.

PRACTICAL APPLICATION

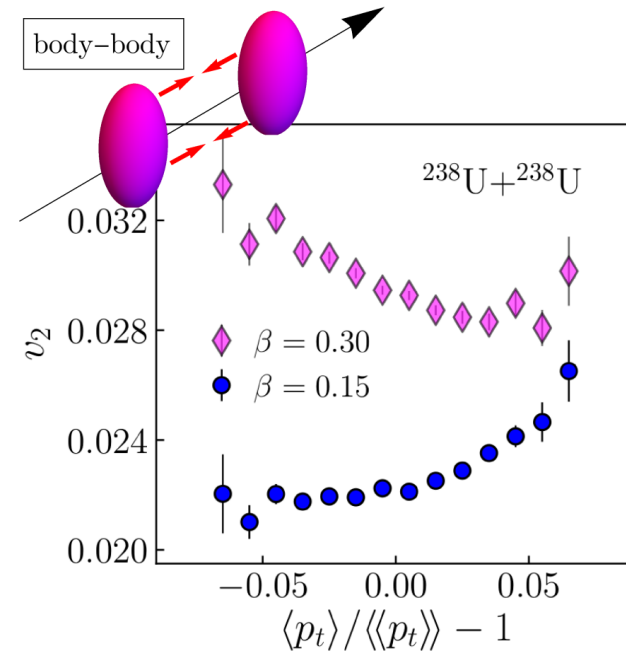
- Ultra-sensitive to the value of β . High-energy experiments as a new probe of the low-energy structure.

DEFORMATION: INTERPRETATION

- Quantum nature of nuclei:

“Body-body collisions” are only a simplification: the nucleus is $J=0$ (spherical!) Exact wavefunction in any basis of (non-deformed) states, should give the same results.

But the QGP and its eccentricity are real entities! Can you get the right $\langle \varepsilon_2 \rangle$ -R correlations without deformed densities?



#3 – Primordial momentum anisotropies.

So far I have considered $V_n \propto \mathcal{E}_n$, where \mathcal{E}_n is the anisotropy of the energy density. The **primordial** energy-momentum tensor contains more structures.

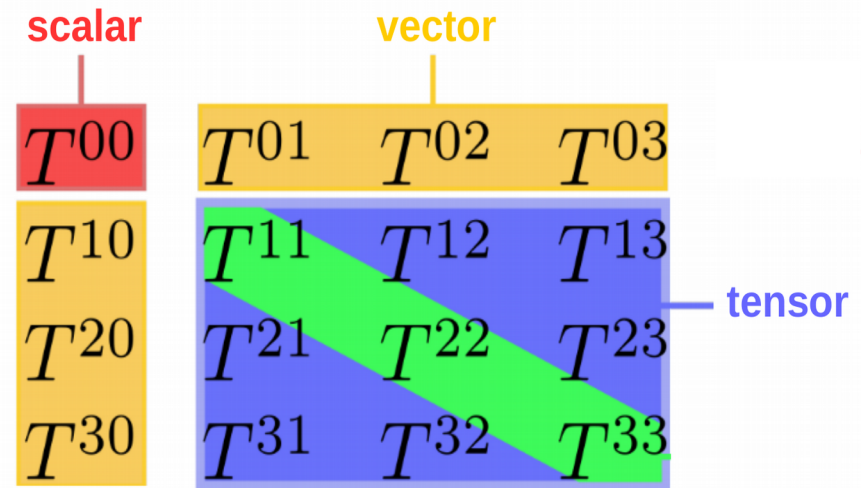
SCALAR

$$\mathcal{E}_2 = \frac{\langle x^2 - y^2 \rangle + i\langle 2xy \rangle}{\langle x^2 + y^2 \rangle}$$

TENSOR

$$\mathcal{E}_p \equiv \frac{\langle T^{xx} - T^{yy} \rangle + i\langle 2T^{xy} \rangle}{\langle T^{xx} + T^{yy} \rangle}$$

(vector is less important)



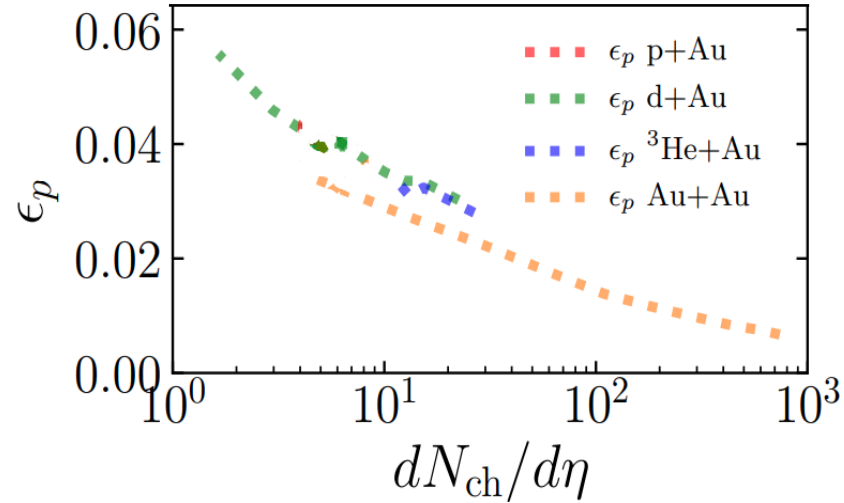
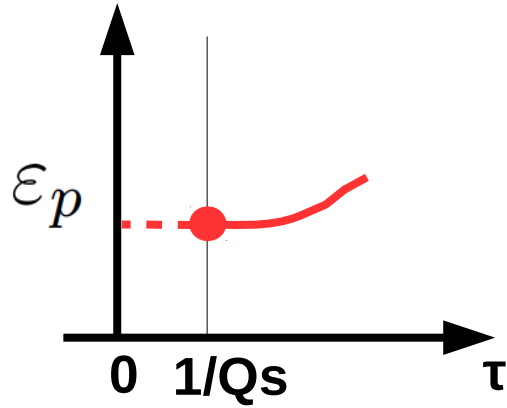
[Sousa, Luzum, Noronha, [2002.12735](#)]

The color glass condensate predicts the ‘off-diagonal’ contribution.

Longstanding question in the field: do we see initial-state CGC anisotropy in the data?

[Altinoluk, Armesto, [2004.08185](#)]

Primordial anisotropy. Evaluations in the IP-GLASMA+MUSIC+urQMD framework.



Assume now the following decomposition:

$$V_2 = \kappa_2 \mathcal{E}_2 + \kappa_p \mathcal{E}_p$$

dominates for
 $dN/d\eta > 10$

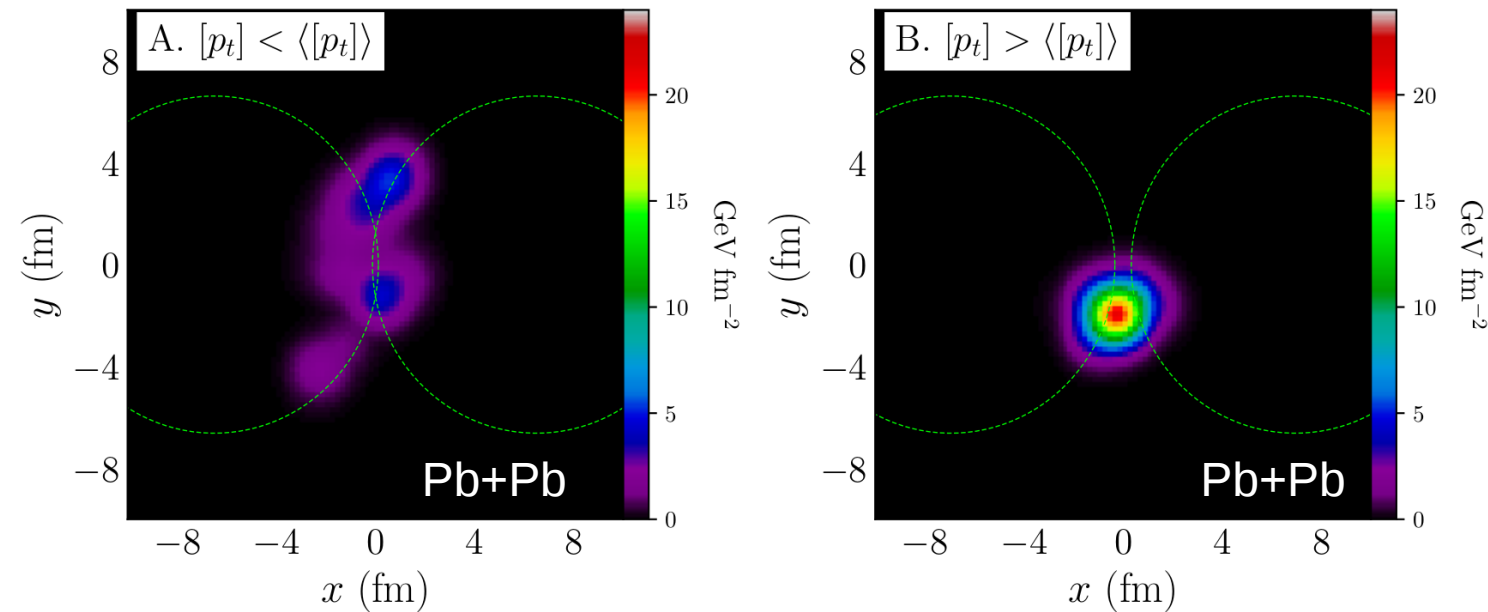
dominates for
 $dN/d\eta < 10$

[Schenke, Shen, Tribedy, [1908.06212](#)]

How to probe the new contribution at small $dN/d\eta$? ... we can use $\langle p_T \rangle$!

Select two **peripheral events** (69-70%) at the **same multiplicity** but very **different $[p_t]$** .

It is an isentropic transformation of the QGP which increases T and reduces R.



event	A	B
N_{ch}	133.5	133.5
b (fm)	13.0	13.9
N_s	392	405
$[p_t]/\langle [p_t] \rangle$	0.907	1.143
R (fm)	2.97	1.34
ε_2	0.675	0.133

@large $[p_t]$: hot spots clustered around one transverse point. Very round system.

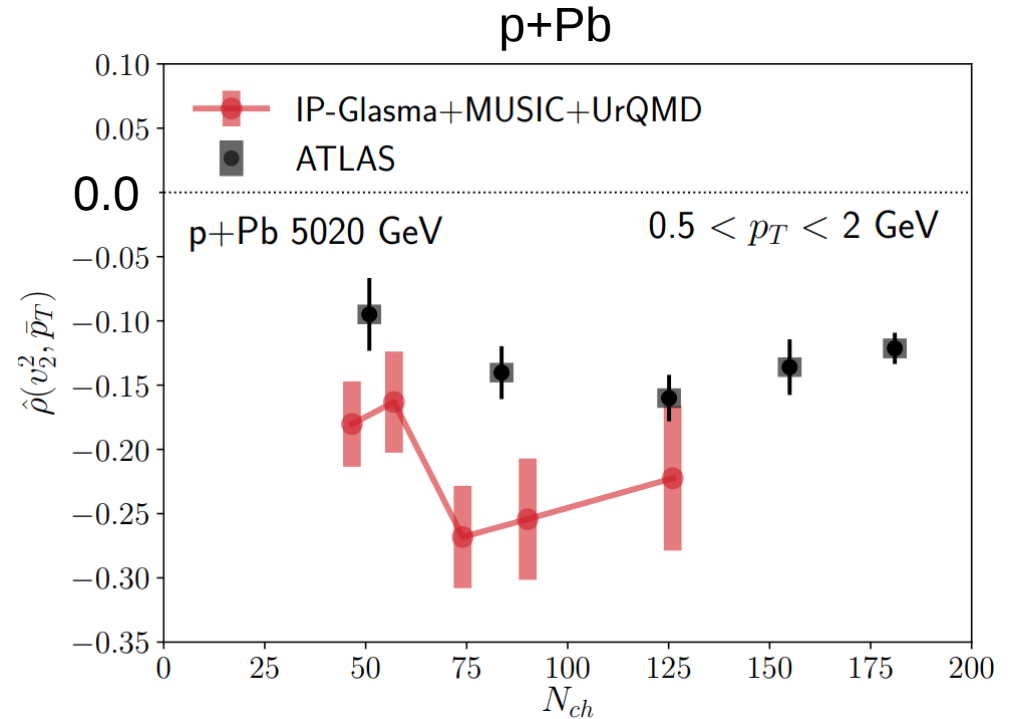
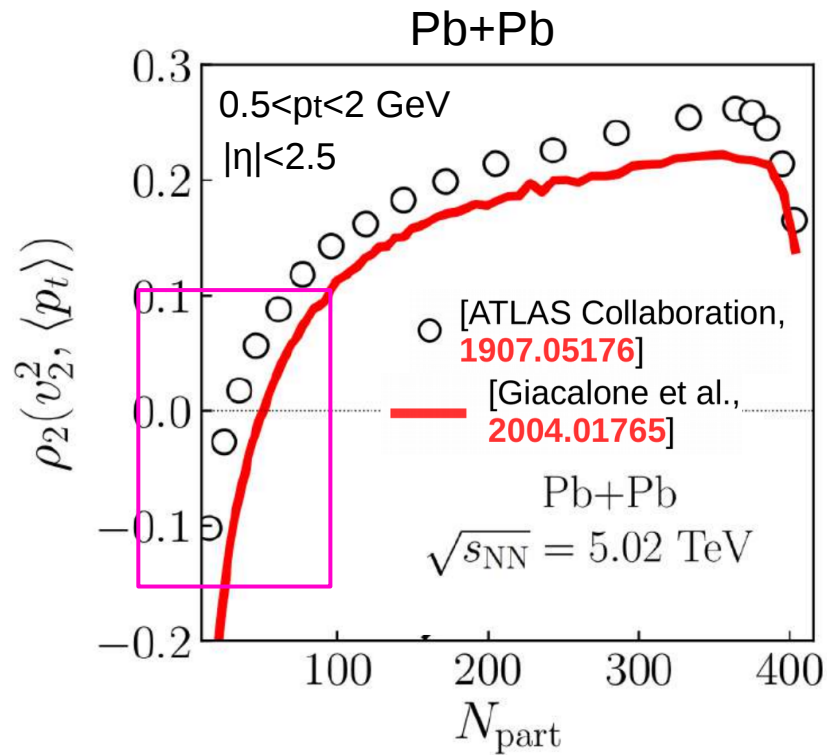
Prediction. In small systems:

$$\rho(v_2^2, [p_t]) < 0$$

[Božek, Mehrabpour, **2002.08832**]

[Schenke, Shen, Teaney, **2004.00690**]

Verified at LHC. Correlation is negative. Captured by hydrodynamic models.

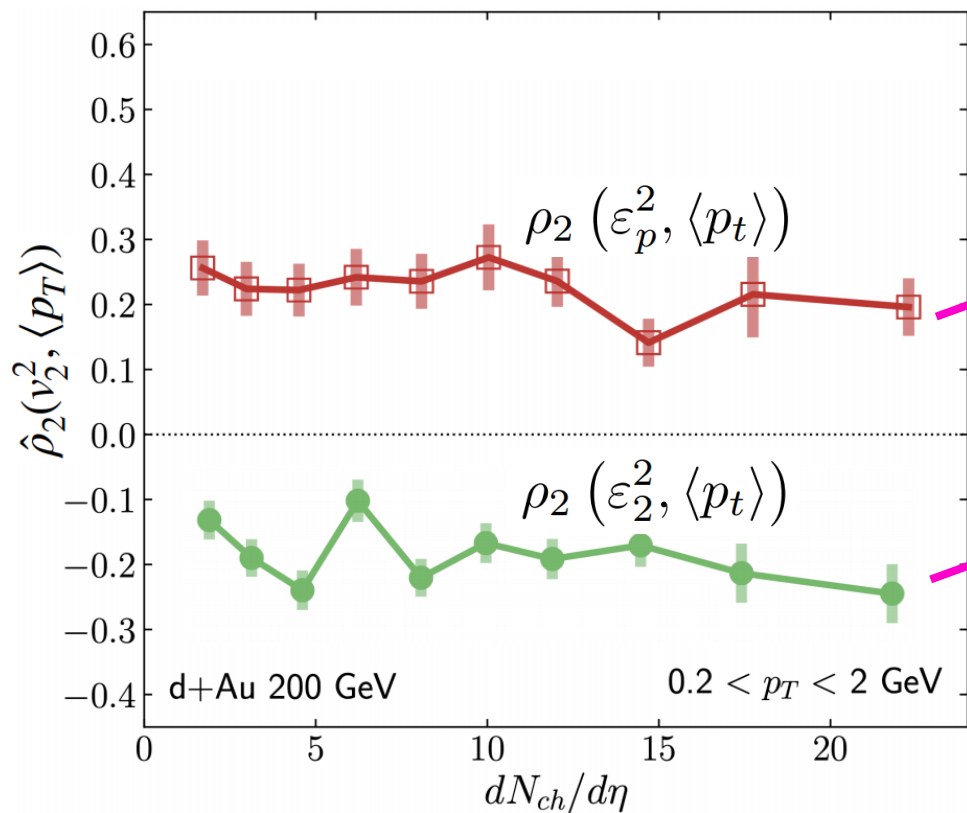


[Schenke, Shen, Teaney, [2004.00690](#)]

What about Au+Au at RHIC? I'll get to that...

But if we consider $V_2 = \kappa_2 \mathcal{E}_2 + \kappa_p \mathcal{E}_p$, we expect:

$$\rho(v_2^2, [p_t]) = \kappa_2^2 \rho(\varepsilon_2^2, [p_t]) + \kappa_p^2 \rho(\varepsilon_p^2, [p_t])$$



Analyze the contributions separately:

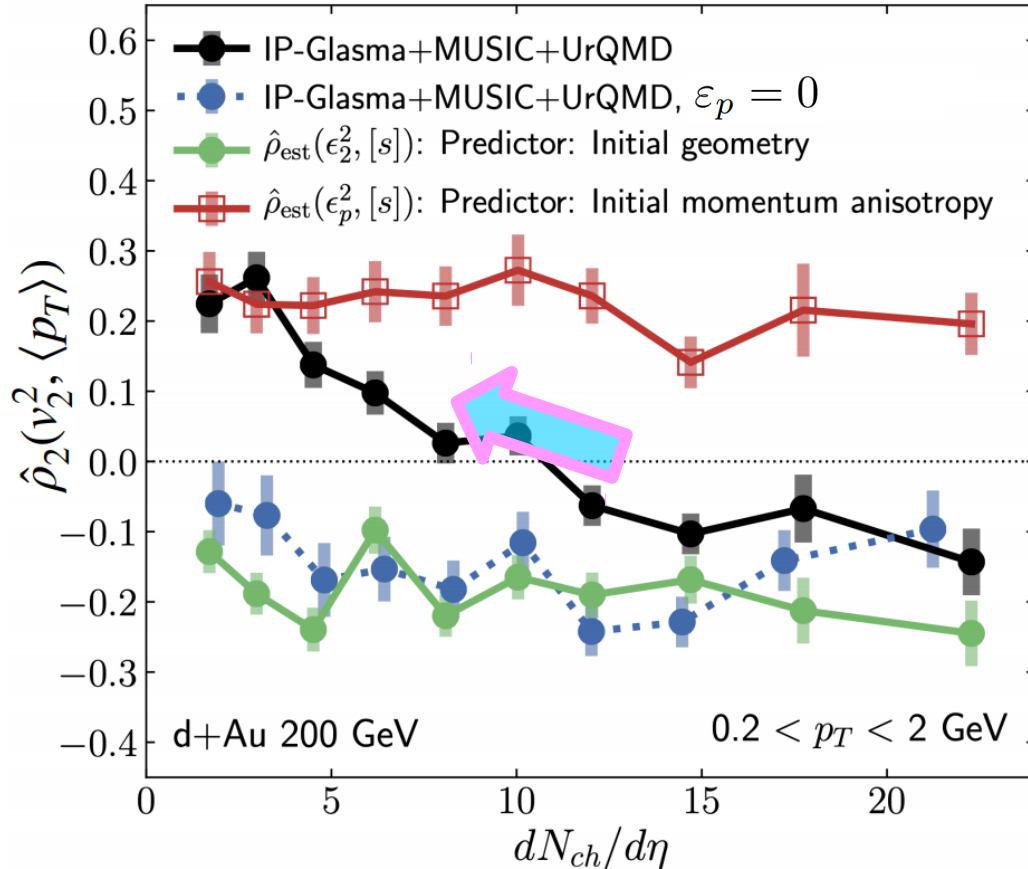
Positive!

A new feature of high-energy QCD.
(consistent with color-domain picture)

Negative. As expected.

The contributions are qualitatively different.

IP-Glasma+Hydro prediction for small systems.



[Giacalone, Schenke, Shen, [2006.15721](#)]

– Sign change occurring as expected around $dN/d\eta=10$.

A neat prediction.

– No sign change if we set $E_p=0$.

– CGC result is solid, but non-flow mimics the signal, and has to be addressed carefully.

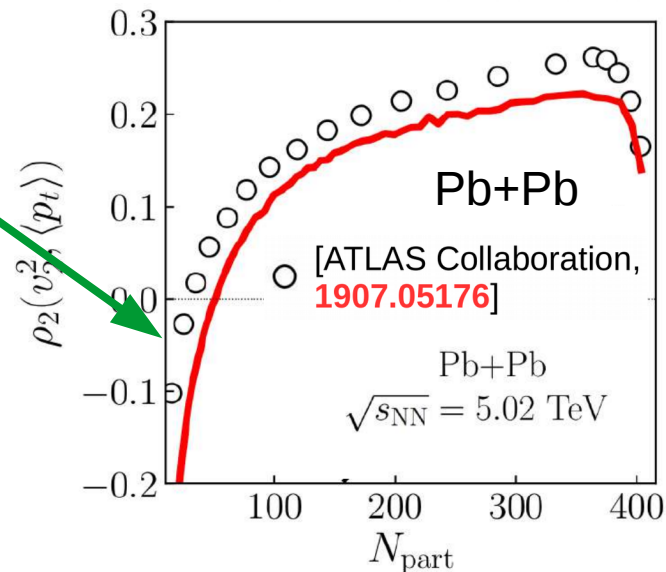
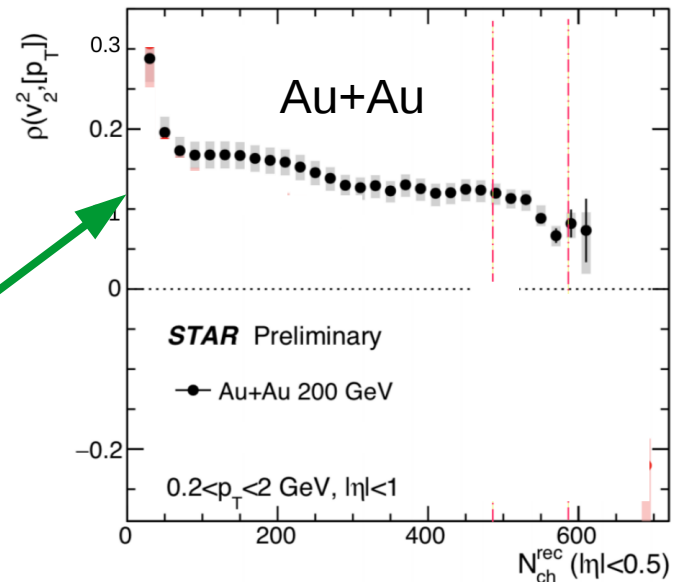
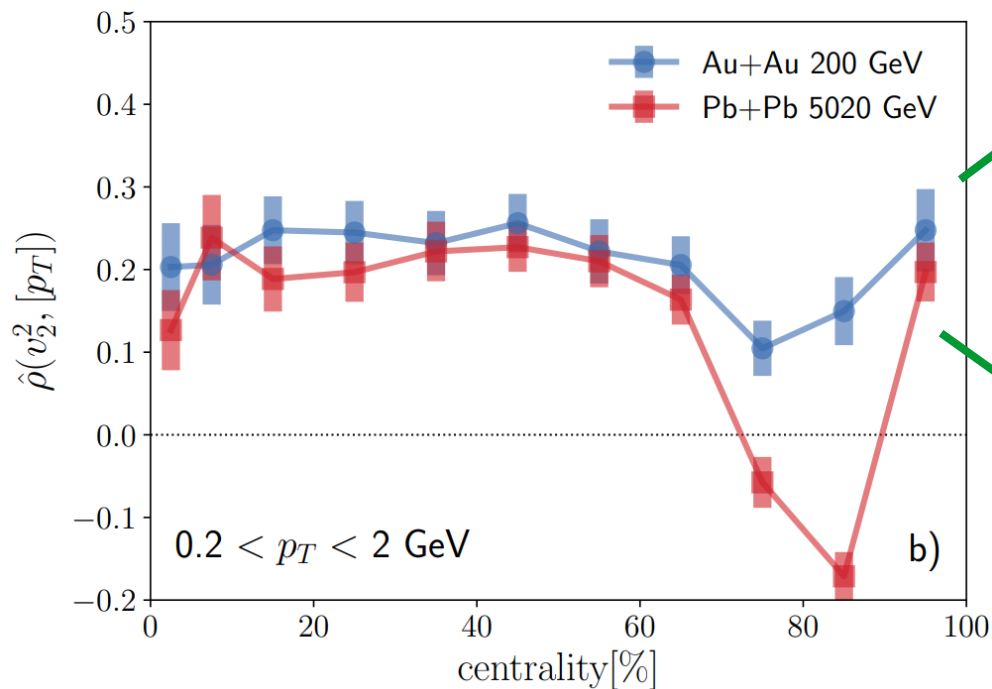
[Behera, Bhatta, Jia, Zhang, [2102.05200](#)]

[Lim, Nagle, [2103.01348](#)]

Prediction for peripheral large systems.

[Giacalone, Schenke, Shen, [2006.15721](#)]

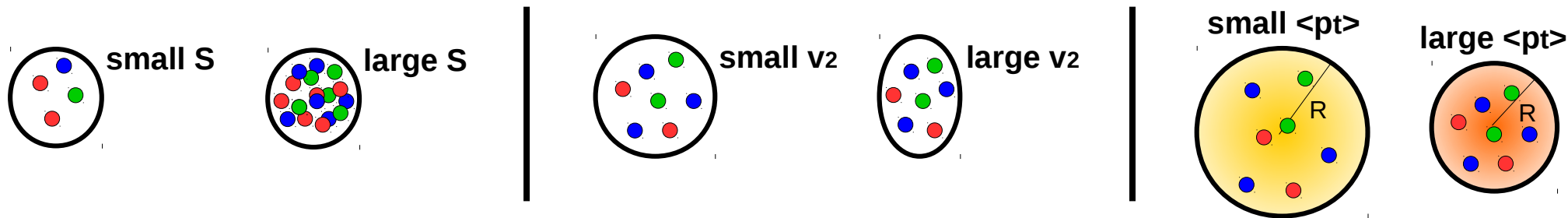
Shorter lifetime makes E_p more important at RHIC.
Sign change does not occur at RHIC energy.



So far consistent with preliminary data.

SUMMARY

– Established picture of the QGP based on hydrodynamics.



– We can study nuclear phenomena beyond QGP by means of $[p_t]$ - v_n correlations:

—► **New probe of strong EM fields:** $\rho^\pm (\langle p_t \rangle, v_1^\pm)$

—► **Polarizing 'deformed' mass densities (body-body):** $\rho(v_2^2, [p_t]) < 0$

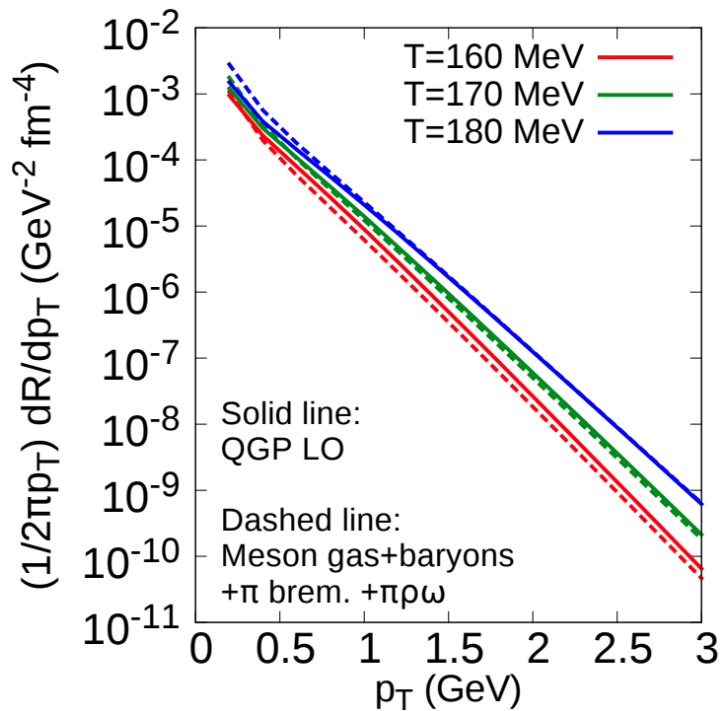
—► **Manifestation of primordial momentum anisotropy:** $\rho(\varepsilon_p^2, [p_t]) > 0$

But there is more...

OUTLOOK

Thermal photon yield, N .
Depends on temperature.

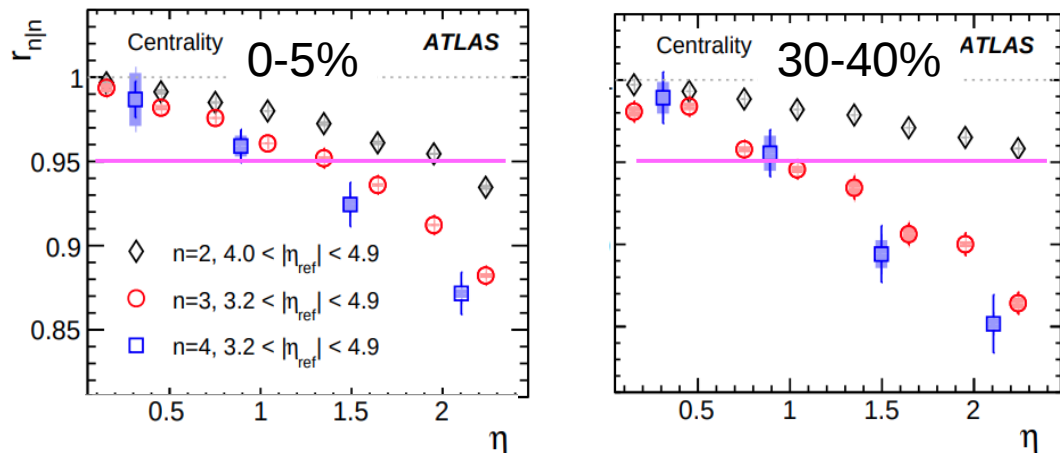
prediction: $\rho(\langle p_T \rangle, N) > 0$.



[Paquet, Shen, Denicol, Luzum, Schenke, Jeon, Gale, [1509.06738](#)]

Longitudinal de-correlations.

Smaller de-correlation at large impact parameters.



[ATLAS Collaboration, [2001.04201](#)]

prediction:

events with large $\langle p_T \rangle$ de-correlate less.

More candidates:

- vorticity?
- energy loss?
- ...
- any other ideas?

**CORRELATIONS WITH $\langle p_T \rangle$:
EXCITING PROSPECTS
FOR THE NEXT DECADE.**

THANK YOU!