Physics at High Baryon Density Region

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— RHIC-BES on-line seminar (Series II) —

Talk Plan

- **Changes of physical degrees of freedom**
  - □ Phenomenological characterization of deconfinement
  - □ Successful applications of HRG: strong $\boldsymbol{B}$ and large ω
  - □ High density behavior of interacting HRG
- **Approaches from perturbative calculations**
  - □ Current status and problems
  - □ Resummation effects and validity extension
- **Topological aspects of dense nuclear matter**
  - □ Emergent symmetry in quark/nuclear matter
  - □ Chiral soliton lattice from the Skyrme crystal

— PHENOMENOLOGY —

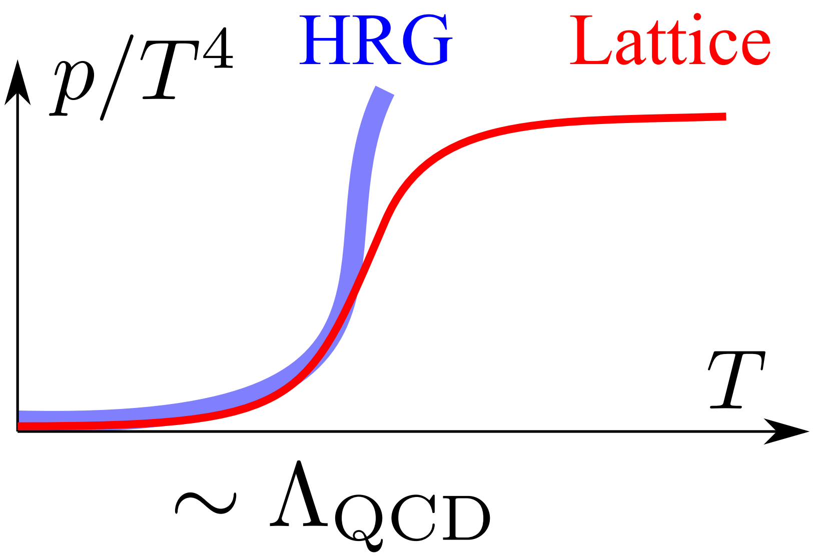
Changes of physical degrees of freedom

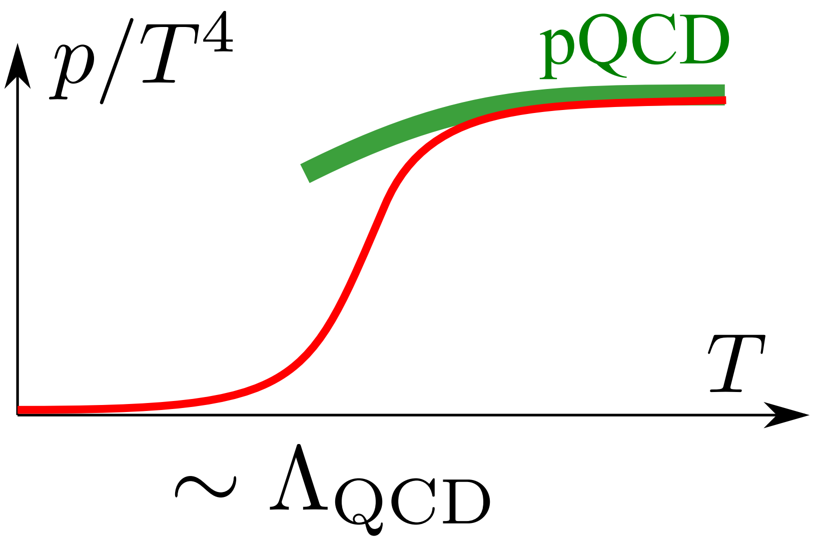
Deconfinement

Polyakov loop — renormalization? interpretation?

Thermodynamics

Talk@QM14

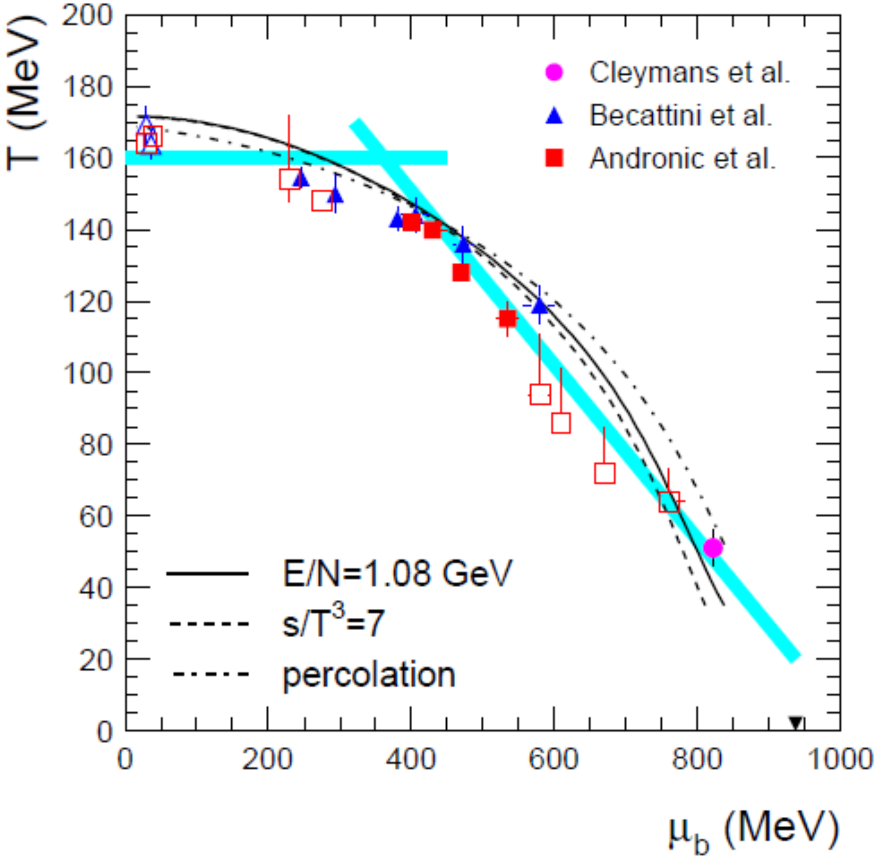




Deconfined where a hadronic description breaks down.
Deconfined where a pQCD description works.

Deconfinement

Phase Diagram = Two Hagedorn Transition Lines



Andronic et al. (2009)

Mesonic Hagedorn Transition

$$Z = N \int dm\, \rho(m)\, e^{-m/T}$$

$$\rho(m) = e^{m/T_H}$$

$$T_c = T_H$$

Baryonic Hagedorn Transition

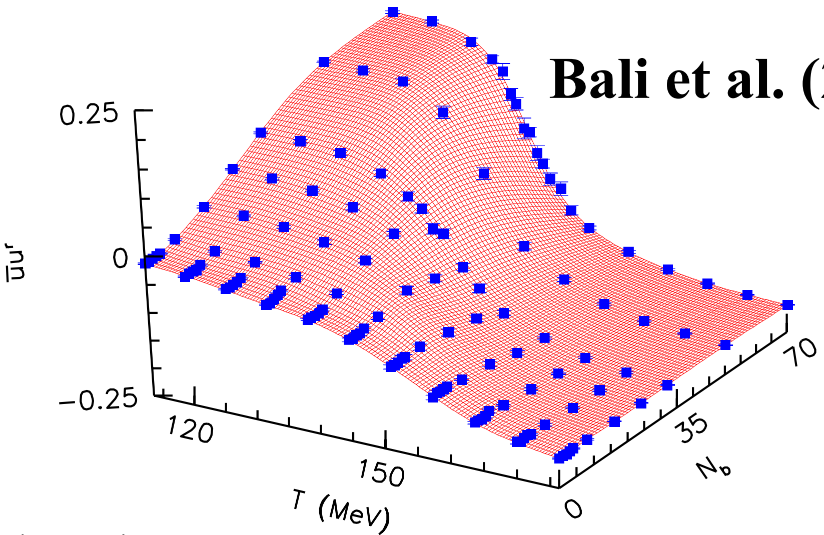
$$Z = N \int dm\, \rho_B(m)\, e^{-(m-\mu_B)/T}$$

$$\rho_B(m) = e^{m_B/T_B}$$

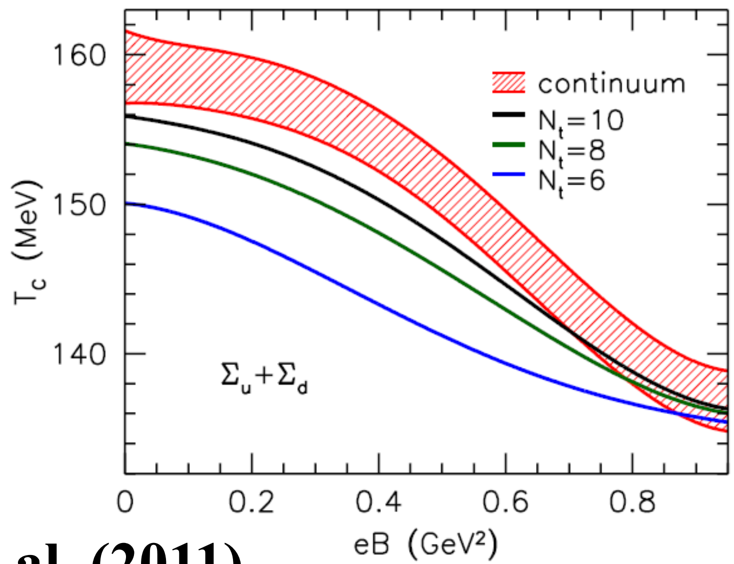
$$T_c = (1 - \mu_B/m_B) T_B$$

Successful Application I

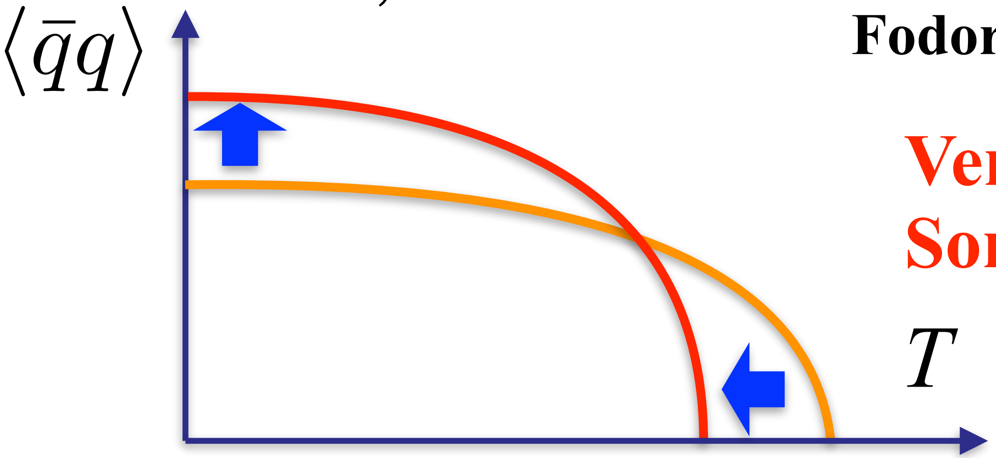
Inverse Magnetic Catalysis (IMC)



Bali et al. (2011)



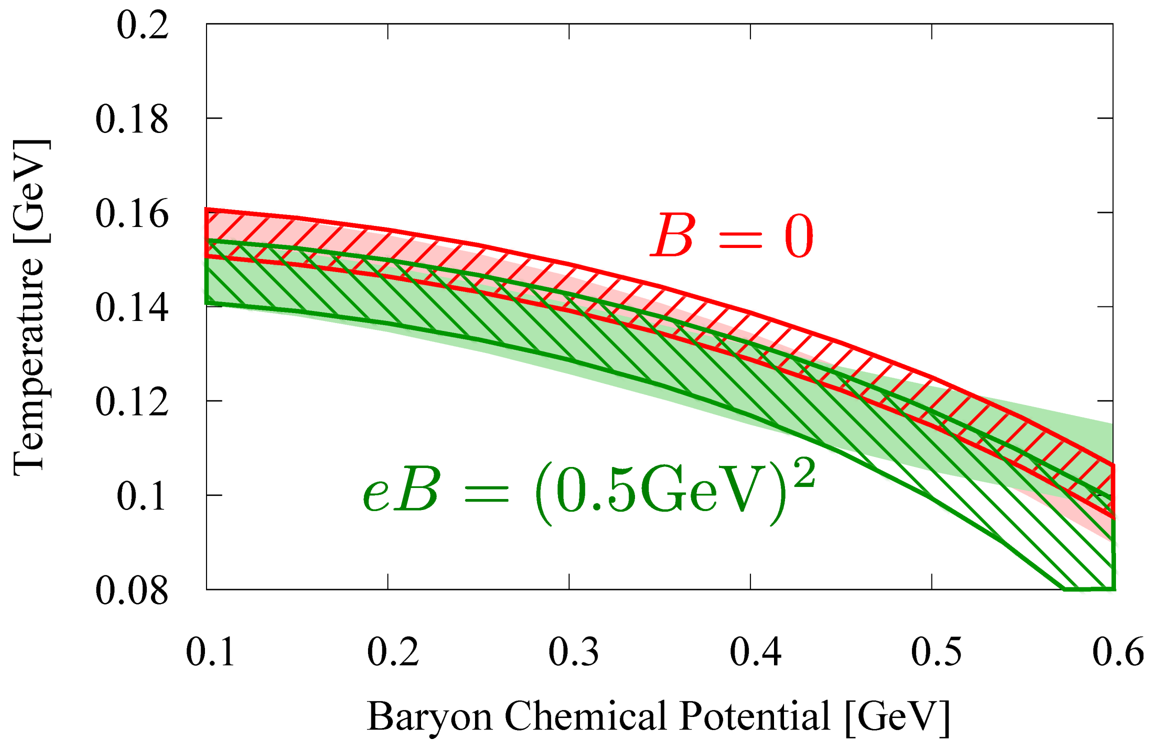
Fodor et al. (2011)



Very different from BCS theory
Something missing
— deconfinement?

Successful Application I

Inverse Magnetic Catalysis (IMC)



Phenomenological freezeout condition

$$E/N \sim 1\mathrm{GeV}$$

E : internal energy

N : particles + antiparticles

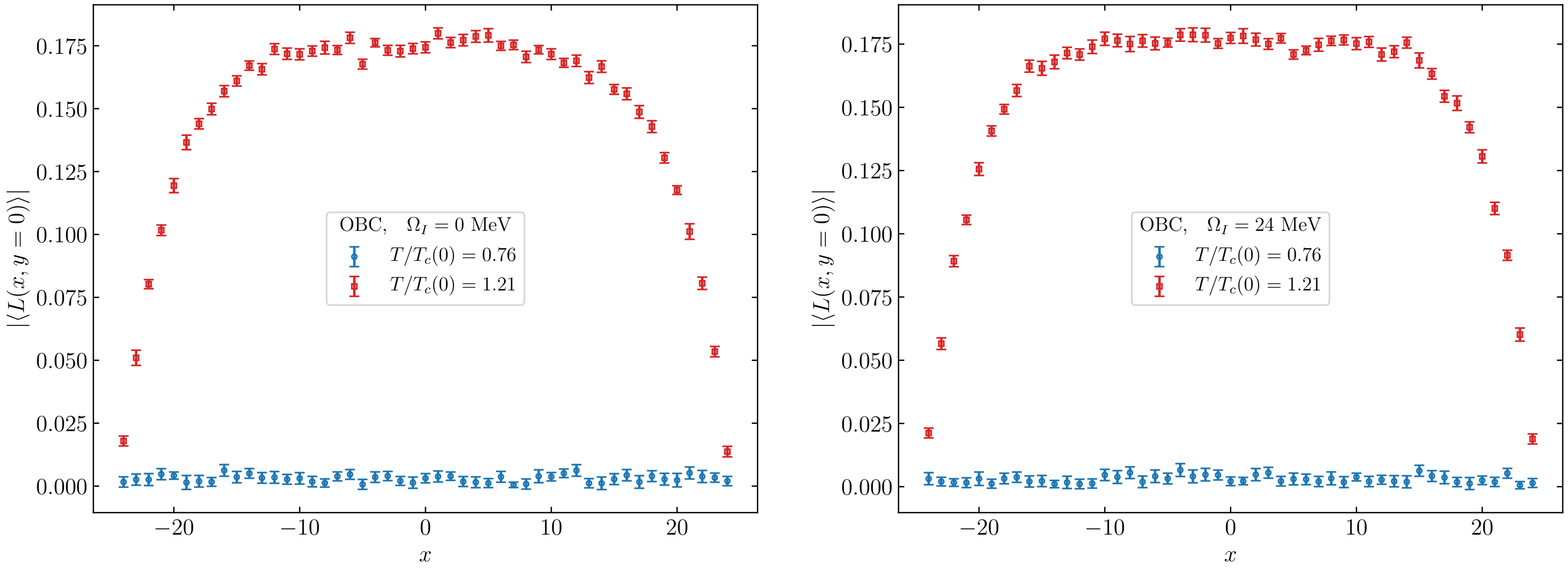
Fukushima-Hidaka (2016)

Ideal Hadron Resonance Gas (IHRG) surprisingly good!

Successful Application II

Controversy on rotational deconfinement

Braguta et al. (2021) — pure gluonic theory on lattice



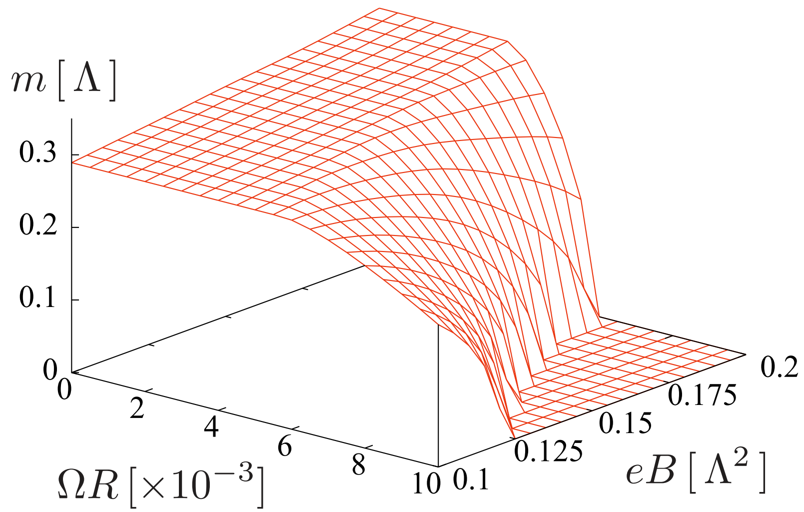
Imaginary rotation enhances the Polyakov loop

Rotation induces more confinement!?

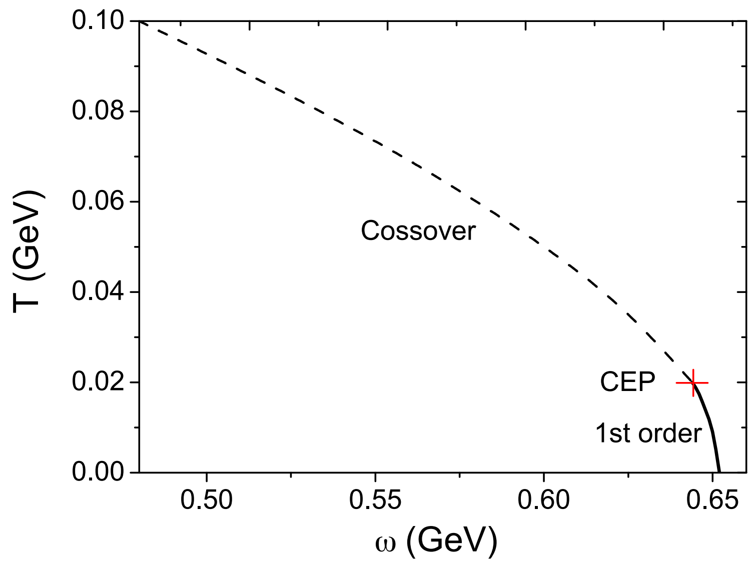
Successful Application II

Controversy on rotational deconfinement

Chiral sector has been well understood: rotation ~ density



Chen et al. (2015)



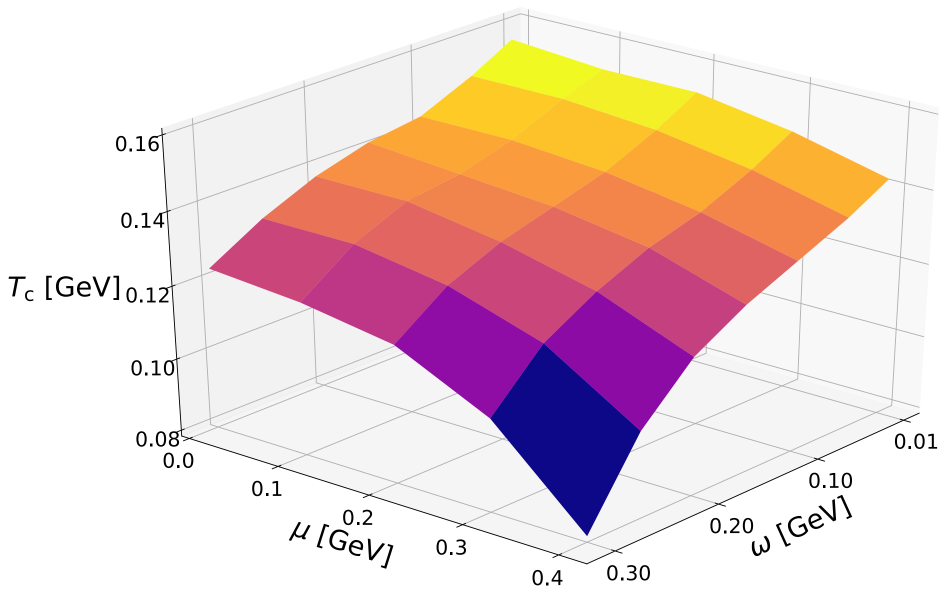
Jiang-Liao (2016)

Chiral restoration is assisted just like density!

Successful Application II

Controversy on rotational deconfinement

Fujimoto-Fukushima-Hidaka (2021)



Also **moment of inertia** estimated (consistent with enthalpy)

IHRG predicts that rotation induces NOT confinement BUT deconfinemet!

Does not falsify Victor's lattice since the system is not the same.

Physics is position dependent.
Polyakov loop?
(private comm. with Maxim)

HRG EoSs further

Andronic-Braun-Munzinger-Stachel-Winn (2012)

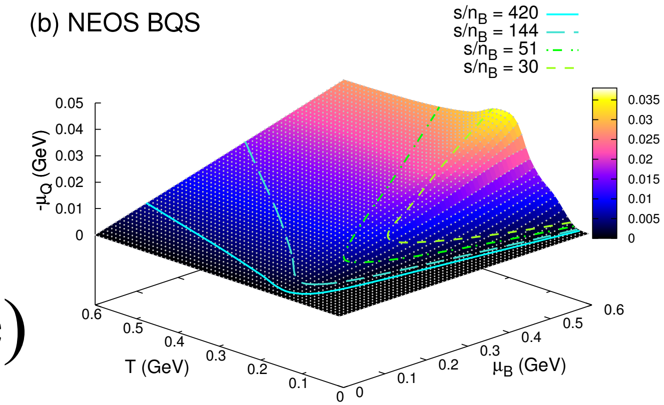
Repulsive interaction through the excluded volume effects
Better agreement with lattice

Vovchenko-Gorenstein-Stoecker (2016)

Van der Waals (excluded volume effect + attractive int.)

Monnai-Schenke-Shen (2019)

NEOS — interpolated between
lattice and IHRG
(HRG is preferable for Cooper-Frye)



HRG EoSs further

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Repulsive interaction through the excluded volume effect
Better agreement with lattice

EV enlarges the validity region

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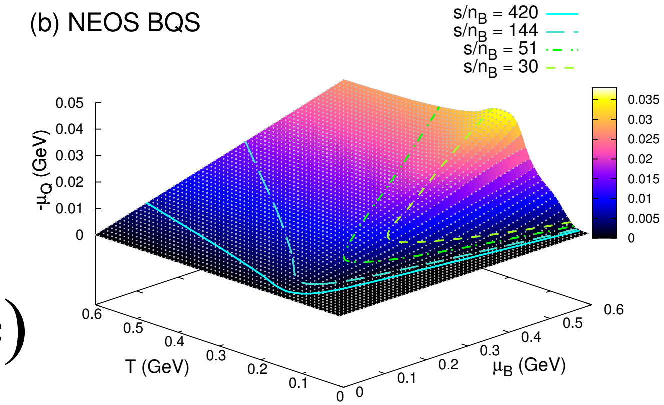
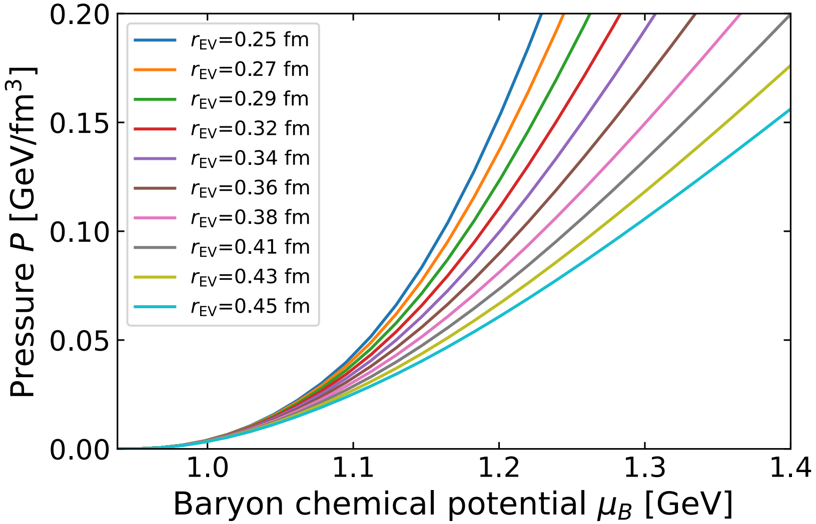
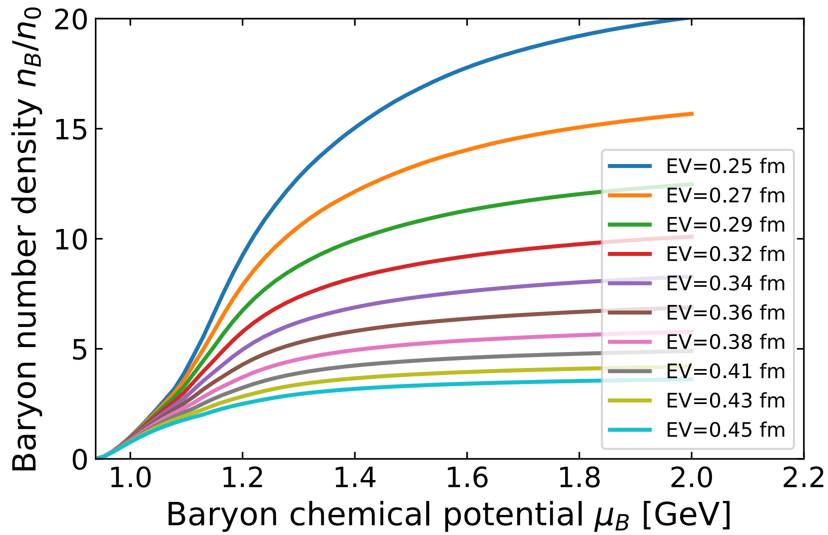


Can we expect EV or VDW works at higher density?

Excluded Volume Effects

If *P* is plotted versus *n*, the behavior blows up abruptly (All the following results are ones with β equilibrium)



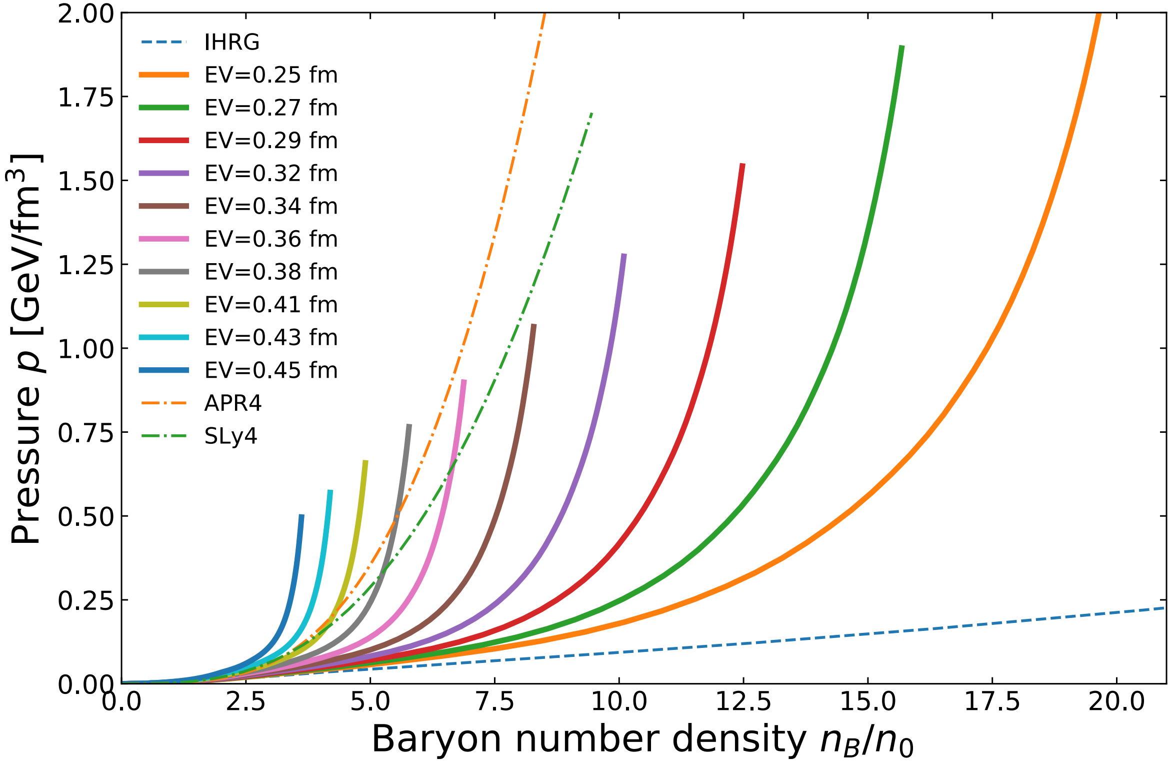


Fujimoto-Fukushima-Hidaka-Iida (in prep)

Density is saturated, while the pressure increases…

$p(\mu_B)$ **then blows up!** Inevitable from $n_B = \dfrac{\partial p}{\partial \mu_B}$

Excluded Volume Effects



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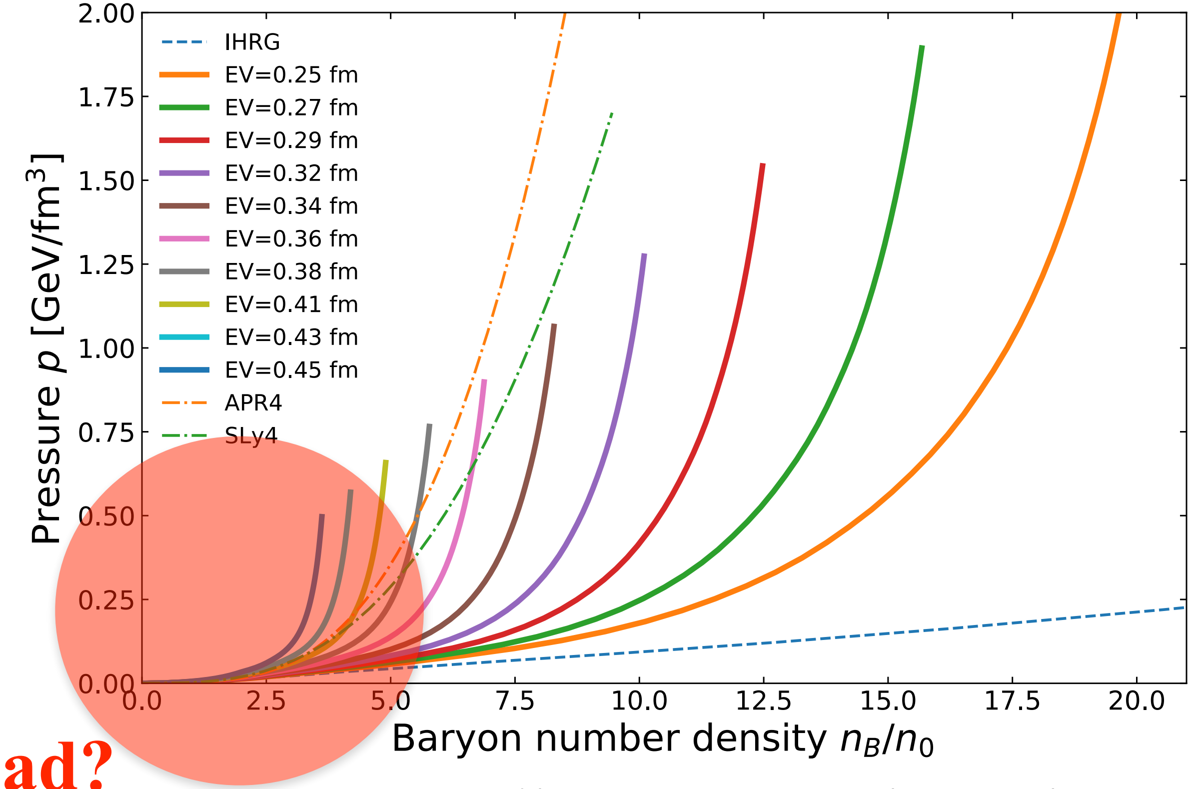


Maybe not so bad?

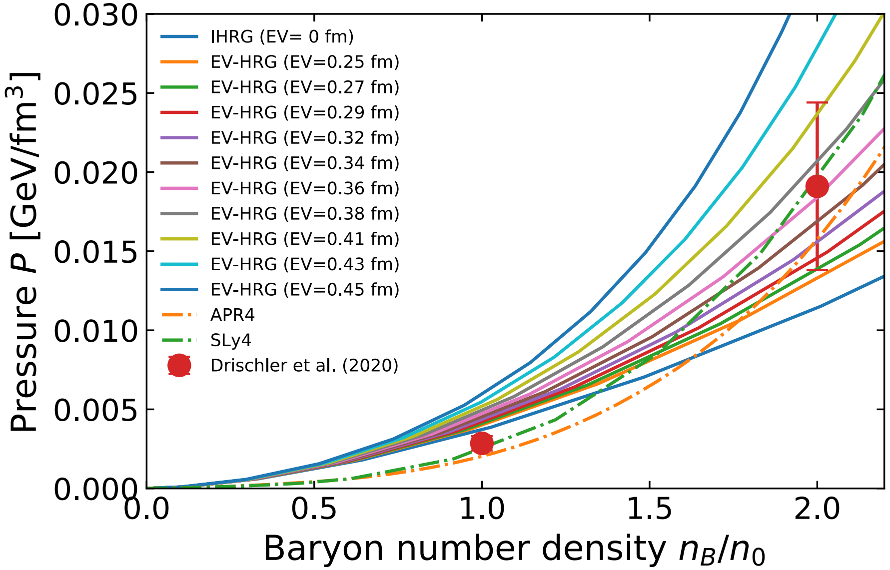
Fujimoto-Fukushima-Hidaka-Iida (in prep)

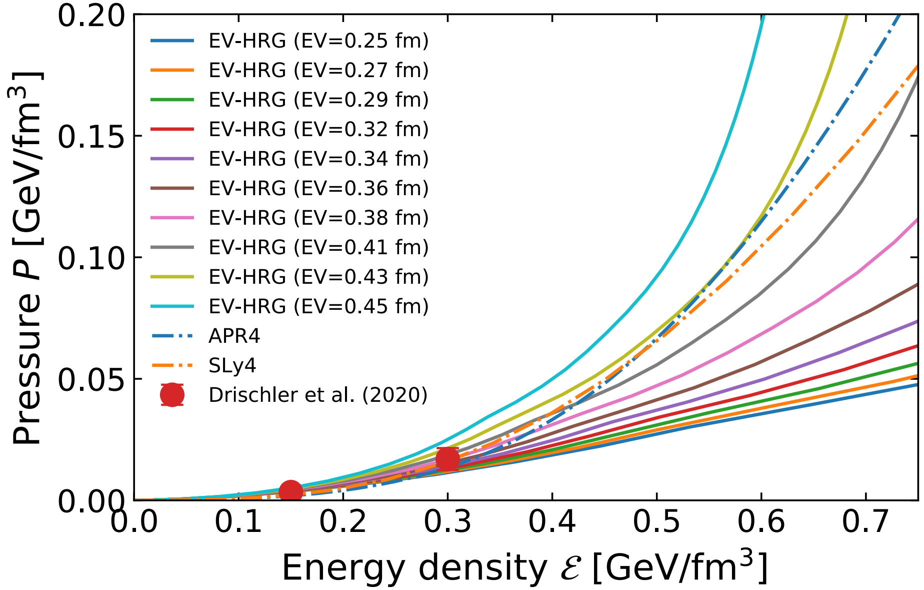
Density is saturated, while the pressure increases…

$p(\mu_B)$ **then blows up!** Inevitable from $n_B = \dfrac{\partial p}{\partial \mu_B}$

Excluded Volume Effects

Comparison to Chiral EFT by Drischler et al. (2020)





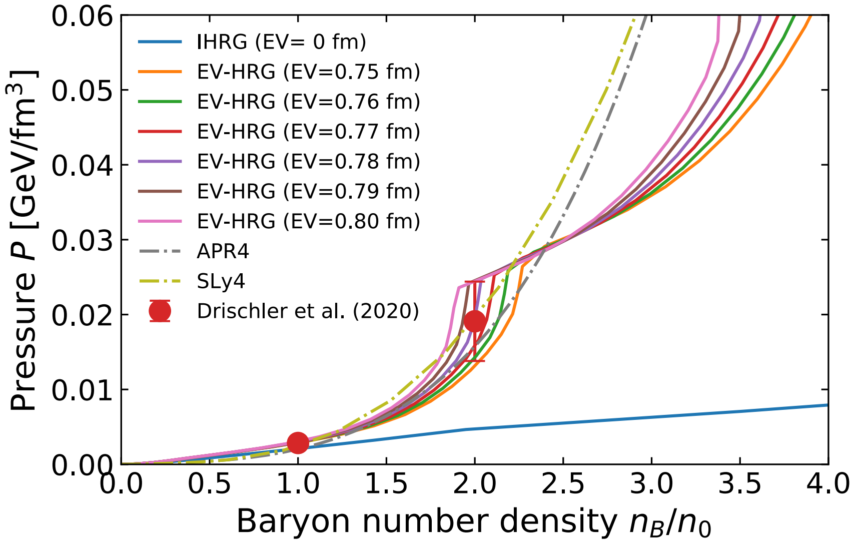
Fujimoto-Fukushima-Hidaka-Iida (in prep)

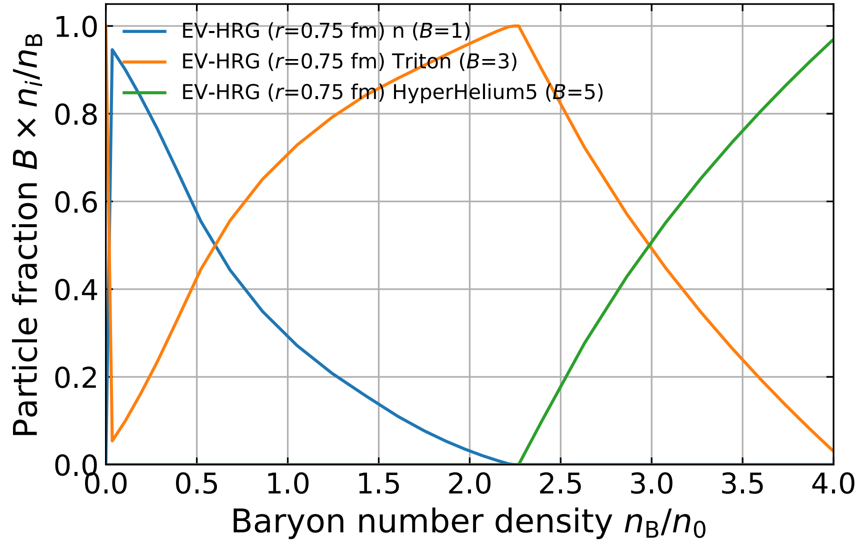
EoS is not a crazy one up to twice saturation density.

$p(\varepsilon)$ may look better at high density though $p(n_B)$ doesn't.

Clustering?

So far, $B > 2$ particles were excluded by hand, and if included, larger EV fits the EoS better (maybe overfitting)



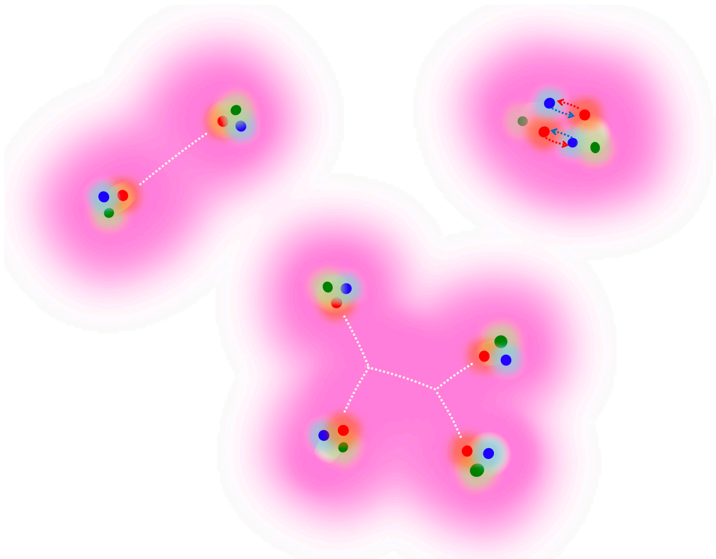


Fujimoto-Fukushima-Hidaka-Iida (in prep)

Quantitatively questionable, but physically reasonable — multi-baryon clustering more and more dominant!

Soft Deconfinement

Fukushima-Kojo-Weise (2020)



Quantum Percolation

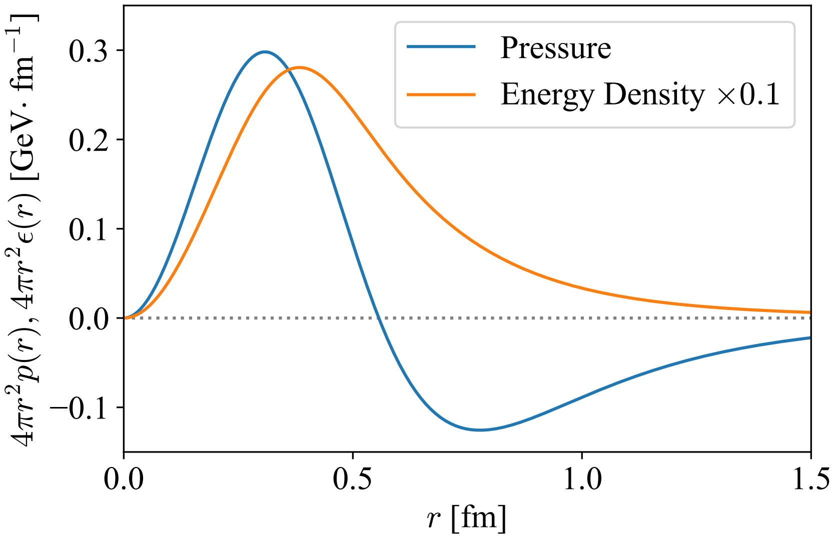
Many-body interactions open connected paths, but it does not necessarily mean connected wave-functions (Anderson localization — insulator).

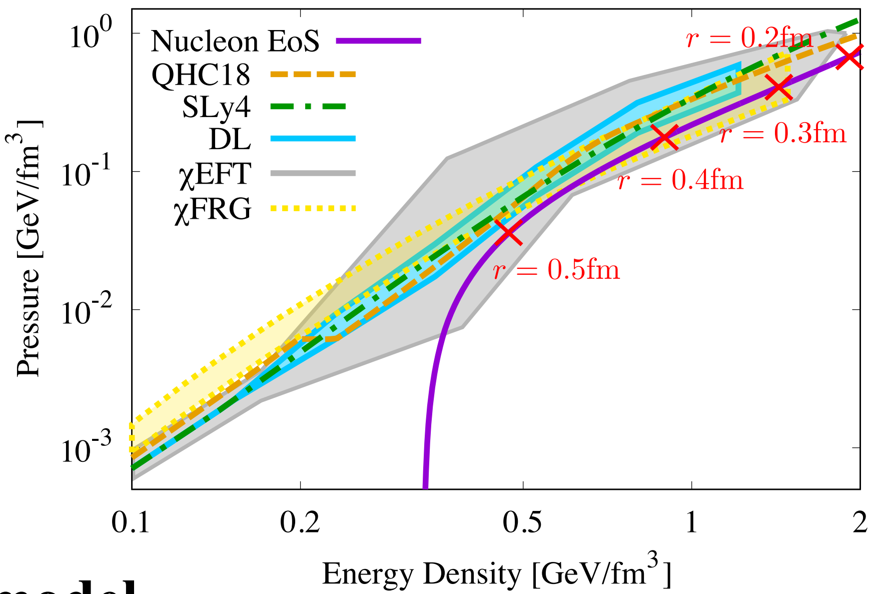
Details of baryon interaction is crucial at high density.
EV (and VDW) is too rigid.

Hard Deconfinement

Fukushima-Kojo-Weise (2020)

Energy-momentum tensor of proton can be probed by the gravitational form factor (at EIC).
If the hydrodynamical pressure is dominated by the nucleon matrix element at high energy density, we can predict:





Calculated in a vectorized Skyrme model

Something to learn

Toward a simple EoS model construction:

* HRG with excluded volume effects may work around twice saturation density.

* VDW (with excluded volume effects and attractive int.) can fit nuclear properties near the saturation density.

* (Soft) deconfinement needs multi-baryon clustering and quantum percolation.

* High density behavior is dominated by the nucleon inner structures (hard deconfinement).

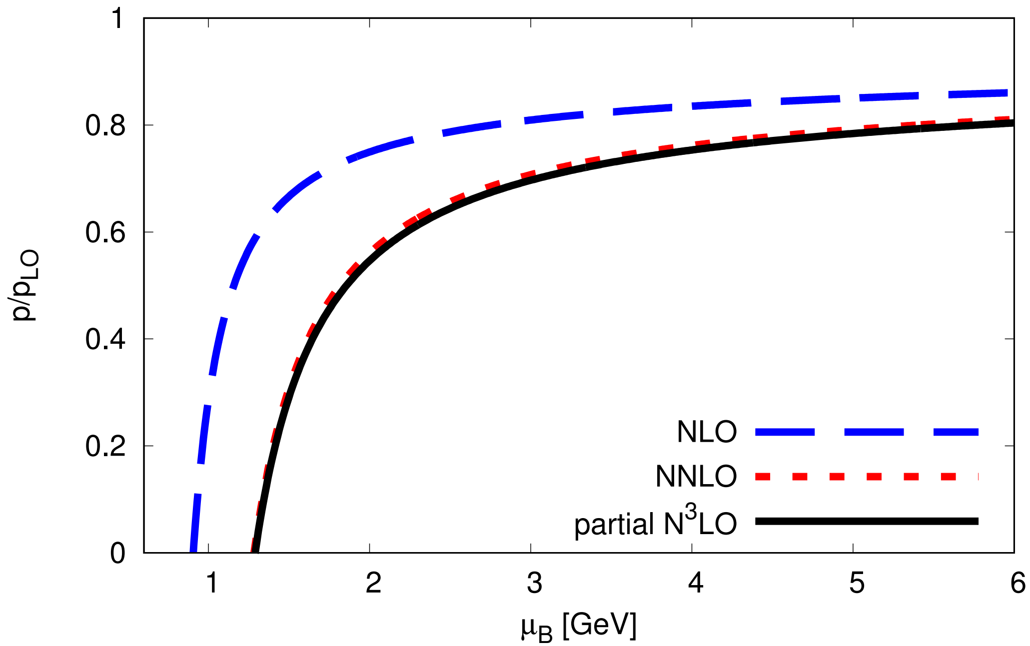
— THEORY —

Approaches from perturbative calculations

Perturbation — Current Status

At high density perturbation theory works!

Gorda-Kurkela-Romatschke-Sappi-Vuorinen (2018)



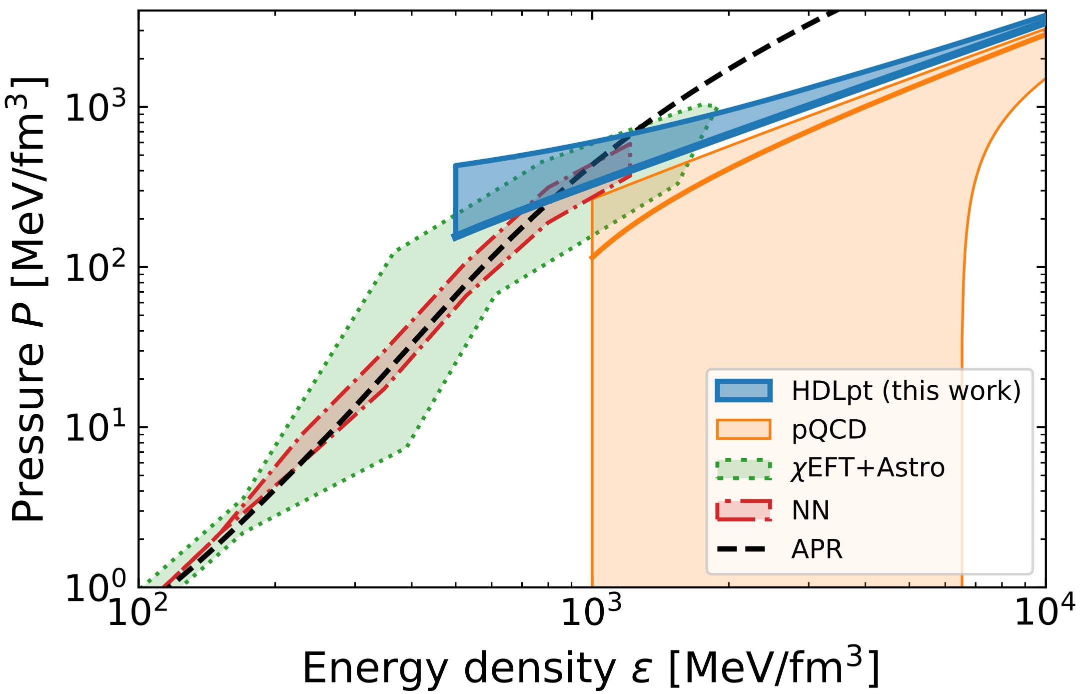
NNLO was known from Freedoman-McLerran (1977)

Convergence is much better than high-*T* perturbation.

No oscillatory divergences of asymptotic series…

Source of Uncertainty

$$\alpha_s(\bar{\Lambda}) = \frac{4\pi}{\beta_0 \ln(\bar{\Lambda}^2/\Lambda_{\overline{\rm MS}}^2)}\left[1 - \frac{2\beta_1}{\beta_0^2}\frac{\ln\ln(\bar{\Lambda}^2/\Lambda_{\overline{\rm MS}}^2)}{\ln(\bar{\Lambda}^2/\Lambda_{\overline{\rm MS}}^2)})\right]$$

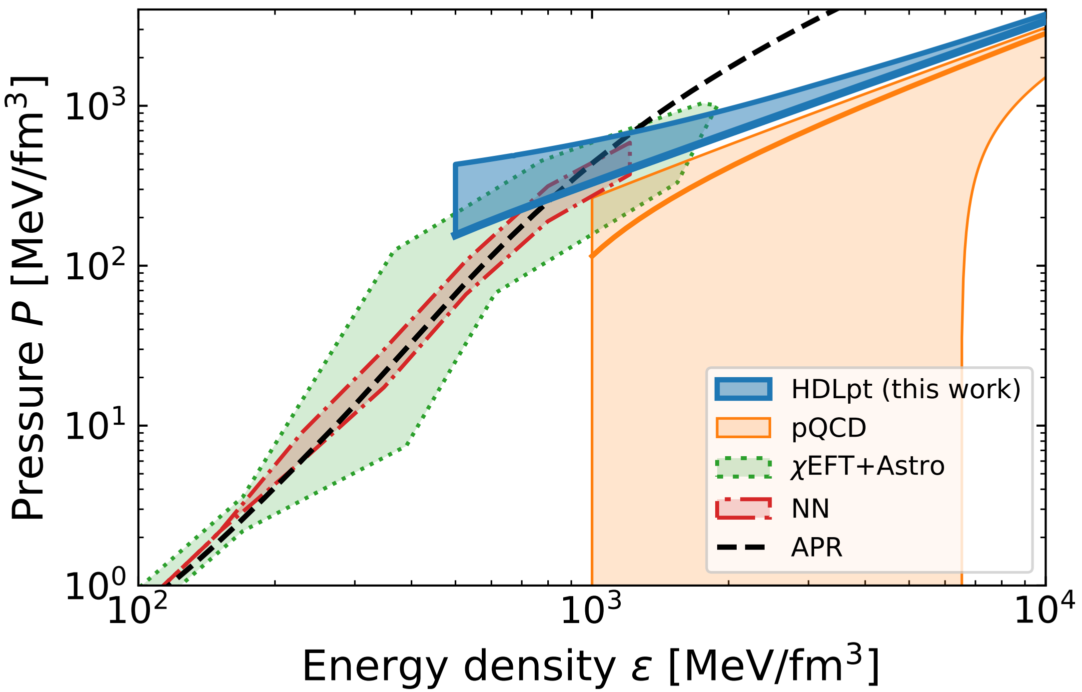


Fujimoto-Fukushima (2020)

$\bar{\Lambda}$ is changed as
μ, 2μ, 4μ

Orange band represents NNLO + strange mass

Source of Uncertainty (closer look)



Fujimoto-Fukushima (2020)

~~pQCD does not work?~~

pQCD works for
$\bar{\Lambda} > 2\mu$ reasonably!

Band is widened
by a single line of $\bar{\Lambda} = \mu$

*** Simply the optimal scale should be set to be larger?**
*** A partial resummation may rescue the situation?**

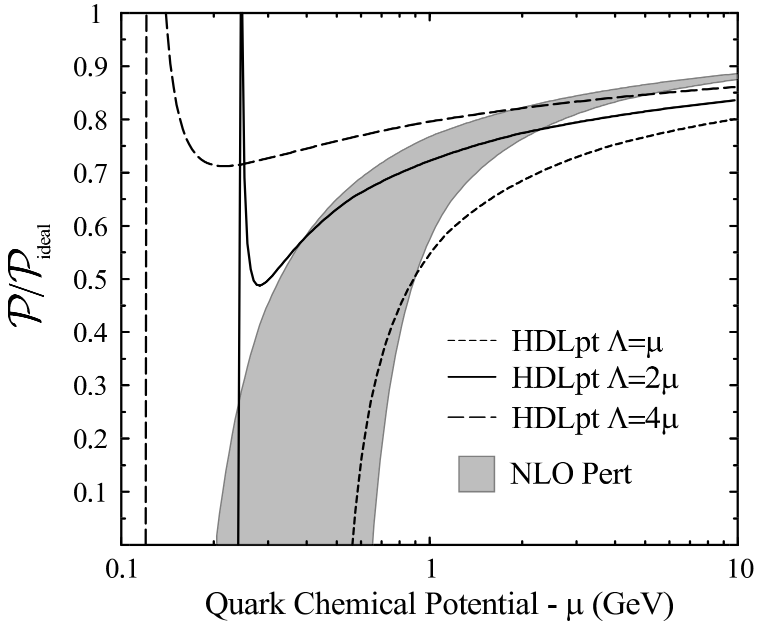
Resummation so far

Baier-Redlich (1999)

Zero temperature limit of HTL resummation (HDL)

Andersen-Strickland (2002)

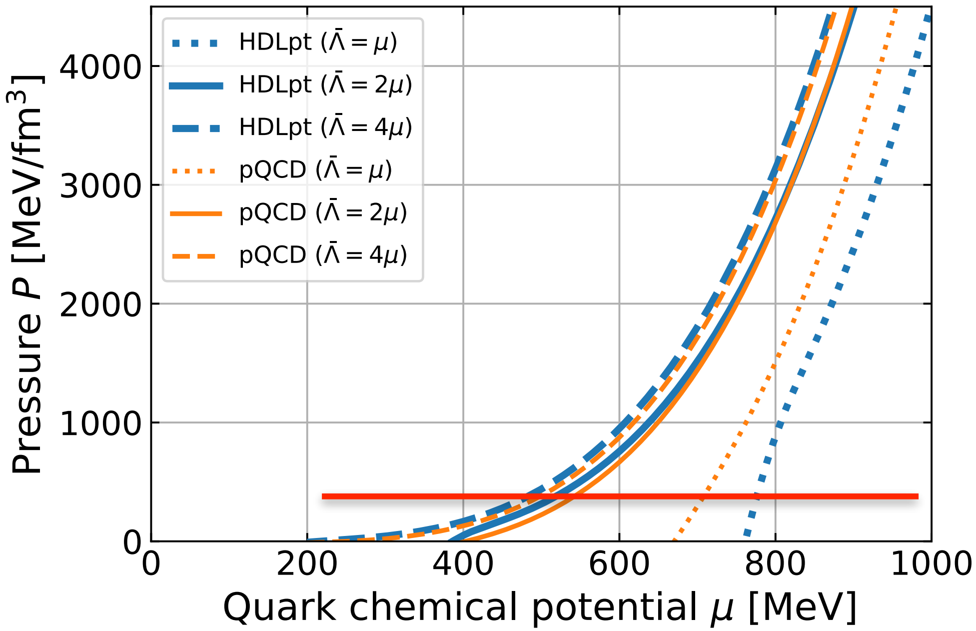
HDL EoS to solve dense quark starts



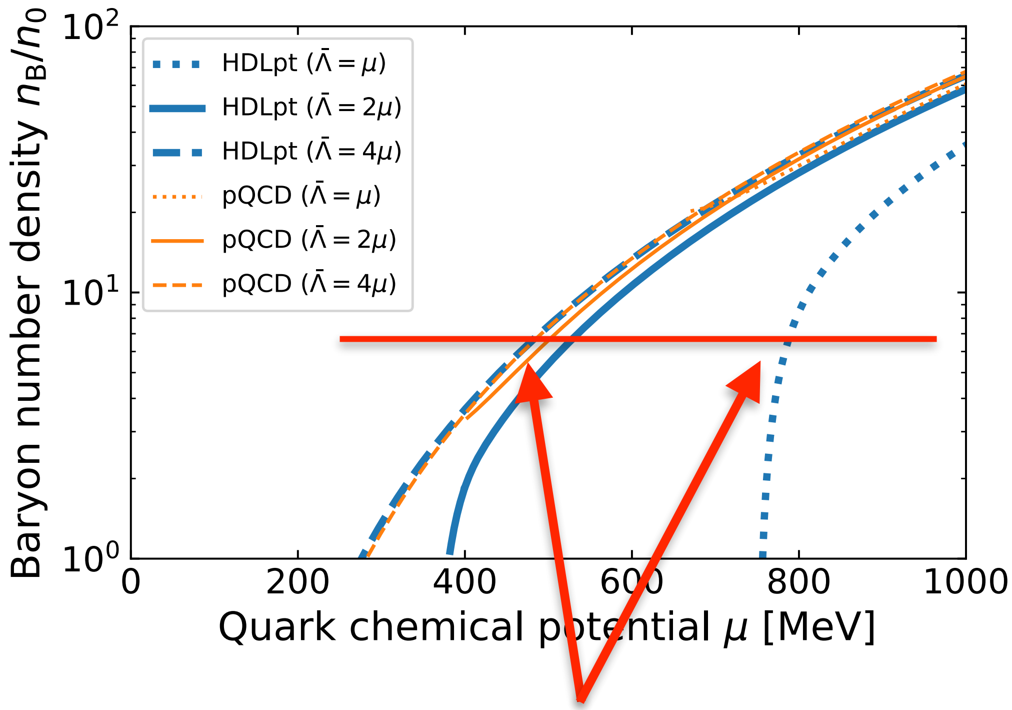
They say that uncertainty is large, but you see that it is driven by $\bar{\Lambda} = \mu$ (dotted) line.

Uncertainty depends on what we see

Fujimoto-Fukushima (2020)



Andersen-Strickland (2002)



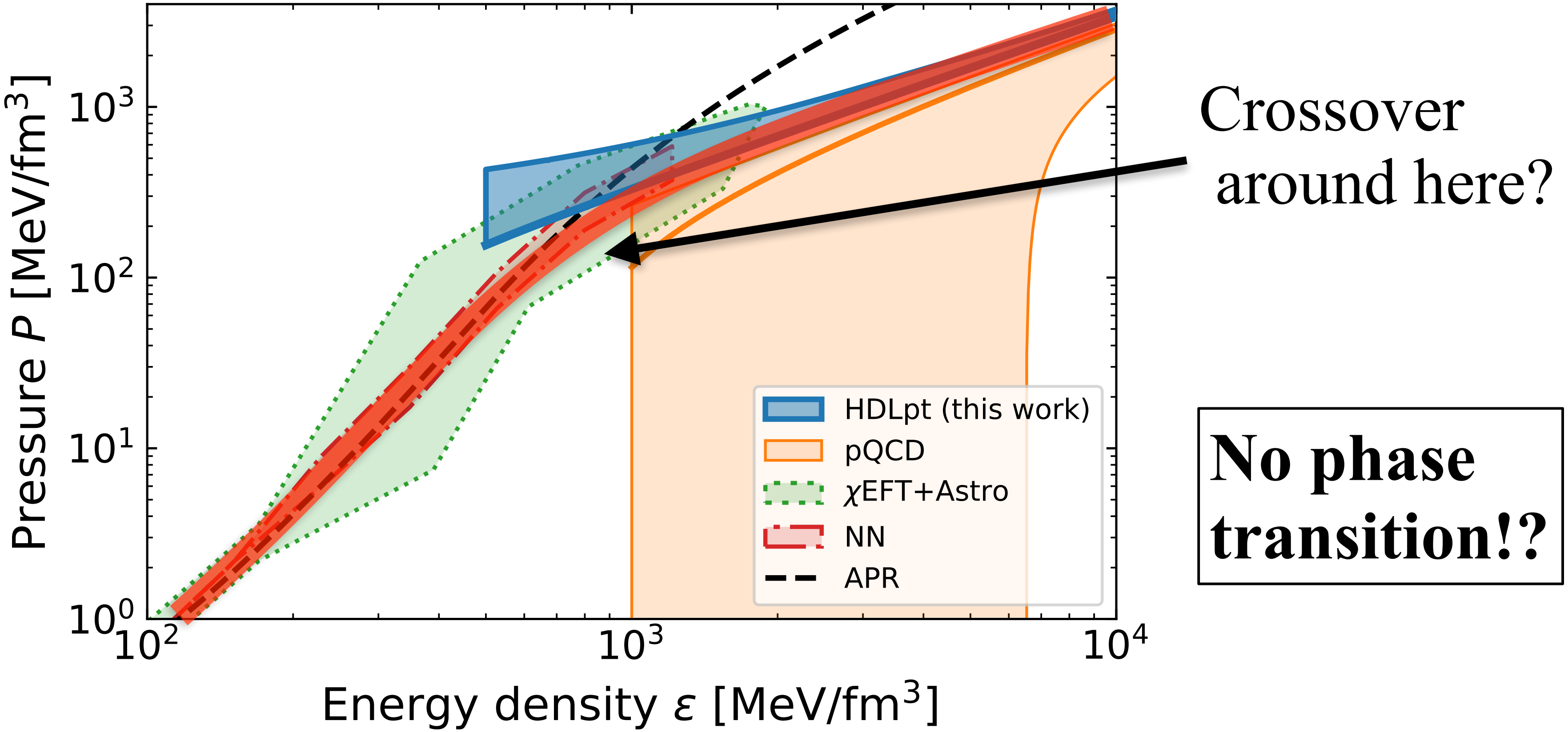
Some (hidden) scaling law???

For a given density the corresponding μ is pushed up by the resummation

Uncertainty depends on what we see

Fujimoto-Fukushima (2020)

Smooth continuation from the nuclear side to the quark side could be possible now!



No phase transition!?

— ACADEMIC —

Topological aspects of dense nuclear matter

Long Standing Puzzle

Is there an order parameter to characterize confinement / deconfinement with quarks?

There are many many works by
DeTar, McLerran, Meyer-Ortmanns, Oleszczuk, Polonyi,
Faber, Borisenko, Zinovjev, …

Nonlocal order parameter?
Maybe no order parameter at all?
(In principle no way to distinguish quarks?)

Quark-Hadron Duality

Heuristic view to understand the duality

Hadronic Matter	Quark Matter
Baryons 8+1 (low-lying)	Quarks 3color × 3flavor = 9

Flavor Triplet

Condensate

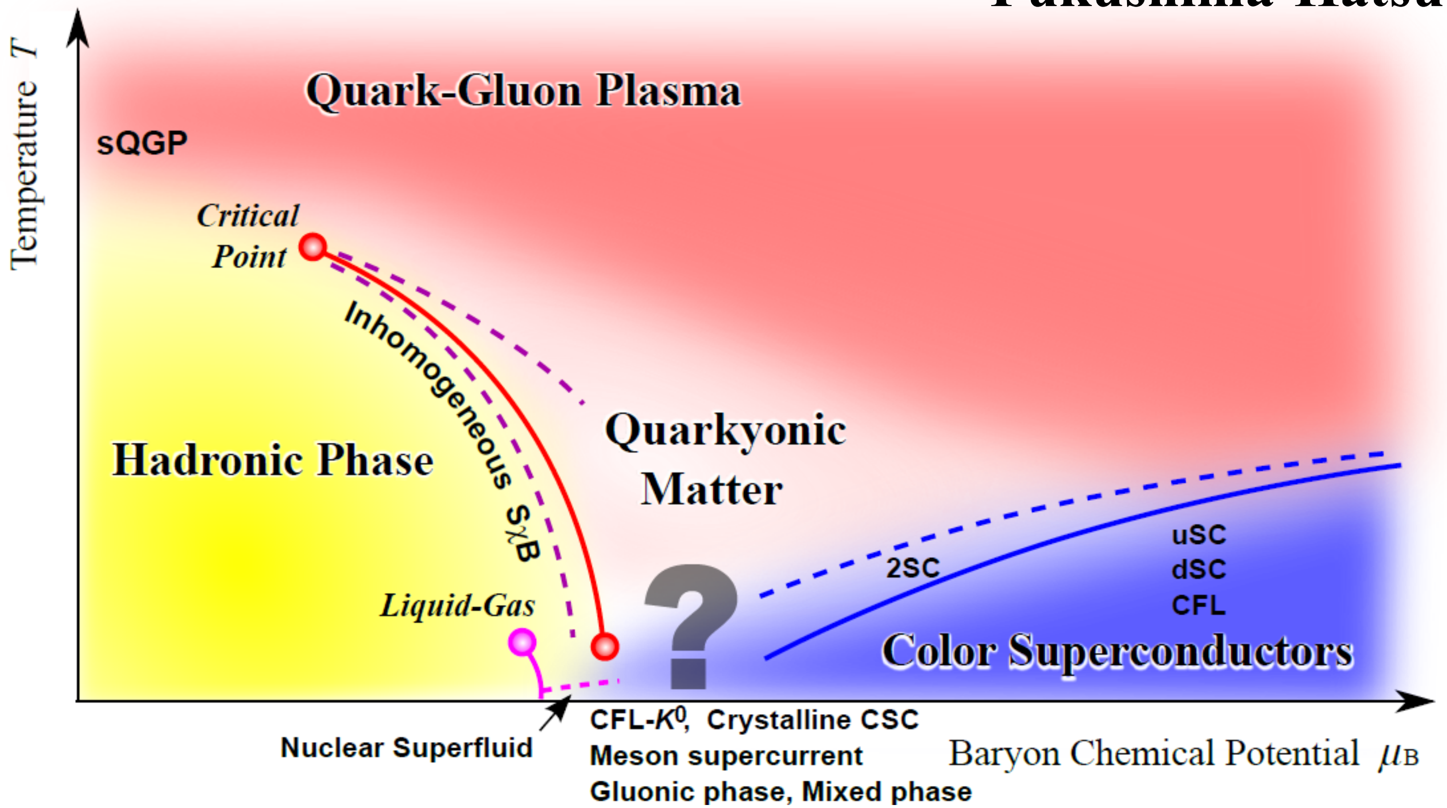
Excitation

All the condensates and excitations have correspondence

8 vector mesons	8 gluons
8 pseudo-scalar mesons	8 tetra-quark mesons

Quark-Hadron Duality

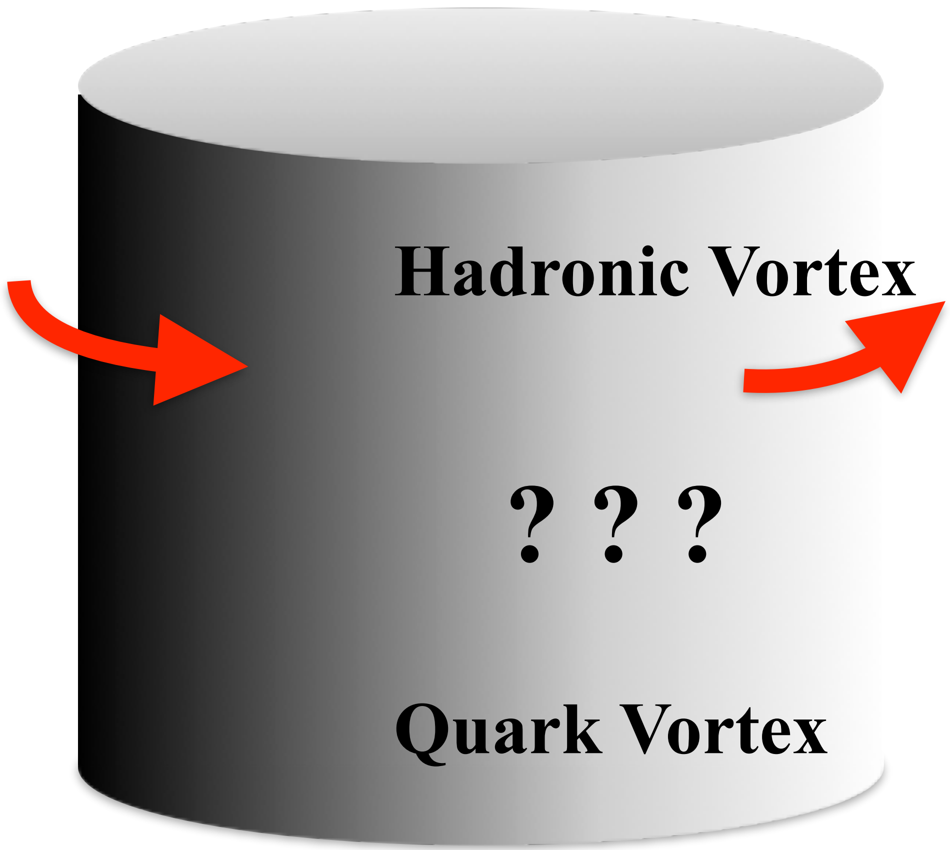
Fukushima-Hatsuda (2010)



No phase transition from the hadronic to the CSC?
Recent works suggest a new possibility!

Quark-Hadron Duality

Thinking experiment



Rotate the bucket filled with quarks

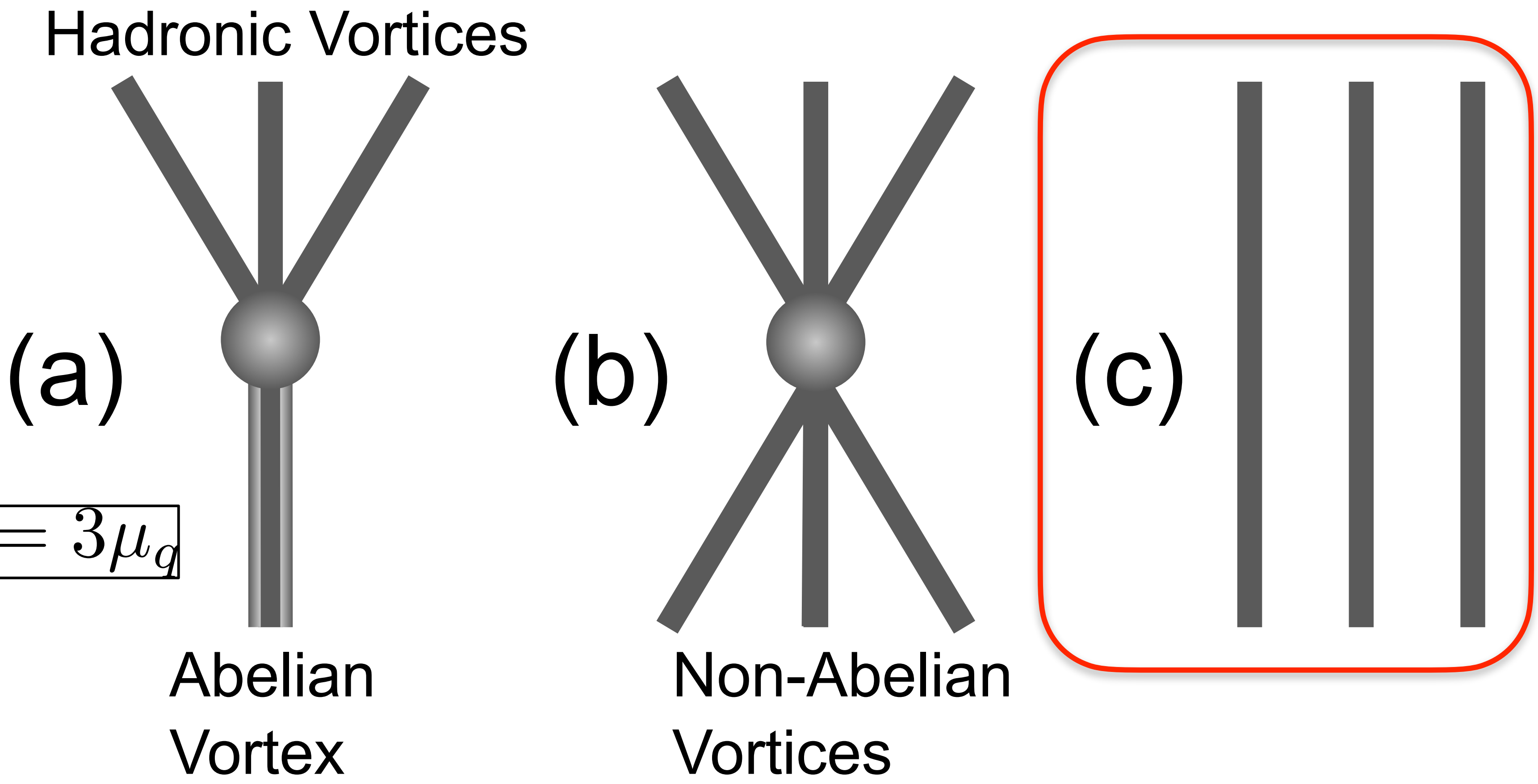
Upper part : Hadronic Vortex

Lower part : Quark Vortex

How can they be connected?

Mismatched Circulation?

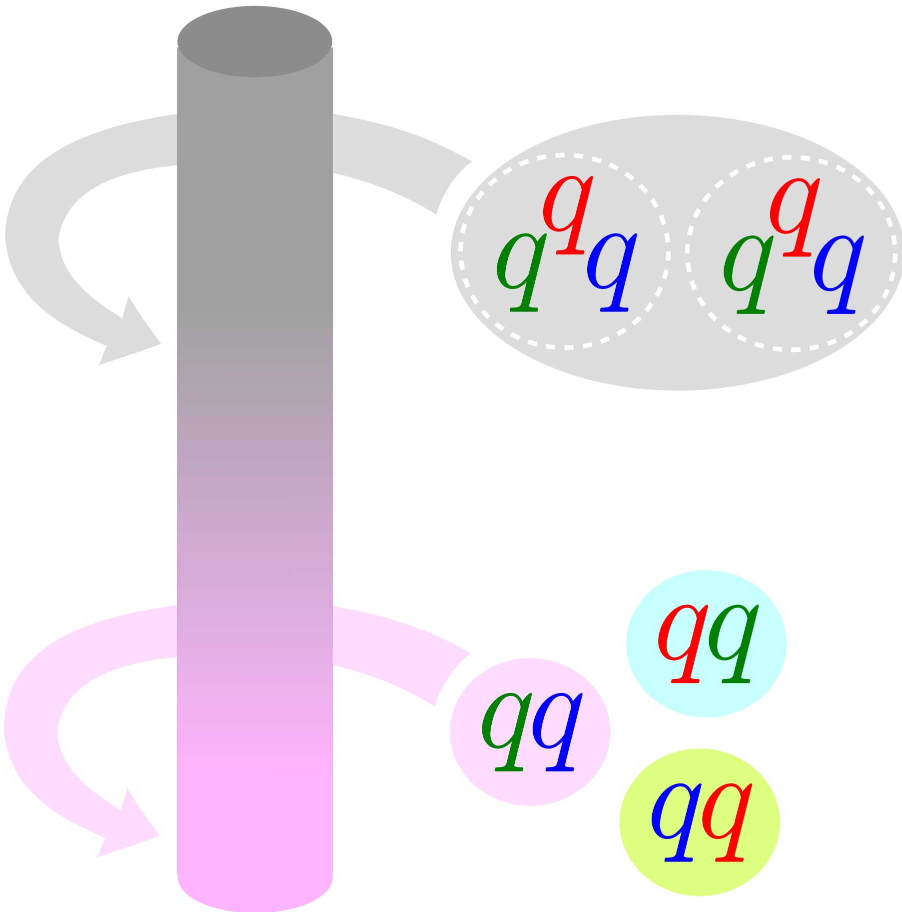
Non-Abelian CFL vortices ~ Hadronic vortices



$$\mu_B = 3\mu_q$$

Vortex Continuity

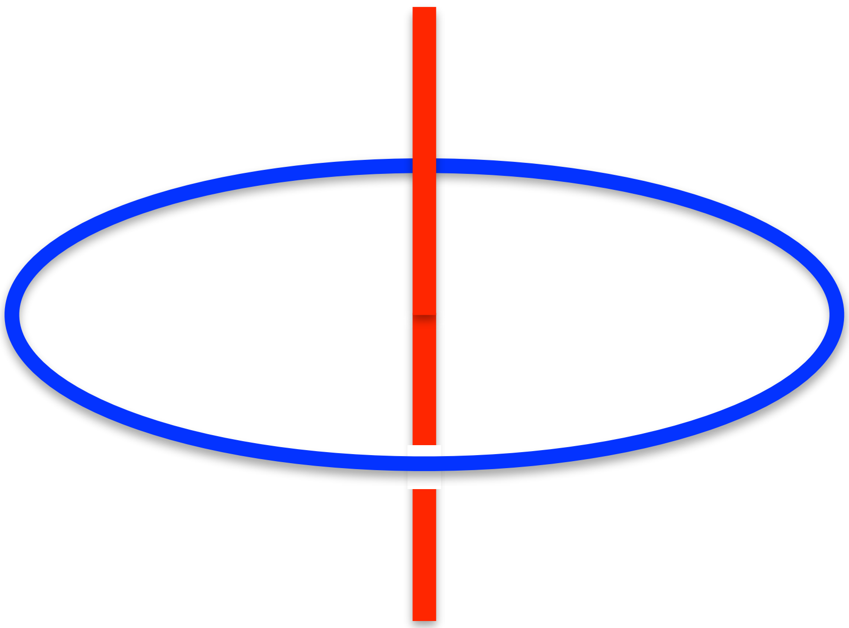
Alford-Baym-Fukushima-Hatsuda-Tachibana (2018)



Vortex Continuity Falsified?

Cherman-Sen-Yaffe (2018)

$$\langle W_3(C)\rangle / \langle W_0(C)\rangle \sim e^{\frac{2\pi i}{3}\nu}$$



Non-abelian vortex solution can be written down explicitly.

"Topological" braiding phase taking Z_3 values discovered!

Wilson line generates 2-form Z_3 symmetry in quark matter

cf. Hirono-Tanizaki (2018)

Vortex Continuity Falsified?

Possible Scenarios

If the braiding phase emerges in the Higgs phase but *not in the confined phase*…. **Cherman-Jacobson-Sen-Yaffe (2020)**

There must be a phase interface (phases distinguished by emergent 2-form symmetry).

If the braiding phase is somehow screened in the Higgs phase and does not emerge in the confined phase….

If the braiding phase emerges both in the Higgs phase and in the confined phase…

There is no manifest distinction and continuity is not falsified yet.

Vortex Continuity Falsified?

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Vortex Continuity Conjecture

If the braiding phase emerges both in the Higgs phase and in the confined phase…

Fujimoto-Fukushima-Hidaka-Hirono (in prep.)

$$\Phi^{(1)} = \Delta_{\mathrm{CFL}} e^{\frac{i}{3}\nu_1\varphi} \begin{pmatrix} e^{\frac{2i}{3}\nu_1\varphi} f(r) & 0 & 0 \\ 0 & e^{-\frac{i}{3}\nu_1\varphi} g(r) & 0 \\ 0 & 0 & e^{-\frac{i}{3}\nu_1\varphi} g(r) \end{pmatrix}$$

U(1) counterpart is needed in the hadronic phase, and a natural candidate is the ω meson.

How the ω meson Wilson loop counts the baryon flux should be (probably?) dictated by the WZW action.

Vortex Continuity Conjecture

If the braiding phase emerges both in the Higgs phase and in the confined phase…

Fujimoto-Fukushima-Hidaka-Hirono (in prep.)

If our scenario is the case, the superfluid vortices in nuclear matter (found in neutron stars) are nontrivially charged by Z_3 generated topologically by the vector meson Wilson loop!?

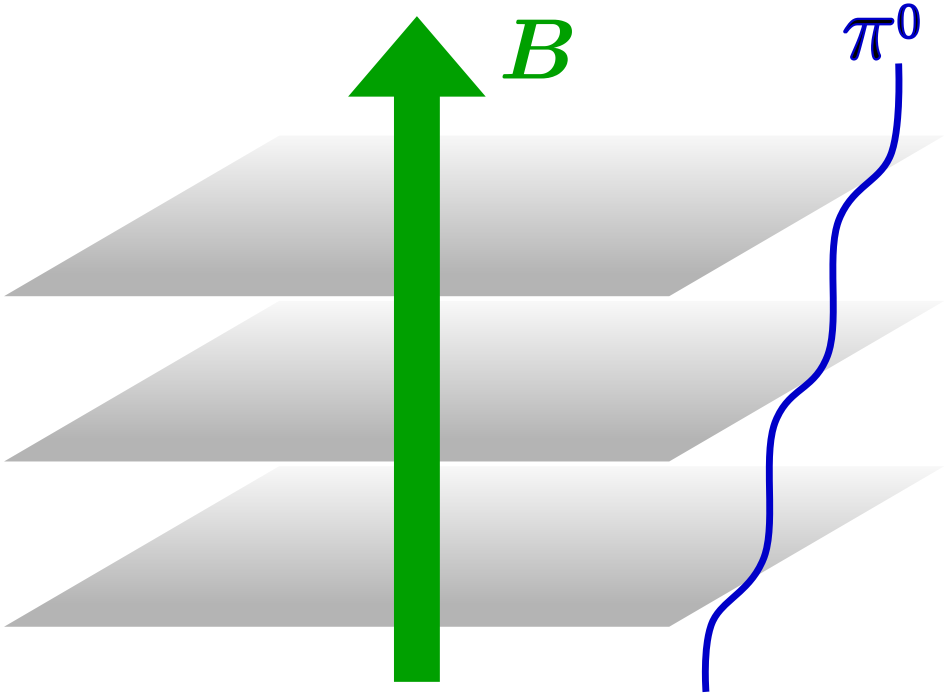
Nuclear matter may be more nontrivial than we may think!

Such nontrivial features might be probed by magnetic field?

Nuclear matter under strong B

Brauner-Yamamoto (2016)

Under strong B $\pi^{\pm}$ are massive and only π^0 remains massless.



π^0 domain-wall layers should be the ground state of nuclear matter.

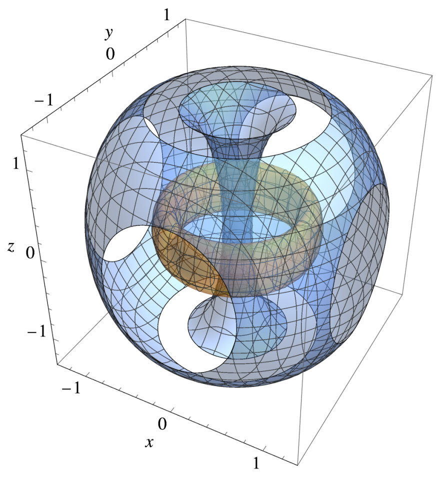
Baryon density is associated with $\pi_1(U(1))$

Single Skyrmion under B

Chen-Fukushima-Qiu (in completion)

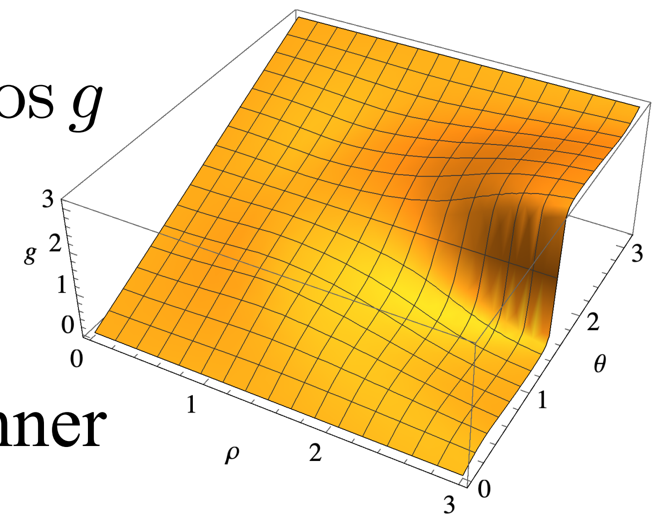
In a Skyrme model the baryon number comes from $\pi_3(SU(2))$

We relaxed the hedgehog Ansatz and studied the deformation.



Topological winding never unwound
(to preserve the baryon number)

$$\pi^0 = f \cos g$$



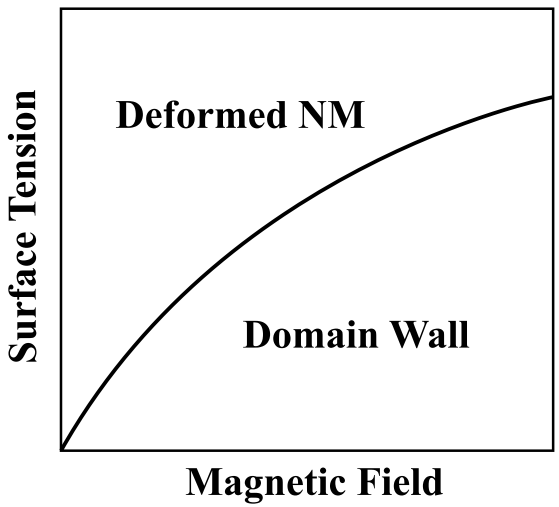
Center keeps hedgehog, but outer is surrounded by π^0 domain wall (whose amplitude is very small)

Skyrme crystal under B

A single Skyrmion can be generalized to a Skyrme crystal by imposing a boundary condition (Klebanov)

We found several consistent boundary conditions and one of them corresponds to the π^0 domain wall!

We are now quantifying the phase diagram: (The criterion to determine the phase boundary is nontrivial)



Topological realizations change at a certain magnetic field. (No continuous deformation)

Take-Home Messages

Phenomenological EoS

- □ Interacting HRG may have validity regions overlapping with a deconfined state of matter.
- □ Excluded volume effects (including van der Waals) are not sufficient but inner structures are crucial.

Perturbative Approach

- □ Convergence is not bad, but the uncertainty appears from the running coupling, which can be improved.

Topological Nuclear Matter

- □ Topological nature of vortices and domain walls in nuclear matter should deserve more investigations!