RHIC-BES seminar, Jan.26,2021

Many-body forces and nucleon clustering near the QCD critical point

Edward Shuryak
Stony Brook

ES and J. M. Torres-Rincon, Phys. Rev. C100, 024903 (2019), arXiv:1805.04444 [hep-ph].

ES and J. M. Torres-Rincon, Phys. Rev. C101, 034914 (2020), arXiv:1910.08119 [nucl-th].

D. DeMartini and ES (2020), arXiv:2007.04863 [nucl-th].

Many-body forces and nucleon clustering near the QCD critical point D. DeMartini and E. Shuryak, e-Print: 2010.02785

1. Introduction: suggestion of BES, critical event-by-event fluctuations
2. The main idea: preclusters may have sizes comparable to corr.length

4. Nucleon clustering using various tools, importance of 4 N systems , kurtosis and viral expansion
5. Repulsive manybody forces near CP, estimates in Landau model
6. universal effective action for Ising universality class
7. Deformed universal effective action
8. Summing all effects near CP
9. EXPERIMENTAL OBSERVABLES: Kurtosis and t*p/d^2 plots: where CP MAY be located?

Two paradoxes:

Paradox 1: Imagine xi is as large as the fireball:
then scalar attractive binary forces get huge O(N^2):
collapse? must be wrong…

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Paradox 2: light nuclei have small bindings, => large sizes and very fragile
Cascades predict they must be destroyed by hundreds of pions.
and yet they are produced

Introduction

The original ideas:
Look for event-by-event fluctuations
Perform beam energy scan
Watch for non-monotonous signals

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Phys. Rev. Lett. 81, 4816 (1998), arXiv:hep- ph/9806219 [hep-ph].

- **Higher moments of the critical field**

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$$\kappa_2 = \langle \phi^2 \rangle, \quad \kappa_3 = \langle \phi^3 \rangle, \quad \kappa_4 = \langle \phi^4 \rangle - 3\langle \phi^2 \rangle^2$$

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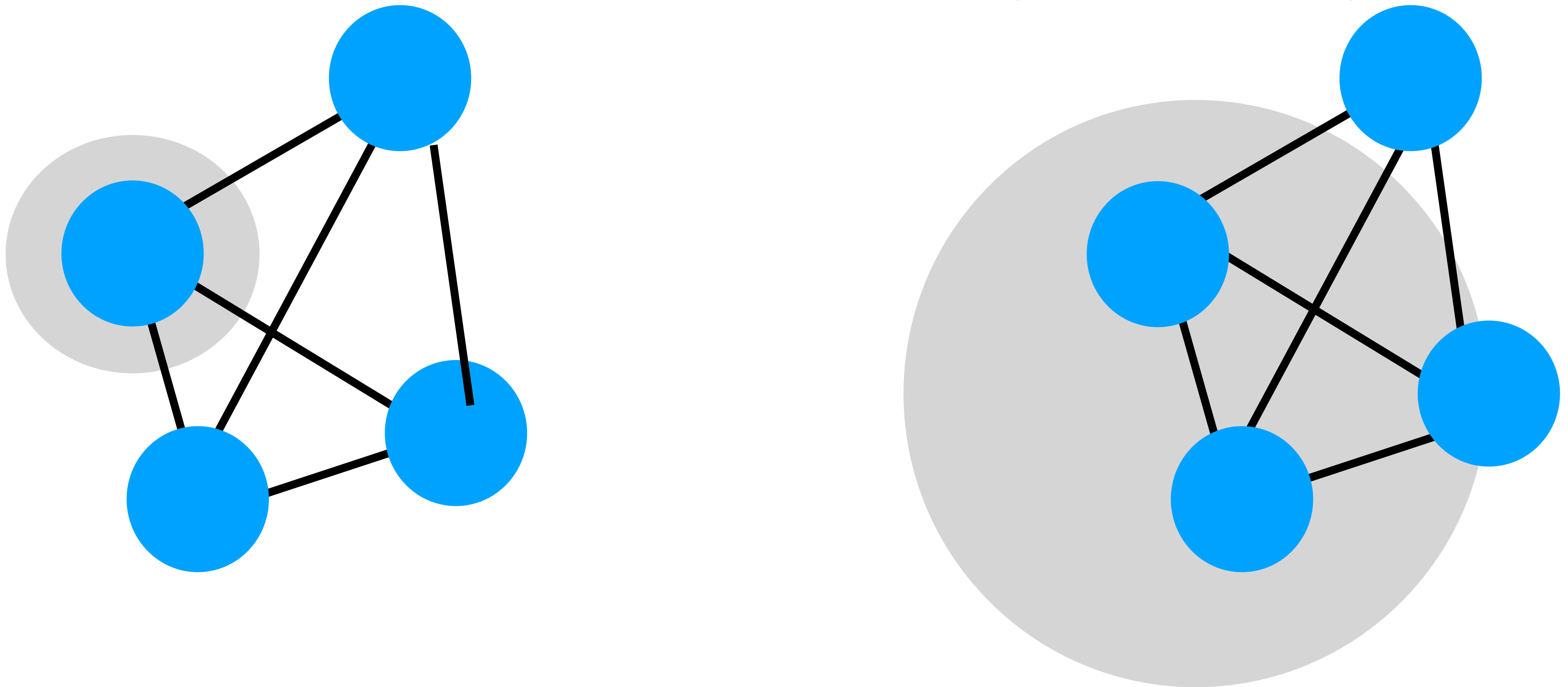
Yet one cannot directly measure moments of phi…
they are related to moments of nucleon multiplicity distribution, but not trivially

So far estimates relied on the assumption that nucleons
are correlated ONLY due to near-CP fluctuations ,
which is of course not the case

•The main idea of this work:

Suppose the CP indeed exists, and is located in the part of the phase diagram near the freezeout line of BES program collisions. Furthermore, while scanning this line, for some specific beam energy one happens to be in a state in which the correlation length reaches a value $\xi max \sim 1.5\text{-}2fm$. What observables are sensitive to such scale of ξ?

$\xi_{min} \sim 1/m_\sigma \sim 0.4\, fm$ $\qquad$ $\xi_{max} \sim 2\, fm$



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Side remark: too many domains. sound waves which we observed have the wavelength much larger than 2 fm, 2piR/m =6fm or more

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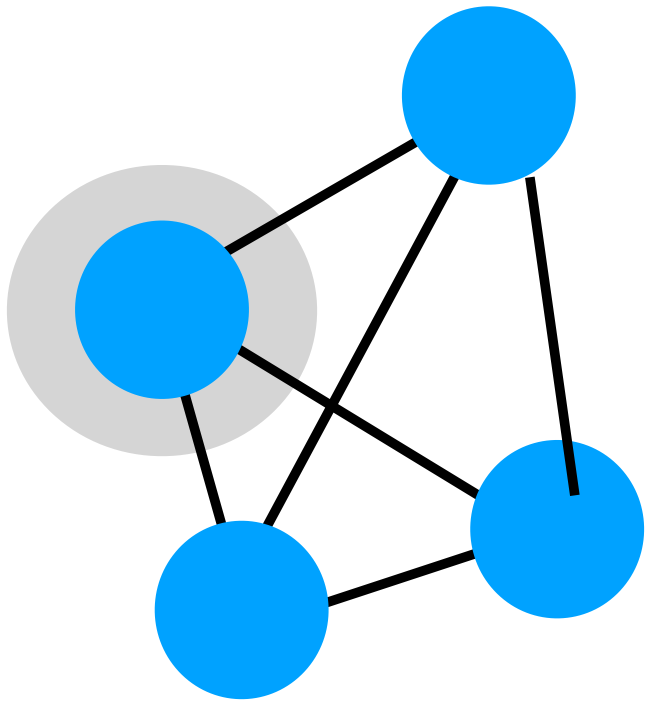

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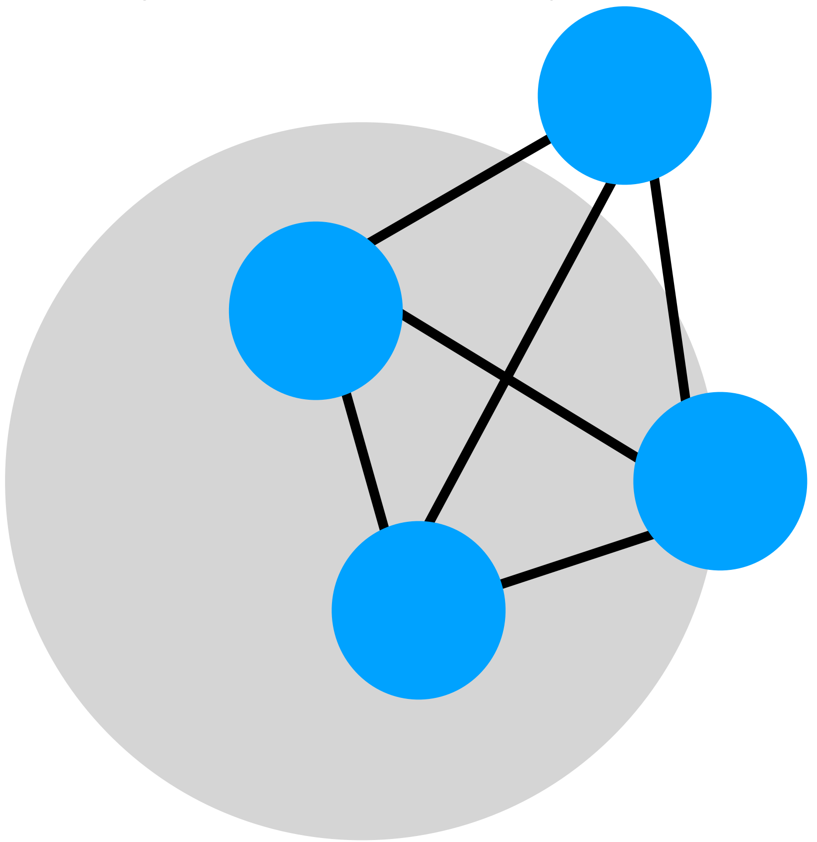

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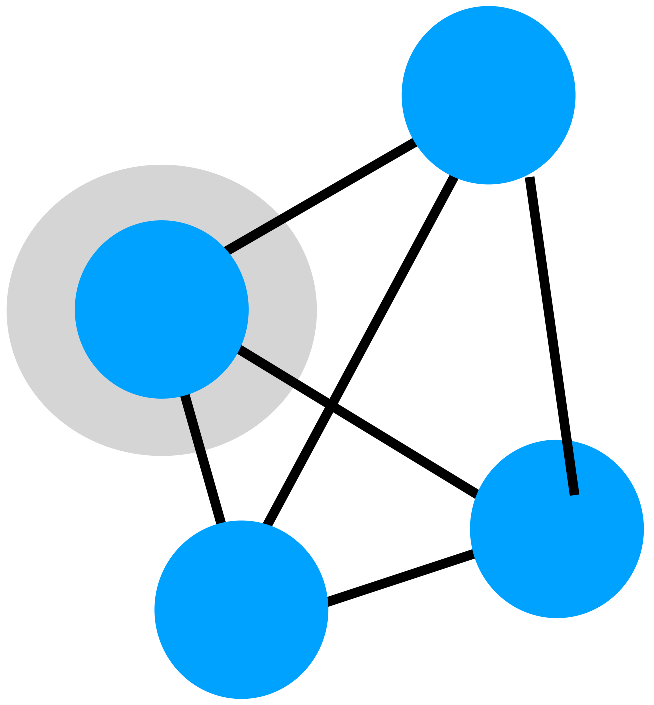

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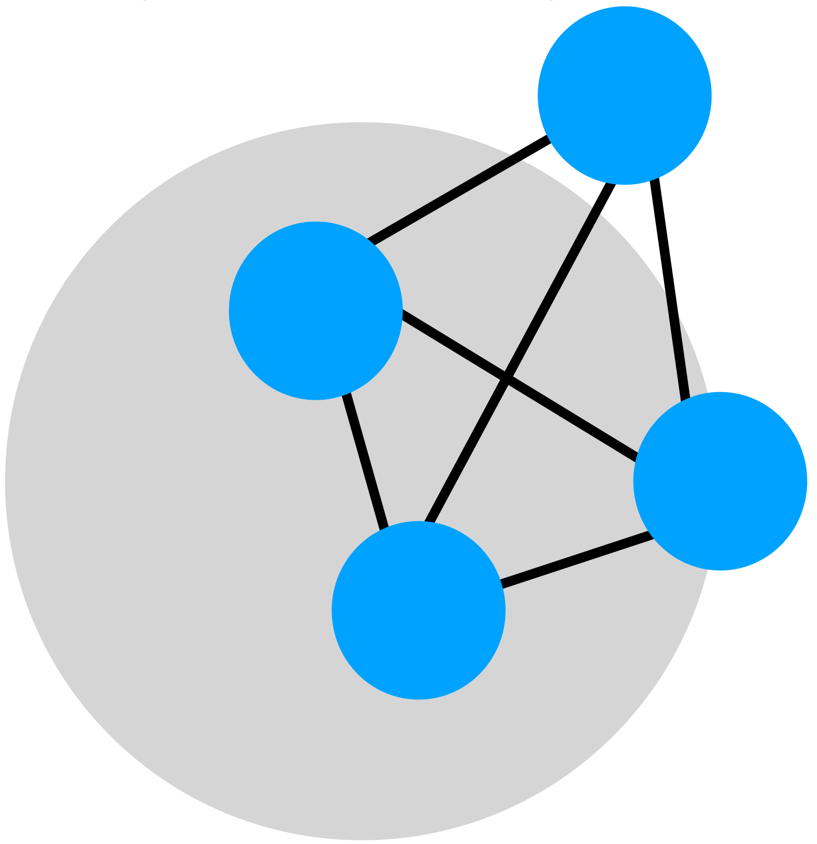

Pre-clustering of nucleons create objects of the right scale !
Their energy — and therefore production yield — is very sensitive to correlation length

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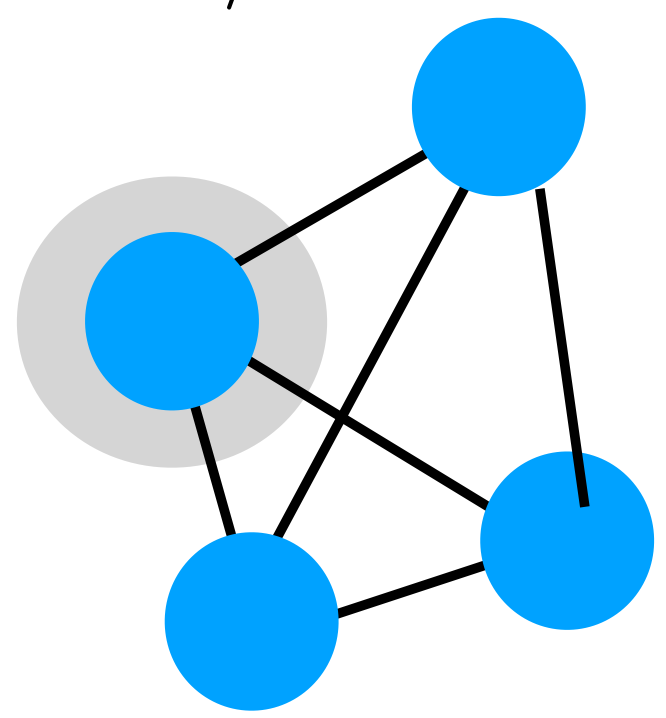

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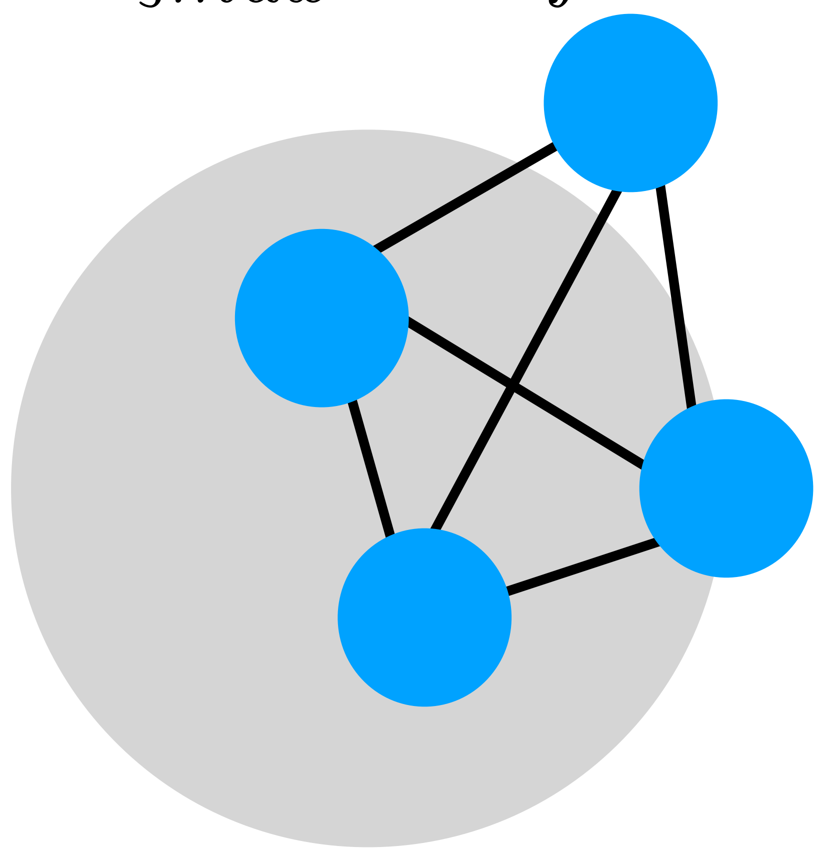


Pre-clustering of nucleons create objects of the right scale !
Their energy — and therefore production yield — is very sensitive to correlation length

As we will show, the interplay of attractive binary And repulsive manybody forces Will lead to very non-monotonous signal

- Studies of few-nucleon pre-clustering at freezeout conditions were done by a number of theoretical approaches

Classical molecular dynamics

- *E. Shuryak and J. M. Torres-Rincon, Phys. Rev. C100, 024903 (2019), arXiv:1805.04444 [hep-ph].*

Semiclassical approximation (fluctons) At finite temperatures

- *E. Shuryak and J. M. Torres-Rincon, Phys. Rev. C101, 034914 (2020),*
- *arXiv:1910.08119 [nucl-th].*

K-harmonics => radial Schreodinger equations in 3*(N-1) dimensions

Direct Path Integral Monte Carlo (PIMC) Numerical simulations

- *D. DeMartini and E. Shuryak, (2020), arXiv:2007.04863 [nucl-th].*

- “clusters” => some bound states, maybe resonances
- “preclusters” are statistical correlations in density matrix

• (the first time ever) testing the flucton method at finite T

Fluctons for anharmonic oscillator at $T \neq 0$

$$S_E = \oint d\tau \left(\frac{\dot{x}^2}{2} + \frac{x^2}{2} + \frac{g}{2} x^4 \right) .$$

•the usual density matrix (line, 60 states)

$$P(x_0) = \sum_i |\psi_i(x_0)|^2 \; e^{-E_i/T}$$

$$P(x_0) \sim exp\big(- S_E[x_{flucton}(\tau)]\big)$$

• (points on the plot)

•so, the method works very well

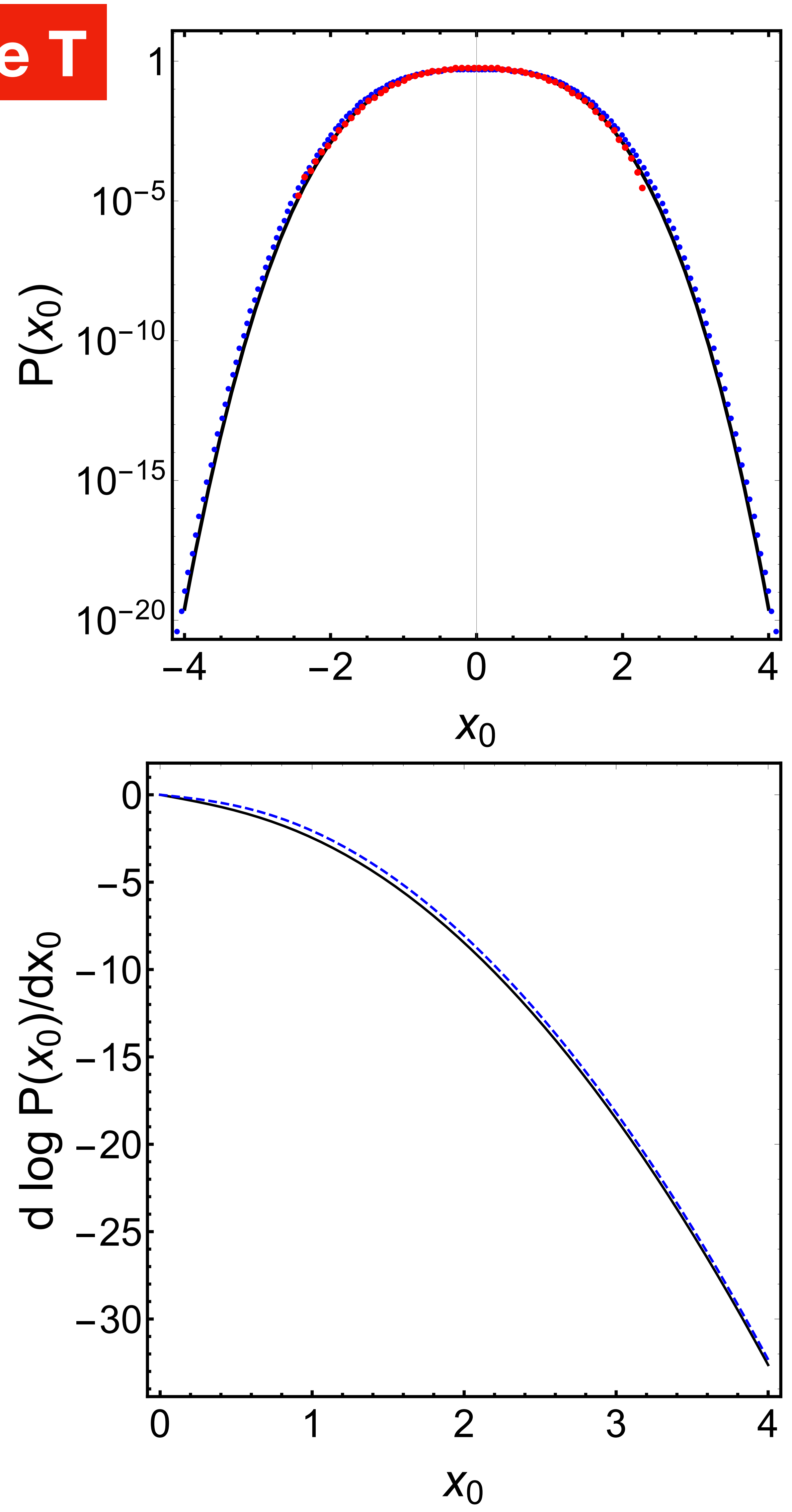


FIG. 3: Top panel: Density matrix $P(x_0)$ vs x_0 for anharmonic oscillator with the coupling $g = 1$, at temperature $T = 1$, calculated via the definition (1) (line) and the flucton method (points). The line is based on 60 lowest state wave functions found numerically. Bottom panel: Comparison of the logarithmic derivative of the density matrix of the upper panel.

- **K-harmonics applied to He4 (not a new method,**
- **and yet we found something new with it…)**

- **9 Jacobi coordinates for 4 particles**

$$\vec{\xi}[1]=\frac{\vec{x}[1]-\vec{x}[2]}{\sqrt{2}},\quad \vec{\xi}[2]=\frac{\vec{x}[1]+\vec{x}[2]-2\vec{x}[3]}{\sqrt{6}},$$

$$\vec{\xi}[3]=\frac{\vec{x}[1]+\vec{x}[2]+\vec{x}[3]-3\vec{x}[4]}{2\sqrt{3}}$$

hyperdistance
in 9 dimensional space
Is sum of squares of all 6
Distances

$$\rho^2=\sum_{m=1}^{3}\vec{\xi}[m]^2=\frac{1}{4}\Big(\sum_{i\neq j}(\vec{x}[i]-\vec{x}[j])^2\Big)$$

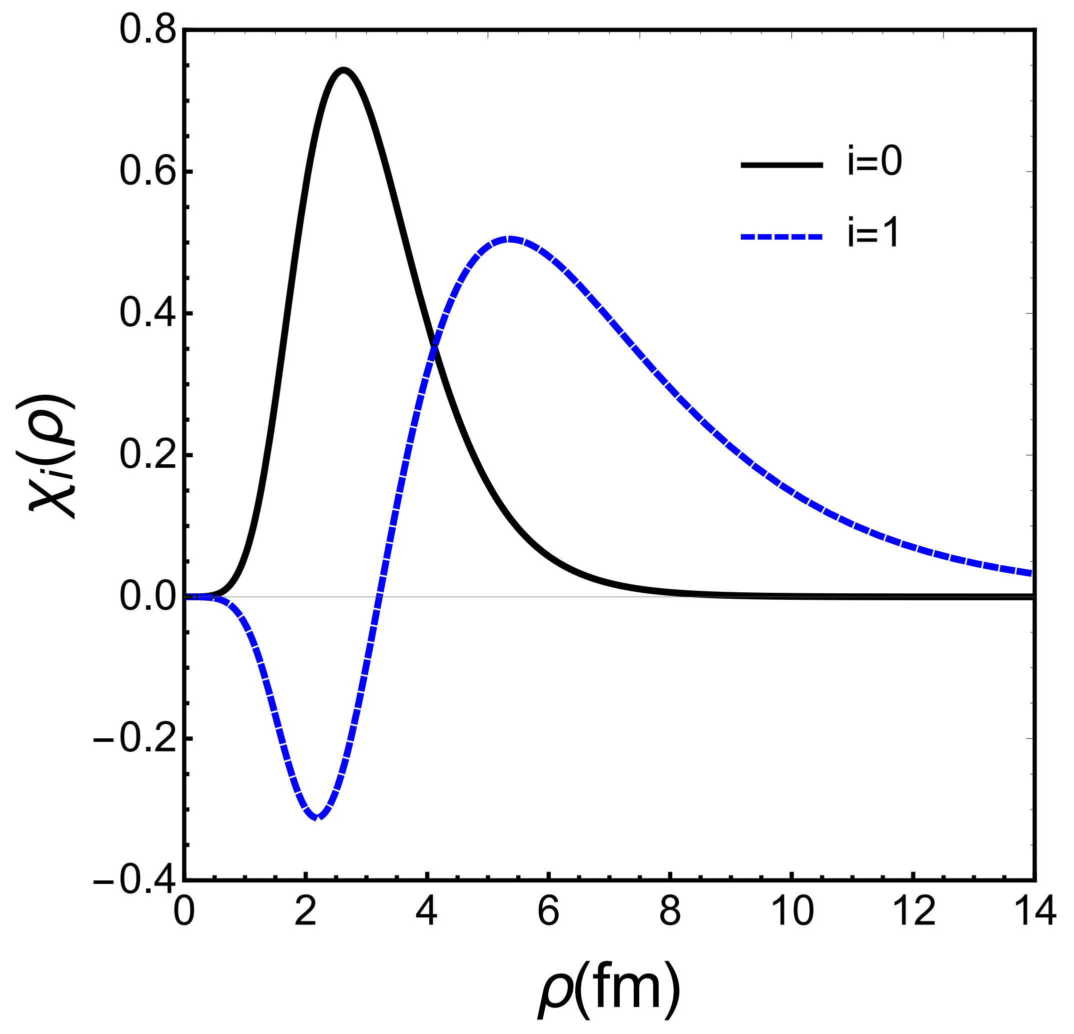
redefining the wave function
and the radial Schreodinger eqn
Note, the first derivative is gone
but some new repulsive
potential remains (not orbital!)

$$\psi(\rho)=\chi(\rho)/\rho^4$$

$$\frac{d^2\chi}{d\rho^2}-\frac{12}{\rho^2}\chi-\frac{2M}{\hbar^2}(W(\rho)+V_C(\rho)-E)\chi=0$$

Solving the eigenvalue problem in App. A we have obtained 40 lowest eigenstates for Eq. (A3) using the simplest potential V_1 from Ref. [17] and the Coulomb term between the two protons. The ground state energy we find is $E_0=-27.8$ MeV, very close to the experimental value of -28.3 MeV.

Rather unexpectedly, we also find a second bound state (missed in [17]) with energy $E_1=-2.8$ MeV. To determine whether this state is physical, we show in Table ?? the excited states of ^{4}He. Among them there is just one 0^+ state, with a binding energy of



$$B=-28.3\text{ MeV}+20.2\text{ MeV}=-8.1\text{ MeV}$$

here are experimentally observed excited states of He4

the first one fits well to our second bound state

Now, getting convinced that we understand quantum mechanics of 4 nucleons in He4 At zero T, we proceed to calculate the density matrix at finite T and check how it changes when the nuclear potential changes

So, people doing stat models For light nuclei Were missing about 50 states!

TABLE I: Low-lying resonances of ^{4}He system, from BNL properties of nuclides listed in *nndc.bnl.gov* web page. J^P is total angular momentum and parity, Γ is the width. The last column is the decay channel branching ratios, in percents. p, n, d correspond to emission of proton, neutron or deuterons.

E (MeV)	J^P	Γ (MeV)	decay modes, in %
20.21	0^+	0.50	p =100
21.01	0^-	0.84	n =24, p =76
21.84	2^-	2.01	n = 37, p = 63
23.33	2^-	5.01	n = 47, p = 53
23.64	1^-	6.20	n = 45, p = 55
24.25	1^-	6.10	n = 47, p = 50 , d=3
25.28	0^-	7.97	n = 48 , p = 52
25.95	1^-	12.66	n = 48 ,p = 52
27.42	2^+	8.69	n = 3 , p = 3 ,d = 94
28.31	1^+	9.89	n = 47 , p = 48 , d = 5
28.37	1^-	3.92	n = 2, p = 2, d = 96
28.39	2^-	8.75	n = 0.2, p = 0.2 , d = 99.6
28.64	0^-	4.89	d=100
28.67	2^+	3.78	d=100
29.89	2^+	9.72	n = 0.4 , p = 0.4, d = 99.2

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Now they are being included: About half of d,t,he3 Come from them!

Paradox 2: light nuclei have small bindings, => large sizes and very fragile
Cascades predict they must be destroyed by hundreds of pions.
and yet they are produced

Resolution: “preclusters” do not have small binding and are not large!
they are wave packages of many states, with $E<0$ AND $E>0$
and do not have large sizes

Their decay into “clusters” (bound states, resonances => final states)
take long time 1/Delta E or 1/Gamma O(100 fm/c)
by that time pions are gone

•I did PIMC simulation of He4 in 1980 already, and managed to put it to NPB

•**Path integral simulations of the few-nucleon clustering at heavy ion collisions freezeot,**

• **(with Dallas DeMartini, SB student)**

•hyperdistance definition

$$\rho^2 = \sum_{m=1}^{3} \vec{\xi}[m]^2 = \frac{1}{4}\Big(\sum_{i\neq j} (\vec{x}[i]-\vec{x}[j])^2\Big)$$

•4 particles, 6 distances

Paths of 4 nucleons
in a Matsubara time
In a periodic box
Only tau discretized
But very many steps needed

The density and
temperature values
correspond to kinetic
Freezeout conditions
at BES 1 energies

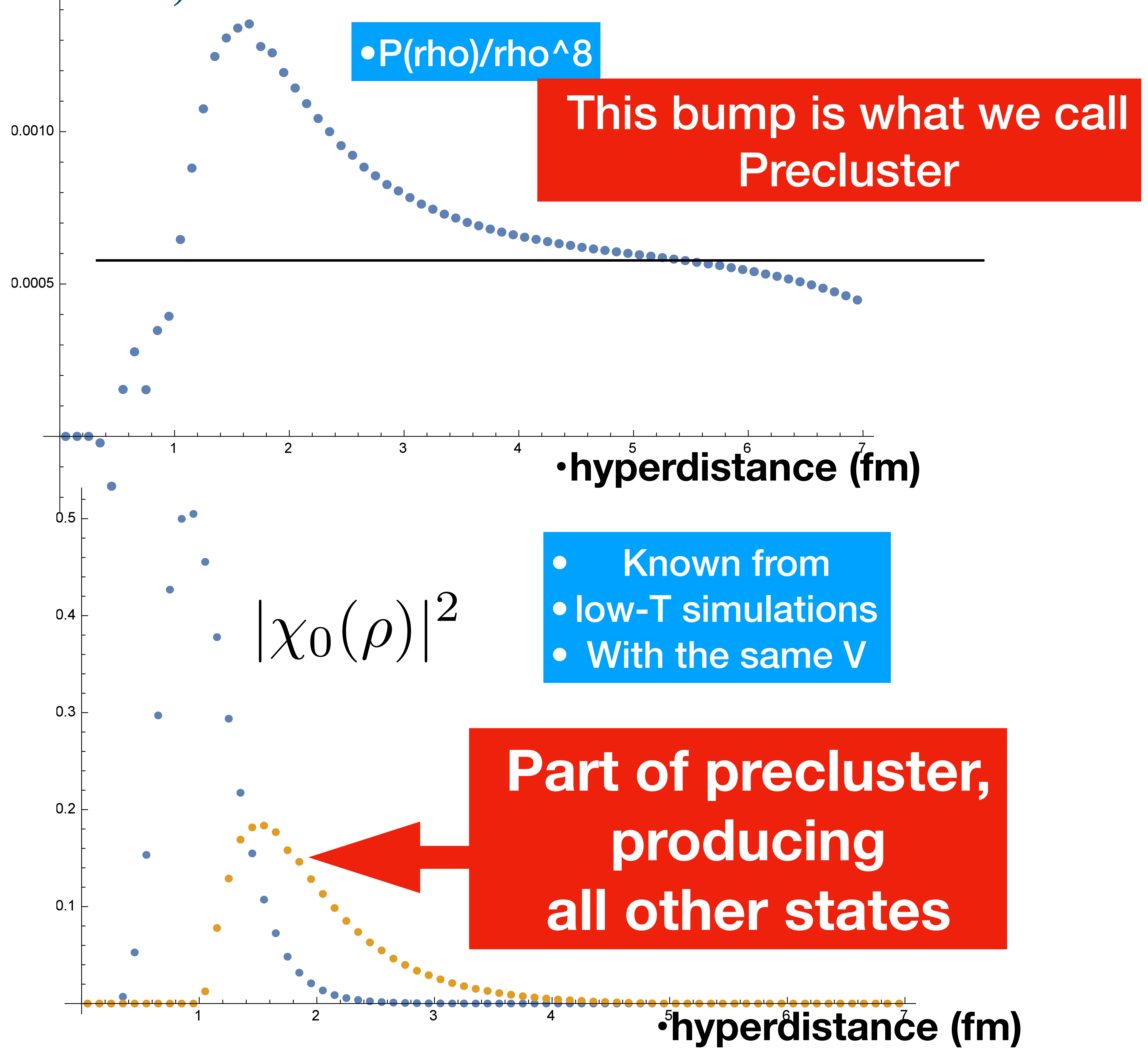

FIG. 6: Ground state and precluster for 19.6 GeV.

- Calculation with
- Conventional nuclear forces

$$Z_{pot} = 1 + \frac{1}{V^N} \int d^3x_1 ... \int d^3x_N \left[e^{\left(-\sum_{i>j} V(\vec{x}_i - \vec{x}_j)/T\right)} - 1 \right]$$

$$Z_{pot} = 1 + \frac{N(N-1)(N-2)(N-3)}{4!} \left(\frac{V_{cor}}{V^3}\right) \approx 1 + n^3 V_{cor}^{(9)} \frac{N}{4!},$$

- definition
- Or 4^4 for nonidentical

$$V_{cor}^{(9)} = \frac{32}{105} \pi^4 \int d\rho \rho^8 (P(\rho) - 1).$$

$$V_{cor}^{(9)}(7.7) \approx 4.3 \cdot 10^4 \, fm^9$$

- From PIMC

$$n_{cl} \equiv \frac{4}{\left(V_{cor}^{(9)}\right)^{1/3}} \approx 0.114 \, / \, fm^3$$

- density in precluster

- about 3 times the density of
- ambient matter $n_B(7.7) \approx 0.037/fm^3$.

$$R_{amb} \equiv n_B^{-1/3} \approx 3.0 \, fm, \quad R_{cl} \equiv n_{cl}^{-1/3} \approx 2.0 \, fm$$

- Will be reached first by the correlation length

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Although 4-clusters contain only fraction of a percent of nucleons In kurtosis it is O(1) at the lowest BES energies

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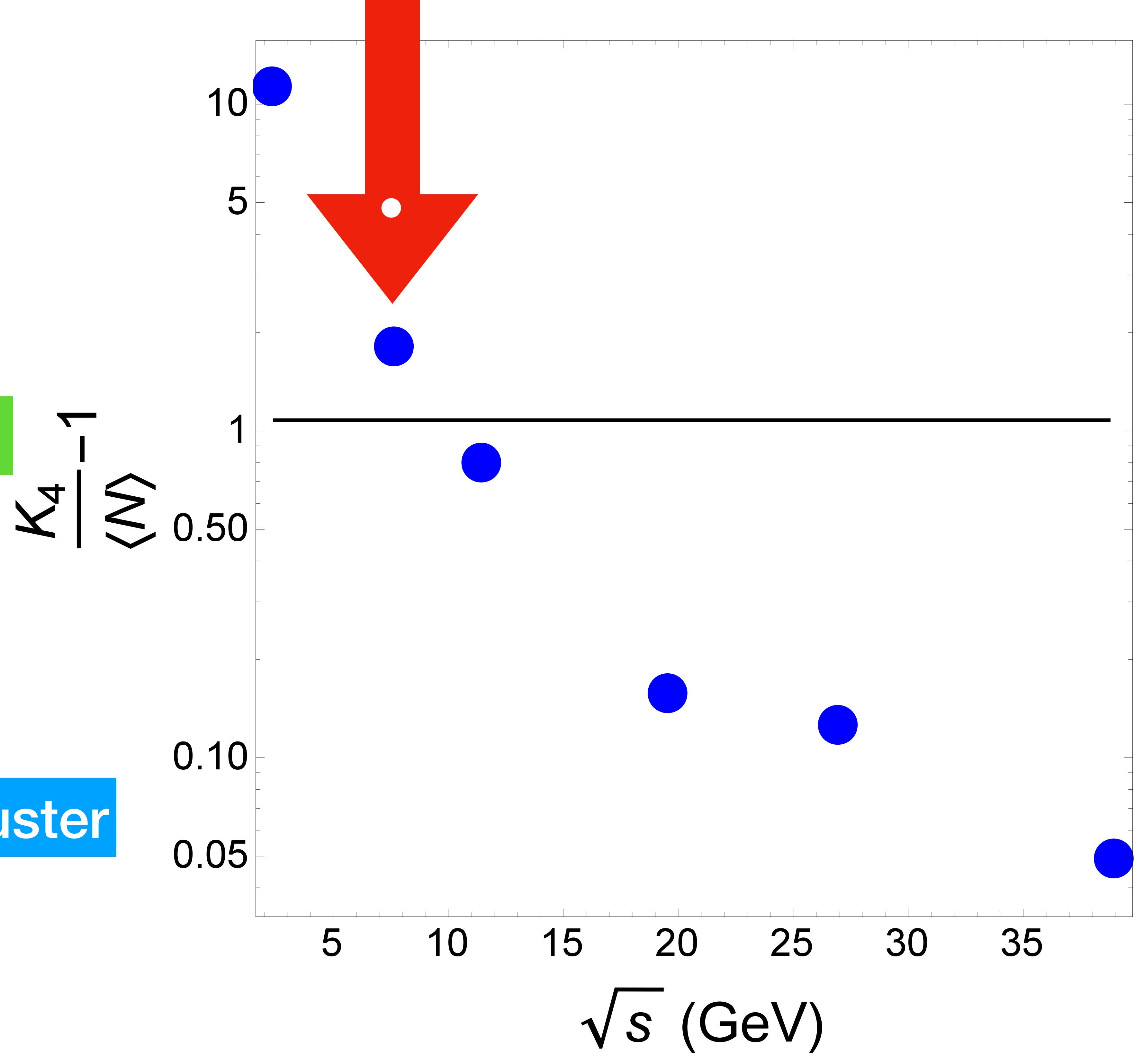


FIG. 10: The 4th cumulant deviation (Eq. (15)) versus $\sqrt{s}$, using the 9-dimensional correlated volume $V_{cor}^{(9)}$ determined from the PIMC simulations.

- On paradox 1

Calculation with
Modified nuclear forces
— reduced sigma mass —
Predicted by chiral transition
produce huge unrealistic effect

- Predicted for
- Chiral transition by RG

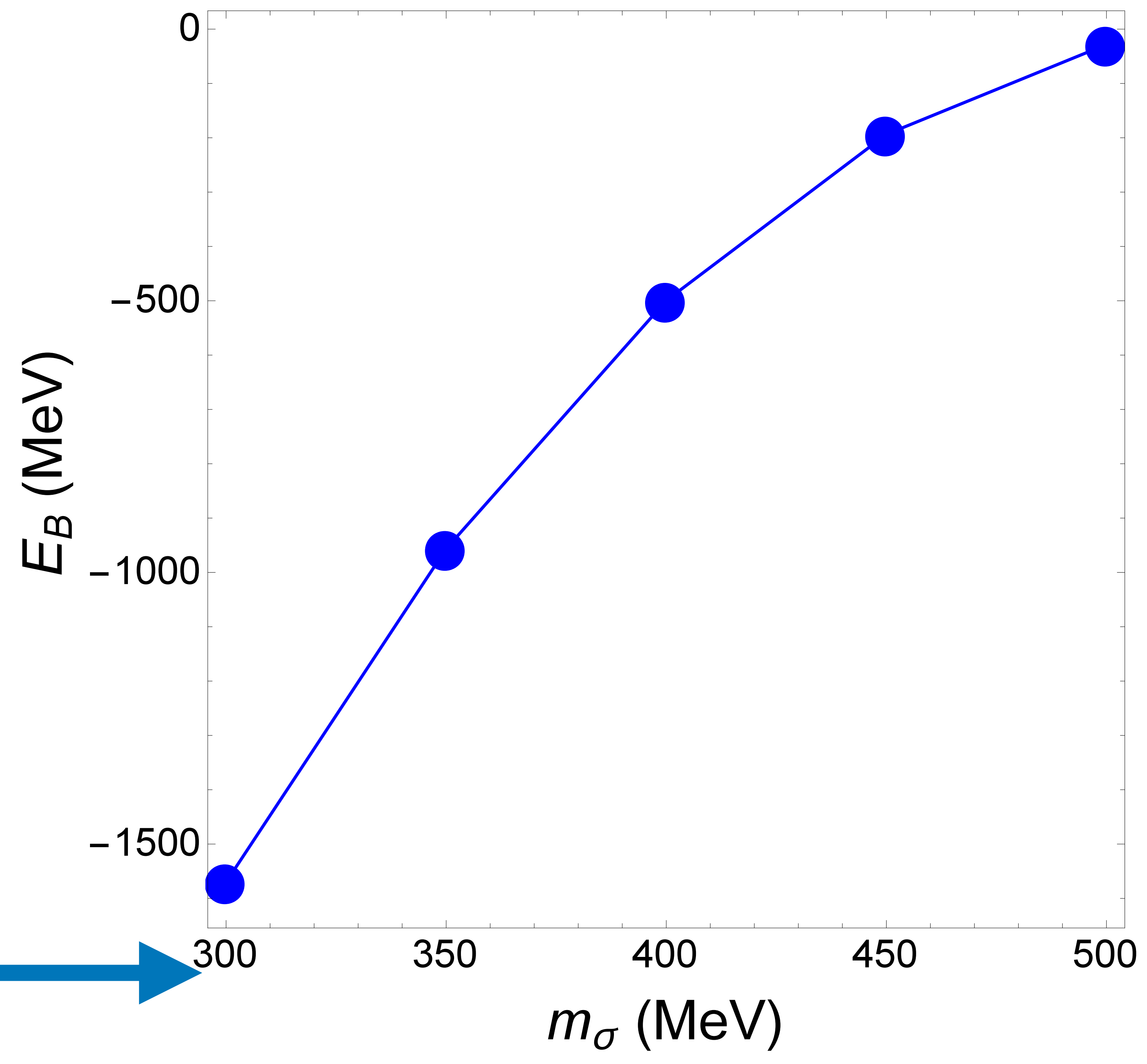
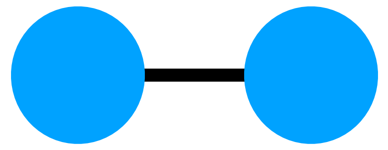


FIG. 8: Binding energy E_B of the 4N system as a function of the σ mass m_σ

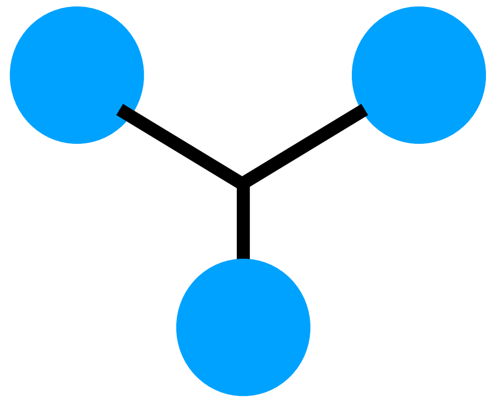
- -28 MeV
- with ordinary
- nuclear forces

the effect of binary forces induced by the critical mode at CP, where $\xi \to \infty$ must be catastrophic. Indeed, if all N (N – 1)/2 ~ 10^4 pairs of nucleons in the fireball be attracted to each other, with a Newton-like long-range potential, the fireball would implode, similar to a gravitational collapse.

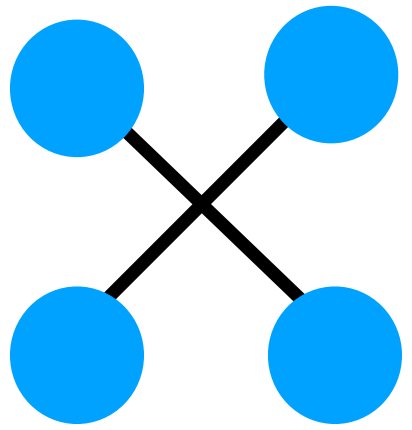
•Binary and manybody forces



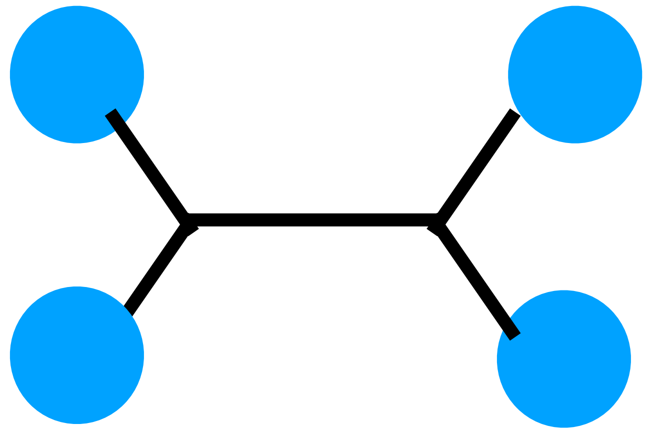
(a)



(b)



(c)



(d)

$$V_a = -\frac{g_c^2}{4\pi}\langle \phi(\vec{r})\phi(0)\rangle = -\frac{g_c^2}{4\pi}\frac{exp(-r/\xi)}{r}$$

•**We introduce the following objects**

$$V_b(\vec{x}_1,\vec{x}_2,\vec{x}_3) \equiv \int d^3u D(\vec{x}_1-\vec{u})D(\vec{x}_2-\vec{u})D(\vec{x}_3-\vec{u})$$

$$V_c(\vec{x}_1,\vec{x}_2,\vec{x}_3,\vec{x}_4) \equiv \int d^3u D(\vec{x}_1-\vec{u})D(\vec{x}_2-\vec{u})D(\vec{x}_3-\vec{u})D(\vec{x}_3-\vec{u}).$$

$$V_d(\vec{x}_1,\vec{x}_2,\vec{x}_3,\vec{x}_4) \equiv \int d^3u d^3v D(\vec{x}_1-\vec{u})D(\vec{x}_2-\vec{u})D(\vec{u}-\vec{v})D(\vec{x}_3-\vec{v})D(\vec{x}_4-\vec{v})$$

$$D(r) = exp(-r/\xi)/r$$

the factor 1/4π present in 3d propagator
will be included later with the couplings.

If nucleons are uncorrelated
They are easy to calculate.
But they are correlated!

These functions depend on 3 or 4 points
should be averaged over manybody
density matrix of the clusters.

•How diagrams depend on the cluster shape ?

• Misha's
•high powers
•

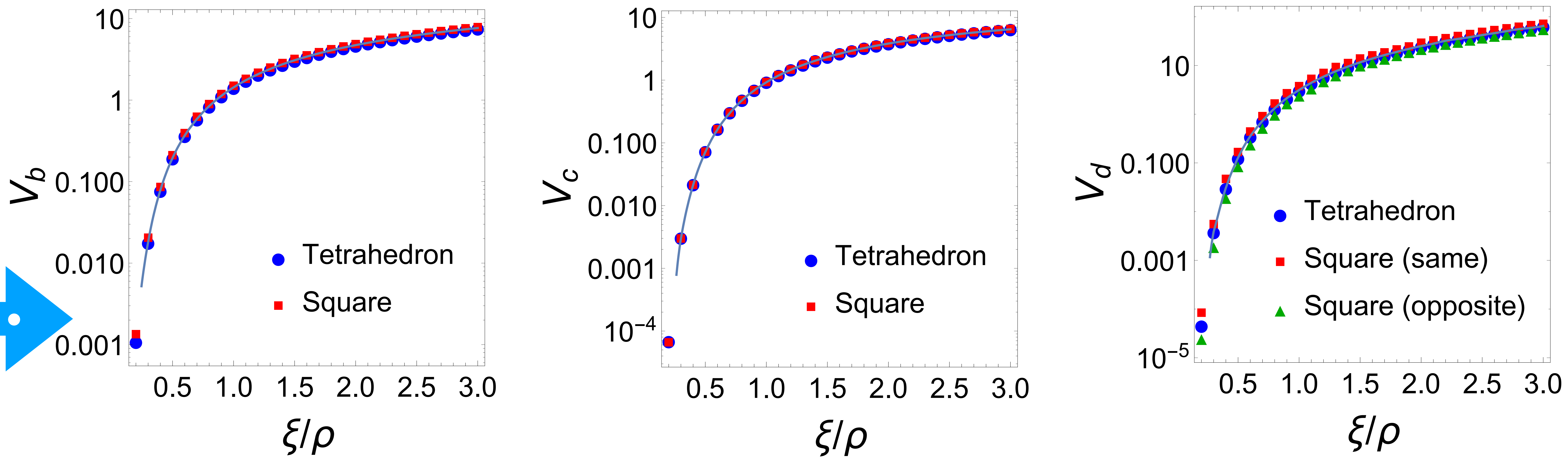


FIG. 3. (Color online) Interactions V_b (left), V_c (center), and V_d (right) corresponding to diagrams (b,c,d) of Fig. 2, respectively, as a function of the correlation-length-to-hyperdistance ratio ξ/ρ for both the tetrahedral and square configurations. The curve is an interpolation of the tetrahedral data points. The distinction between the 'same' and 'opposite' square configurations for diagram (d) is explained in the text.

At small correlation length very strong dependence on xi
But it is moderate at xi/rho>1

Rather weak dependence
on shape if rho is the same

- How diagrams depend on clustering
- Averaging diagrams over snapshots from PIMC simulation

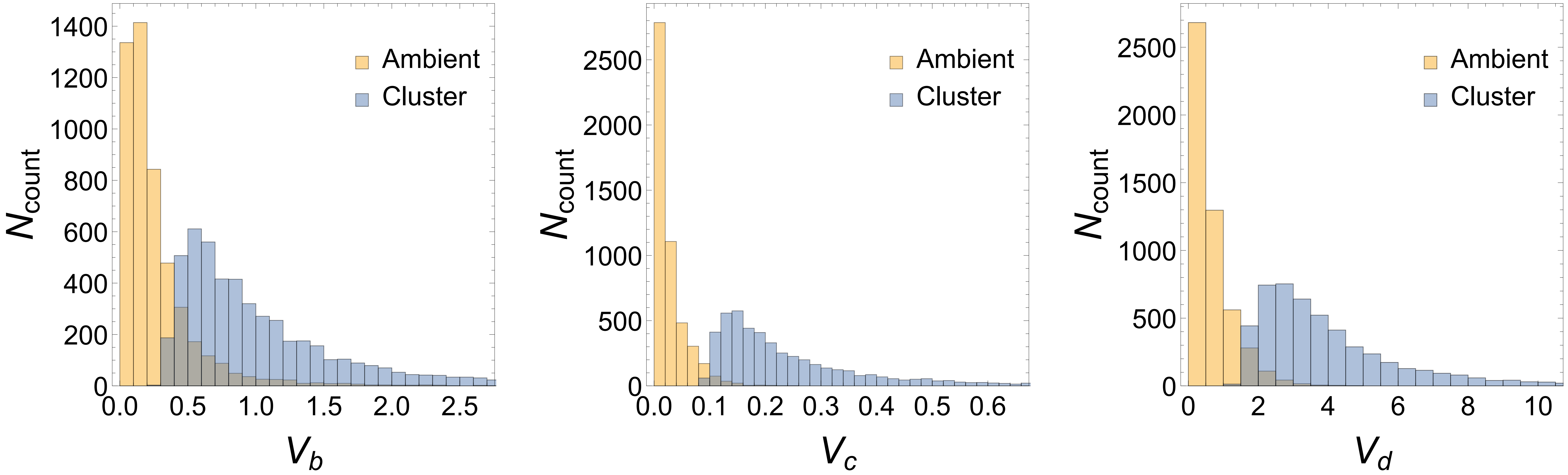


FIG. 6. Distribution of values of the multibody interactions V_b (left), V_c (center), and V_d (right) corresponding to diagrams (b,c,d) of Fig. 2, respectively, in 5000 configurations each for the cluster ($\rho < 3$ fm) and ambient nucleon matter ($\rho > 3$ fm) generated in PIMC simulation. Calculation performed with $\xi = 2$ fm.

- **Diagrams are significantly larger for clusters**
- **Tails to the right are due to very small clusters: but those will be killed**

•Preliminary estimate: Landau phi^4 model

$$V_{tet} = -6\frac{g_c^2}{4\pi}\langle V_a\rangle_{tet} + 4!\lambda_4(\frac{g_c}{4\pi})^4\langle V_c\rangle_{tet}$$

$$\frac{g_\sigma^2}{4\pi} = 6.04, \quad \frac{g_\omega^2}{4\pi} = 15.17.$$

Walecka model of
Relativistic mean field
For nuclear matter

Critical mode is their mixture
Stephanov used 10
as some round average
And so do we

But we do not know
The value of
The quartic coupling

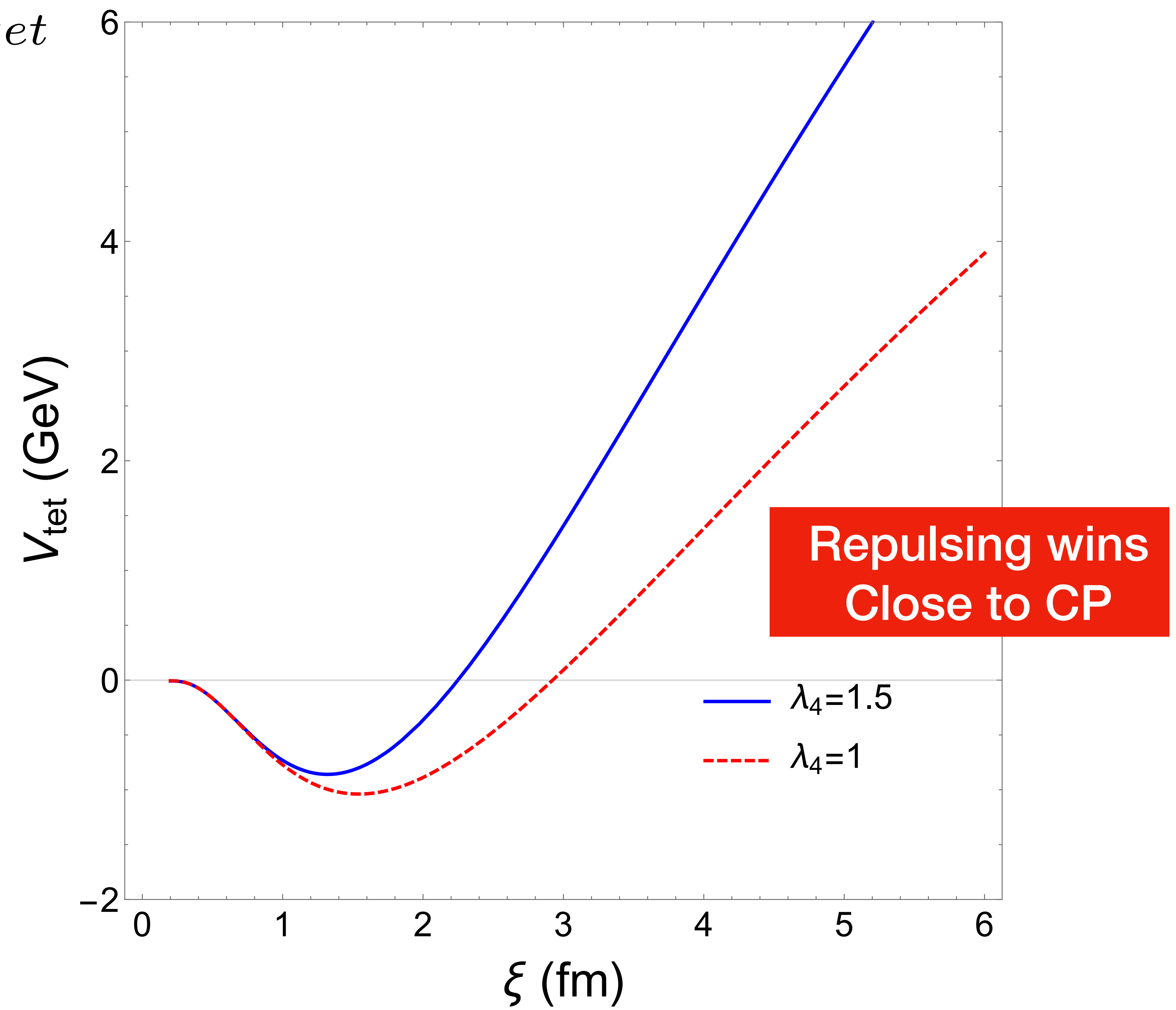


FIG. 4. Energy of four-nucleon tetrahedral cluster (in GeV) as a function of correlation length ξ (fm)
The critical mode-nucleon coupling is taken to be equal to nucleon-sigma meson coupling of the Waleck
model (20), and the values of quartic coupling $\lambda_4 = 1.5$ (upper curve) and $\lambda_4 = 1$ (lower curve).

THE UNIVERSAL EFFECTIVE ACTION FOR ISING-TYPE CRITICAL FLUCTUATIONS

The Landau model, used as an initial approximation, does *not* however represent correct behavior near Ising-like critical points. Wilson's epsilon expansion – in $\epsilon = 4-d$ where d is space dimension– has found that under the renormalization group flow the Landau model goes into the fixed-point regime in infrared, with small coupling at small ϵ. While Wilson famously calculated approximate values of the critical indices, one might still doubt whether ϵ-expansion gives an accurate account at $\epsilon = 1, d = 3$.

Three arguments suggesting that at the critical point Ω ~ φ6.

- 1. $\dfrac{d\Omega}{d\phi} = J. \qquad \langle\phi\rangle(J) \sim J^{1/\delta}, \;\; \delta = \dfrac{d+2-\eta}{d-2+\eta} \approx 4.78$ •**Much closer to 5 than to 3**

- 2. including a φ6 term – but not higher powers – can be justified
- because this term is the renormalizable one, in d = 3 case

- 3. there are numerical studies showing this ansatz for Ω gives good fits of lattice data.

Probing effective action at large phi is done by doing simulations with different J
At critical line parameterized by t=T/Tc-1

$$\Omega(\phi) = \int d^3x \left[\frac{(\phi_{,\mu})^2}{2} + \frac{m^2\phi^2}{2} + m g_4 \phi^4 + g_6 \phi^6\right]$$

Note that at $m \to 0, \xi \to \infty$ it indeed has only the last ϕ^6 term.

Dimensionless couplings are fixed

$$m = \frac{1}{\xi} \approx M t^{\nu} \qquad \tilde{\phi} \equiv \frac{\phi}{M}$$

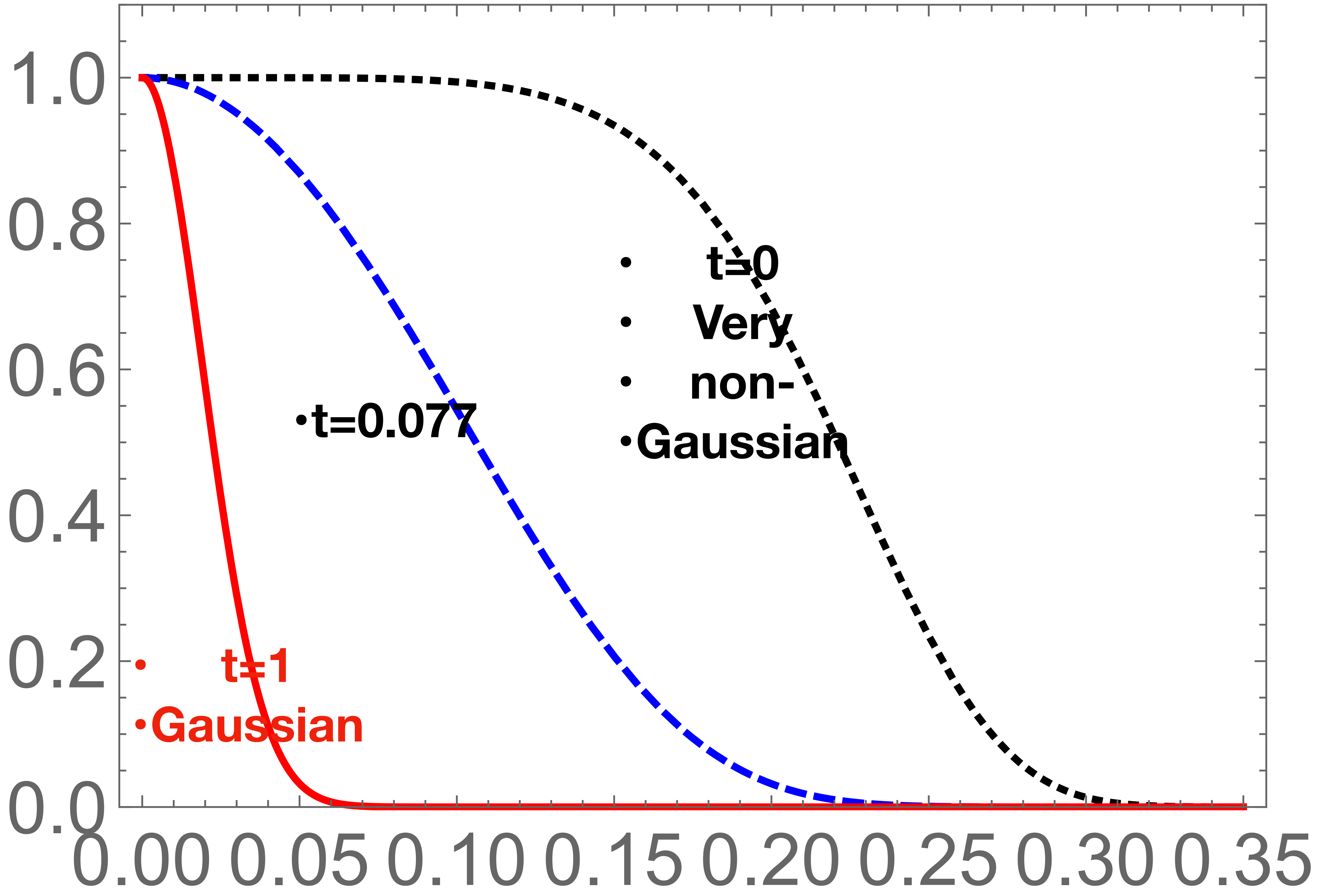
at CP, ξ = ∞, t = 0, only the last term survives.
Only one dimensional parameter M
Which we take to be sigma mass

$$Z = \int D\phi e^{-(\Omega(\phi) + J(x)\phi(x))V_3/T}$$

- **g4=0.97, g6-2.05 from fit**

- M. M. Tsypin, (1994), arXiv:hep-lat/9401034 [hep-lat].
- Agrees also with RG calculation by Heidelberg group

- **P=exp(-VM^3 Omega)**



DEFORMED EFFECTIVE POTENTIAL NEAR THE CRITICAL LINE

$$\frac{\partial \Omega}{\partial \tilde{\phi}}(\tilde{\phi}_0) = J$$

As example we use dimensionless J =1/100
Then solve the 5-th order eqn for maximum
Then re-center the distribution by

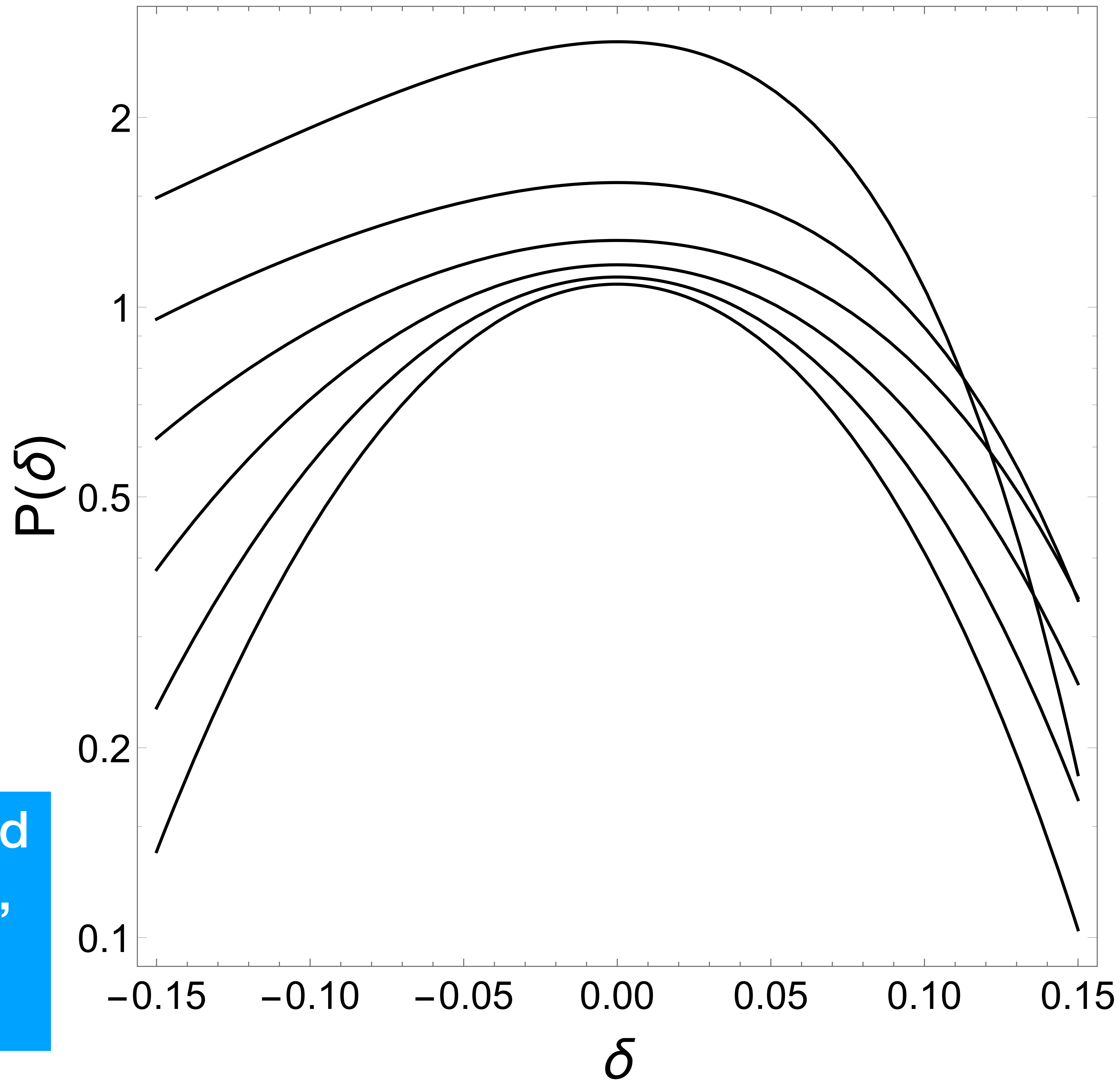
$$\tilde{\phi}_0(t=0.01) \approx 0.224, \;\; \tilde{\phi}_0(t=0.41) \approx 0.031$$

And get new action in terms of delta
Which has all powers of it except the first

$$\tilde{\phi} = \tilde{\phi}_0 + \delta$$

$$\Omega_{def}(t=0.01) \approx -0.0017 + 0.095\delta^2 + 0.51\delta^3 + 1.60\delta^4 + 2.75\delta^5 + 2.05\delta^6$$

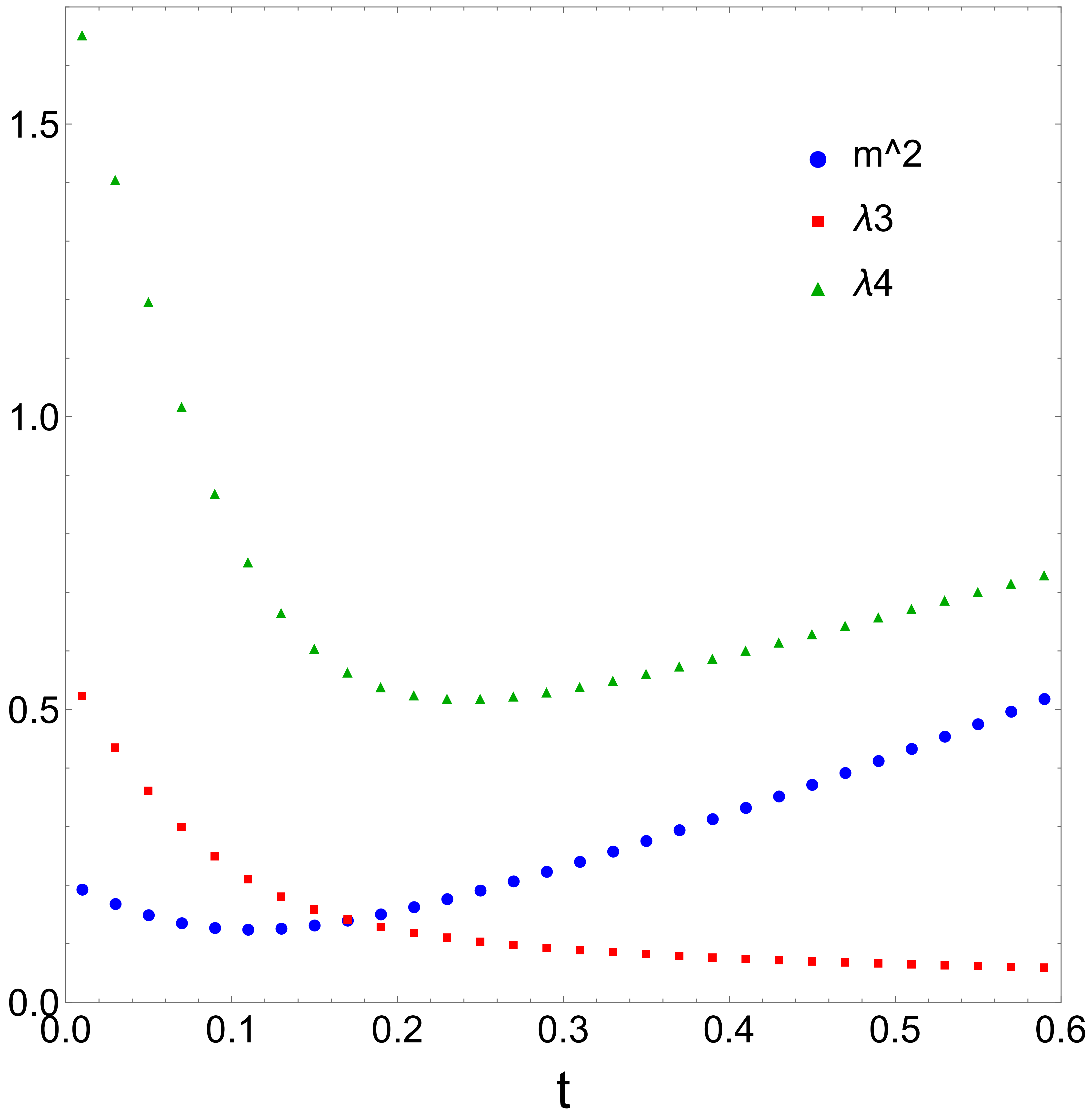
- **No linear term, small quadratic one => \xi not infinite even at t=0**



Six curves, top to bottom, correspond to values of t = 0.01, 0.09, 0.19, 0.29, 0.39, 0.49, 0.59.

DEFORMED EFFECTIVE POTENTIAL NEAR THE CRITICAL LINE

Triple and Quartic couplings Strongly grow Near CP, t->0

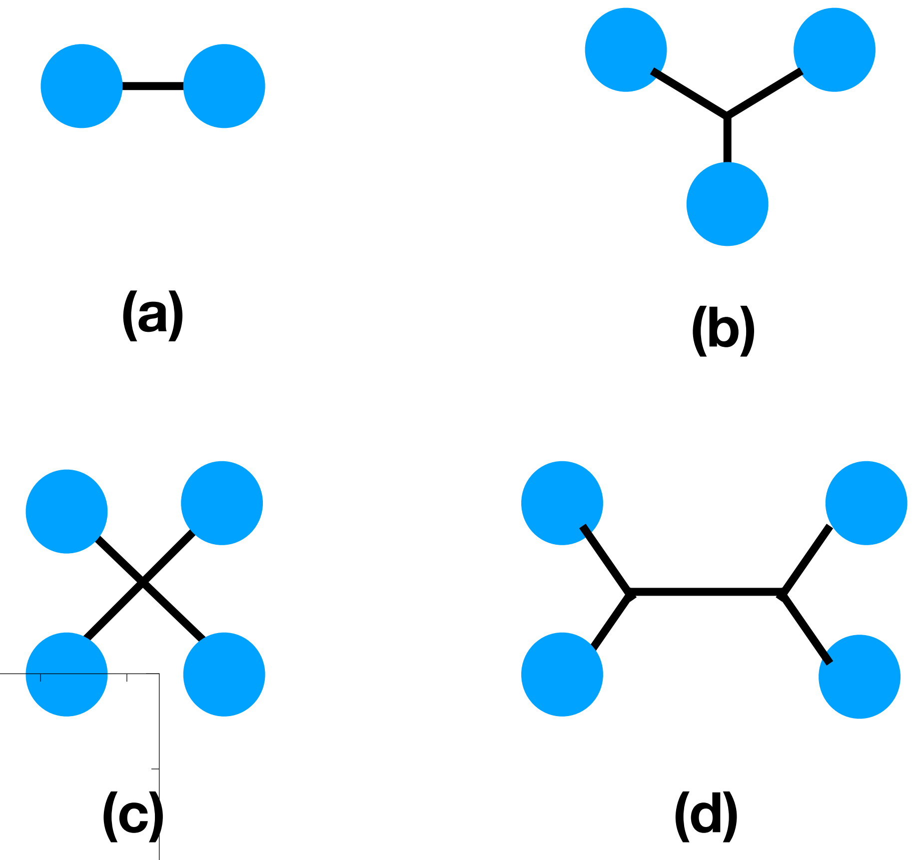


m=1/xi does not vanish Near CP but remains small

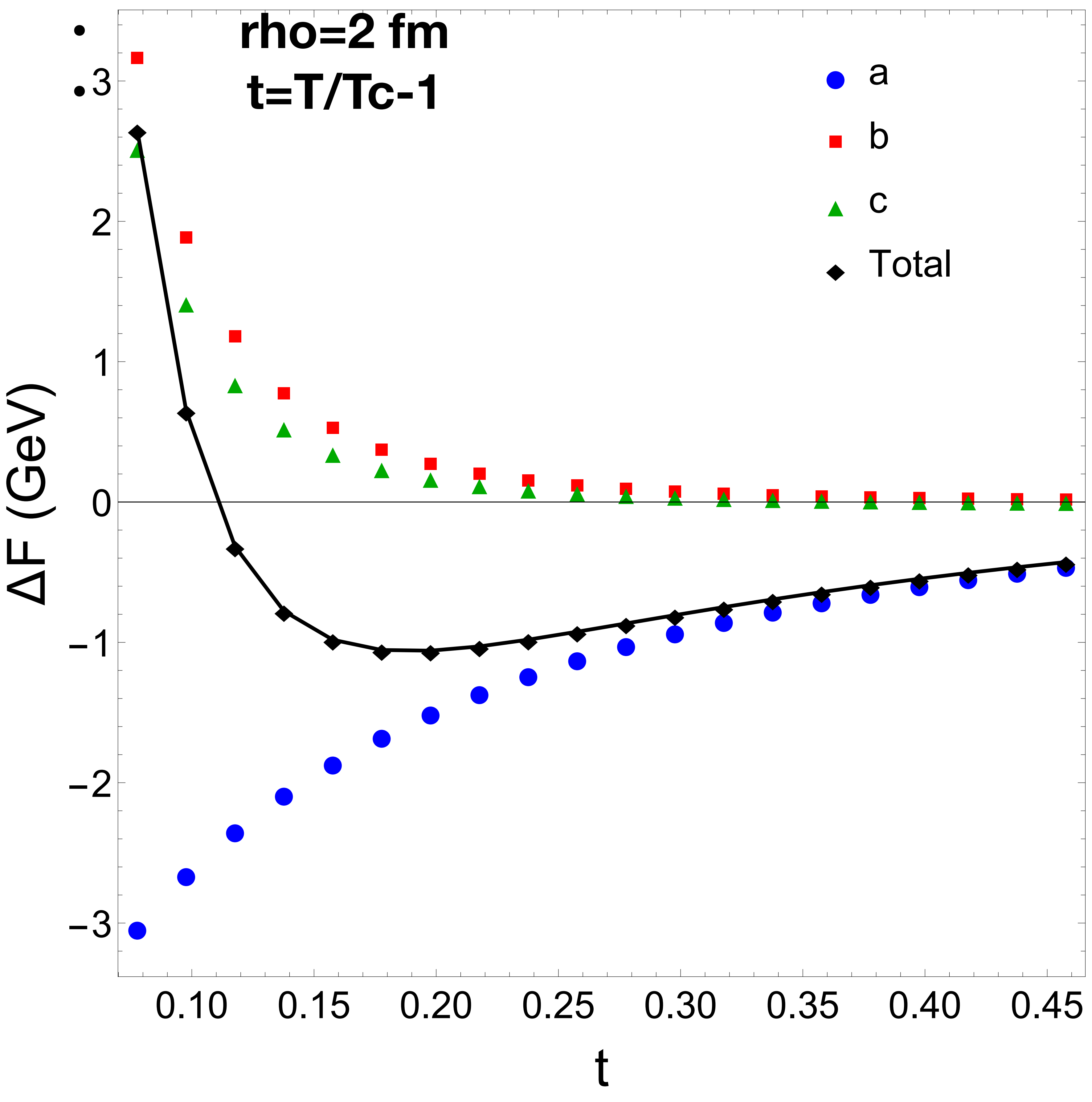
• Taking all effects together

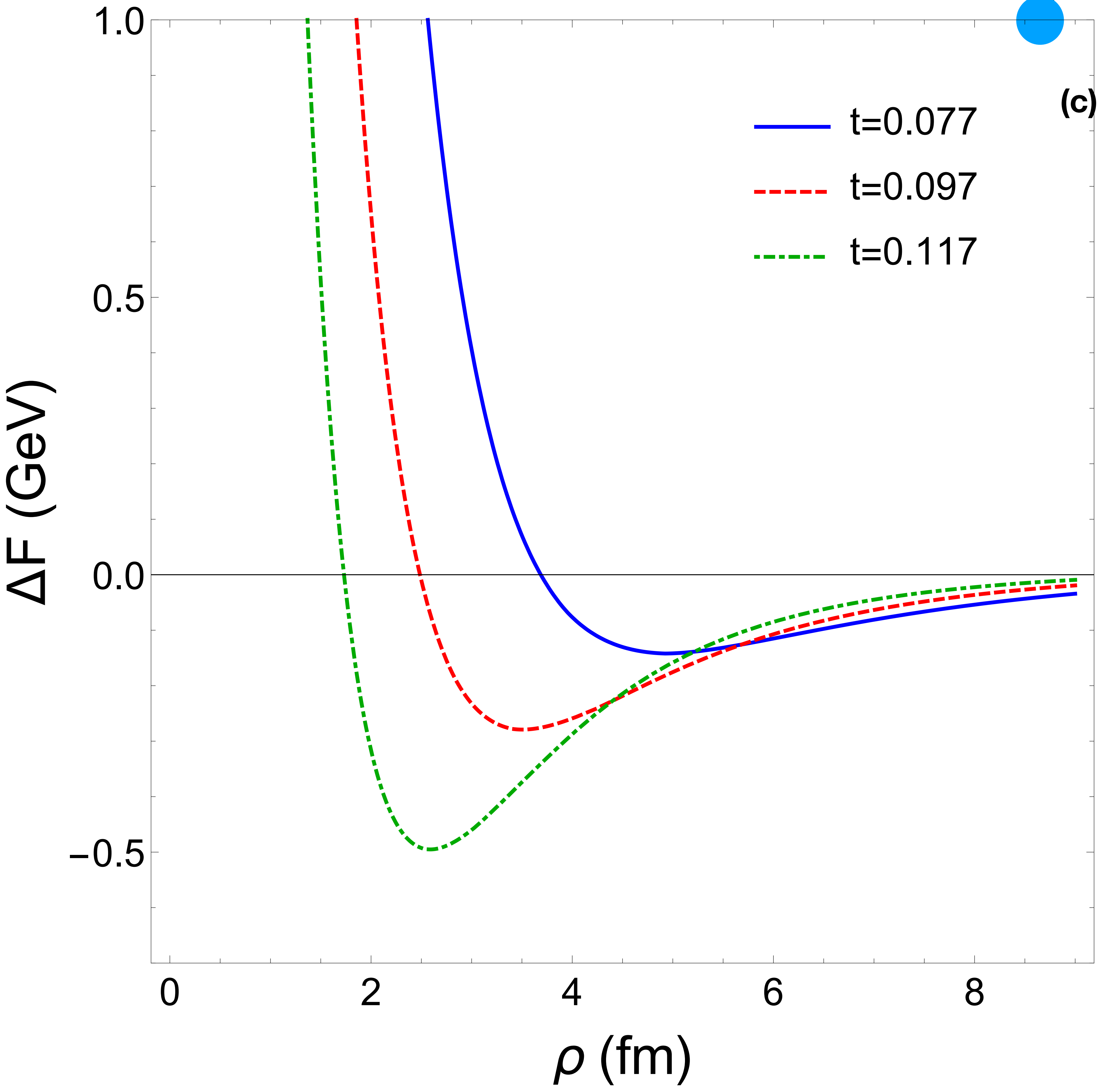
Repulsive three and four Body forces overcome Attraction near CP

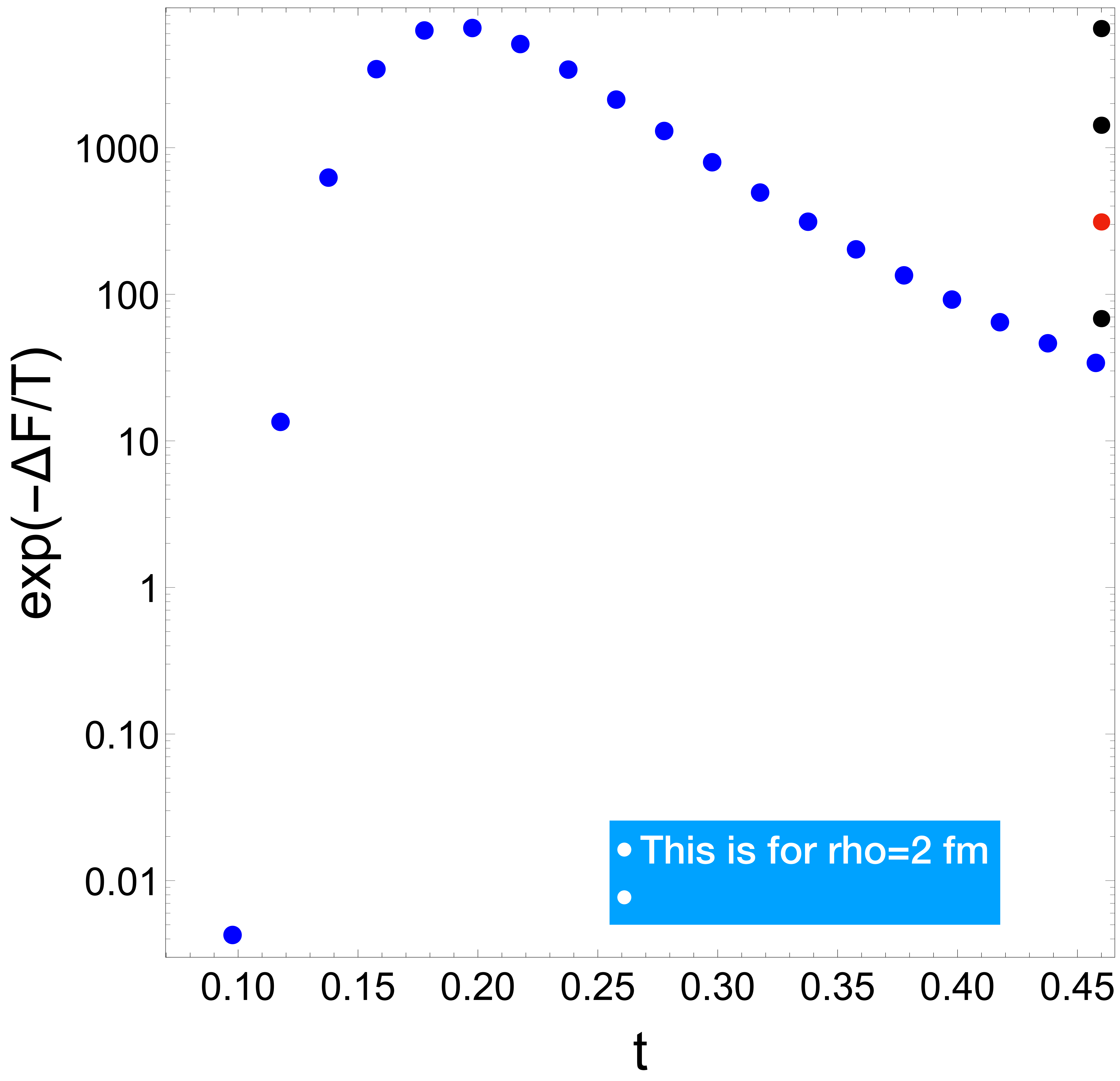
$$V = -\frac{4\cdot 3}{2}\frac{g_c^2}{4\pi}\frac{exp(-r_{ij}/\xi)}{r_{ij}} + 4\cdot 3!\lambda_3(\frac{g_c}{4\pi})^3 V_b + 4!\lambda_4(\frac{g_c}{4\pi})^4 V_c - 4!\frac{\lambda_3^2}{8\pi}(\frac{g_c}{4\pi})^4 V_d$$



- **Energy of 4-N cluster**
- **rho=2 fm**
- **t=T/Tc-1**







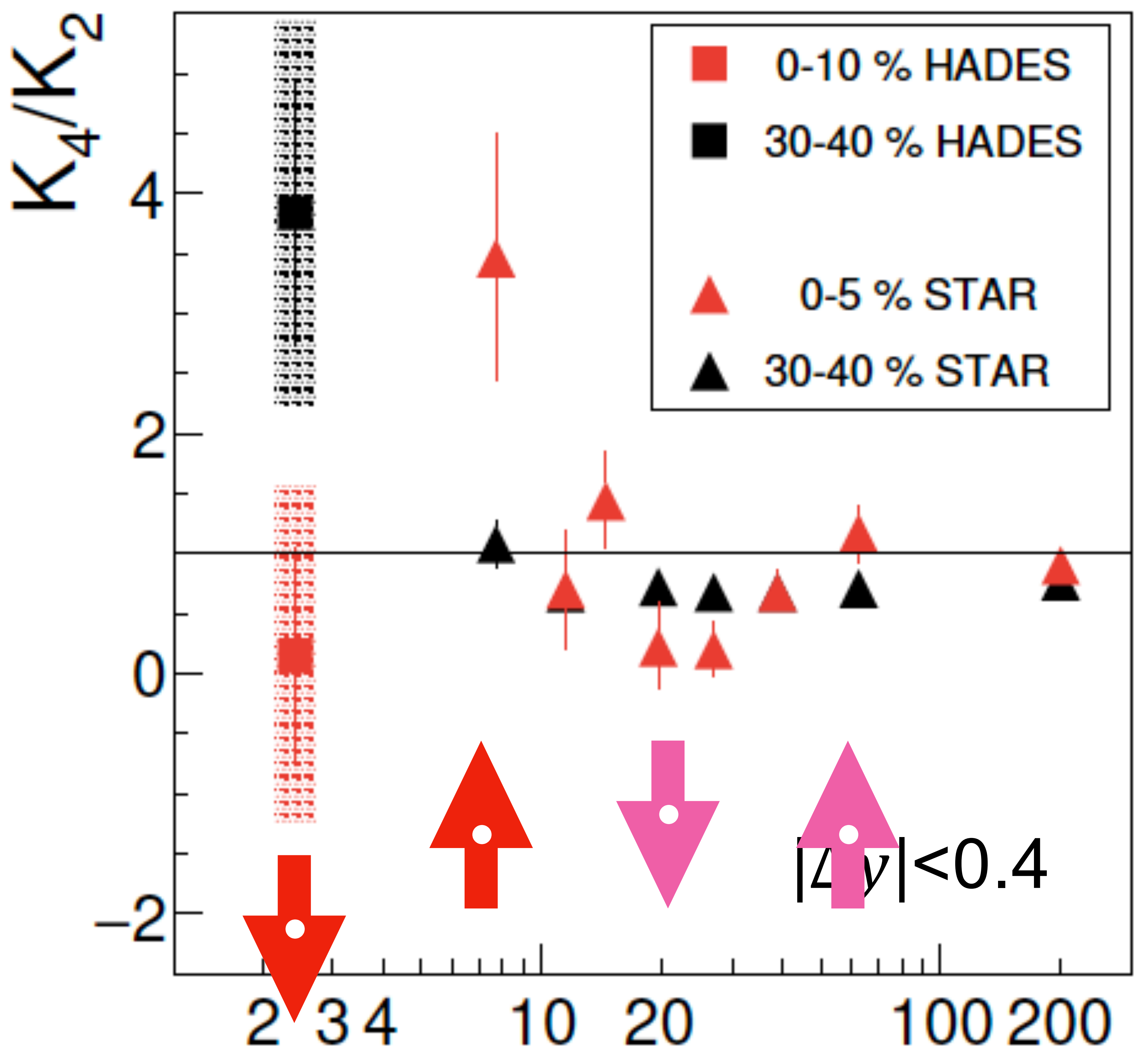
- **Plotting exp(-V/T)**
- **One gets dramatic**
- **non-monotonous signal**
- **for cluster formation**

Perhaps the nucleon coupling to critical mode Is not that large as assumed But qualitative shape is now clear

One needs to look for a dip In clustering

•Let us now look at experimental kurtosis

•Older STAR data have shown large effect



Two dips for central bins
large at 2 and smaller at 20 GeV?
Errors still large => BESII

• e-Print: 2001.02852

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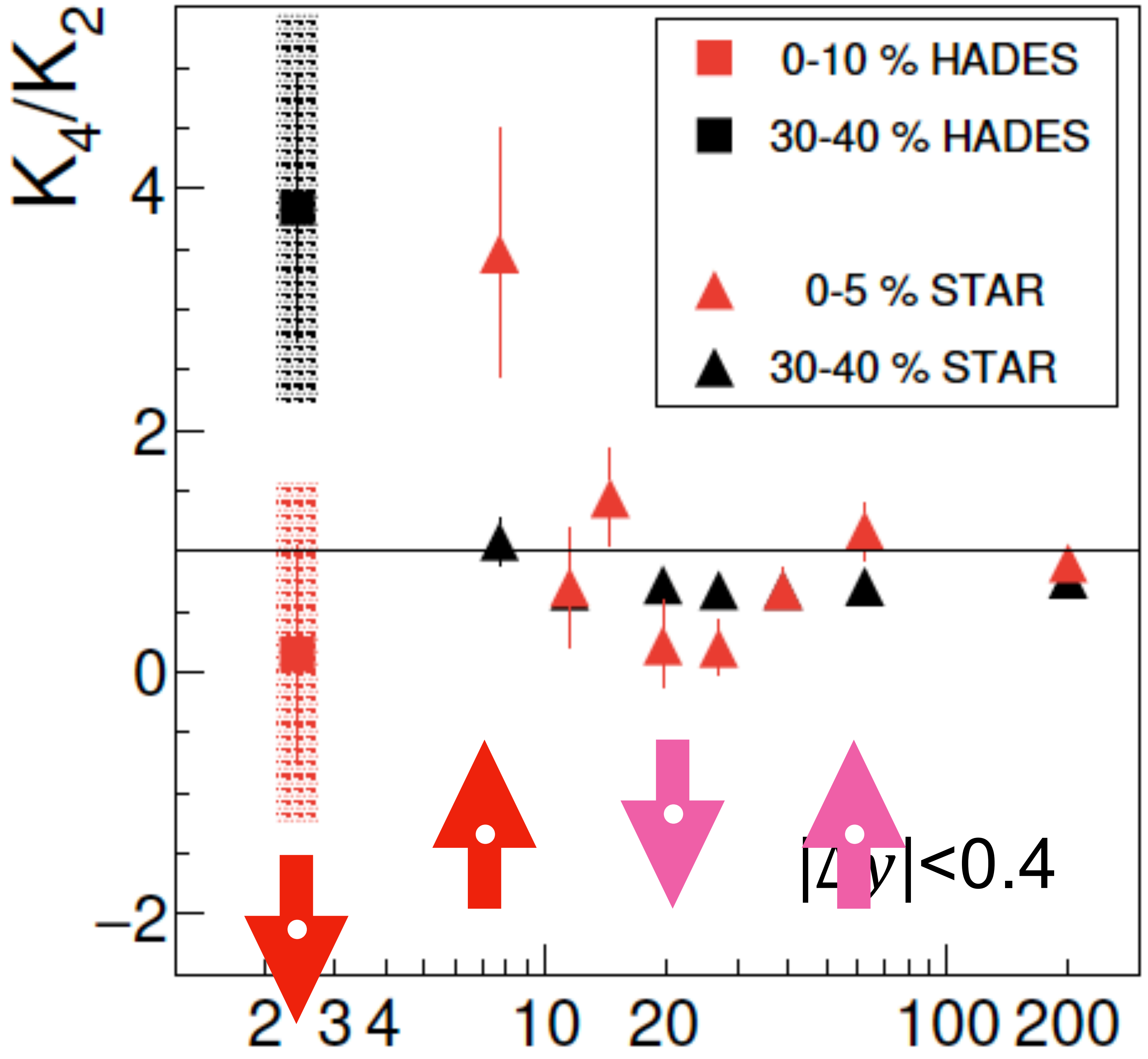
Which was recently found to be partly due to small set of defective events for central bin



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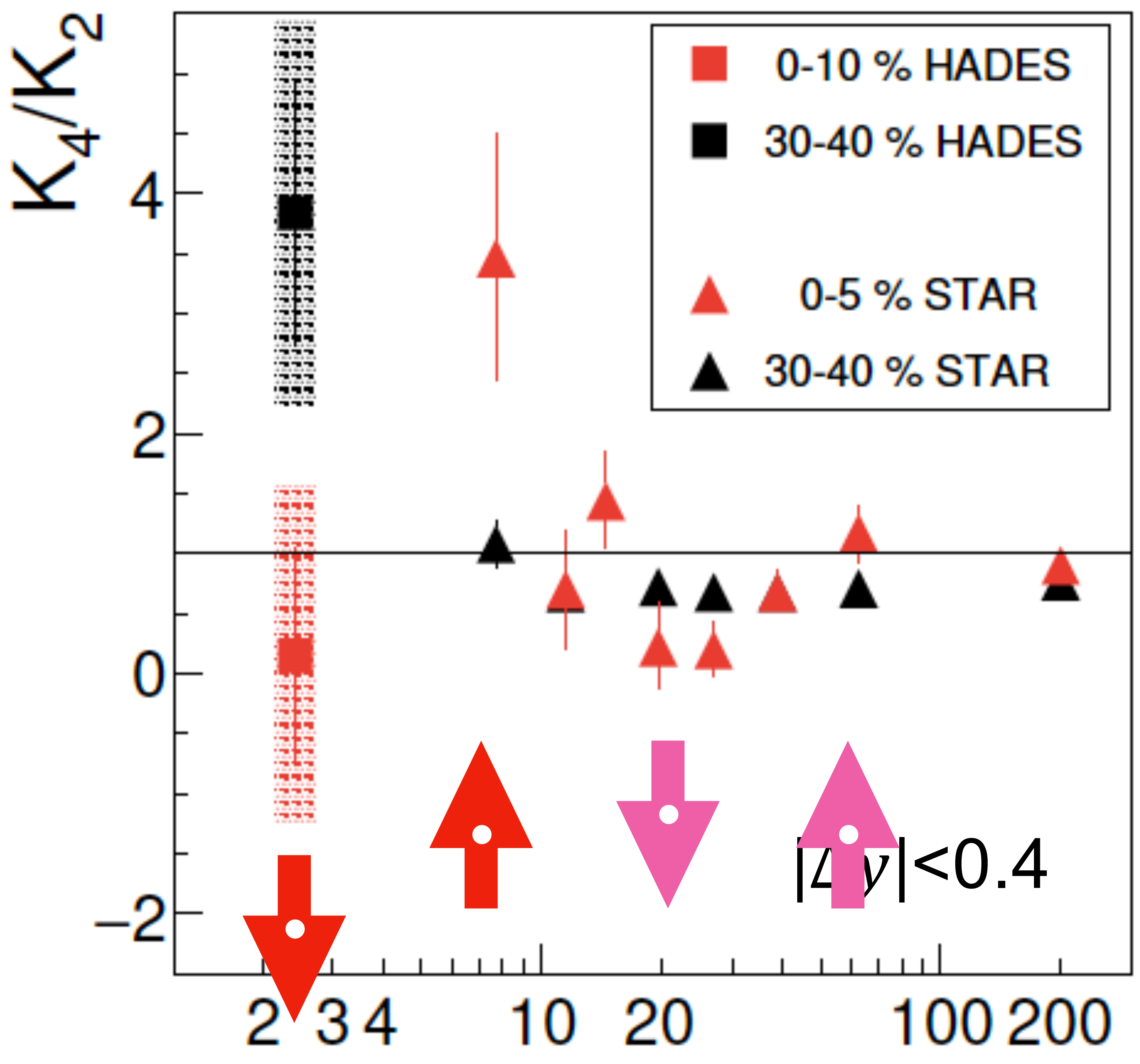

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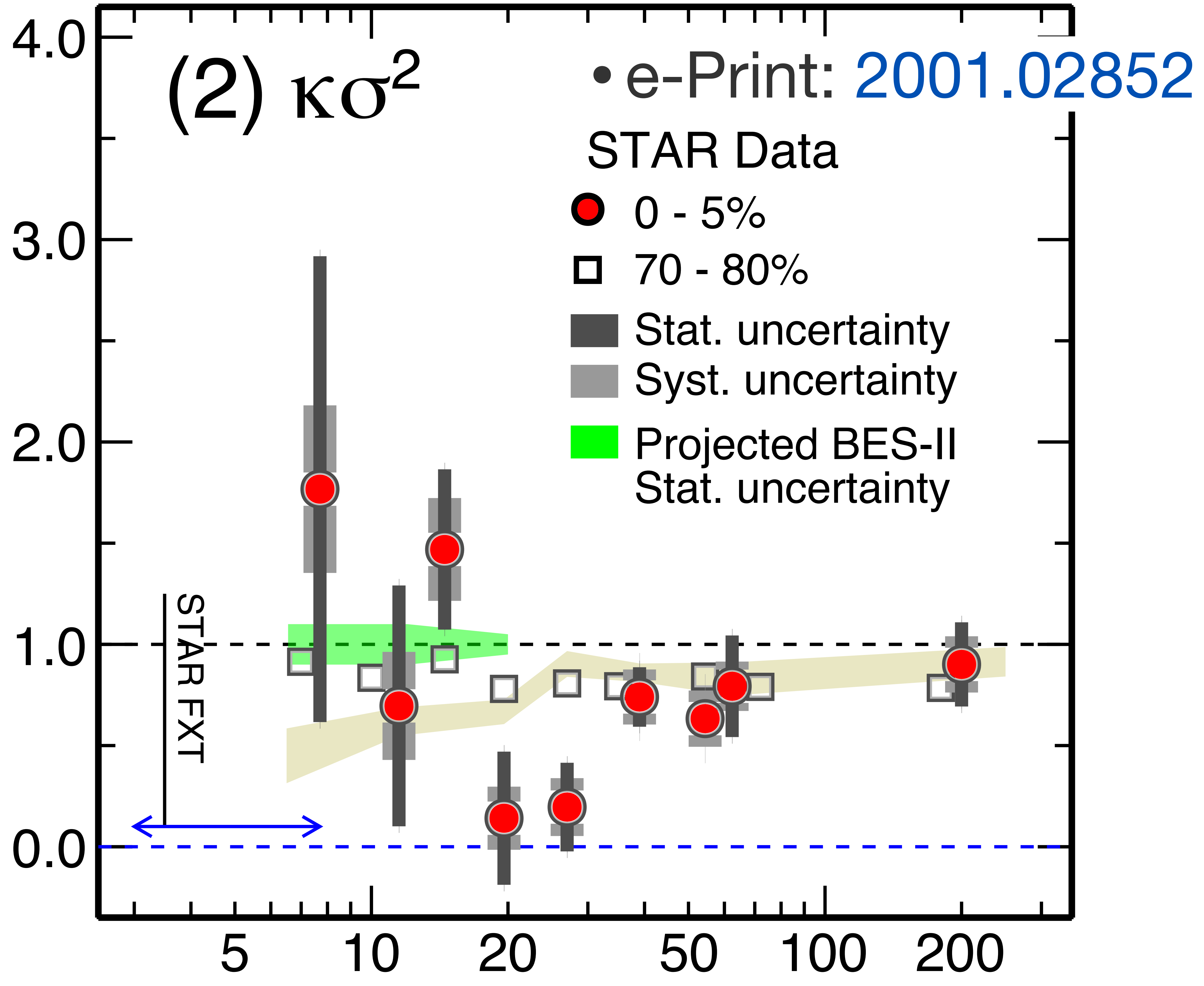

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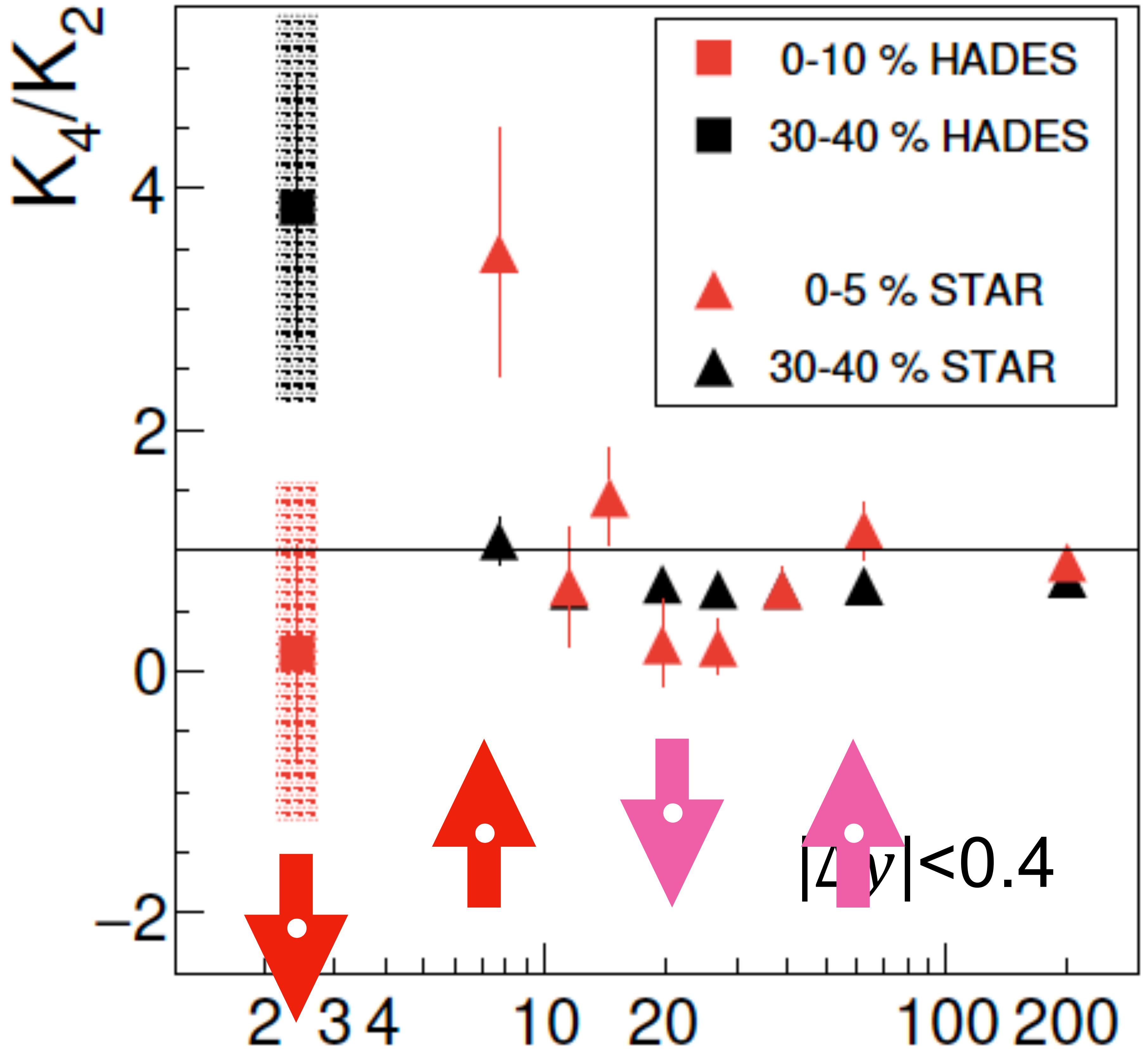

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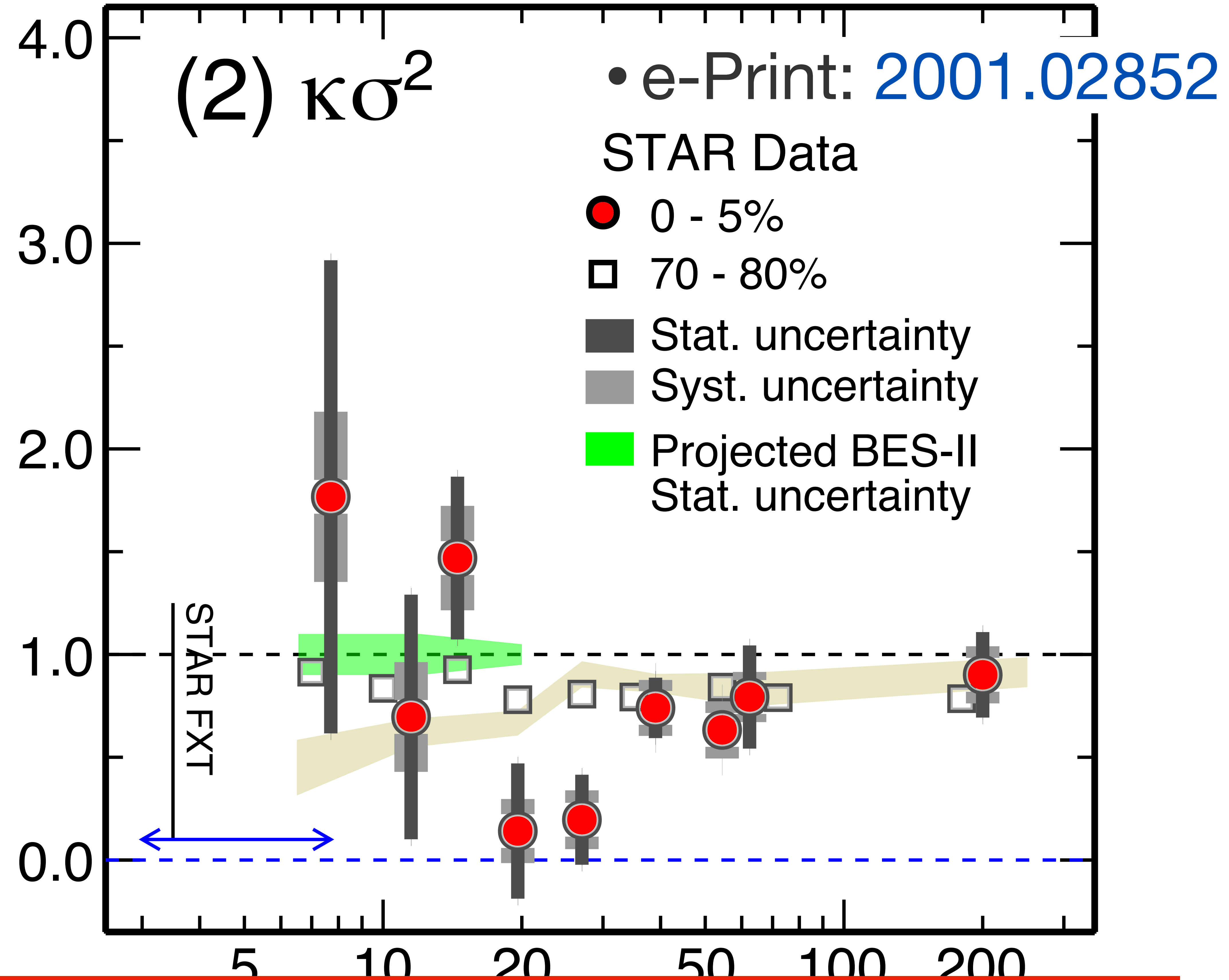
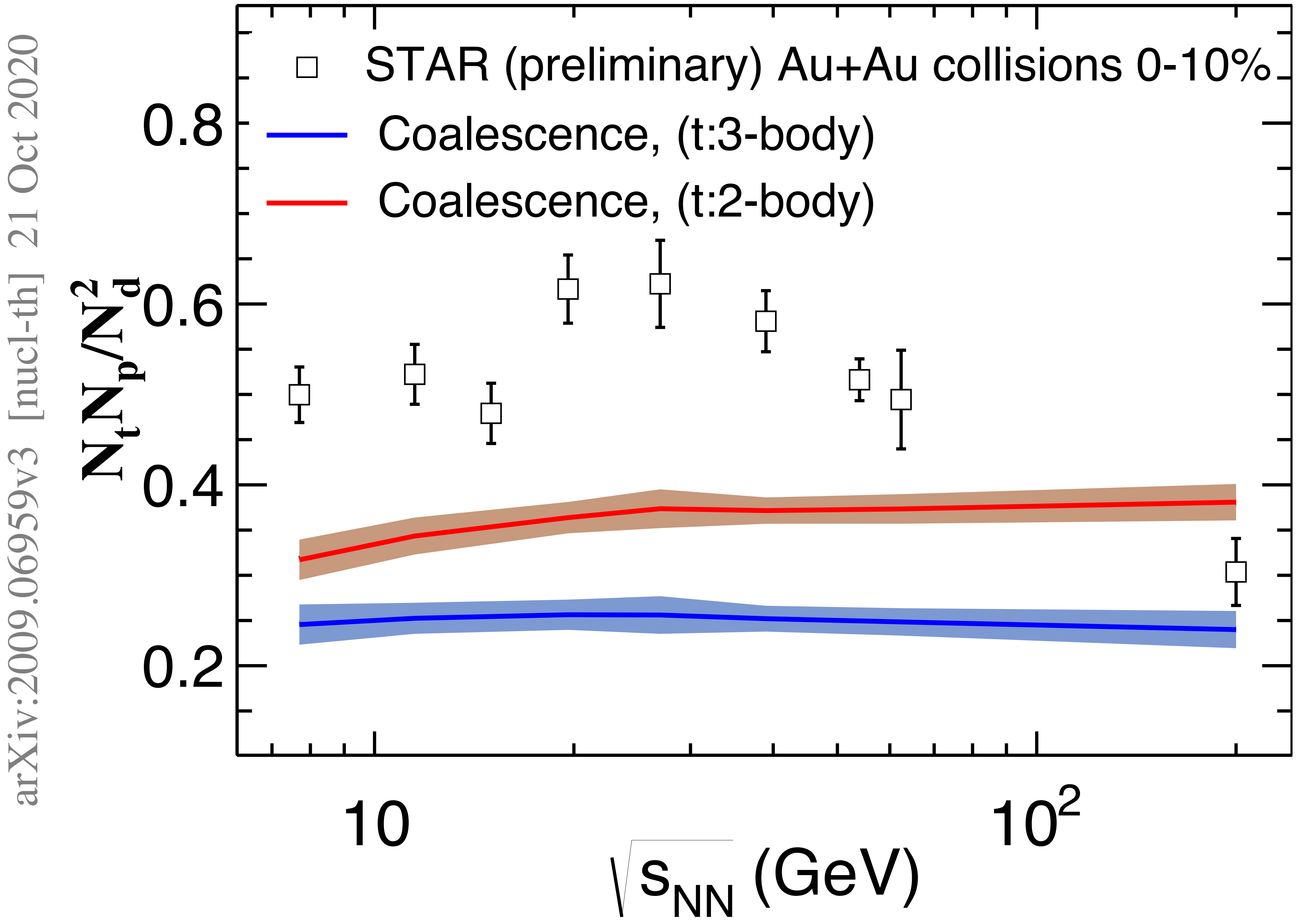


Still non-monotonous signal?
Clearly much more accurate measurements from BeS-II are needed

•Let us now look at light nuclei production: the tritium ratio

In this ratio the main driver
— fugacity exp(mu/T) -
Cancels out



Wenbin Zhao,[1,2,3,4] Chun Shen,[5,6] Che Ming Ko,[7] Quansheng Liu,[1,2] and Huichao Song[1,2,3]

Extra source of t is needed:
4-N reclusters =>
50 states of He4 =>
feeding

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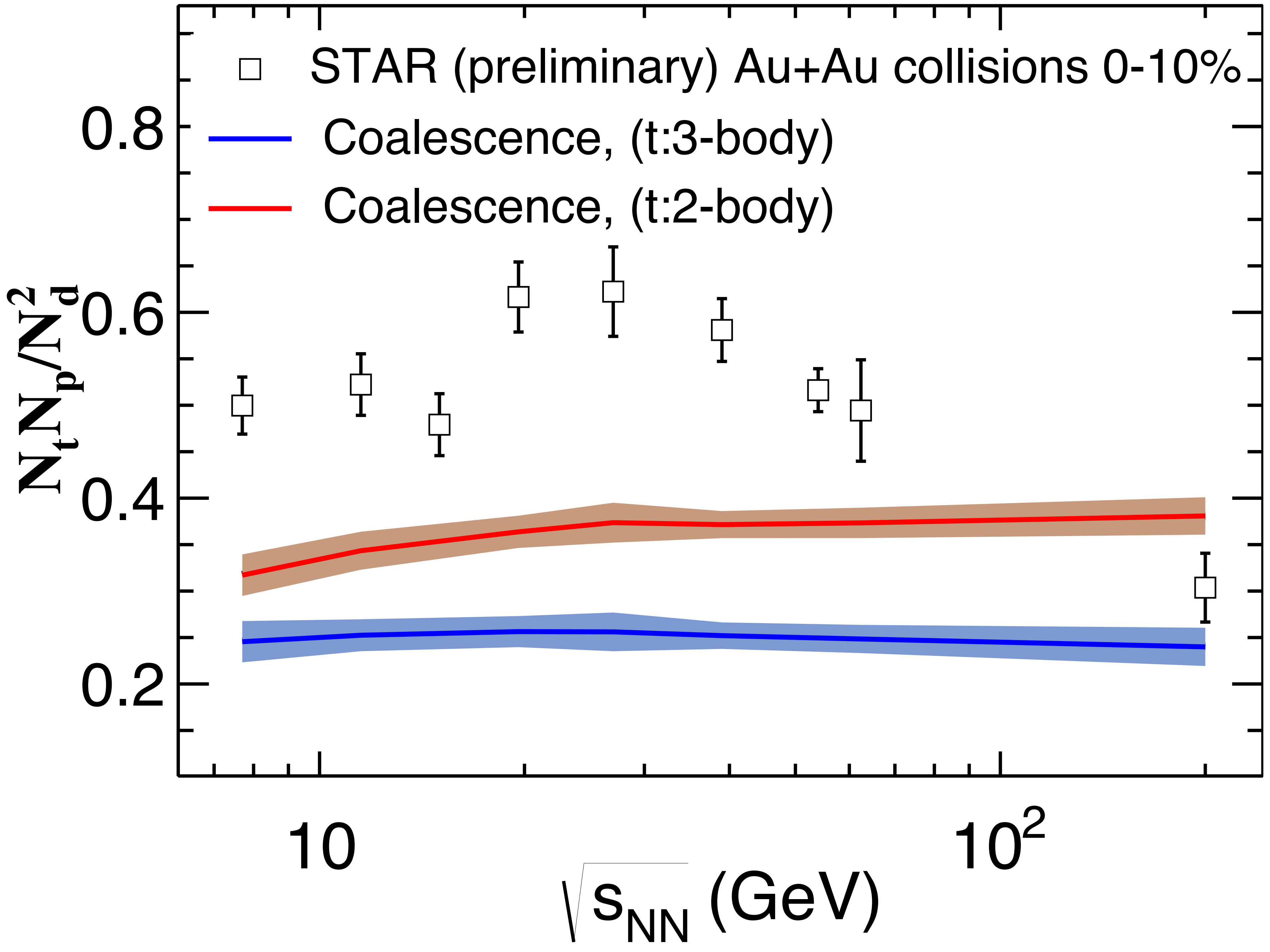

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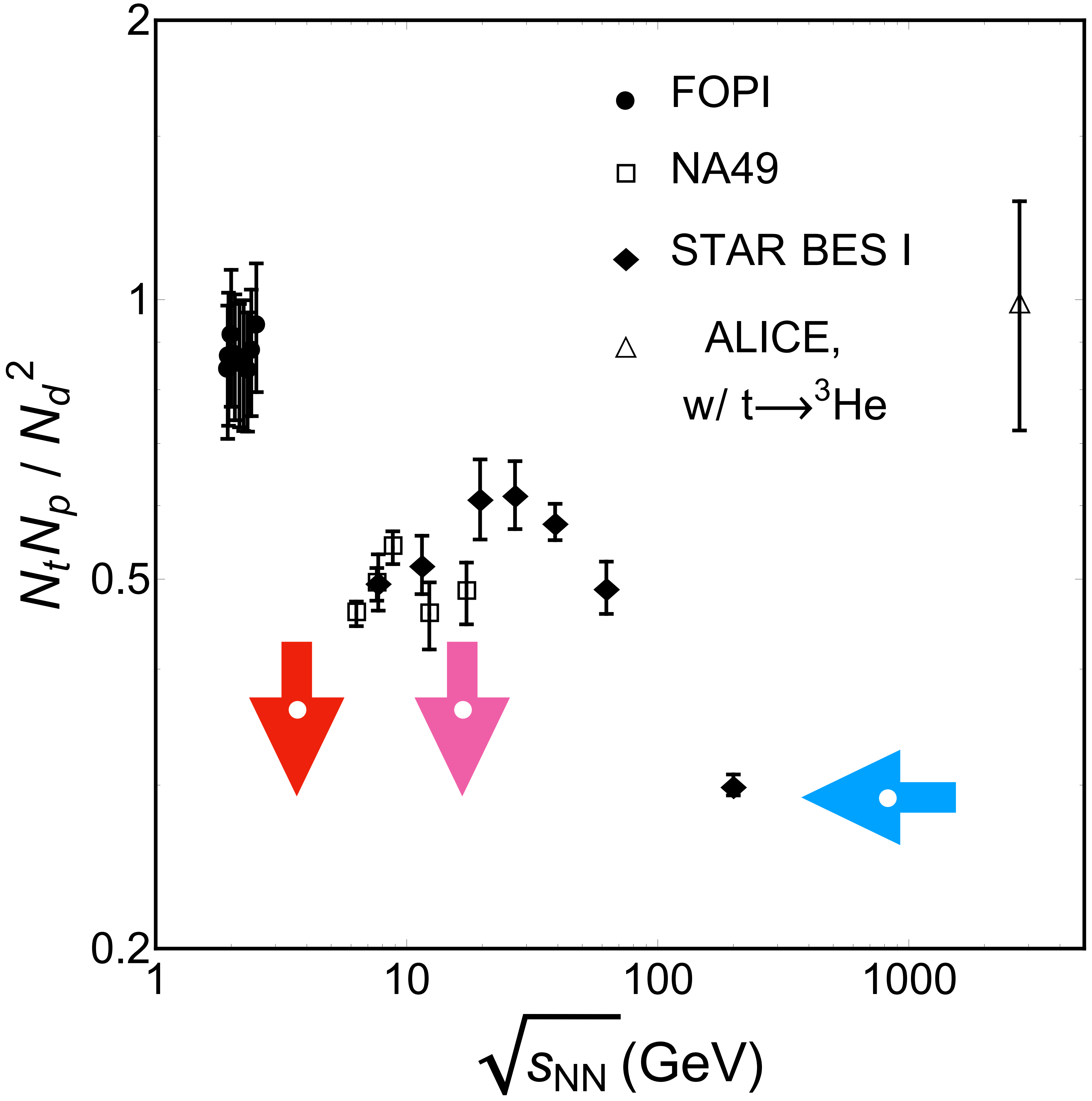

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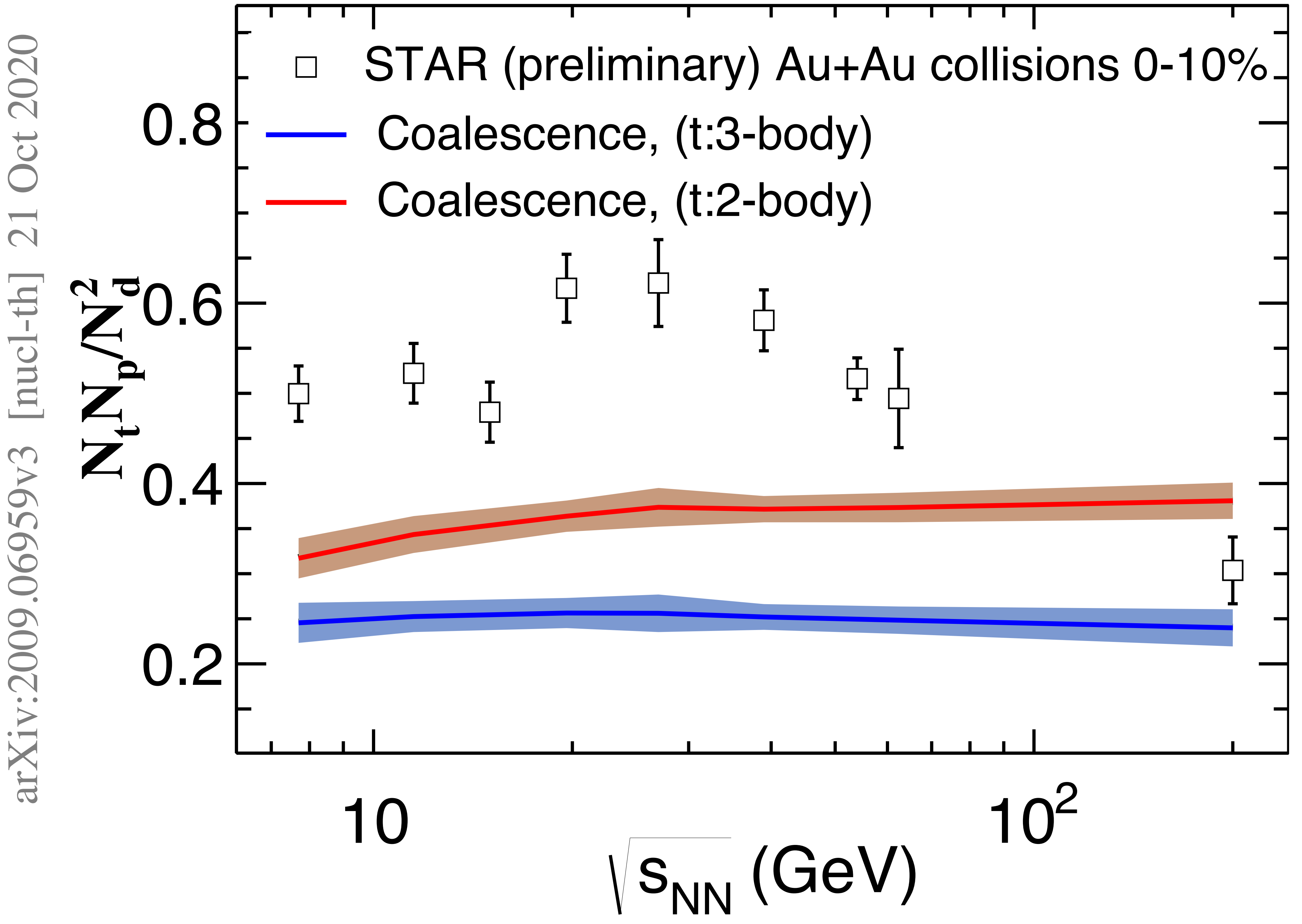

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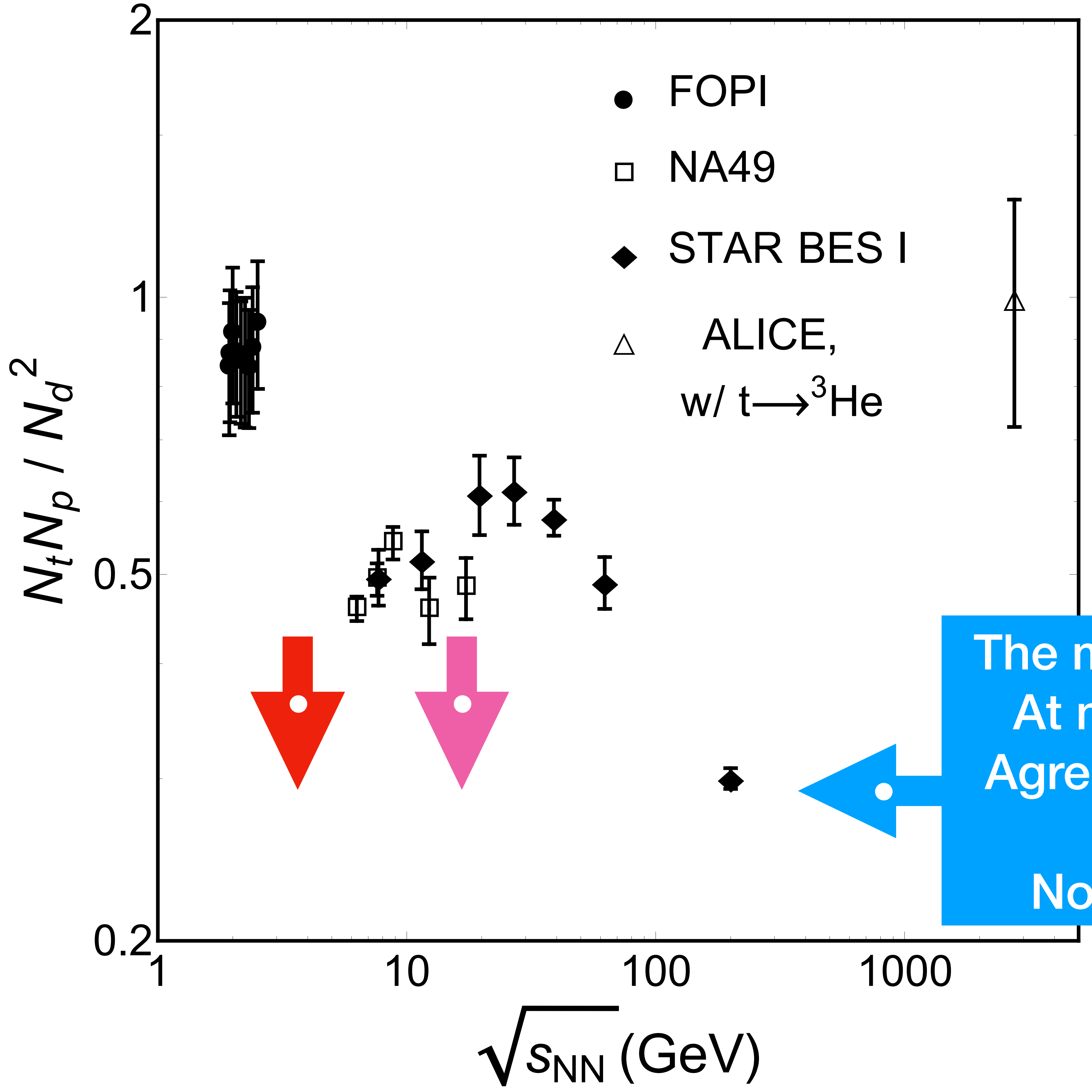

The most accurate point
At main RHIC energy
Agrees with stat weight
Ratio 0.29 =>
No feedowns here?

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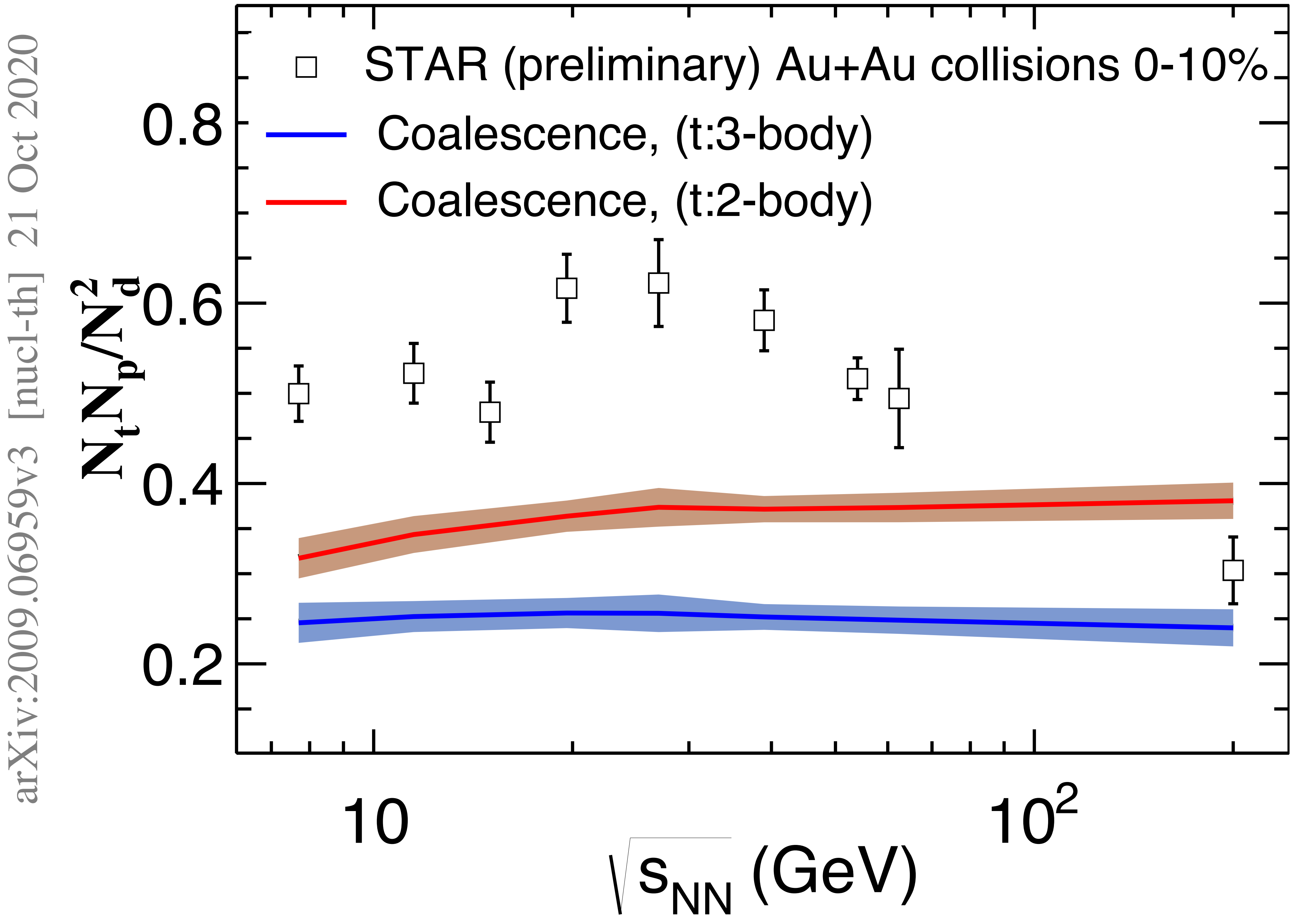

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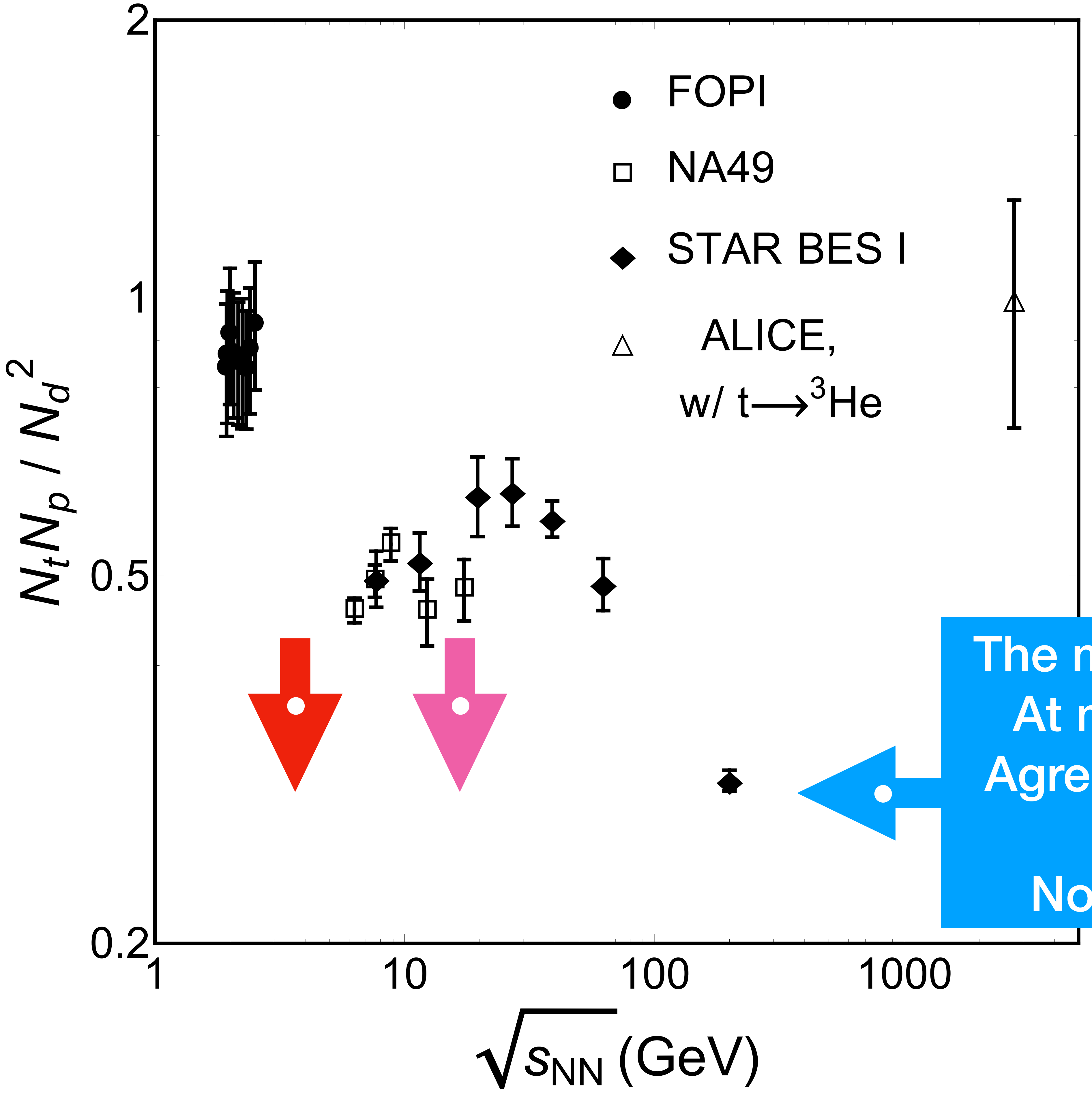

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Extra source of t is needed: 4-N reclusters => 50 states of He4 => feeding

Also two dips for central bins large at 2 and smaller at 20 GeV? Errors still large => BESII

Summary

- *Paradox 1 (ES,2006) at CP is not there, even at xi->infinity there is no implosion*
- *because of ulti-body repulsive forces*

- *Before xi reaches inter-nucleon distances in ambient matter, it does so for clusters*
- *So, watching the clusters (which are very sensitive) is a better signal than ambient EOS*

- *We calculated 3,4-body forces for different shapes and sizes of clusters, using universal Ising fluctuation potential, deformed because freeze out is away from critical line*
- *And get temperature dependence of effective triple and quartic couplings*

- *The results predict strong dip of clustering near TC*

- *Experimental data hint to TWO (?) correlated dips in TWO (very different) observables, kurtosis and tritium ratio*