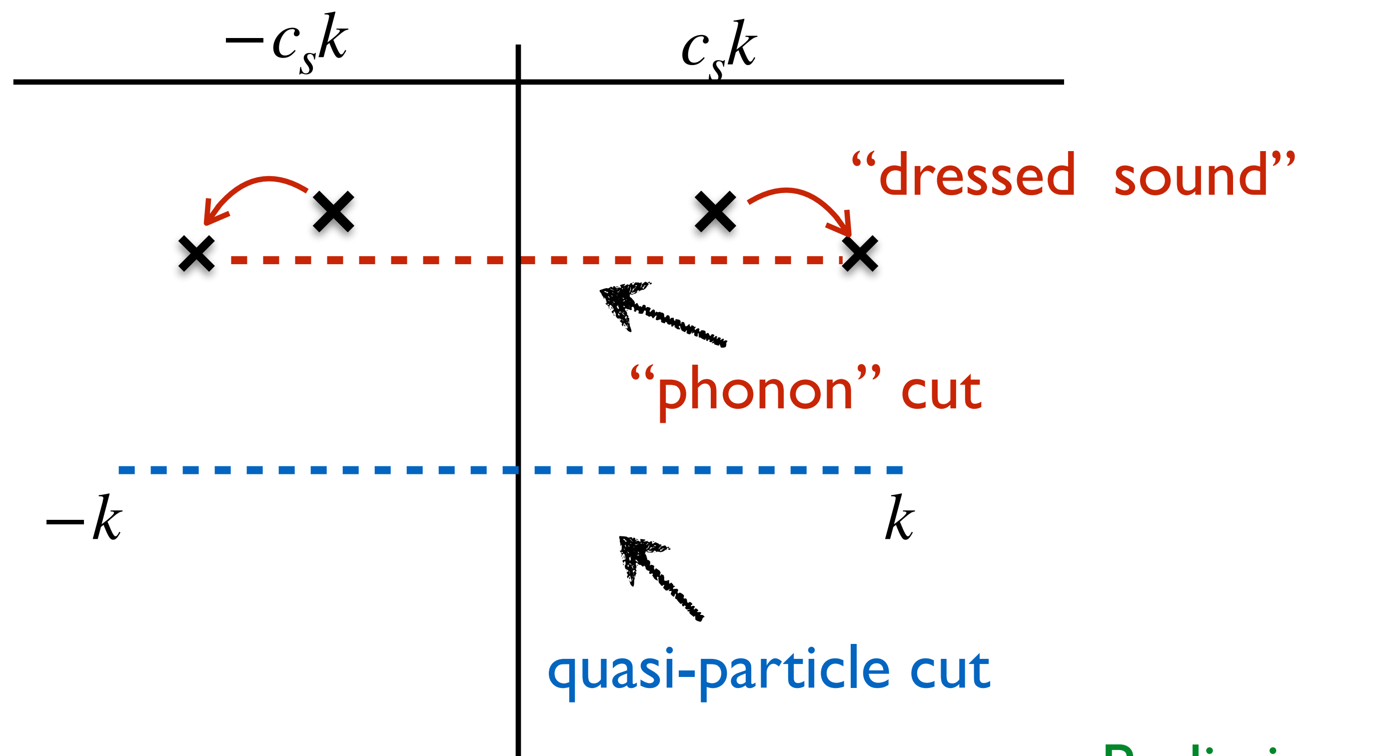
# Non-equilibrium EFT and hydro. fluctuations



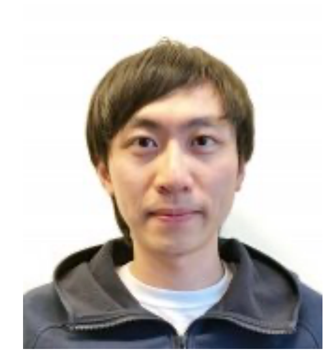

Preliminary


Yi Yin

*Based on: Chris Lau-Hong Liu-YY, in preparation;*

*Sogabe-Yamamoto-YY, to appear.*


*Institute of Modern Physics, Chinese academy of sciences*



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BES seminar series, Mar.2, 2021

# Outline

Very recently, the non-equilibrium effective field theory (EFT) of fluctuating hydrodynamics has been formulated based on the principle of symmetry.

*Glorioso-Crossley-Liu, JHEP 17';*

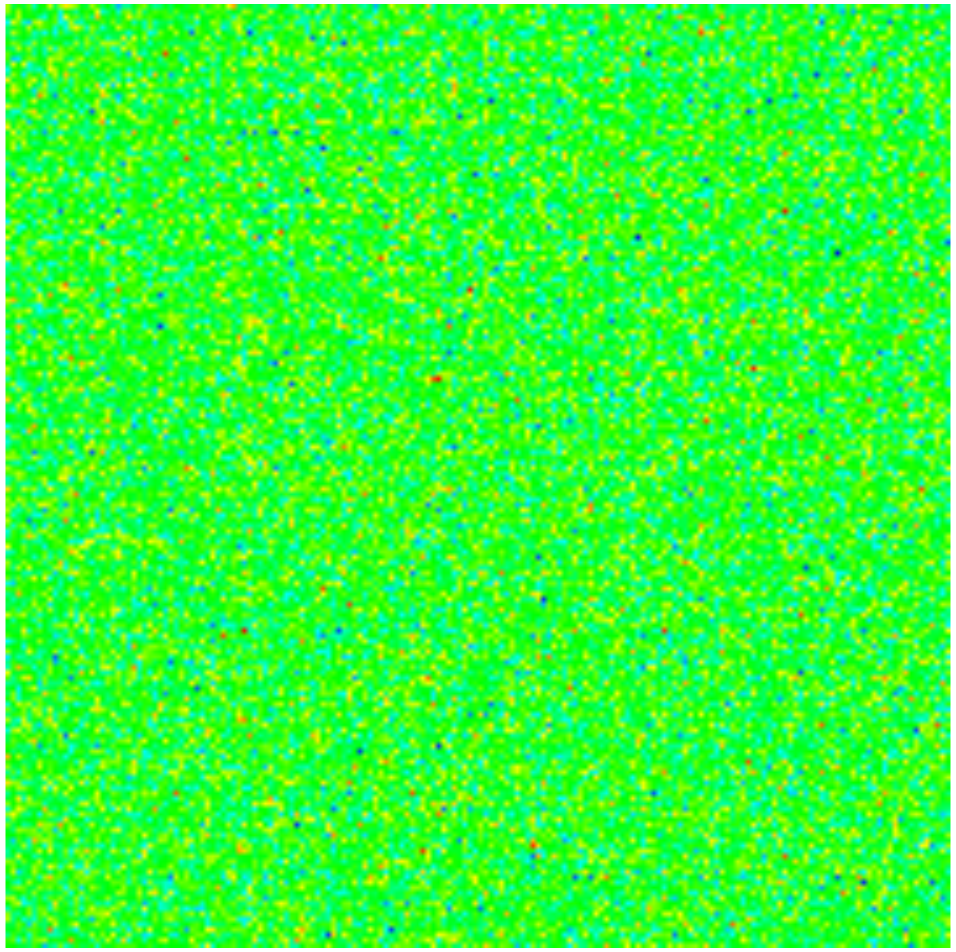
This talk: the application of non-equilibrium EFT to relativistic hydrodynamics

Stress-energy tensor correlator for a neutral fluid. *Chris Lau-Hong Liu-YY, in preparation*

The modification of conductivity, bulk viscosity due to chiral magnetic effect (CME) and fluctuations. *Sogabe-Yamamoto-YY, to appear.*

Outlook: e.g. implementation of hydro. EFT on quantum computer.

# *Hydrodynamic fluctuations*



*From https://ccse.lbl.gov/Research/MuMSS/index.html*

## *Physical consequence of hydro. fluctuations*

controlling non-equilibrium fluctuation of energy density, momentum density etc => important near the phase boundary

inducing finite frequency and momentum correction to transport coefficients (back-reaction)=> describing the evolution of medium properties as a function of scale.

Setting the scale(s) for hydrodynamization/thermalization => deserves further attention when studying initial stages and/or small colliding systems.

# Approaches to fluctuating hydro.

Stochastic approach: adding noise to hydro. eqns

$$\partial_t \vec{u} = -(\vec{u}\cdot\vec{\partial})\vec{u} - \nu\nabla^2\vec{u} + \vec{F} \qquad \langle F(t,\vec{x})F(t',\vec{x}')\rangle \sim 2T\eta\delta(t-t')\delta(\vec{x}-\vec{x}')$$

Landau-Lifshitz; Kapusta-Mueller-Stephanov; ..

Deterministic approach: formulating and solving a set of deterministic equations, which couple fluctuations with hydro modes.

$$\partial_\mu \left[T^{\mu\nu}_{\rm ave}(\epsilon, n, u^\mu) + \Delta T^{\mu\nu}(\text{2pt,3pt}, \ldots)\right] = 0$$

E.o.Ms for 2pt, 3pt,…

*Kawasaki, Ann. Phys. '70; Andreev, JTEP, '1971; …*

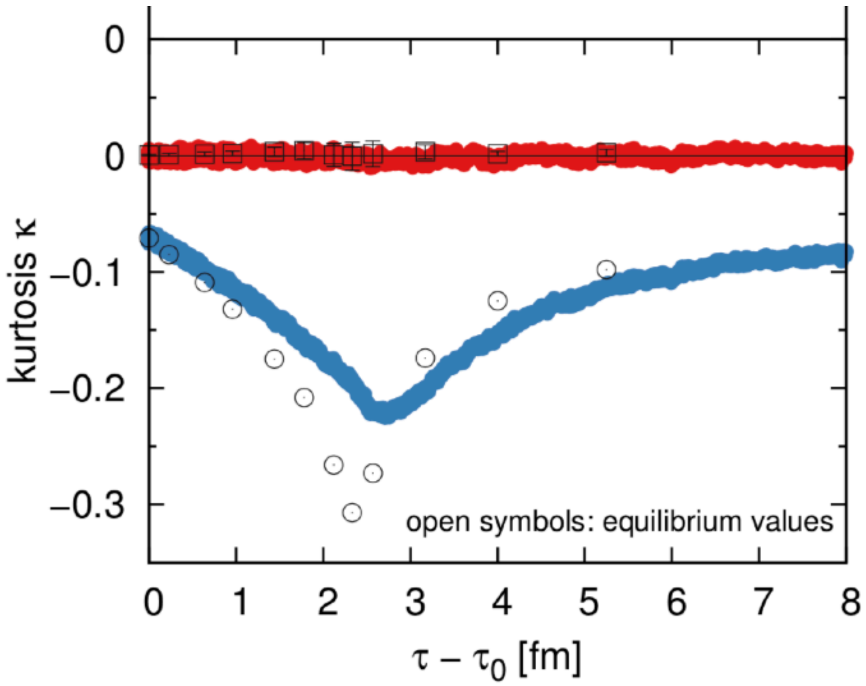
*Akamatsu-Mazeliauskas-Teaney, PRC 16' & 18'; Stephanov-YY PRD 18'; Mauricio-Schaefer PRC 19; Xin An-Basar-Stephanov -H.-U. Yee, PRC19'&20'& 2009.10742;*

EFT approaches: based on action principle

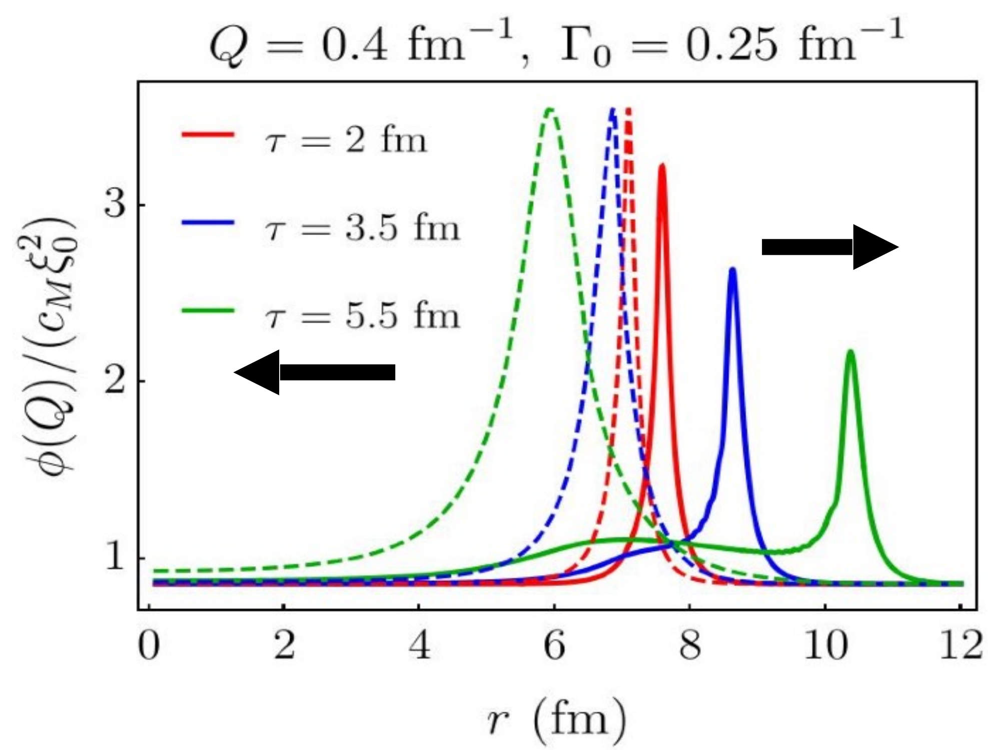
$$Z = \int D\psi_{\rm hydro}\, e^{iI_{hydro}[\psi_{\rm hydro}]}$$

*Kovtun-Moore-Romatschke, JHEP 14'; Glorioso-Crossley-Liu, JHEP 17'; Haehl-Loganayagam-Rangamani, 1803.11155, …*

# Fluctuating hydro. and dynamical modeling for BES




*Stochastic simulation by Nahrgang-Bluhm-Schaefer-Bass, PRD 19'; see also Singh-Shen-Jeon-Gale, to appear*




*Simulation of Hydro+, from Rajagopal-Ridgway-Weller-YY, PRD 20'; see also Lipei Du-Heinz-Rajagopal-YY, PRC 20'*

Broad view: we are entering the quantitative era of BES dynamics. (see other talks in this seminar series)

*for more detail, "The BEST framework for the search of the QCD critical point and the chiral magnetic effect", in preparation, BEST collaboration,*

*Why study EFT approach?*

# Motivation for studying EFT approach:

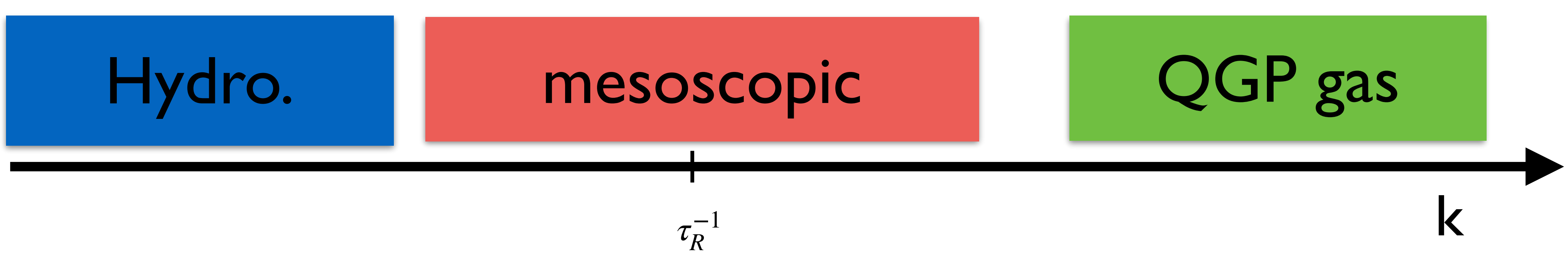
A systematic treatment which makes theoretical structures transparent.

With this new formulation, a number of novel phenomena have been uncovered, even in problems that have been studied extensively before.

A new framework for the future (personal view): *Xinyi Chen Li-Delacrétaz-Hartnoll PRL 18'; Delacrétaz-Glorioso PRL 20';*

suitable for implementation on quantum computing.

the formalism can be extended in a way which does not depend on any long wavelength expansion. (a basis for studying QGP in non-hydrodynamic yet non-perturbative regime)



# EFT action for fuctuating hydro.

integrating out fast modes

$$Z = \int D\psi_{\mathrm{mic}}\, e^{iI[\psi_{\mathrm{mic}}]} \longrightarrow Z = \int D\psi_{\mathrm{hydro}}\, e^{iI_{hydro}[\psi_{\mathrm{hydro}}]}$$

Hydro EFT: action describing long wavelength and long time limit of many-body systems.

Saddle point=> Hydro. E. o. M ; path-integral: accounts for fluct.

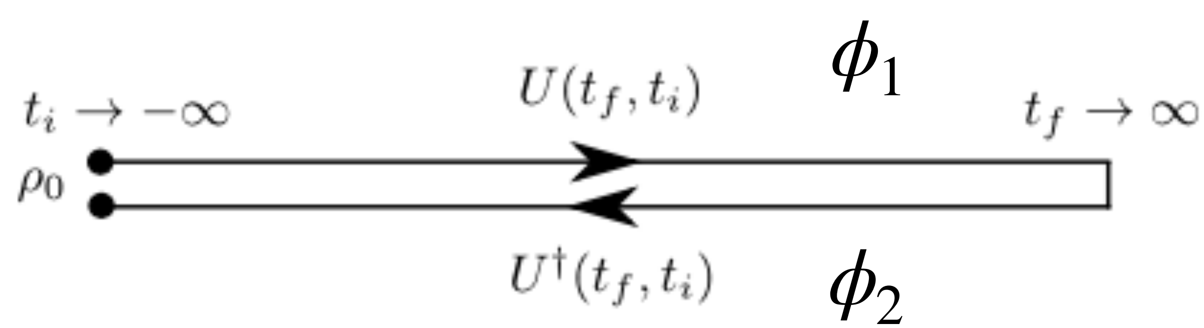
In reality, the procedure of integrating out fast modes can not be done directly. Instead, one can construct the EFT action consistent with symmetries and constraints of the system.

We shall use the recent "top-down" formulation. *Glorioso-Crossley-Liu, JHEP 17';*

## The double life in non-equilibrium systems: Schwinger-Keldysh formalism

Consider the density matrix of a non-equilibrium system

$$\rho(t_f) = U(t_f, t_i)\rho_I U^+(t_f, t_i)$$




The path integral representation is naturally formulated on the Schwinger-Keldysh contour

Introducing r and a-variable, representing (average) observables and noise respectively.

$$\phi_r \equiv \frac{1}{2}\left(\phi_1 + \phi_2\right) \ , \qquad \phi_a \equiv \phi_1 - \phi_2$$

# *EFT Action for a neutral fluid.*

*Glorioso-Crossley-Liu, JHEP 17'*

EFT action is organized by the double expansion in gradient and a-variable X. Here $\beta^\mu = \beta u^\mu$ is the r-variable.

$$\mathscr{L}_{\text{Hydro}} = \left(\epsilon u^\mu u^\nu + p\Delta^{\mu\nu} - \beta^{-1}H^{\mu\nu\alpha\lambda}(\partial_\alpha\beta_\lambda)\right)\,\nabla_\nu X_\mu$$

$$+i\beta^{-1}H^{\mu\nu\alpha\beta}\nabla_\alpha X_\beta \nabla_\mu X_\nu + \text{higher order in X or grad.}$$

$$H^{\mu\nu\alpha\beta} = \eta\left(\Delta^{\mu\alpha}\Delta^{\nu\beta} + \dots\right) + \zeta(\dots)$$

The action at second order in X is fixed by the viscous part of $T^{\mu\nu}_{\text{hydro.}}$ via local KMS $Z_2$ symmetry (*defining local equilibrium through symmetry*).

X (a-variable) is conjugate to force acting on r-variables, i.e. noise in the stochastic formulation.

$$\delta_X I_{\text{Hydro}} \to \partial_\mu T^{\mu\nu}_{\text{hydro}} = F^\nu(X)$$

In contrast to stochastic approach, X is viewed as dynamical variables whose evolution is coupled with hydro. modes.

$$\delta_\beta I_{\text{Hydro}} \to \text{E.o.M for X}$$

## *Stress-energy tensor correlator*

Describing the fluctuation, propagation and dissipation of stress-energy tensor disturbance.

$$G_R^{\mu\nu;\alpha\beta}(\omega, k) \sim \langle T_r^{\mu\nu} T_a^{\alpha\beta} \rangle$$

The analytic structure reflects the properties of medium excitations.

Related to E.o.S, transport coefficients in long wavelength, long time limit.

For QGP, their behaviors are only known in limiting cases so far:

in high frequency limit can be computed perturbatively,

its form in low frequency limit is universally fixed by (deterministic) hydro.

The study of the effects of hydro. fluctuations to stress-energy correlator has a long history, *no complete answer* is known to date.

Our goal: a complete one-loop analysis of stress-energy correlator for general $\omega, k$ based on hydro. EFT

## Correlators and tree level results

We put five independent $G_R^{\mu\nu\alpha\beta}$ into two categories. (setting momentum along z-direction).

Category A: no poles in hydro regime but related to viscosity.

$$\langle T_r^{xy} T_a^{xy} \rangle \qquad \langle \tilde{\Theta}_r \tilde{\Theta}_a \rangle \qquad \langle T_r^{00} \tilde{\Theta}_a \rangle \qquad \tilde{\Theta} = \sum_i T^{ii} - 3c_s^2 T^{00}$$

$$\text{e.g.}: \lim_{\omega, \vec{k} \to 0} \langle T_r^{xy} T_a^{xy} \rangle / \omega = \eta \, , \lim_{\omega, \vec{k} \to 0} \langle \tilde{\Theta}_r \tilde{\Theta}_a \rangle / \omega = \zeta$$

Category B: has poles in hydro regime.

$$\langle T_r^{0z} T_a^{0z} \rangle = \frac{w\left(c_s^2 k^2 - i\nu_L \omega k^2\right)}{\omega^2 - c_s^2 + i\nu_L \omega k^2} \qquad \langle T_r^{0x} T_a^{0x} \rangle = \frac{-\eta k^2}{-i\omega + \nu_T k^2}$$

## Set-up for loop calculation

Dynamical fields: X and the fluctuations of energy-momentum density.

$$\lambda^0 = -\frac{\delta T^{00}_r}{c_s w_0}\,, \qquad \lambda^i = \frac{\delta T^{0i}_r}{w_0}\,.$$

Propagators: $G^{ra} \sim \langle \lambda^\mu X^\nu \rangle$ $\qquad G^{rr} \sim \langle \lambda^\mu \lambda^\nu \rangle$ *(can be related to $G^{ra}$ )*

$$G^{ra}_L \sim \frac{1}{\omega^2 - c_s^2 k + i\nu_L\,\omega\,k^2} \qquad G^{ra}_{TT} \sim \frac{1}{\omega + i\nu_T\,k^2} \qquad \nu_T = \frac{\eta}{w}\,, \qquad \nu_L = \frac{(4/3)\eta + \zeta}{w}\,.$$

Cubic action: $\mathscr{L} = \lambda\lambda X + \lambda X X \Rightarrow$ ideal vertices+viscous vertices

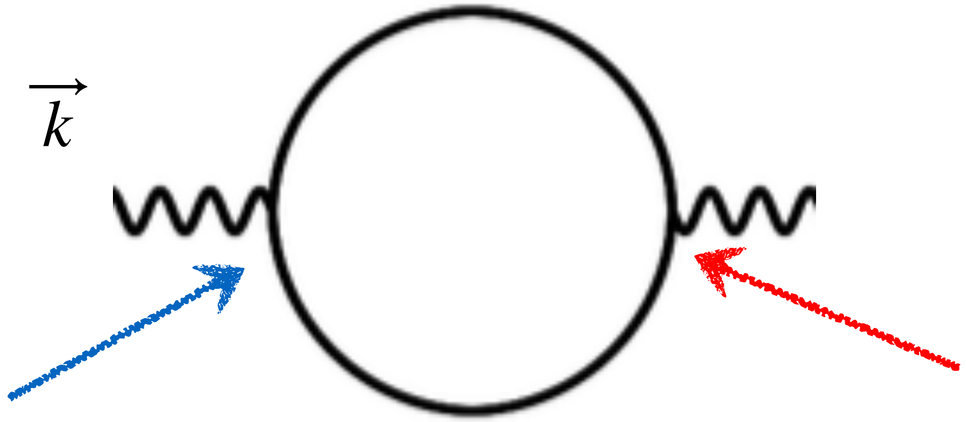
Connection to stochastic approach

$$\partial_t \overrightarrow{u} = -(\overrightarrow{u}\cdot\overrightarrow{\partial})\overrightarrow{u} - \nu(T+\delta T)\,\nabla^2\overrightarrow{u} + (\text{a-field})$$

*ideal vertices* $\qquad$ *example of viscous vertices*

NB: $T^{\mu\nu}_r, T^{\mu\nu}_a$ can de computed explicitly from the variation of hydro. action.
(not obvious how to do so in “bottom-up” MSR approach. )

# Characteristic feature of one-loop computations




Hydro. modes A,B=$\pm, T$

Sound: $\Omega_\pm(k) = \pm\, c_s\, k - i\nu_L k^2\,,$

Shear: $\Omega_T(k) = -\, i\nu_T k^2\,,$

$$I(\omega, \vec{k}\,) \propto \sum_{A,B=\pm,T} \int_{\vec{p}} (\text{vertices})^2 \frac{1}{\omega - \Omega_A(p_+) - \Omega_B(p_-)}$$

$\vec{p}_\pm = \pm\,\vec{p} + \frac{\vec{k}}{2}$

Propagation time of the "mode pair"

$$\Rightarrow \quad \boxed{I_{AB}(\omega, \vec{k}\,) \sim (V^*)^2_{AB}\,(p^*_{AB})^3\,\tau^*_{AB}}$$

Here, the typical momentum of "fluctuation modes" $p^*$ and typical propagation time of "mode pair" $\tau^*$ depends on the combination $(A, B)$ and $(\omega, k)$.

# Characteristic scale

p* and $\tau^*$ can be estimated by looking for poles in complex p plane:

T-T: $$\int_{\vec{p}} \frac{1}{\omega + 2i\nu_T(p^2 + \frac{k^2}{4})} \to$$

$$p^*_{TT} \sim \sqrt{\frac{\omega}{2\nu_T}} \qquad k < \sqrt{\frac{\omega}{\nu}}$$

$$p^*_{TT} \sim k \qquad k > \sqrt{\frac{\omega}{\nu}}$$

*Akamatsu-Mazeliauskas-Teaney, PRC 16,*

s-s: $$\int_{\vec{p}} \frac{1}{\omega - c_s(p_+ - p_-) + i\nu_L(p^2 + \frac{k^2}{4})} \to$$

$$p^*_{ss} \sim \sqrt{\frac{\omega}{2\nu_L}} \sim p^*_{TT} \qquad c_s k < \omega$$

$$p^*_{ss} \sim \sqrt{\frac{c_s k}{2\nu_L}} \gg p^*_{TT} \qquad c_s k > \omega$$

Shear-shear and sound-sound are more important than other combinations.

When $c_s k > \omega$, the main contribution comes from "sound-sound" .

# Results for correlators in category A.

Category A: related to viscosity but do not have poles in hydro regime:

$$\langle T_r^{xy} T_a^{xy} \rangle \qquad \langle \tilde{\Theta}_r \tilde{\Theta}_a \rangle \qquad \langle T_r^{00} \tilde{\Theta}_a \rangle \qquad \tilde{\Theta} = \sum_i T^{ii} - 3c_s^2 T^{00}$$

Consider tensor correlator as an example.

$$(\langle T_r^{xy} T_a^{xy} \rangle)_{\text{tree}} = P - i\eta\omega$$

Zero k: both "sound-sound" and "T-T" contribution are important.

$$\mathrm{Im}\Delta G_R^{xyxy}(\omega,0)/\omega = \frac{-T}{60\pi\sqrt{2}} \left( \sqrt{\frac{\omega}{\nu_L}}\frac{1}{\nu_L} + \sqrt{\frac{\omega}{2\nu_T}}\frac{7}{2\nu_T} \right)$$

*Kovtun-Yaffe, 2003; Kovtun-Moore-Romatschke, 2011*

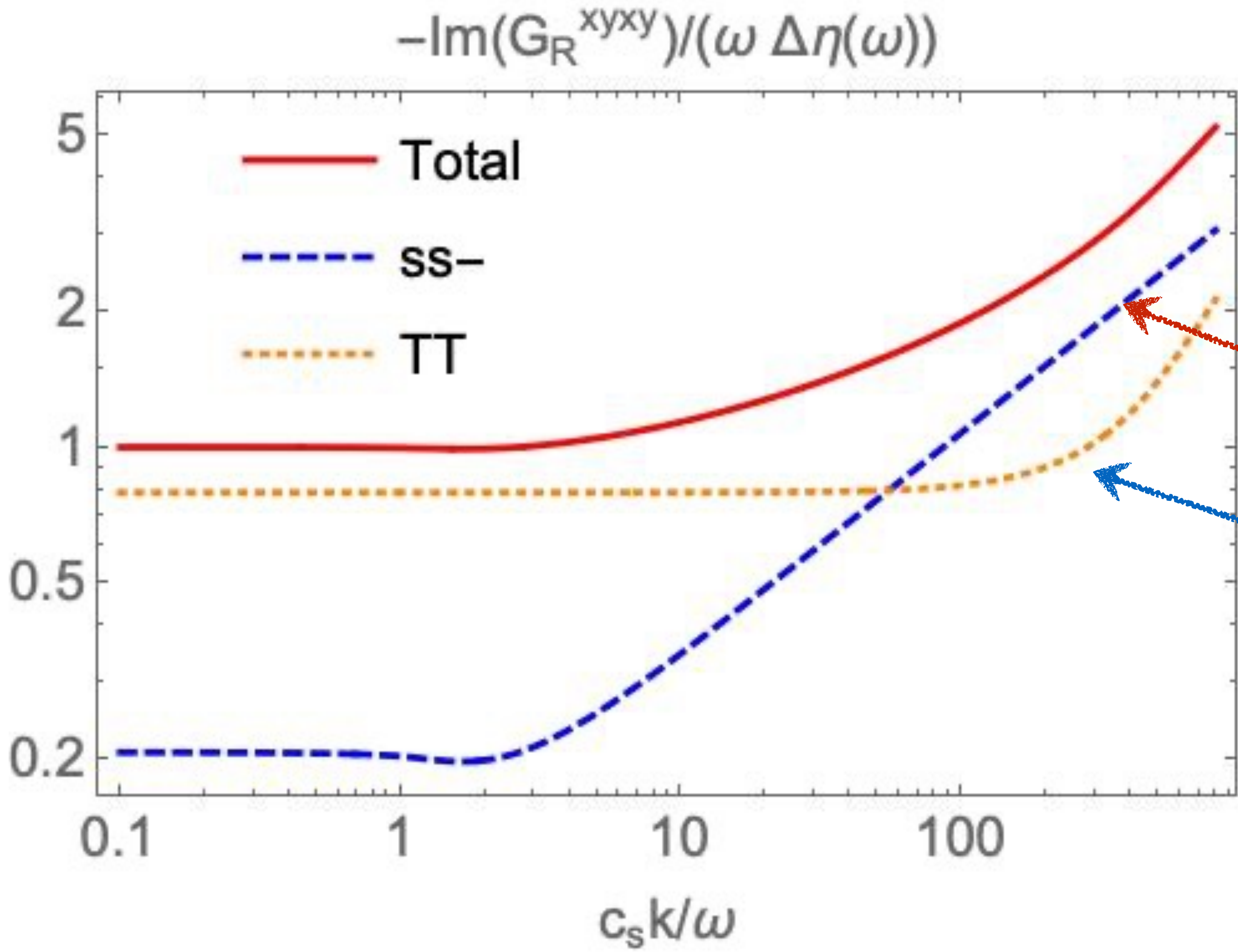
When $c_s k > \omega$: "sound-sound" pair dominates over "shear-shear" pair.

$$\lim_{c_s k \gg \omega} \mathrm{Im}\Delta G_R^{xyxy}(\omega, k)/\omega = \frac{-T}{231\sqrt{2}\pi} \sqrt{\frac{c_s k}{\nu_L}}\frac{1}{\nu_L}$$

N.B.: for shear-shear pair, one will obtain $G_R^{xyxy} \sim T\omega k/\nu_T$ which is parametrically smaller in large k regime.

*(e.g Foster et al, PRA 1977; see also Jain-Kovtun-Ritz-Shukla)*

# The renormalization of shear viscosity at finite k




$$\lim_{c_s k > \omega} \mathrm{Im} G_R^{xyxy} \sim \sqrt{\frac{c_s k}{\nu_L}}$$

$$\lim_{k \gg \sqrt{\frac{\omega}{\nu}}} \mathrm{Im}\, G_R^{xyxy} \sim k$$

(*Foster et al, PRA 1977*)

## Results for sound and shear channel

$$\langle T_r^{0z} T_a^{0z} \rangle = \frac{w\left(c_s^2 k^2 - i\nu_L \omega k^2\right)}{\omega^2 - c_s^2 + i\nu_L \omega k^2} \qquad \langle T_r^{0x} T_a^{0x} \rangle = \frac{-\eta k^2}{-i\omega + \nu_T k^2}$$

At loop, both the dispersion of hydro. modes and the corresponding residues are modified.

$$\frac{-\eta k^2}{-i\omega + \nu_T k^2} \to \frac{-\eta(1 + \delta R(\omega, k))k^2}{\omega + i(\nu_T k^2 + \Sigma_{ar}(\omega, k))}$$

Focus on the self energy

$$(G^{ra})^{-1}(\omega, k) - \Sigma^{ar}(\omega, k) = 0$$ ➡ *"dressed" sound/shear modes*

## The role of viscous vertices in $\Sigma^{ar}$

For sound-sound contribution to $\Sigma^{ar}$, the loop with viscous vertices must be included for completeness.

Generically: gradient expansion organizes the hierarchy among one-point functions, but not necessarily that of two-point functions.

For example: consider two "composite operators".

$$O_1(x) = (\lambda \partial X)(x) \qquad O_2(x) = (\nu \partial \lambda \partial X)(x)\rangle$$

For one point function:

$$\langle O_1(x) \rangle \gg \langle O_2(x) \rangle$$

For correlation, it could happen

$$\langle O(x)\, O_1(0) \rangle \sim \langle O(x) O_2(0) \rangle$$

# More details

$$\langle(\lambda\lambda)(K)(\lambda\partial X)(-K)\rangle \sim \int_{\vec{p}} \left[\vec{p}_+ G_+(p_+)G_-(p_+) + \vec{p}_- G_-(p_+)G_+(p_-)\right]$$

ideal-ideal vertice

$$\sim \int_{\vec{p}} (\vec{p}_+ + \vec{p}_-)\, G_+(p)G_-(-p) \sim \boxed{k p_*^3\, \tau_*}$$

$$\vec{p}_\pm = \pm\,\vec{p} + \frac{\vec{k}}{2}$$

$$\langle(\lambda\lambda)(K)(\nu\partial\lambda\partial X)(-K)\rangle \sim \int_{\vec{p}} \left[\nu \vec{p}_+^{\,2} G_+(p_+)G_-(p_+) + \nu \vec{p}_-^{\,2} G_-(p_+)G_+(p_-)\right]$$

ideal-viscous vertice

$$\sim \int_{\vec{p}} (\vec{p}_+ - \vec{p}_-)\, G_+(p)G_-(-p) \sim \boxed{(\nu p_*^2)\, p_*^3 \tau_*}$$

In this case

$$\langle O(x)\, O_1(0)\rangle \sim k p_*^3\, \tau_* \sim \langle O(x) O_2(0)\rangle \sim (\nu p_*^2)\, p_*^3 \tau_*$$

NB: $\nu p_*^2 = c_s k$

## *KMS symmetry saves the day*

Viscous vertices are many ( $> 10$), and depend on $\eta', \zeta', \eta p'', \zeta p''$ etc.

However, since KMS symmetry imposing strong constraint on propagators and vertices , it turns out that only two independent vertices contribute.

the final contribution to self-energies is proportional to

$$\frac{p''(\beta)}{w_0}\eta\,, \qquad (c_s^2(\beta))' \times \zeta$$

# Dressed hydro. modes (in 3d)

The dispersion of shear modes:

$$\omega_{\rm sh} = -i(\nu_T + \Delta\nu)k^2 \qquad \Delta\nu = -\frac{1}{4\sqrt{2}\pi}\frac{8c_s^2}{77s\nu_L}\sqrt{\frac{c_sk}{\nu_L}}(1-c_1)$$

From Ideal vertices

F r o m   v i s c o u s   v e r t i c e s

$$c_1 = \frac{\beta_0^2 p''}{2w_0} = 5/2\textit{(conformal fluid)}$$

Sound dispersion :

$$\delta\omega_\pm = \pm\left[c_sk - \frac{k^2}{4\sqrt{2}\pi s}\left((0.073-0.332)\sqrt{\frac{c_sk}{\nu_L^3}} - 0.311\sqrt{\frac{c_sk}{2\nu_T^3}}\right)\right]$$

From Ideal vertices From viscous vertices

$$-i\left[\nu k^2 + \frac{k^2}{4\sqrt{2}\pi s}\left((0.073+0.332)\sqrt{\frac{c_sk}{\nu_L^3}} - 0.311\sqrt{\frac{c_sk}{2\nu_T^3}}\right)\right]$$

# Discussion and implication

# Parametrically behavior of fluctuation correction

$$\frac{\Delta\eta}{\eta} \sim \frac{(\nu p_*^2)^{-1}}{\nu} \times (\frac{p_*^d}{s})$$

$$\nu_T = \frac{\eta}{w}, \qquad \nu_L = \frac{(4/3)\eta + \zeta}{w}.$$

*Ratio between life time of "fluctuation mode-pair" and micro. m.f.t . (Recall $\nu \propto \tau_{\rm mft}$)*

*Ratio between phase space volume of "fluctuation modes" and micro. d.o.f.*

The fluctuation contribution:

be suppressed by the density of microscopic modes.

be enhanced if viscosity is small.

# *Implication for dynamical modelling for BES*

On stochastic approach: fluctuation back-reacts on stochastic eqn.

$$\partial_t v^x = -\left(\nu k^2 + \Sigma_{ar}(t,k)\right) v^x + F\,, \qquad \langle F(t,\overrightarrow{k})F(t',\overrightarrow{k}')\rangle \sim 2(\eta + \delta\eta(t,k))\delta(t-t')$$

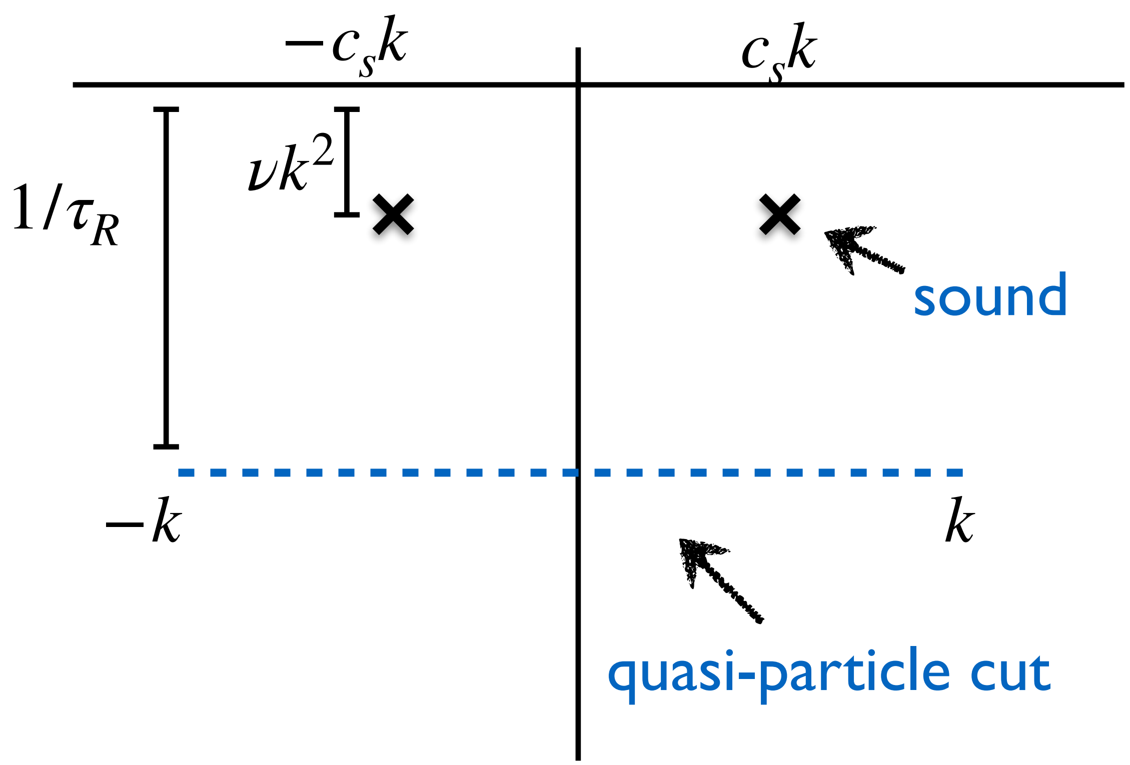
On deterministic approach: self-energies are expected to lead to "collision kernel" in "kinetic eqn" for two-point fluctuations.

Recall Kadanoff-Baym eqn. for scalar theories: *c.f. Mueller-Son, PLB 04'*

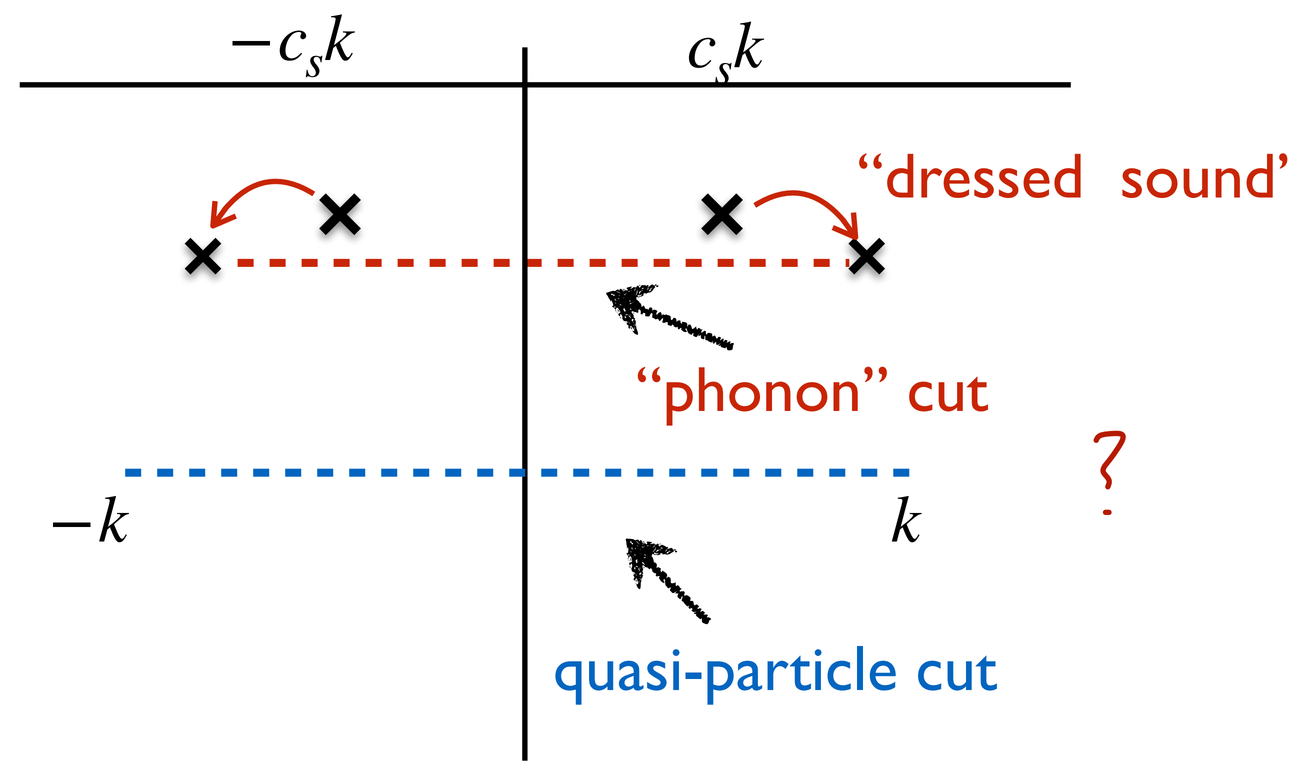
$$\partial_t G_{rr}(t,x,p) + p\frac{\partial}{\partial x}G(t,x,p) = -\underbrace{(\Sigma_{ar}-\Sigma_{ra})G_{rr} - \Sigma_{aa}(G_{ra}-G_{ra})}_{\text{collision integral}}$$

Self-energies computed here can serve as input for the future development of hydro-kinetic equation.

# Analytic structure at finite k (preliminary)




without hydro. fluct. from kinetic theory



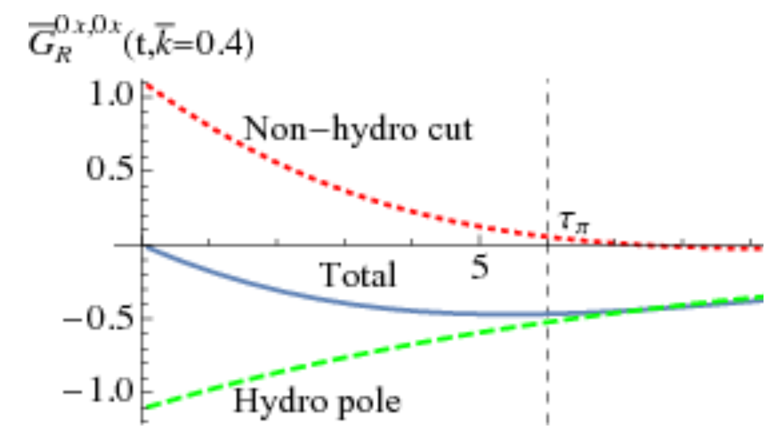

with hydro. fluct.

*Hydro. flucts. leads to rich non-analytic structure in correlators; we show samples of them above.*

A step forward towards understanding non-hydro. yet non-perturbative regime of QGP.

# Hydro. fluctuation and hydrodynamization

2nd order hydro: $\langle T_r^{xy} T_a^{xy} \rangle = P - i\eta\omega + \eta\tau_\pi\omega^2 - \frac{\kappa}{2}(\omega^2 - k^2)$




Physical interpretation of $\tau_\pi$ : hydrodynamization time

$$T^{xy} - T^{xy}_{\text{vis}} \sim e^{-\tau_\pi t}$$

Kurkela-Wiedemann, EPJC 17’

Kubo formula for 2nd hydro. is divergent !

$$\eta\tau_\pi = \lim_{\omega,k\to 0} \left(\partial_\omega^2 - \partial_k^2\right) G_{ra}^{xy;xy}(\omega, k)$$

Moore-Sohrabi, PRL 11’

$$\Delta G^{xyxy} \sim \omega^{3/2}, \omega\sqrt{k}$$

Interpretation: hydro. fluctuation sets the scale of hydrodynamization.

$$T^{xy} - T^{xy}_{\text{vis}} \sim e^{-\tau_\pi t} \quad ➡. \quad t^{-1/2}$$

Decay of fluctuations modes

Another thought: $\tau_\pi$ as well as other second hydro. coefficient might be viewed as transport coefficient defined at non-hydro scale.

see also Ghiglieri-Moore-Teaney, PRL 18’

# Hydrodynamic fluctuation and chiral magnetic effect

Sogabe, postdoc@IMP

## Set-up

For a chiral fluid (consisting massless fermions): chiral anomaly induced a number of interesting transport phenomenon, such as chiral magnetic effect (CME).

$$\vec{j} = C_{\mathrm{anom}} \mu_A \overrightarrow{B}$$

CME couples vector charge density with axial charge density, and induces new types of collective modes.

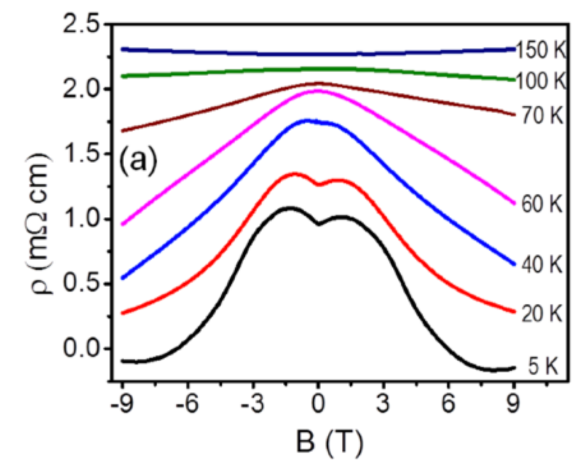
Based on hydro. EFT including anomaly effects, we study the contribution of fluctuations to conductivity.

Glorios-Hong Liu-S. Rajagopal, JHEP 18'

# Negative magento-resistivity

Consider the conductivity in a chiral medium in the presence of a magnetic field.

$$\sigma^{ij} = \sigma_L \hat{B}^i \hat{B}^j + \sigma_T(\delta^{ij} - \hat{B}^i \hat{B}^j)$$




*measurement of conductivity in Weyl semimetal, from1412.6543*

CME togeher with the relaxation of axial density leads to negative magneto-resitvity , i.e., a positive contribution to longitudinal conductivity (tree level results).

$$\sigma_L = \sigma_0 + \frac{C_{\rm anom}^2 B^2}{\chi^2}\Gamma_A^{-1}$$

Finite and negative one-loop correction:

$$(\Delta\sigma_L)_{\rm loop} \sim -\, C_{\rm anom}^2 B^2 \sqrt{\frac{\Gamma_A}{D}}$$

*Non-linearity from CME*

*Emergent scale due to the competition between diffusion and axial relaxation*

# The correction to conductivity and bulk viscosity at zero axial damping

The collective modes: chiral magnetic wave

$$\omega_{\mathrm{CMW}}(\overrightarrow{k}) = \pm v_B \hat{B} \cdot k - iDk^2 \qquad v_B = \frac{CB}{\chi_0}$$

Non-linearity due to advection: $\langle (n_B v^i)(t,x)(n_B v^i)(0) \rangle$

$$(\Delta\sigma)_{\mathrm{loop}} \sim -\frac{\chi}{32(D+\nu_T)w_0\pi}\left(\frac{v_B}{D+\nu_T}\right)$$

*applicable when $\nu_B/(D+\nu_T) \ll l_{\mathrm{mfp}}^{-1}$*

*Emergent scale due to the competition between diffusion and CMW*

Similar for bulk viscosity

$$(\Delta\zeta)_{\mathrm{loop}} \propto -\frac{c^2\chi^2}{D}\left(\frac{v_B}{D}\right)$$

$(\delta p = c\delta n^2)$

Extension to the critical point (preliminary):

$$\Delta\sigma \sim \xi \qquad (\xi^{-1} \gg \frac{v_B}{\nu_T}) \qquad\qquad \Delta\sigma \sim \left(\frac{\nu_T}{\nu_B}\right) \qquad (\xi^{-1} \ll \frac{v_B}{\nu_T})$$

Even a weak magnetic field could lead to dramatic changes in transport coefficients through hydro. fluctuations !

# Summary and outlook

## Summary and outlook

Hydrodynamic fluctuations lead to rich physics.

The newly-developed EFT approach is a powerful tool for studying fluctuations dynamics.

First complete analysis of stress-energy tensor correlator.

Hydrodynamic fluctuations and chiral anomaly.

Outlook: first-order transition, hydrodynamization, non-hydro. yet non-perturbative regime of QGP. (QGP mesoscopy).

# Perspective on quantum computing

Simulating stochastic hydro. proves to be numerically demanding. Taking advantage of quantum computer?

Hydro EFT might be a useful starting point:

$$I_{\rm hydro} \to \mathscr{T} \exp(iH_{\rm hydro} t)$$

A recent breakthrough: the quantum computing algorithm for non-linear, non-Hermitian "Hamiltonian".

*Lloyd et al, 2011.06571*

A small step forward: understanding this algorithm in a simple (Bjorken expanding) fluid model.

*Bao Xiangrun (undergrad. of Peking U.), Huichao Song, YY, in progress*

Many exciting physics ahead. Stay tuned!

# Back-up