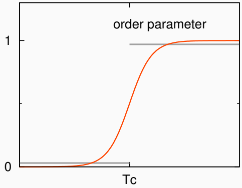
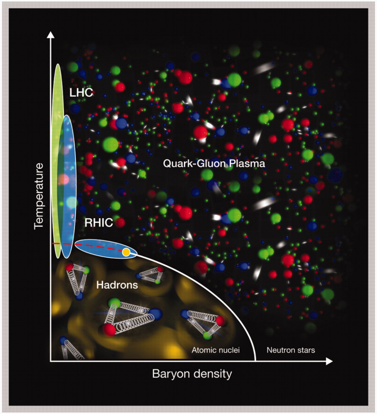
Dynamical treatment of fluctuations

Marlene Nahrgang, Marcus Bluhm, Christoph Herold, Grégoire Pihan

RHIC-Beam Energy Scan Seminar Series
4 May 2021

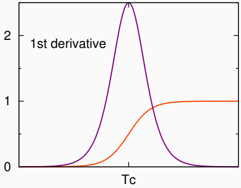
The phase transition in heavy-ion collisions

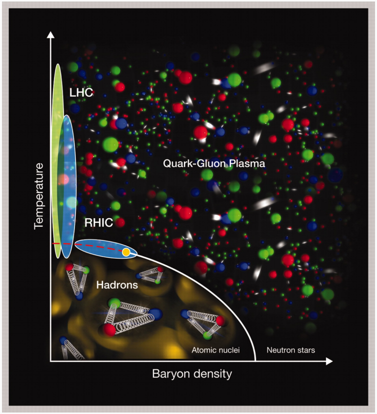




B. Jacak and B. Müller Science 337 (2012)

The phase transition in heavy-ion collisions





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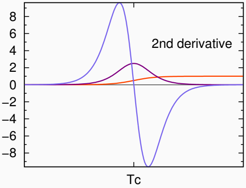

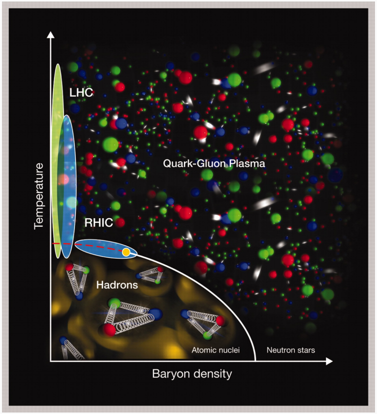

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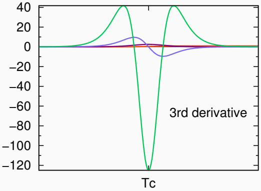

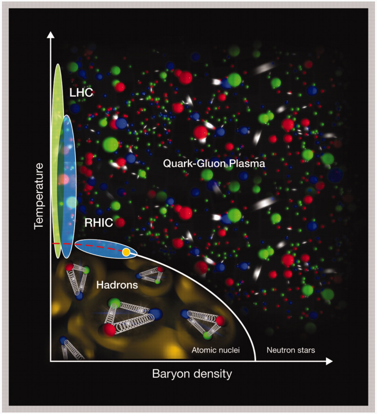

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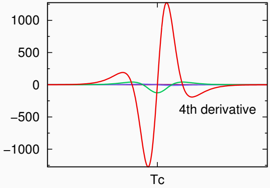
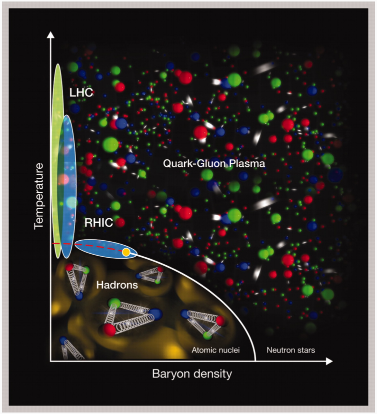


- Derivatives reveal more details!
- Derivatives of thermodynamic quantities (susceptibilities χ) are related to fluctuations!
- To zeroth-order in volume fluctuations:

$$\frac{\chi_2}{\chi_1} = \frac{\sigma^2}{M}, \quad \frac{\chi_3}{\chi_2} = S\sigma, \quad \frac{\chi_4}{\chi_2} = \kappa\sigma^2$$

variance Skewness Kurtosis



B. Jacak and B. Müller Science 337 (2012)

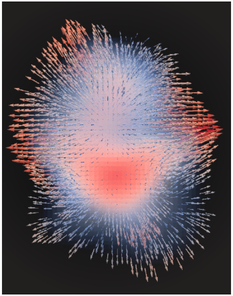
Why is dynamical modeling important?

In a grand-canonical ensemble, the system is

- in thermal equilibrium (= long-lived),
- in equilibrium with a particle heat bath,
- spatially infinite
- and static.

Systems created in heavy-ion collisions are

- short-lived,
- spatially small,
- inhomogeneous,
- and highly dynamical!



plot by H. Petersen, madai.us

Solution: Develop dynamical models to describe the phase transition in heavy-ion collisions!

Event-by-event dynamical modeling allows us in addition to study different particle species, experimental cuts, hadronic final interactions, etc.

EMMI Rapid Reaction Task Force: M.Bluhm et al. NPA 1003 (2020), 2001.08831

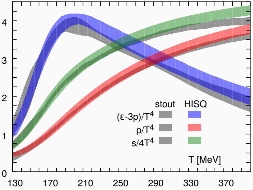
The phase transition in fluid dynamics

- The discovery of RHIC: The QGP is an almost ideal strongly coupled fluid.

  P. Kolb, U. Heinz, QGP (2003)

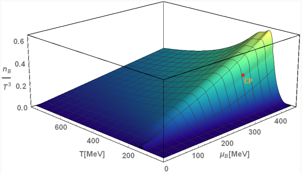
- Including the phase transition in fluid dynamics is easy: just need to know the equation of state!

HotQCD Coll. PRD90 (2014); WB Coll. PLB730 (2014)



EoS at $\mu_B = 0$ from lattice QCD.

P. Parotto et al., PRC101 (2020)



EoS with a 3D Ising critical point

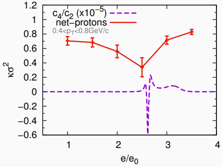
- But fluctuations matter at the phase transition, and including fluctuations in fluid dynamcis is challenging!

K. Murase et al., NPA956 (2016); MN et al., APP 10 (2017); M. Singh et al., NPA982 (2019)

Dynamical effects are very important...

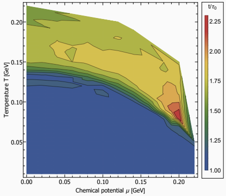
- **At the critical point:** $\xi \to \infty \Rightarrow$ fluctuations of the critical mode diverge!
- Relaxation time $\tau_{\mathrm{rel}} \propto \xi^z$ diverges $\Rightarrow$ critical slowing down!

NχFD



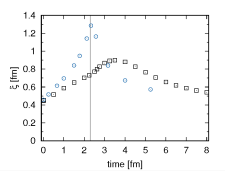
C. Herold, MN, Y. Yan and C. Kobdaj PRC93 (2016) no.2

QCD-assisted transport



M. Bluhm et al., NPA982 (2019)

diffusive n_B dynamics



MN et al., PRD99(2019), PRD102(2020)

$\Rightarrow$ Study fluctuations coupled to a fluid dynamical medium!

Nonequilibrium chiral fluid dynamics (NχFD)

Nonequilibrium chiral fluid dynamics (NχFD)

Propagate the critical mode σ coupled to a fluid dynamical expansion

- Relaxational equation for the critical mode: damping and noise from the interaction with the fermions/fast modes

$$\partial_\mu \partial^\mu \sigma + \frac{\delta V_{\rm eff}(\sigma)}{\delta \sigma} + \eta \partial_t \sigma = \xi$$

- Phenomenological dynamics for the Polyakov-loop

$$\eta_\ell \partial_t \ell T^2 + \frac{\partial V_{\rm eff}(\ell)}{\partial \ell} = \xi_\ell$$

- Fluid dynamical expansion = heat bath, including energy-momentum exchange

$$\partial_\mu T^{\mu\nu}_{\rm fluid} = S^\nu = -\partial_\mu T^{\mu\nu}_\sigma \,, \quad \partial_\mu N^\mu_{\rm q} = 0$$

$\Rightarrow$ includes a stochastic source term!

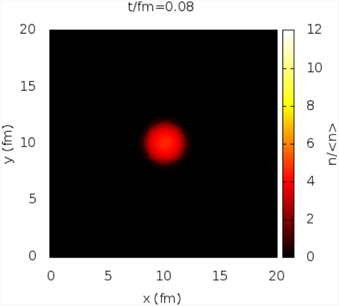
- Nonequilibrium equation of state $p = p(e, \sigma)$

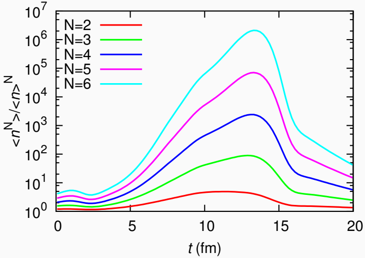
MN, S. Leupold, I. Mishustin, C. Herold, M. Bleicher, PRC 84 (2011); PLB 711 (2012); JPG 40 (2013); C. Herold, MN, I. Mishustin, M. Bleicher PRC 87 (2013); NPA925 (2014), C. Herold, MN, Y. Yan, C. Kobdaj JPG 41 (2014); MN and C. Herold, EPJA 52 (2016); C. Herold, MN, Y. Yan and C. Kobdaj, PRC93 (2016) no.2, PLB790 (2019)

Droplet formation & decay at the QH phase transition

- Chiral effective model with correct low-temperature degrees of freedom in NχFD!

V. Dexheimer, S. Schramm, PRC81 (2010); M. Hempel, V. Dexheimer, S. Schramm, I. Iosilevskiy PRC88 (2013)



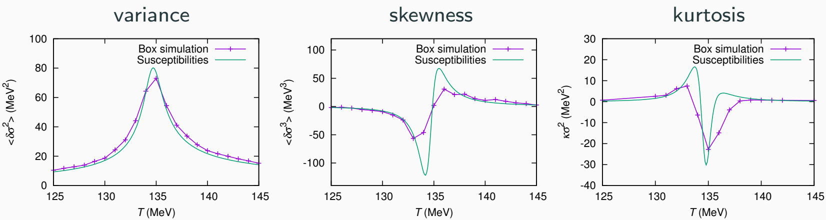


- Droplets of quark density form dynamically at the phase transition.
- Droplets of quark density decay in the hadronic phase due to non-vanishing large pressure

MN and C. Herold, EPJA 52 (2016); cf, but no dynamical fluctuations: J. Steinheimer et al., PRC89 (2014)

Sigma field Fluctuations/Susceptibilities in the critical region

- static, finite-size medium, periodic boundary conditions
- fixed temperature (no back-coupling) at $\mu_q = 100$ MeV

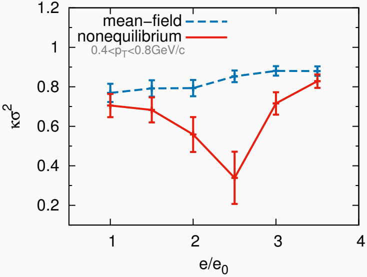


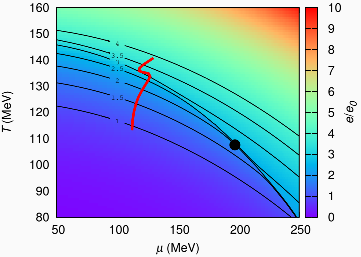
- Shape is well reproduced, some discrepancy for higher-order cumulants (renormalization of the EoS? coarse-graining of the noise?)!

QM 2017 talk by C. Herold

Net-Proton fluctuations near the critical point

- UrQMD initial conditions rescaled to the EoS of the effective model.
- From densities to particles via Cooper-Frye particlization.
- Couple the densities of the sigma field with the fluid dynamical densities





C. Herold, MN, Y. Yan and C. Kobdaj, PRC93 (2016) no.2

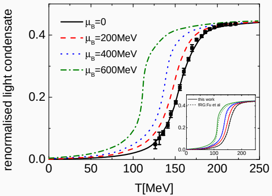
- No non-monotonic behavior in pure mean-field equilibrium calculations.
- Clear signal for criticality in net-proton fluctuations at the transition energy density!
- Overall decreasing trend probably due to net-baryon number conservation cf. MN et al, EPJC72 (2012), 0903.2911.

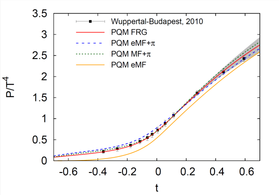
NχFD + FRG $\Rightarrow$ QCD assisted transport

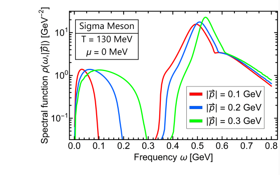
- Include effective potential beyond mean field, momentum dependent equilibrium sigma spectral function $\Rightarrow$ linear response regime of QCD.

First-principle approach to QCD from the Functional Renormalization Group (FRG)

Cyrol, Mitter, Pawlowski, Strodthoff PRD97 (2018)

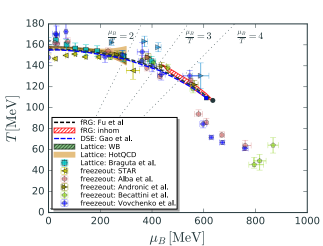






F. Gao, J. Pawlowski, 2010.13705; T. Herbst et al, PLB731 (2014); T. Herbst PRD88 (2013); F. Rennecke, J. Pawlowski, N. Wink

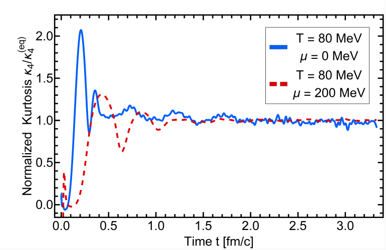
- Excellent description of phase structure at vanishing chemical potential.
- Phase structure qualitatively similar to the conjectured QCD phase diagram.
- Obtain spectral functions from analytical continuation.



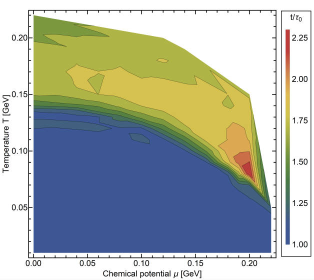
M. Bluhm et al., NPA982 (2019)

Transport equation: $\frac{\delta\Gamma}{\delta\sigma} = \xi$, where $\{\Re\Gamma^{(2)}_\sigma(\omega,\vec{p}), \Im\Gamma^{(2)}_\sigma(\omega,\vec{p}), U\} \in \Gamma$

Normalized Kurtosis:



Equilibration time:



- Critical end point and the phase structure are clearly identifiable.
- Critical slowing down in the vicinity of the critical point, but no dramatic enhancement of τ_{relax} in a dynamic setup!

Diffusive dynamics of net-baryon density fluctuations

Diffusive dynamics of the net-baryon density

$$\partial_\mu N_B^\mu = 0 \quad \text{net-baryon charge conservation}$$

The diffusive dynamics follows the minimized free energy $\mathcal{F}$:

$$\partial_t n_B(t,x) = \kappa \nabla^2 \left(\frac{\delta \mathcal{F}[n_B]}{\delta n_B} \right) + \nabla \mathbf{J}(\mathbf{t},\mathbf{x})$$

To study intrinsic fluctuations include a stochastic current:

$$J(t,x) = \sqrt{2T\kappa}\,\zeta(t,x), \quad \kappa = \frac{Dn_c}{T}$$

$\rightarrow$ $\zeta(t,x)$ is Gaussian and uncorrelated (white noise):

$$\langle \zeta(x,t)\zeta(0,0) \rangle = \delta(x)\delta(t)$$

$\Rightarrow$ respects the fluctuation-dissipation theorem:

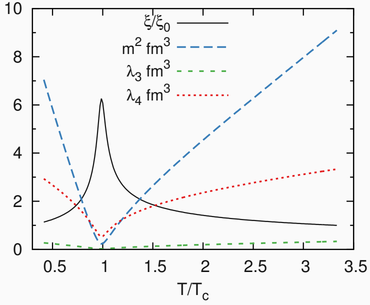
$$P_{\mathrm{eq}}[n_B] = \tfrac{1}{\mathcal{Z}} \exp\left(-\mathcal{F}[n_B]/T\right)$$

$$\mathcal{F}[n_B] = T\int \mathrm{d}^3 r \left(\frac{m^2}{2n_c^2}\Delta n_B^2 + \frac{K}{2n_c^2}(\nabla \Delta n_B)^2 + \frac{\lambda_3}{3n_c^3}\Delta n_B^3 + \frac{\lambda_4}{4n_c^4}\Delta n_B^4 + \frac{\lambda_6}{6n_c^6}\Delta n_B^6 \right)$$

The couplings depend on temperature via the correlation length $\xi(T)$:

$$m^2 = 1/(\xi_0 \xi^2)$$
$$K = \tilde{K}/\xi_0$$
$$\lambda_3 = n_c\, \tilde{\lambda}_3\, (\xi/\xi_0)^{-3/2}$$
$$\lambda_4 = n_c\, \tilde{\lambda}_4\, (\xi/\xi_0)^{-1}$$
$$\lambda_6 = n_c\, \tilde{\lambda}_6$$

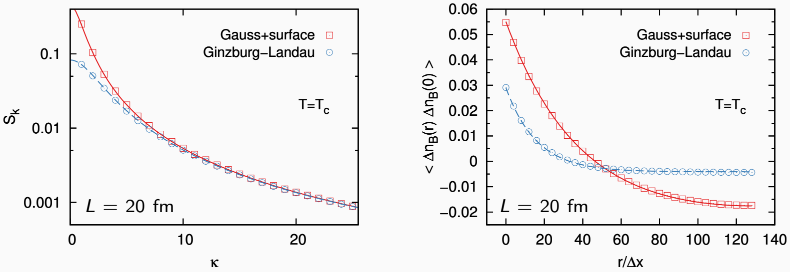
M. Tsypin PRL73 (1994); PRB55 (1997)



in this Fig.: $\tilde{\lambda}_3 = 1$, $\tilde{\lambda}_4 = 10$

parameter choice: $\Delta n_B = n_B - n_c$
$\xi_0 \sim 0.5$ fm, $T_c = 0.15$ GeV, $n_c = 1/3$ fm^{-3}
$K = 1/\xi_0$ (surface tension)
$\tilde{\lambda}_3$, $\tilde{\lambda}_4$, $\tilde{\lambda}_6$ (universal, but mapping to QCD)

Ginzburg-Landau model in equilibrium



- Important for any algorithm of fluctuations: test fluctuations observables vs analytical expectations (in the appropriate limit)!
- Here: structure factor and the equal-time correlation function are well reproduced, discretization and baryon conservation effects under control.
- Nonlinear interactions reduce S_k for long-wavelength fluctuations!
- At the level of 2-point correlations, results of Ginzburg-Landau model can be described by a modified m^2 coupling for the Gauss+surface model.

Scaling of equilibrium cumulants

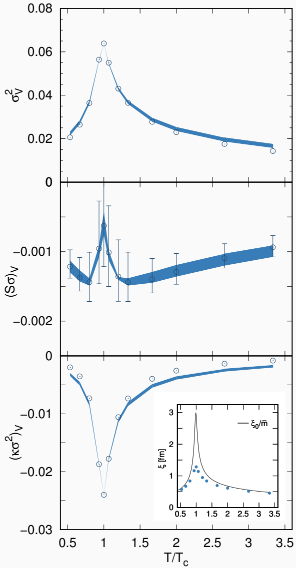
- Expected scaling in an infinite system ($\xi \ll V$): M. Stephanov, PRL102 (2009)

  $$\sigma_V^2 \propto \xi^2, \quad (S\sigma)_V \propto \xi^{2.5}, \quad (\kappa\sigma^2)_V \propto \xi^5$$

- Here, a finite system with exact baryon conservation ($\xi \lesssim V$)! Can be systematically studied in $\xi/V \Rightarrow$ affects equilibrium scaling!

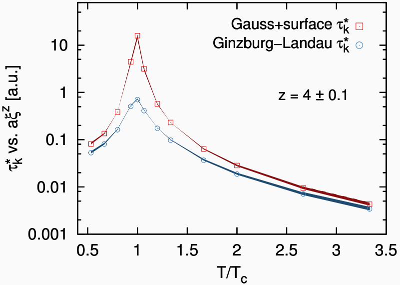
- E.g. for the skewness terms $\propto \lambda_3\lambda_4$ and $\propto \lambda_3\lambda_6$ contribute with opposite sign.

  $$\sigma_V^2 \propto \xi^{1.3\pm0.05}$$
  $$(S\sigma)_V \propto -\#\xi^{1.47\pm0.05} + \#\xi^{2.4\pm0.05}$$
  $$(\kappa\sigma^2)_V \propto \xi^{2.5\pm0.1}$$



Dynamical critical scaling

- Dynamical structure factor for Gaussian model in continuum:
  $S(k,t) = S(k)\exp\left(-t/\tau_k\right)$ with $\tau_k^{-1} = \frac{Dm^2}{n_c}\left(1 + \frac{K}{m^2}k^2\right)k^2$
- Analyze ξ-dependence of relaxation time for modes with $k^* = 1/\xi$:



for both models: $\tau_k^* = a\,\xi^z$ with

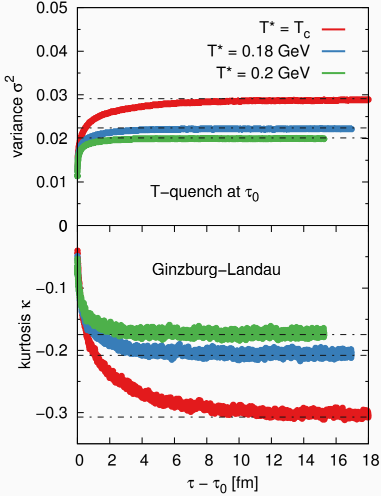
$$z = 4 \pm 0.1$$

$$a = \frac{n_c \xi_0}{D(1+\tilde{K})}$$

$\Rightarrow$ Simulations reproduce scaling of model B!

(for the dynamics of a HIC, couple to fluctuations in the momentum density
$\Rightarrow$ model H Hohenberg, Halperin, Rev.Mod.Phys.49 (1977))

Dynamics: temperature quench and equilibration

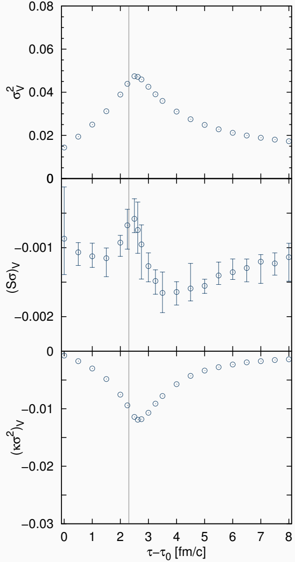


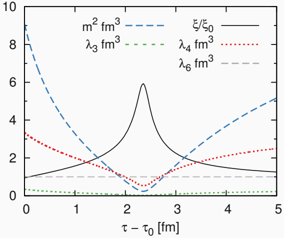
- temperature quench: at τ_0 temperature drops from $T_0 = 0.5$ GeV to T^*
- fast initial relaxation
- variance approaches equilibrium value faster than kurtosis
- long relaxation times near T_c

B. Berdnikov, K. Rajagopal PRD61 (2000)

Dynamics: time evolution of critical fluctuations

For a Bjorken-like temperature evolution:





- shift of extrema for variance/kurtosis (retardation effects) to later times corresponding to $T(\tau) < T_c$
- |extremal values| in dyn simulations < equilibrium values (nonequilibrium effects):

$$(\sigma_V^2)^{\mathrm{max}}_{\mathrm{dyn}} \approx 0.75\,(\sigma_V^2)^{\mathrm{max}}_{\mathrm{eq}}$$
$$((\kappa\sigma^2)_V)^{\mathrm{min}}_{\mathrm{dyn}} \approx 0.5\,(\kappa\sigma_V^2)^{\mathrm{min}}_{\mathrm{eq}}$$

- expected behavior with varying D and c_s^2 (expansion rate)

MN, M. Bluhm, T. Schaefer, S. Bass, PRD99 (2019), MN, M. Bluhm, PRD102 (2020)

Diffusive dynamics of net-baryon density fluctuations in expanding systems

Grégoire Pihan, M. Bluhm, M. Kitazawa, T. Sami, MN, in preparation

From Cartesian to Milne coordinates

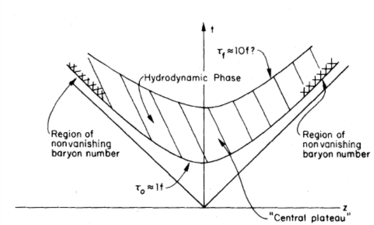
Choose an appropriate coordinate system for the geometry of a HIC

$$\tau = \sqrt{t^2 - z^2}$$

$$y = \frac{1}{2}\ln\left(\frac{t+z}{t-z}\right)$$

$$\frac{n(y,\tau)}{\tau} = n(x,t)$$

fluctuations studied in an expanding background, e.g.: J. Kapusta et al, PRC85 (2012) Y. Akamatsu et al, PRC95 (2017), M. Martinez et al, PRC99 (2019),...



J. Bjorken, PRD27 (1983)

The nonlinear stochastic diffusion equation transforms as:

$$\partial_\tau n_B = \frac{Dm^2}{n_c\tau}\partial_y^2 n_B - \frac{DK}{n_c\tau^3}\partial_y^4 n_B + \sum_{i=3,4,6}\frac{D\lambda_i}{n_c^{i-1}}\frac{1}{\tau}\partial_y^2 n_B^{i-1} + \partial_y\zeta$$

$$\langle \zeta^y(Y)\zeta^y(Y')\rangle = \frac{2Dn_c}{\tau}\delta^2(Y-Y')$$

in Gauss limit: M. Sakaida, M. Asakawa, H. Fuji, M. Kitazawa, PRC95 (2017);

nonlinear (only critical): M. Kitazawa, G. Pihan, N. Touroux, M. Bluhm, M. Nahrgang, NPA1005 (2021)

Singular and regular susceptibilities

- Parametrize the susceptibilities $\chi_2(\tau)$ and $\chi_4(\tau)$ with a regular part using the argument in M. Asakawa, U. Heinz, B. Muller, PRL85 (2000)

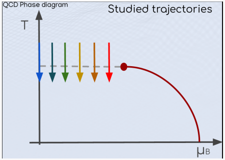
$$\chi_n(\tau) = \frac{\langle \Delta N_B^n \rangle}{S}\Big|_{\mathrm{QGP/HRG}} = \frac{\chi_{\mathrm{B}}^n}{s/T^3}\Big|_{\mathrm{QGP/HRG}}$$

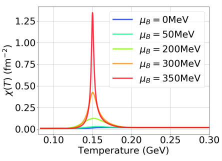
  with χ_{B}^n and the entropy density fixed to lattice results at $T = 280$ MeV for the QGP and $T = 130$ MeV for the HRG, match via a tanh function.
- Couple with the singular contribution (3D Ising as before) via

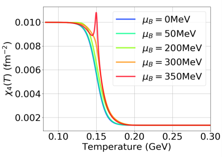
$$\chi_n(T) = \chi_n^{sing}(T) + \chi_n^{reg}(T)$$

- Match to the coefficients in the expansion of the free energy density functional.

$$\chi_n(\tau) = \tau \left(\frac{\delta^n \mathcal{F}}{\delta n_B^n}\Big|_{\Delta n_B = 0} \right)^{-1}$$

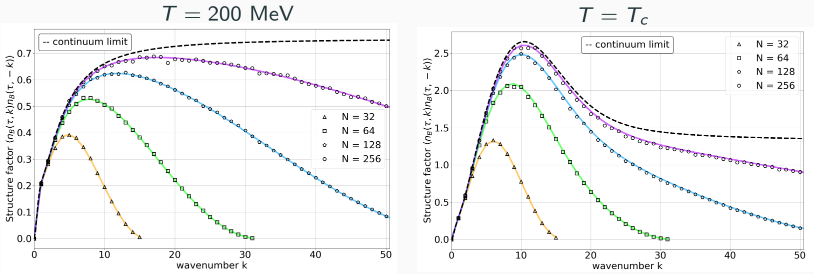






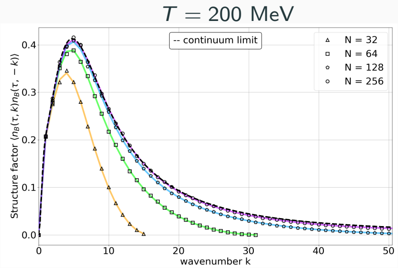
- Investigate several trajectories in the QCD phase diagram.

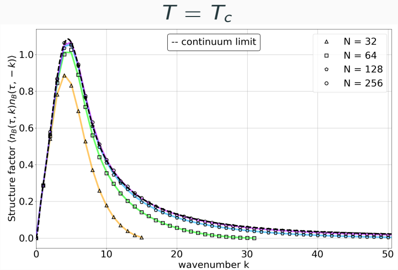
Benchmarking in Gauss model: structure factor



- Numerics in perfect agreement with discretized analytical results.
- Approach to continuum as resolution is increased.
- Lower wavenumbers well described with the maximal resolution chosen for this work.
- Enhancement of fluctuations with low wavenumbers at T_c.

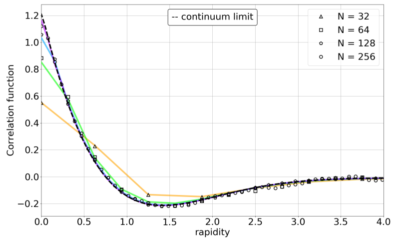
Benchmarking in Gauss+surface model: structure factor



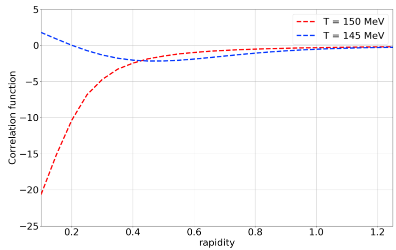


- Numerics in perfect agreement with discretized analytical results.
- Approach to continuum as resolution is increased.
- **All** wavenumbers well described with the maximal resolution chosen for this work, **perfect reproduction of continuum results**.
- Enhancement of fluctuations with low wavenumbers at T_c.

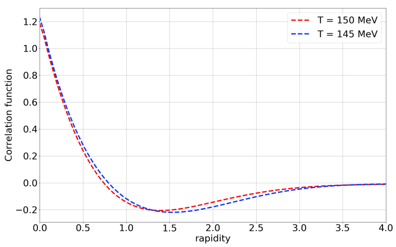
Benchmarking in Gaussian models: correlation function



Gauss:



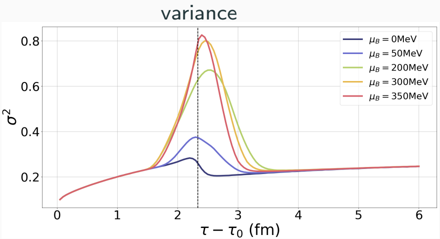
Gauss+surface:

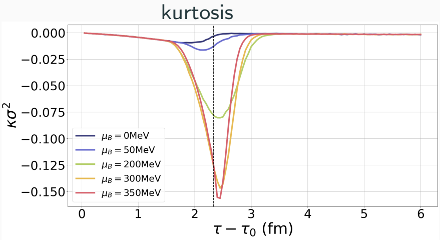


- Numerics in perfect agreement with discretized analytical results.
- Qualitatively different shape of the correlation function in Gauss and Gauss+surface models.
- Strong signal of criticality washed out in Gauss+surface model.

Time evolution of variance and kurtosis

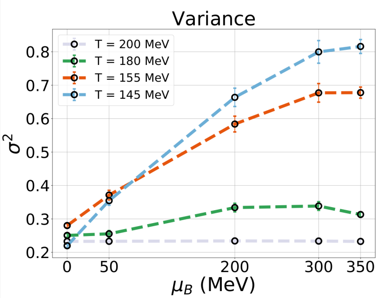
- Include the fourth-order coupling.
- Variance and kurtosis show the critical point signal for trajectories at larger μ_B.
- Quickly decrease to hadronic values for $T > T_c$.
- Final values depend strongly on the freeze-out temperature.

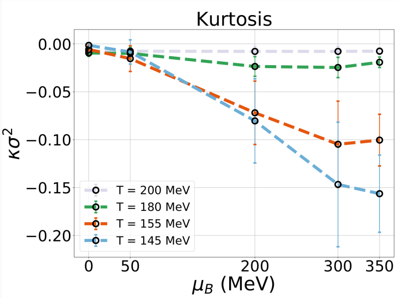




Variance and kurtosis at freeze-out

- As the trajectory closest to the critical point is approached the critical signal in the variance and kurtosis increases for temperatures around T_c.
- Despite the rapid expansion the critical signal survives if the system freezes-out close to T_c.



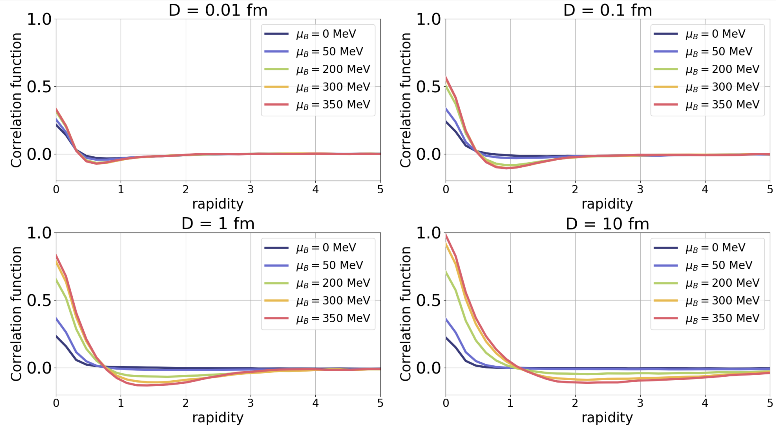


Dependence on the diffusion length: correlation function

As the diffusion length D is increased, diffusion wins over expansion:

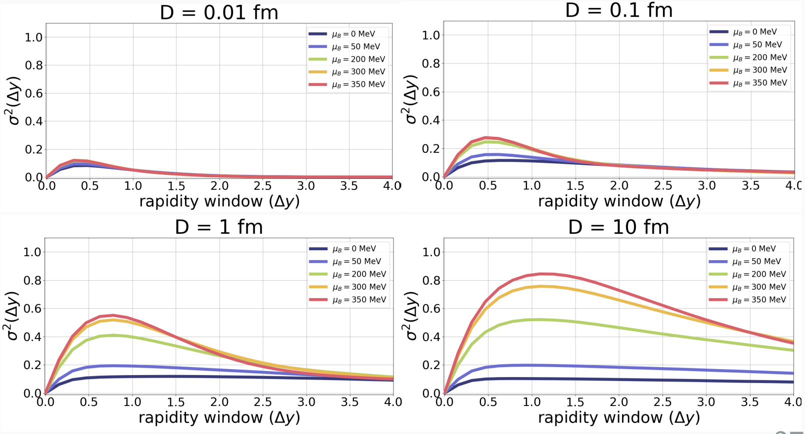
$\Rightarrow$ Fluctuations are balanced over larger and larger distances.

$\Rightarrow$ The correlation function ressembles the equilibrium CF in a static system.



Dependence on the diffusion length: integrated variance

- Take the variance over $\Delta y >$ the numerical resolution.
- Increases as D is increased, maximum moves to larger Δy.
- For trajectories farther away from the critical point this integrated variance quickly saturates.



Conclusions

Many promising approaches to treating the dynamics of critical fluctuations!

Ongoing work (with Master and PhD students):

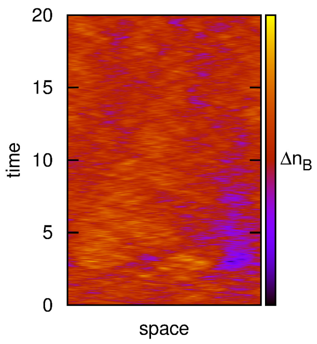
NχFD:

- Renormalization due to lattice spacing important? with Nadine Attieh
- Use of realistic EoS and damping coefficients $\rightarrow$ FRG input

Stochastic net-baryon diffusion:

- Coupling to energy- and momentum density with Grégoire Pihan
- Renormalization in 3d with Nathan Touroux

Fluctuating Dissipative Fluid Dynamics with Nathan Touroux and Grégoire Pihan



Thanks to: