S.P. & C.Plumberg PRC(2019)
S.P. & C.Plumberg PRC(2018)
S.P. PRC (2020)
S.P. & R.Steinhorst, PRC (2020)
S.P. & C.Plumberg, submitted to PRC (2020)

Extracting the Diffusivity of the QGP

Scott Pratt

Department of Physics & Astronomy …

Properties of the QGP

1. Eq. of State
2. Chemistry (charge fluctuations)
3. Chiral Symmetry Restoration

Transport Coefficients

4. Viscosity (shear & bulk)
5. Diffusivity & Conductivity (light / heavy quark)
6. Electromagnetic Opacity & Emissivity
7. Gluonic Opacity and Emissivity (jet quenching)

Properties of the QGP

1. Eq. of State
2. Chemistry (charge fluctuations)
3. Chiral Symmetry Restoration

Transport Coefficients

experimental progress

4. Viscosity (shear & bulk)
5. Diffusivity & Conductivity (light / heavy quark)
6. Electromagnetic Opacity & Emissivity
7. Gluonic Opacity and Emissivity (jet quenching)

GOAL: Determine diffusivity / conductivity of light quarks

Properties of the QGP

1. Eq. of State
2. Chemistry (charge fluctuations)
3. Chiral Symmetry Restoration

Charge balance functions are principal tool

Transport Coefficients

4. Viscosity (shear & bulk)
5. Diffusivity & Conductivity (light / heavy quark)
6. Electromagnetic Opacity & Emissivity
7. Gluonic Opacity and Emissivity (jet quenching)

Charge balance functions also important for:

- CME background
- Background for fluctuations for phase transitions

Definition of Diffusivity

$$\vec{j}_a = -D_{ab} \nabla \rho_b,$$

3x3 matrix (colors)

$$= -\sigma_{ab} \nabla (\mu_b / T),$$

$$\sigma = \chi D,$$

$$\chi_{ab} = \langle \delta Q_a \delta Q_b \rangle / V = \partial \rho_a / \partial (\mu_b / T)$$

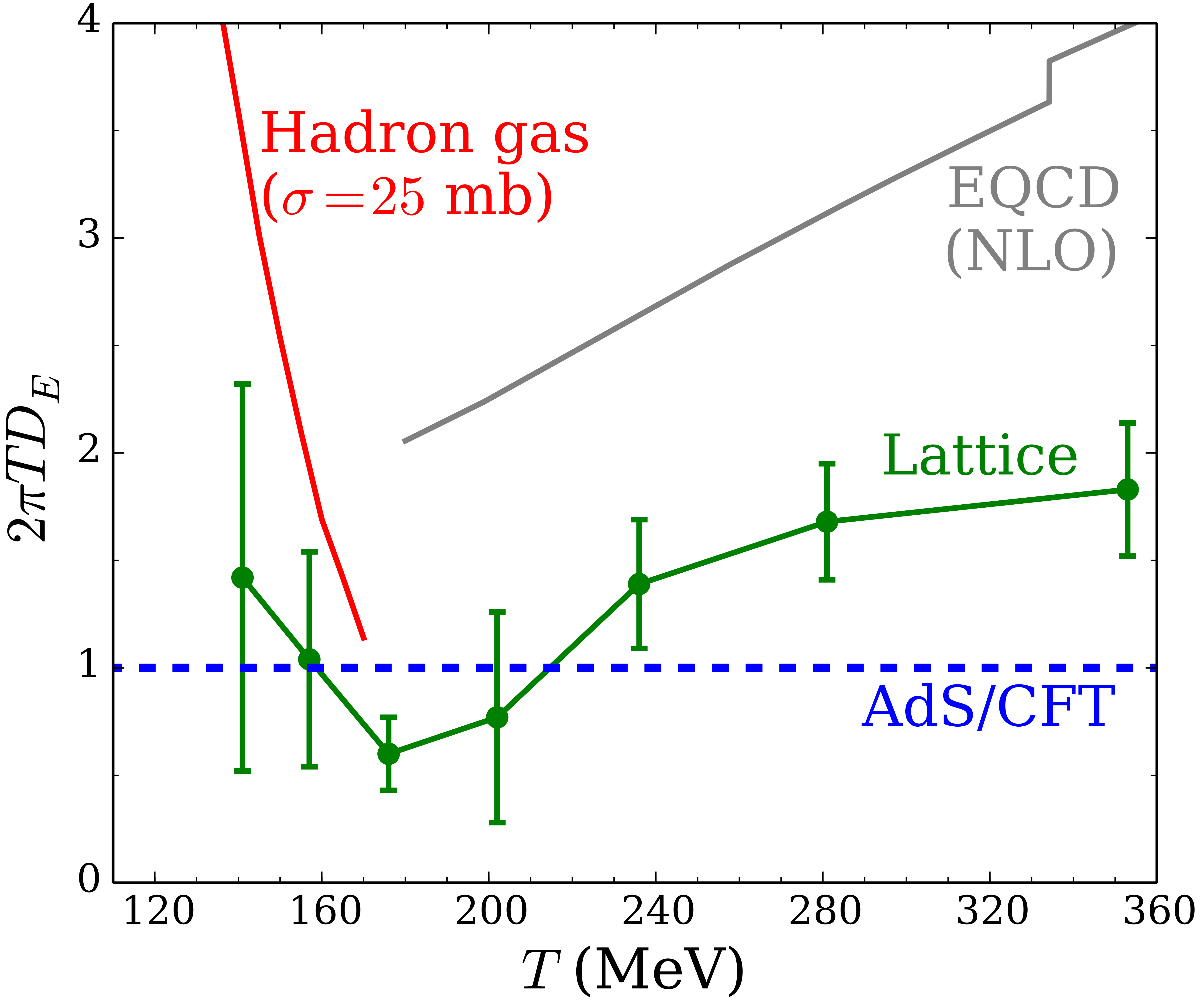
susceptibility

Kubo Relation

$$\sigma_{ab} = \frac{1}{2T} \int d^4x \, \langle \{ j_a(0), j_b(x) \} \rangle$$

difficult for lattice gauge theory

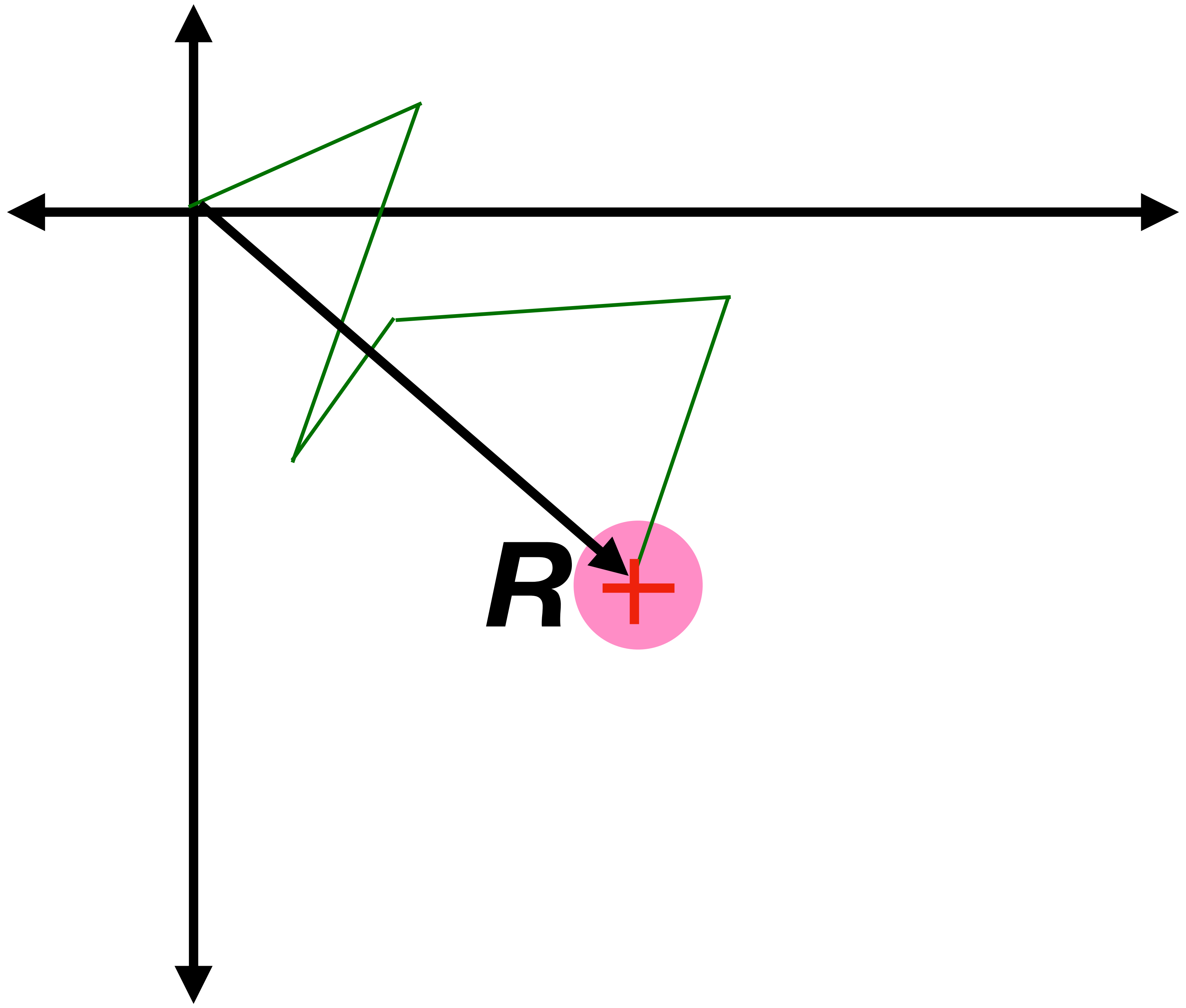
No Clear Consensus



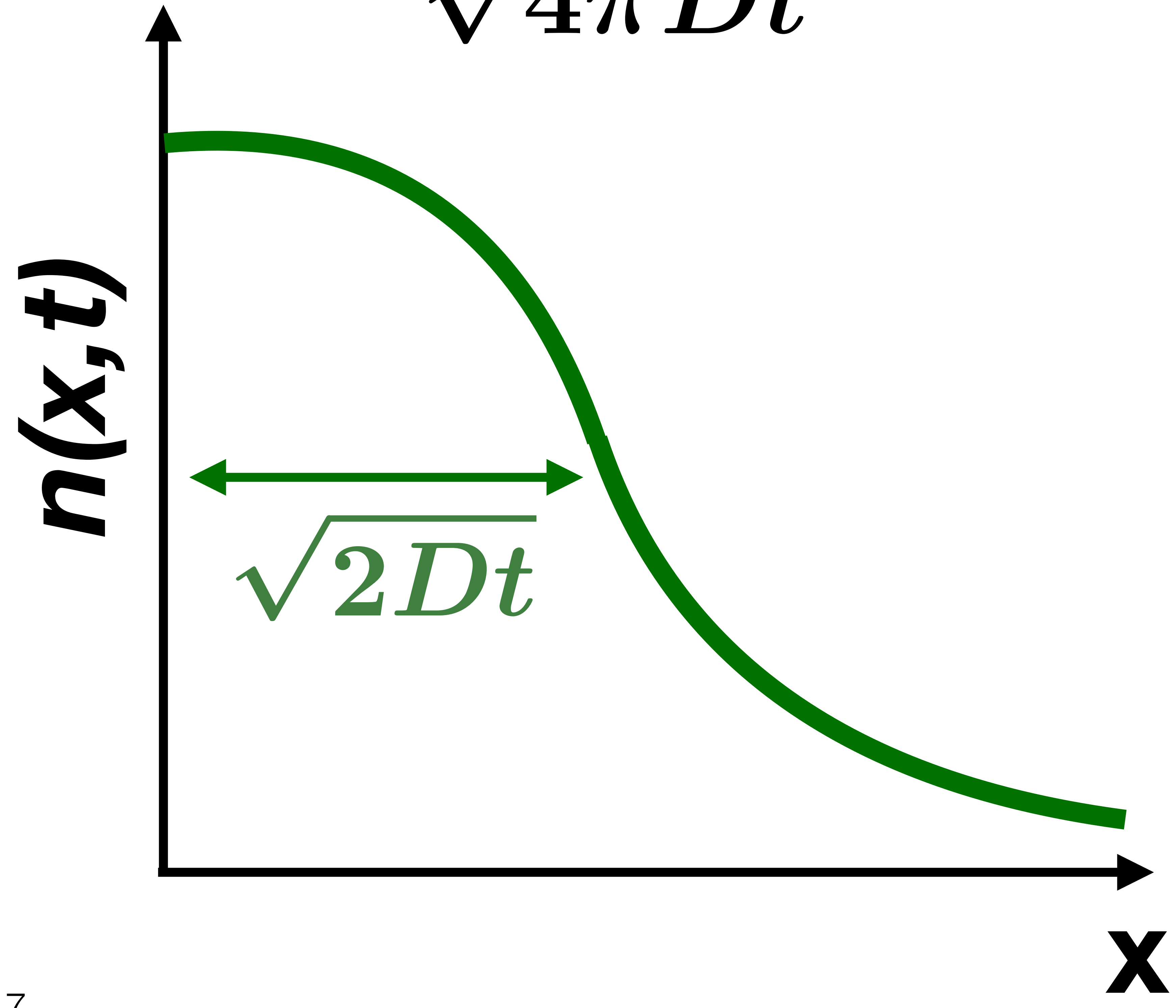
G.Aarts et al, JHEP (2015)
J.Ghiglieri et al, JHEP (2018)
G.Policastro et al, JHEP (2002)

Measuring Diffusivity

$$\partial_t n(\vec{r},t) - D\nabla^2 n(\vec{r},t) = S(t,\vec{r}) = \delta(t)\delta(\vec{r})$$

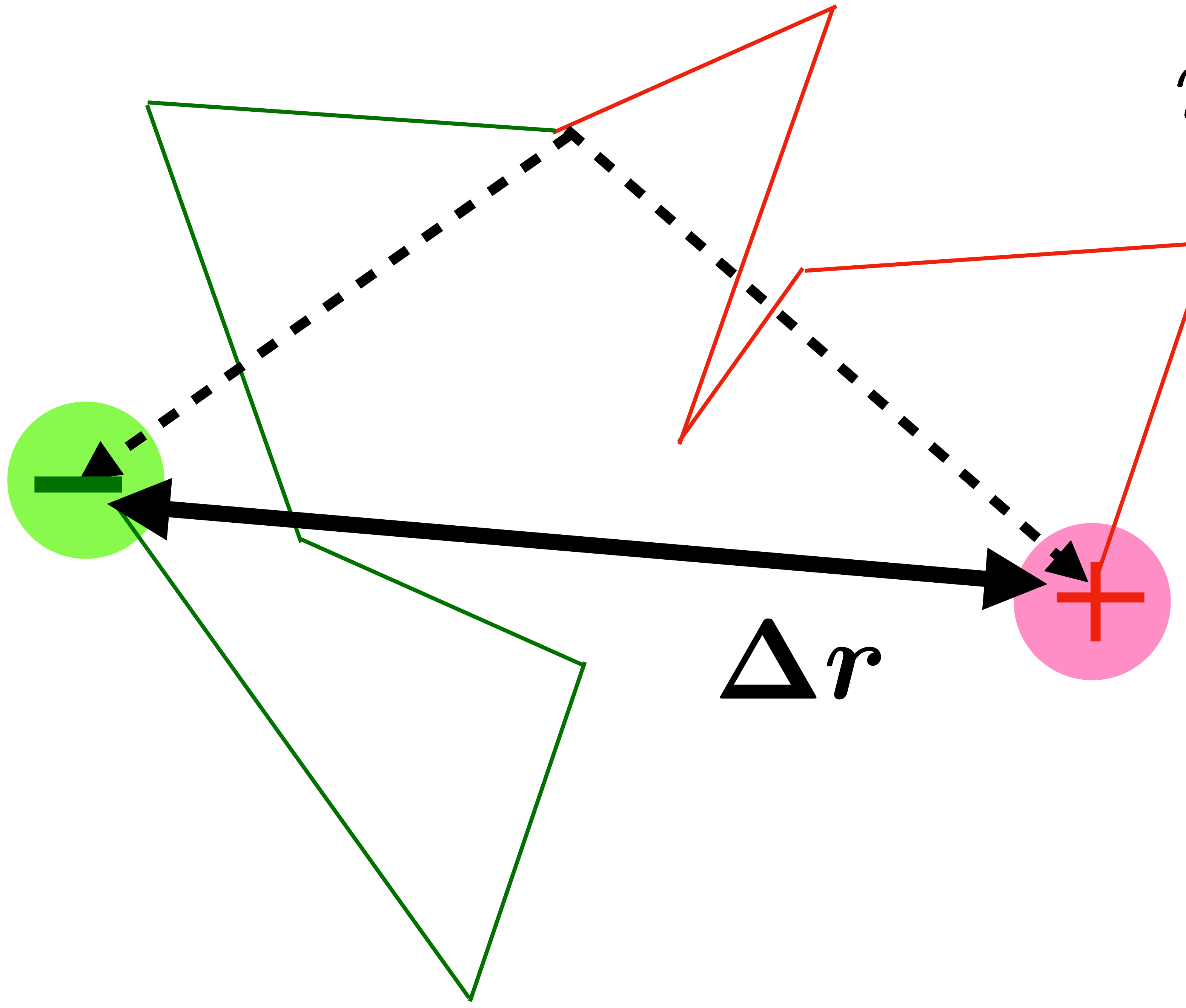


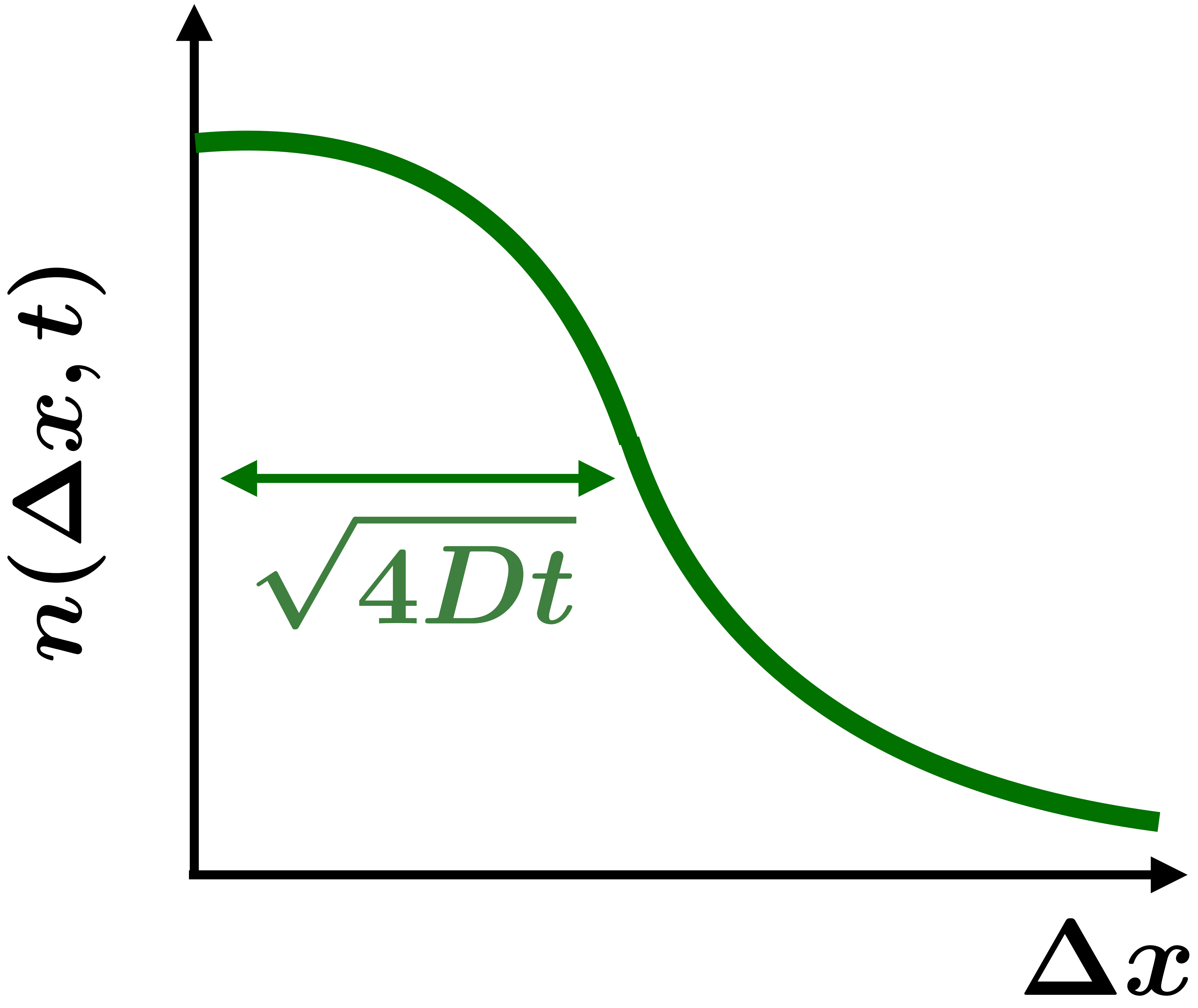
$$n(\vec{r},t) = \frac{1}{\sqrt{4\pi Dt}} e^{-r^2/4Dt}$$



Don't know origin?

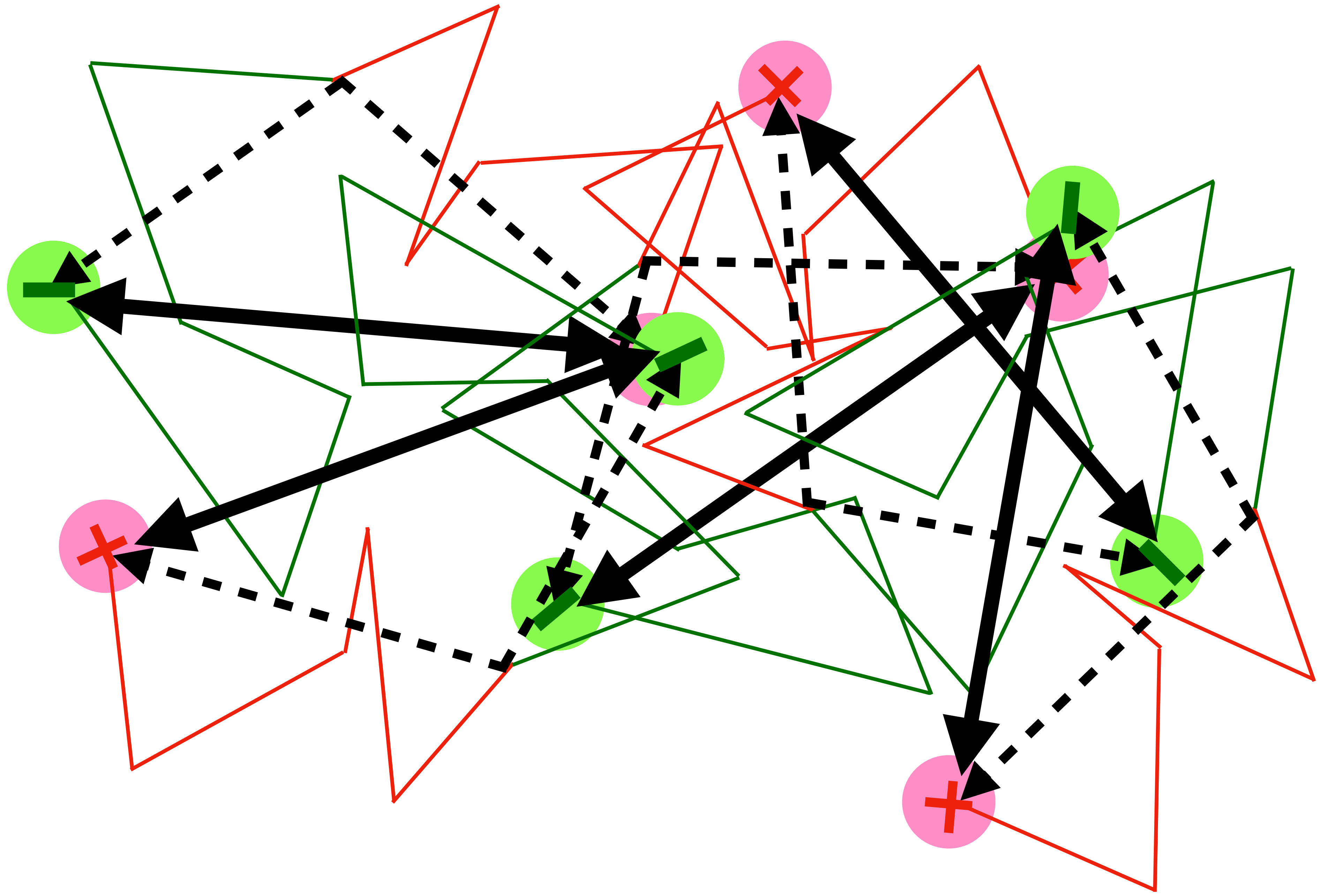
$$n(\Delta\vec{r},t) = \frac{1}{\sqrt{4\pi Dt}}e^{-\Delta r^2/8Dt}$$





Still must know time!

Many pairs?

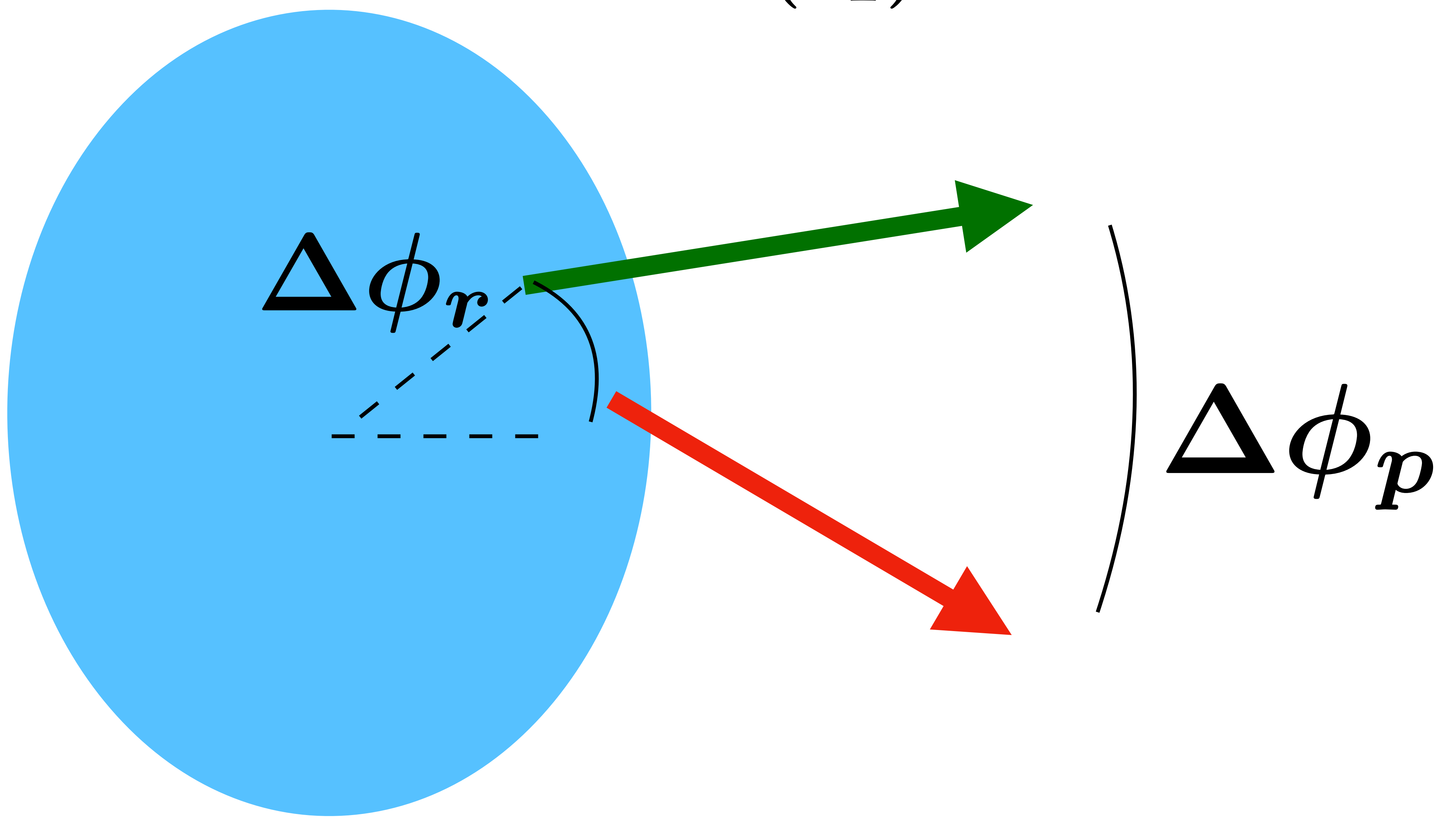


Construct Balance Function (like-sign subtraction)

$$B(r_2|r_1) \equiv \frac{N_{+-}(r_1,r_2) - N_{++}(r_1,r_2)}{2N_+(r_1)} + \frac{N_{-+}(r_1,r_2) - N_{--}(r_1,r_2)}{2N_-(r_1)}$$

Can't measure positions?

$$B(r_2|r_1) \equiv \frac{N_{+-}(r_1,r_2) - N_{++}(r_1,r_2)}{2N_+(r_1)} + \frac{N_{-+}(r_1,r_2) - N_{--}(r_1,r_2)}{2N_-(r_1)}$$



$$\Delta\vec{r} \rightarrow \Delta\vec{p}$$

$$\Delta\vec{p} \rightarrow \Delta\phi, \Delta y, \Delta\eta, M_{\rm inv}, Q_{\rm inv}, (Q_{\rm out}, Q_{\rm side}, Q_{\rm long})$$

mapping relies on collective flow

thermally smearing of spatial info

Define C'_{ab} (ignore self-correlation)

$$\begin{aligned} C_{ab}(\vec{r}_1,\vec{r}_2) &= \langle \delta\rho_a(\vec{r}_1)\delta\rho_b(\vec{r}_2)\rangle \\ &= \chi_{ab}\delta(\vec{r}_1-\vec{r}_2)+C'_{ab}(\vec{r}_1,\vec{r}_2), \end{aligned}$$

C'_{ab} is balance function numerator

charge conservation:

$$0 = \int d^3r_1 \; [C'_{ab}(\vec{r}_1,\vec{r}_2)+\chi_{ab}(\vec{r}_1)\delta(\vec{r}_1-\vec{r}_2)]$$

χ_{ab} from lattice for local chemical equilibrium

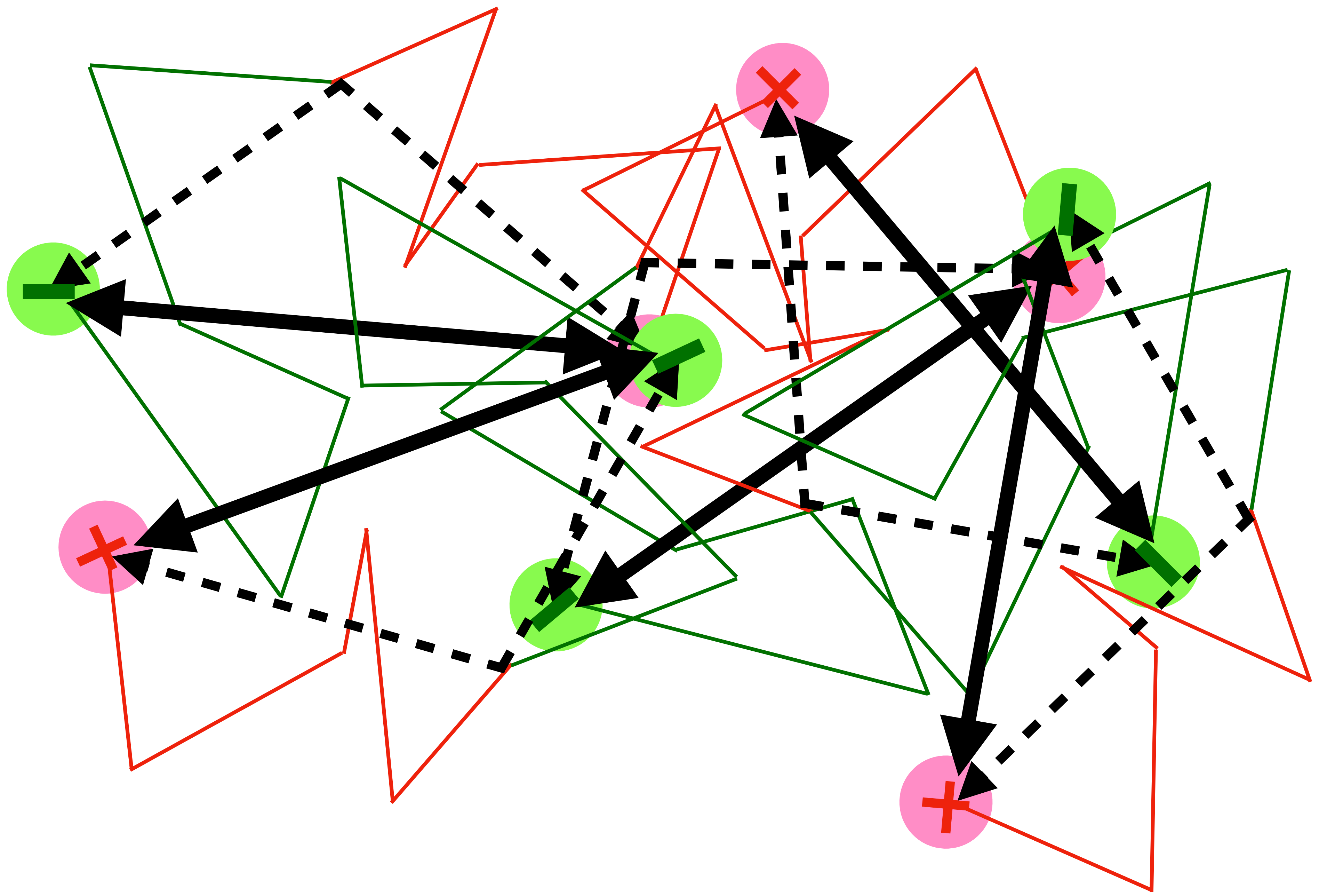
(could be set dynamically)

Source Function for C'_{ab}

$$(\partial_t - D_1 \nabla_1^2 - D_2 \nabla_2^2) C'_{ab}(t, \vec{r}_1, \vec{r}_2) = S_{ab}(t, \vec{r}_1, \vec{r}_2)$$

$$S_{ab}(t, r_1, r_2) = -\delta(\vec{r}_1 - \vec{r}_2)[\partial_t + \nabla \cdot \vec{v} + \vec{v} \cdot \nabla]\chi_{ab}(t, \vec{r})$$

$$\approx s D_t \frac{\chi_{ab}(t, \vec{r})}{s}$$



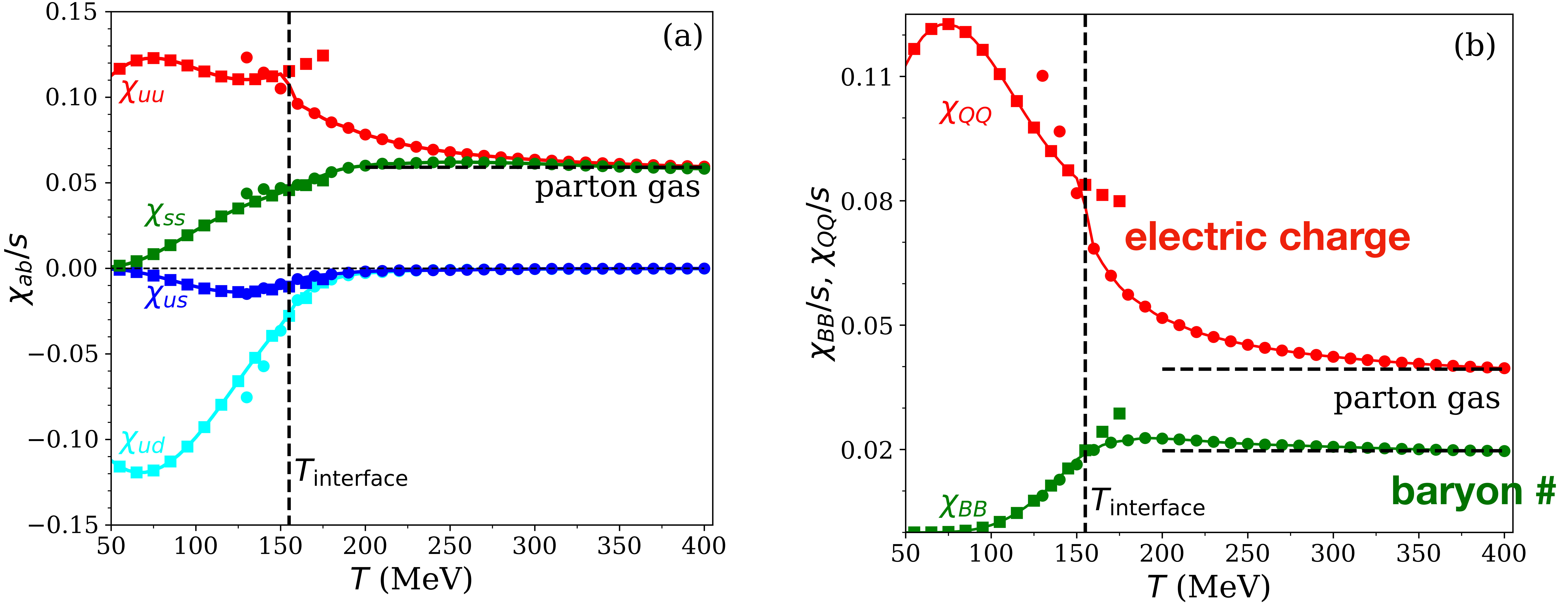
represent correlation by weighted pairs (Monte Carlo) undergoing random walk

Susceptibility

For hadron gas: $\chi_{ab} = \sum_h n_h q_{ha} q_{hb}$

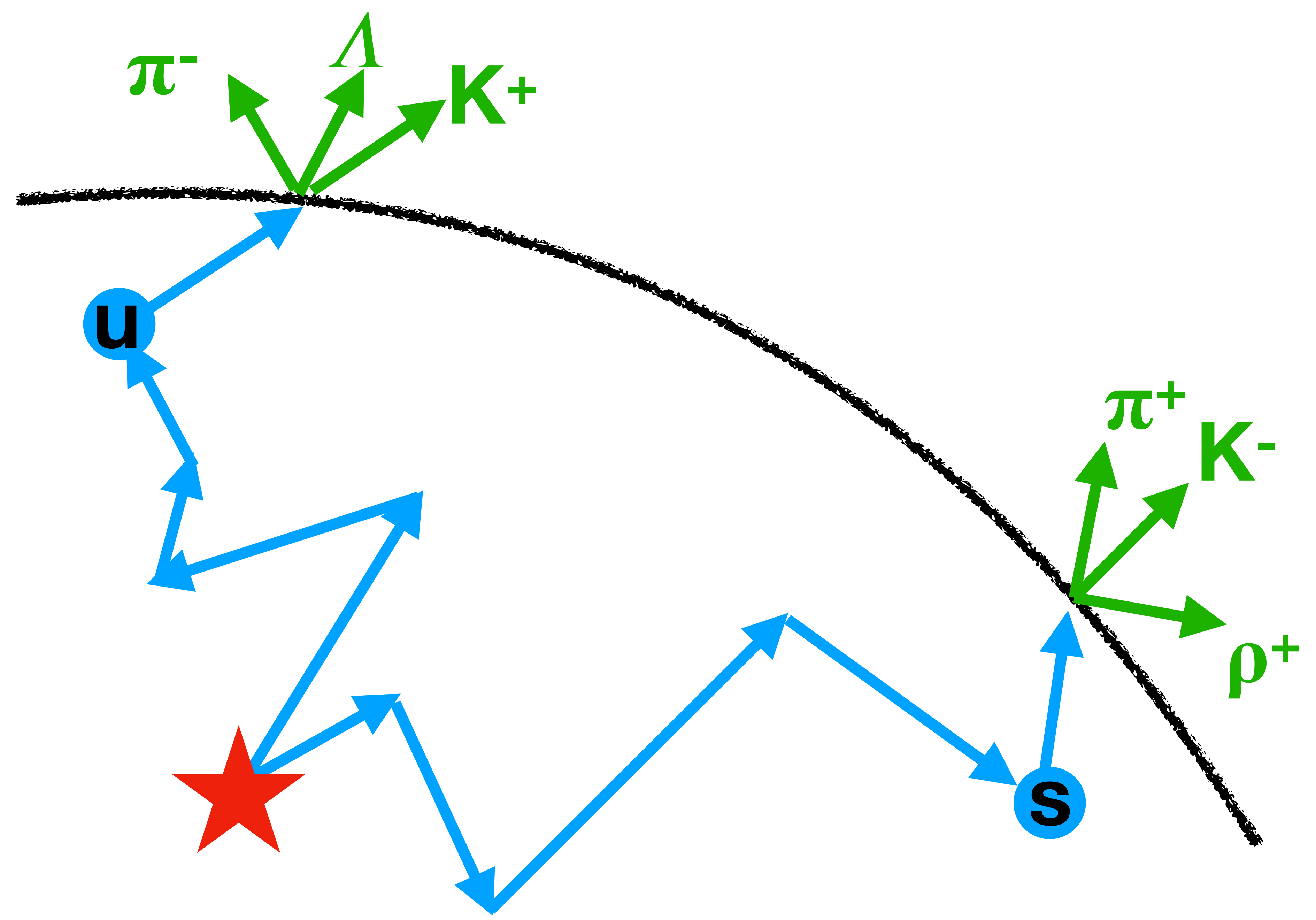
a=(u,d,s)

For parton gas: $\chi_{ab} = \sum_a (n_a + n_{\bar{a}})\delta_{ab}$



Hydro/Simulation Interface

In hadron gas: π,K,p….



$$\delta N_h = n_h q_{h,a} \chi^{-1}_{ab}(T_{\rm interface})\delta Q_b$$

Translates charge indices

to hadron indices

Given *T*, flow and susceptibility at QGP/hadron interface,

$$C_{ab}(r_1, r_2) \rightarrow B_{hh'}(p_1, p_2)$$

$$B_{hh'}(p_2|p_1) \equiv \frac{N_{h'\bar{h}}(p_1,p_2) - N_{h'h}(p_1,p_2)}{2N_{h'}(p_1)} + \frac{N_{\bar{h}'h}(p_1,p_2) - N_{\bar{h}'\bar{h}}(p_1,p_2)}{2N_{\bar{h}'}(p_1)}$$

Adjustable Parameters

1. **Diffusion Constant $D(T)$ (multiples of lattice values)**
2. **σ_0 — spread in rapidity at τ_0 = 0.6 fm/c**
3. **T_h = 155 MeV**

ALGORITHM

Diffusion = Random walk

Monte Carlo procedure:

A) Overlay with hydro evolution to create $S_{ab}(t,\vec{r})$

B) Generate partners (uu,dd,ss,ud,us,ss) proportional to $S_{ab}(t,\vec{r})$ with weights

C) Move particles in random directions punctuated by re-directioning according to $\tau_{\rm coll}$

D) Translate δQ_a to δN_h at hyper surface

E) Collide (fixed σ) and decay particles

F) Combine decay products with those from partner

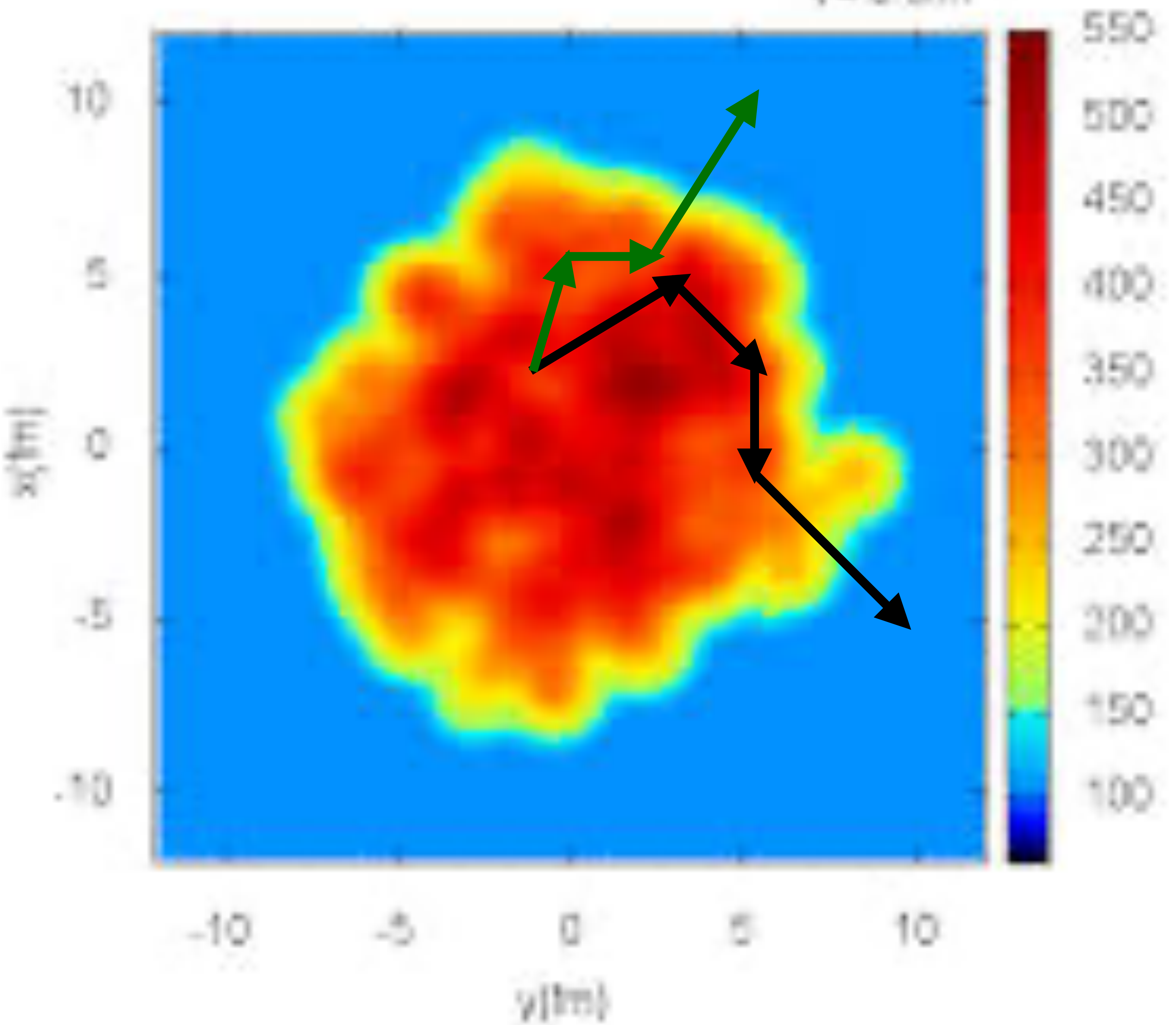
G) Correlations created during hadronic phase: create uncorrelated hadrons, run through cascade, combine ALL particles to create BF

H) Add contributions from (E) and (F)

I) Fold with acceptance/efficiency

J) Test sum rules

$$\tau_{\rm coll} = 6D$$



ALGORITHM

Correlations from Hydro:

— Depends of D and σ_0

— Only a few hours of CPU

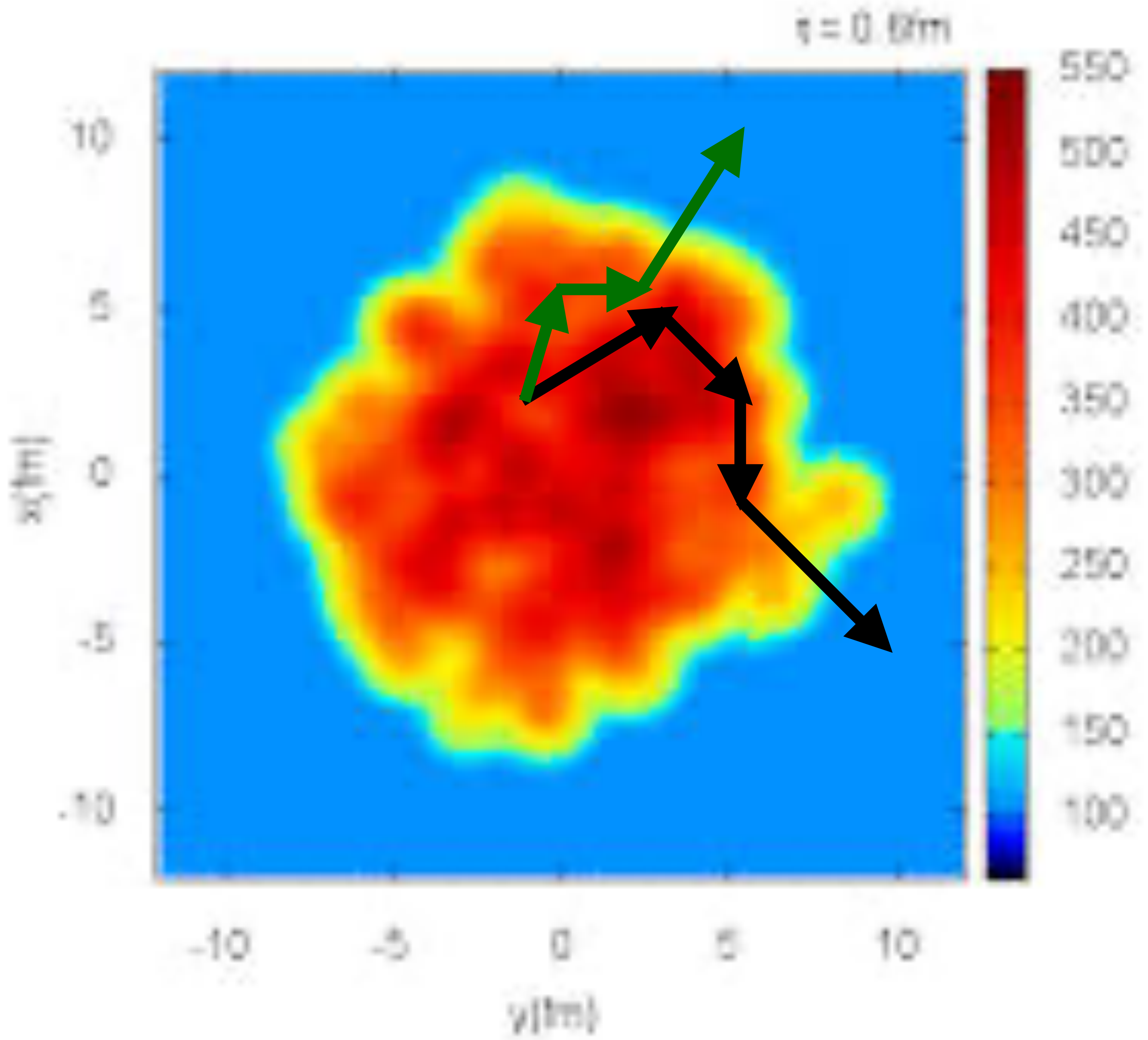
— track charges from same source point

Correlations from Cascade

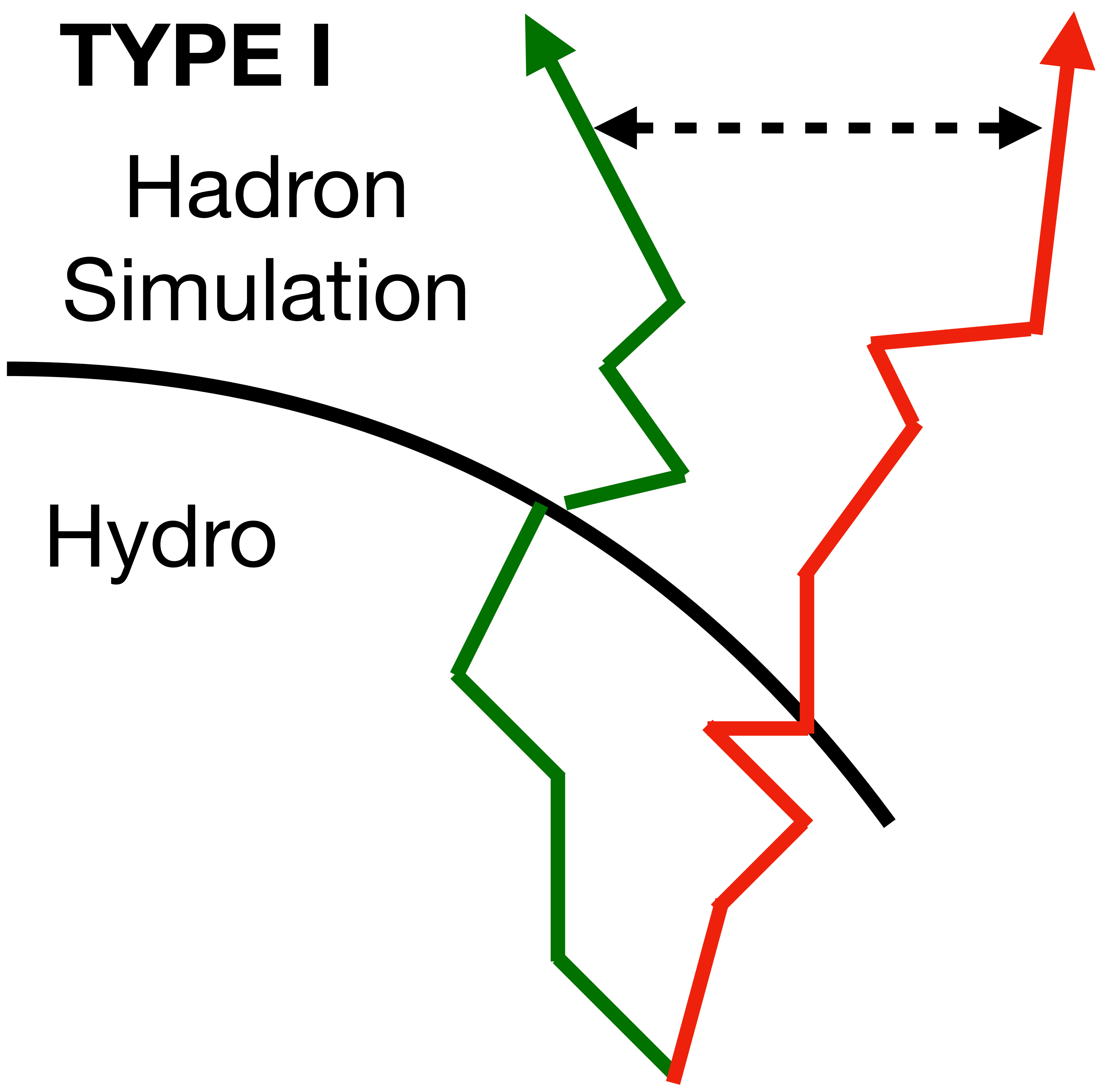
— Weeks of CPU

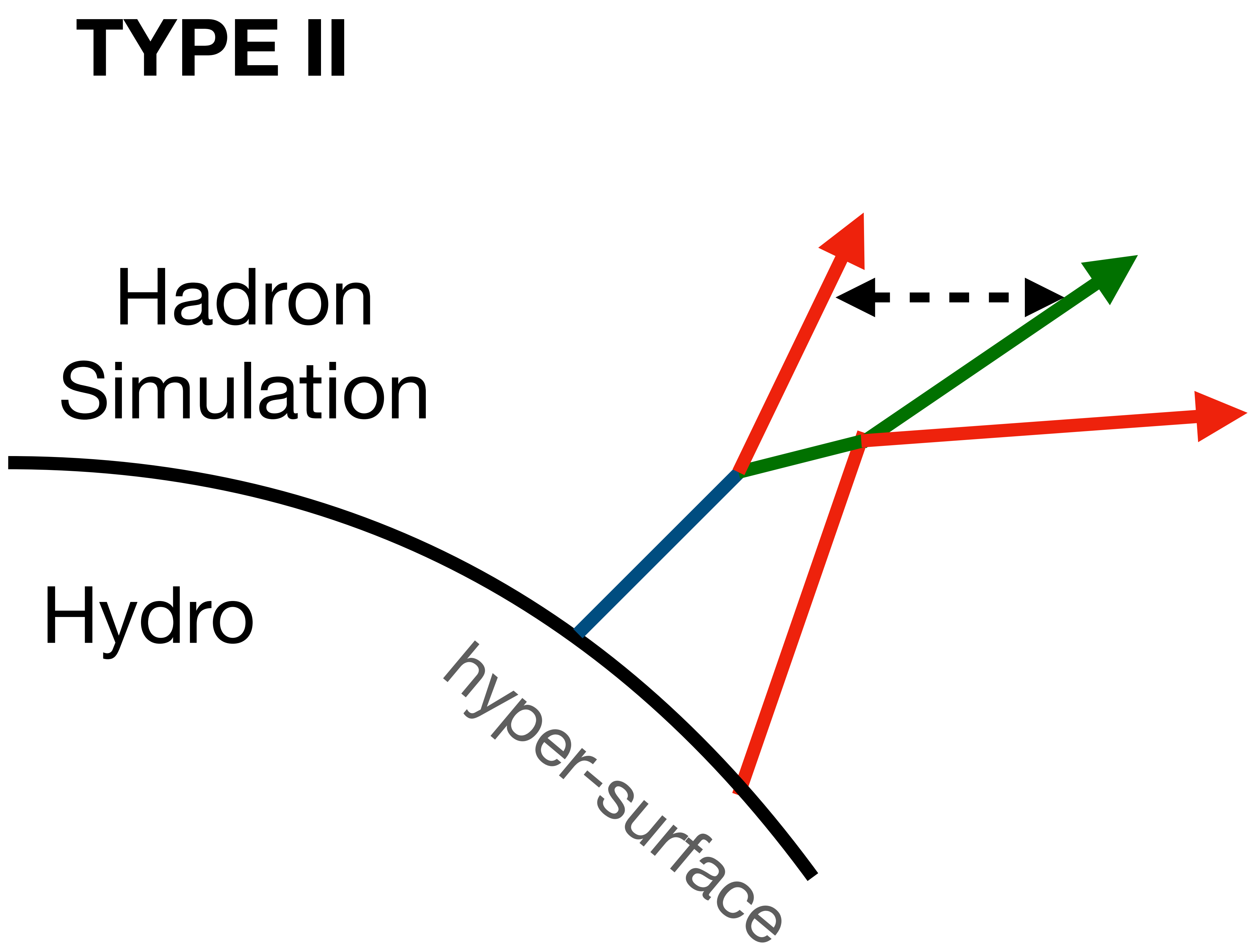
— One hydro event (independent of D, σ_0)

— Millions of cascade events



ALGORITHM


TYPE I
Hadron
Simulation
Hydro

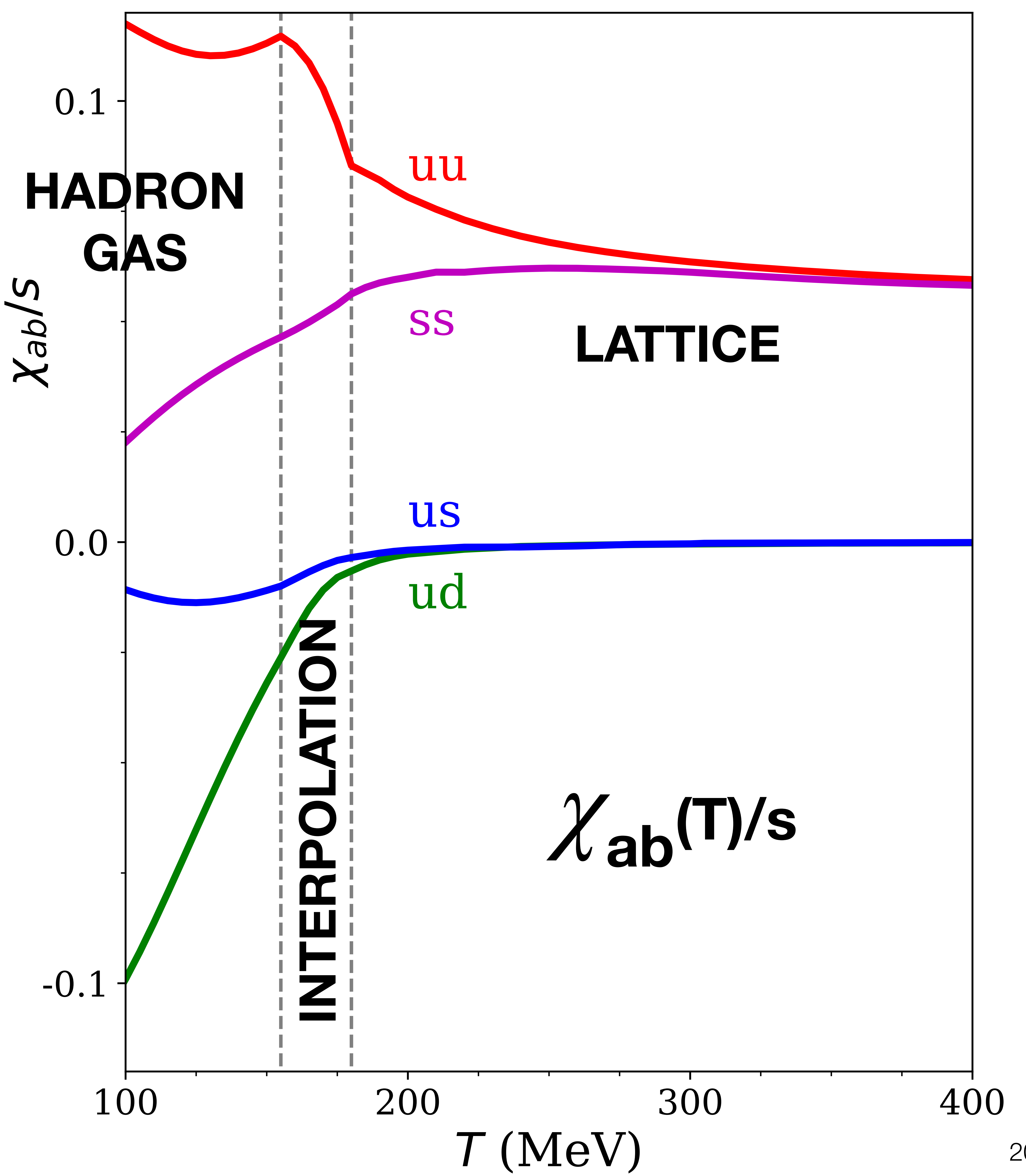

TYPE II
Hadron
Simulation
Hydro
hyper-surface

Model input
Hydro history

Chris Plumberg

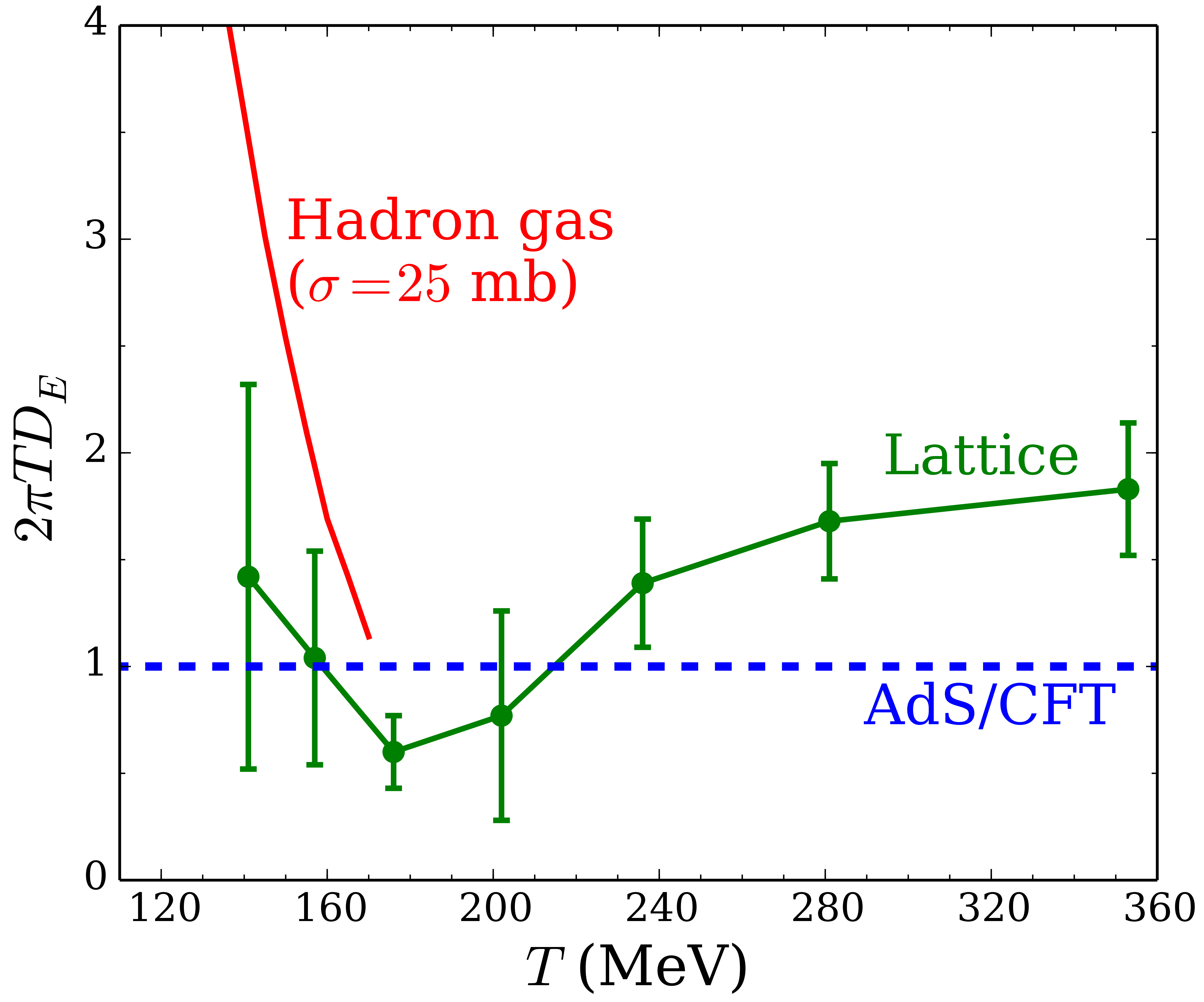
VISHNU Hydro, Au+Au (200A GeV)

Model input Susceptibility



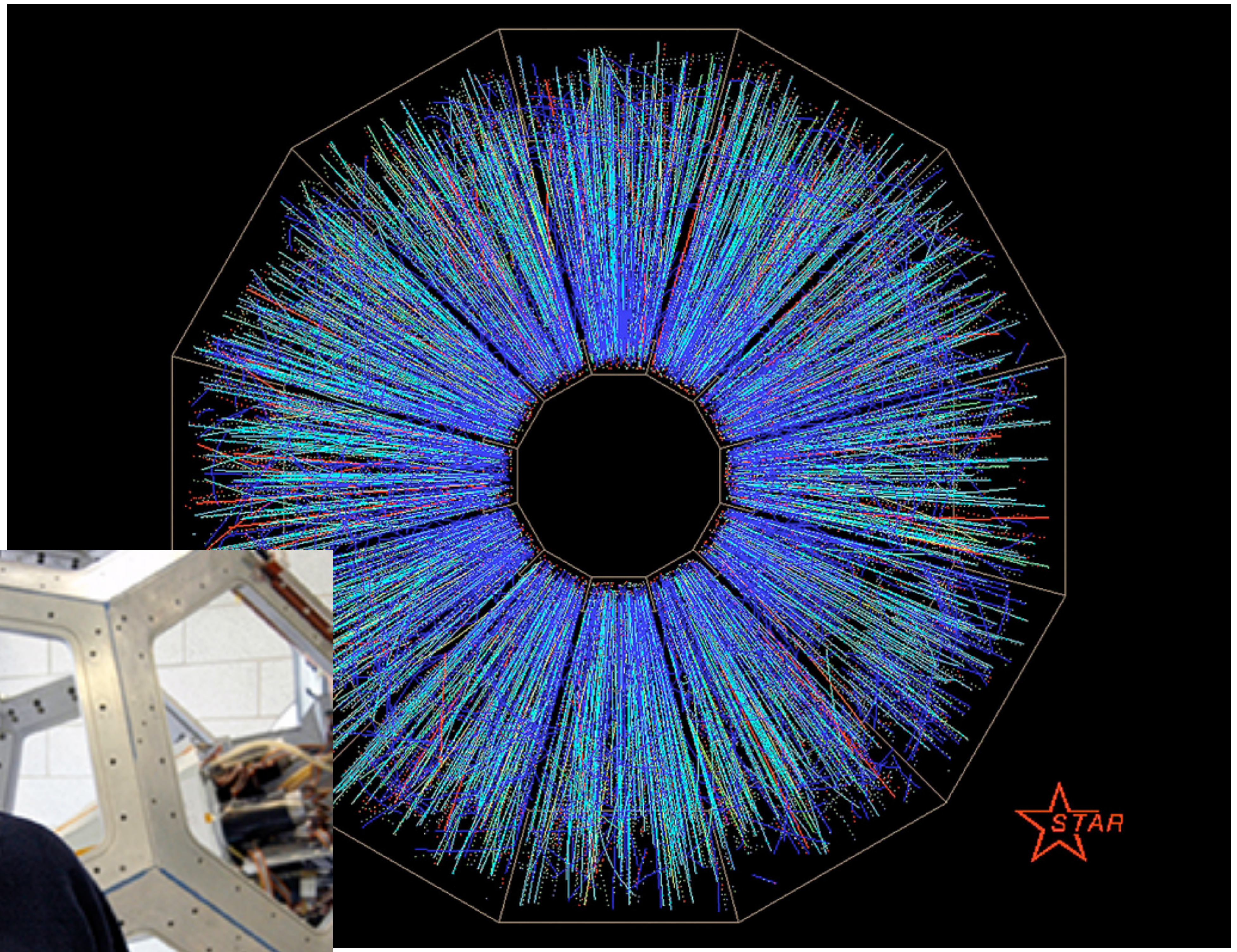
Claudia Ratti
BW Collaboration

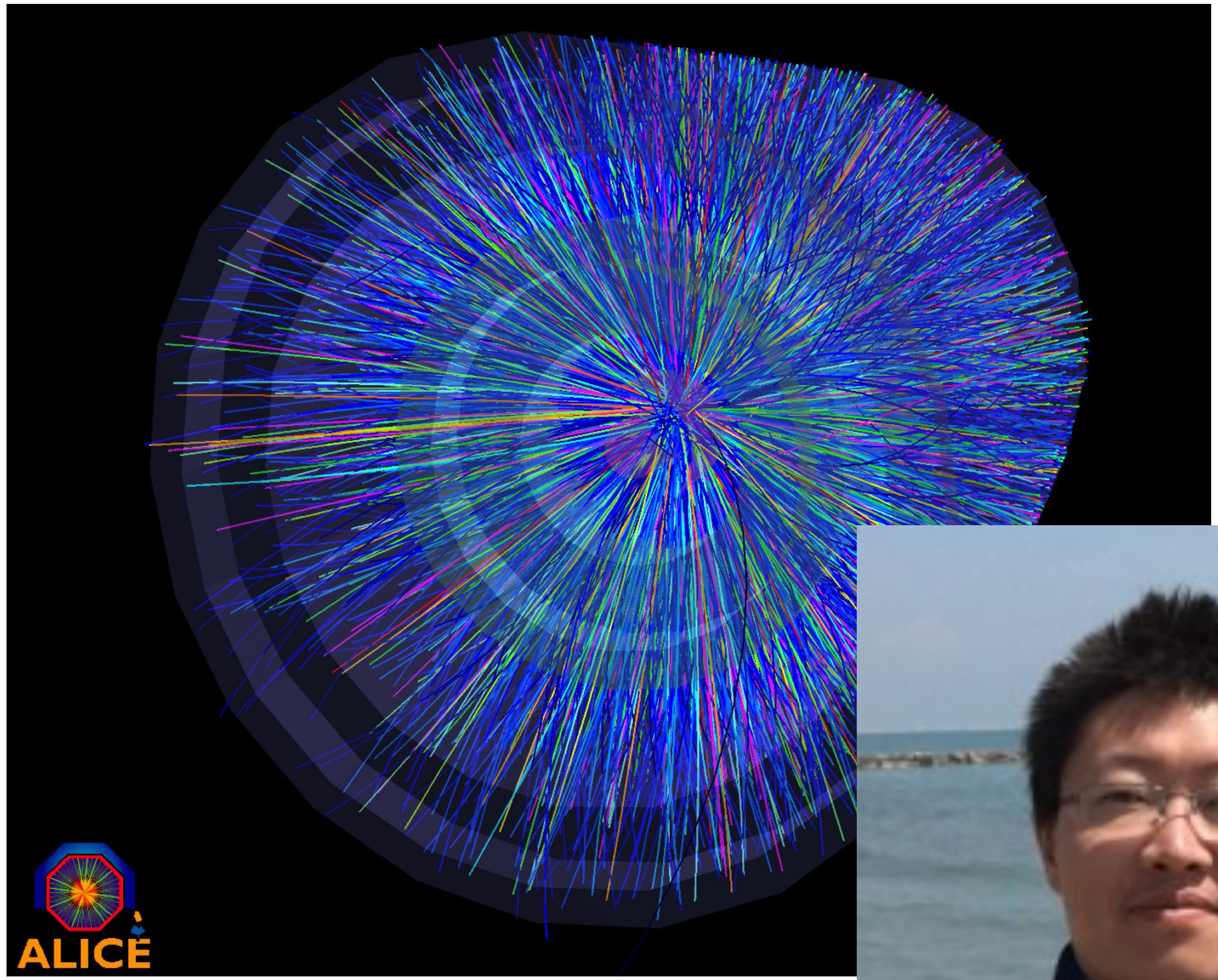
Model input Diffusivity

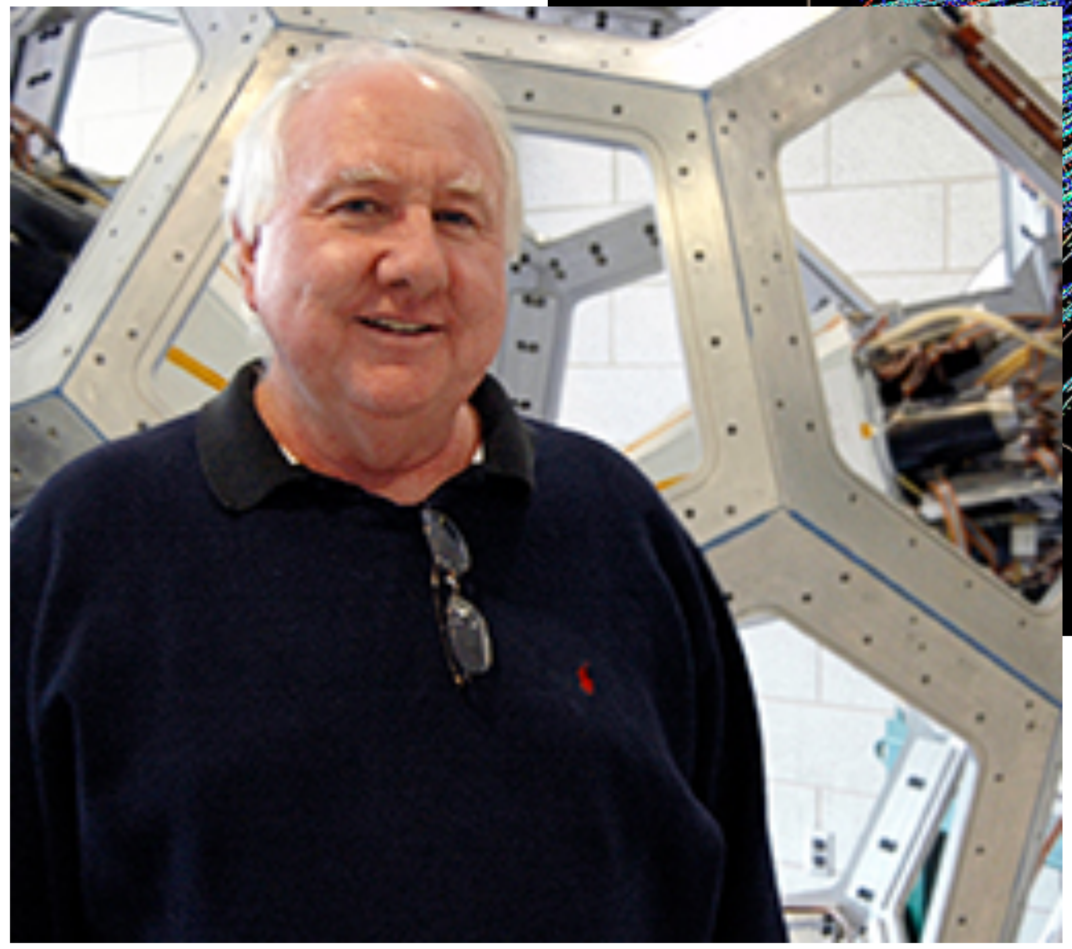


Lattice: Diffusivity/Conductivity, $D_E(T)$, $\sigma(T)$
G.Aarts, C.Allton, A.Amato, P.Guidice, S.Hands & J.I.Skullerud, JHEP(2015)

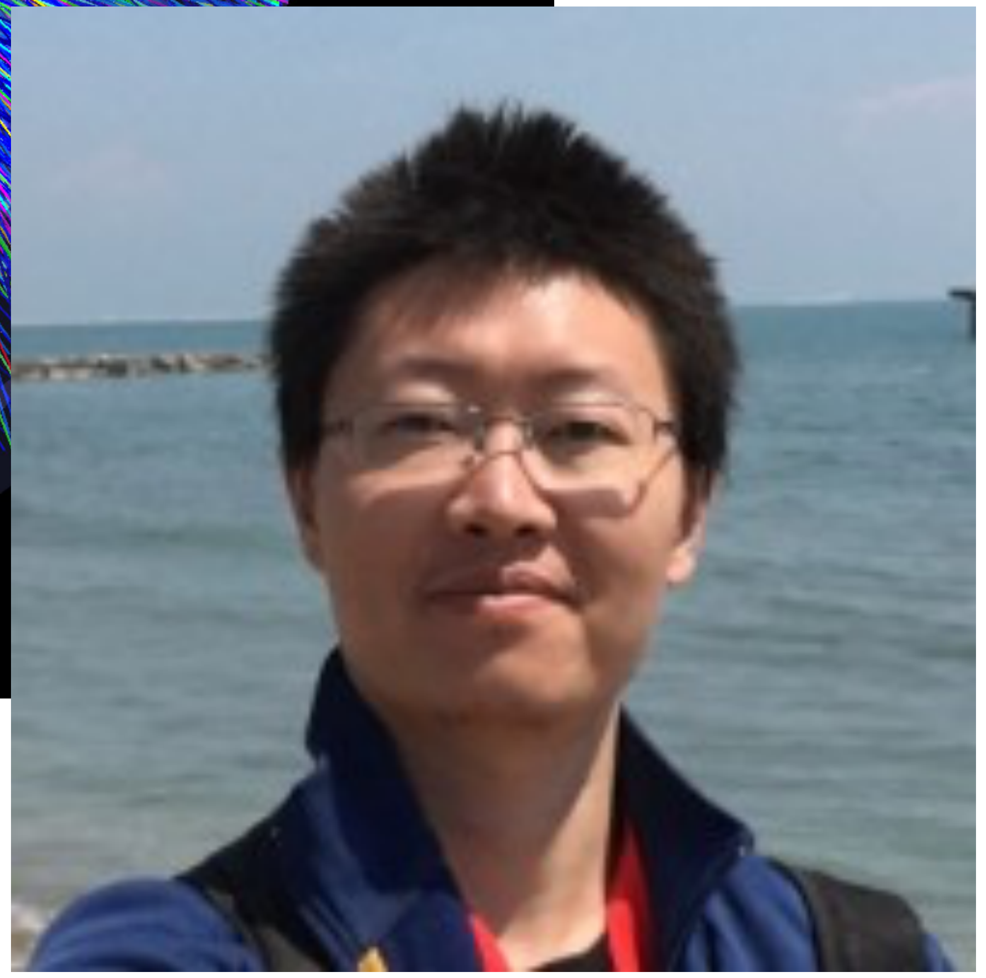
Experimental Acceptance/Efficiency



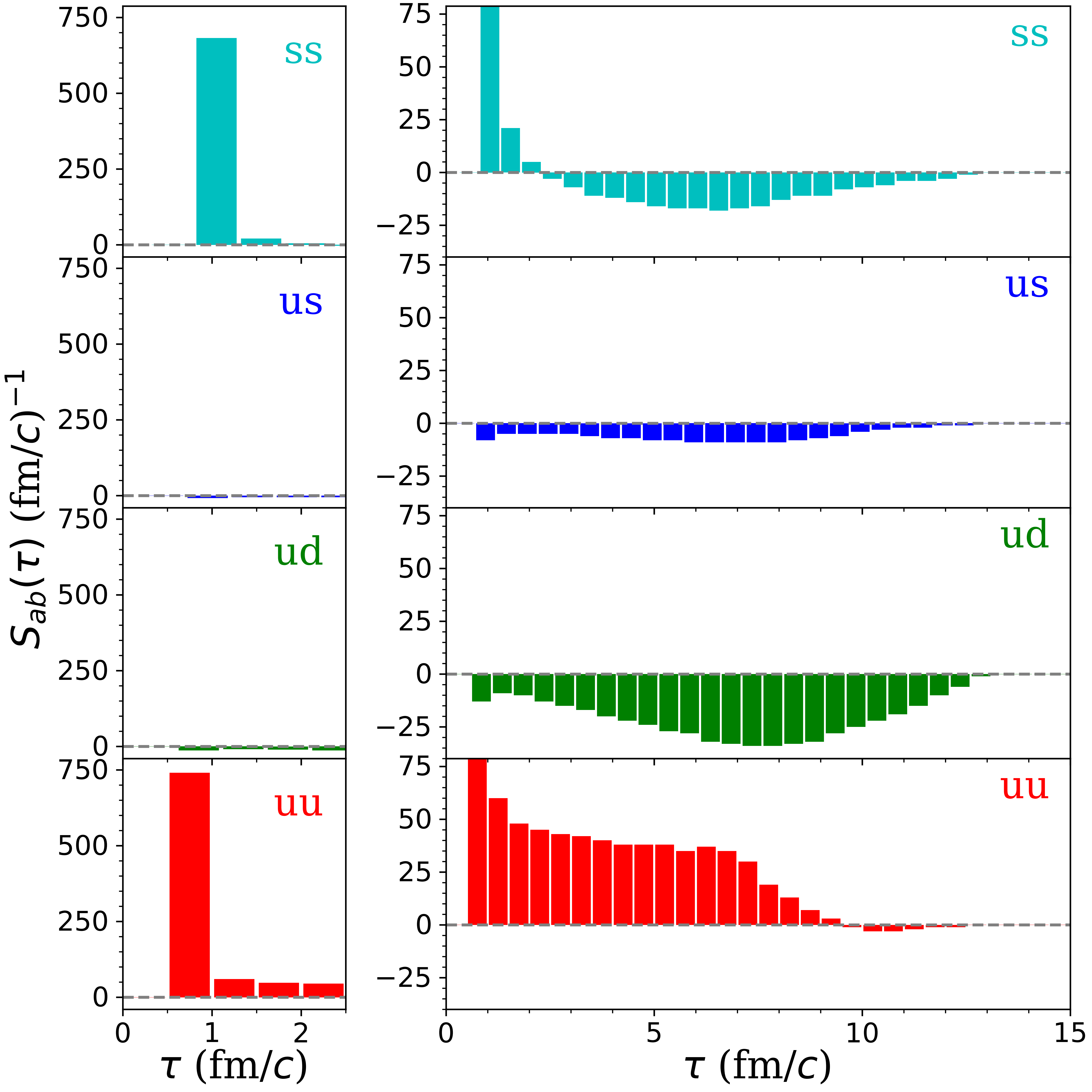




Gary Westfall
MSU



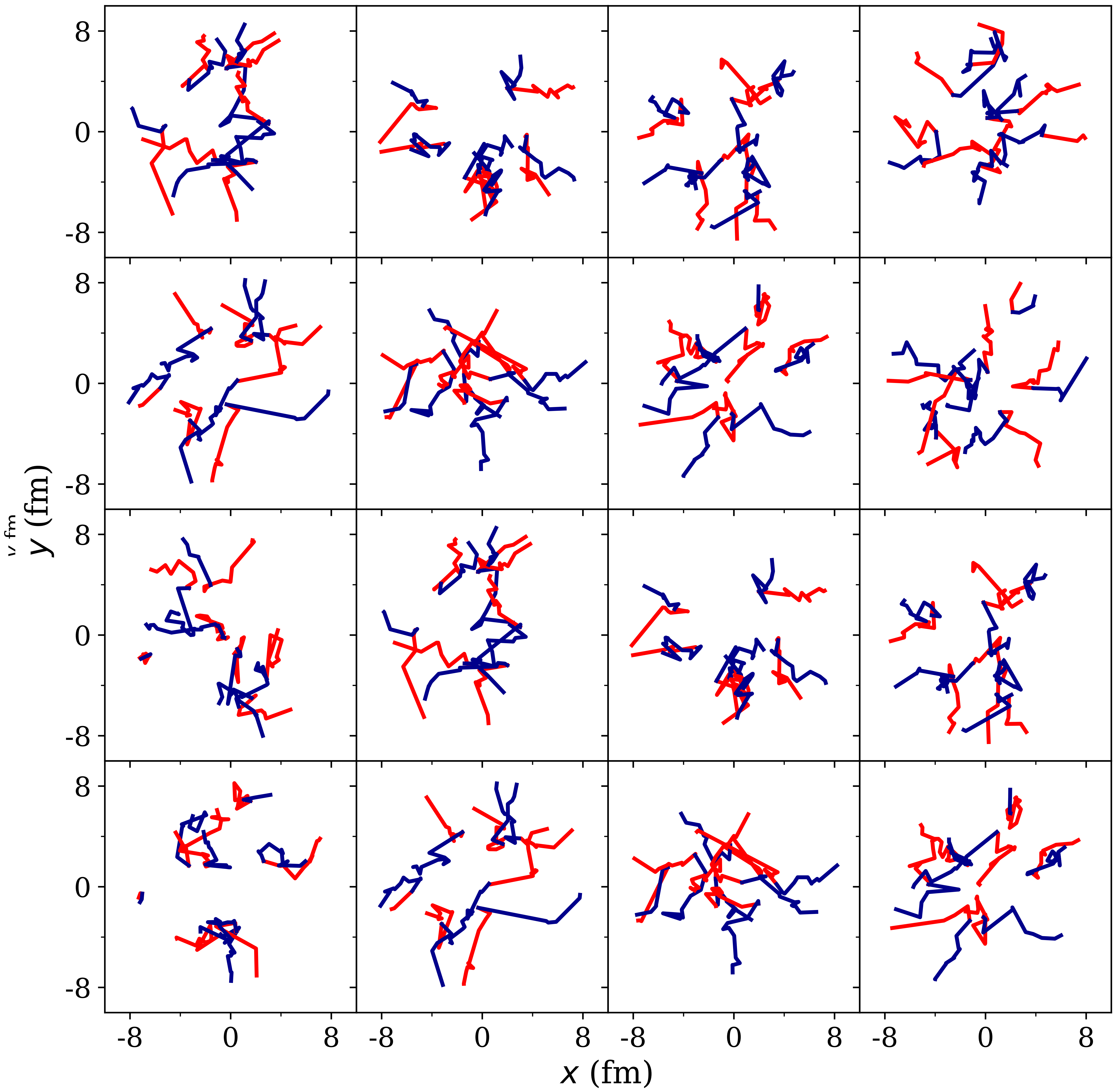
Jinjin Pan
Wayne State



Source Function

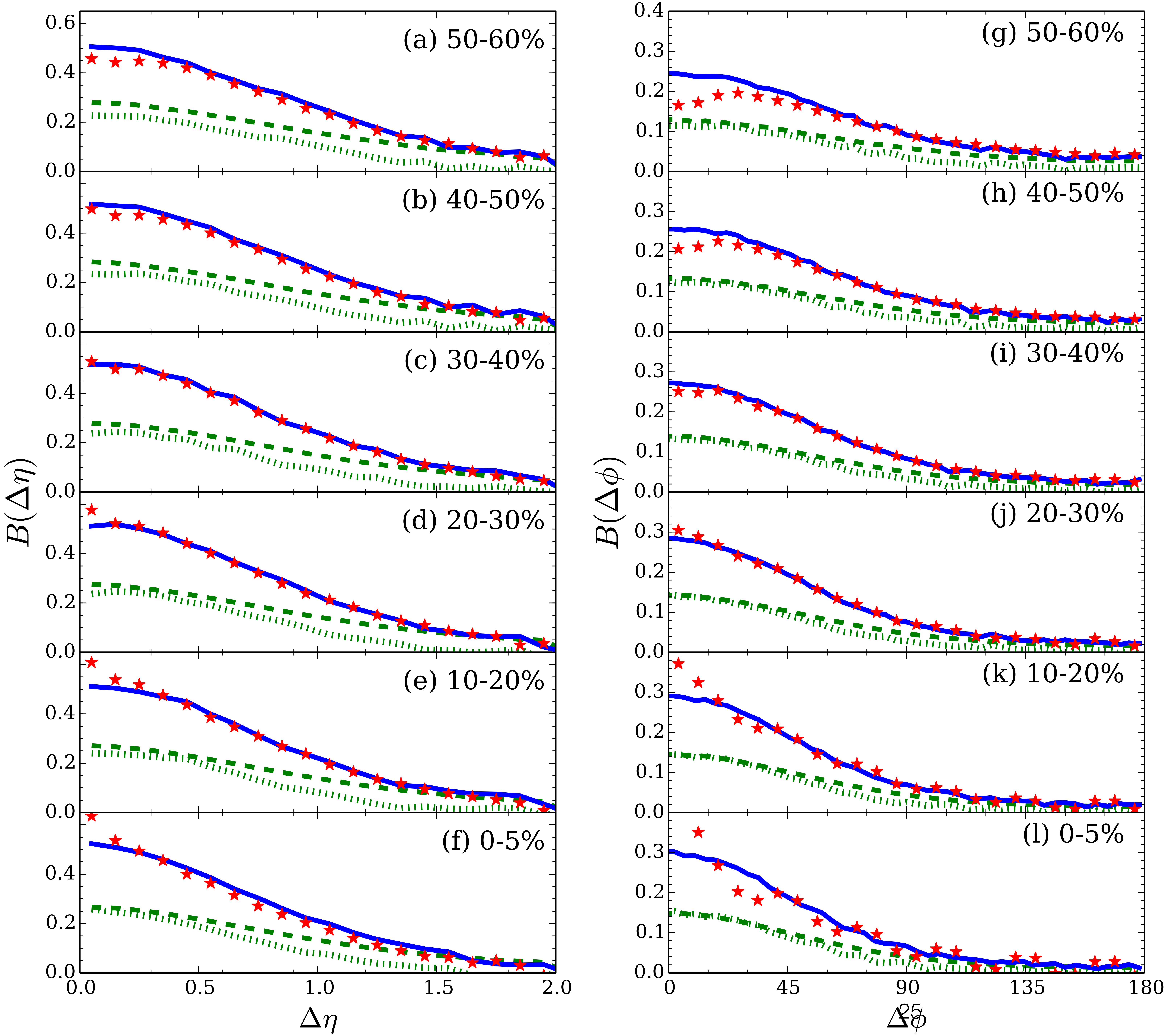
- First surge when QGP is created
- uu,dd continuously created
- ss nearly steady
- ud,us,ds at hadronization

Diffusive Trajectories



Model vs. STAR

Unidentified Particles



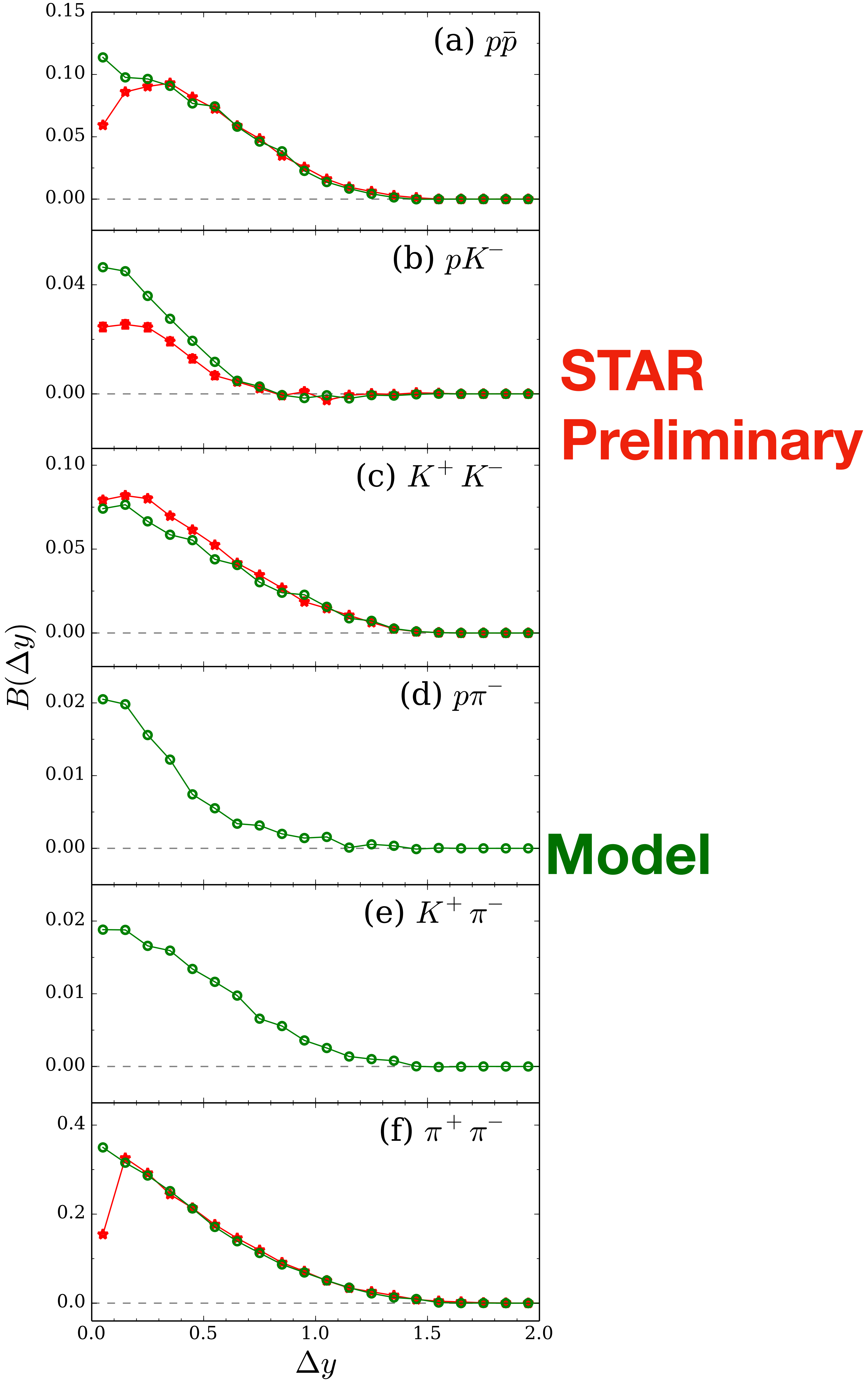
★ STAR Preliminary

Model, Type 1 + Type 2

Type 1 (dashes, hydro)

Type 2 (dots, cascade)

Model vs. STAR



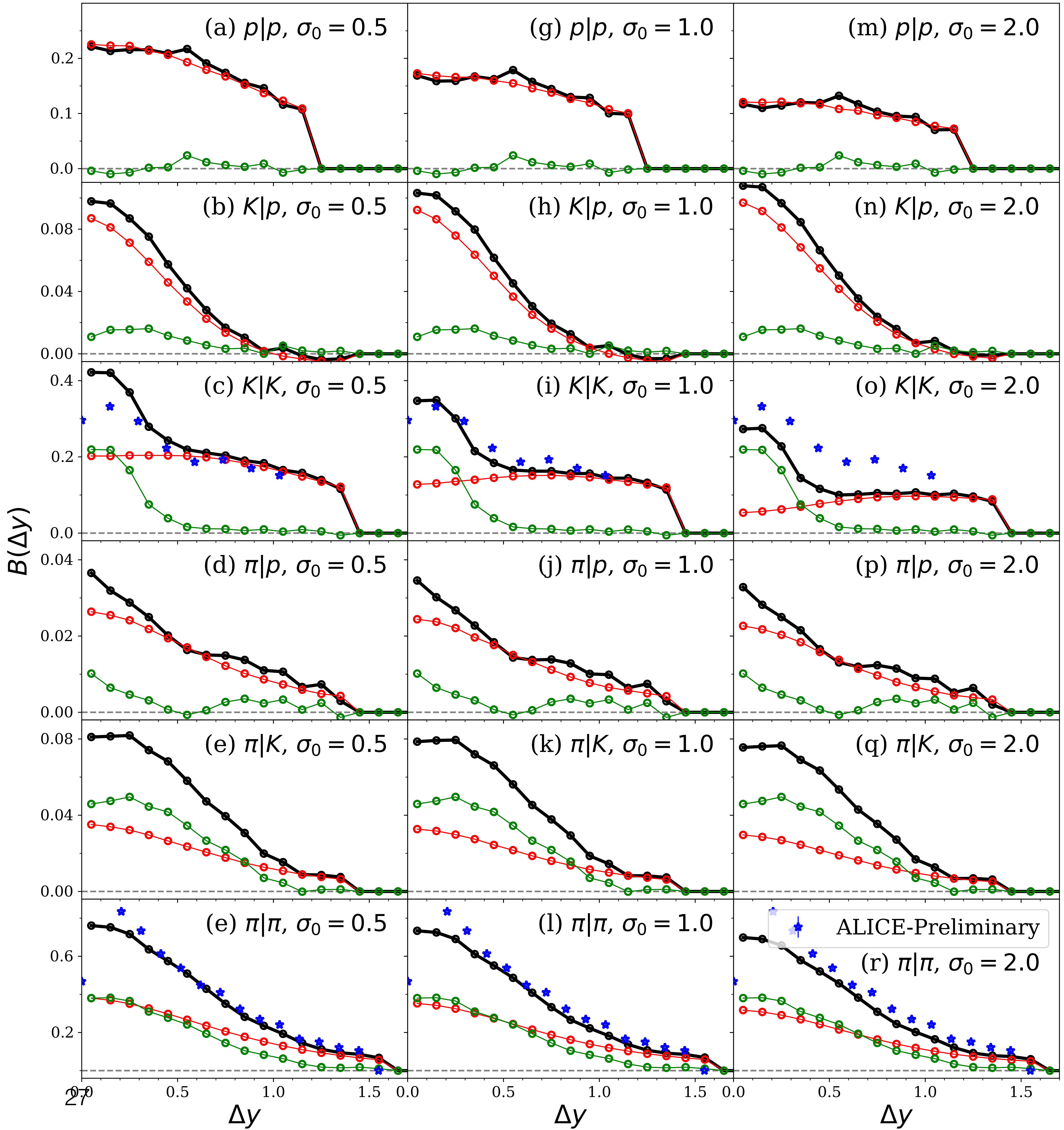
- Identified particles (vs. Δy)
- pK is off
- pp is off (annihilation missing)

Model vs ALICE

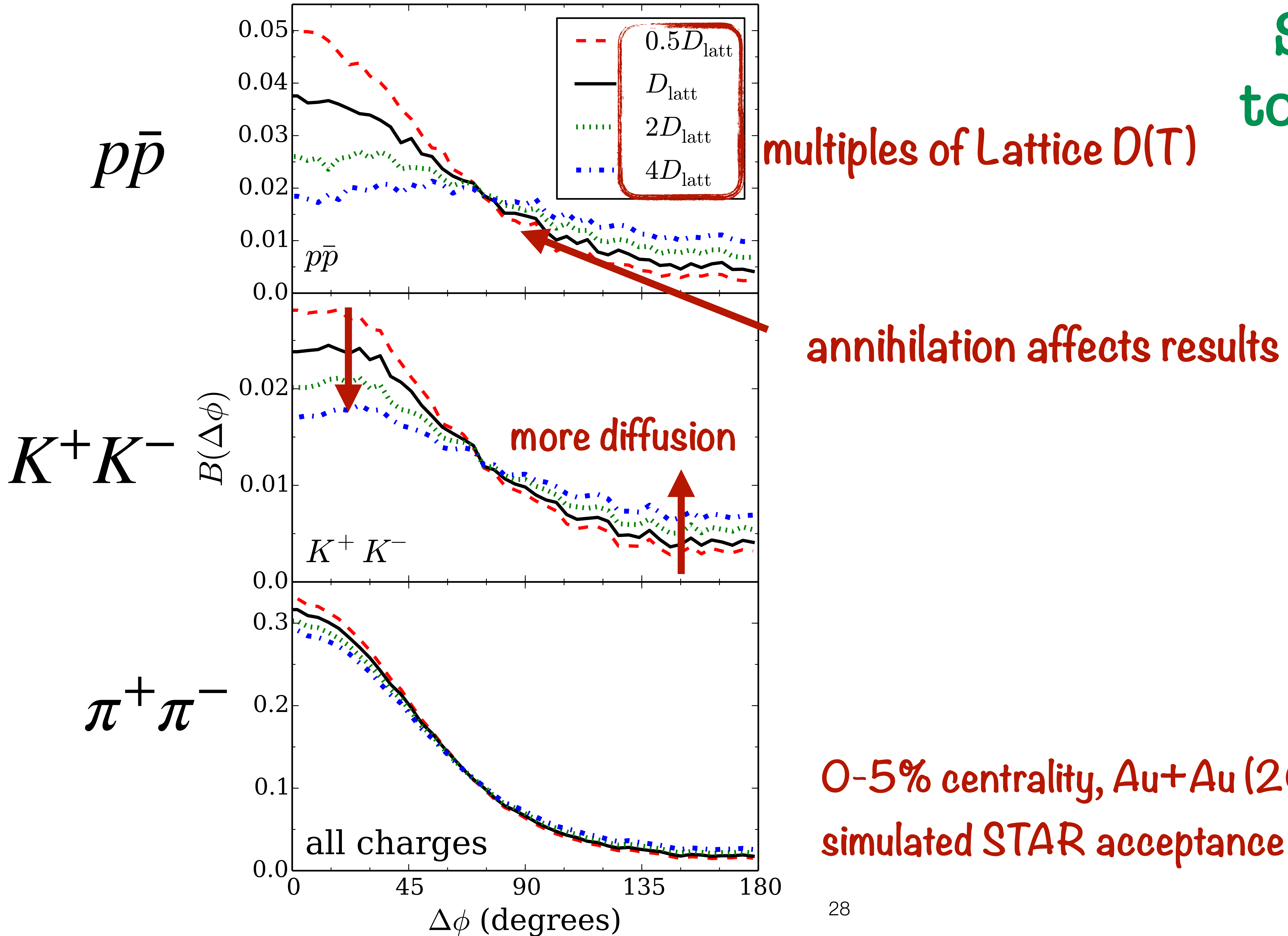
Thesis of Jin-Jin Pan

Binned by Δy

Type 1 + Type 2
Type 1 (hydro)
Type 2 (cascade)



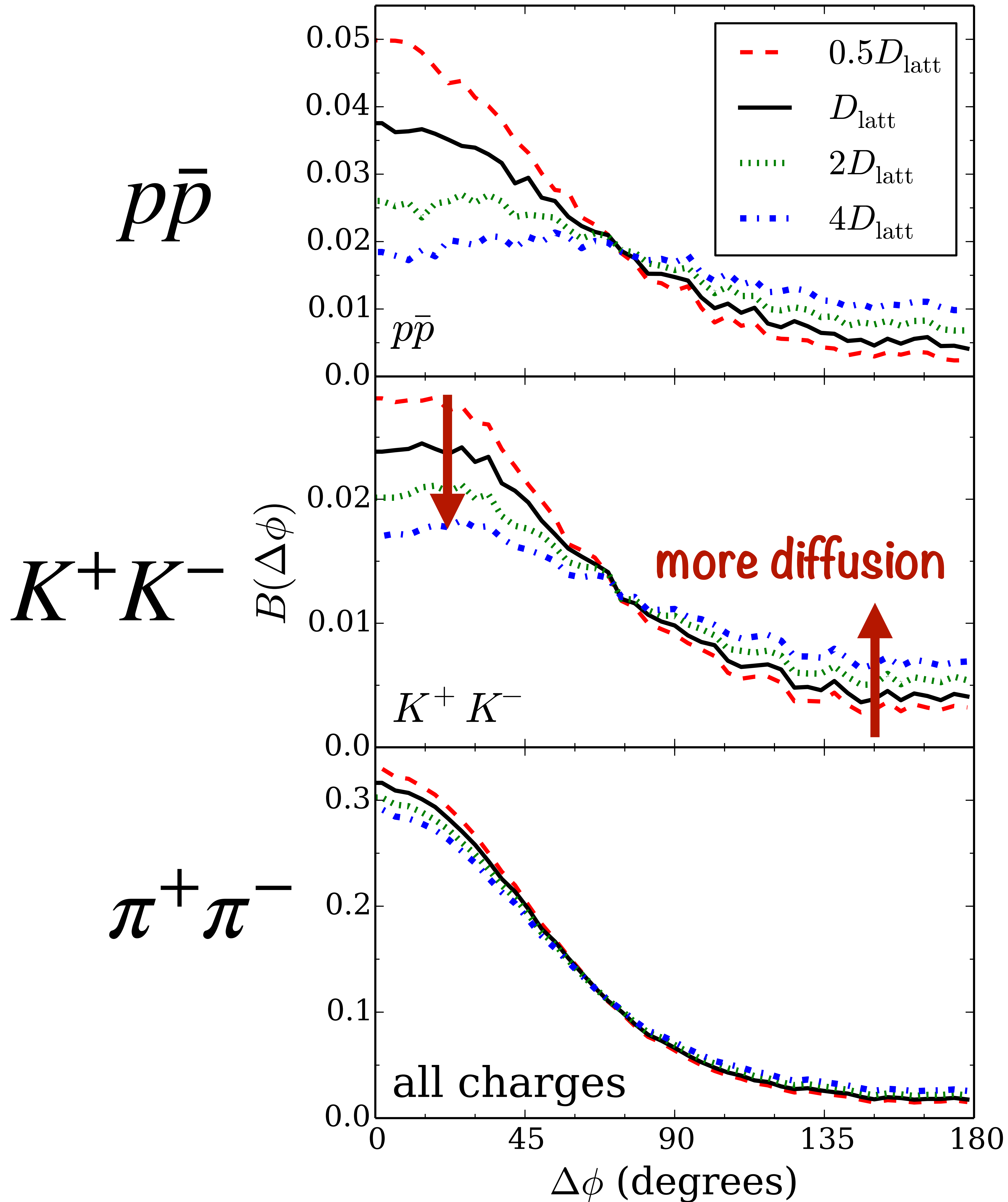
Sensitivity to Diffusivity



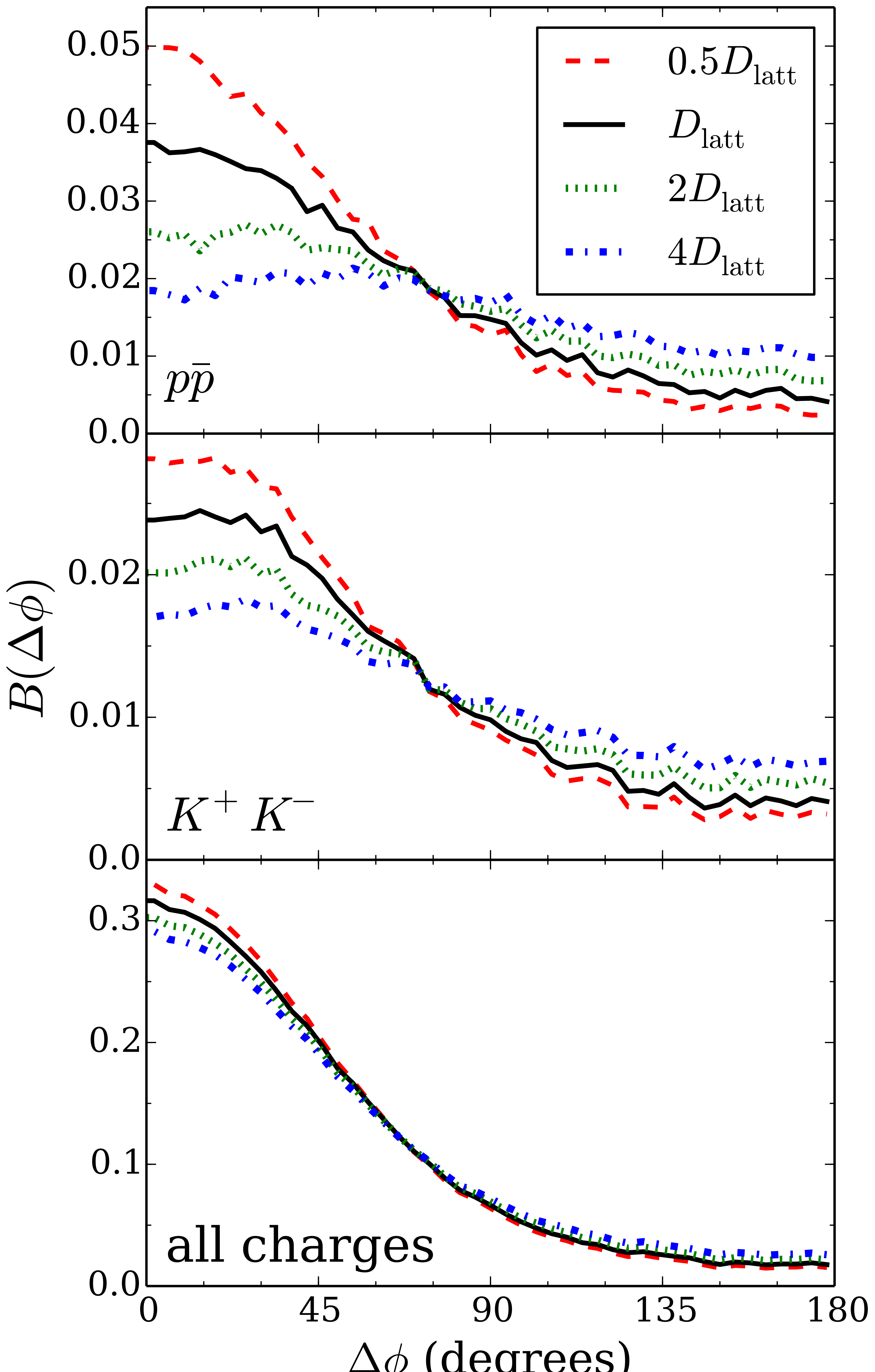
0-5% centrality, Au+Au (200A GeV)

simulated STAR acceptance

Sensitivity to Diffusivity



- $\Delta\phi$ binning reduces dependence on σ_0
- kaons or protons best suited:
- χ_{ss}/s roughly constant
  $\approx$ only phi contributes from final state
- χ_{BB}/s roughly constant
  annihilation an issue



Sensitivity to Diffusivity

Extract D ~ ±50 % ?

But much work needed:

- φ contribution to kaon B.F.
  — BF binned by Q_{inv}
- absorption of strangeness into baryons
  — look at pK, KΛ BFs
- strangeness annihilation
  — multiplicities and BF vs Δy

Model vs. ALICE

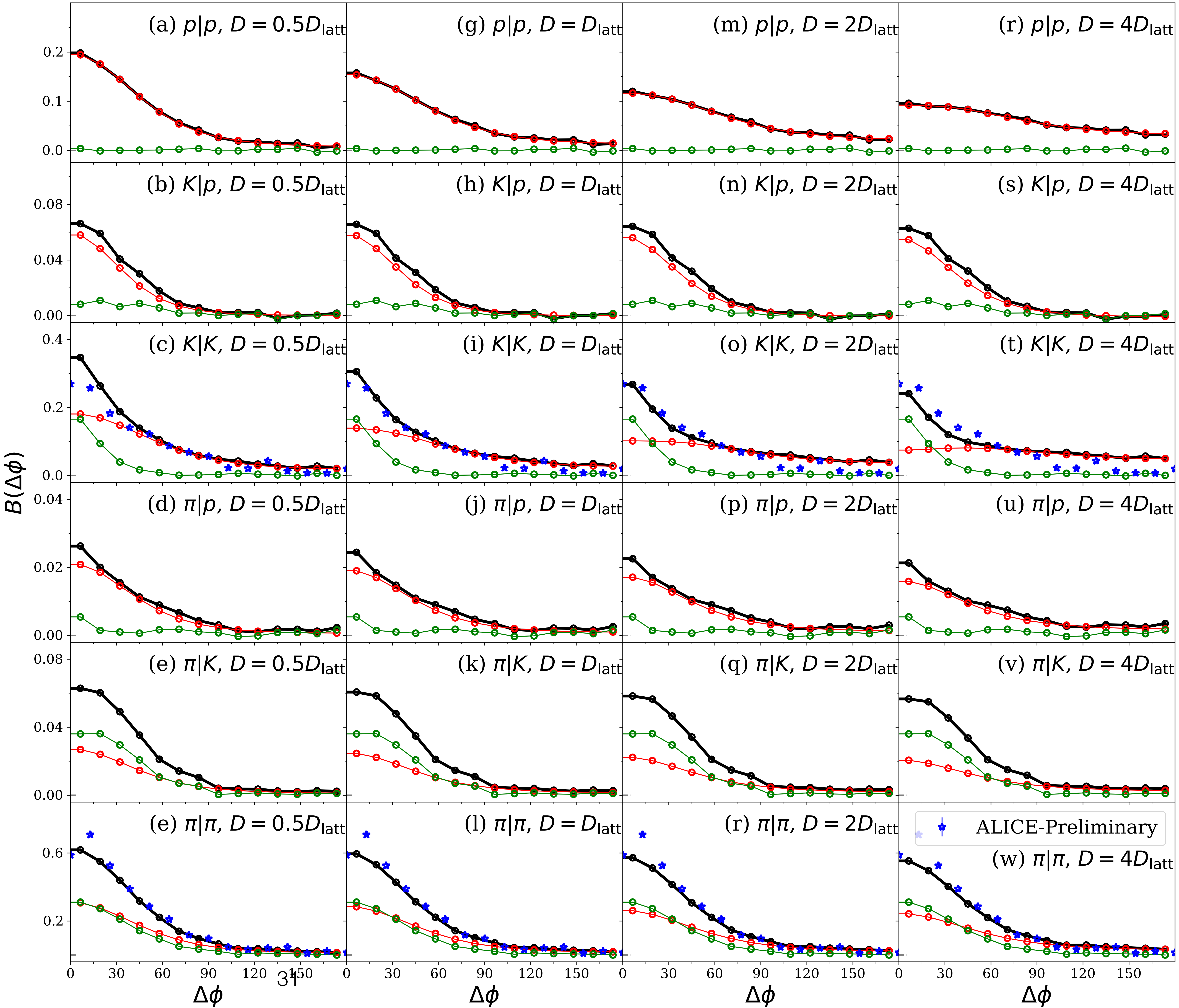
Binned by $\Delta\phi$

Lattice diffusion looks OK

Type 1 + Type 2

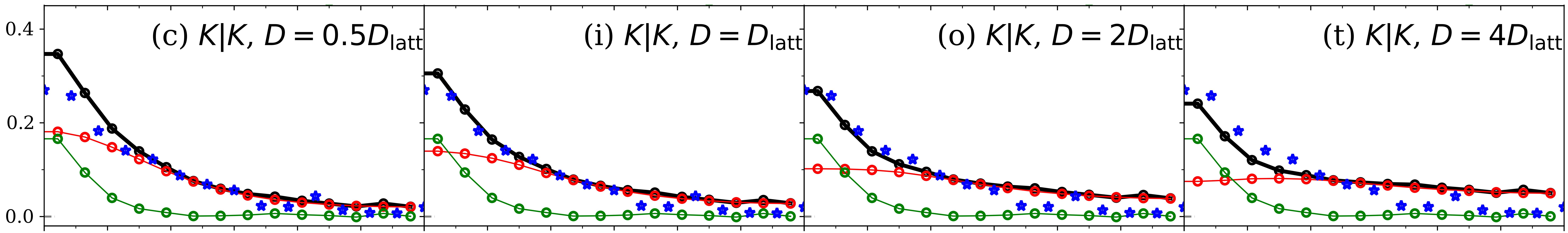
Type 1 (hydro)

Type 2 (cascade)



Model vs. ALICE

Type 1 + Type 2
Type 1 (hydro)
Type 2 (cascade)



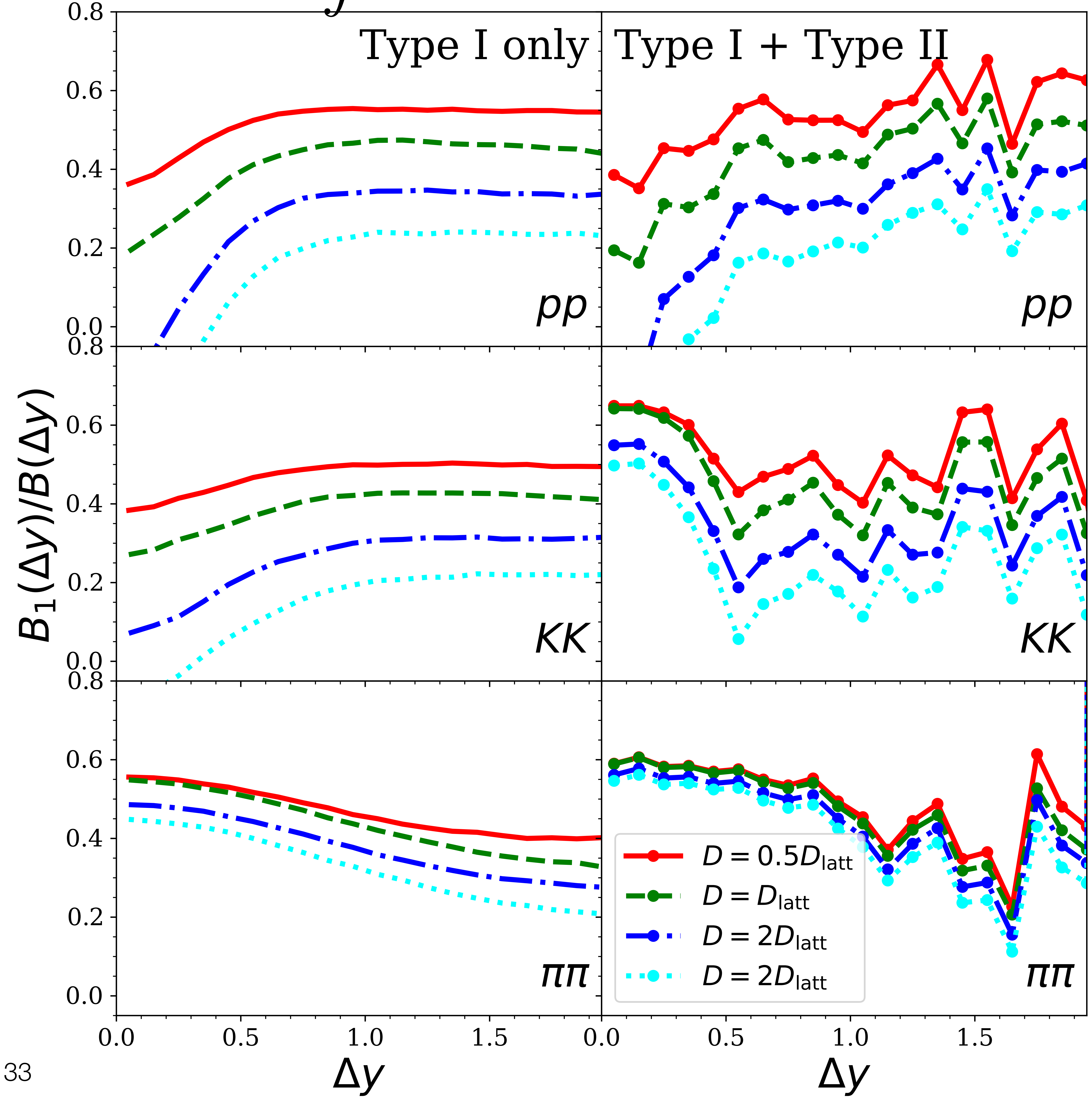
Lower diffusivities look better

Better Focus on Diffusivity
Analyze $B(\Delta\phi)$,
Cutting on large Δy

Eliminate Effects from:

- HBT
- Resonant Decays
- Annihilation
- Experimental 2-track resolution
- $\Delta y \gtrsim 0.75$ should be good enough

$$B_1(\Delta y) \equiv \int d\Delta\phi B(\Delta y, \Delta\phi)\cos(\Delta\phi)$$



Analyze $B(\Delta\phi)$
Cutting on large Δy

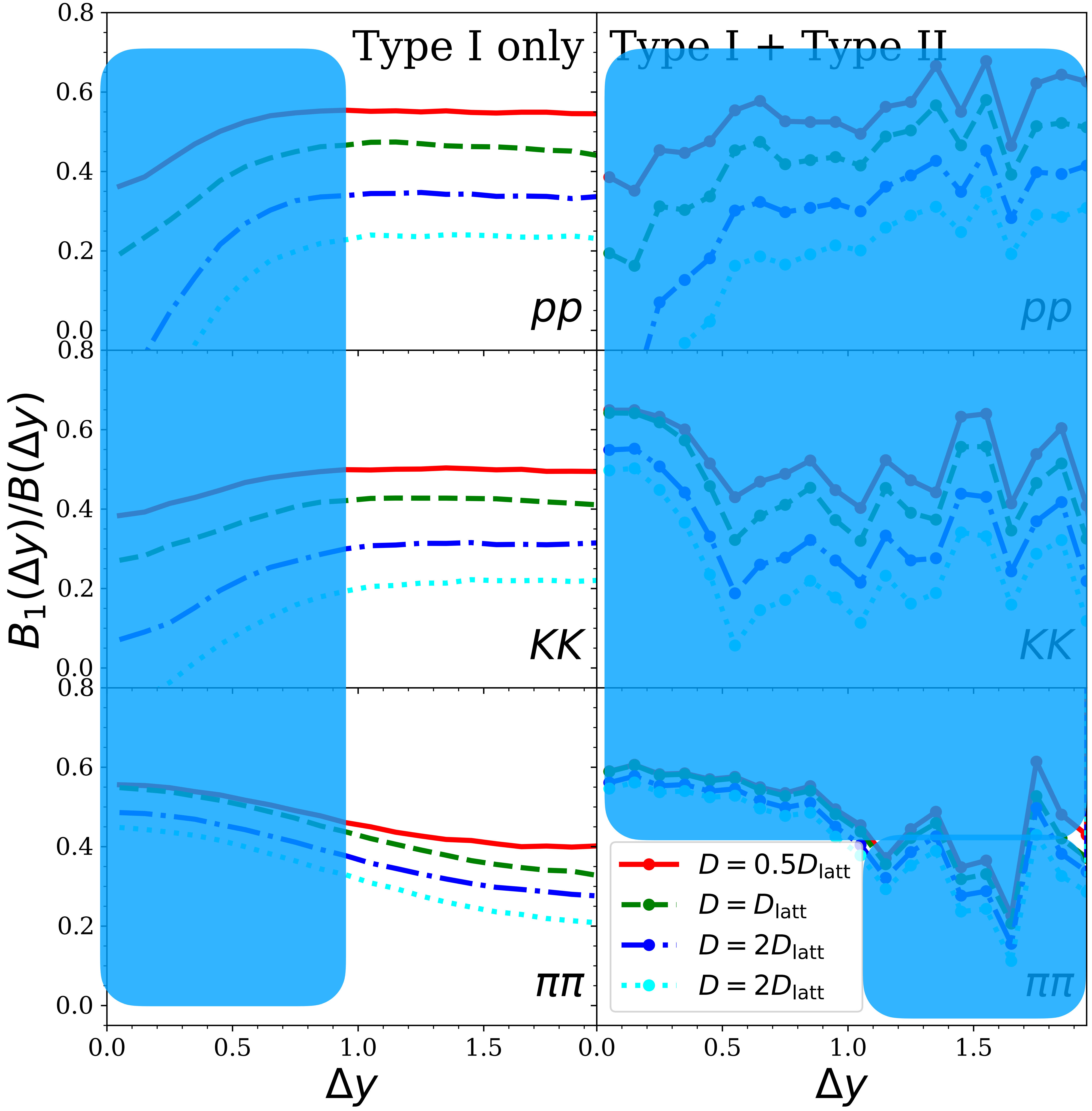
$$B_1(\Delta y) \equiv \int d\Delta\phi B(\Delta y, \Delta\phi)\cos(\Delta\phi),$$

$$B(\Delta y) = \int d\Delta\phi B(\Delta y, \Delta\phi)$$

Type II only provides noise for $\Delta y \gtrsim 1$

Robust extraction of diffusivity

for this window



Summary

- Charge correlations (order Q^2) calculated in "standard model"
- STAR/ALICE data consistent (mostly) with early chemical equilibration
  $K^+K^-, p\bar{p}, \pi^+\pi^-$ systematics reproduced
  (STAR pK normalization off & ALICE KK normalization off)
- Diffusivity can be extracted from BFs binned by $\Delta\phi$ cut on large Δy
  High statistics STAR & ALICE data coming
- Many opportunities for progress
  Both theoretical and experimental
  Both for diffusivity and for chemistry
  Similar to femtoscopy

U.S. DEPARTMENT OF ENERGY
Office of Science

Bonus Slides

- CME background
- Skewness/kurtosis background
- Theory for higher-order charge fluctuations

Model vs. STAR

Effect of Elliptic Flow

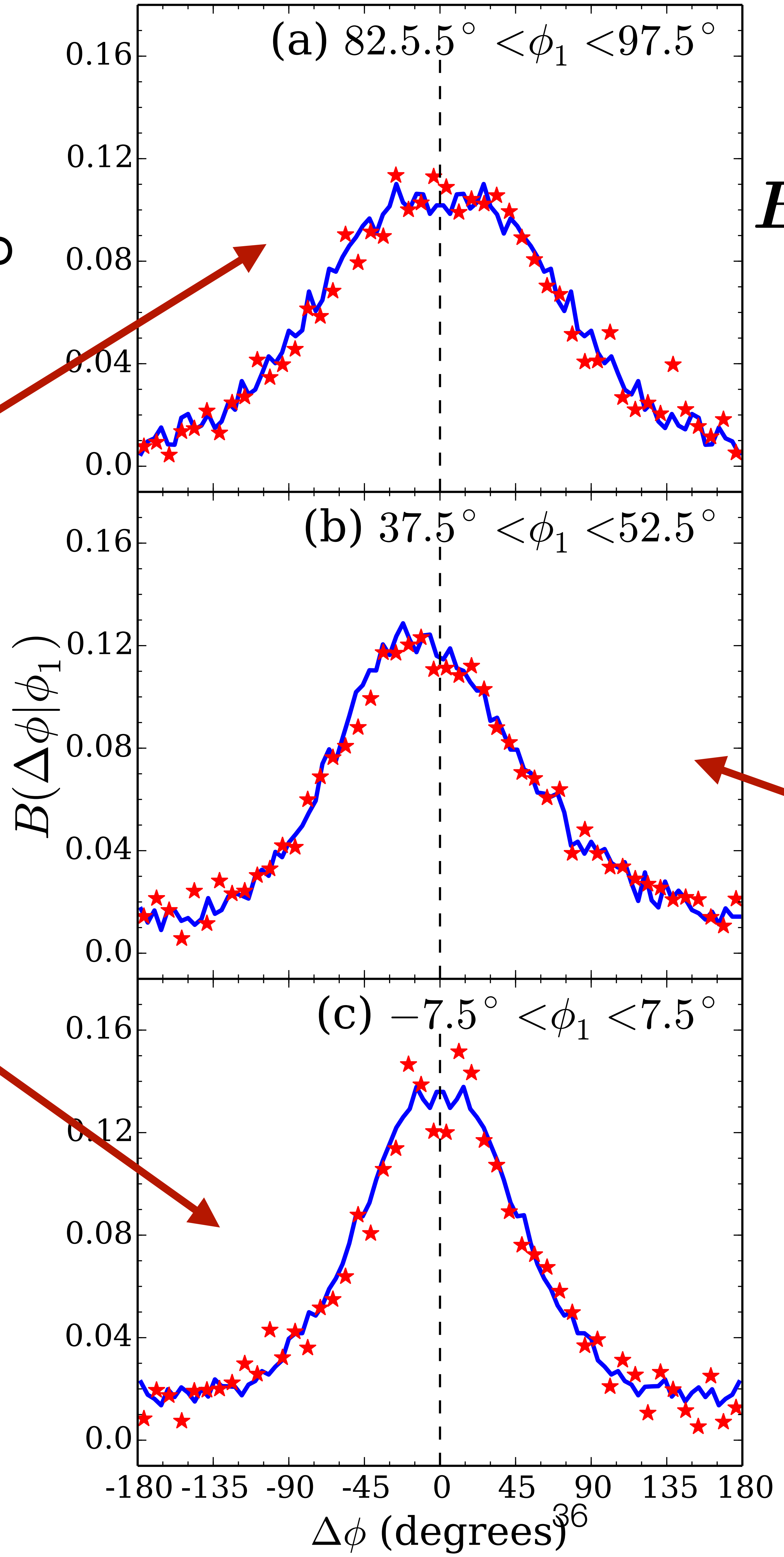
$$B(\Delta\phi|\phi_1) = \int d\phi_2 \; B(\phi_1,\phi_2)\delta(\Delta\phi - \phi_1 + \phi_2)$$

$\phi_1 \sim 90^\circ$

$\phi_1 \sim 45^\circ$

$\phi_1 \sim 0^\circ$

correlation tighter in-plane due to elliptic flow



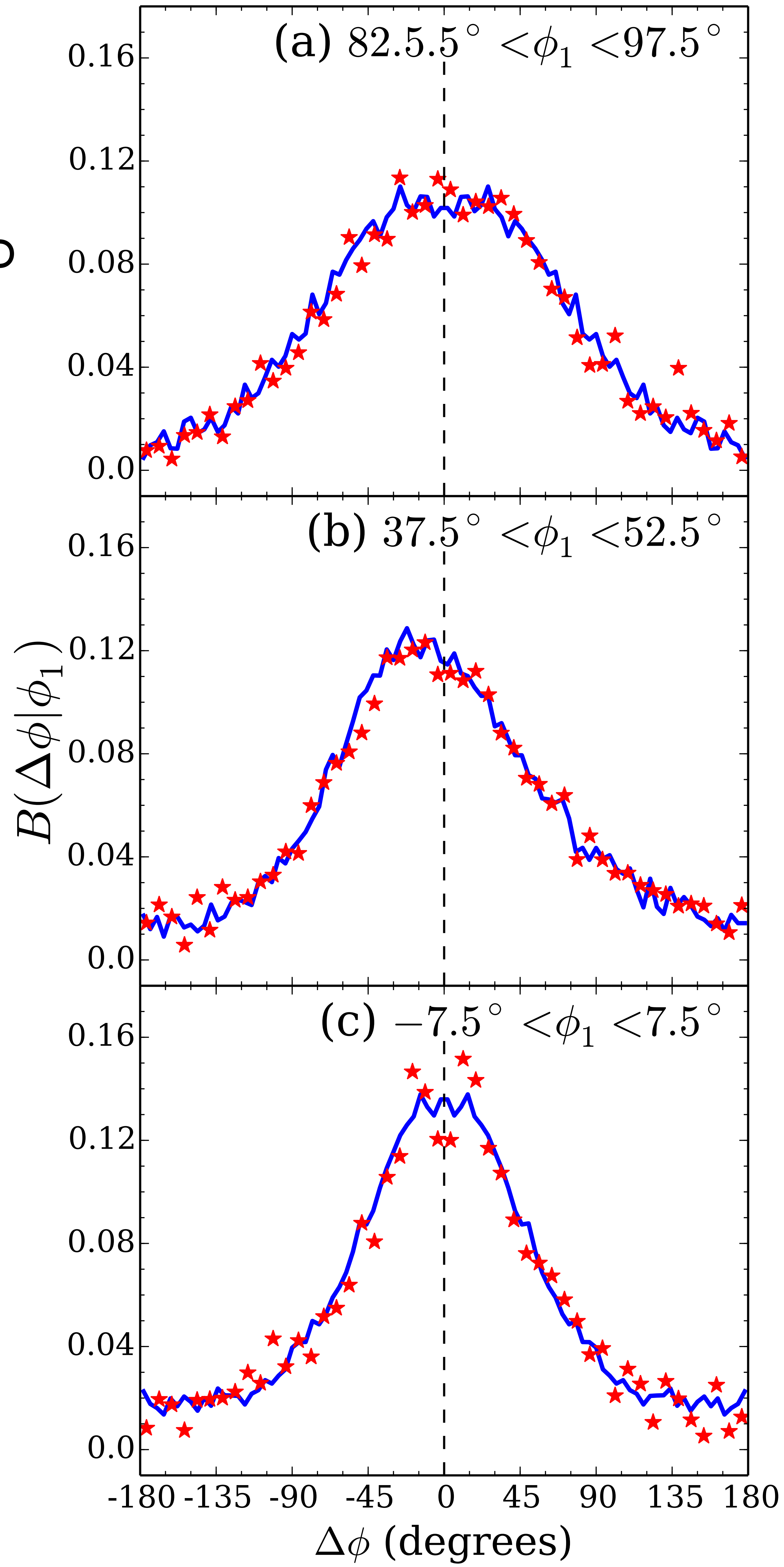
balancing charge more likely to be in-plane

ASIDE: CME correlator

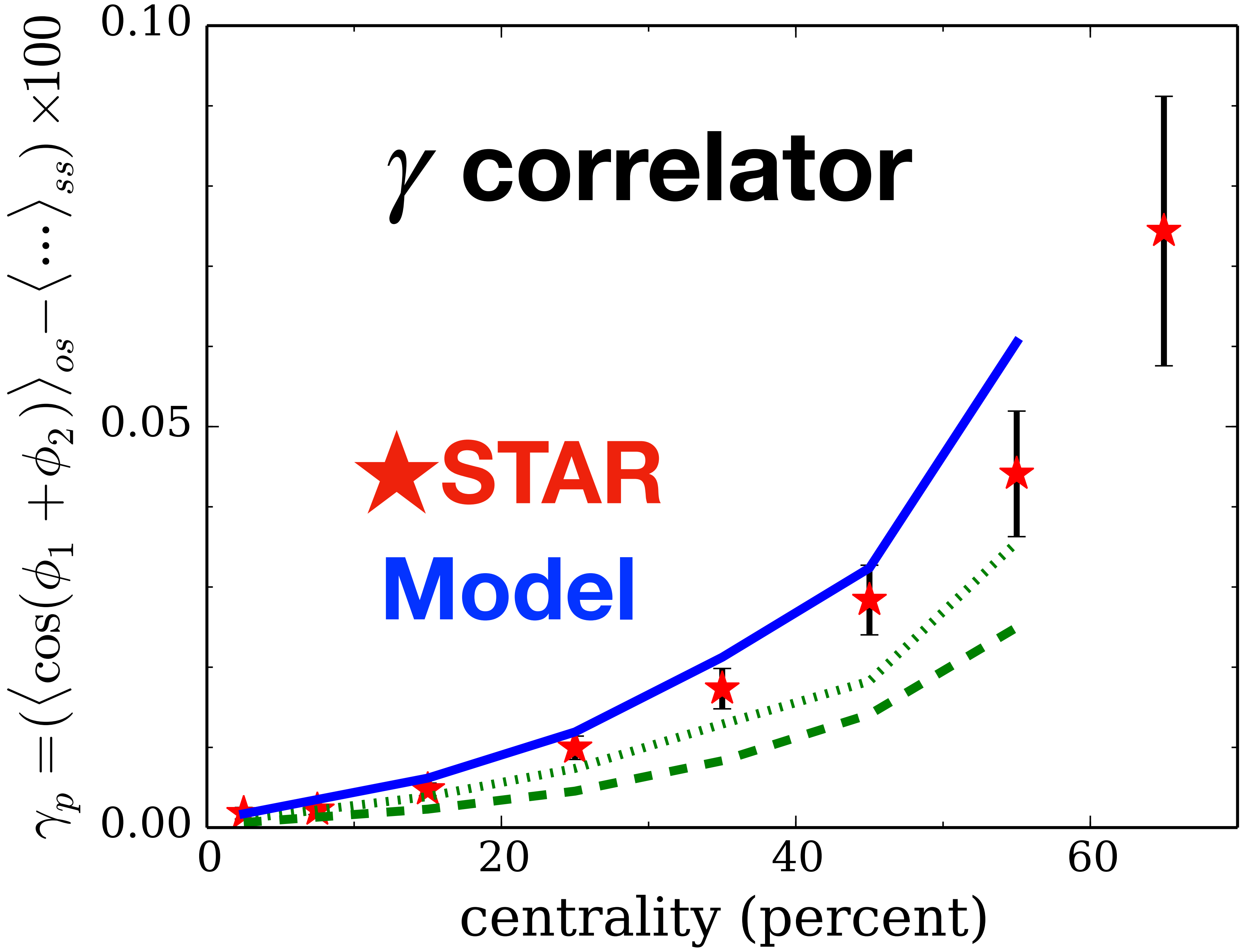
$\phi_1 \sim 90^\circ$

$\phi_1 \sim 45^\circ$

$\phi_1 \sim 0^\circ$



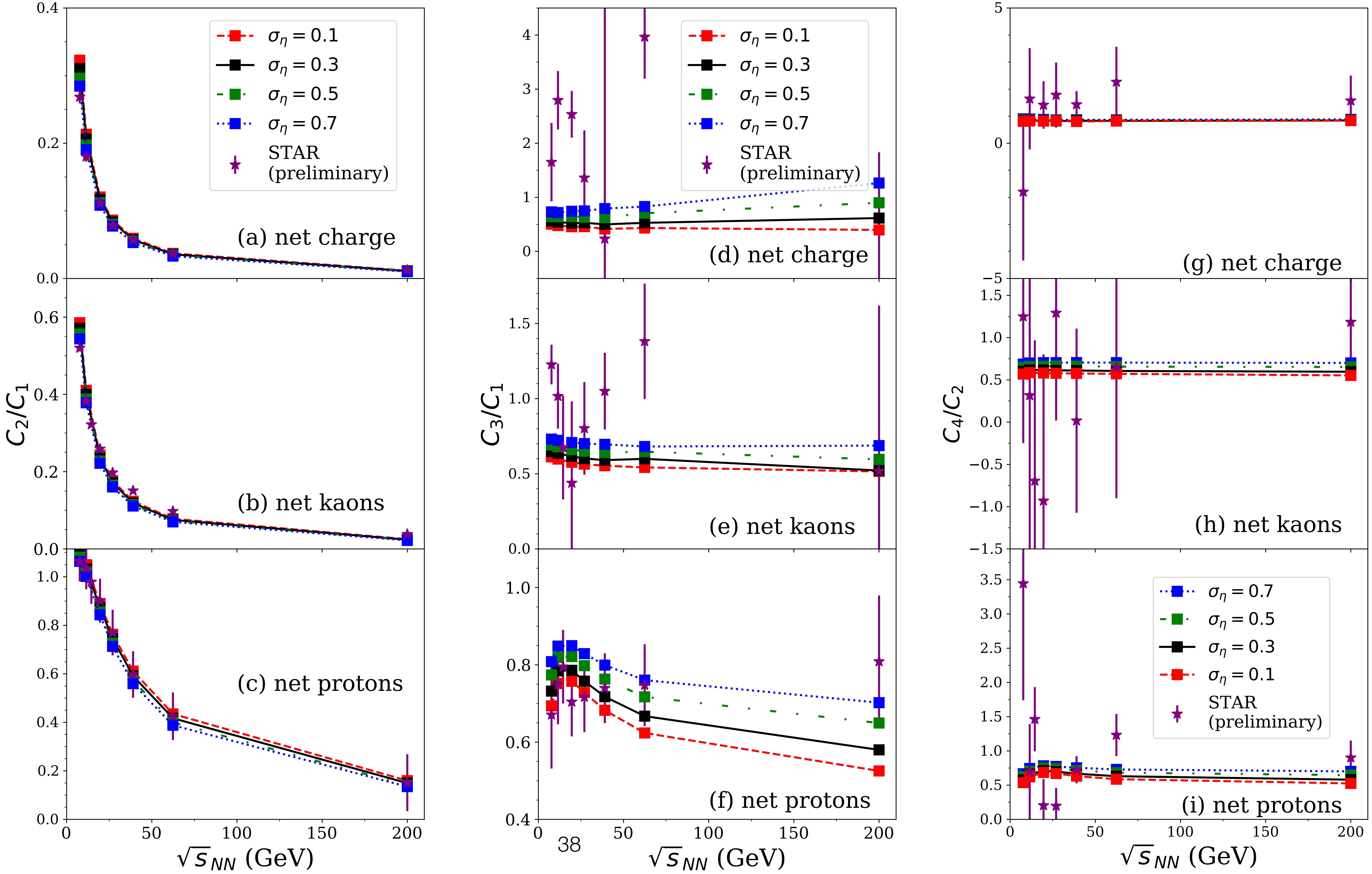
$$\gamma = \frac{1}{2}\{\langle\cos(\phi_1)\cos(\phi_2) - \sin(\phi_1)\sin(\phi_2)\rangle_{\mathrm{opp.sign}}\}$$
$$- \frac{1}{2}\{\langle\cos(\phi_1)\cos(\phi_2) - \sin(\phi_1)\sin(\phi_2)\rangle_{\mathrm{same.sign}}\}$$
$$= \frac{1}{M^2}\int d\phi_1 d\Delta\phi\, \frac{dM}{d\phi_1} B(\Delta\phi|\phi_1)\cos(2\phi_1 + \Delta\phi)$$



Model predicts ~ 115% of signal

Charge Conservation and Q^3, Q^4 correlations

b) Perform canonical ensemble on sub-volumes & superimpose on blast wave (crude)

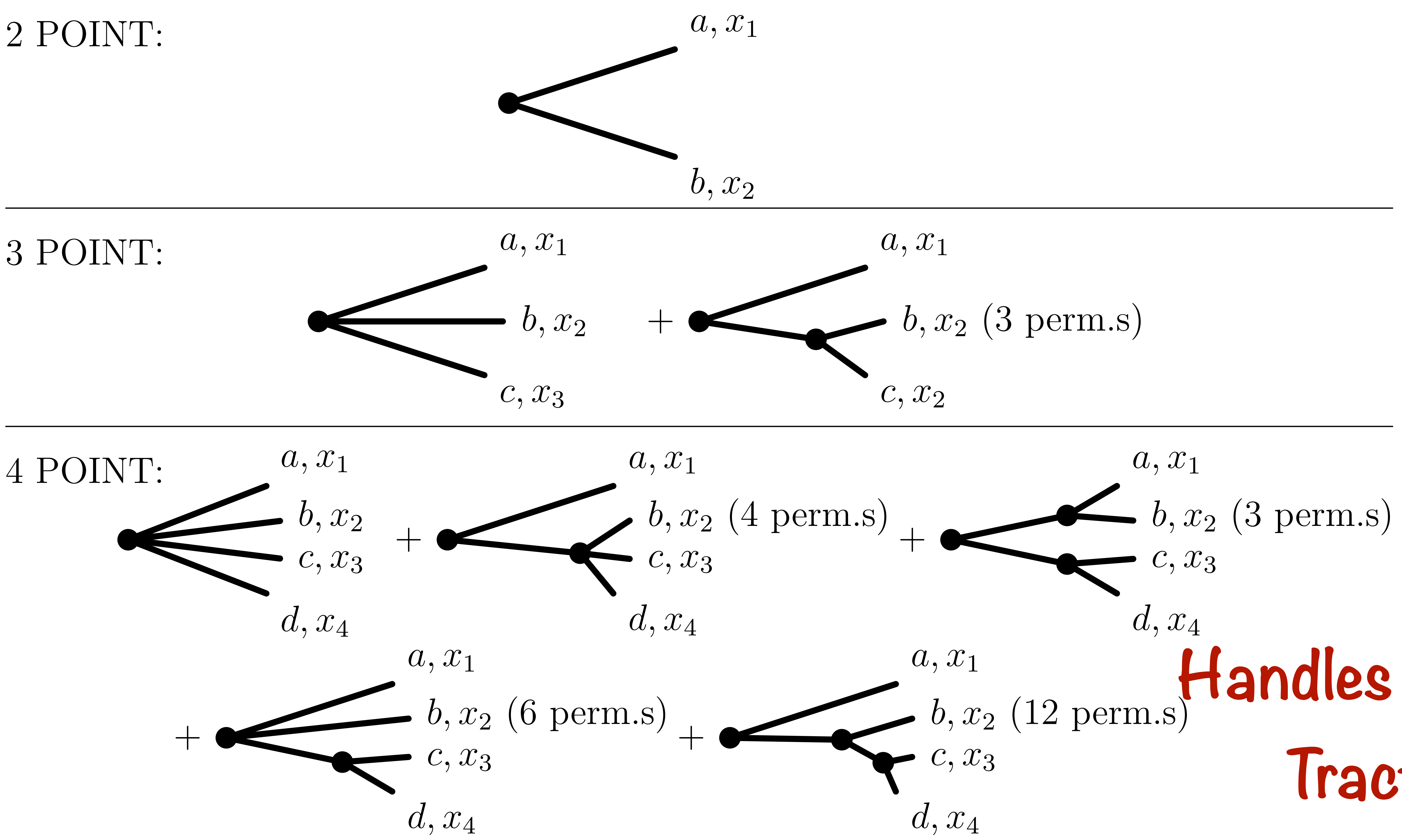


S.P. and R.Steinhorst PRC (2020)

BONUS: Charge conservation and Q^3, Q^4 correlations (formalism)

a) Integrate n-point correlations to obtain skewness & kurtosis

S.P., PRC (2020)



Handles full 3x3 flavor dynamics

Tractable, but DIFFICULT

Evidence of early chemical equilibrium

- $p\bar{p}$, K^+K^- **BFs broader than** $\pi^+\pi^-$ **BFs**
- $\sigma_0 > 0$