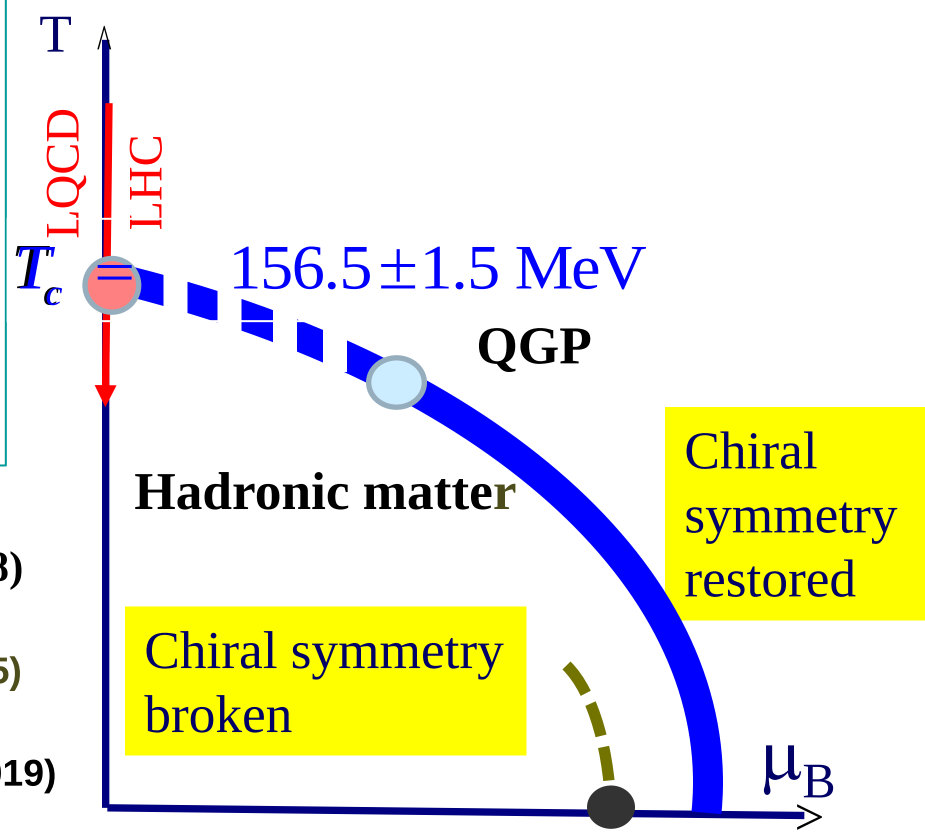
Probing of the QCD phase boundary within heavy ion collision

Krzysztof Redlich Uni. Wroclaw, Poland

- Linking ALICE yields data and 2nd order fluctuations from LQCD

- Modelling LQCD equation of state:
  From S-matrix => Hadron Resonance Gas
  => LQCD EqS => Particle Yields
- Net proton fluctuations:
  Establishing the non-critical baseline for fluctuation measurements



A. Andronic, P. Braun-Munzinger, J. Stachel & K.R., **Nature 561, 302-309 (2018)**

P. Braun-Munzinger, A. Kalweit, J. Stachel & K.R., **Phys.Lett.B 747, 292 (2015)**

A. Andronic, P. Braun-Munzinger, Pok Man Lo, B. Friman J. Stachel & K.R **Phys. Lett. B 792, 304 (2019)**

Pok Man Lo, B. Friman, C. Sasaki & K.R. **Phys.Lett. B778 (2018)**

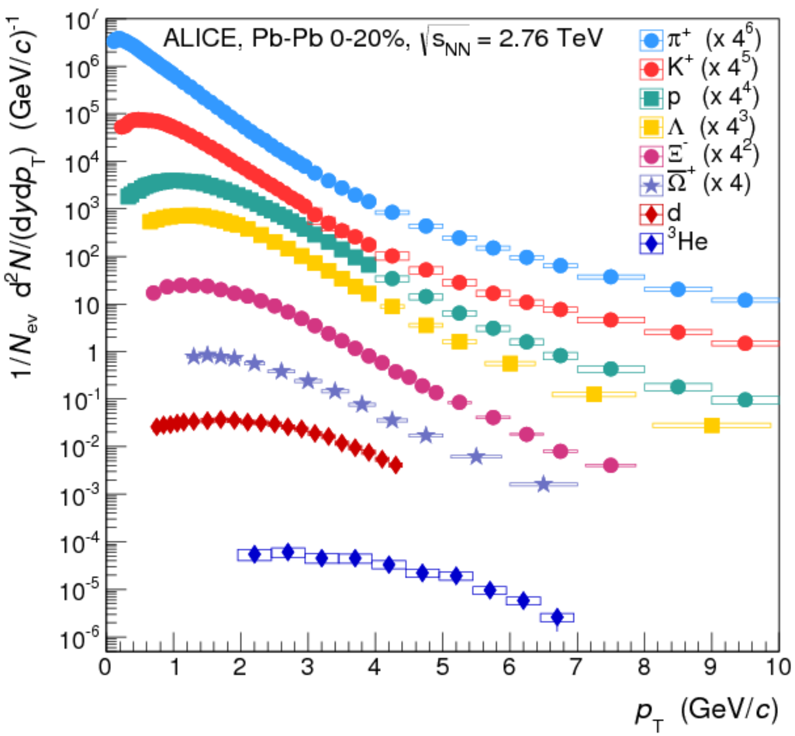
P. Braun-Munzinger, B. Friman, A. Rustamov, J. Stachel & K.R. 2007.02463 [nucl-th] **Nucl. Phys. A (2021)**

J. Cleymans, Pok Man Lo, N. Sharma &K.R. **Phys. Rev. C103 (2021)**

Direct link between ALICE data and fluctuations in LQCD

Can the thermal nature and composition of the collision fireball in HIC be verified ?

HIC ⇔ **LQCD**



Excellent data of LHC experiments on p_T-distributions and particle pseudo-rapidity densities

- **The strategy:**

Compare directly measured fluctuations and correlations of conserved charges, (B,Q,S) în HIC at the LHC and LQCD results

$$\chi_{ijk}^{BQS} = \frac{\partial^{(i+j+k)} P(T,\mu)}{\partial \mu_B^i \mu_B^i \mu_B^i}\Big|_{\vec{\mu}=0} = < B^i Q^j S^k > + ..$$

F. Karsch and K. R, Phys. Lett. B 695, 136 (2011) A. Bazavov et al., Phys. Rev. Lett. 109, 192302 (2012):

P. Braun-Munzinger, A. Kalweit, J. Stachel & K.R., Phys.Lett.B 747, 292 (2015)

P. Braun-Munzinger, et al. Nucl.Phys. A956, 805 (2016)

Consider 2nd order fluctuations and correlations of conserved charges to be compared with LQCD

Excellent probe of:

- QCD χ-criticality

A. Asakawa at. al.
F.Karsch,S. Ejiri et. al.,
M. Stephanov et al.,
K. Rajagopal,
E. Shuryak
B. Frimann et al.

- EQS in HIC

F. Karsch &
S. Mukherjee et al.,
C. Ratti et al.
P. Braun-Munzinger et al., V. Koch et al.

- They are quantified by susceptibilities:

If $P(T,\mu_B,\mu_Q,\mu_S)$ denotes pressure, then

$$\frac{\chi_N}{T^2} = \frac{\partial^2 (P)}{\partial(\mu_N)^2} \qquad \frac{\chi_{NM}}{T^2} = \frac{\partial^2 (P)}{\partial \mu_N \partial \mu_M}$$

$$N = N_q - N_{-q}, \quad N,M = (B,S,Q), \quad \mu = \mu / T, \quad P = P/T^4$$

- Susceptibility χ_N is connected with variance σ_N^2

$$\frac{\chi_N}{T^2} = \frac{1}{VT^3}(< N^2 > - < N >^2) \qquad \frac{\chi_{NM}}{T^2} = \frac{1}{VT^3} < NM >$$

- If $\mathrm{P}(N)$ probability distribution of N then

$$< N^n > = \sum_N N^n \mathrm{P}(N)$$

In experiment σ_N^2 are measured from the event by event net charge distributions

Consider special case: the Skellam distribution

- Charge $\mathrm{P}(N_q)$ and anti-charge $\mathrm{P}(N_{\bar{q}})$ Poisson distributed, then for $N = N_q - N_{\bar{q}}$
- $\mathrm{P}(N)$ is the Skellam distribution

$$\mathrm{P}(N) = \left(\frac{< N_q >}{< N_{-q} >} \right)^{N/2} I_N(2\sqrt{< N_q N_{-q} >}) \exp[-(< N_q > + < N_{-q} >)]$$

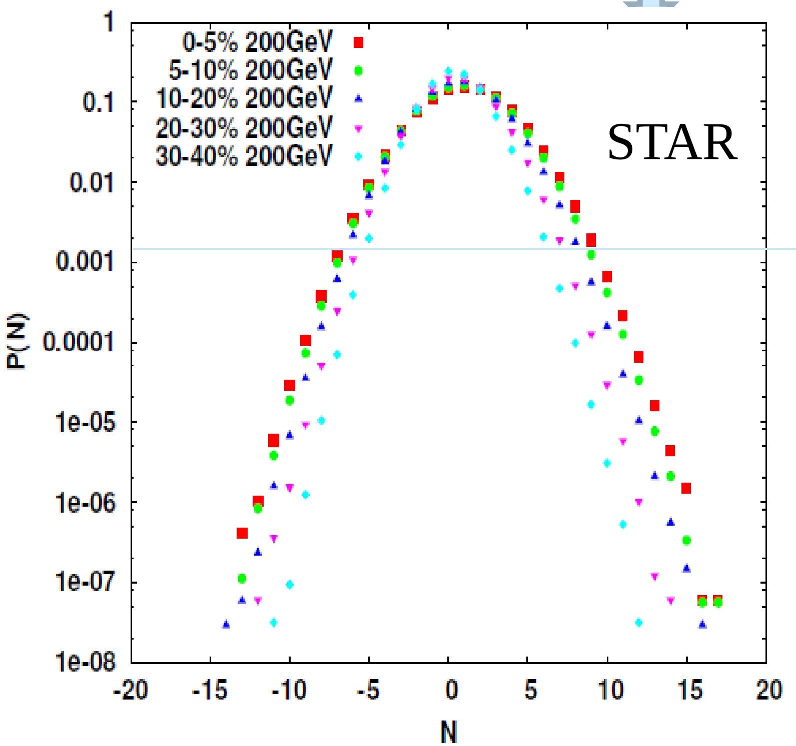
- Then, the susceptibility

$$\frac{\chi_N}{T^2} = \frac{1}{VT^3}(< N_q > + < N_{-q} >)$$

expressed by yields of particles and antiparticles carrying the conserved charge $q = \pm 1$

$$< N_q > = \sum_i < N_q^i > \qquad < N_{\bar{q}} > = \sum_i < N_{\bar{q}}^i >$$

Variance of net-proton in AA central coll. at LHC

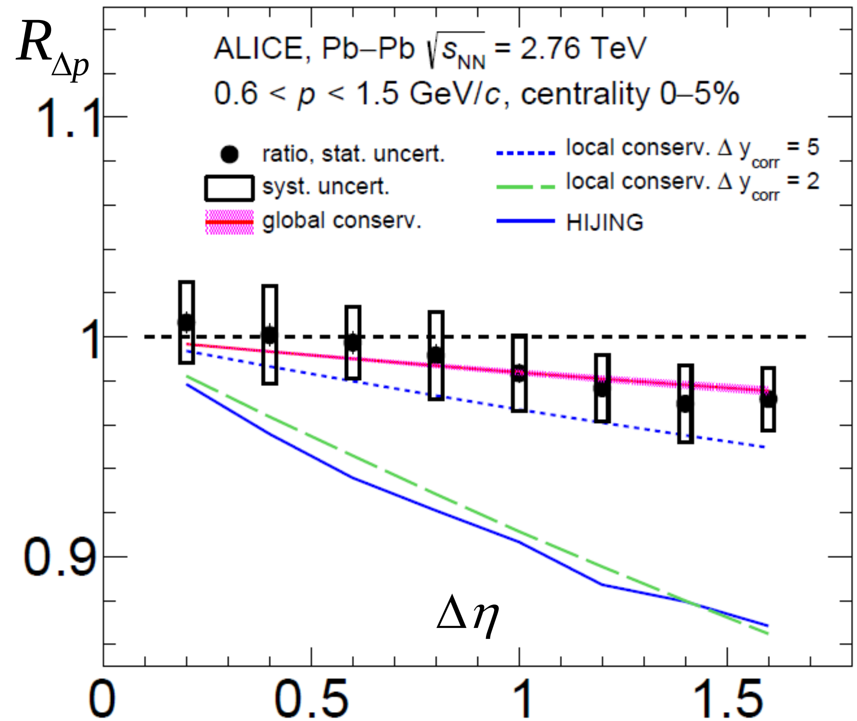


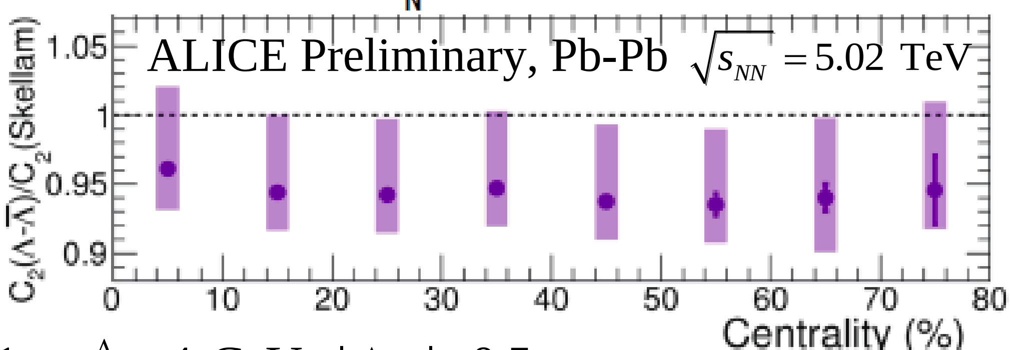
ALICE data for 2[nd] net proton fluctuations

$$N = N_p - N_{\bar{p}} \quad \Longrightarrow \quad \sigma^2(N) = \langle N^2 \rangle - \langle N \rangle^2$$

consistent with Skellam distribution i.e.

$$R_{\Delta p} = \sigma^2(N) / (\langle N_p \rangle + \langle N_{\bar{p}} \rangle) \approx 1$$





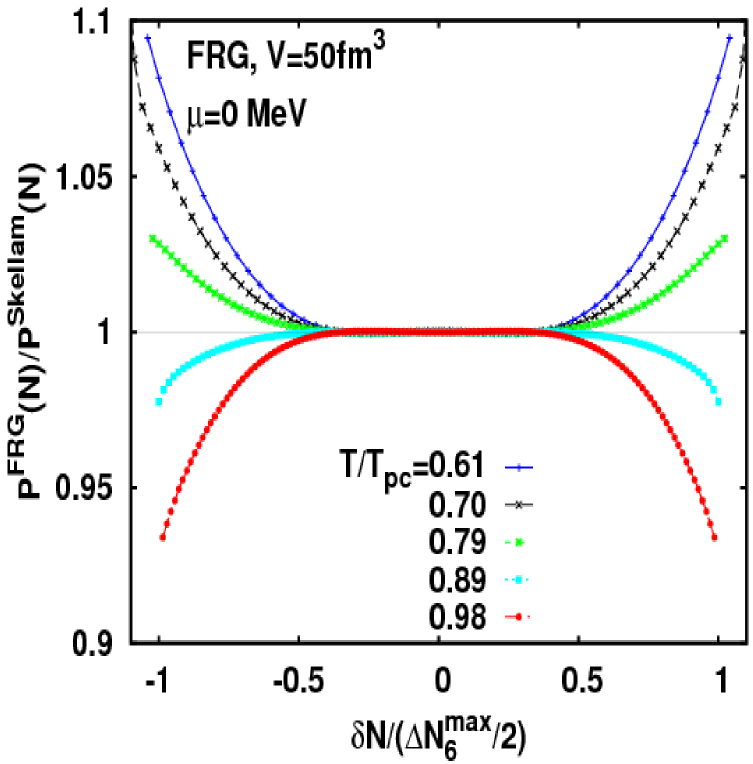
$1 < p_T^{\Lambda} < 4$ GeV $\quad |\Delta\eta| < 0.5$

P. Braun-Munzinger, A. Rustamov,

The influence of baryon number conservation: J. Stachel. Nucl Phys. A960 (2017) 114

Variance $\sigma^2_{\Delta p}$ in AA central collisions at RHIC

K. Morita, B. Friman and K.R.
Phys.Lett. B741 (2015) 178

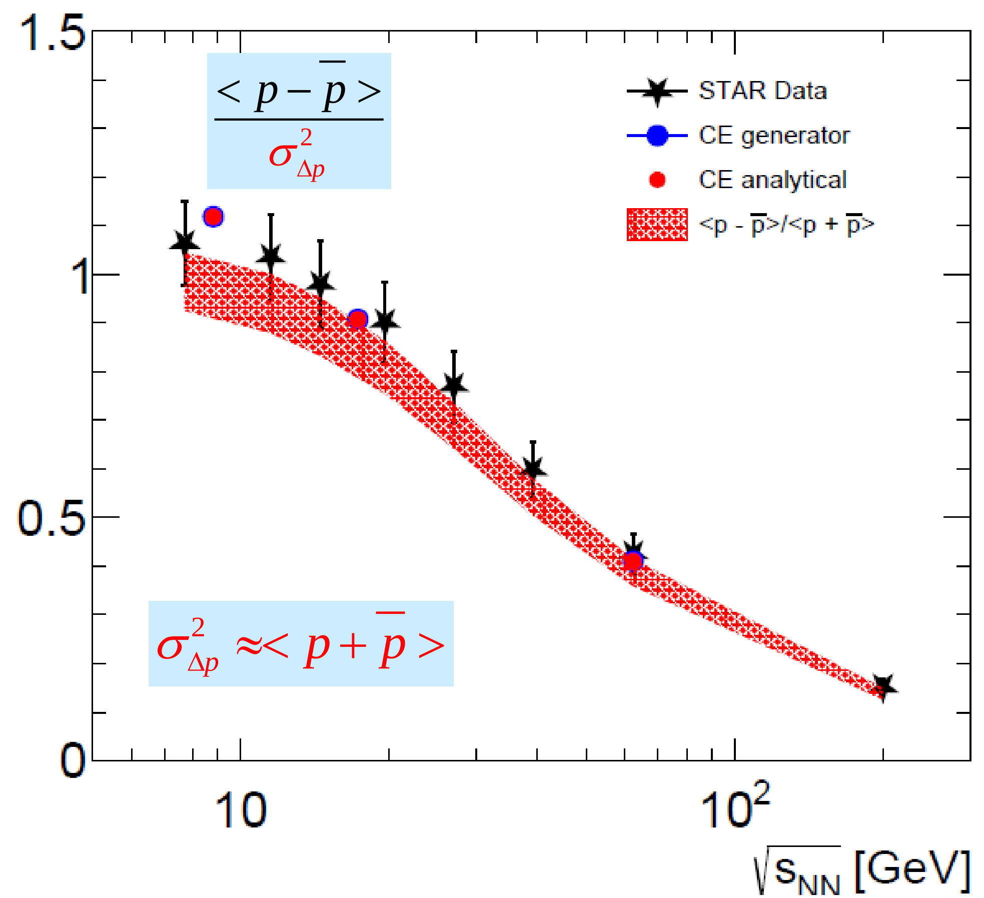


Skellam distribution provides good approximation of P(N) that exhibits O(4) criticality only for the second order cumulants. This is however not the case for higher order cumulants

STAR $\sigma^2_{\Delta p}$ data in central AA for diff. $\sqrt{s}$

- also consistent with Skellam distribution

P. Braun-Munzinger, B. Friman, A. Rustamov
J. Stachel & K.R. Nucl. Phys. (2021)



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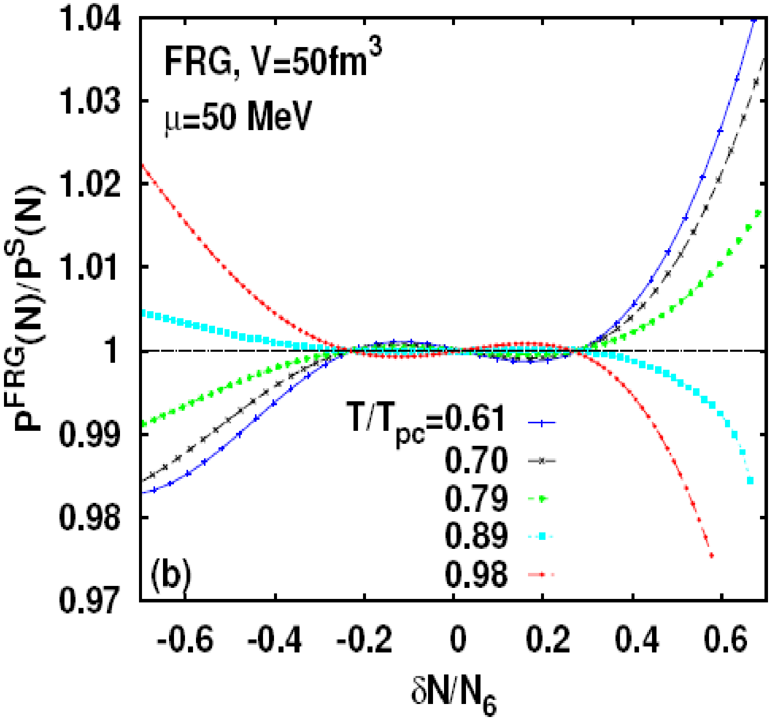

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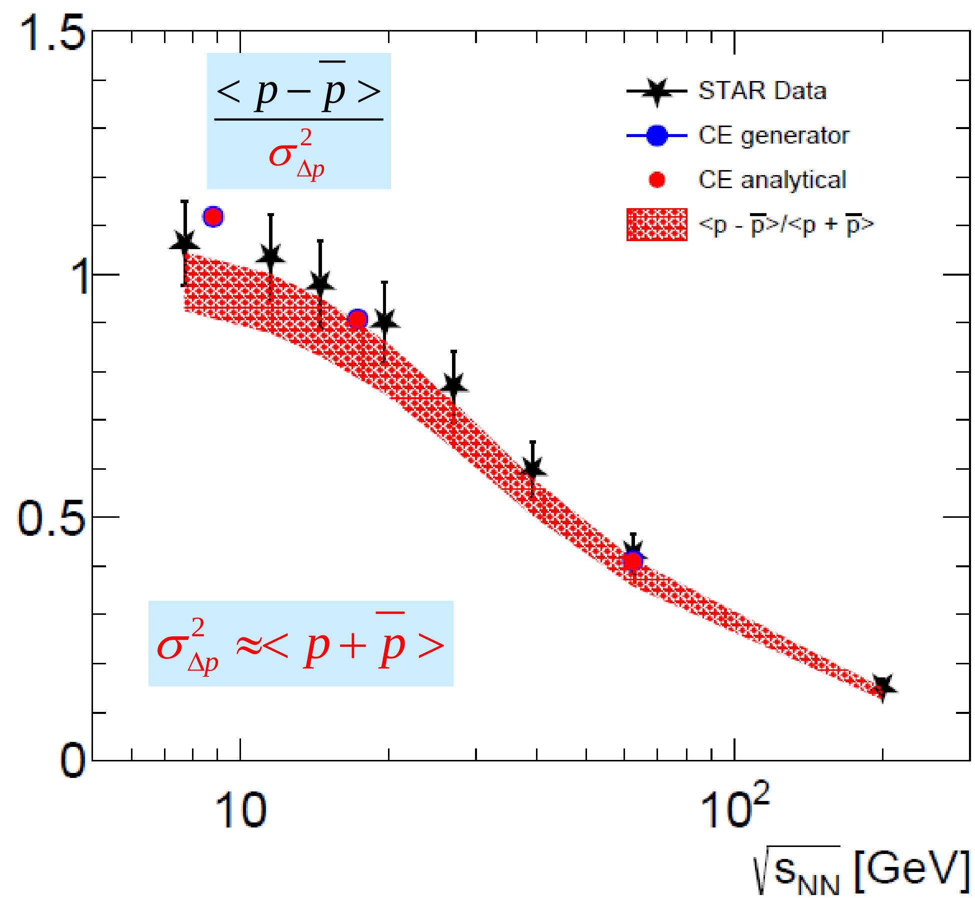


Skellam distribution provides good approximation of P(N) that exhibits O(4) criticality only for the second order cumulants. This is however not the case for higher order cumulants

Constructing net charge fluctuations and correlation from ALICE data

$$\frac{\chi_Q}{T^2} \approx \frac{1}{VT^3}(<N_q> + <N_{-q}>)$$

P. Braun-Munzinger, A. Kalweit, J. Stachel, & K.R. Phys. Lett. B747, 292 (2015)

- Net baryon number susceptibility

$$\frac{\chi_B}{T^2} \approx \frac{1}{VT^3}(\langle p\rangle + \langle N\rangle + \langle \Lambda + \Sigma_0\rangle + \langle \Sigma^+\rangle + \langle \Sigma^-\rangle + \langle \Xi^-\rangle + \langle \Xi^0\rangle + \langle \Omega^-\rangle + \overline{par}\ )$$

- Net strangeness

$$\frac{\chi_S}{T^2} \approx \frac{1}{VT^3}(\langle K^+\rangle + \langle K_S^0\rangle + \langle \Lambda + \Sigma_0\rangle + \langle \Sigma^+\rangle + \langle \Sigma^-\rangle + 4\langle \Xi^-\rangle + 4\langle \Xi^0\rangle + 9\langle \Omega^-\rangle + \overline{par}$$
$$-(\Gamma_{\varphi\to K^+} + \Gamma_{\varphi\to K^-} + \Gamma_{\varphi\to K_S^0} + \Gamma_{\varphi\to K_L^0})\langle \varphi\rangle\ )$$

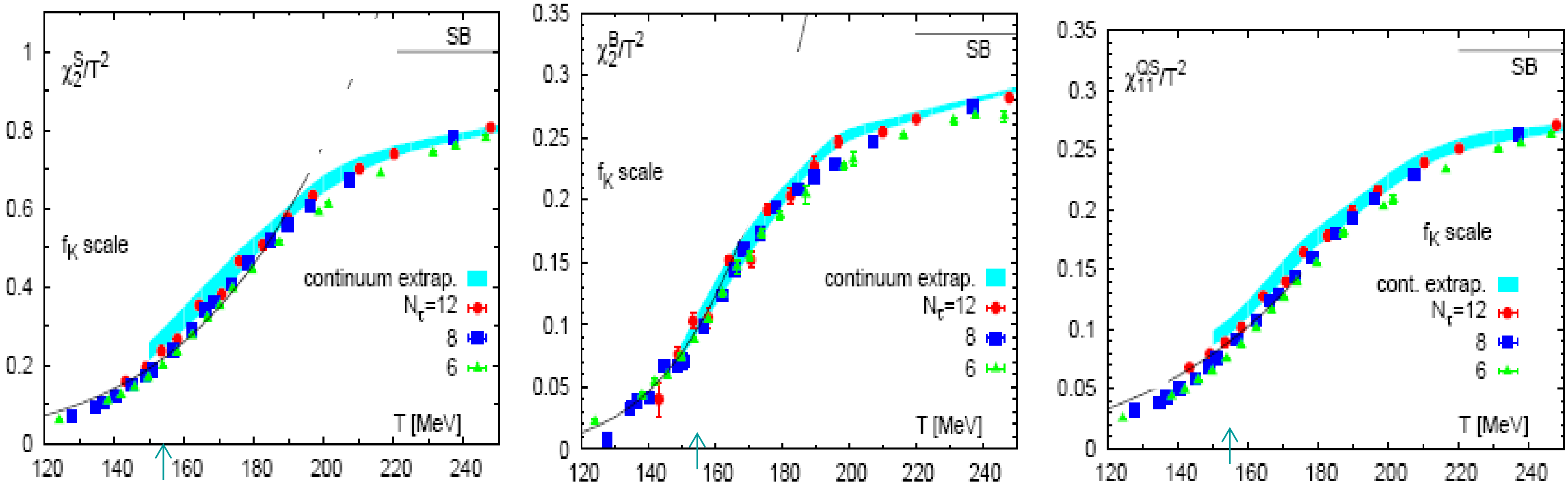
- Charge-strangeness correlation

$$\frac{\chi_{QS}}{T^2} \approx \frac{1}{VT^3}(\langle K^+\rangle + 2\langle \Xi^-\rangle + 3\langle \Omega^-\rangle + \overline{par}$$
$$-(\Gamma_{\varphi\to K^+} + \Gamma_{\varphi\to K^-})\langle \varphi\rangle - (\Gamma_{K_0^*\to K^+} + \Gamma_{K_0^*\to K^-})\langle K_0^*\rangle\ )$$

2^{nd} order fluctuations in LQCD

- 2^{nd} order cumulants of strangeness and net-baryon number fluctuations and charge-strangeness correlation calculated on the lattice and extrapolated to the continuum limit. (see also see also: J. Goswami, et al., 2011.02812 [hep-lat])

A. Bazavov, et al. HotQCD Coll (2014).



- Is there a common temperature where all 2^{nd} order cumulants obtained from ALICE data agree with LQCD result?

$$\frac{\chi_N}{T^2} = \frac{1}{VT^3}(< N_q > + < N_{-q} >)$$

Direct comparison of Heavy ion data at LHC with LQCD

χ_{NM} with $N, M = \{B, Q, S\}$ are expressed by particle yields: for Skellam distribution

- Is there a common temperature where all 2nd order cumulants constructed from ALICE data agree with LQCD result?

LQCD From ALICE DATA

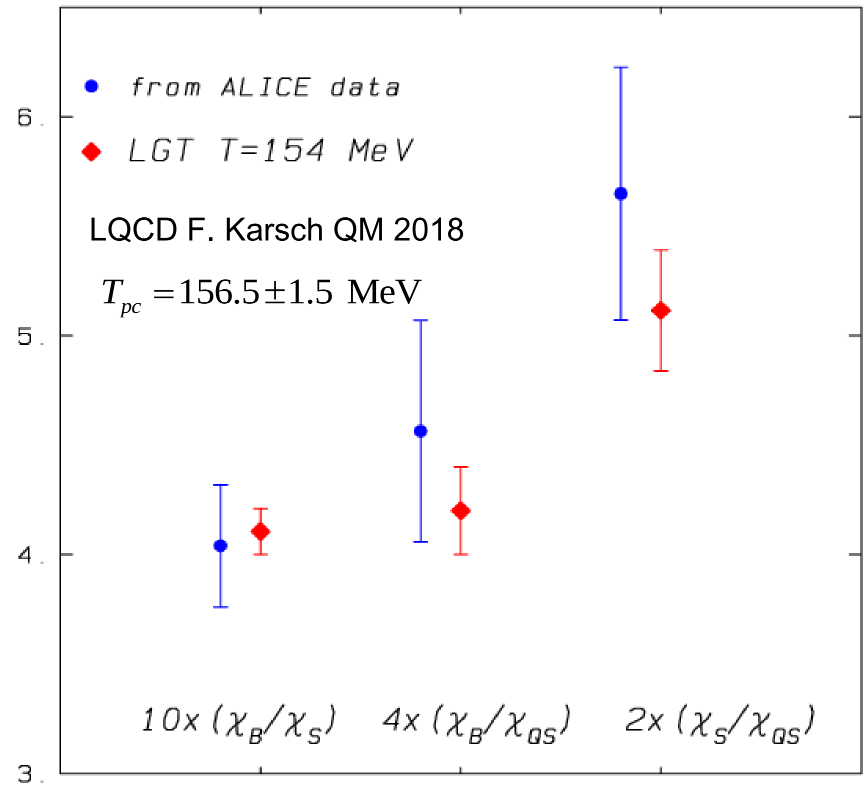
$$\frac{\chi_B}{T^2} = \frac{1}{VT^3}(203.7 \pm 11.4)$$

$$\frac{\chi_S}{T^2} = \frac{1}{VT^3}(504.2 \pm 16.8)$$

$$\frac{\chi_{QS}}{T^2} = \frac{1}{VT^3}(191.1 \pm 12)$$

- The Volume at $T \approx 154\ MeV$

$$V_{T_f} = 3800 \pm 500\ \ fm^3$$

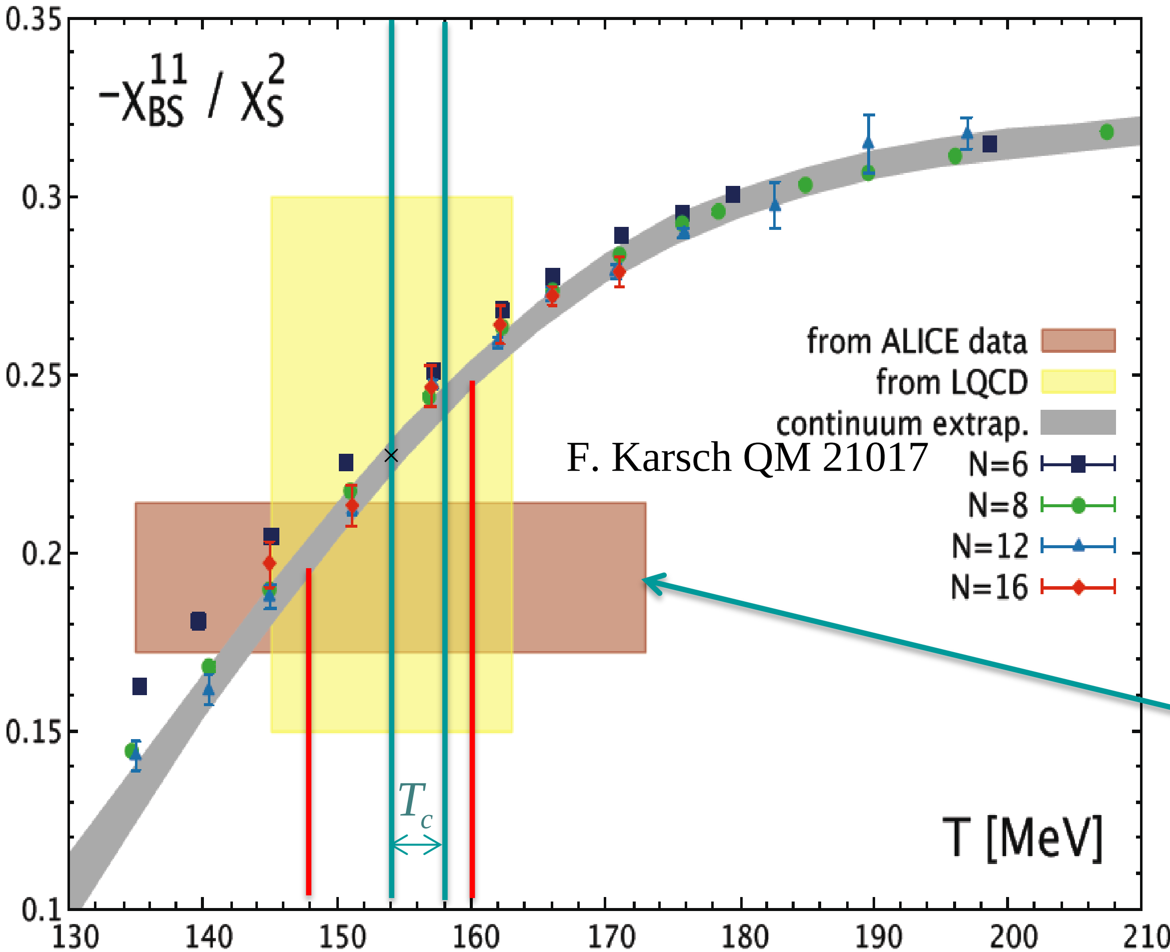


The 2nd cumulant ratios extracted from ALICE data are consistent with LQCD results at

$$T_f = 154 \pm 6\ \ MeV$$

Evidence for thermalization and saturation of the 2nd order fluctuations near the QCD phase boundary

Constraining chemical freezeout temperature at the LHC



At the LHC the fireball created in HIC is QCD medium at the chiral crossover temperature.

$$C_{BS} = -\frac{<(\delta B)(\delta S)>}{<(\delta S)^2>} = -\frac{\chi_{BS}}{\chi_S}$$

- Excellent observable to fix the temperature

$$-\frac{\chi_{BS}}{T^2} \approx \frac{1}{VT^3}[2<\Lambda+\Sigma^0>+4<\Sigma^+>$$
$$+8<\Xi>+6<\Omega^->] = (97.4\pm5.8)/VT^3$$

This is the lower limit since e.g. $\Sigma^*(\geq 1660) \to N\overline{K}$ $\Lambda^*(\geq 1520) \to N\overline{K}$ are not included

- Data on $\chi_B, \chi_S, \chi_{QS}$ are consistent with LQCD at

$$T_f = 154 \pm 6 \;\; MeV$$

Modelling QCD thermodynamic potential in hadronic phase

Pressure of an interacting, a+b ⇔ a+b, hadron gas in equilibrium

$$P(T) \approx P_a^{id} + P_b^{id} + P_{ab}^{\text{int}}$$

The leading order interactions, determined by the two-body scattering phase shift, which is equivalent to the second virial coefficient

$$P^{\text{int}} = \sum_{I,j} \int_{m_{th}}^{\infty} dM \; B_j^I(M) P^{id}(T,M)$$

$$B_j^I(M) = \frac{1}{\pi} \frac{d}{dM} \delta_j^I(M)$$

↓ *Effective weight function* ↓ *Scattering phase shift*

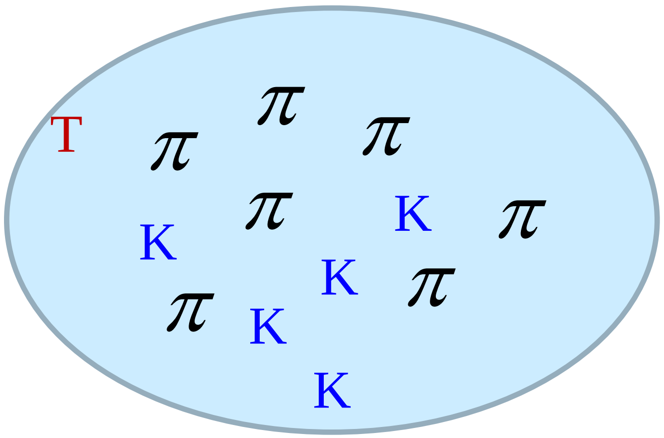
R. Dashen, S. K. Ma and H. J. Bernstein, Phys. Rev. 187, 345 (1969)
R.Venugopalan, and M. Prakash, Nucl. Phys. A 546 (1992) 718.
W. Weinhold,, and B. Friman, Phys. Lett. B 433, 236 (1998).
Pok Man Lo, Eur. Phys.J. C77 (2017) no.8, 533

- Interactions driven by narrow resonance of mass M_R

  $B(M) = \delta(M^2 - M_R^2) \quad \Rightarrow \quad P^{\text{int}} = P^{id}(T, M_R) \; \Rightarrow HRG$

  For finite and small width of resonance, B(M) => Breit-Wigner form
- For non-resonance interactions or for broad resonances $P_{ab}^{\text{int}}(T)$ should be linked to the phase shifts

S-MATRIX APPROACH and Hadron Resonance Gas



- Consider interacting pions and kaons gas in thermal equilibrium at temperature T
- Due to *K*π scattering resonances are formed

  I =1/2, s -wave : κ(800), K0*(1430) [*JP* = 0+]

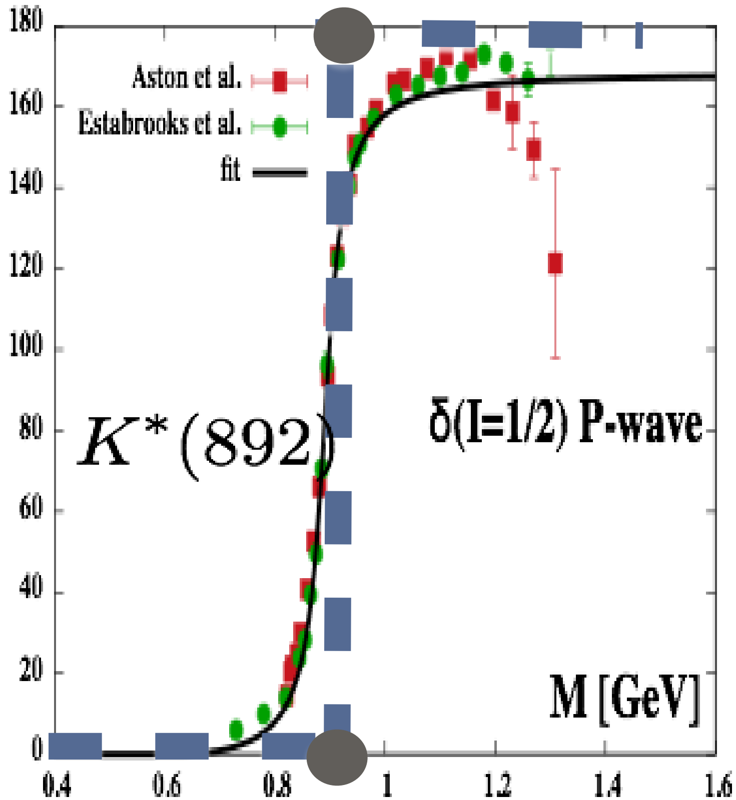
  I =1/2, p -wave : *K**(892), *K**(1410), *K**(1680) [*JP* =1−]

  I =3/2 purely repulsive interactions
- In the S-matrix approach the thermodynamic pressure in the low density approximation

$$P(T) \approx P_{\pi}^{id} + P_{K}^{id} + P_{\pi K}^{\text{int}}$$

Experimental phase shift in the P-wave channel

δ_M



B. Friman, P. M. Lo, M. Marczenko, K. Redlich and C. Sasaki, Phys. Rev. D 92, no. 7, 074003 (2015)

For narrow resonances

$$B(M) = 2\frac{d}{dM}\delta_M$$

well described by Breit-Wigner form

$$B(M) \approx M\frac{2M\Gamma_{BW}}{(M^2 - M_R^2) + M^2\Gamma_{BW}^2}$$

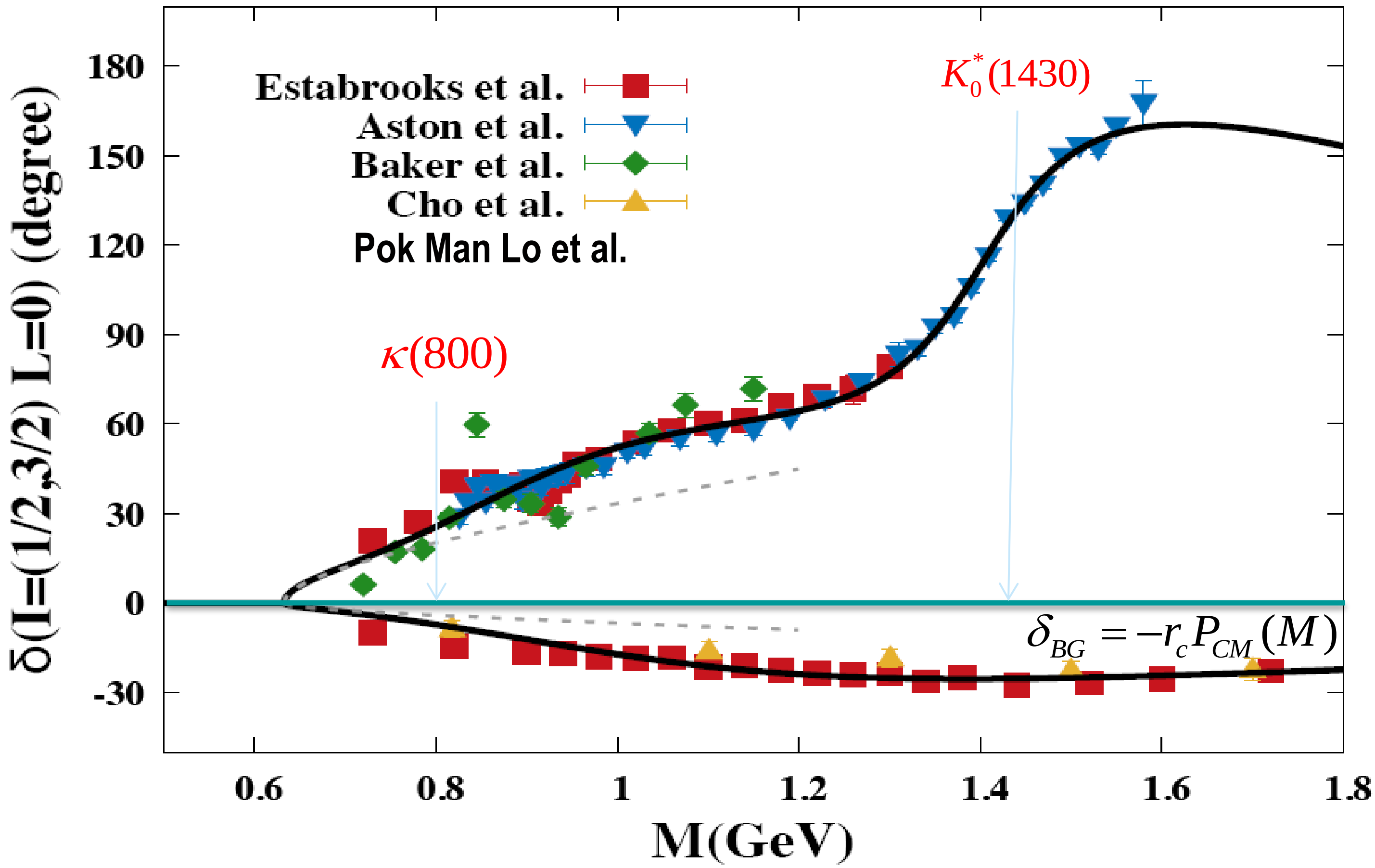
for $\Gamma_{BW} \to 0$

$$B(M) = \delta(M^2 - M_0^2)$$

and consequently interacting part of pressure contributes as an ideal gas Of resonances

$$P_{\pi K}^{\text{int}}(T) \approx P_{K^*}^{id}(T)$$

Non-resonance contribution- negative phase shift in S-wave channel



$$\delta_M = \delta_\kappa + \delta_{K^*} + \delta_{BG} \Rightarrow B(M) = 2\frac{d}{dM}\delta_M \Rightarrow P_{\pi K}^{\text{int}}(T)$$

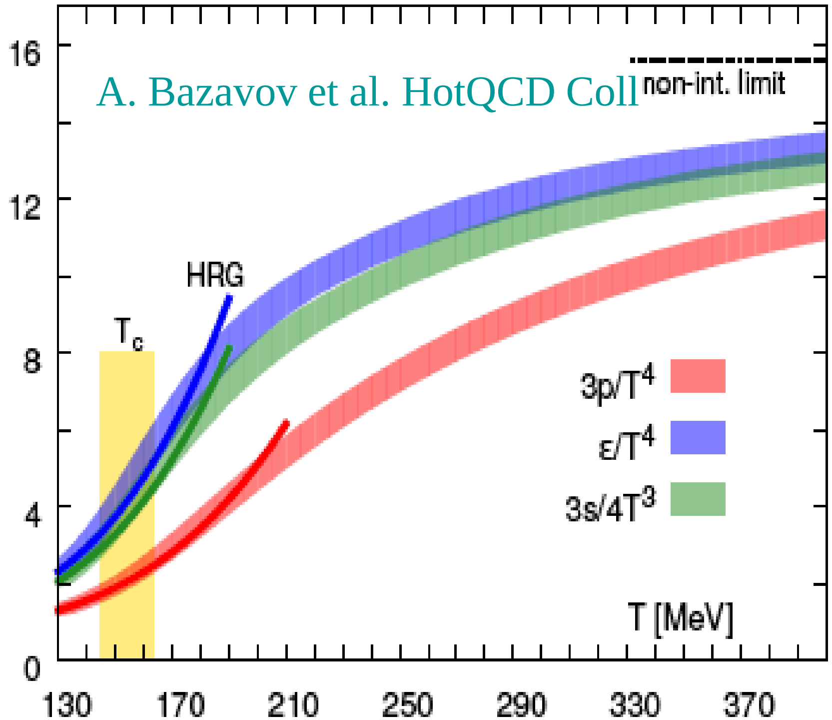
Quark-Hadron duality near the QCD phase boundary

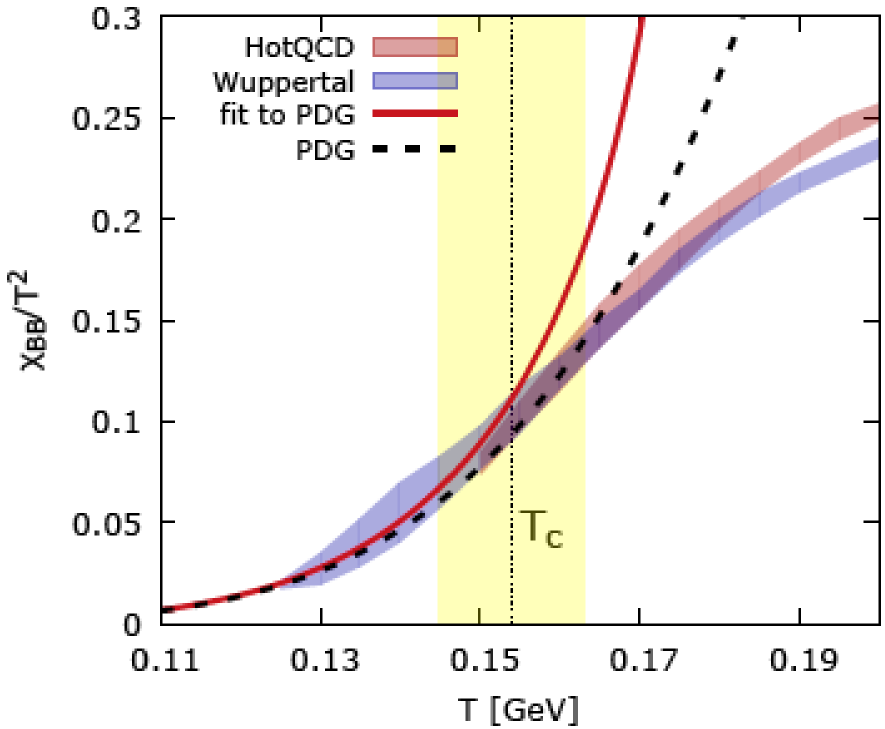
The HRG is a 1st order approximation of the QCD EQS in confined phase

$$P(T,\vec{\mu}) \approx \sum_H P_H^{id} + \sum_R P_R^i$$

$$P_R^{\,i} = \pm \frac{Tg_i}{2\pi^2}\int p^2 dp \int dM \ln(1 \pm e^{-\beta(E_i - \vec{q}_i \vec{\mu}_i)}) F_R^{BW}(M)$$

see also: J. Goswami, et al., 2011.02812 [hep-lat]





- Hadron Resonace Gas thermodynamic potential provides an excellent approximation of the QCD equation of states in confined phase
- Good description of net- baryon number fluctuations and in further sectors of hadronic quantum number on correlations and fluctuations

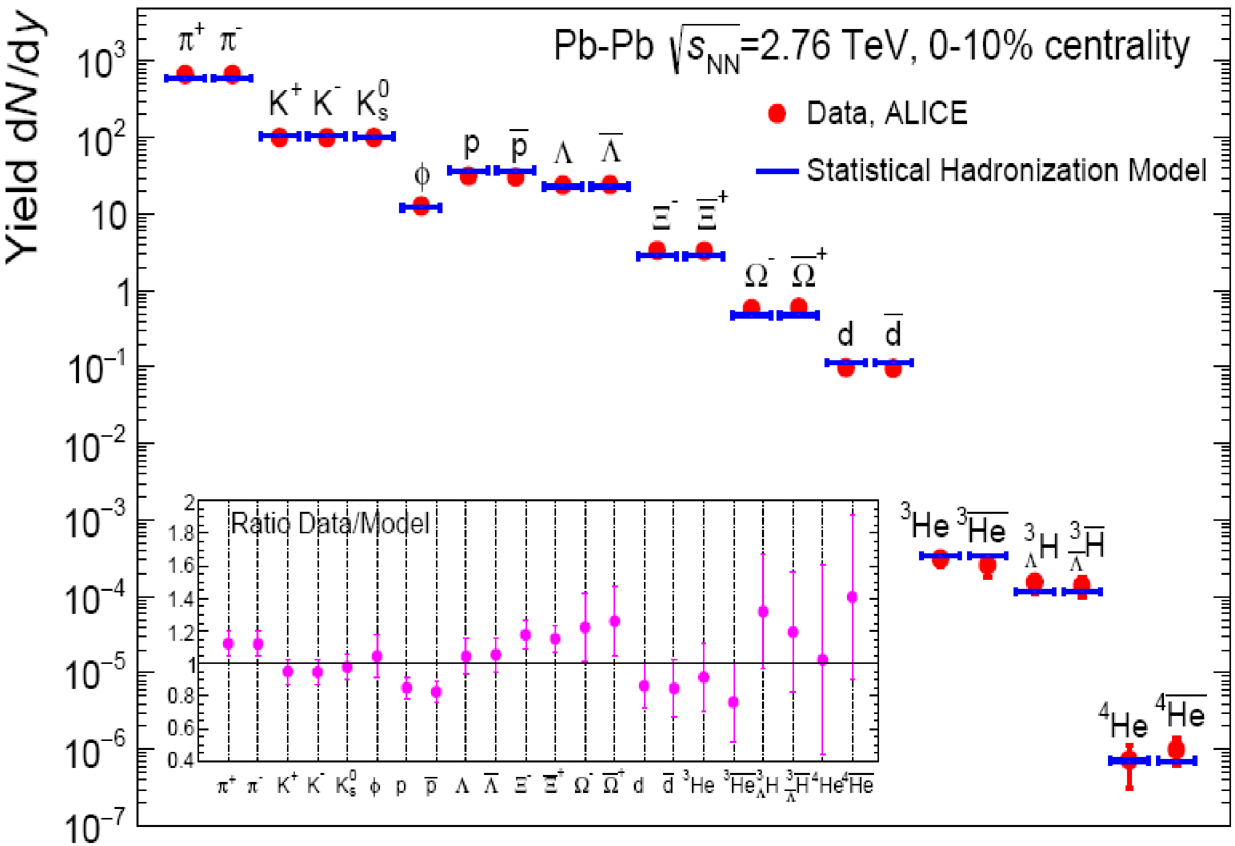
Thermal origin of particle yields and production at T_c

Apply the Hadron Resonance Gas (HRG) partition function as an excellent approximation of QCD statistical operator in the hadronic phase,

$$P^{regular}(T,\vec{\mu}) \approx \sum_H P_H^{id} + \sum_R P_R^i$$

A. Andronic, P. Braun-Munzinger, J. Stachel & K.R., Nature 561 (2018)

$$\frac{\langle N_i \rangle}{V} = n_i^{th}(T,\vec{\mu}) + \sum_K \Gamma_{K\to i} n_i^{Res.}(T,\vec{\mu})$$



- Measured yields are well reproduced at $T \approx 156$ MeV

$\mu_B = 0.7 \pm 3.8$ MeV

$\chi^2/\mathrm{dof} = 29.3/19$

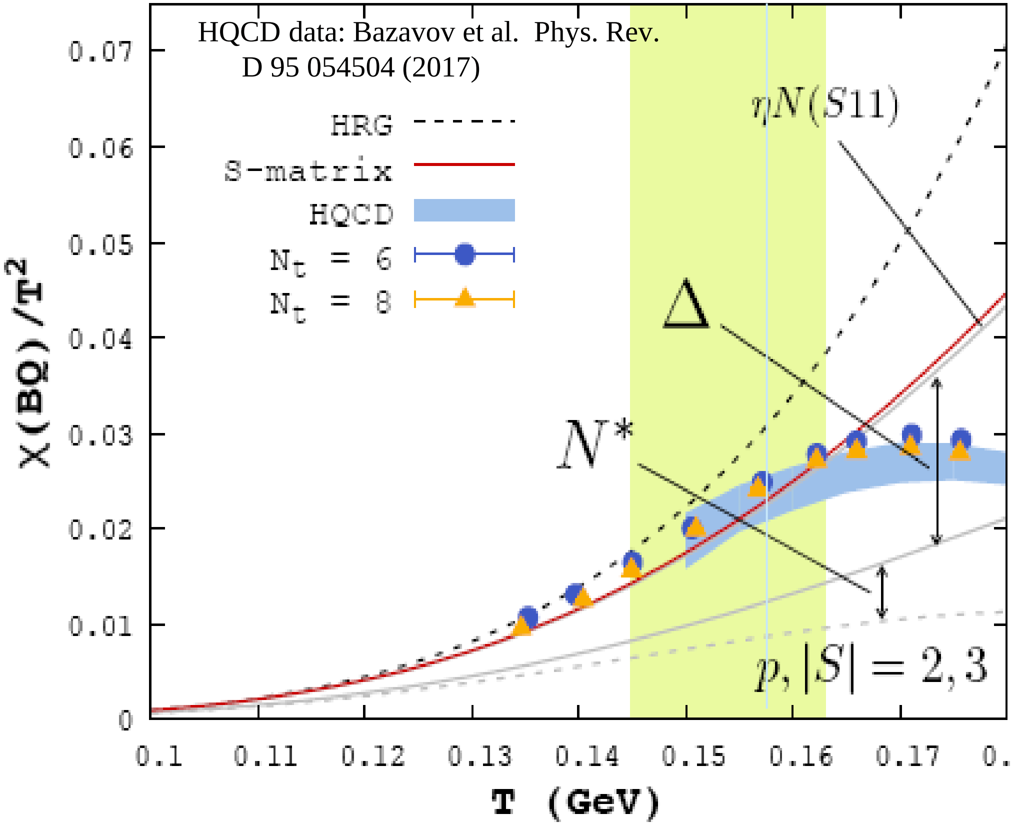
- The fireball in central A-A is QCD matter in equilibrium at T_c

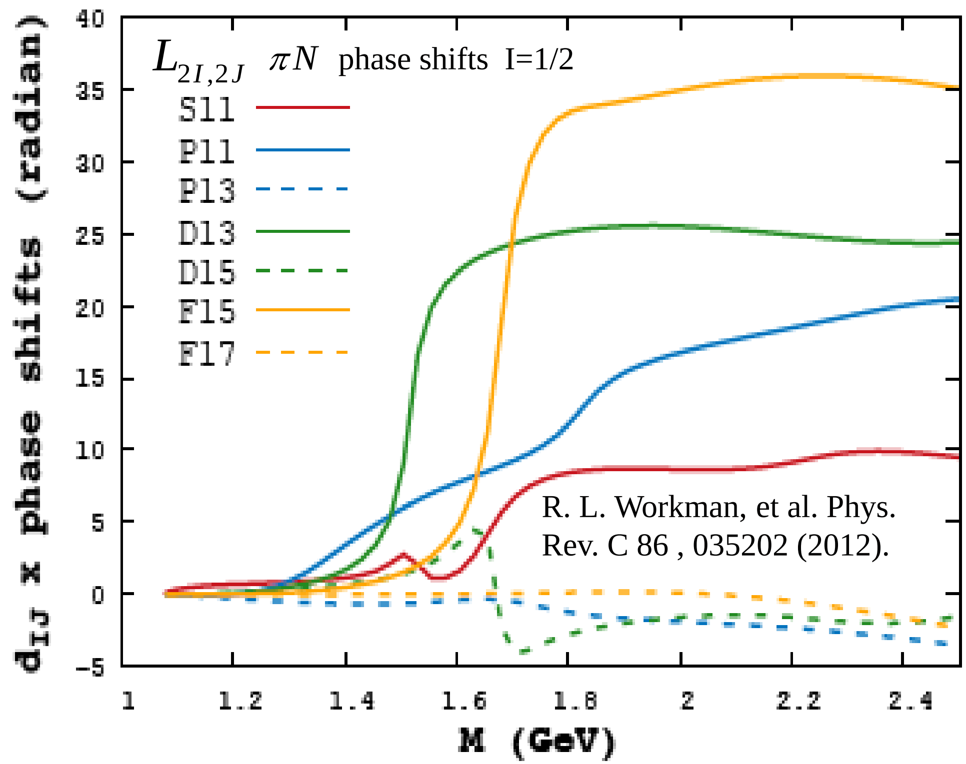
- However, too many protons (25%) relative to data: proton puzzle?

Probing non-strange baryon sector in πN- system

Pok Man Lo, B. Friman, C. Sasaki & K.R. , Phys.Lett. B778 (2018)

$$\chi_{BQ} = (\chi_{BB} - |\chi_{BS}|)/2$$



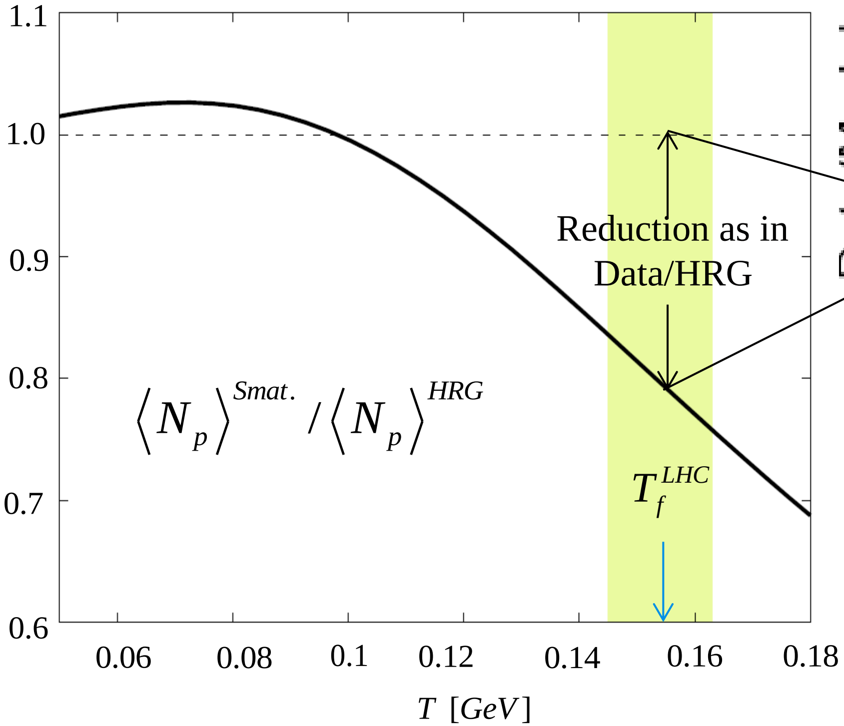


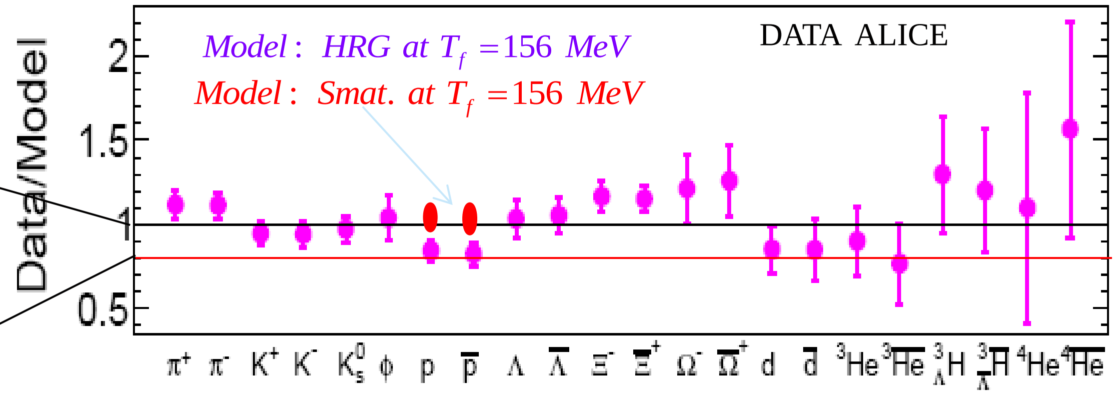
$$\Delta\chi_{BQ} \approx \sum_{I_z, j, B} d_j BQ \int dM \int d^3p \frac{1}{T} \frac{d\delta_j^I}{dM} \times e^{-\beta\sqrt{p^2+M^2}} (1+e^{-\beta\sqrt{p^2+M^2}})^{-2}$$

- Considering contributions of all πN $\delta_j^{I=(1/2),\ (3/2)}$ $(N^*, \Delta^*$ resonances) to χ_{BQ} within S-matrix approach, reduces the HRG predictions towards the LQCD in the chiral crossover $0.15 < T < 0.16\ GeV$

S-matrix Phenomenological consequences: proton production

see: A. Andronic, P. Braun-Munzinger, B. Friman, Pok Man Lo, J. Stachel & K.R. **Phys.Lett. B792 (2019) 304**

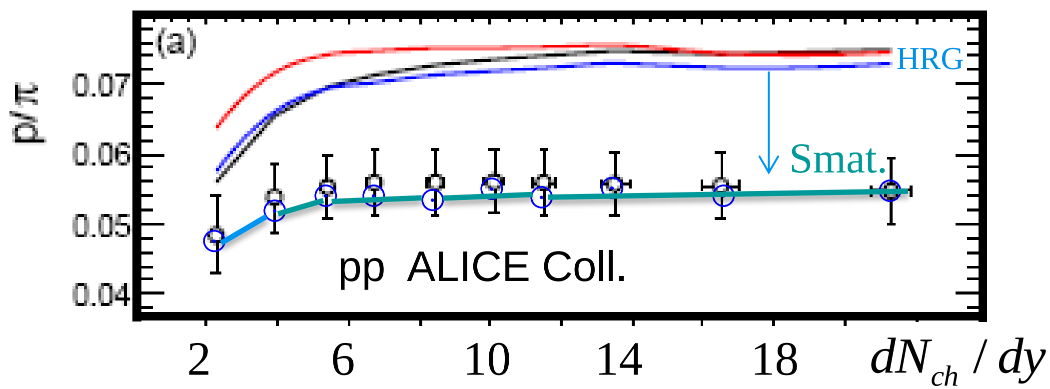




- Yields of protons in the S-matrix is suppressed relative to HRG

- Yields of protons in AA collisions at LHC is consistent with S-matrix result within 1σ

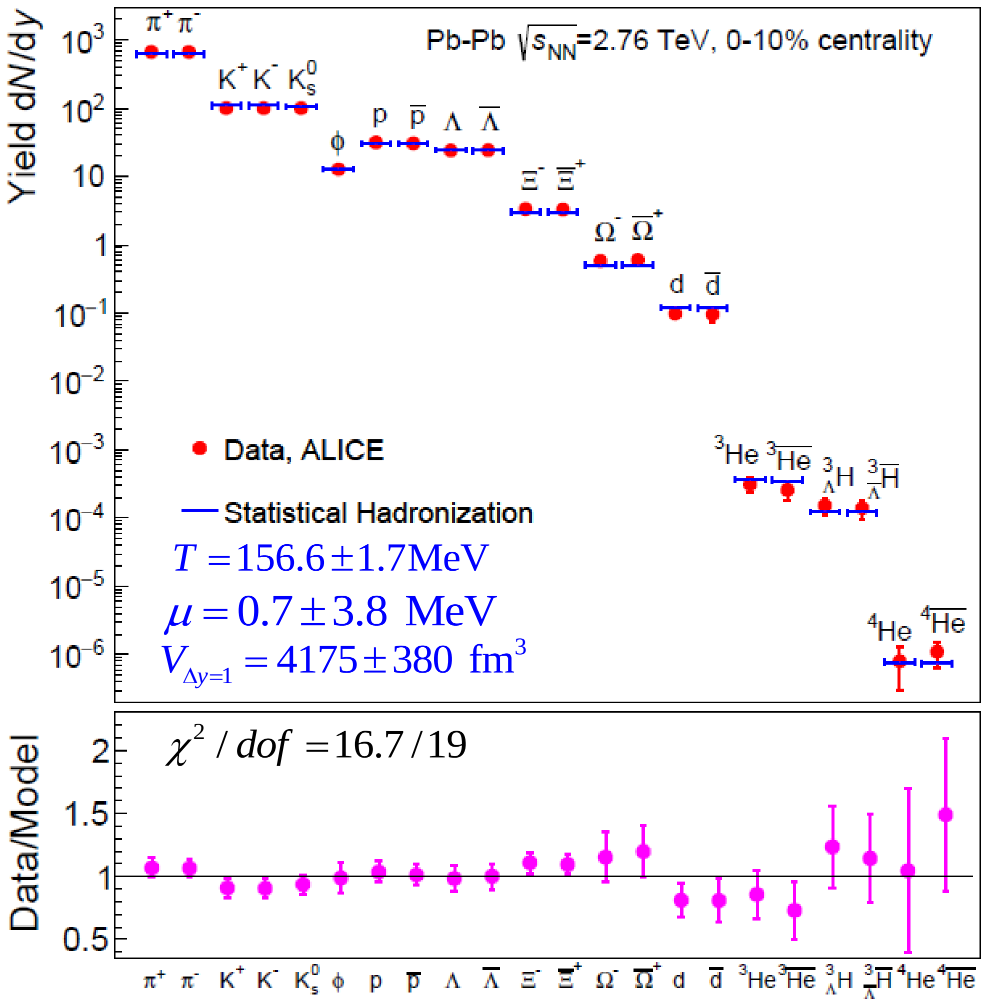
HRG: N. Sharma, J. Cleymans, B. Hippolite, arXiv: 1803.05409



For further consequences of smat. See also: P. Huovinen, P. Petreczky Phys. Lett. B77 (2018)

- S-matrix results well consistent with pp data

S-matrix corrections to HRG and particle yields at LHC



A. Andronic, P. Braun-Munzinger, J. Stachel & K.R

The S-matrix corrections due to pion-nucleon scattering in HRG provide still better description of LHC data, solving the problem of too large yield of protons and antiprotons

However:

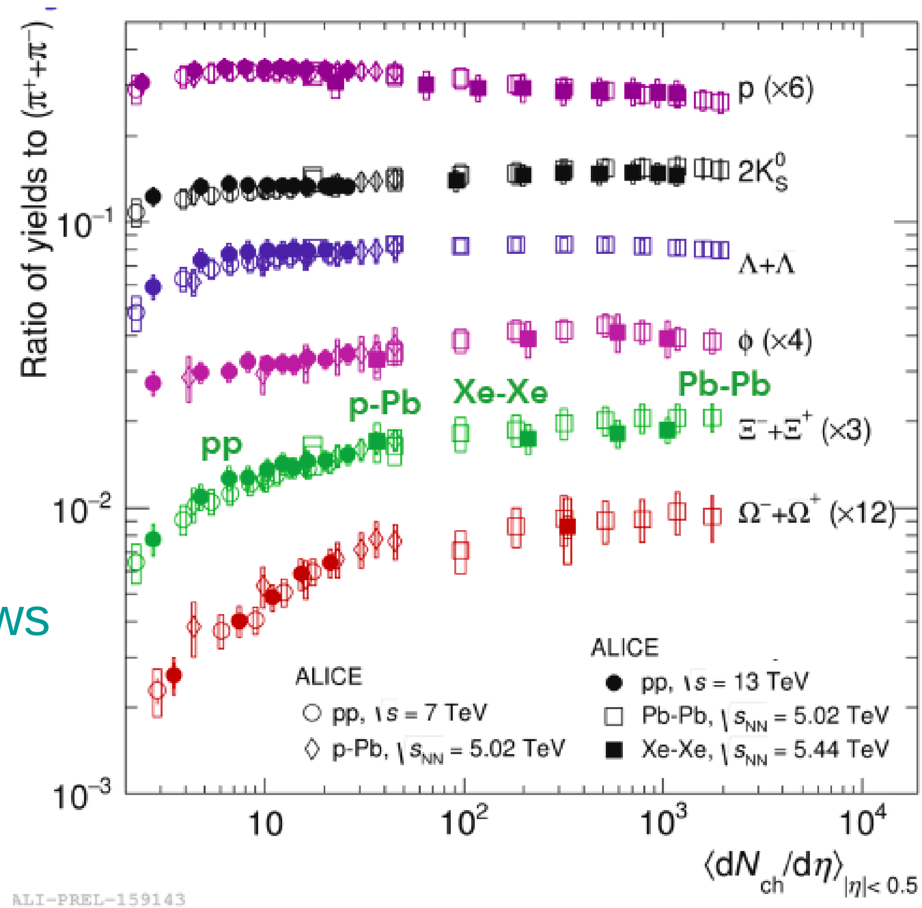
Particle yields linked to $dN_{ch}/d\eta$: from pp, pA to AA

- Increase of strangeness production with increasing multiplicity until saturation,

  as well as,

  its dependence on strange quantum number of hadrons can be linked to "canonical suppression effect" i.e. constraints imposed on thermal particle yields due to exact strangeness conservation. This requires canonical ensemble formulation of conservation laws

Smooth evolution of particle yields as function of charged particle multiplicity, and strangeness suppression



S. Hamieh, A. Tounsi & K.R. Phys. Lett. B486 (2000),

Eur.Phys.J. C24 (2002) , J. Cleymans, H. Oeschler & K.R. Phys. Rev. C59 (1999) 1663

Strangeness canonical suppression with yields of charged particles

- If the number of s-particles is small then strangeness conservation must be exact

J. Cleymans, Pok Man Lo, N. Sharma & K.R.
Phys. Rev. C103 014904 (2021)

$$Z^{GC}(\mu) = Tr[e^{-\beta(H-\mu S)}] \Rightarrow Z_S^C = Tr[e^{-\beta H}\delta_S]$$

$$Z^{GC}(\lambda) = \sum_{S=-\infty}^{\infty} \lambda^S Z_S^C \Rightarrow Z_S^C \simeq \int_{-\pi}^{\pi} d\varphi e^{i\phi S} e^{\mathrm{Ln}(Z^{GC}(\mu \to i\varphi))}$$

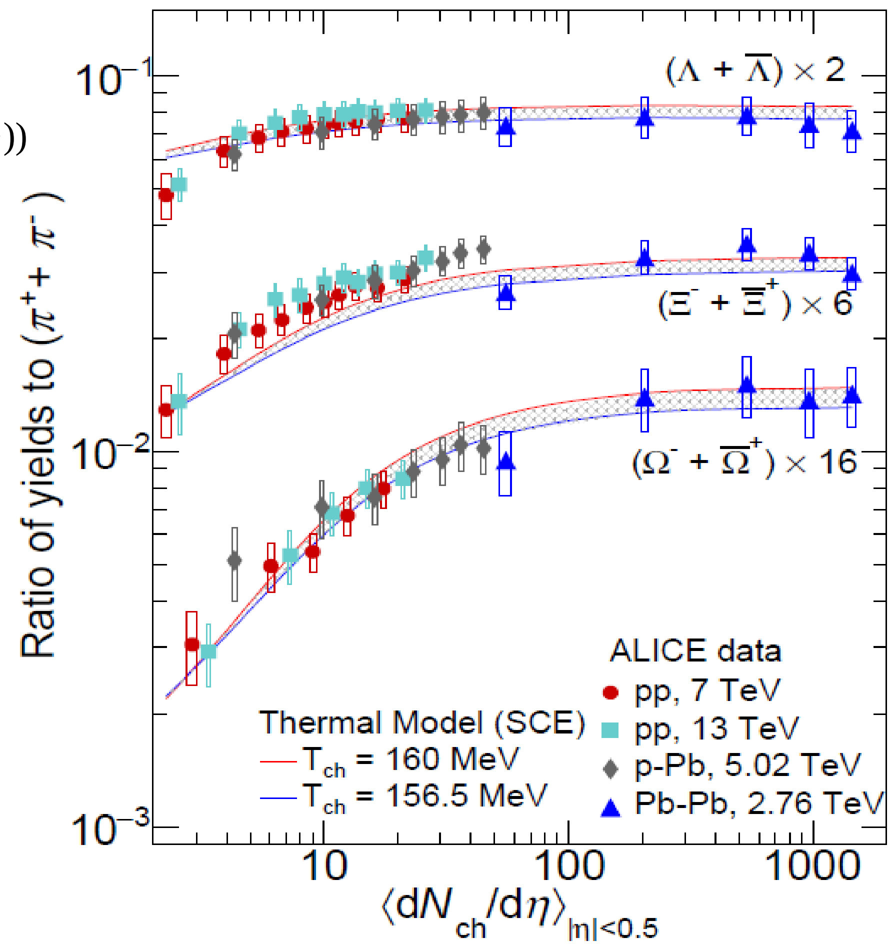
- This implies strangeness suppression effect

$$< N_s >_A^C \approx V_A n^{GC} \bullet \frac{I_s(2V_C n_{s=1}^{th}(T))}{I_0(2V_C n_{s=1}^{th}(T))}$$

where volume parameters $V_{A(C)} \sim dN_{ch}/d\eta$

V_C - full phase-space volume where S is exactly conserved

V_A - effective fireball volume in the acceptance



The suppression factor $I_S(x)/I_0(x) \leq 1$ decreases with decreasing x, and increasing strange s-quantum number of hadron.

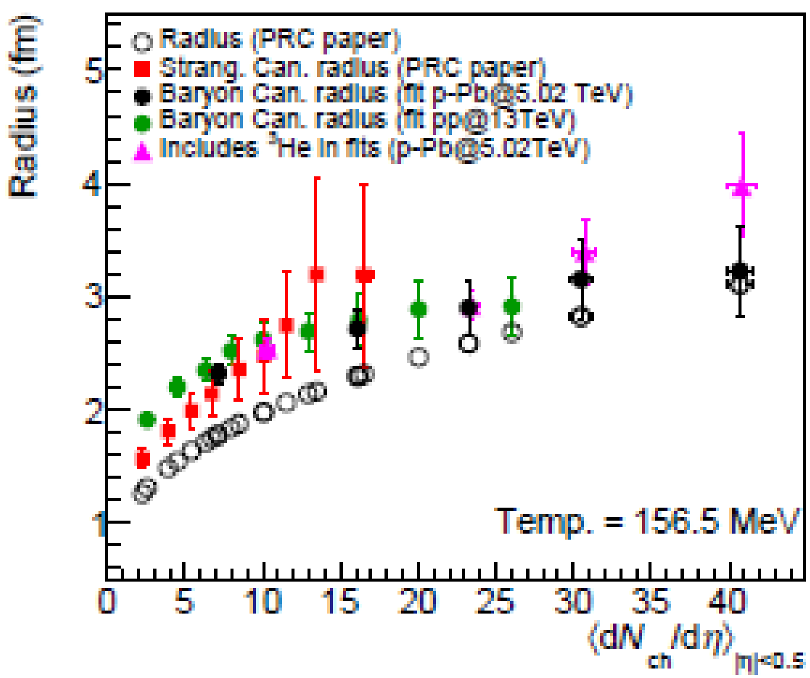
Canonical suppression in baryonic sector in pp and pA collisions

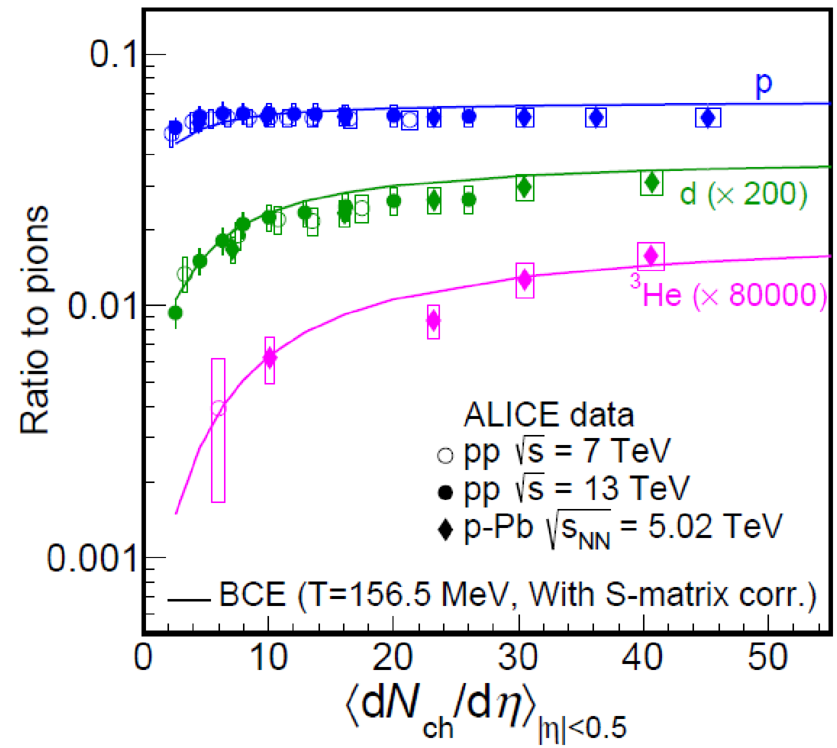
- Baryon canonical suppression effect

$$< N_b >_A^C \approx V_A n^{GC} \bullet \frac{I_b(2V_C^B n_{s=1}^{th}(T))}{I_0(2V_C^B n_{s=1}^{th}(T))} \times (R_N)^b$$

J. Cleymans, Pok Man Lo, N. Sharma & K.R. too appear (2021)

Where T, and V_A as obtained from strangeness
The only fitted parameter is: $V_C^B = 4/3\pi R^3$





The S-matrix correction factor R_N of nucleons due to dressing by mesons is also needed for Deuteron as R_N^2 and for ^3He as R_N^3 to describe yields data in pp and pA collisions at the LHC as obtained by ALICE coll.

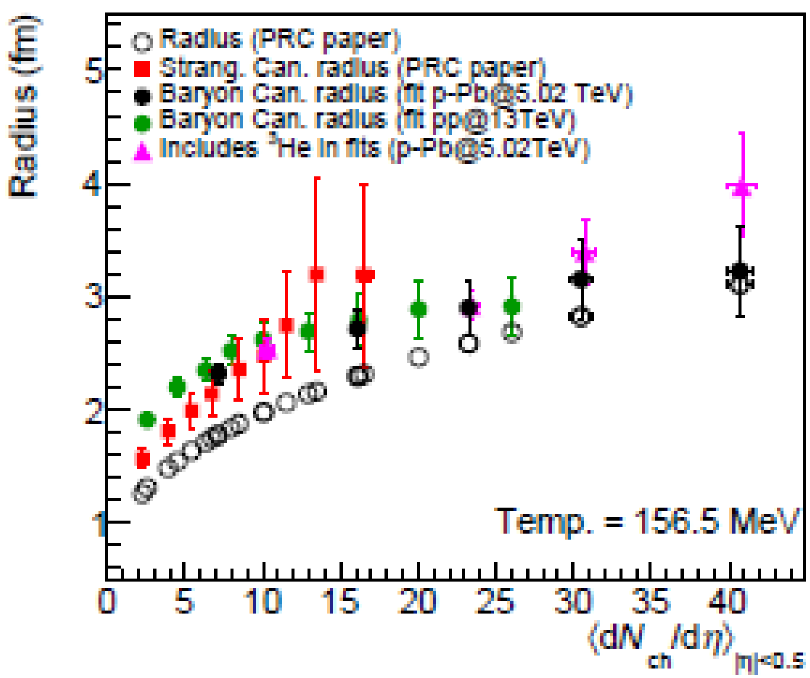
Canonical suppression in baryonic sector in pp and pA collisions

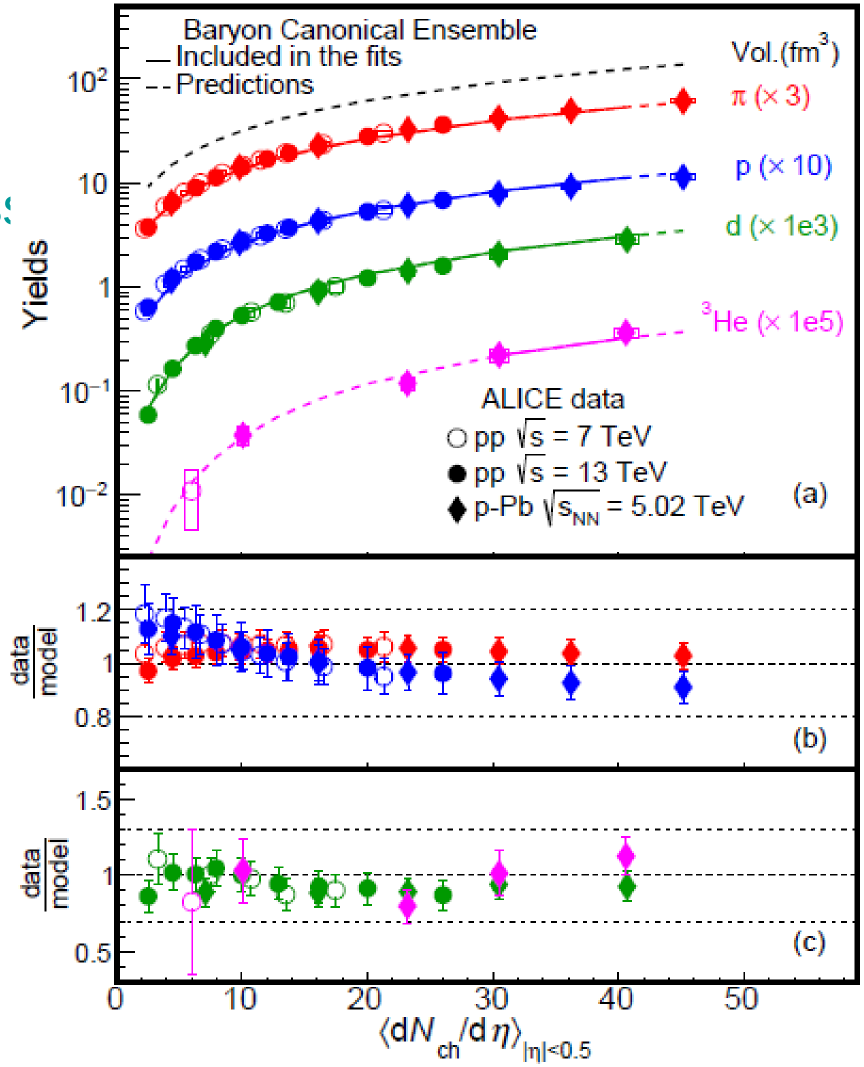
- Baryon canonical suppression effect

J. Cleymans, Pok Man Lo, N. Sharma & K.R. to appear (2021)

$$< N_b >_A^C \approx V_A n^{GC} \bullet \frac{I_b(2V_C^B n_{s=1}^{th}(T))}{I_0(2V_C^B n_{s=1}^{th}(T))}$$

Where T, and V_A as obtained from strangeness
The only fitted parameter is: $V_C^B = 4/3\pi R^3$





Probing chiral criticality with charge fluctuations

- Due to expected O(4) scaling of QCD free energy:

$$F = F_R(T, \mu_q, \mu_I) + b^{-1} F_S(b^{(2-\alpha)^{-1}} t(\mu), b^{\beta\delta/\nu} h) \qquad t(\mu) = \frac{T - T_c^o}{T_c^o} + \kappa(\frac{\mu}{T})^2$$

- Direct delineation of chiral symmetry restoration via higher order fluctuations of conserved charges. Consider e.g. net baryon number suscep.

$$\chi_B^{(n)} = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n} = \chi_R^{(n)} + \chi_S^{(n)} \quad \text{with} \quad \begin{cases} \chi_s^{(n)}|_{\mu=0} \approx h^{(2-\alpha-n/2)/\beta\delta} f^{(n)}(z) \\ \chi_s^{(n)}|_{\mu\neq 0} \approx h^{(2-\alpha-n)/\beta\delta} f^{(n)}(z) \end{cases}$$

- At $\mu = 0$ only $\chi_B^{(n)}$ with $n \geq 6$ receive contribution from $\chi_S^{(n)}$
- At $\mu \neq 0$ only $\chi_B^{(n)}$ with $n \geq 3$ receive contribution from $\chi_S^{(n)}$

At LHC the 6th order cumulant should carry direct information on chiral symmetry restoration due to remnant of O(4) criticality

Modelling $P^{s-gular}(T,\mu_B)$ in the O(4)/Z(2) universality class

Gabor Almasi, Bengt Friman & K.R., Phys. Rev. D96 (2017) no.1, 014027

$$\mathcal{L} = \bar{q}\left(i\gamma^\mu D_\mu - g(\sigma + i\gamma_5\vec{\tau}\vec{\pi}) - g_\omega\gamma^\mu\omega_\mu\right)q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 - U_m(\sigma,\vec{\pi}) - \mathcal{U}(\Phi,\bar{\Phi};T) - \frac{1}{2}m_\omega^2\omega^2 + \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Effective potential is obtained by solving *the exact flow equation* (Wetterich eq.) with the approximations resulting in the O(4)/Z(2) critical exponents

$$\partial_k\Omega_k(\sigma) = \frac{Vk^4}{12\pi^2}\left[\sum_{i=\pi,\sigma}\frac{d_i}{E_{i,k}}[1+2n_B(E_{i,k})] - \frac{2\nu_q}{E_{q,k}}[1-n_F(E^+_{q,k})-n_F(E^-_{q,k})]\right]$$

Full propagators with $k < q < \Lambda$

$$E_{\pi,k} = \sqrt{k^2 + \Omega'_k}$$

$$E_{\sigma,k} = \sqrt{k^2 + \Omega'_k + 2\rho\Omega''_k}$$

$$E_{q,k} = \sqrt{k^2 + 2g^2\rho}$$

$\Gamma_\Lambda = S_{\text{classical}}$

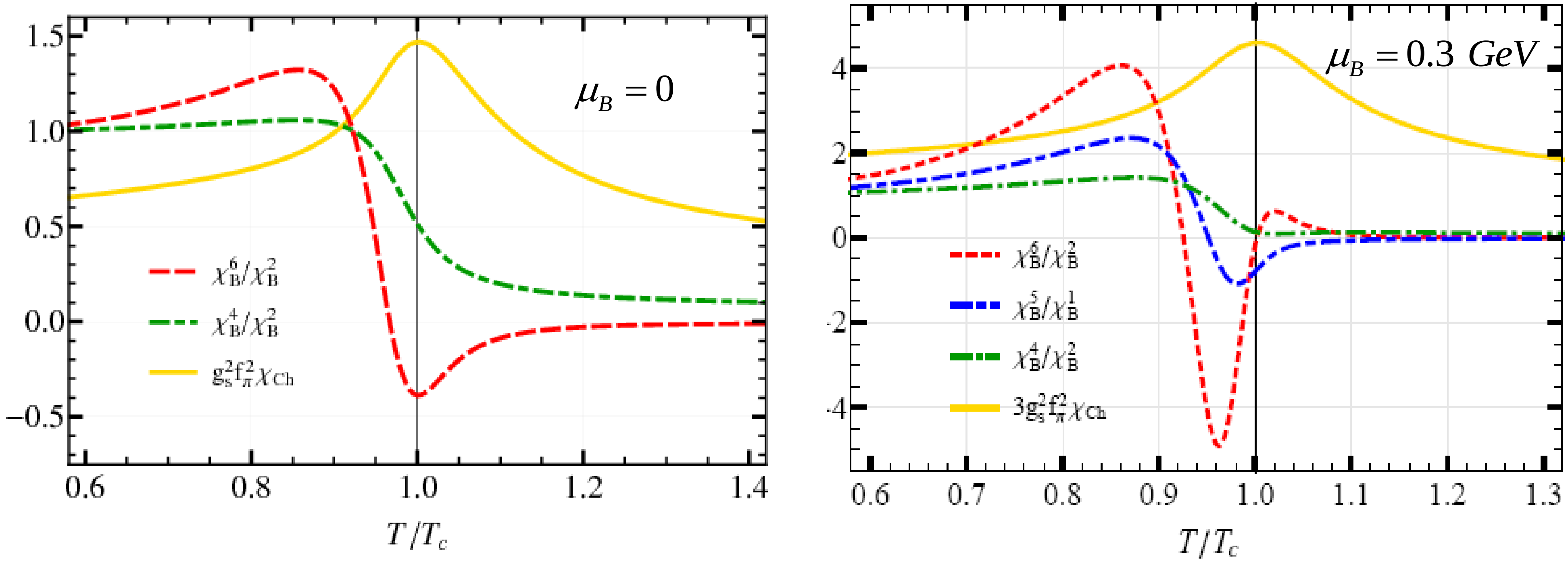
Integrating from $k=\Lambda$ to $k=0$ gives full quantum effective potential

$$\Omega'_k \equiv \frac{\partial\Omega_k}{\partial(\sigma^2/2)}$$

Higher order cumulants in effective chiral model within FRG approach, belongs to the O(4)/Z(2) universality class

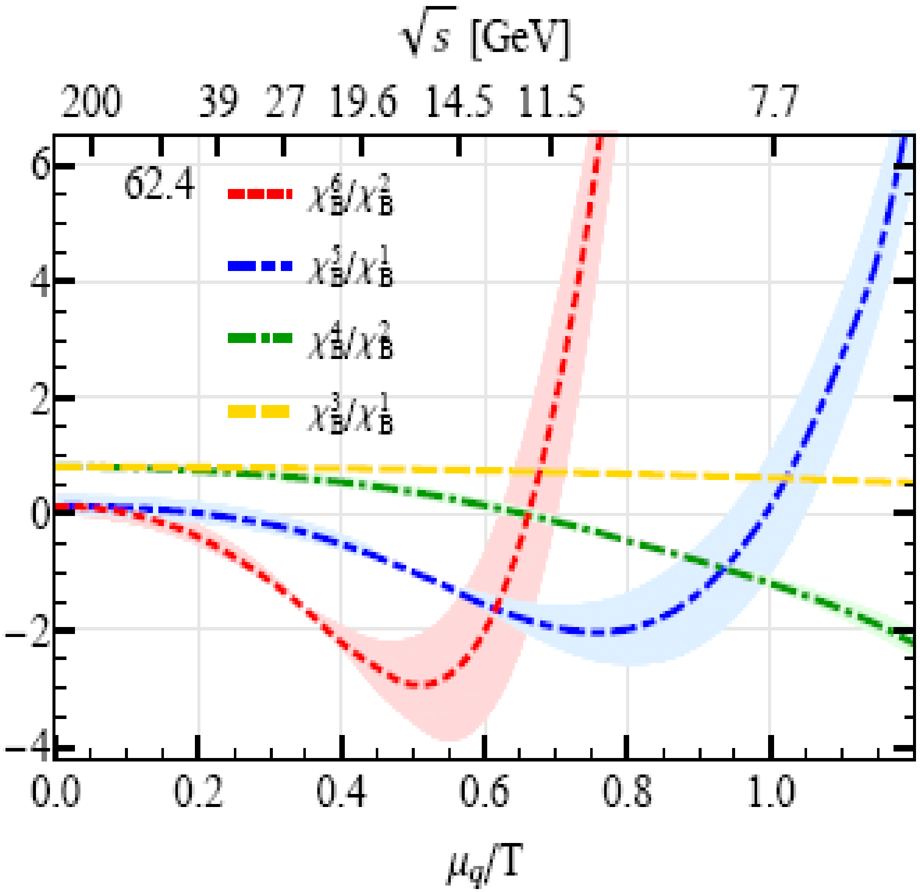
B. Friman, V. Skokov &K.R. Phys. Rev. C83 (2011) 054904

G. Almasi, B. Friman &K.R. Phys. Rev. D96 (2017) no.1, 014027



Deviations of cumulant ratios from Skellam distribution are increasing with the order of the cumulants and can be used to identify the chiral QCD phase boundary in HIC

Higher order cumulants - energy dependence



- Strong non-monotonic variation of higher order cumulants at lower $\sqrt{s}$
- Equality of different ratios excellent probes of equilibrium evolution in HIC
- Is the Skellam distribution appropriate as the noncritical baseline for baryon number fluctuations?
- What is the influence of exact baryon number conservation in full phase space on fluctuation observables in the acceptance window?

However, to make final conclusions the influence of non-critical fluctuations must be analyzed:

See e.g. P. Braun-Munzinger, A. Rustamov and J. Stachel Nucl. Phys. A 960, 114 (2017),
A. Bzdak, V. Koch, V. Skokov, Eur.Phys.J. C77 (2017) 288.

M. Kitazawa et al. (2015,16,17)

Fluctuation of the net-baryon number in the acceptance window

- Calculate probability distribution in the acceptance $P_A(B_A)$ from B-can. Ensemble

$$Z_B^C = Tr[e^{-\beta H}\delta_B] = \sum_{N_B,N_{\bar B}} \frac{z_B^{N_B}}{N_B!}\frac{z_{\bar B}^{N_{\bar B}}}{N_{\bar B}!}\delta(N_B - N_{\bar B} - B)$$

- Resulting in: mean number of $\pm B$ baryons

$$N_{\pm B} = z\frac{I_{B\mp 1}(2z)}{I_B(2z)} \quad \text{and} \quad N_{\pm B}^A = \alpha_{\pm B} z\frac{I_{B\mp 1}(2z)}{I_B(2z)}$$

- probability to find net baryon B_A in acceptance

$$P_A(B_A) = I_B(2z)(\alpha_B/\alpha_{\bar B})^{B_A/2}((1-\alpha_B)/(1-\alpha_{\bar B}))^{(B-B_A)/2}$$
$$\times I_{B_A}(2z\sqrt{\alpha_B\alpha_{\bar B}})I_{B-B_A}(2z\sqrt{(1-\alpha_B)(1-\alpha_{\bar B})})$$

- Moments of the net baryon number in acceptance

$$\mu_n = \left\langle (N_B^A - N_{\bar B}^A)^n \right\rangle = \sum_{B_A}(B_A)^n P_A(B_A)$$

Full phase space

$N_B, N_{\bar B} \Rightarrow B = N_B - N_{\bar B}$

$z_B, z_{\bar B} \Rightarrow z = \sqrt{z_B z_{\bar B}}$

$z_{\pm B} = \int d^3x \int \frac{d^3p}{(2\pi)^3} e^{-\beta(E\mp q\vec{\mu_q})}$

$\chi_B^{(n)} = 0$ for $n > 1$

Acceptance subspace

$N_B^A, N_{\bar B}^A \Rightarrow B_A = N_B^A - N_{\bar B}^A$

$z_B^A, z_{\bar B}^A \quad \alpha_{\bar B} = \frac{N_{\bar B}^A}{N_{\bar B}} \quad \alpha_B = \frac{N_B^A}{N_B}$

$P_A(B_A) = f(B_A, \alpha_B, \alpha_{\bar B}, B, z)$

$\chi_{B_A}^{(n)} \neq 0$

due to exact baryon number conservation

A.Bzdak, V. Koch and V. Skokov, Phys. Rev. C87, 014901 (2013)

P. Braun-Munzinger, B. Friman, A. Rustamov, J. Stachel & K.R. Nucl. Phys. A (2021)

2007.02463 [nucl-th]

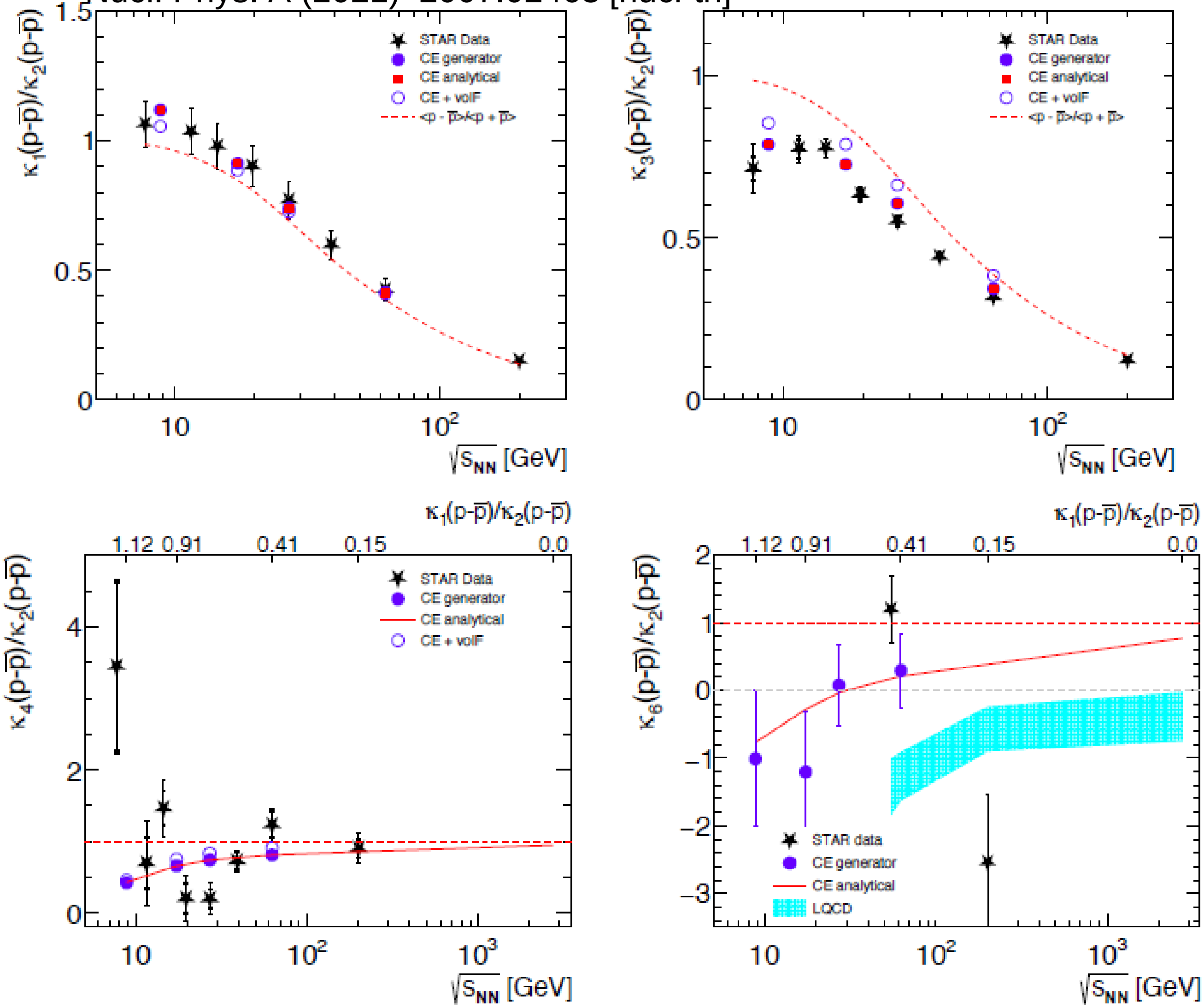
- The corresponding cumulants are Bell-polynomials in the moments

$$\chi_{B_A}^{(n)} = \sum_{k=1}^{n}(-1)^{k-1}(k-1)!B_{n,k}(\mu_1,..,\mu_{n-k+1})$$

Non-critical baseline for net-proton fluctuations

P. Braun-Munzinger, B. Friman, A. Rustamov, J. Stachel & K.R.

Nucl. Phys. A (2021) 2007.02463 [nucl-th]



- Fixing the model parameters: $N_B, N_{\bar{B}}$ in full phase space and $\alpha_p = N_p^A / N_B$, $\alpha_{\bar{p}} = N_{\bar{p}}^A / N_{\bar{B}}$ from data at different $\sqrt{s_{NN}}$, as well as the number of baryon-anti-baryon pairs z in the thermal system in full phase space from

$$N_B = z\frac{I_{B-1}(2z)}{I_B(2z)}$$

the cumulants of any order can be calculated

- The cumulants up to $n \leq 3$ order follow the SATR data
- Kurtosis data exhibit interesting deviations!

CONCLUSIONS:

- QCD thermodynamic potential is encoded in nuclear collision data
- S-matrix (Hadron Resonance Gas) thermodynamic potential provides an excellent approximation of the QCD equation of states in confined phase
- Hadrons are produced at QCD phase boundary with yields and 2^{nd} order fluctuations and correlations of conserved charges which are consistent with LQCD predictions
- The exact conservation of net strange and baryon number is essential to quantify the particle yields and their observed scaling with charged particle multiplicities
- To establish a noncritical background for the net proton (baryon) number fluctuations in the acceptance, the canonical formulation of the conservation law has to be accounted for in a full phase space